

Mirror Surface Contamination Specification Derived from Coronagraphy Scatter Error Budget Allocation

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ABSTRACT

Near-Angle Scatter (NAS) of the host star's light may limit the ability of a potential Habitable Worlds Observatory (HWO) to detect and characterize an Earth-like planet around a Sun-like star via coronagraphy. NAS from each optical surface before the coronagraph's focal plane mask produces an E-field across the dark hole that is coherent. These E-fields sum and could be as large or larger than the coronagraph mask leakage E-field. This paper assumes an error budget allocation for scatter of 20 ppt. NAS E-fields contribute to the dark hole noise floor via both shot noise and heterodyne amplification of the wavefront instability. While previous papers have developed specifications for scatter from surface scatter, this paper develops specifications for scatter from surface contamination and micrometeoroid impacts. The development process utilizes an expression that predicts scatter throughput into the dark hole based on surface BRDF. Analysis does not include scatter from coating columnar structure, edges, contamination, micrometeoroid impacts, or polarization.

Keywords: near angle scatter, space telescopes, coronagraphy

1.0 INTRODUCTION

Near Angle Scatter (NAS) may limit the ability of a potential Habitable Worlds Observatory (HWO) to detect and characterize a potential exo-Earth located 50 to 100 milli-arc-seconds (mas) from its host star via coronagraphy. NAS of the host star's light from optical surfaces before the coronagraph's focal plane mask produces an E-field across the dark hole that is coherent. (Scatter from surfaces after the focal plane mask is too small amplitude to be significant.) These E-fields sum – they do not RSS – and they might be as large or larger than the coronagraph mask leakage E-field. NAS E-fields contribute to the dark hole noise floor via both shot noise and heterodyne amplification of the wavefront instability.

In 2023, we published a first-order single-surface analysis that derived 'placeholder' specifications for the primary mirror using Rayleigh-Rice vector scatter theory, generalized Harvey-Shack scalar scatter theory, and Greynolds approximation.

[1] For a 10 ppt throughput error budget allocation, the primary mirror specification was: static surface roughness < 1 nm rms, and dynamic 'post-correction' surface structure < 1 pm PV. These specifications are consistent with other analyses.

[2-3] The 2023 paper did not include scatter from other surfaces, primary mirror segment edges, coating structure, contamination, micrometeoroid impacts, etc.

In June 2024 [4], we derived an expression that specifies the surface statistics required to achieve a given coronagraph error budget NAS throughput allocation. We did this by deriving a direct connection between coronagraph contrast throughput and mirror surface BRDF. The amount of NAS is determined entirely by the statistical properties of the optical surface microroughness and the operating wavelength. Surface properties include not only the rms roughness, but also correlation length, and the functional form of the distribution itself. Alternatively, the surface microstructure can be defined via a slope specification. Please note that Appendix A contains a correction to the June 2024 paper.

Also, in June 2024 [5], leveraging the ROMAN error budget approach, we developed a potential raw contrast error budget for detecting an Earth-like planet around a Sun-like star that defined a 'post-correction' NAS allocation.

This paper reviews the BRDF to Coronagraph Throughput derivation and its relevance to coronagraph contrast. And, it reviews how the raw contrast error budget is defined and the flow-down to a scatter allocation. We then use the BRDF to Throughput relationship to define a potential specification for surface contamination and maximum micrometeoroid impact size. Also, Appendix A describes a change to the June 2024 paper's derived specification for surface microstructure. Finally, this paper does not include scatter from coating columnar structure, edges, or polarization.

2.0 SCIENCE REQUIREMENT DRIVES SPECIFICATIONS

Direct imaging of exoplanets requires coronagraph/telescope systems capable of rejecting the light from the host star to below the brightness of its companions. Habitable Worlds Observatory (HWO) plans to detect and characterize Earth-like planets in the habitable zone of sun-like stars. Viewed at a distance of 10-pc, such planets would have a maximum angular separation of 100 mas (0.1"). (Fig 1) Taking into account albedo, phase angle and coronagraph working angle, we adopt, as a nominal value for error budgeting, a flux ratio (i.e. ratio of planet's reflected light to its host star's direct light) of 150 ppt. (parts-per-trillion). [5]

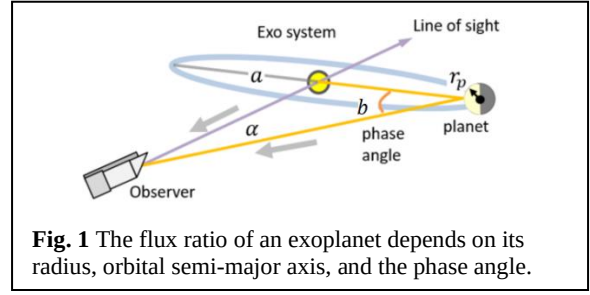


Fig. 1 The flux ratio of an exoplanet depends on its radius, orbital semi-major axis, and the phase angle.

Coronagraphs (Fig 2) achieve this flux ratio by blocking the star's light with a focal plane mask (FPM). The planet's light, which comes in at a slight angle, misses this mask and proceeds. After recollimation, a 'Lyot' mask removes most (but not all) of the remaining on-axis light. The beam is then focused onto the image plane of a detector, creating what is usually referred to as a 'dark hole,' a zone where host star light has been strongly suppressed. The key attributes of any coronagraph needed to derive a direct-imaging error budget are core throughput, inner and outer working angles, and raw contrast. [2] For the purpose of our NAS study, the key parameter is contrast.

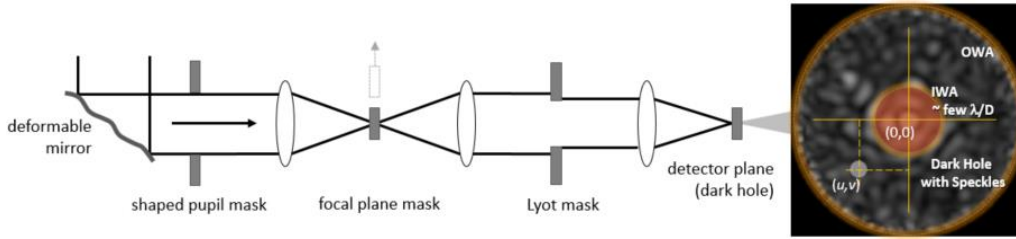


Fig. 2 Typical coronagraph: incoming light from the left is shaped in phase by a deformable mirror then sent through a focal plane and Lyot mask. The result is a 'dark hole' where (on-axis) starlight is strongly suppressed relative to the (off-axis) planet light. The inner and outer working angles (IWA, OWA, respectively, denote the limits of the coronagraph suppression region (the "dark hole").

Contrast is the measure of the effectiveness of the coronagraph in suppressing starlight near the planet and is defined as the ratio of two throughputs.

$$C(u, v) \equiv \frac{\tau(u, v)}{\tau_{pk}(u, v)},$$

where $\tau_{pk}(u, v)$ is the "peak" throughput or fraction of the star's light, if it were located at (u, v) , detected within a solid angle region of interest Ω_r centered at (u, v) relative to the total light incident upon the primary mirror. And, $\tau(u, v)$ is throughput, which is defined as the ratio of power at position (u, v) divided by total incident power from the star, in its nominal position of $(0, 0)$.

$$\tau(u, v) \equiv \frac{\Phi(u, v)}{\Phi_i(0, 0)},$$

If a coronagraph were perfect, then $\tau(u, v)$ would be zero and $\tau_{pk}(u, v)$ would be 1. But coronagraphs are not perfect. Even after wavefront control gives the coronagraph its final level of starlight suppression, a small amount of light (E-field) will 'leak' around the coronagraph mask due to design inefficiency and wavefront instability. The contribution of this light to the dark hole is quantified by the 'speckle' throughput, τ_{sp} . The dark hole also receives stray light from many phenomena and sources. One source is near-angle scatter (NAS), which is quantified by NAS throughput, τ_{NAS} . NAS is largely coherent with the diffractive contribution. There could also be other sources. Total throughput is the sum of all sources:

$$\tau(u, v) = \tau_{sp} + \tau_{NAS} + \dots$$

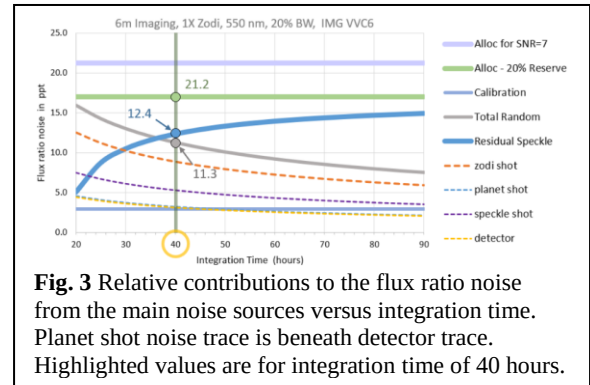
The importance of this 'leaked' E-field is that it produces a speckle intensity pattern given by $|E(u, v)|^2$ in the dark hole. When a disturbance or drift error occurs in the opto-mechanical configuration of the telescope or coronagraph, the E-field changes by a small amount – coherently adding a perturbation field $\Delta E(u, v, t)$ to the 'initial' or 'static' part of the field

$E(u, v)$. The coherent mixing of the original field E and perturbation field ΔE creates a speckle pattern, the intensity of which is given by:

$$|E + \Delta E|^2 = |E|^2 + |\Delta E|^2 + 2 \mathcal{R}\{E^* \cdot \Delta E\}.$$

It is this mixed or heterodyned term for which error budgets allocate a maximum value. Typical error budgets assume that the static E-field is entirely caused by coronagraph mask inefficiency when deriving a wavefront instability specification. But near-angle scatter can increase the static E-field. And consequently, to keep the heterodyne term constant, the wavefront instability must become smaller. Also, if NAS increases the static E-field, then it also increases its shot noise.

An error budget is required to ensure that the coronagraph will satisfy its mission. For a 150 ppt exo-Earth target to be observed with SNR = 7, the total error from all sources combined must be less than 21.2 ppt. (Because a practical error budget must have some reserve.) The primary categories of error are calibration errors, random noise, and residual speckle. The last of these is the most difficult to estimate. But if we budget for all the errors combined and can budget the other two branches, then the remainder must be the residual speckle. Fig. 3 plots total random noise and its constituents as a function of integration time. Also plotted is the maximum allowable systematic error from residual speckle, i.e. the quadrature difference between the total allowable error (21.2 ppt) and total random error. If we choose an integration time of 40 hours, total random error is 11.3 ppt. Assuming a 2% allocation to calibration errors, that branch must total to 3 ppt. That leaves 12.4 ppt for the residual speckle branch. [5]



With the three branch allocations complete, lower-level allocations can be made. Contrast contributes both to the random noise branch and to contrast stability. To date, the best achieved contrast in testbeds has been at the level of $\sim 4 \times 10^{-10}$, or 400 ppt. We assume for the error budget a slightly better total average raw contrast of 300 ppt after including all expected on-orbit sources of background in the dark hole. Fig. 4 shows the suballocations to contrast, including scattered light, which is the topic of this paper. The scattered light allocation is seen to be 20 ppt. [5]

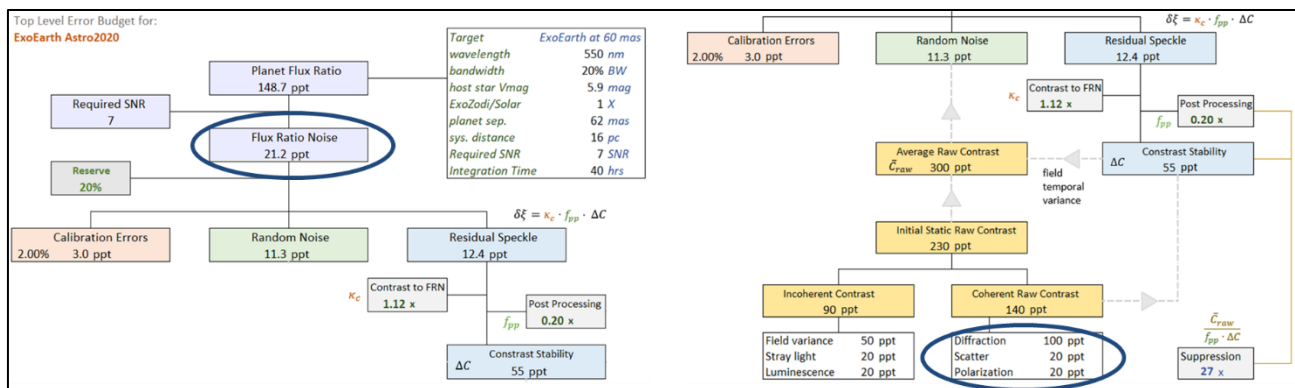


Fig 4: Scattered Light error budget allocation flows from flux ratio to detect exoplanet with SNR = 7 and 40 hr. integration time.

The error budget allocation of 20 ppt needs to be sub-divided between multiple scattered light sources. Our 2023 paper showed that Rayleigh-Rice Vector Scatter Theory could model long-spatial period ‘post-correction’ surface structure. That paper allocated 10 ppt to uncorrectable long-spatial surface structure. [1] Our June 2024 paper [4], derived an expression for specifying the surface microstructure required to meet a 5 ppt error budget allocation. This paper allocates 2.5 ppt to contamination and 2.5 ppt to micrometeoroid impacts.

3.0 SCATTERED LIGHT: THROUGHPUT = BRDF

In our June 2024 paper [4], we derived an expression that relates near-angle scatter throughput to BRDF which can be used to define a surface microstructure specification to meet a given error budget scatter allocation.

Near-Angle Scatter Throughput is defined as the ratio of power scattered off an optical surface (e.g. primary mirror, PM) and imaged into the dark hole relative to the power incident on optical surface.

$$\text{NAS Throughput} = \tau_{\text{NAS}} = \frac{\Phi_{\text{NAS}}(u, v)}{\Phi_i}$$

BRDF (defined by Nicodemus in 1977 [6]) is a geometric relationship to describe surface scatter in radiometric terms. (Fig 5) BRDF is defined as the reflected radiance, L , (power per unit solid angle per unit projected area) divide by the incident irradiance, E , (power per unit area). (Please note that the international standard (SI) radiometric irradiance E notation is different from electric field E vector.)

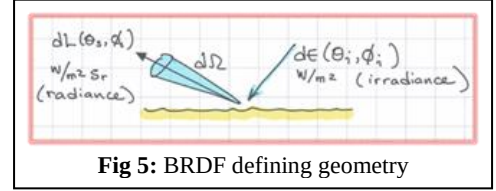


Fig 5: BRDF defining geometry

$$\text{BRDF}(\theta_s, \varphi_s; \theta_i, \varphi_i) = \frac{dL(\theta_s, \varphi_s; \theta_i, \varphi_i)}{dE(\theta_s, \varphi_s; \theta_i, \varphi_i)} = \frac{\Phi_s/\Omega}{\Phi_i \cos(\theta_s, \varphi_s)} = \frac{\Phi_s/\Omega}{\Phi_i}$$

where:

$$\begin{aligned} \Phi_i &= \text{Total Power Incident on Entrance Pupil, i.e. Primary Mirror Area, } = E_i \times A_{\text{PM}} \\ \Phi_s/\Omega &= \text{Power per unit Solid Angle Scattered from Entrance Pupil, i.e. PM Area} \\ (\theta_s, \varphi_s; \theta_i, \varphi_i) &= (m\lambda/D, n\lambda/D; 0, 0) \text{ where } m \text{ and } n \text{ span } 1 \text{ to } 30, \text{ thus } (\theta_s, \varphi_s) \text{ are very small} \end{aligned}$$

Thus, via radiometric definition, NAS throughput is directly proportional to BRDF

$$\tau_{\text{NAS}} = \frac{\Phi_{\text{NAS}}(u, v)}{\Phi_i} = \text{BRDF} \cdot \Omega_{\text{Core}} = \text{BRDF} \cdot \pi (\lambda/D)^2$$

where $\Omega_{\text{Core}} = \text{Airy Disc solid angle} = \pi (\theta_{\text{Core}})^2 = \pi (\lambda/D)^2$. Appendix A describes the choice of core diameter.

4.0 SURFACE CONTAMINATION SCATTER

While contamination particles are not typically spherical, particulate contamination is typically modeled via Mie scatter. Mie scatter is direction dependent and the larger the particle the narrower and more directional its scatter cone (Fig. 6). Also, while we are not considering polarization, it is proper to note that each polarization state has its own scatter profile. As illustrated in Fig 7, a scattering particle on the primary mirror of a telescope/coronagraph produces a spherical wavefront that is focused (by the secondary and aft-optics) into the conjugate pupil. The coronagraph collimates this focus such that most of its light passes the focal plane mask unobscured and is reimaged into the dark hole as a collimated E-field. Because the particle is located on the PM, its E-Field is coherent and in phase with the host star's light – thus, it increases the heterodyne amplification of any wavefront instability.

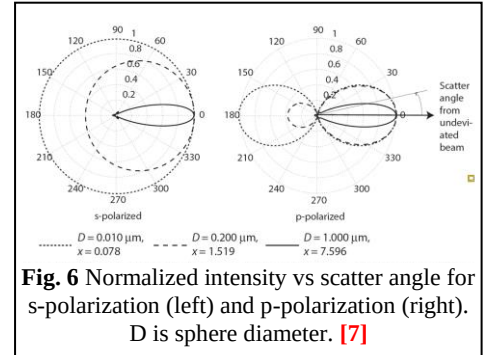


Fig. 6 Normalized intensity vs scatter angle for s-polarization (left) and p-polarization (right). D is sphere diameter. [7]

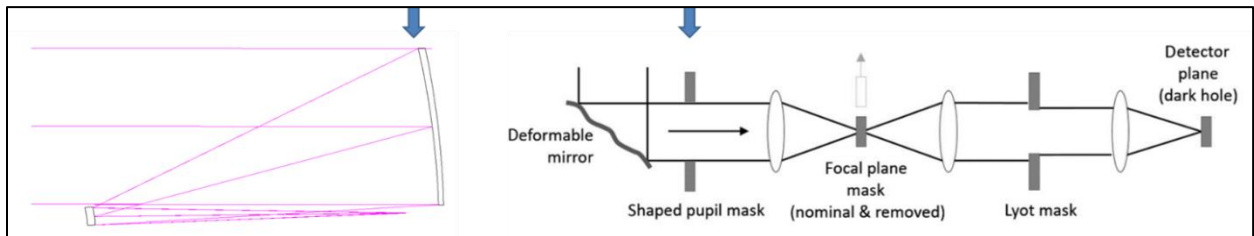


Fig 7. Each scattering particle on the primary mirror produces a collimated coherent E-Field in the dark hole.

Similarly, a particle on the secondary mirror produces a spherical E-field in the dark hole that adds with the primary mirror E-field and E-fields from all other surfaces (Fig 8). Thus, because there are 10 or more surfaces in the optical chain, the 2.5 ppt allocation for contamination must be sub-allocated, to say 0.25 ppt per surface.

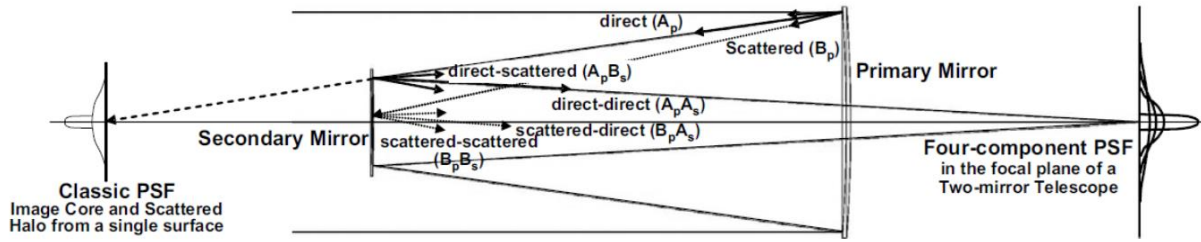


Fig. 8 Each surface in telescope/coronagraph optical train is an ‘in-field’ source of near-angle scatter which sums in focal plane. [8]

All surfaces have contamination, but because each contamination population is unique it is impossible to deterministically model contamination scatter. Thus, contamination is described statistically via Cleanliness Level (CL) and BRDF. [9] CL corresponds to the average particle size per 0.1-m². (Fig 9) Thus, a CL-200 Surface has average of one ‘equivalent’ 200-μm particle per 0.1-m².

BRDF varies with particle diameter to the 4th power (Fig 10). Thus, large particles are more important than small particles. But it is assumed that standard cleaning processes can remove particles larger than 50 μm. [10] Also, Fig 10 illustrates how small amplitude surface microstructure creates a near-angle BRDF floor.

Peak BRDF values can be derived for each cleanliness level. [7, 9] Thus, using the BRDF to NAS throughput equation, a surface contamination specification can be defined for a given per surface scatter allocation. For a 6-m telescope and a 0.25 ppt per surface allocation, the peak BRDF is:

$$\text{BRDF}_0 = \frac{\tau_{\text{NAS}}}{\Omega_{\text{core}}} = \frac{0.25 \cdot E^{-12}}{\pi \cdot E^{-14}} \approx 8 \text{ [1/sr]}$$

From Fig 11, a peak BRDF of 8 corresponds to a cleanliness level of ~ CL-400 for a wavelength of 633 nm. It is important to note that the surfaces will need to be cleaner if the coronagraph operates at shorter wavelengths – because particles scatter more at shorter wavelengths. For comparison, Webb had a CL-600 specification. Also please note that it is not necessary to divide the contamination scatter allocation equally between surfaces. Because scatter sums, one could allocate more contamination to the telescope mirrors and require the coronagraph mirrors to be cleaner.

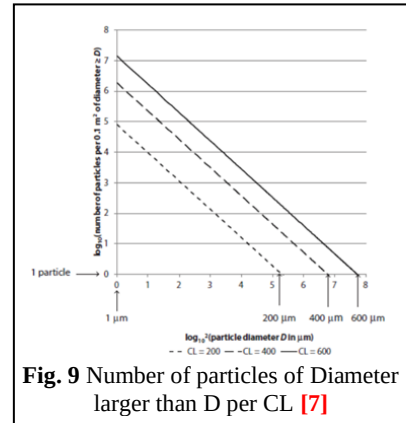


Fig. 9 Number of particles of Diameter larger than D per CL [7]

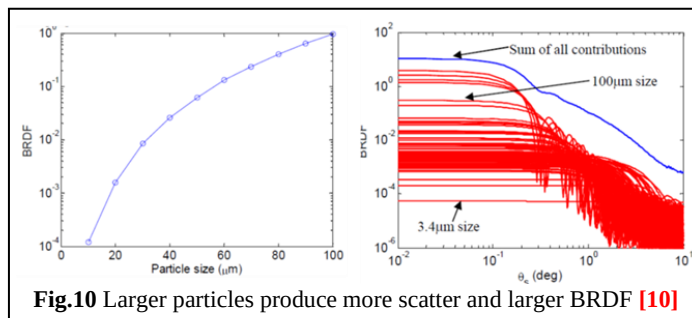


Fig.10 Larger particles produce more scatter and larger BRDF [10]

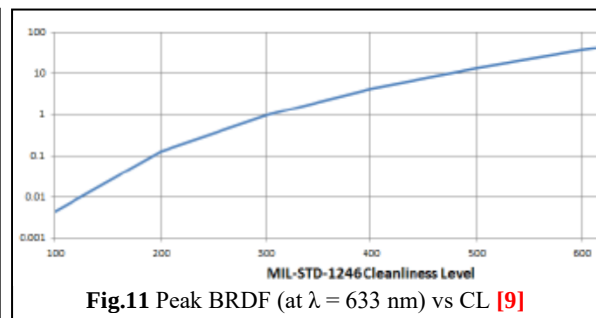


Fig.11 Peak BRDF (at $\lambda = 633$ nm) vs CL [9]

5.0 MICROMETEOROID IMPACT SCATTER

While contamination particles are small, micrometeoroid impacts are large. Therefore, a micrometeoroid impact's scatter will be very narrow. And, while all surfaces will have contamination, micrometeoroid impacts should only occur on the primary mirror. Therefore, the entire micrometeoroid impact scattered light 2.5 ppt allocation can be assigned to the PM.

Because it is a crew safety risk, micrometeoroid impact on glass has been studied extensively, both via ground and space experiments. The result is that high-velocity impacts (HVIs) of glass surfaces result in spalling that is ~11X larger in diameter than the central pit diameter and ~10X larger in diameter than the pit depth (Fig 12). [11]

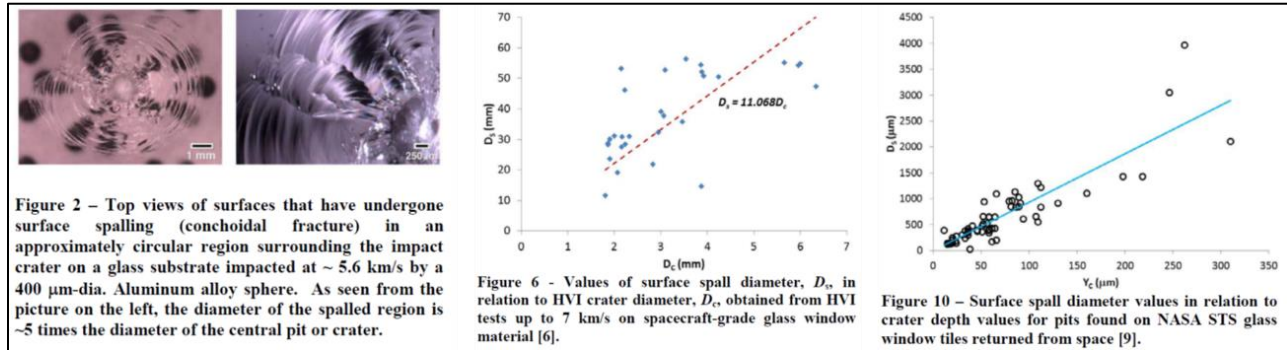


Fig 12. Figures from REF 11 showing that micrometeoroid impacts produce a central pit surrounded by spall (fracture zone). On average the spall is ~11X larger than the pit diameter and ~10X larger than the pit depth. [11]

The size of the impact pit and spall diameter are a function of the kinetic energy of the impact – not the size of the impacting particle. Fig 13 shows that for the highest energy impacts, the spall diameter is in the 1 to 100 mm range. Thus, for this analysis we will analyze the effect of four different impact sizes (Table 1).

So, how does one model micrometeoroid impact scatter: Cleanliness Level, BRDF, or Diffraction?

5.1 Micrometeoroid Scatter via Cleanliness Level

Cleanliness Level (CL) is defined as the average particle size per 0.1-m². A potential 6-m diameter HWO primary mirror has an area of ~28-m². Thus, for a given CL, the PM will have a total of 280 particles of that size.

Section 4.0 found that to contribute < 0.25 ppt of scatter, the primary mirror needs to be ~ CL-400, i.e. no more than 280 particles of size 400 μm; which is equivalent to one particle of 6.7 mm diameter. But, because micrometeoroids should only impact the primary mirror, we can assign it the full 2.5 ppt scatter micrometeoroid allocation. Since BRDF increases to the 4th power with particle size (Fig 10), increasing the BRDF allocation by 10X allows a 1.77X larger particle resulting in a micrometeoroid impact specification of < CL-700. A single impact spall of ~12 mm diameter has the approximate 'effective' diameter of 280 0.7-mm diameter particles (CL-700).

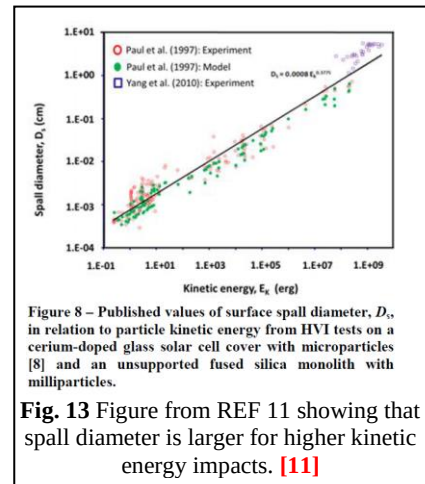
Again, the allowed impact size will decrease if the coronagraph is operated at a wavelength shorter than 633 nm.

5.2 Micrometeoroid Scatter via BRDF

The Webb Program Office has impacted glass samples with hypervelocity particles to study micrometeoroid impacts. GSFC measured the BRDF of a sample with a ~5-mm central crater and ~24-mm spall from 5 to 60 deg at 633 nm. [12]

Using an admitted back-of-envelope method, the GSFC BRDF data was extrapolated along a slope consistent with published CL BRDF curves [7], to estimate a 'peak' BRDF for that sample of ~1E+3. Scaling by the ratio of the 24-mm spall area to a 6-m primary mirror yields an effective BRDF = 0.02 and an estimated scatter throughput of ~10 ppt.

If this calculation is even close to correct, then to reduce the scatter throughput to 2.5 ppt requires reducing the spall diameter by 0.7X to ~17-mm – close enough to the ~12-mm size derived from the CL specification.



Given that it is nearly impossible to measure near-angle BRDF, what is required to make this analysis more precise is a PSD of the spall's surface structure as measured by a micro-profilometer.

5.3 Micrometeoroid Scatter via Diffraction Analysis

Using 1D Gaskill notation [13], if one assumes that the micrometeoroid impact pit is a circle with a 1D 'rect' function profile of diameter d , then its scattered light will have a 1D 'sinc' amplitude profile.

$$\text{Rect}(x/d) \langle \mathcal{F} \rangle d \cdot \text{sinc}(d \xi) = \frac{d \sin(d \xi)}{\pi \xi}$$

where ξ = spatial frequency. To calculate scattered power at $2\lambda/D = 0.2 \mu\text{rad}$ (for $\lambda = 600 \text{ nm}$, $D = 6\text{m}$) in the dark hole, we calculate the ratio of the area of the impact pit to that of the primary mirror then multiply by the 'sinc' function's peak value of $\sim d^2/\pi$ (because for small angles $\sin(d \xi) = d \xi$).

Table 1: Relative Power in Dark hole from Micrometeoroid Impact					
Impact Diameter [mm]	Input Power Ratio	Scatter Angular Spread [μrad]	Scatter Angular Spread [λ/D]	Peak Scatter Amplitude	Relative Power at $2\lambda/D$ [ppt]
10	2.8E-06	60	300	3E-05	88
8	1.8E-06	75	375	2E-05	36
6	1.0E-06	100	500	1E-05	11
4	4.4E-07	150	750	5E-06	2.3

One micrometeoroid impact pit of 4-mm diameter may contribute the entire 2.5 ppt scattered light allocation.

Repeating the analysis, assuming the pit has a 'gaussian' profile, will yield a similar result.

5.4 Micrometeoroid Mitigation

Webb is seeing approximately 1 micrometeoroid impact per day. Most are small but some are large enough to be visible in diagnostic imagery – which implies that their scatter will be significant. For a potential HWO mission, impacts can be reduced from ~ 1 per day to ~ 1 per 100 days by placing the telescope inside of a barrel. Nevertheless, a 100-day impact may be large enough to significantly degrade the coronagraph's contrast noise floor.

6.0 CONCLUSIONS

Near Angle Scatter (NAS) of the host star's light from optical surfaces before the coronagraph's focal plane mask produces an E-field across the dark hole that is coherent. These E-fields sum and might be as large or larger than the coronagraph mask leakage E-field. NAS E-fields contribute to the dark hole noise floor via both shot noise and heterodyne amplification of the wavefront instability. There are multiple sources of near-angle scatter, including: long-scale surface structure, surface microstructure, contamination, and micrometeoroid impacts. This paper assumes an error budget allocation for scatter of 20 ppt and develops specifications for scatter from surface contamination and micrometeoroid impacts using an expression that predicts scatter throughput into the dark hole based on surface BRDF. Previous papers developed specifications for scatter from surface scatter. Additionally, Appendix A contains an ERRATA to the June 2024 paper's derived specification for surface microstructure. Finally, this analysis does not include scatter from coating columnar structure, edges, or polarization.

REFERENCES

1. H. Philip Stahl, "Assessment of near-angle scatter on exo-Earth coronagraphy," Proc. SPIE 12676, (4 October 2023); <https://doi.org/10.1117/12.2676543>
2. Nemati, Bijan, H. Philip Stahl, Mark T. Stahl, Garreth J. Ruane, Leah J. Sheldon, "Method for deriving optical telescope performance specifications for Earth-detecting coronagraphs," J. Astron. Telesc. Instrum. Syst. 6(3), 039002 (2020), doi: 10.1117/1.JATIS.6.3.039002.
3. Laurent Pueyo, Roser Juanola-Parramon, Jason Tumlinson, Rémi Soummer, Iva Laginja, Heidi B. Hammel, C. Mattias Mountain, "Coronagraphic detection of Earth-like planets with large, actively controlled space telescopes," J. Astron. Telesc. Instrum. Syst. 8(4) 049002 (25 October 2022) <https://doi.org/10.1117/1.JATIS.8.4.049002>
4. Stahl, H Philip, David D. Smith, and Bijan Nemati, "Impact of Near-Angle Scatter on Exo-Earth Coronagraphy", SPIE Proc. 13092, (June 2024).
5. Bijan Nemati and H. Philip Stahl, "Leveraging the Roman Coronagraph Approach for the HWO Error Budget", SPIE Proc. 13092, (June 2024).
6. Nicodemus, F E, et al "Geometric Considerations and Nomenclature for Reflectance", US Dept of Commerce, Washington DC, NBS Monograph 160, 1977
7. Fest, Eric C., Stray Light Analysis and Control, SPIE Press, 2013
8. James E. Harvey, Narak Choi, Andrey Krywonos, Gary L. Peterson, Marilyn E. Bruner, "Image degradation due to scattering effects in two-mirror telescopes," Opt. Eng. 49(6) 063202 (1 June 2010) <https://doi.org/10.1117/1.3454382>
9. Pfisterer, Richard N., James E. Harvey, James B. Breckinridge, "The role of narrow-angle forward surface scatter and particulate scatter in exoplanet exploration," Proc. SPIE 10698, Space Telescopes and Instrumentation 2018: Optical, Infrared, and Millimeter Wave, 106985F (6 July 2018); <https://doi.org/10.1117/12.2314229>
10. Kunjithapatham Balasubramanian, Stuart Shaklan, Amir Give'on, "Stellar coronagraph performance impact due to particulate contamination and scatter," Proc. SPIE 7440, Techniques and Instrumentation for Detection of Exoplanets IV, 74400T (25 August 2009); <https://doi.org/10.1117/12.825182>
11. Iyer, et. al., "Glass Surface Spall Size Resulting from Interplanetary Dust Impacts", IEEE, 978-1-4799-5380-6, 2015.
12. Georgi Georgiev, Private Communication, 22 July 24
13. J. D. Gaskill, "Linear Systems, Fourier Transforms, and Optics," John Wiley & Sons, Inc., New York, 1978.

APPENDIX A: REVISED SPECIFICATION

REFERENCE: H. Philip Stahl, David Smith, Bijan Nemati, “Impact of near-angle scatter on exo-Earth coronagraphy,” SPIE Proc.13092, (June 2024)

Assuming Gaussian statistics, the June 2024 paper [4] derived an upper-bound limit relationship between scatter throughput and surface microstructure roughness requirement [in Å] that depends on λ and ℓ [both in μm].

$$\sigma[\text{\AA}] < \frac{\lambda^2 \cdot 10^4}{4\pi\ell} \left[\frac{\tau_{NAS}}{\pi\Omega_{core}} \right]^{1/2},$$

In that paper, following the assumptions of the Roman Space Telescope Coronagraph error budget, we defined the core angular radius to be the Airy disc’s angular half power width:

$$\theta_{core} = \frac{\lambda}{D} \rightarrow \Omega_{core} = \pi\theta_{core}^2/4 = \pi(\lambda/D)^2/4.$$

After some consideration, to be more conservative, we are changing to the Airy disc’s first zero crossing radius:

$$\theta_{core} = 2 \frac{\lambda}{D} \rightarrow \Omega_{core} = \pi\theta_{core}^2 = \pi(\lambda/D)^2.$$

Please note: the choice of what constitutes the PSF core must be informed by a photometric analysis of the planet PSF and the noise background, including the applicable detector noise. In general, as the noise levels go higher there is less advantage to choosing a larger photometric aperture, where there is very little signal above the noise.

Table A-1 gives the maximum allowed microstructure roughness, σ , (in Å) as a function of wavelength, λ , and microstructure correlation length, ℓ , (both in μm) for a single surface to meet a 0.5 ppt near-angle scatter throughput, τ_{NAS} , specification.

Table A-1: Surface RMS Microstructure [Å] to have NAS Throughput < 0.5 ppt				
	Wavelength (λ) [μm]			
Correlation Length (ℓ) [μm]	0.300	0.680	1.000	1.800
10	16.2	83.2	180.0	583.2
100	1.6	8.3	18.0	58.3
1,000	0.2	0.8	1.8	5.8
10,000	0.0	0.1	0.2	0.6

Again, the shortest coronagraph science wavelength drives the surface specification. And correlation length is important.

This change in Core Solid Angle also changes the allowed slope specification.

$$RMS\ Slope = \sqrt{2} \frac{\sigma}{\ell}$$

Table A-2 shows the maximum allowed slope (in μrad) as a function of wavelength, λ , and microstructure correlation length, ℓ , (both in μm) for a single surface to meet a 0.5 ppt near-angle scatter throughput, τ_{NAS} , specification.

Table A-2: Surface RMS Slope [μrad] to have NAS Throughput < 0.5 ppt				
	Wavelength (λ) [μm]			
Correlation Length (ℓ) [μm]	0.300	0.680	1.000	1.800
10	229.1	1177.1	2545.6	8247.7
100	2.3	11.8	25.5	82.5
1,000	0.0	0.1	0.3	0.8
10,000	0.0	0.0	0.0	0.0