(Preprint) AAS XX-XXX

METRICS ON PREDICTIVE PROBABILITY OF COLLISION ASSESSMENTS RELIANT ON OBJECT STATE UPDATES

Travis Lechtenberg,[*] Aroni Gupta,[†] and William Todd Cerven[‡]

The probability of collision (Pc) metric is the dominant method of risk characterization for evaluating satellite close approaches, and has become of particular interest following major debris production incidents. However, confidence in the use of Pc for actionability has been questioned due in part due to its non-intuitive and often significant corrections following messaging updates. This paper assesses predictively computed probabilities and confidence bounds on Pc values corresponding to orbit determination updates by comparing these predictive values to observed operational values. This paper also presents use cases for this methodology in assessing event actionability.

EXTENDED ABSTRACT

The probability of collision (Pc) calculation represents the probability of two objects colliding by evaluating the integral of the combined position uncertainty distribution of the two objects over their combined projected area centered at the predicted miss geometry. This process is particularly relevant in space situational awareness as it gives a quantitative value to the risk of collision between two resident space objects (RSOs). Pc is typically approximated by evaluating this integral in 2-dimensional space in the encounter plane as the vast majority of conjunction events in space (particularly in LEO) are over short encounter times where the two RSO's relative motion can be approximated as linear, and their covariance matrices as static. This two-dimensional probability of collision is reliant on several assumptions to address the problem:

1. Statistical independence: The two objects' uncertainty distributions are independent of each other.
2. Rectilinear motion: The conjunction can be approximated by linear motion.
3. Gaussian position uncertainty distributions: The position uncertainties for both objects are accurately approximated with Gaussian distributions.

---

[*] Engineering Specialist, Space Safety and Situational Awareness Department, The Aerospace Corporation, 14745 Lee Road, Chantilly VA 20151.

[†] † Graduate Intern, Software Implementation and Design, The Aerospace Corporation, 14745 Lee Road, Chantilly VA 20151.

[‡] ‡ Director, Space Safety and Situational Awareness Department, The Aerospace Corporation, 14745 Lee Road, Chantilly VA 20151.

4. Temporal isolation: The conjunction represents a discrete event, where the two objects do not remain in close proximity for an extended period of time.

Beyond this formulation, additional, more robust methods of determining the probability of collision have been developed to speed computation times and better accommodate stressing conjunction cases where the above assumptions do not hold, such as low-speed conjunctions. For the joint position uncertainty projected into the conjunction plane, the two-dimensional Pc calculation can be characterized by the semi-major axis, semi-minor axis and correlation coefficient of the projected ellipse as follows:

$$\mathrm{Pc} = \iint_A \frac{1}{2\pi\sigma_x\sigma_z\sqrt{1-\rho_{xz}^{2}}}\, e^{-\left[\left(\frac{x}{\sigma_x}\right)^2 - 2\rho_{xz}\left(\frac{x}{\sigma_x}\right)\left(\frac{z}{\sigma_z}\right)+\left(\frac{z}{\sigma_z}\right)^2\right] \Big/ 2\left(1-\rho_{xz}^{2}\right)}\, dx\,dz$$

The 18th Space Defense Squadron generates orbit determination (OD) assessments consisting of a state estimate and associated uncertainty for all catalogued resident space objects. These OD solutions are propagated forward in time to yield predictive trajectories which are screened against each other to find close approaches. These close approaches are reported if they violate a specified screening volume, the definitions of which are found in the Spaceflight Safety Handbook for Satellite Operators [1]. These close approaches are then reported to operators in the form of Conjunction Data Messages (CDMs). A mission operator can then choose a course of action to mitigate the collision risk posed by these events.

An operator will often be notified of a potential conjunction several days in advance of the actual event and receive regular event updates from the notifying agency as orbit determination solutions are revised in the face of updated measurements from either ground-based observers or space-born instruments. These updated OD solutions result in shorter propagation times with more confident propagated states at the event Time of Closest Approach (TCA). The practical effect of this is that messages reporting a conjunction event several days in advance can instill in an operator that the event poses significant risk to a protected spacecraft, but that assessment only holds if no additional data is acquired. As additional data is acquired, the event solution is revised, often resulting in a much lower risk profile, and occasionally a higher risk profile for the protected object. The open question for operators then becomes whether collision risk assessments are likely to increase or decrease with subsequent state updates. A visualization of this can be seen by examining the Pc dilution curve which examines the relationship between the ratio of the covariance size to miss distance against the calculated Pc as seen in the following figure:

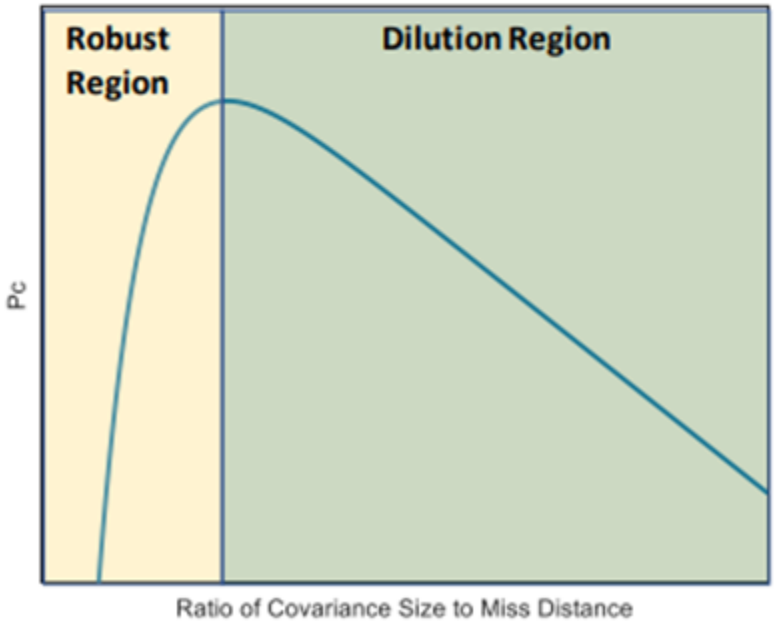

Figure 1: Pc vs the Ratio of Covariance Size to Miss Distance, Hejduk [2]

Pc assessments several days prior to an event tend to be in the dilution region. Should the miss geometry remain constant with subsequent updates, the Pc value would move to the left along the Pc dilution curve as the state estimates become more confident. However, there is also an even chance that the state update seen in these subsequent updates will increase the miss distance between the objects, pushing the event even further to the left of the dilution curve. For these two reasons, operators often see high risk events drop off in risk assessment metrics as additional state estimates are acquired.

Previous work by Cerven [3] established a method for determining whether a hypothesized covariance update at TCA resulted in Pc greater than that of the previous update. This value was denoted as the Exceedance Probability (Pexc).

$$\mathrm{Pexc} = \iint_A \frac{1}{2\pi\sigma_{x'''}\sigma_{z'''}\sqrt{1-\rho_{x'''z'''}^{\,2}}}\, e^{-\left[\left(\frac{x'''}{\sigma_{x'''}}\right)^2 - 2\rho_{x'''z'''}\left(\frac{x'''}{\sigma_{x'''}}\right)\left(\frac{z'''}{\sigma_{z'''}}\right)+\left(\frac{z'''}{\sigma_{z'''}}\right)^2\right] \Big/ 2\left(1-\rho_{x'''z'''}^{\,2}\right)}\, dx'''dz'''$$

This method was examined using an estimated covariance growth model characterized by quadratic growth curves and by application to an example event between a SPOT satellite and a FENGYUN 1-C Debris object. Similar behavior to what is observed in the dilution curve was seen in these examples, although with the axes reversed as the author used time from TCA as the independent variable.

This work focuses on validating this technique in a more holistic sense by examining a large operational data set of conjunction messages provided by the NASA Conjunction Assessment Risk Analysis (CARA) program. The examination was limited to events that had at least one update where the Pc was of a relevant quantity with preliminary results limited to Pc >1e-12. The events are examined over their entire update cycle, calculating the probability of subsequent data updates for consistency. Comparing the cumulative distribution of these Pexc values to the expected distribution, which would ideally be directly proportional, yielded a qualitative assessment of the consistency of this approach. Preliminary results in the figure below correspond to a limited subset (1000 samples) of the total entries that indicate that update consistency in covariance modelling does not perfectly represent the reality of state updates, and that covariance projections are, in truth, marginally conservative.

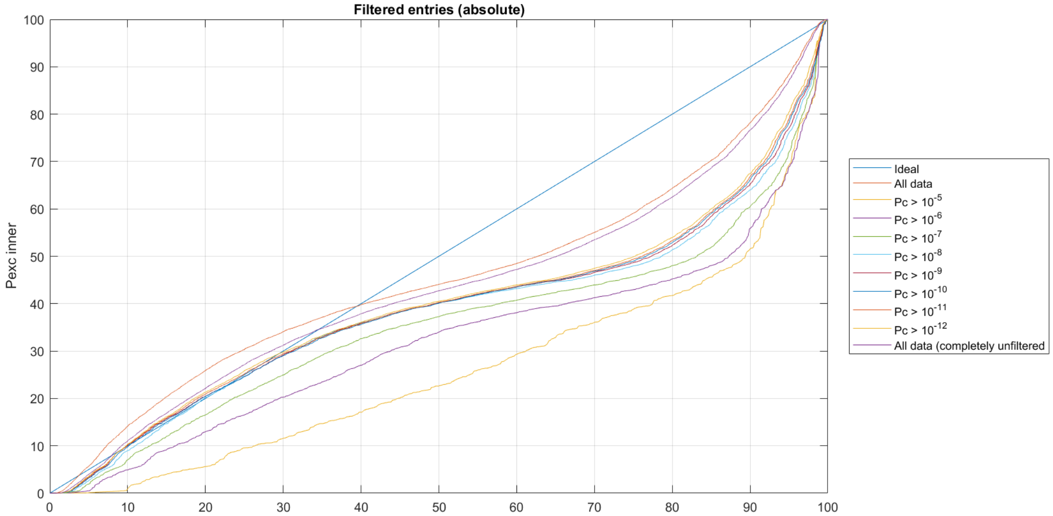

Figure 2: Preliminary Results of Pexc Consistency to Idealized Distributions

This methodology is further extended to examining the update cycles with regards to the frequency with which high interest events (HIEs) are identified at any point in the update cycle. HIEs are typically identified as events with Pc>1E-4, though operators may opt for more or less risk averse postures. This allows for insights to be derived with regards to collision avoidance planning based on the time to TCA. These insights can be used to determine at what time prior to TCA to begin collision avoidance planning. Future work may implement Kalman gain updates to predicted covariances in conjunction with the proposed strategies to calculate Pexc to aid operators in decision making for specific events and whether to initiate collision avoidance planning activities. Metrics on these frequencies are delivered relating to rates of actionable HIE events.

REFERENCES

[1] Spaceflight Safety Handbook for Satellite Operators." 18th Space Defense Squadron. V 1.5. Aug 2020. <https://www.space-track.org/documents/Spaceflight_Safety_Handbook_for_Operators.pdf>

[2] Hejduk, M. *"Satellite Conjunction Assessment Risk Analysis for "Dilution Region" Events: Issues and Operational Approaches" (2019)* Space Traffic Management Conference. 28.

[3] Cerven, William Todd. "Bounding Collision Probability Updates." AAS/AIAA Space Flight Mechanics Meeting. Williamsburg, VA. 2015. AAS 15-571.