



94th Shock and Vibration Symposium



Use of Spectral Analysis of Singular Values as a Test Metric for Impedance Matched Multi-Axis Test Trials

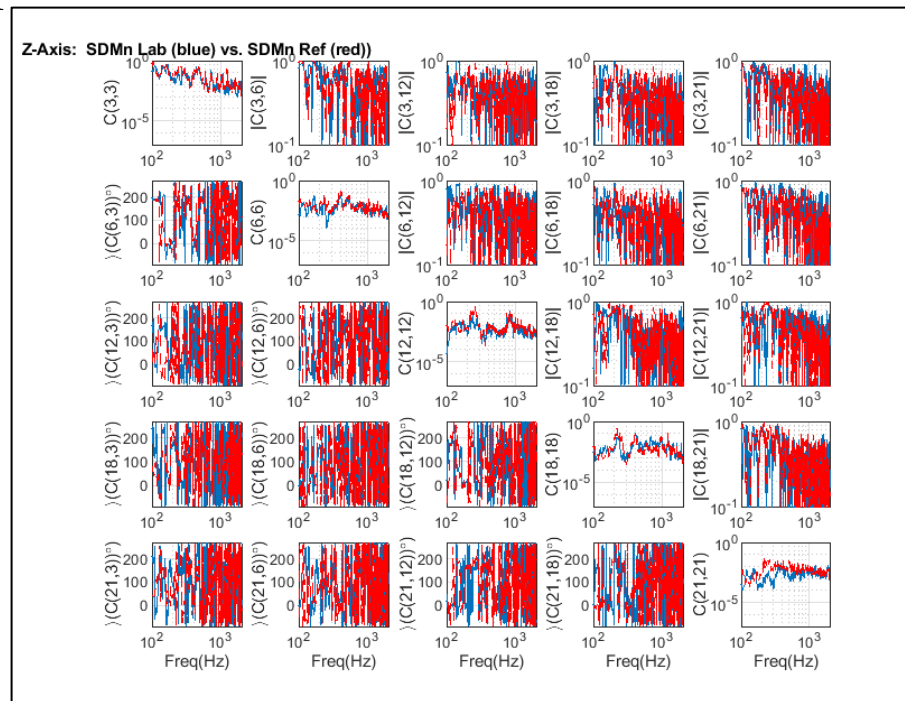
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Michael T. Hale - US Army RTC, Trideum Corporation

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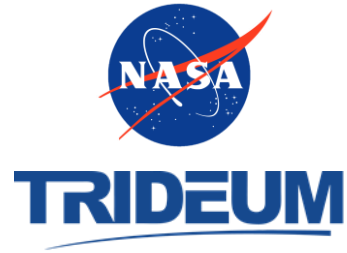
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- Multi-exciter testing (MET) continues to grow across the dynamic test community.
- Shown below is a standard view of a spectral density matrix (SDM) for a MET test with multiple control locations.
- The SDM shown is only a subset of the total control points.
- Even with this subset of the overall data, it is difficult to develop a measure of the overall quality of the test.





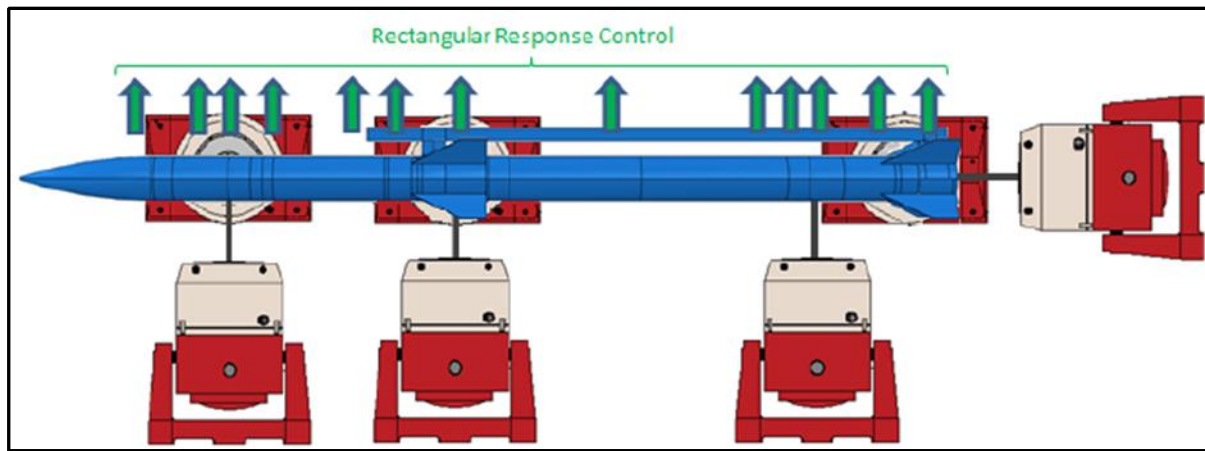
Background



- While applying classical single degree-of freedom (SDOF) test tolerances to multiple excitation points remains common practice, it becomes impractical to analyze the overall test quality as the number of control locations increases.
- The challenge then becomes how to process the test data into a compact view that provides meaningful information.
 - Method must work for a large number of control locations.
- The proposed method uses singular value decomposition (SVD) of the reference and laboratory SDMs to establish a measure of overall test quality.

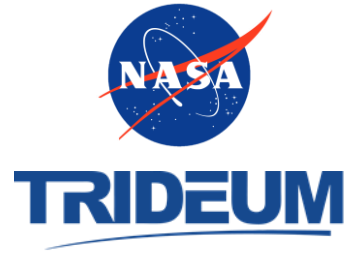
IMMAT Overview

- The Impedance Matched Multi-Axis Test (IMMAT), a special class of MET testing, is a unique approach to addressing high frequency (>500 Hz) dynamic environments.
- The IMMAT method employs multiple small exciters configured to optimally replicate measured reference (field) spectral density matrices (SDM) or an ensemble of reference time histories while minimizing the difference in mechanical impedance between the laboratory and reference environments.





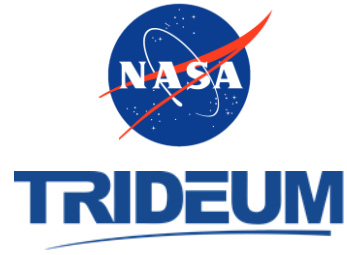
IMMAT Overview



- IMMAT testing is best employed for replicating environments where the input forces are spatially distributed over the surface of the test article.
 - A traditional large single-axis exciter using a rigid fixture is known to do a poor job of accurately replicating this type of environment.
- Free flight vibration and captive carry vibration are ideal candidates for IMMAT testing.
 - Input forces are a spatially distributed aero-mechanical load.
 - The associated velocity and displacement environments are relatively low (when compared to transportation environments) and thus do not require large exciters.



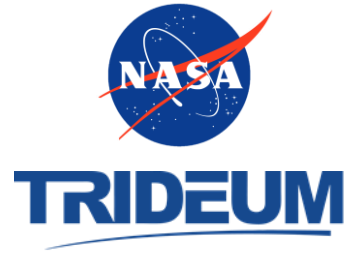
Operational Modal Analysis



- The use of computing the SVD on a SDM of test measured responses and plotting the singular values is a common frequency-domain modal identification technique used in Operational Modal Analysis (OMA).
 - The peaks in the largest singular value can indicate structural modes of the system.
- In Experimental Modal Analysis (EMA), the plot of the singular values from the SVD of the frequency response function $H(f)$ yields the Complex Mode Indicator (CMIF) function that is also used for identification of modal parameters.
 - Peaks in the CMIF plot indicate structural modes.



SVD Methodology

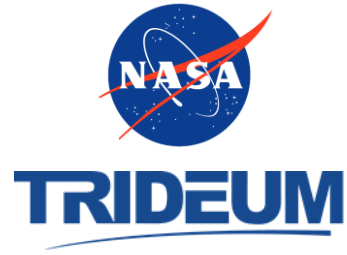


1. Establish a positive definite spectral based reference criteria $SDM_{ref}(f)$ of dimension $m \times m$, where m is the number of control channels for frequencies f_1, \dots, f_d .
2. Conduct the random vibration test and record acceleration time histories of all control locations as well of other channels of interest.
 - Note: The control channels should directly correlate to the locations from which SDM_{ref} was based.
3. Compute the laboratory test result $m \times m$ $SDM_{lab}(f)$ at the same frequency resolution as $SDM_{ref}(f)$
4. Compute the singular value decompositions of the $m \times m$ $SDM_{ref}(f)$ and $SDM_{lab}(f)$ matrices at each of the frequencies f_1, \dots, f_d as shown below.
 - $SVD[SDM_{ref}(f)] = U_{ref}(f)S_{ref}(f)V_{ref}^*(f), f = f_1, \dots, f_d$

$$- S_{ref}(f) = \begin{bmatrix} \sigma_{ref\ 1}(f) & & & \\ & \ddots & & \\ & & \sigma_{ref\ m}(f) & \\ & & & \end{bmatrix}, \sigma_{ref\ 1}(f) \geq \dots \geq \sigma_{ref\ m}(f) > 0$$



SVD Methodology



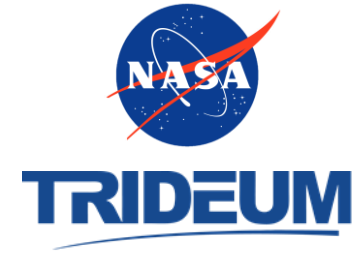
5. For each singular value from the reference criteria and the laboratory test results, compute the root mean square (RMS) value

$$- \sigma_{ref\ i\ rms} = \sqrt{\frac{1}{d} \sum_{k=1}^d \sigma_{ref\ i}^2(f_k)}$$

6. For each singular value of the S matrix across dimension d, starting with the dominant (first) singular value:
 - Compute the ratio of the RMS of the i^{th} laboratory test results singular value to the RMS of the i^{th} reference criteria singular value.
 - Compute the sum of the RMS of each reference criteria singular value.
 - Compute the ratio of the RMS of the i^{th} reference criteria singular value to that of the sum of the RMS of each reference criteria singular value.
 - Finally compute the ratio of the cumulative sum of the RMS of the first j spectral based reference criteria singular values to the sum of the RMS of each spectral based reference criteria singular value



SVD Methodology



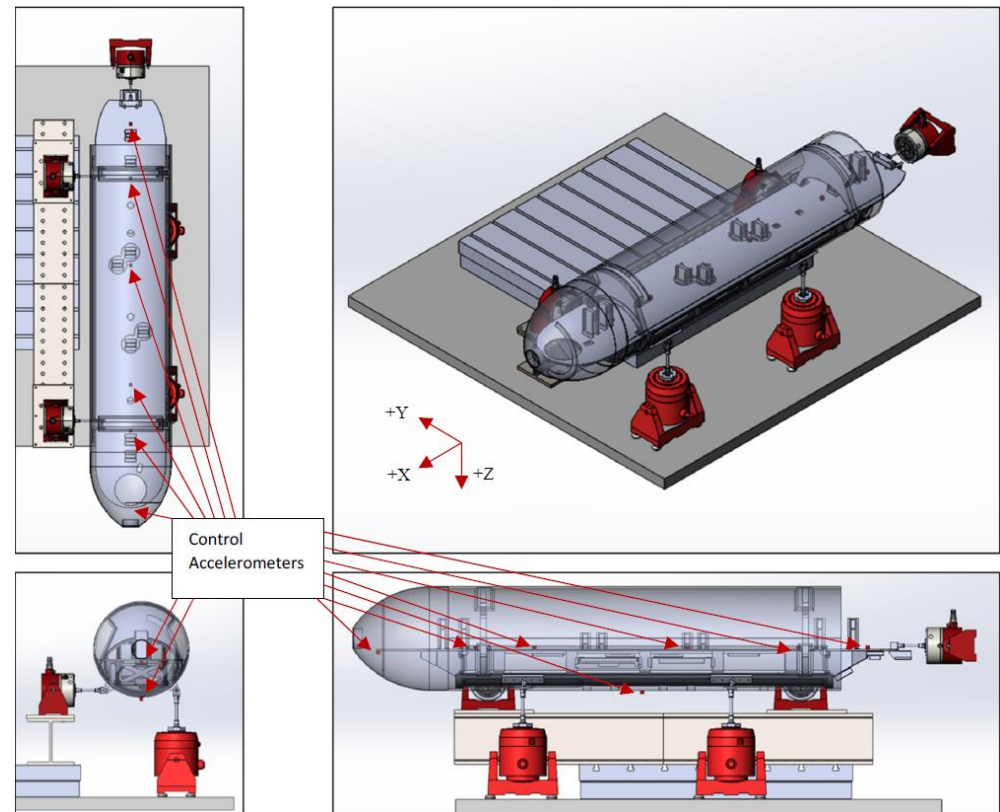
7. Populate the following table with the results of Steps 5 and 6.

Singular Value #	$\frac{\sigma_{lab\ i\ rms}}{\sigma_{ref\ i\ rms}}$ (%)	$\frac{\sigma_{ref\ i\ rms}}{Ref_{rms\ sum}}$ (%)	$\frac{\sum_{k=1}^j \sigma_{ref\ k\ rms}}{Ref_{rms\ sum}}$ (%)
1			
2			
3			
4			
5			
⋮			

8. To get a spectral view of the SVD data, plot the reference criteria and laboratory test result singular value ASD's, with specific tolerance bands relative to the reference criteria singular values, for the dominant reference criteria singular values that were identified in Step 7.

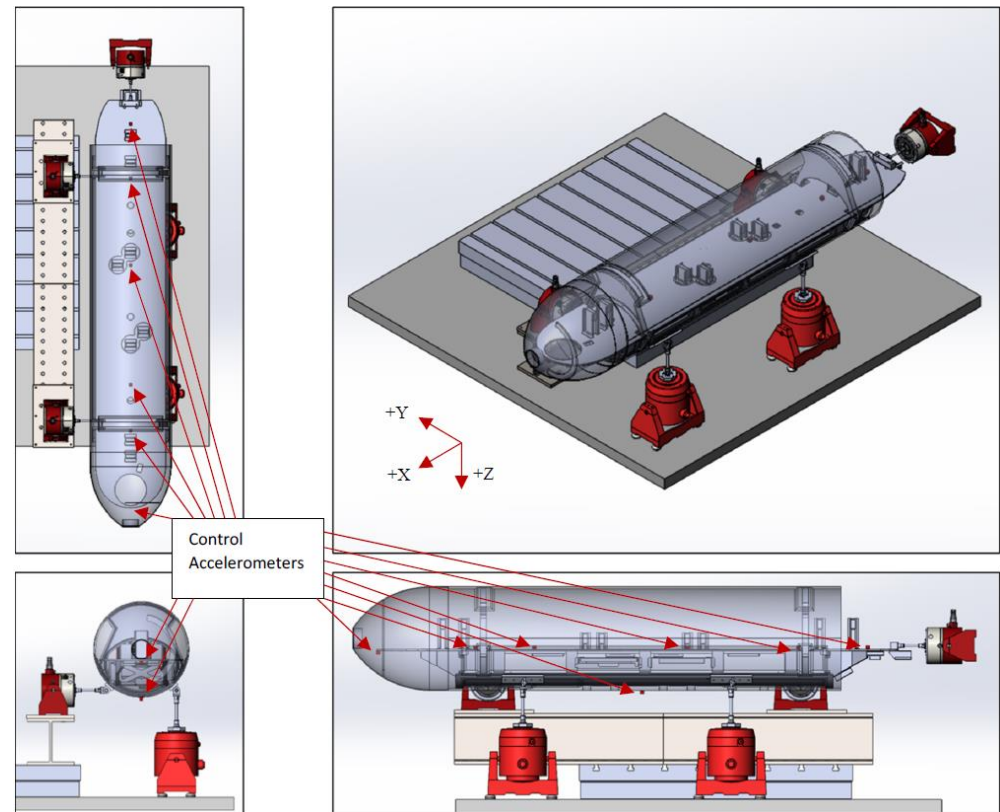
Test Configuration

- The test item in the example shown is a ~1800 lb class instrumentation pod, flown underneath a fixed wing jet.
- The external fiberglass shell contains an internal carbon fiber structure that supports a large steel platform that supports various pieces of equipment



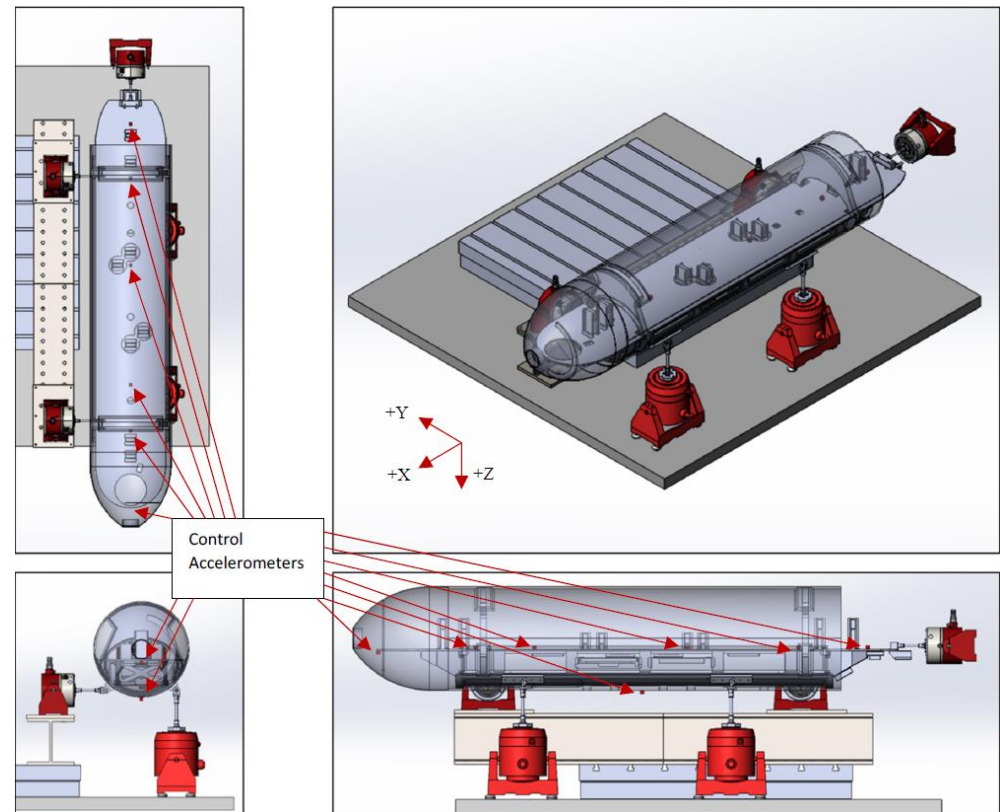
Test Configuration

- This figure shows the five exciter locations employed in the initial trials.
- The locations were chosen based on engineering judgement and available connection points of the structure to the shell.



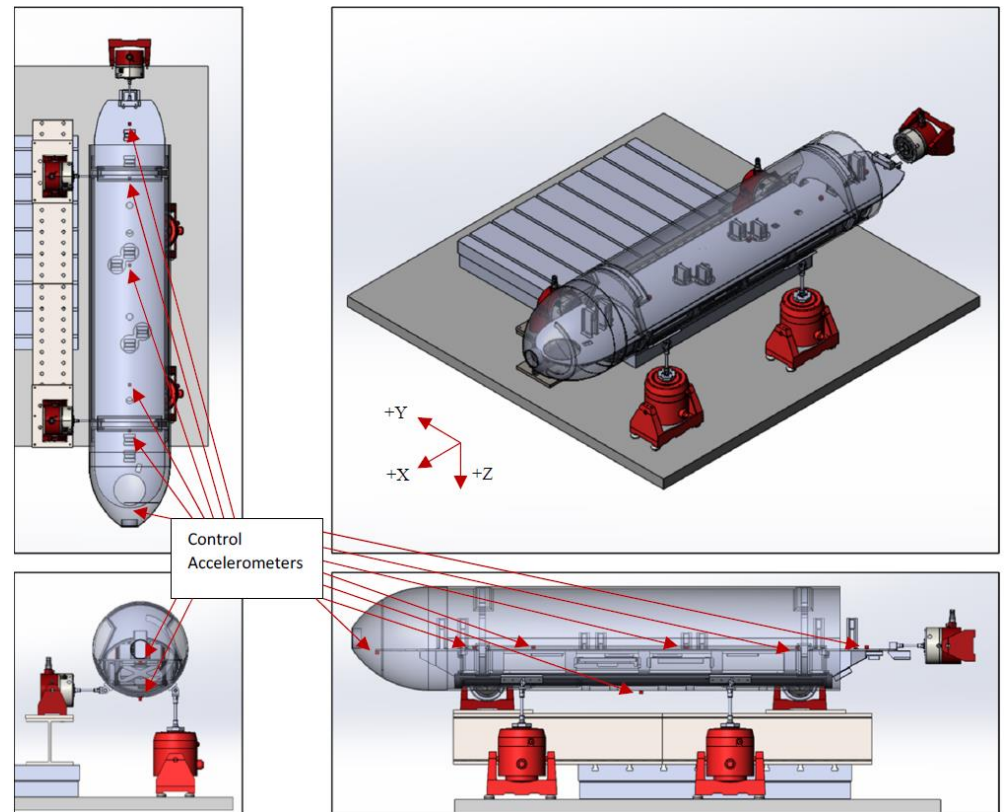
Test Configuration

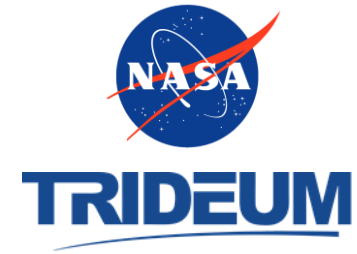
- As is often the case in developmental tests, particularly early on during development, actual flight data was not available.
- As an alternative, the test reference criteria SDM_{ref} employed was computed from acceleration time histories acquired on the instrumented pod while it was exposed to high intensity acoustics in a reverberant chamber at levels characteristic of non-buffet jet aircraft environments.



Test Configuration

- The pod arrived with preselected internal instrumentation locations based on the position of critical equipment within the pod.
 - 7 triaxial accelerometers for a total of 21 total measurement channels.
 - Post test reviews of the acoustic test results indicated several measurement channels to be suspect. Therefore, only the 16 channels identified as being good were selected for control in the laboratory trials.





Laboratory Test Results Follow



SVD RMS Results



- The table below shows the ratios of the RMS ratios of the singular value ASD's for the first 6 singular values
 - The 2nd column shows the laboratory test result RMS levels tend to be lower than the reference criteria
 - The 3rd column shows that the first singular value dominates the sum of the RMS levels
 - The 6th singular value only comprises 5% of the sum
 - The first six singular values comprise 84% of the sum of the reference criteria

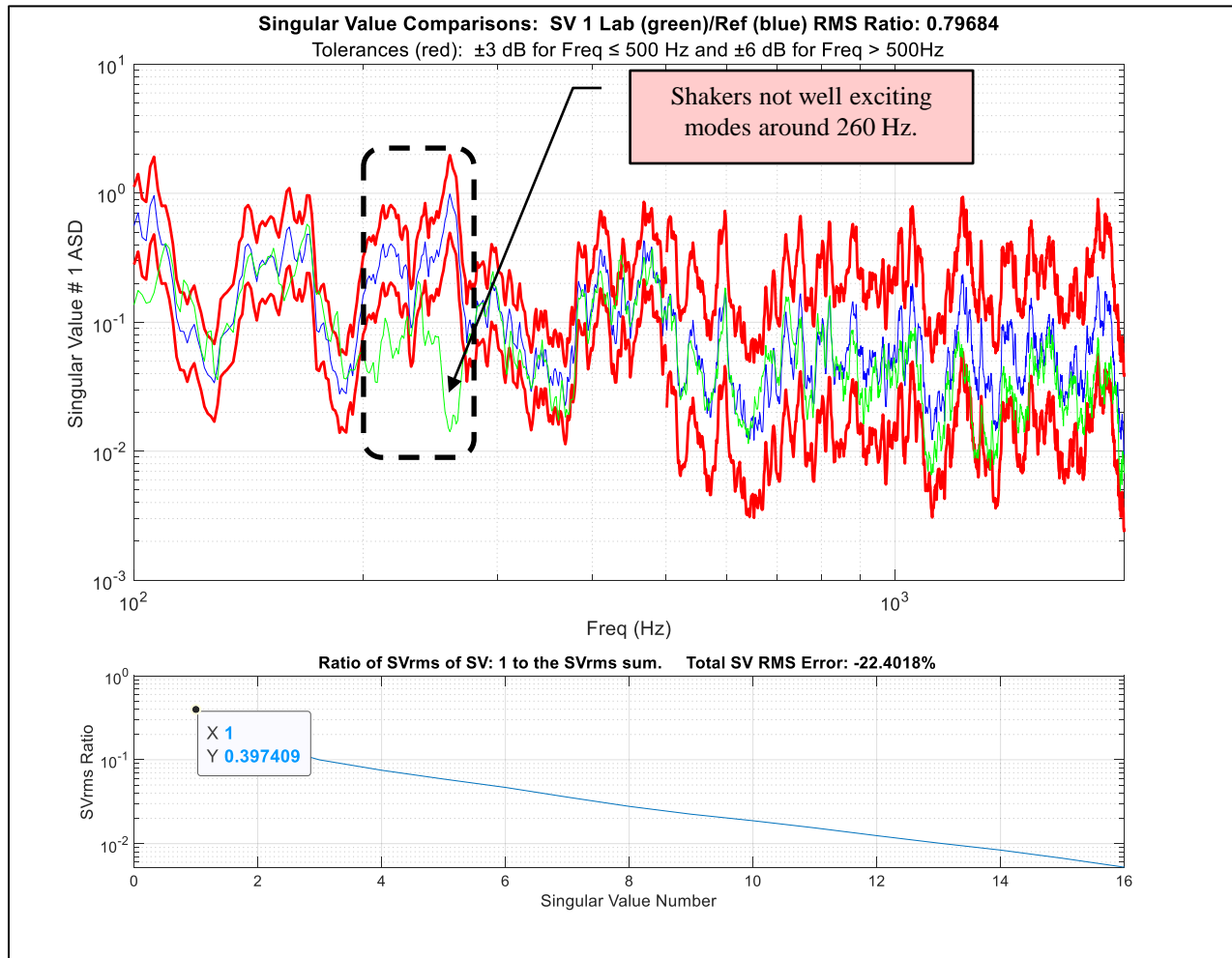
Singular Value #	$\frac{\sigma_{lab\ i\ rms}}{\sigma_{ref\ i\ rms}}$ (%)	$\frac{\sigma_{ref\ i\ rms}}{Ref_{rms\ sum}}$ (%)	$\frac{\sum_{k=1}^j \sigma_{ref\ k\ rms}}{Ref_{rms\ sum}}$ (%)
1	80	40	40
2	91	16	56
3	96	10	66
4	90	7	73
5	81	6	79
6	65	5	84

First Six Singular Values

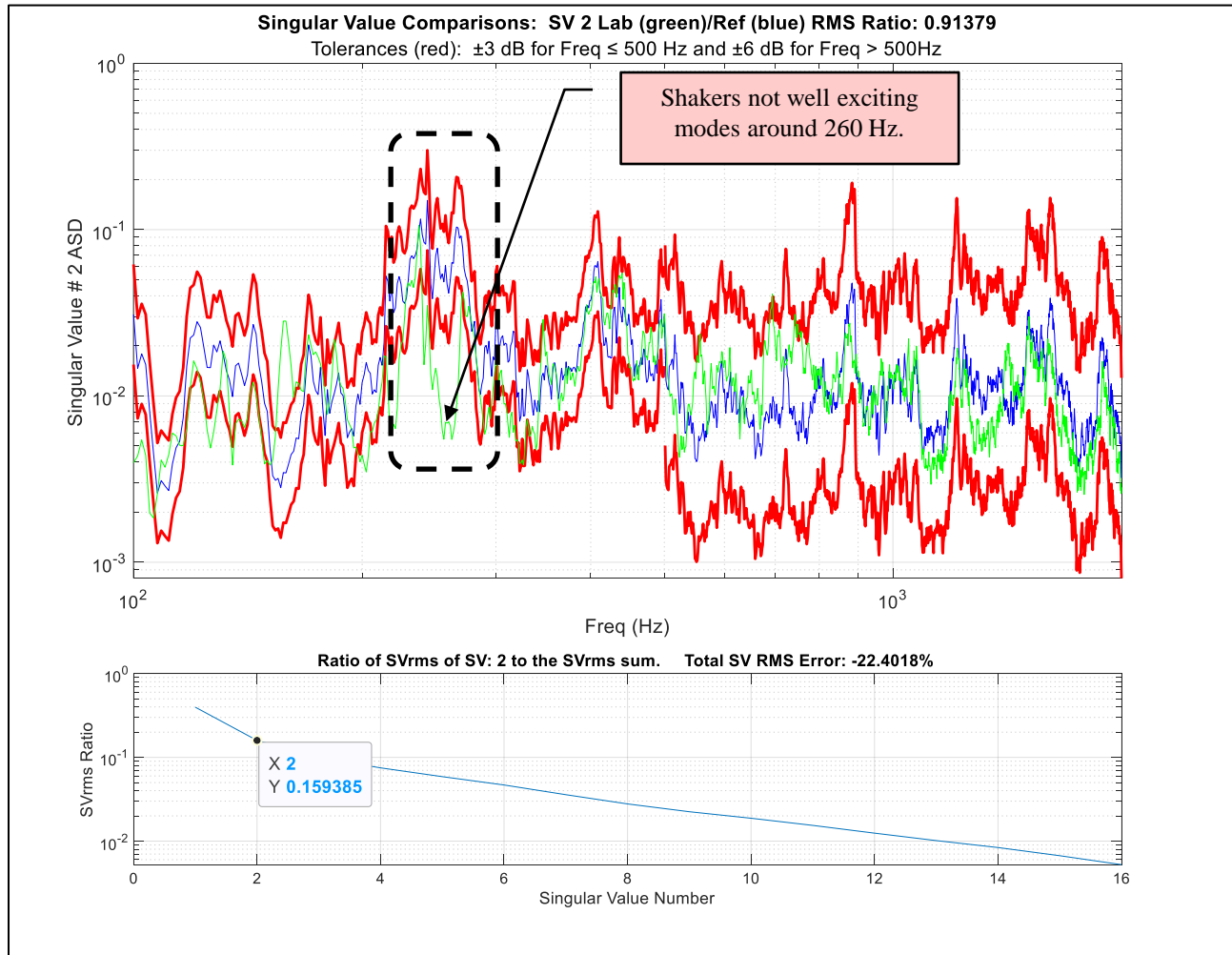
- The first singular value dominates, but the second through sixth singular values are significant.



SV1 Results



SV2 Results



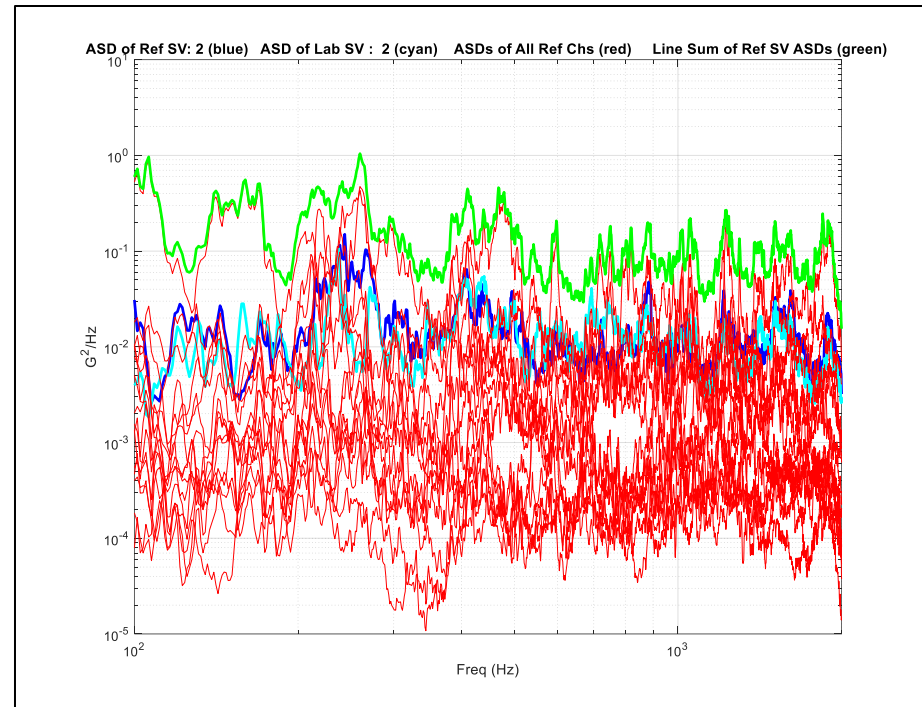
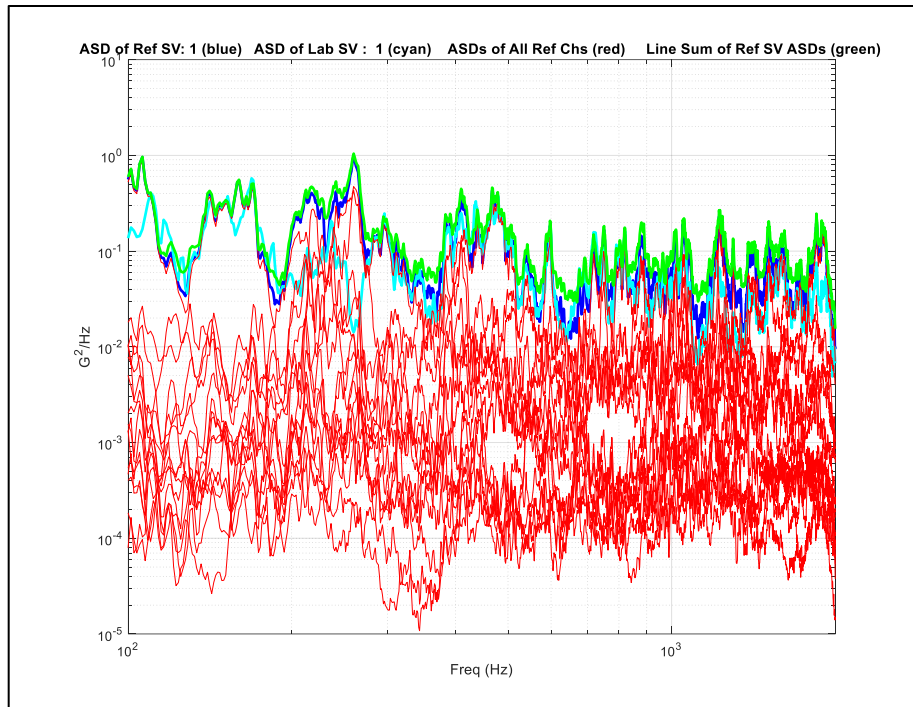


SV Combined Results



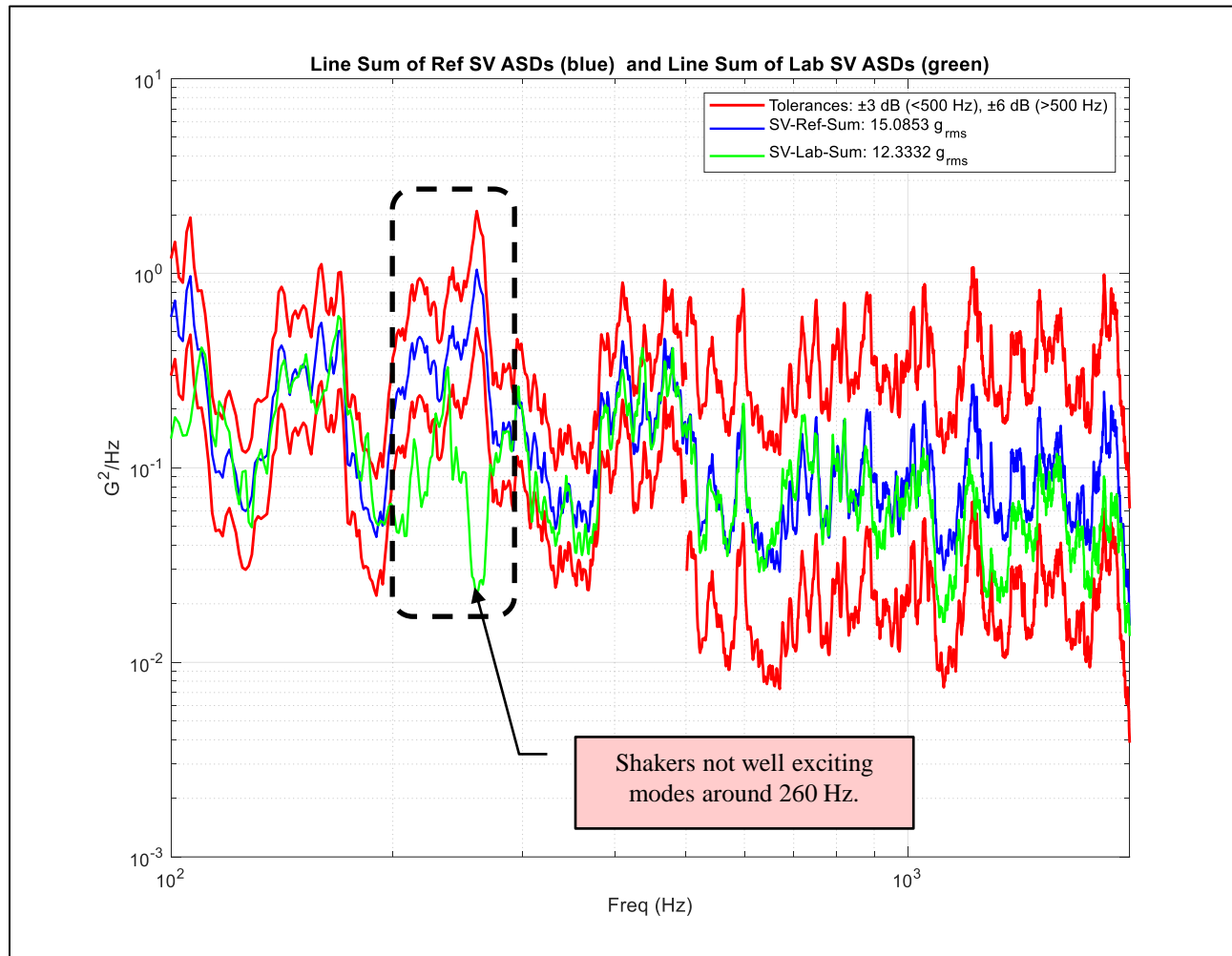
SV1_{ref} (Dark Blue)
SV1_{lab} (Light Blue)

SV2_{ref} (Dark Blue)
SV2_{lab} (Light Blue)



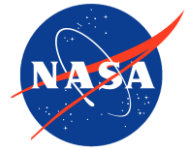
SV Sum (Green)
Other SVs (Red)

SV Sum Results

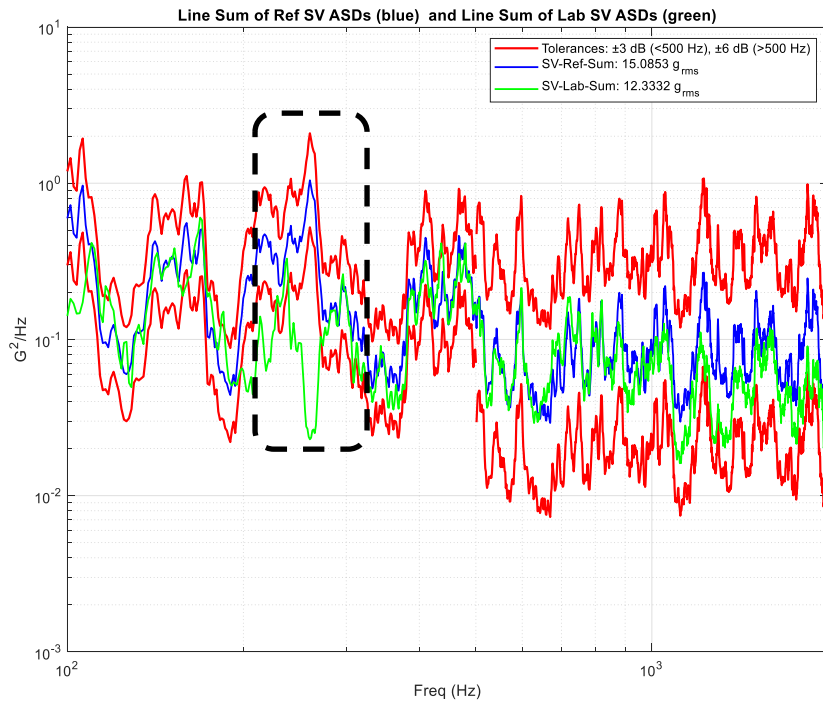




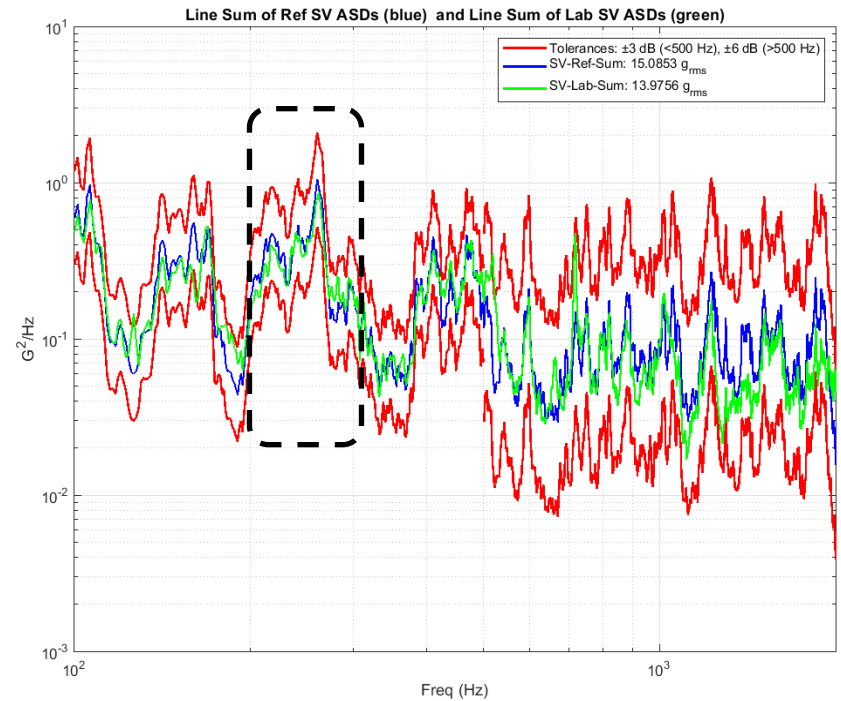
SV Sum Results



Original



Updated Setup





SVD Summary



- The results from the reference and laboratory SVD analysis were shown to serve well as a compact test metric for a MET random vibration test.
- The discussion presented here focused on SVD analysis of Spectral Density Matrices.
 - The first 6 singular values were showed to have significant contributions based upon RMS results.
 - The singular values showed there was an issue with selected shaker locations being able to adequately excite modes around 260 Hz.
 - Shaker placements for this original trial were based on judgement and positions on the test payload framework that were strong enough to connect a shaker and the instrumentation locations were preselected by a test customer.
- In this example, the number and placement of accelerometers was not sufficient to extract mode shape information.



Conclusions



- Future work includes looking at changes that can be made to the test setup that could significantly improve its controllability.
 - Finish the finite element model and review modal analysis results to investigate alternate or additional shaker locations to yield better control.
 - Investigate alternate or additional instrumentation locations that capture all significant modes.
- Further research into the modal interpretation of the individual singular value ASDs and the associated singular vectors may provide more insight into identified deficiencies and may help optimize test setups.
- This technique also showed a possible correlation between the number of significant singular values and the number of drive points. This will be studied in future work.