Comparison of Multisine Peak Factor Minimization Algorithms for Aircraft System Identification

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Outline

- Introduction
- Multisine Signals
- Multisine Design and Optimization
- Sample Results
- Discussion and Applications
- Conclusions

Introduction

- Efficient experiment design saves time and money
- Phase-optimized multisines have proven very effective for efficient system identification testing
  - Optimized to minimize **peak factor**
- Existing optimization methods can be time-consuming depending on application
- Two peak factor minimization algorithms are evaluated and compared against typical current approach

Multisine Signals

- Multisines can be expressed in time domain:

$$u(t) = \sum_{i=1}^{N_k} A_i \sin\left(\frac{2\pi k_i t}{T} + \phi_i\right)$$

or frequency domain:

$$U(k) = \frac{N}{2} A(k) e^{j\left[\phi(k) - \frac{\pi}{2}\right]}$$

- Peak factor measures compactness of signal:

$$PF(\boldsymbol{u}) = \frac{\max(|\boldsymbol{u}|)}{\mathrm{rms}(\boldsymbol{u})} = \frac{\max(|\boldsymbol{u}|)}{\sqrt{\sum A_i^2/2}}$$

- Relative peak factor defined as:

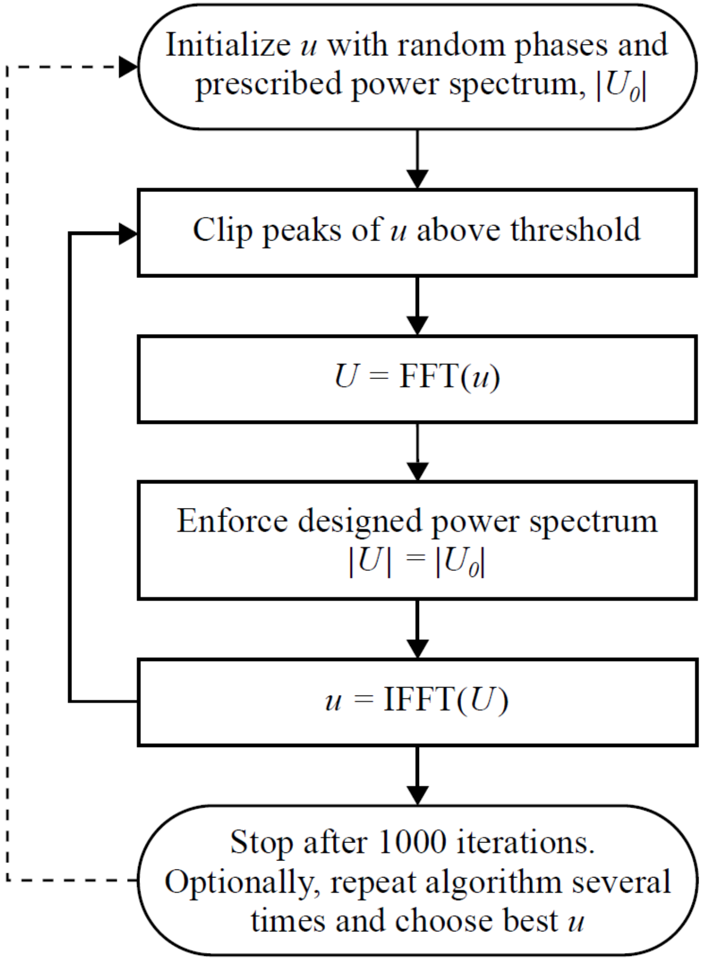
$$RPF(\boldsymbol{u}) = PF(\boldsymbol{u})/\sqrt{2}$$

Objective of optimization:
For given amplitudes, $\boldsymbol{A}$, and frequencies, $\boldsymbol{k}$, find phase angles, $\boldsymbol{\phi}$, that minimize $\max(|\boldsymbol{u}|)$ (and thus PF/RPF)

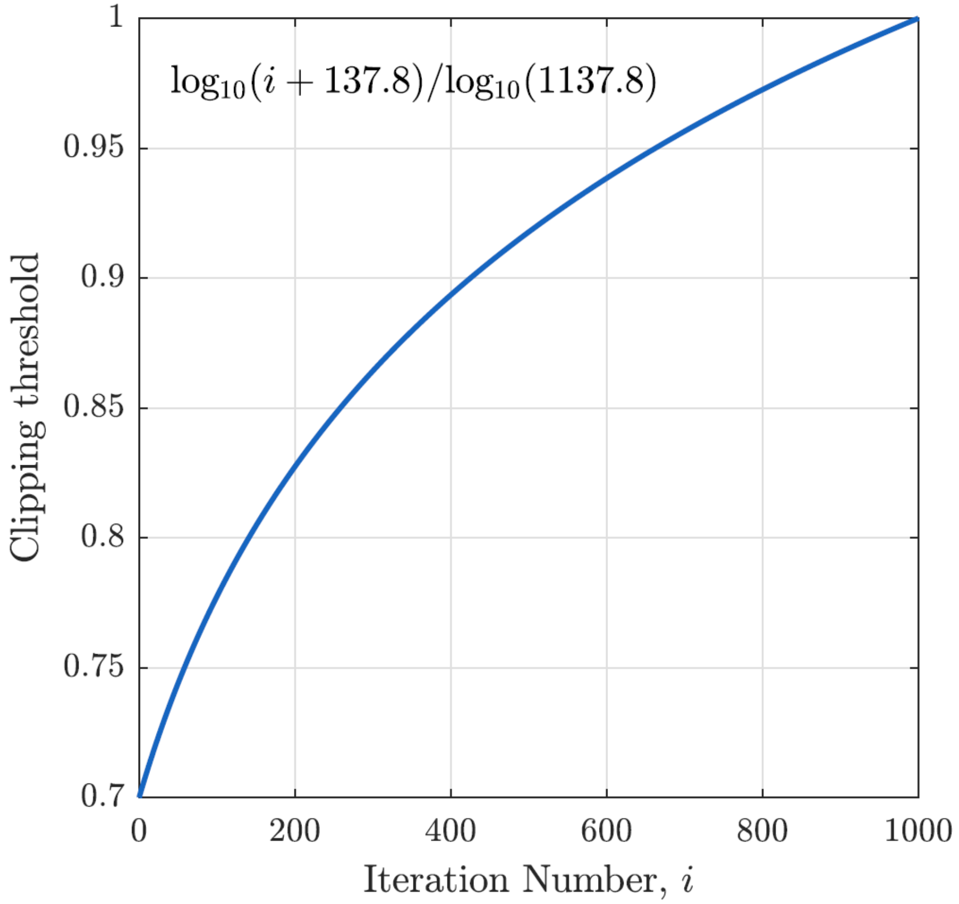
Multisine Design and Optimization

- Peak Factor Minimization Algorithms
  - Clipping Algorithm
  - Infinity Norm Algorithm
- Additional Considerations
  - "Snowing"
  - Simultaneous Input-Output Peak Factor Minimization
  - Effect of Sampling Rate

Clipping Algorithm

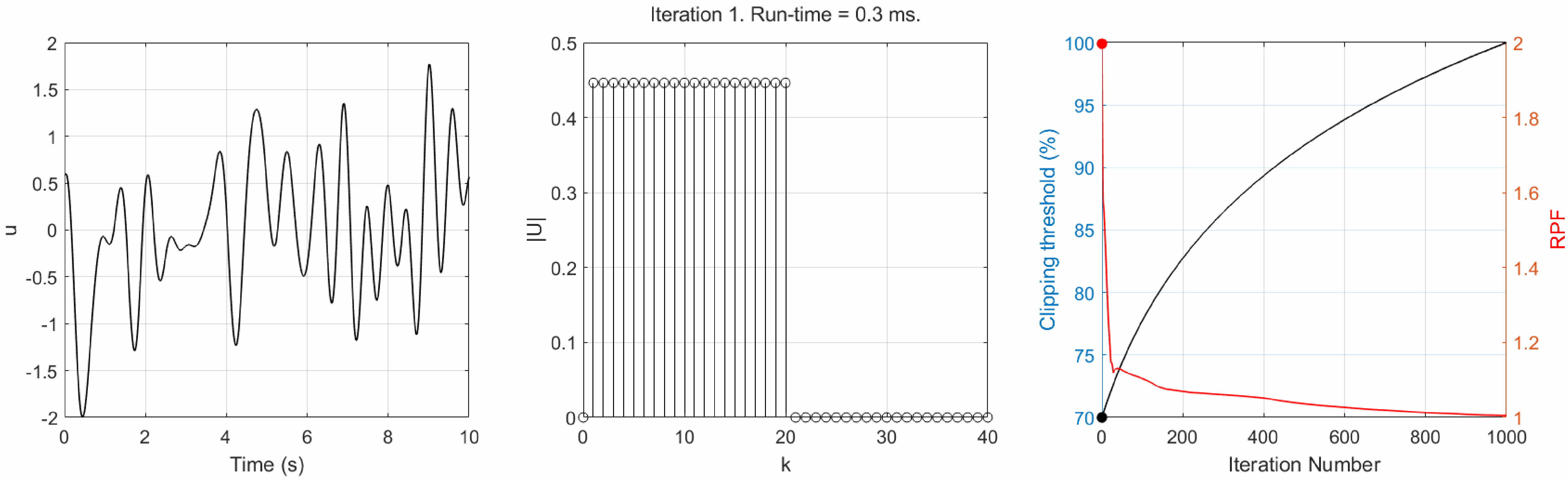


Clipping algorithm.



Clipping threshold function.

Clipping Algorithm



Example of clipping algorithm iterating over time.

Infinity Norm Algorithm

Prerequisites:

- The Chebyshev or infinity norm of a function $u(t)$ is the maximum absolute value:

$$||u||_\infty = \max_{t\in[0,T]} |u(t)|$$

- p-norm of a continuous function:

$$||u(t)||_\mathrm{p} = \left(\frac{1}{T}\int_0^T |u(t)|^p dt\right)^{1/p}$$

- Infinity norm is related to the p-norm of a function by

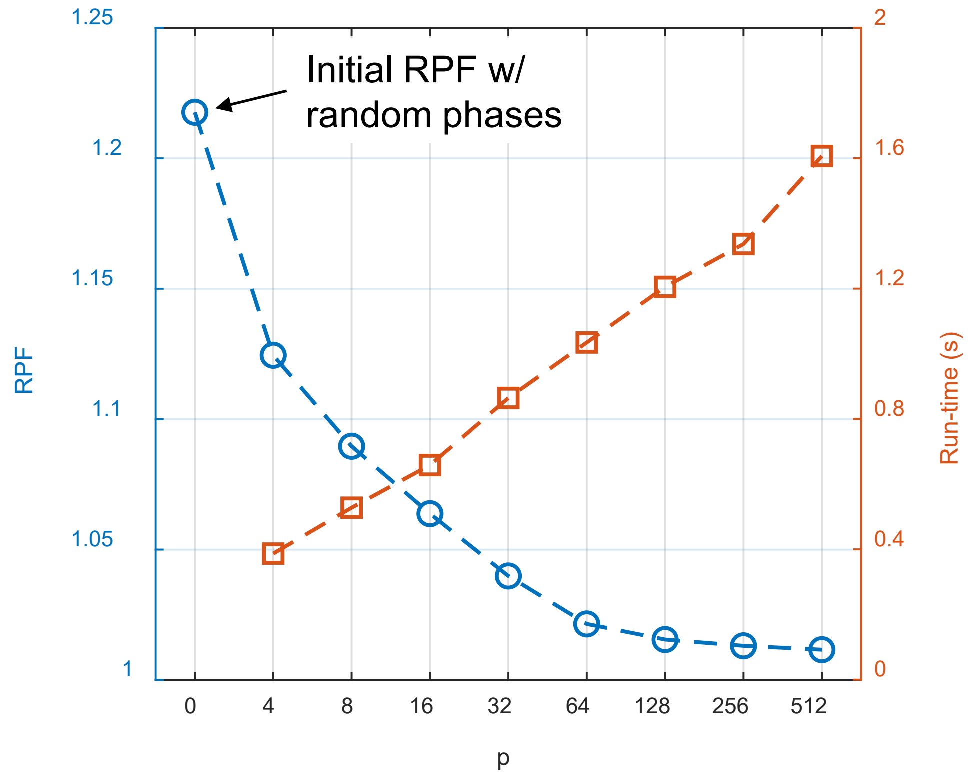
$$\lim_{p\to\infty} ||u||_p = ||u||_\infty$$

- Recall objective is to minimize peak factor – can achieve by minimizing $||\boldsymbol{u}||_\infty$

$$PF(\boldsymbol{u}) = \frac{\max(|\boldsymbol{u}|)}{\mathrm{rms}(\boldsymbol{u})} = \frac{||\boldsymbol{u}||_\infty}{\sqrt{\sum A_i^2/2}}$$

Infinity Norm Algorithm

- Find phase angles, ϕ_p, that minimize p-norm for a sequence of p such as
$$p = 4,8,16, \dots, 512$$
  - Last solution is used for next iteration (i.e., $\phi_{p=4}$ is starting value for solving $\phi_{p=8}$)
- Converges to optimal solution*:
$$\lim_{p\to\infty} \phi_p = \phi_\infty$$
- Can be solved efficiently as nonlinear least squares problem



RPF and run-time over duration of algorithm for a sequence $p = 4, 8, 16, \dots, 512$

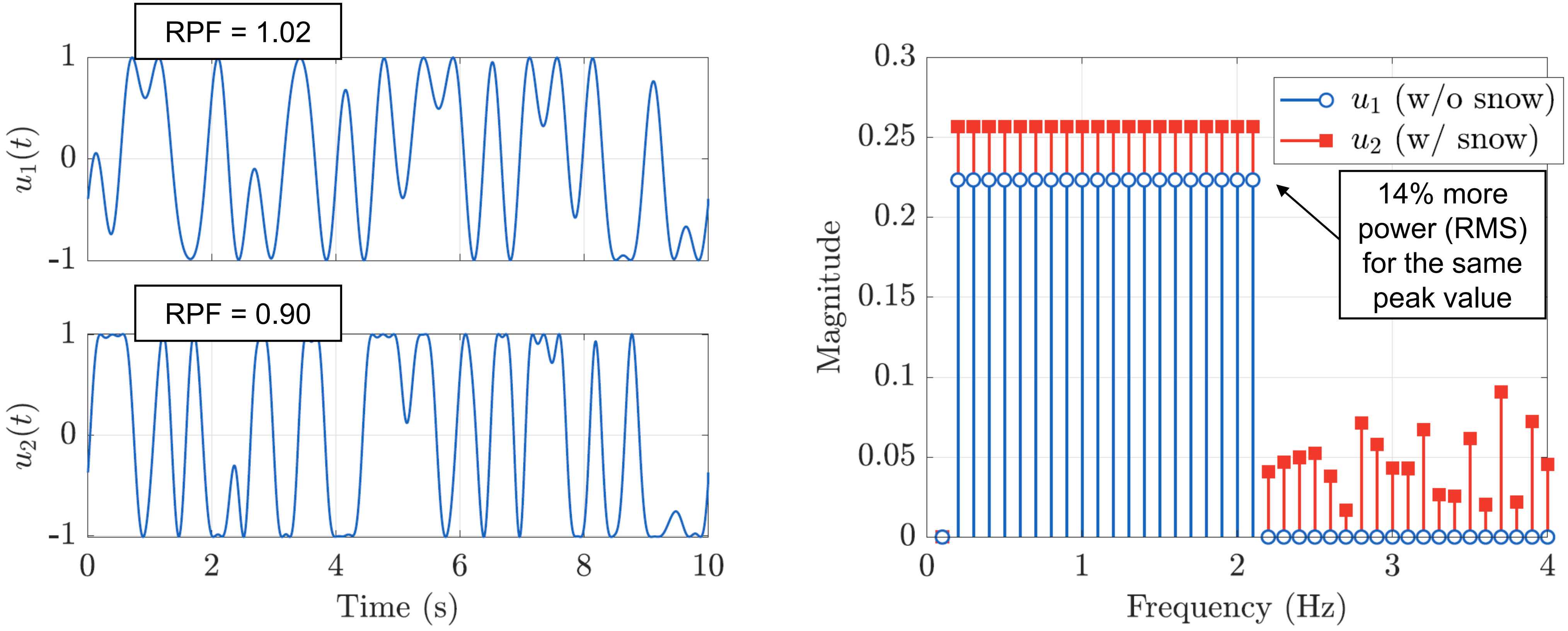
Subject to some regularity conditions. See paper.

Additional Considerations

1. “Snowing”
2. Simultaneous input-output peak factor minimization
3. Effect of sampling rate on algorithm performance

Additional Considerations

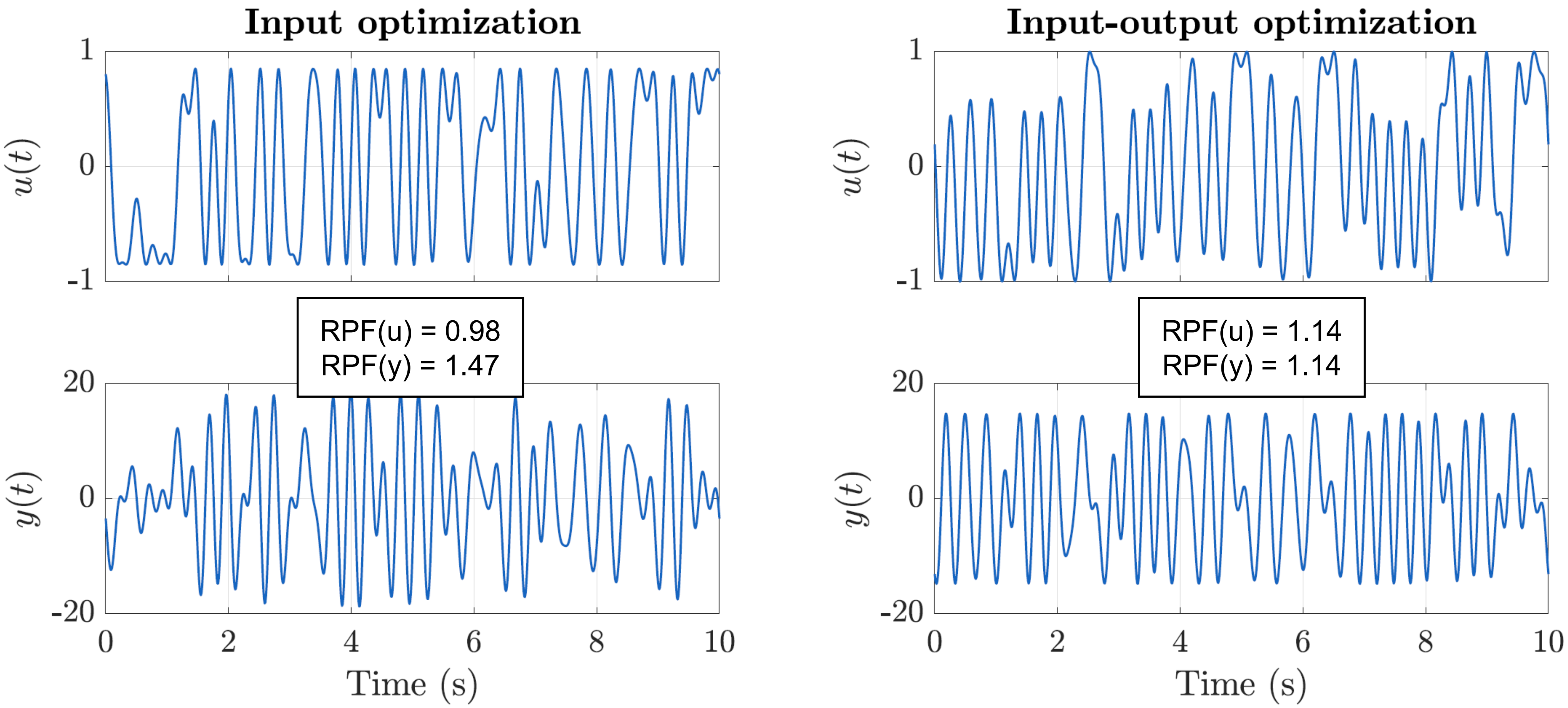
Snowing is the process of adding power at additional harmonics in order to further reduce the peak factor of the signal



Example of optimized multisine signals without snow, $u_1(t)$, and with snow, $u_2(t)$, and their respective power spectra

Additional Considerations

Input-Output Peak Factor Minimization



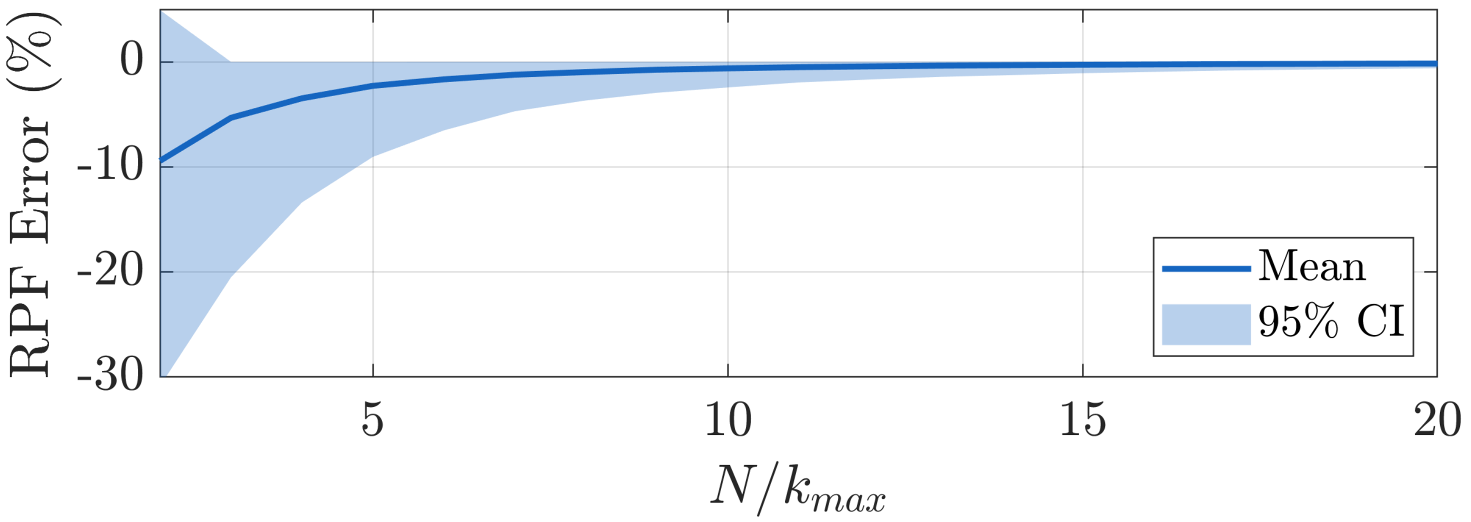
Comparison of a phase-optimized multisine signal, $u(t)$, and its time derivative, $y(t)$, after input optimization (left) and simultaneous input-output optimization (right)

Additional Considerations
Sampling Rate

- Number of samples in signal will affect run-time of any algorithm
- Sample rate for algorithm comparisons selected to have less than 1% error when comparing the discrete signal with the equivalent *continuous* multisine signal
- Less than 1% error (95% CI) for

$$N/k_{\max} \geq 16$$

N: number of samples $k_{\max}$: maximum harmonic number

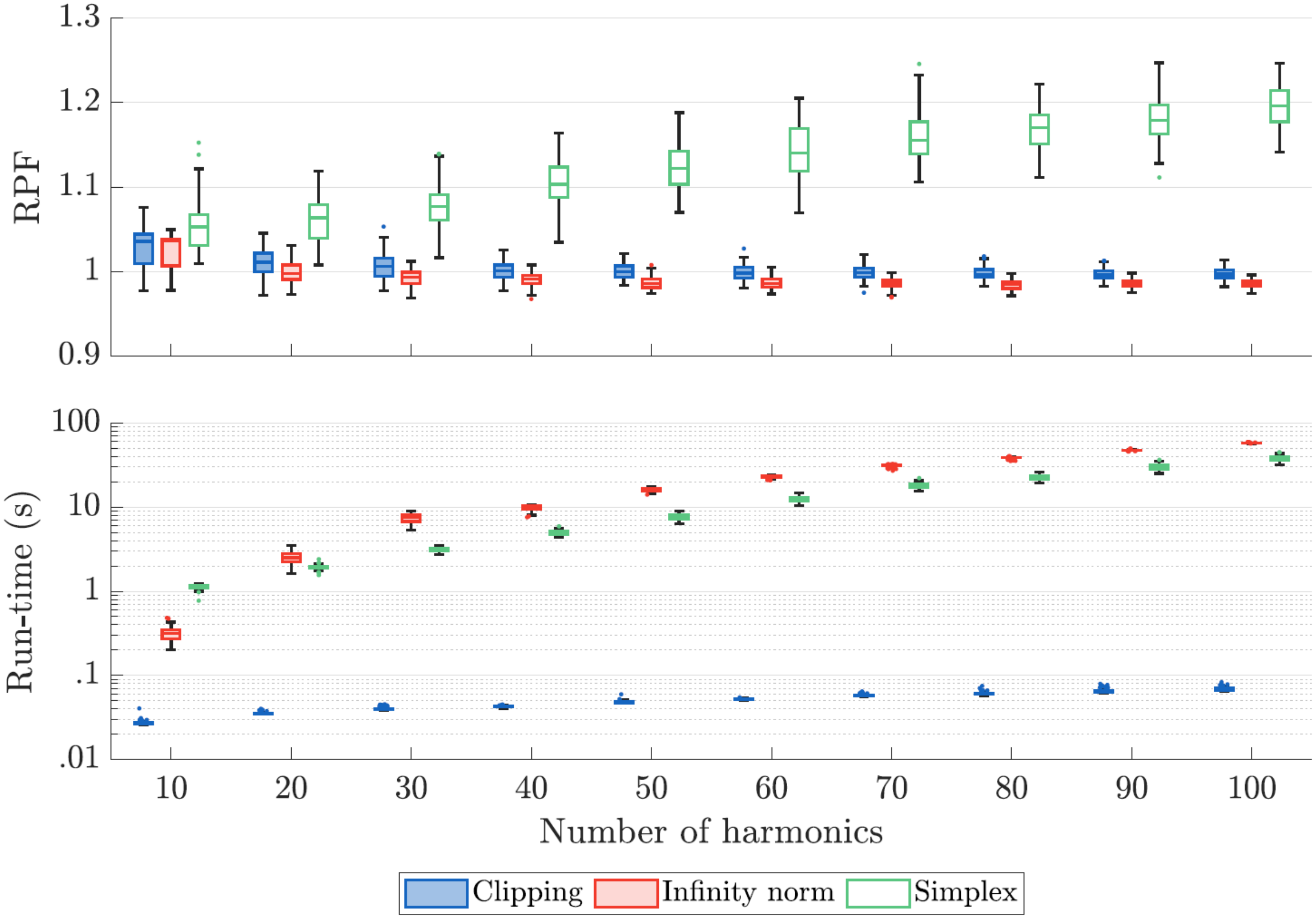


Effect of sample-to-maximum harmonic ratio on the accuracy of the computed peak factor

Results

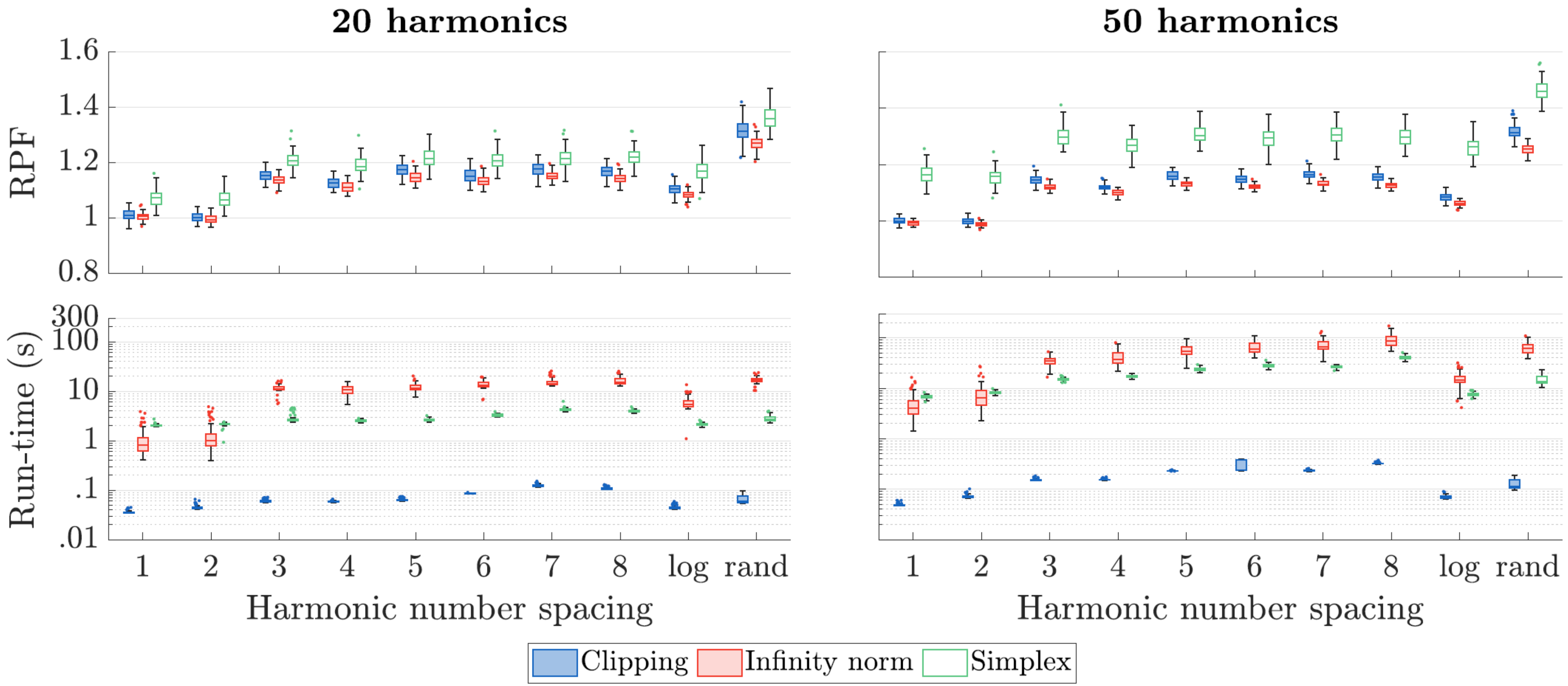
- Evaluate both algorithms as function of multisine properties
  - Number of harmonics
  - Harmonic number spacing
  - Inclusion of snow harmonics
- Compare against traditional approach (SIDPAC) of direct optimization using simplex algorithm
  - Modified to generate signals in frequency domain then IFFT – faster
- Evaluate effect of repeating clipping algorithm from multiple initial conditions

Results



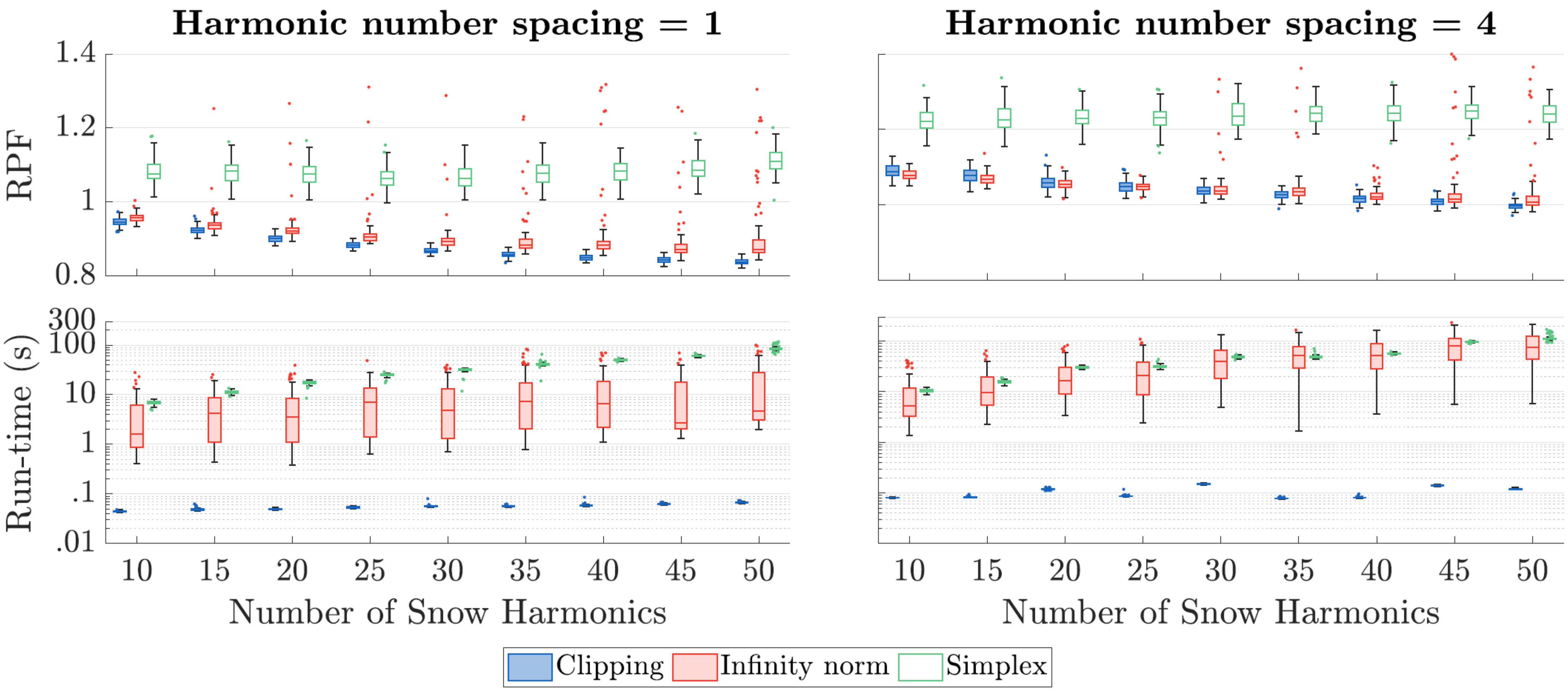
Comparison of peak factor and run-time between algorithms for different numbers of harmonics

Results



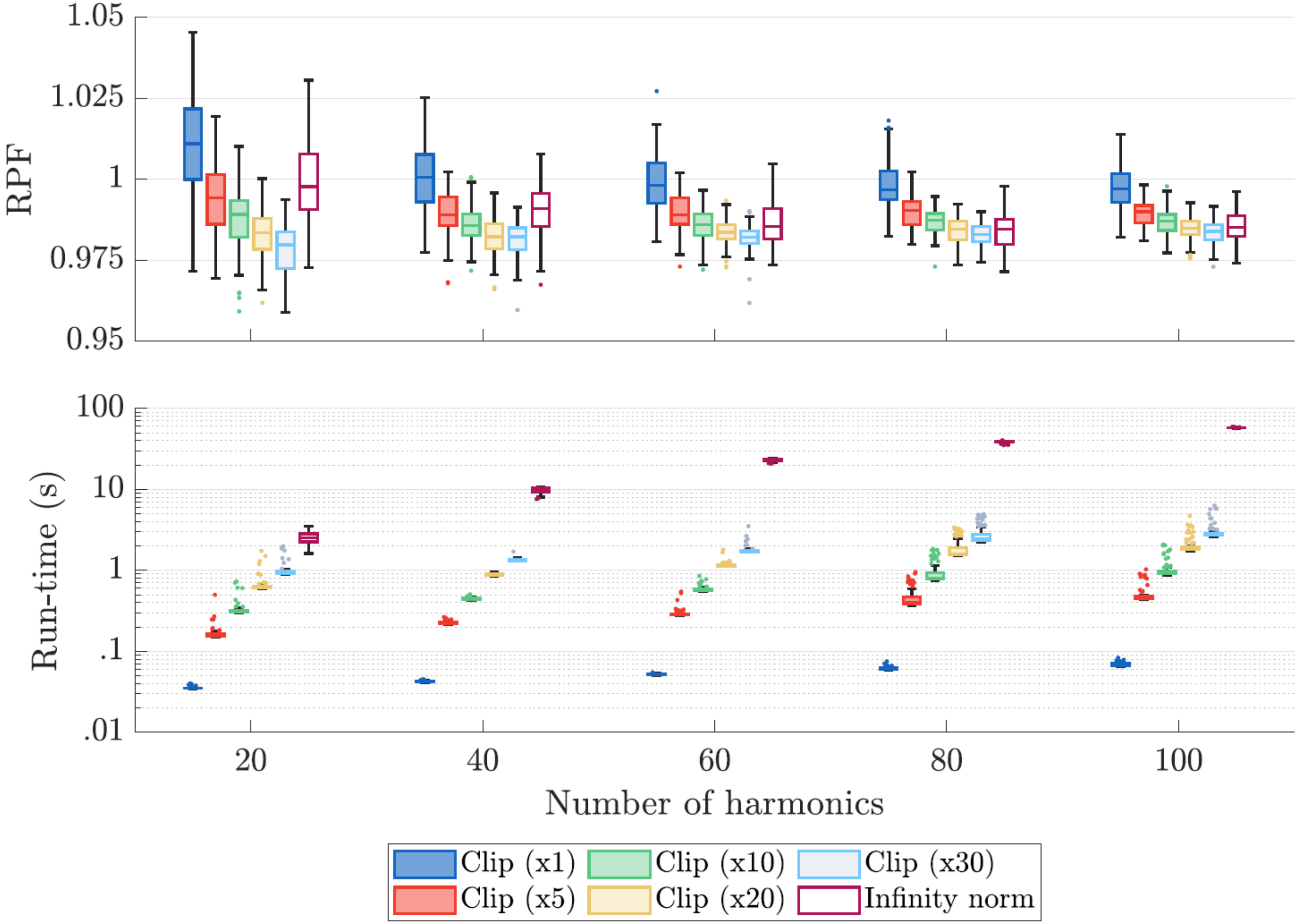
Comparison of peak factor and run-time between algorithms for different harmonic number spacings.

Results



Comparison of peak factor and run-time between algorithms for different numbers of snow harmonics.

Results



Peak factor and run-time of clipping algorithm when repeating the routine from a varying number of initial conditions.

Discussion and Applications

- Infinity norm and clipping algorithm resulted in comparable RPFs
  - Both lower than simplex algorithm
  - If clipping algorithm is run from multiple initial conditions, it also resulted in the lowest RPFs
- Clipping algorithm runs fastest, nearly instantly
- Effectiveness and speed of clipping algorithm opens door for potential applications:
  - Designing signals with hundreds of thousands of harmonics for structural mode testing
  - Optimization of signals in real-time based on observations or real-time modeling results

Conclusions

- Two multisine peak factor minimization algorithms were presented, evaluated and compared
- Clipping algorithm is best choice for rapid design of phase-optimized multisine signals
  - Very fast (typically <1s)
  - Most effective at minimizing RPF
- Clipping algorithm can reduce time spent designing and optimizing multisine signals for flight tests and other experiments
- High-speed of clipping algorithm could facilitate new test techniques
  - Real-time optimization of signals
  - Experiment design for aeroelastic model identification

Thank You!

- Thanks to Gene Morelli and Ben Simmons for assistance with this report
- Questions?