Direct Reconstruction of Ablative Thermal Protection System Aeroheating Using a Green's Function Approach



Kenneth McAfee [1], Hannah Alpert [2], Peter Sunderland [1], Oded Rabin [1]

[1]University of Maryland, College Park, MD 20742

[2]NASA Ames Research Center, Moffett Field, CA 94035

Abstract: A Green's function **inverse heat transfer (IHT)** approach is used to reconstruct the surface heating conditions on ablative **thermal protection system (TPS)** materials from embedded heat flux sensor and temperature probe measurements. The approach models the temperature time-history at the measurement location as a discrete linear system, allowing for the heat flux boundary condition to be recovered directly without the need for time-marching schemes. The effects of material decomposition and pyrolysis gas transport are modeled using an energy source/sink analogue. The performance of the reconstruction approach is analyzed on a 1D test case representative of an atmospheric entry heating scenario. The approach can recover the TPS surface heat flux to within 4% of the input heating condition with a computation time of 2-3 seconds (>3 orders of magnitude faster than current time-marching IHT methods). As a byproduct of the surface heating reconstruction, the algorithm also captures the surface pyrolysis gas mass flux and solid decomposition at multiple through-thickness locations within the TPS.

Motivation



https://mars.nasa.gov/mer/gallery/artwork/entry_br.html

- Reconstructions of the heat loads experienced by spacecraft during atmospheric entry are critical to evaluate the performance of thermal protection systems (TPS) and validate computational models.
- Current reconstruction methods are costly and demand significant computing resources.

Conventional IHT Methods *vs* *Green's Functions*

- Conventional inverse heat transfer (IHT) methods rely on time-marching finite volume/element schemes in an objective function minimization approach[2].
- The full mesh solution must be computed to calculate the objective function (i.e., temperature at a single location).
- Nonlinear optimization methods are required, which scale poorly with increasing sample size.

Total computation time $>$ **30 minutes**

Requires $>$ **1000** evaluations

Real computational cost $(N_{samples} = 600)$

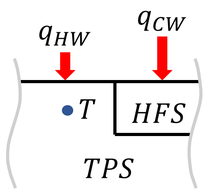
- Green's function-based methods (mesh-less) only model the temperature response at a *single* location, but for the *entire* time-series simultaneously.
- The material response is modeled as a linear system.
- The heat flux boundary condition can be recovered via regularized inversion of the linear system.

Total computation time $\approx$ **3 seconds**

Requires $<$ **10** evaluations (**1** for linear systems)

Inverse Heat Transfer

- IHT methods are often used for post-flight assessment of TPS thermal loading, as they directly calculate the heat flux absorbed by the hot-wall TPS q_{HW}.
  - ➢ Total heat flux sensors, while offering a more stable and direct measurement, absorb an augmented cold-wall heat flux q_{CW}.
- Sensor fusion methodologies—combining measurements from collocated heat flux and temperature sensors—yield substantial robustness for IHT–based aerothermal reconstruction efforts[1].



Schematic of Collocated Heat Flux Sensor (HFS) and Temperature Probe [1]

The Reconstruction Algorithm

Objective: Reconstruct the net hot-wall and incident surface heat flux on ablative TPS using measurements from embedded temperature and heat flux sensor instrumentation.

Conservation of Energy

$$\underbrace{\rho_s C_p \frac{\partial T}{\partial t} - \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)}_{\text{Homogeneous heat conduction equation}} = \underbrace{\left(h_g - \frac{\rho_c h_c - \rho_v h_v}{\rho_c - \rho_v}\right)\frac{\partial \rho_s}{\partial t}}_{\text{Enthalpy change due to material decomposition}} \underbrace{- \frac{\partial}{\partial t}\left(\phi \rho_g e_g\right)}_{\text{Gaseous energy storage}} \underbrace{+ \frac{\partial h_g}{\partial x}\int_0^x \frac{\partial \rho_s}{\partial t} dx'}_{\text{Pyrolysis gas transport}}$$

- Material properties are both functions of temperature and composition (char fraction).
- Pyrolysis effects (RHS of energy conservation) condensed into a single term → g_{pyro}
- g_{pyro} is a function of the local temperature, temperature gradient, and composition.
  - ➢ Analogous to a volumetric energy source/sink.

Green's Function Solution Equation

- Green's function formalism used to connect temperature measurement, effects of pyrolysis, and unknown hot-wall heat flux boundary condition.

$$\underbrace{T(x,t)}_{\text{Measured temperature}} = \underbrace{-\int_0^t \frac{\alpha}{k} q_{HW}(\tau) G(x,t,L,\tau) d\tau}_{\text{Effects of heat flux boundary condition (unknown)}} + \underbrace{\int_0^t \int_0^L \frac{\alpha}{k} g_{pyro}(x_0,\tau) G(x,t,x_0,\tau)\, dx_0\, d\tau}_{\textbf{Effects of pyrolysis} \equiv \boldsymbol{g_{tot}}}$$

- Quadrature scheme used to compute the volume integral in $\boldsymbol{g_{tot}}$.
  - ➢ Pyrolysis effects (g_{pyro}) calculated at select (~10) through-thickness locations.

Recovery of the Net Hot-Wall Heat Flux

- For discrete measurements, Green's function solution equation becomes a linear system.
- $\boldsymbol{g_{tot}}$ accounts for the *net* effects of pyrolysis on the temperature measurement.
- Inversion of linear system stabilized using collocated heat flux sensor measurement q_{CW}. Smoothness enforced in the time evolution of $C(t)$.

$$\{T\} = -\frac{\alpha}{A}[G]\{q_{HW}\} + \{\boldsymbol{g_{tot}}\}$$

$$q_{HW} = C(t) q_{CW}$$

- Unknown hot-wall heat flux recovered via minimization of the measurement residual.

$$\min\left\{\left\|\{T\} - \{\boldsymbol{g_{tot}}\} + \frac{\alpha}{k}[G]\,[diag(q_{CW})\,]\{C\}\right\|_2^2 + \|\lambda_2[L_2]\{C\}\|_2^2\right\}$$

Measurement Residual — 2nd derivative penalty term to enforce smoothness in C

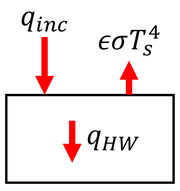
- $\boldsymbol{g_{tot}}$ calculated simultaneously with $C(t)$ (i.e., q_{HW}).

Incident Heat Flux Calculation

- Incident heat flux q_{inc} from external environment linked with net hot-wall heat flux q_{HW} via surface energy balance.

$$q_{inc} = q_{HW} + \epsilon\sigma T_s^4$$

- Incident heat flux q_{inc} includes heat transfer mechanisms from shock layer radiation and boundary layer convection/advection.



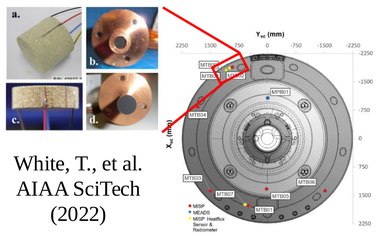
Key

q	*Heat Flux*	k	*Thermal Conductivity*	ρ	*Density*	h	*Enthalpy*	e	*Internal Energy*
T	*Temperature*	C_p	*Specific Heat Capacity*	ϕ	*Porosity*	L	*Thickness*	G	*Green's Function*
α	*Thermal Diffusivity*	g_{pyro}	*Pyro. Sources/Sinks*	g_{tot}	*Net Pyro. Effects*	ϵ	*Emissivity*	λ_2	*Regularization Coeff.*

Numerical Verification

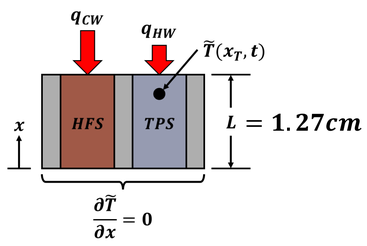
Objective: Verify the accuracy of the Green's function-based reconstruction approach on a 1D ablative model system.

1D Model System



White, T., et al. AIAA SciTech (2022)

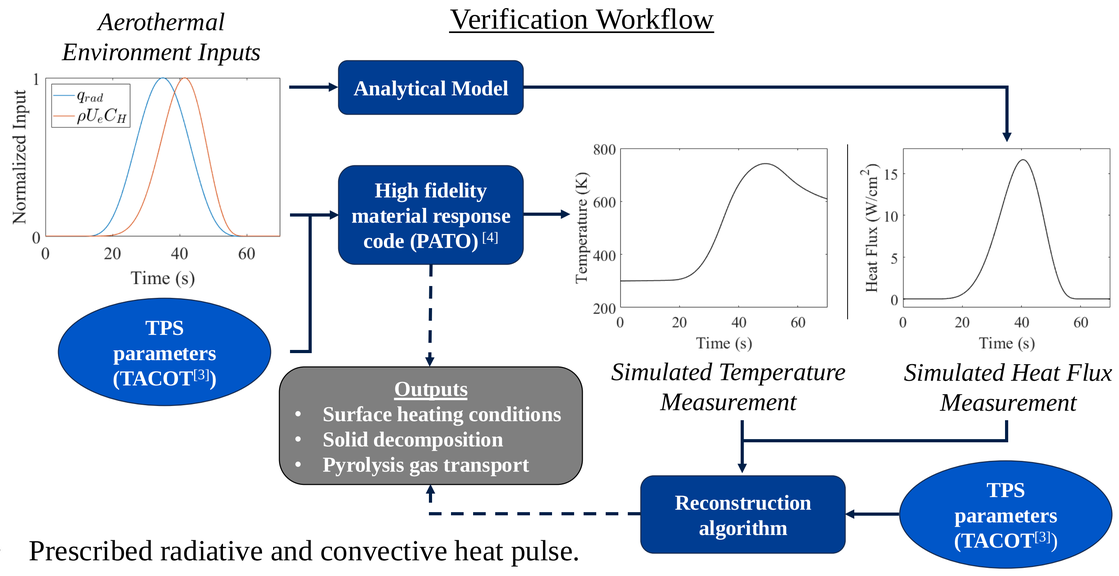
MEDLI2 Backshell Instrumentation



Model System Schematic

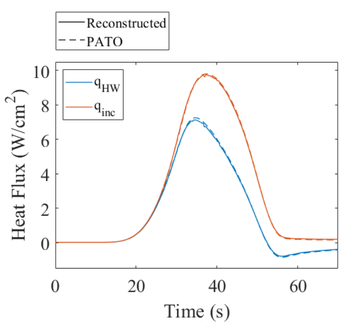
- Model system simulates heat flux sensor and thermocouple plug as separate 1D domains.
- Thermocouple plug geometry representative of Mars 2020 backshell instrumentation[2].
- Backface modeled as insulating.

Verification Workflow

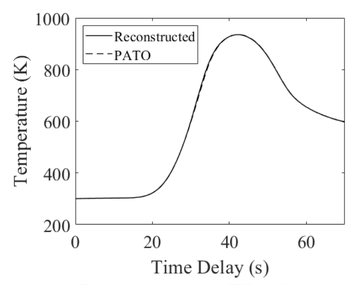


- Prescribed radiative and convective heat pulse.
  - ➢ Heat transfer coefficient $\rho U_e C_H$ pulsed with constant recovery enthalpy.
  - ➢ B'-type ablation without recession.
- Reconstructed heating conditions and pyrolysis effects compared with PATO outputs.

Results

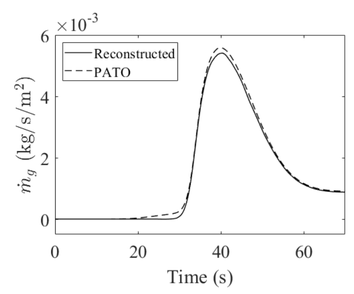


Reconstructed Net Hot-Wall and Incident Heat Flux Values

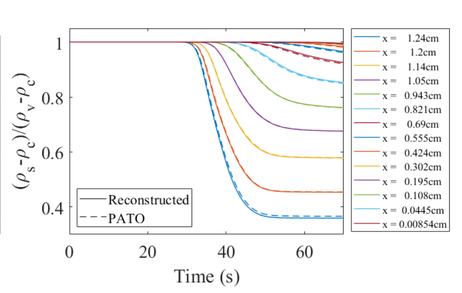


Reconstructed TPS Surface Temperature

- Reconstruction algorithm recovers both the net hot-wall and incident heat flux values to within 4% of PATO results.
- Reconstruction algorithm recovers TPS surface temperature nearly identical to PATO results.



Reconstructed Pyrolysis Gas Mass Flux



Reconstructed Through-Thickness Solid Decomposition

- Recovered pyrolysis gas mass flux at TPS surface is resolved to within 4% of PATO results.
- Through-thickness solid decomposition results show great agreement with PATO-generated results.

Conclusions:

1. We have demonstrated Green's functions as a viable approach to reconstruct the surface heating on atmospheric entry spacecraft using thermal instrumentation embedded in ablative TPS.
2. The Green's function-based reconstruction algorithm can generate reconstructed heating conditions in a few seconds, compared to tens of minutes for current IHT methods.
3. In a 1D test case, the reconstruction algorithm recovers the prescribed surface heating conditions and through-thickness TPS material response to within 4% of PATO-generated solutions.

Acknowledgements:
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References:

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[2] H. S. Alpert, *J Spacecr Rockets* (2023)

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[4] N. Mansour, *J Thermophys Heat Trans* (2014)