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Stabilized mascon solutions through combinations of GRACE and SLR normal equations

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SUMMARY

A new global mascon solution using GRACE and GRACE Follow-On data is co-estimated with Satellite Laser Ranging (SLR) measurements to seven major geodetic satellites. This combined solution is compared with an otherwise similar GRACE-only solution to determine improvements in the estimate. We find similar performance between both solutions in the recovered mass change, but significant improvements in the associated errors in the combination solution. Errors in recovered basin mass change are 10-20% better for the combination solution across all basin sizes, with the greatest improvements in high latitude ice sheets. These results lead to our recommendation that all GRACE Level 3 mascon and spherical harmonic user-oriented gridded solutions should include SLR information during the solution inversion. As an ancillary contribution, we also provide validation of the choice of truncated spherical harmonics used in determining GRACE Technical Note 14, the current recommended mechanism for including SLR information with GRACE solutions in post-processing.

Key words: satellite geodesy; time variable gravity; global change from geodesy

1 INTRODUCTION

The GRACE and GRACE Follow-On missions, hereafter “GRACE(-FO),” have recorded spatiotemporal changes in Earth’s gravity field since 2002 (Tapley et al., 2004; Landerer et

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al., 2020). These missions have been instrumental in providing global records of changes in terrestrial water storage, ice sheets and glaciers, solid earth, and more, resolving changes on monthly timescales at approximately 300 km spatial resolution (Landerer et al., 2020; Tapley et al., 2019). However, both missions have shown poor performance recovering changes in Earth's oblateness, J_2 (or $C_{2,0}$ in spherical harmonics), possibly due to ocean tidal aliasing or β' sun-orbital plane angle effects (Loomis et al., 2019b), and since late in the GRACE mission both have likewise had difficulty recovering changes in the “pear shape” of the Earth, J_3 (or $C_{3,0}$) (Landerer et al., 2020).

Satellite Laser Ranging (SLR) has long provided an independent estimate of changes in $C_{2,0}$, with records dating to the launch of Starlette in 1975. As additional SLR satellites have launched (including Stella, Ajisai, LAGEOS-1 and -2, LARETS, and LARES), SLR estimates of low degree/order spherical harmonics have improved, and SLR estimates of $C_{2,0}$ have exceeded the quality of those from GRACE(-FO). For this reason, it has long been recommended that GRACE(-FO) estimates of $C_{2,0}$ be discarded and replaced by those from SLR. More recently, high-quality SLR estimates of $C_{3,0}$ since the launch of LARES in 2012 have been demonstrated to outperform worsening GRACE(-FO) estimates. As a result, it is usually recommended that GRACE(-FO) estimates of $C_{2,0}$ for all months and of $C_{3,0}$ after June 2016 be discarded and replaced by SLR using a product termed GRACE Technical Note 14 (TN-14) (Loomis et al., 2019a; Landerer et al., 2020). This “GRACE and Replace” method has been widely performed with both Level 2 spherical harmonic solutions and Level 3 mass concentration (mascon) products (e.g., from NASA Goddard Space Flight Center (GSFC), the Jet Propulsion Laboratory (JPL), and the University of Texas at Austin's Center for Space Research (CSR)). While this replacement improves the overall quality of all of these time-variable gravity solutions, such replacement is not robust. Errors in the recovered time-variable gravity field associated with recovered $C_{2,0}$ and $C_{3,0}$ including both noise and correlations between these and other recovered terms, are maintained within the resulting gravity field after

replacing these terms with SLR, and errors from SLR are likewise not properly mapped into the solution.

In order to robustly include SLR as part of a solution, a combined solution must be estimated incorporating all information from both GRACE and SLR. The inclusion of SLR has a stabilizing effect on the long-wavelength portion of the spectrum, where SLR is most sensitive (Kvas et al., 2021). Past studies have demonstrated that combinations of spherical harmonic gravity coefficients from GRACE, SLR, and other missions at the normal equation level further have positive impacts on recovered harmonics across many degrees and orders (Meyer et al., 2019; Kang et al., 2022; Weigelt et al., 2024, for example). While some available solutions have been designed to combine these data types, including the CNES/GRGS monthly and 10-day solutions and the GOCO06S time-variable gravity model (Lemoine et al., 2019; Kvas et al., 2021), this has not become common practice and no mascon solution to date has incorporated SLR in a robust way. Here, we develop a first of its kind mascon solution robustly estimated from both GRACE(-FO) and SLR and fully incorporating the information content from each. We then show that this solution, which benefits from both the combination of GRACE(-FO) and SLR and the parametrization and regularization of the gravity field as surface mascons, shows 10-20% improvements in basin mass change recovery for all types of basins across all observable basin sizes. Finally, we recommend all GRACE(-FO) mascon solutions adopt similar GRACE+SLR combination strategies.

2 METHODS AND DATA

In this study, we compute two monthly mascon solutions, one using only information from GRACE(-FO) in the solution inversion (referred to as “GRACE-only” from here) and one combining GRACE(-FO) and SLR information in the solution inversion (referred to as “GRACE+SLR” from here). Both solutions are computed using the fast mascon technique (Croteau et al., 2021), as explained in Section 2.1. For each solution after the mascon inversion, we apply the necessary post-processing steps to match each solution

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77 to the same standards as the widely available GSFC, JPL, and CSR mascon solutions.
 78 For the GRACE-only solution, this includes replacing the solution's $C_{2,0}$ terms for all
 79 times and $C_{3,0}$ terms for all times beginning with July 2016 with those from SLR using
 80 TN-14 (Loomis et al., 2020). For the GRACE+SLR solution, we do not need to do this
 81 step since SLR and GRACE spherical harmonics are combined at the normal equation
 82 level prior to mascon estimation (as explained in more detail in Section 2.2). For both
 83 solutions, we restore the forward modeled ocean bottom pressure signal and subtract the
 84 mean atmospheric mass over the ocean, meaning ocean results reflect sea level anomaly.

85 2.1 Fast Mascon Estimation

86 Mascon solutions are determined using the “fast mascon” method (Croteau et al., 2021).
 87 This method estimates mascons from converged spherical harmonic solutions and their
 88 full normal equation information, and is equivalent to robust mascon estimation methods
 89 from Level-1B ranging observations as done for the GSFC and CSR mascon solutions
 90 (Loomis et al., 2019a; Save et al., 2016), while also seeing significant gains in simplicity
 91 and computational efficiency. The fast mascon method solves the equation

$$35 \quad \hat{\mathbf{m}} = \mathbf{P}^{-1} \mathbf{L}^T (\lambda \mathbf{N}^{-1} + \mathbf{L} \mathbf{P}^{-1} \mathbf{L}^T)^{-1} \mathbf{c}, \quad (1)$$

36 where $\hat{\mathbf{m}}$ is the estimated set of mascons, \mathbf{c} is a converged estimate of spherical harmon-
 37 ics, \mathbf{N} is the normal equation matrix associated with \mathbf{c} , \mathbf{L} is the partial derivatives of
 38 the differential spherical harmonics coefficients with respect to the mascon parameters,
 39 \mathbf{P} is the mascon regularization matrix, and λ is a weighting factor applied to the regu-
 40 larization. A full derivation and computationally stable formulation of this equation is in
 41 (Croteau et al., 2021).
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49 For this study, we follow the formulation and all solution parametrization methods
 50 and choices in (Croteau et al., 2021), using the same set of 41,168 mascons (also used
 51 in the GSFC mascon solution), the same mascon regularization, and the same method
 52 for determining the regularization scaling λ . Core details of that formulation have been
 53 reproduced in Section S1 of the included supplementary materials for convenience. We
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103 extend the GRACE-only solution from that paper forward in time to now include through
 104 July 2023 (previously it only extended through April 2021) and compare it with a new
 105 GRACE+SLR solution also through July 2023. This new GRACE+SLR solution first
 106 combines GRACE and SLR spherical harmonic normal equations (see Section [2.2](#)) and
 107 then applies the fast mascon method (using the same parametrization as the GRACE-
 108 only solution) to estimate regularized GRACE+SLR mascons. This allows this study to
 109 isolate the effects of the GRACE+SLR combination, and we can make direct comparisons
 110 between the GRACE-only and GRACE+SLR solutions to evaluate any impacts of the
 111 inclusion of SLR.

112 **2.2 Combination of GRACE and SLR Spherical Harmonic Normal** 113 **Equations**

114 We combine the GRACE and SLR solutions at the normal equation level by solving the
 115 equation

$$(w_G \mathbf{N}_G + w_S \mathbf{N}_S) \mathbf{c} = w_G \mathbf{n}_G + w_S \mathbf{n}_S \quad (2)$$

116 for the combined spherical harmonics from the sets of GRACE(-FO) and SLR normal
 117 matrices, \mathbf{N}_G and \mathbf{N}_S , and right hand side vectors of these normal equation systems,
 118 \mathbf{n}_G and \mathbf{n}_S , as well as their respective data weights, w_G and w_S . This method is well
 119 understood and has been shown by previous studies to effectively combine solutions from
 120 GRACE, SLR, and other missions ([Meyer et al., 2019](#); [Kang et al., 2022](#); [Weigelt et al.,](#)
 121 [2024](#)). To select the GRACE and SLR weighting factors, we set w_G to 1.0 and tested
 122 values for w_S from 0.01 to 1000. Ultimately, $w_S = 10$ was selected because this was the
 123 smallest weighing that allowed SLR to drive recovery $C_{2,0}$ for all times and $C_{3,0}$ during
 124 the GRACE-FO era, which matches the method typically employed using TN-14 where
 125 SLR replaces GRACE for these terms. GRACE information is included for all spherical
 126 harmonics from degree 2 through 96, while SLR information is included for all spherical
 127 harmonics from degree 2 through 10. Note that $C_{2,0}$ and $C_{3,0}$ in TN-14 are estimated as

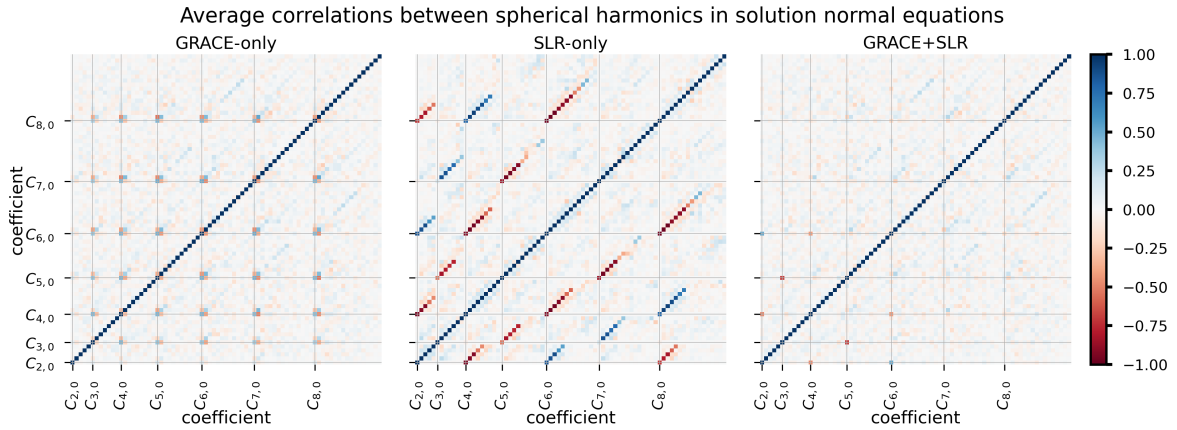


Figure 1. Average correlations between spherical harmonic coefficients in monthly gravity fields from GRACE-only (left), SLR-only (center), and GRACE+SLR (right) solutions. Correlations between all pairs of coefficient are significantly reduced except for between $C_{3,0}$ and $C_{5,0}$. The x- and y-axes are ordered by degree, then order, then type (i.e., $C_{2,0}$, $C_{2,1}$, $S_{2,1}$, $C_{2,2}$, $S_{2,2}$, $C_{3,0}$, $C_{3,1}$, etc.)

part of a set of spherical harmonics that includes only coefficients for degrees 2 through 5 plus $C_{6,1}$ and $S_{6,1}$. Fig. S2 in the supplementary materials depicts this breakdown.

GRACE-only solutions contain correlations between estimated spherical harmonic coefficients due to limitations of the observing platform at even the lowest degrees and orders, manifest as clusters of correlated coefficients around pairs of zonal coefficients as shown in Fig. 1 (left). In the SLR-only solution, higher degree/order harmonics are very noisy and introduce correlation issues with lower-degree terms (e.g, between even zonals and their successive tesseral terms) as shown in Fig. 1 (center). By combining GRACE(-FO) and SLR normal equations, however, we are able to mitigate correlations in each data type while allowing both data types to contribute whatever information they contain to the estimate for all coefficients, as shown in Fig. 1 (right) and Section 3. In fact, we see from Fig. 1 that correlations between individual coefficients are reduced for all coefficients with the exception of the $C_{3,0}/C_{5,0}$ pair, where already large negative correlations from the SLR solution are made slightly worse in the GRACE+SLR solution, and all clusters of correlated coefficients are eliminated for degree 2-10 terms.

2.3 Data

The fast mascon method requires the use of a converged spherical harmonic solution and its associated normal equations. In this study, the GRACE(-FO) solution used is the ITSG-Grace2018 spherical harmonic solution and full normal equations to degree/order 96 (Kvas et al., 2019; Mayer-Gürr et al., 2018). We use this solution because it matches the quality of the GRACE Level 2 spherical harmonic solutions released by the GRACE Science Data System team while additionally providing the full normal equation information freely and openly. We combine this GRACE(-FO) solution with the normal equations for the satellite laser ranging solution (to degree/order 10) from GSFC that is used in the production of TN-14 (though that solution only includes degrees 2 through 5 plus $C_{6,1}$ and $S_{6,1}$). Additionally, comparisons are made to the GSFC RL06v2.0 Global Mascons (Loomis et al., 2019a), the JPL RL06.1v03 Global Mascons (Watkins et al., 2015; Wiese et al., 2018), the CSR RL06.2 Global Mascons (Save et al., 2016; Save, 2020), and TN-14. As with the fast mascon solutions estimated here, the mean atmospheric mass over the oceans has been subtracted from the GSFC, JPL, and CSR mascon solutions, meaning ocean results for those solutions also reflect sea level anomaly.

3 RESULTS

While this paper primarily focuses on extending GRACE+SLR combinations to mascon solutions, the necessary first step in the fast mascon approach is to combine GRACE(-FO) and SLR spherical normal equations, which allows us to examine the effects of this combination in addition to the effects of applying mascon regularization. Fig. 2 shows a comparison of estimates for $C_{2,0}$ and $C_{3,0}$ from GRACE(-FO), SLR (10x10 spherical harmonic expansion), GRACE+SLR, and TN-14 ($5 \times 5 + C_{6,1}/S_{6,1}$). Interestingly, the combined GRACE+SLR estimate for $C_{2,0}$ matches the estimate for $C_{2,0}$ from TN-14 extremely well, despite these two solutions having significant differences in information content. The SLR solution in the combination uses a 10x10 expansion that includes significant correlations between multiple harmonics (see Fig. 1, center) due to observability

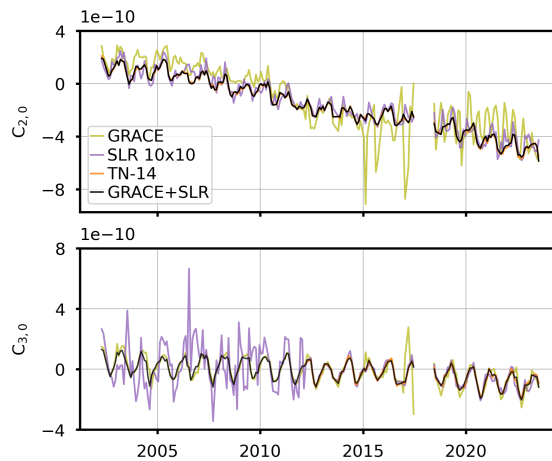


Figure 2. Comparison of recovered $C_{2,0}$ and $C_{3,0}$ terms from GRACE(-FO), SLR 10x10, TN-14, and combined GRACE+SLR spherical harmonics. The GRACE+SLR solution nearly perfectly recovers the $C_{2,0}$ and $C_{3,0}$ results from TN-14 without needing to limit contributing SLR terms.

issues as the harmonic expansion is increased in size, while TN-14 uses a truncated expansion omitting $C_{6,0}$ and all terms higher than $C_{6,1}$ and $S_{6,1}$ to eliminate the harmful effects of these correlations. These differences explain why $C_{2,0}$ and $C_{3,0}$ in the SLR 10x10 are relatively noisier than for TN-14 in Fig. 2. The reduced noise in the GRACE+SLR solution then demonstrates that this combination uses GRACE information to similarly help break correlations between such terms without needing to selectively eliminate certain harmonics from the estimate, and also provides validation for the system employed in estimating the TN-14 SLR-only solution.

After final estimation of our GRACE+SLR mascon solution, we can study the impact of regularization on the gravity field in spherical terms by converting the set of mascons back to spherical harmonics. We found that the regularization further stabilized the solution, reducing errors across many spherical terms for challenging months and for correlated harmonics, which matches findings by GRACE-only mascon solutions. Fig. 3 shows estimates of nine sample zonal harmonics from $C_{2,0}$ to $C_{60,0}$ from SLR-only, GRACE-only, and GRACE+SLR spherical harmonics, as well as GRACE+SLR mascons. Fig. 3 demonstrates that low-degree harmonics are most improved by the spherical combination and not impacted much by the mascon regularization while higher-degree

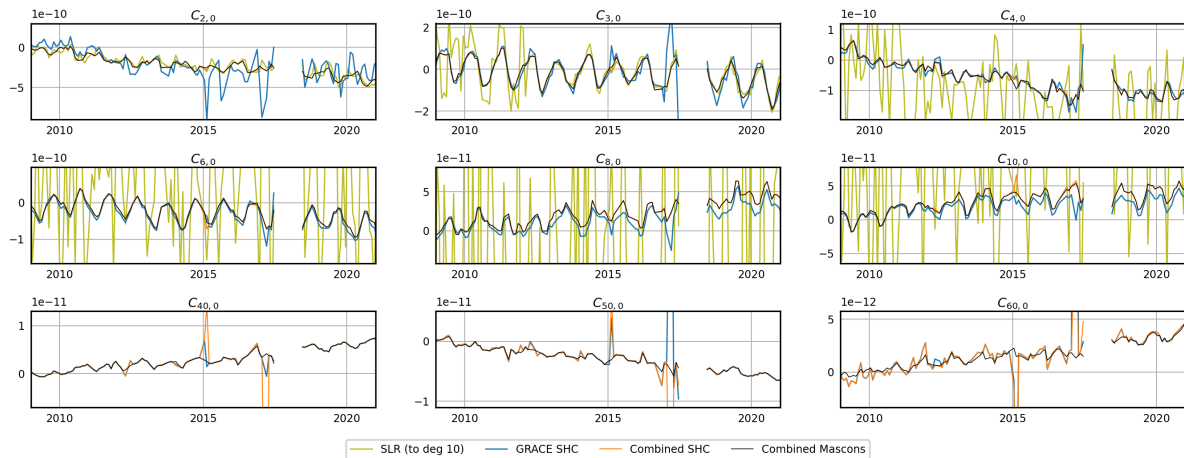


Figure 3. Recovered spherical harmonic time series from SLR, GRACE, GRACE+SLR combined spherical harmonics, and GRACE+SLR combined regularized mascons. The GRACE+SLR combination greatly improves the longer-wavelength portion of the spectrum for all times, while the mascon regularization further improves the shorter-wavelength portion for all times as well as the longer-wavelength portion for certain challenging months in a complimentary manner.

harmonics are most improved by the mascon regularization and only somewhat improved by the spherical combination. We can also see that, in addition to stabilizing $C_{2,0}$ and $C_{3,0}$, the inclusion of SLR has noticeably changed the trend in other low-degree zonal harmonics such as $C_{8,0}$ and $C_{10,0}$. This is likely not due to the SLR-only estimates of these harmonics themselves, which are not resolved well, but rather is due to the reduction of correlations between coefficients in the GRACE+SLR solution (as discussed in Section 2.2). Similar improvements are seen for non-zonal harmonics, as in other studies.

Finally, basin mass change was computed for 20 basins varying in size and type (including ice covered, land, and seas) for both the GRACE-only and GRACE+SLR mascons. Table 1, as well as Fig. S3 in the supplementary materials, shows few substantial changes in the recovered basin trends, annual signals, or inter-annual signals using the GRACE+SLR solution compared to the GRACE-only solution, though solutions for individual months in Antarctica in particular do show notable improvements. We additionally see very good agreement with both solutions for all basins when comparing to the commonly available mascon solutions from GSFC, JPL, and CSR, as had been previously

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Table 1. Basin mass recovered for GRACE-only (A) and GRACE+SLR (B) mascons, with comparisons to GSFC (G), JPL (J), and CSR (C) mascon solutions. The temporal mean of the total basin error ($\bar{\epsilon}$) is shown for the GRACE-only and GRACE+SLR solutions, along with the percent reduction in error ($\Delta\%$) due to the GRACE+SLR combination.

	Area [km ²]	Trend [Gt/yr]					Annual Amplitude [Gt]					$\bar{\epsilon}$ [Gt]		
		A	B	G	J	C	A	B	G	J	C	A	B	$\Delta\%$
Antarctica	13,043,374	-129	-127	-139	-140	-145	118	105	114	92	151	110.0	87.1	-20.8
Greenland	2,451,975	-267	-268	-266	-271	-260	154	153	156	126	132	112.3	93.2	-17.0
Gulf of Alaska	520,064	-69	-70	-68	-66	-66	144	142	147	155	133	41.6	34.9	-16.0
Iceland	161,073	-9	-9	-8	-8	-7	18	18	17	4	11	15.0	12.9	-14.6
Sahara	5,528,176	-12	-12	-9	-10	-10	30	26	27	12	22	53.8	47.3	-12.0
Congo	3,791,292	16	15	20	19	17	174	175	188	161	176	51.0	43.5	-14.6
Mississippi	3,308,286	8	8	8	5	6	189	194	203	193	202	44.0	38.6	-12.2
La Plata	2,999,402	-3	-3	-4	-2	-4	151	152	168	162	163	52.2	45.3	-13.2
St. Lawrence	1,276,012	12	12	10	11	11	53	55	59	67	64	29.6	25.9	-12.4
Murray	966,794	3	3	2	3	3	12	15	19	16	19	23.8	20.3	-14.6
Japan	619,584	3	3	-5	-1	0	29	28	30	11	13	25.9	22.7	-12.4
California	495,538	-7	-6	-7	-5	-4	48	47	43	39	40	19.4	17.3	-10.8
Lake Balkash	446,146	-2	-2	-3	-2	-3	20	20	20	16	19	14.4	13.4	-7.1
Volta	433,881	4	4	4	4	4	47	47	48	47	45	18.2	15.6	-14.4
Yucatán	260,222	1	1	1	0	1	23	23	22	19	19	14.4	12.7	-11.4
Oder	123,737	-1	-1	-1	-1	-1	7	7	7	6	6	6.2	5.3	-14.3
Hudson Bay	879,243	1	1	-0	2	0	5	7	7	11	14	37.5	32.0	-14.7
Red Sea	532,897	-0	-0	1	3	1	83	83	87	77	73	28.1	24.7	-12.2
Caspian Sea	458,489	-34	-33	-32	-32	-20	55	55	39	53	33	47.0	41.2	-12.3
Black Sea	347,114	-1	-1	-0	0	-1	18	18	17	26	25	28.7	25.6	-10.8

202 shown for the GRACE-only solution in (Croteau et al., 2021). However, we importantly
 203 find that the GRACE+SLR solution shows substantially reduced errors for all basins
 204 when compared to the GRACE-only solution. These errors were computed following the
 205 same methods as in the GSFC monthly solution (Loomis et al., 2019a). Improvements
 206 range from 15-20% for ice covered basins and 10-15% for nearly all hydrologic basins and
 207 seas, and this finding holds regardless of the size of the basin or the total trend or annual
 208 signal recovered in the basin.

4 DISCUSSION

This study demonstrates the impacts of including SLR information with GRACE(-FO) in estimating Level 3 mascon solutions. Following past studies illustrating positive impacts of similar combinations for Level 2 spherical harmonic solutions, we have demonstrated that significant improvements of 10-20% can be made in the reduction of the overall errors in basin mass change recovery from mascon solutions by allowing the long-wavelength portion of the gravity field to be informed by SLR during the solution inversion. Additional improvements are seen in individual months where GRACE(-FO) solutions are more challenging, as well as in Antarctic mass change recovery. The greatest basin error reductions are seen for the major ice sheets, which are located at high latitudes and thus are most impacted by errors in $C_{2,0}$ and $C_{3,0}$. These 15-20% improvements in glacial ice mass change errors are significant in better accounting for ice mass contributions to sea level rise and other climate change processes. Error reductions of 10-15% for all other basins suggest that these improvements will benefit almost all GRACE applications, regardless of basin size or location. Because of these improvements in mascon solution errors, we recommend all Centers producing GRACE Level 3 mascons or Level 3 spherical harmonic-based gridded solutions similarly incorporate SLR information into their solution inversion.

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235 expertise and assistance with SLR processing and the inclusion of the GSFC SLR normal
236 equations.

237 DATA AVAILABILITY STATEMENT

238 ITSG-Grace2018 normal equations are available from GFZ Data Services and the Interna-
239 tional Center for Global Earth Models (ICGEM), <https://doi.org/10.5880/ICGEM.2018.003>.
240 The JPL RL06.1 v03 mascon solution and GRACE Technical Note 14 are available
241 from JPL's Physical Oceanography Distributed Active Archive Center (PO.DAAC),
242 <https://podaac.jpl.nasa.gov>. The GSFC RL06 v2.0 mascon solution is available from
243 GSFC, <https://earth.gsfc.nasa.gov/geo/data/grace-mascons>. The CSR RL06.2 mascon
244 solution is available from CSR, <https://doi.org/10.15781/cgq9-nh24>.

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Supplemental Information: Stabilized mascon solutions through combinations of GRACE and SLR normal equations

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S1 FAST MASCON FORMULATION AND PARAMETERIZATION

In this study, we followed the same mascon formulation and parameterization as in (Croteau et al., 2021) and estimate GRACE-only and GRACE+SLR solutions for April 2002 through July 2023. This means that the GRACE-only solution examined here is simply an extension of the mascon solution presented in that paper to include 25 additional months. Since both solutions use the same regularization design as that paper, differences in the recovered solutions will show the effects of including SLR information while holding other variables fixed. In this section, we repeat the major design decisions from that paper for reference here.

As in (Croteau et al., 2021), we define the regularization matrix in covariance space as \mathbf{P}^{-1} , and define the individual terms for each row/mascon i and column/mascon j as $P_{i,j} = \rho_{ij}\sigma_i\sigma_j$. As before, we use the same set of 41,168 1-arc-degree mascons as already commonly used in the GSFC RL06 mascon solution. We use the same map of expected uncertainties, σ_i , shown in Figure S1. Correlations for each mascon i, j pair, ρ_{ij} , are defined using the Gaussian smoothing function for a 300 km radius, with regional constraints applied to set $\rho_{ij} = 0$ if mascons i and j are in separate geographic regions. Regions included in the regularization design include: (1) Greenland Ice Sheet, low elevations (< 2000 m), (2) Greenland Ice Sheet, high elevations (> 2000 m), (3) Antarctic Ice Sheet including the Ronne and Ross ice shelves, (4) Gulf of Alaska, (5) Land, (6) Ocean, including smaller ice shelves, (7) Mediterranean Sea, (8) Black Sea, (9) Red Sea, (10) Caspian Sea, and (11) Hudson Bay.

S2 SPHERICAL HARMONIC COEFFICIENT BREAKDOWN

The combined GRACE+SLR solution includes estimates of gravity field spherical harmonic coefficients from GRACE(-FO) for degrees 2-96 and from SLR for degrees 2-10. This means the combination includes a larger spherical harmonic expansion from SLR than is used in developing the SLR-only “GRACE Technical Note 14” estimates of $C_{2,0}$ and $C_{3,0}$, which is estimated from only coefficients in degrees 2-5 plus $C_{6,1}$ and $S_{6,1}$ for stability reasons. Degree 0 (Earth’s total mass) and 1 (Earth’s geocenter) coefficients are not included in the estimate, as typical for GRACE solutions, and Degree 1 terms are instead restored after the estimation step following (Sun et al., 2016). This breakdown is shown in Figure S2.

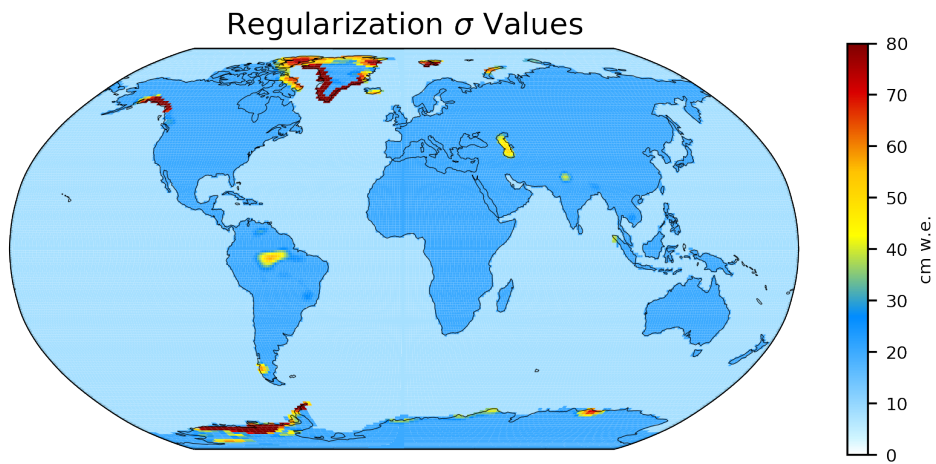
S3 BASIN MASS CHANGE COMPARISONS

Basin mass change was computed for numerous basins varying in size and type (including ice covered, land, and seas) for both the GRACE-only and GRACE+SLR mascons. Figure S3 show few substantial changes in the recovered basin trends, annual signals, or inter-annual signals using the GRACE+SLR solution compared to the GRACE-only solution, though signals in Antarctica in particular do show notable improvements for individual months. We additionally see very good agreement with both solutions for all basins when comparing to the commonly available mascon solutions from GSFC, JPL, and CSR.

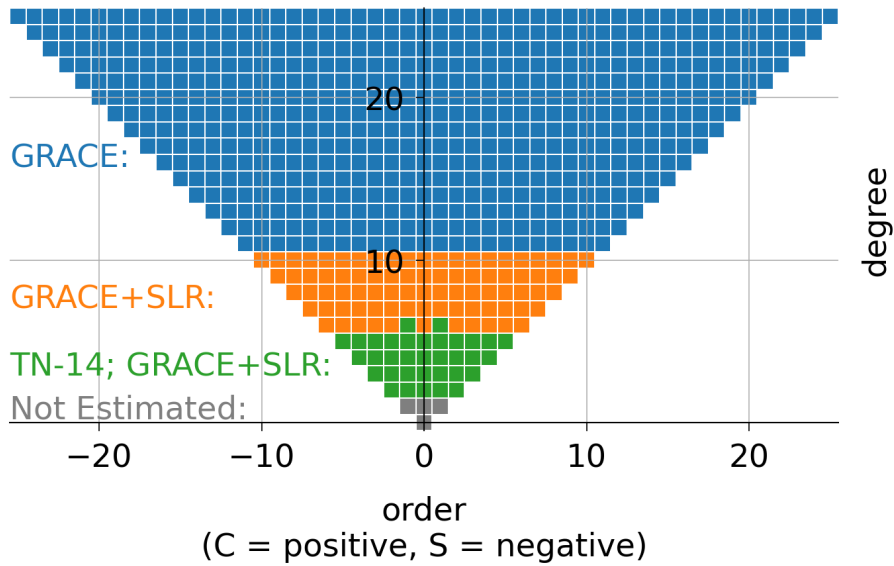
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2 *Croteau et al.*



18 **Figure S1.** Expected uncertainties σ used to populated the regularization matrix (from Croteau et al., 2021).



41 **Figure S2.** Spherical harmonic coefficients included in the estimate per source (through degree/order 25). Also depicted are those
 42 coefficients included in the TN-14 SLR-only estimate of $C_{2,0}$ and $C_{3,0}$.

Supplemental Information: Stabilized mascon solutions through combinations of GRACE and SLR normal equ

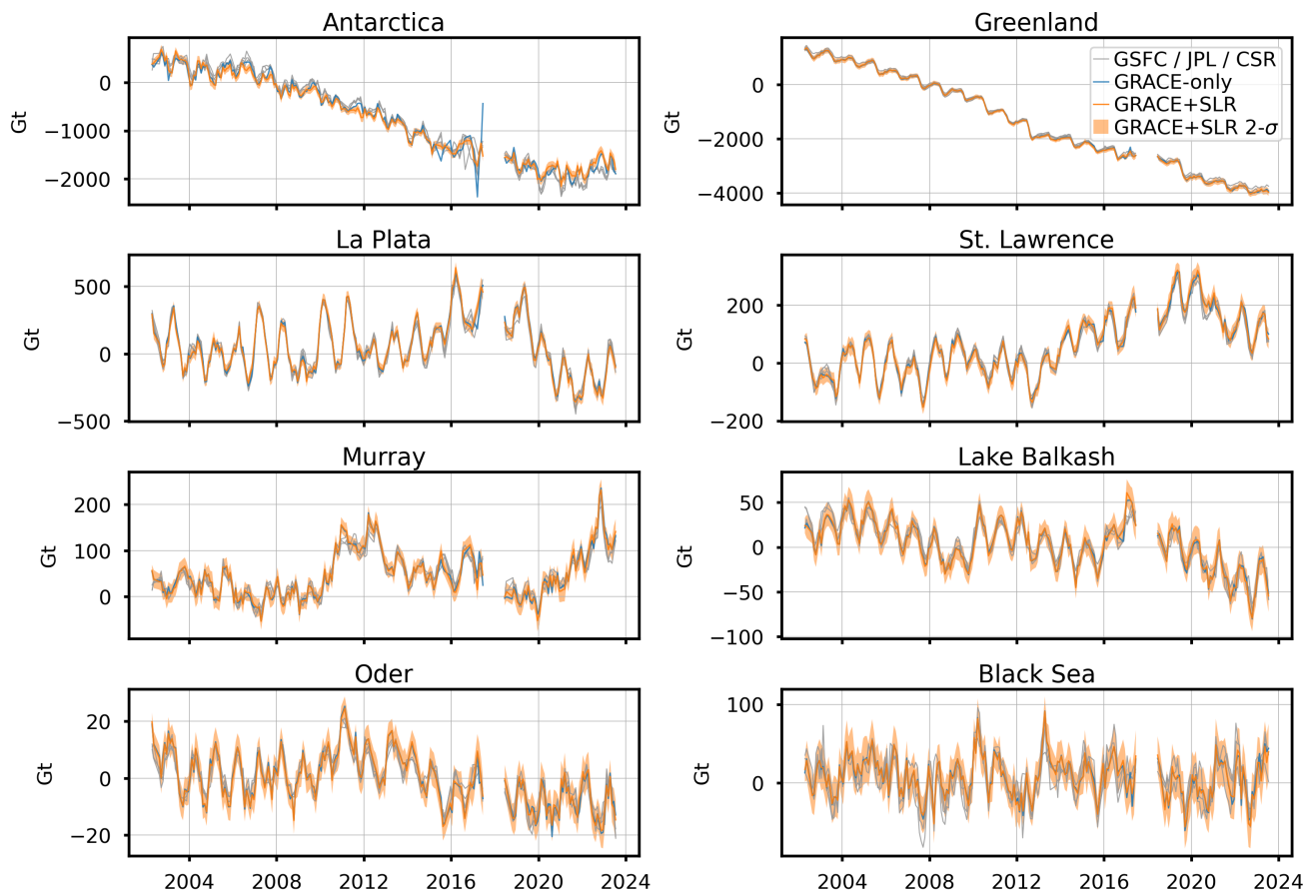


Figure S3. Basin mass change recovered from GRACE+SLR and GRACE-only solutions, with the GSFC, JPL, and CSR mascon solutions shown in gray for comparison. For most months and basins, the inclusion of SLR has only small impacts on the recovered signal but has larger impacts in the reduction of the error of that signal.