Using Global Market Demand Analysis to Guide Conceptual Design of Low-Boom Supersonic Transports

Wu Li*, Karl Geiselhart†, Ryan Palma†, and Michael Patterson#
NASA Langley Research Center, Hampton, VA 23681, USA

Samuel Dollyhigh‡
Analytical Mechanics Associates, Hampton, VA 23681, USA

Zhou Wang§, Nicolas Hinze¶, and Antonio Trani⁑
Virginia Polytechnic Institute and State University, Blacksburg, VA 24061, USA

This paper uses a mathematical framework to identify the interdependencies of key variables in supersonic transportation demand analysis. The existing quantitative models for supersonic transportation demand analysis are compared for consistency in modeling the interdependencies. Unlike other quantitative models, the Low-Boom Systems Analysis Model (LBSAM2) can propagate important design characteristics of a supersonic transport aircraft concept to the level of economic metrics (such as the number of future supersonic passengers), with a consistent coupling of the market demand analysis, detailed mission analysis, and low-boom constraint. This enables the use of the detailed demand analysis results from LBSAM2 to maximize the economic viability of a supersonic transport aircraft by finding favorable system-level trades between weight, range, fuel burn, and an assumed sonic boom ground noise limit for supersonic overland flight. In this paper, LBSAM2 is integrated with conceptual low-boom design to improve the economic viability of low-boom supersonic transport aircraft. The LBSAM2 analysis results show that the range, cruise speed, and weight could each become the dominant driver for higher demand under some unique condition. A brief discussion of uncertainties in the LBSAM2 analysis is also included, focusing on their impacts on the relative economic advantages between low-boom concepts.

Nomenclature

$\|\cdot\|$	=	2-norm of a function f on interval [0, l_e] defined by $\|f\|$, which equals $\left(\int_0^{l_e} \|f(t)\|^2\, dt\, / l_e\right)^{\frac{1}{2}}$
$A_{\mathrm{e,r}}$	=	reversed equivalent area of supersonic configuration, ft^2
$A_{\mathrm{e,r}}^{\mathrm{target}}$	=	target equivalent area for $A_{\mathrm{e,r}}$, ft^2
C_{hour}	=	airline operating cost per revenue flight hour excluding fuel cost and C_{flight}, $/hr
C_{flight}	=	various fees per flight such as ground handling, navigation, landing, emission, and noise fees, $
C_{nmi}	=	passenger cost per nmi including airline profit and US excise tax, $/nmi
C_{sub}	=	premium ticket price for nonstop subsonic flight between OD pair, $
C_{sup}	=	ticket price for supersonic flight between OD pair, $
$\hat{\boldsymbol{D}}_{\mathrm{LoT}}$	=	label for a low-boom supersonic transport in a reference paper
FW_{overland}	=	fuel weight burned for low-boom supersonic flight at overland cruise Mach between OD pair, lb
$FW_{\mathrm{overwater}}$	=	fuel weight burned for unrestricted supersonic flight at overwater cruise Mach between OD pair, lb

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* Senior Research Engineer, Aeronautics Systems Analysis Branch
† Aerospace Engineer, Aeronautics Systems Analysis Branch
# Aerospace Engineer, Aeronautics Systems Analysis Branch, AIAA Associate Fellow
‡ Aerospace Engineer, Analytical Mechanics Associates
§ Graduate research assistant, Air Transportation Systems Laboratory
¶ Senior research associate, Air Transportation Systems Laboratory
⁑ Professor, Air Transportation Systems Laboratory, AIAA Senior Member

FW_{sup} = fuel weight burned flying between OD pair with one refueling stop if needed, lb
F_{scale} = scaling factor
l_{e} = effective length of a configuration, which is the largest effective distance where Mach angle cut plane intersects the configuration, ft
N_{aircraft} = number of manufactured low-boom supersonic transport aircraft to meet market demand
N_{flight} = number of flights for OD pair in two days
N_{pax} = number of supersonic passengers served for OD pair
N_{sub} = subsonic premium passenger demand for OD pair
N_{sup} = supersonic passenger demand for OD pair
OD = origin-destination (referring to the airports between which a flight takes place)
$\mathrm{PLdB}(A_{\mathrm{e,r}}^{\mathrm{target}})$ = perceived level in decibels of undertrack sonic boom ground signature for $A_{\mathrm{e,r}}^{\mathrm{target}}$
$R_{\mathrm{max,land}}$ = maximum range for low-boom overland flight of supersonic transport, nmi
$R_{\mathrm{max,water}}$ = maximum range for unrestricted overwater flight of supersonic transport, nmi
R_{sub2sup} = switching percentage of subsonic premium passengers to supersonic flight for OD pair
T_{overland} = travel time for low-boom supersonic flight using overland cruise Mach between OD pair, hr
$T_{\mathrm{overwater}}$ = travel time for unrestricted supersonic flight using overwater cruise Mach between OD pair, hr
T_{sub} = travel time for typical subsonic nonstop flight between OD pair, hr
T_{sup} = travel time of supersonic transport between OD pair with one refueling stop if needed, hr
VOT = value of time for premium air travel passengers, \$/hr
$\mathrm{VOT}_{\mathrm{OD}}$ = value of saved time for supersonic flight between OD pair compared to subsonic flight, \$
ε = tolerance for acceptable low-boom inverse design objective value, ft^2
τ = (total distance of overland segments)/(total distance for flight path)
σ_{max} = limit for sonic boom ground noise level, perceived level in decibels

I. Introduction

ANY prediction of a future event has intrinsic uncertainty; however, important investment decisions must be made based on these uncertain prediction results. For example, the NASA Aeronautics Research Mission Directorate (ARMD) uses benefit assessments of future aviation technologies for investment decisions that could lead to transformative improvements in mobility, efficiency, sustainability, and safety for air travel. Technologies for supersonic commercial flight are typically in ARMD's investment portfolio. Historically, these technologies have had strategic value for the nation. With the last commercial supersonic airplane, the Concorde, terminating service in 2003, the economic viability of commercial, passenger-carrying supersonic flight is an unavoidable question for any significant investment in commercial supersonic flight technologies. The ban on supersonic overland flight (see Federal Aviation Regulations, specifically 14 CFR §91.817) is considered detrimental to the profitability of a supersonic transport aircraft. The X-59 low-boom flight demonstrator [1], under development by NASA and Lockheed Martin, aims to demonstrate the technological feasibility of reducing a loud sonic boom to a soft sonic "thump" on the ground. The NASA Quesst mission [1] will collect ground noise data for the X-59 and the corresponding community responses to its sonic thump to support a potential future regulation for supersonic overland flight. However, even if the ban on supersonic overland flight can be lifted, the economic viability of a low-boom supersonic transport aircraft is still an important research topic to support future civil supersonic technology investment decisions.

Several studies, both qualitative [2] and quantitative [3-14], have analyzed future demand for commercial supersonic transport aircraft. This paper introduces a mathematical framework for assessing how a market demand analysis model captures the interdependence among the key analysis variables. Unlike other quantitative prediction models [3-5,7-14], the Low-Boom Systems Analysis Model (LBSAM2) [6] can propagate important design characteristics of a supersonic transport aircraft concept to the level of economic metrics (such as the number of future supersonic passengers), with a consistent coupling of all the key supply and demand variables. This enables the use of the detailed demand analysis results from LBSAM2 to find favorable system-level trades between aircraft weight, range, and fuel burn for maximizing economic viability. This paper focuses on applications of LBSAM2 to improve the economic viability of low-boom supersonic transport aircraft under a low-boom constraint.

The "low-boom" qualifier is widely used in the literature to indicate a design feature of the aircraft shape to reduce the sonic boom on the ground. There is no specific standard to define precisely what "low-boom" means, so various definitions can be found in the literature. For the X-59, the low-boom qualifier means an outer mold line (OML) shape that is predicted [15,16] to attain sonic boom ground noise levels of about 75 perceived level in decibels (PLdB) based on the Mark VII calculation procedure in Ref. [17]. For an aircraft concept in this paper, the low-boom qualifier means that the aircraft satisfies the conceptual-level low-boom constraint given in Eqs. (1a) and (1b) [18,19].

$$\left\| A_{\mathrm{e,r}} - A_{\mathrm{e,r}}^{\mathrm{target}} \right\| \le \epsilon \tag{1a}$$

$$\mathrm{PLdB}\left(A_{\mathrm{e,r}}^{\mathrm{target}}\right) \le \sigma_{\max} \tag{1b}$$

The purpose of Eqs. (1a) and (1b) is to make the reversed equivalent area ($A_{\mathrm{e,r}}$) of the underlying supersonic aircraft concept closely match the low-boom target ($A_{\mathrm{e,r}}^{\mathrm{target}}$) with an undertrack sonic boom ground noise level of no more than $\sigma_{\max}$. The reversed equivalent area, $A_{\mathrm{e,r}}$, was first introduced in Ref. [20] using the undertrack pressure distribution at three body lengths below the aircraft. A choice of $\sigma_{\max}$ = 70 PLdB conceptually enforces the NASA N+3 low-boom goal for sonic boom ground noise level below 70 PLdB [21]. The tolerance (ε) for the inverse design error is a user-specified positive number, generally less than 1% of the maximum value of $A_{\mathrm{e,r}}^{\mathrm{target}}$. For convenience, the value of PLdB($A_{\mathrm{e,r}}^{\mathrm{target}}$) is referred to as the target PLdB for the underlying supersonic aircraft configuration that satisfies Eq. (1a). It quantifies the quality of the low-boom characteristics of the underlying supersonic concept. A lower value of PLdB($A_{\mathrm{e,r}}^{\mathrm{target}}$) means a more stringent low-boom requirement for the aircraft. The Mach, altitude, and angle of attack (AoA) for evaluating Eqs. (1a) and (1b) are determined by the start of overland cruise (SOC) conditions for the overland mission with the maximum overland range.

Supersonic market demand analysis tends to be considered an art instead of a science due to the complexity and uncertainty of subjective assumptions typically used in analysis. The scientific value of a supersonic market demand analysis method is in its fidelity capturing the market dynamics. For example, the "gold standard" for assessing the scientific value of any market prediction tool is set by the Black-Merton-Sholes model [22] for pricing stock option contracts and other derivative investment instruments. Although a fair value for an option contract predicted by the Black-Merton-Sholes model might not precisely match a future value in reality, the Black-Merton-Sholes model provides a useful scientific tool using stochastic differential equations to capture the financial market dynamics based on data-driven parameters for quantifying the uncertain behaviors of the underlying financial assets. In the same vein as this prediction model, a scientific process to capture the market dynamics for supersonic transportation demand is needed for conceptual design of supersonic transport aircraft. LBSAM2 takes the first step in this direction by using mathematical models to capture the interdependencies of key analysis variables in the supersonic market dynamics. It enables an unbiased comparison between two supersonic transport aircraft of economic viability (in terms of market demand) based on their performance characteristics in terms of weight, range, and fuel burn, with the overland supersonic flight feasibility quantified by PLdB($A_{\mathrm{e,r}}^{\mathrm{target}}$) for $A_{\mathrm{e,r}}^{\mathrm{target}}$ satisfying Eq. (1a).

Due to the ban on supersonic overland flight in the United States (US), aircraft are not allowed to fly at a Mach number greater than one in the US unless approved by the Federal Aviation Administration (FAA). This currently prevents commercial supersonic flight operations over land in the US, which needs to be considered in a market demand analysis tool by limiting supersonic flight to flight segments over water. If 14 CFR §91.817 were replaced with a rule dictating that no sonic boom be produced over land, aircraft could fly up to the *Mach cutoff speed* for the overland segments of a flight. A Mach cutoff speed is the maximum speed that a supersonic aircraft can fly without creating a sonic boom on the ground. For convenience, such an operating pattern that avoids creating an overland sonic boom is referred to as a *sonic-boom-restricted mission*.

There have been several studies estimating market demand for sonic-boom-restricted missions. The selected route for a sonic-boom-restricted mission is usually optimized for minimum travel time [5,6,9-12]. However, optimization for productivity of passengers is used in Ref. [4], which leads to an optimized route for the sonic-boom-restricted mission that also depends on the specified value of time (VOT) for passengers. An explicit trade formula between fuel burn and travel time is proposed in Refs. [7,13]. This formulation results in a variety of optimized routes for each OD pair by changing a trade-off parameter to produce a minimum time route (for the trade-off parameter at zero), a minimum fuel burn route (for the trade-off parameter at one), or something between these. A value of 0.4 for the trade-off parameter is used in Ref. [7] for an approximately optimal route for a sonic-boom-restricted mission that generates the maximum demand for the corresponding OD pair.

A market demand analysis for JAXA's S4 low-boom supersonic transport aircraft with 50 passengers and a cruise Mach of 1.7 is documented in Ref. [12]. Even though the S4 has a well-shaped undertrack ground signature imitating a sine wave (see Fig. 5 in Ref. [12]), its undertrack ground noise level is about 87.6 PLdB, which is much higher than the X-59's design target of 75 PLdB [15,16]. Market demand for JAXA's S4 is analyzed assuming both great circle paths with a cruise Mach of 1.7 and sonic-boom-restricted missions. The results show that the sonic-boom-restricted missions reduce the demand by about 50% (see Table 5 in Ref. [12]), illustrating the severe penalty of not being able to fly over land at the designed supersonic cruise speed. However, since the S4 was not designed for efficient performance in sonic-boom-restricted missions, it is possible that an aircraft designed for these sonic-boom-restricted missions might show a much less severe loss of market share.

Credible low-boom supersonic transport aircraft concepts are difficult to obtain because it is difficult to design an aircraft that can achieve both low-boom and mission performance goals. A systematic approach for integrating the conceptual-level low-boom constraint, Eqs. (1a) and (1b), in conceptual design of supersonic aircraft was developed in Refs. [18,19]. These references describe the conceptual design of two low-boom supersonic aircraft carrying 40 passengers with a seat pitch of 48 inches that satisfy Eq. (1a) with PLdB($A_{\mathrm{e,r}}^{\mathrm{target}}$) = 69.9 PLdB. The first was designed to a cruise Mach of 1.8 and has a lift-to-drag ratio (*L/D*) of 7.0 for overland cruise. The second aircraft was designed to be more efficient, flying with an overland cruise *L/D* of 7.73 at a speed of Mach 1.7.

Market demand analysis with modified versions of the two aircraft from Refs. [18,19] is described in Ref. [6]. Specifically, the two low-boom concepts were repackaged with 50 passengers and a seat pitch of 38 inches for the market demand analysis because the number of passengers (PAX) is a key driver for economic viability of a supersonic transport. The resulting low-boom aircraft were published in Ref. [6] without trade analysis details. The market demand analysis assumes that a low-boom mission profile [23] is used for the aircraft to cruise over land at the designed supersonic overland cruise Mach with a fixed AoA along a great circle route for any OD pair. A high-performance supersonic transport concept with 50 passengers was also designed to fly sonic-boom-restricted missions with the minimum travel time. Two different overland cruise speeds of Mach 0.95 and 1.15 were used to evaluate the corresponding market demand for the high-performance supersonic aircraft. The analysis results show that, in 2040, the high-performance supersonic concept could have 16.6% more passenger demand than the low-boom concept with an overland cruise Mach of 1.7 but 21.6% less passenger demand than the one with an overland cruise Mach of 1.8 (see Table 8 in Ref. [6]). Although the demand predictions are uncertain, these analysis results were derived with the same assumed values for the input variables, such as fuel price and VOT. There were no subjective assumptions that hindered the propagation of the aircraft performance data to the level of the passenger demand.

This paper is motivated by the market analysis results in Ref. [6], which were not aligned with our expectations. Prior to analysis, we believed that a low-boom aircraft with a lower maximum takeoff gross weight (MTOGW) and higher cruise efficiency would lead to lower ticket prices and induce a higher demand for supersonic flight. However, LBSAM2 predicts that the more-fuel-efficient low-boom aircraft has only about 2/3 of the passenger demand of the less-fuel-efficient, longer-range low-boom concept [6]. We will use LBSAM2 to explore different trades between design range, cruise speed, cruise efficiency, and PLdB($A_{\mathrm{e,r}}^{\mathrm{target}}$) in order to improve economic viability of a low-boom supersonic transport concept.

The paper is organized as follows. Section II.A introduces the mathematical framework that is used to identify what interdependencies of the key variables in the supersonic transportation demand analysis are consistently modeled. The coupling consistency of LBSAM2 for the market demand analysis, detailed mission analysis, and low-boom constraint is discussed in Sec. II.B. The system-level trade methods for low-boom aircraft concepts with a consistent coupling of the low-boom constraint are documented in Sec. III. Section IV demonstrates that LBSAM2 is capable of capturing non-intuitive supersonic market demand dynamics, and that the detailed market demand analysis results from LBSAM2 can be used to extract knowledge about a favorable design trade to improve the economics of the underlying low-boom aircraft concept. Section V discusses uncertainties in the LBSAM2 analysis, along with a laminar flow technology benefit assessment and a sensitivity analysis for fuel price. The summary and conclusions are in the final section.

II. Global Market Demand Analysis for Supersonic Transports

There are different ways to perform market demand analysis, such as a straightforward profitability analysis for an airline with an assumed market size [14] or an estimation of the market size using an assumption about the percentage of the subsonic passengers that will switch to supersonic flight [10-13]. The authors prefer an economic analysis for an equilibrium state of the supersonic transportation market with one airline in a future year under an assumed profit margin for the airline [6-9]. The equilibrium state means that the number of supersonic aircraft serving the market exactly meets the predicted passenger demand in the future year under the specified economic and profit margin assumptions. The primary benefit of this equilibrium analysis is that it provides a quantitative estimate of the market demand that reflects the performance merits of the supersonic transport aircraft: a “better” aircraft will induce a higher passenger demand under the same assumptions.

The development of a supersonic transport aircraft market will involve many dynamic market forces. One past study has captured these dynamics with an agent-based and discrete event simulation to model the supersonic aircraft development, gradual acquisition and deployment of aircraft, market expansion, and multiple market participants over a 35-year period [3]. This approach has some limitations for market demand analysis of low-boom supersonic transports, such as assuming that each aircraft only serves two airports for one over-ocean route to avoid fleet scheduling[1] and that the percentage of subsonic premium passengers who are willing to switch to supersonic flights for a given supersonic ticket price is available (see Figs. 5.3 and 5.4 in Ref. [3]) for each OD pair. In contrast, the equilibrium analysis we employ simplifies the dynamic aspects of the market and focuses on a fully coupled market analysis for the anticipated future year, with the airline achieving the specified profit margin. The implicit assumption in the equilibrium analysis is that venture capital will support the market development until the airline can achieve the specified profit margin in the future year. The equilibrium analysis focuses on the economic viability once the supersonic transportation market is stabilized. The user-specified profit margin can be treated as a reward-to-risk factor for the initial venture capital investments to create the market. Additionally, the single-airline and single-manufacturer assumptions of the equilibrium analysis represent a simplified market dynamic that may not reflect an anticipated future state. These simplifying assumptions provide an optimistic economic scenario for these companies; so, if only a very small market or no market is predicted with these assumptions, it is highly unlikely any market would be practically realizable and there is no need to model the additional market dynamics from multiple competing companies. Ultimately, the details of these market dynamics are not our concern; we desire to focus on how vehicle design changes generally impact potential demand, which does not require modeling all of the market dynamics.

In our modeling, we evaluate market equilibrium in the year 2045, which implies an unmodeled timeline for a commercial supersonic aircraft to be developed and enter the market prior to 2045. The simulation provides the number of required supersonic aircraft in the specified future year to achieve the supply and demand balance for that single year. This is not the number of aircraft needed for the supersonic transportation market over the expected full production life cycle of the aircraft. A business decision on whether to invest in the development of such an aircraft would need to consider the full lifecycle, but for our analysis in comparing the relative attractiveness of various aircraft designs, the full lifecycle modeling is not inherently required.

A supersonic market demand analysis must use many assumptions and approximations in modeling the future demand for supersonic flight. It is difficult to compare two sets of market demand analysis results without fully understanding the effects of those assumptions and approximations. A mathematical framework is provided in Sec. II.A with six groups of the key analysis variables that quantify the market supply and demand. This framework enables a direct comparison of how the interdependencies between these variables are modeled in different market demand analysis methods. Section II.B shows how a consistent coupling of the supersonic market demand analysis, detailed mission analysis, and low-boom constraint is implemented in LBSAM2. To our knowledge, LBSAM2 is the only market demand analysis model with a consistent coupling of all the key analysis variables for the supersonic transportation market.

A. Consistency of Modeling Interdependencies in Supersonic Transportation Market

The equilibrium analysis of the supersonic transportation market with one airline can be quantified by the following six groups of data:

(II.1) Number of existing premium subsonic passengers and ticket price distribution for each selected OD pair in a base year.
(II.2) Number of subsonic premium passengers for each selected OD pair in a future year.
(II.3) Percentage of subsonic premium passengers who will switch to supersonic flight for each selected OD pair.
(II.4) Number of supersonic passengers for each selected OD pair in the future year.
(II.5) Annual revenue/non-revenue flight hours and load factor per aircraft in the future year.
(II.6) Number of aircraft needed to serve the supersonic passengers in the future year.

The methods for computing the quantities in (II.1)-(II.6) are briefly discussed in the remainder of this subsection. The data in (II.1) are usually based on some existing database about air travel, such as the air travel data from the Official Airline Guide [24]. The information about the premium passengers and other relevant data for OD pairs is extracted from the existing database with some user-specified filters to screen out demand that would not likely be served by a commercial supersonic aircraft, such as airport runway length and minimum distance between an OD pair. The variables in (II.1) do not require any modeling and are the outputs of a data screening process.

Multiple existing forecast models for future air travel demand can be used to compute (II.2). The Boeing forecast model was used in Refs. [3,4,7,10-13] and the International Civil Aviation Organization (ICAO) forecast model was used in Refs. [6,8]. We leverage ICAO's forecast in this paper.

[1] Our results below indicate that fleet scheduling may have a notable impact on desirable aircraft design parameters—most notably cruise speed—that produce higher passenger demand.

The most complex modeling problem for supersonic demand analysis is (II.3). The simplest approach is to assume the percentage of subsonic premium passengers who are willing to switch to supersonic flights ($R_{sub2sup}$), computing the number of supersonic passengers (II.4) from Eq. (2) [10-13].

$$N_{sup} = N_{sub} \cdot R_{sub2sup} \cdot F_{scale} \tag{2}$$

Here, N_{sub} and N_{sup} are the predicted subsonic premium and supersonic passenger demands for each OD pair in the future year, respectively. The scaling factor (F_{scale}) is usually based on an empirical formula that reflects the demand change with respect to some key drivers, such as the saved travel time when compared to subsonic flight. Equation (2) with an assumed $R_{sub2sup}$ does not accurately capture how the supersonic passenger demand in (II.4) depends on the weight and performance of the supersonic transport aircraft.

A higher fidelity modeling approach is to use passengers' VOT and the estimated operating costs of a supersonic transport to model the interdependency between (II.3) and the aircraft performance data. The accepted value proposition [4-9] is that a subsonic passenger will become a supersonic passenger if the value the passenger places on the time saved by flying supersonically outweighs the increase in ticket cost, which is expressed in Eq. (3).

$$C_{sup} - C_{sub} \leq \mathrm{VOT} \cdot (T_{sub} - T_{sup}) \tag{3}$$

Often, VOT in Eq. (3) takes a single, assumed value [4,6], or values published by the Department of Transportation [5]; however, other approaches are also employed. Some estimate VOT for each OD pair using the ticket prices of nonstop and one-stop subsonic flights [8], others assume the hourly income of a passenger is each passenger's VOT [7], and a proprietary model is used in Ref. [9] to determine VOT and the number of passengers satisfying Eq. (3). The values of subsonic ticket price (C_{sub}) and travel time (T_{sub}) can be derived from the air travel data used in (II.1). The supersonic travel time (T_{sup}) is computed from either an optimized sonic-boom-restricted mission [4-7,9-12] or the great circle travel time for a low-boom supersonic transport [6,8,12]. The supersonic ticket price (C_{sup}) is the most difficult variable in Eq. (3) to model. The value of C_{sup} depends on the life cycle cost (LCC) of the supersonic aircraft, but no standard LCC model exists, causing estimates for C_{sup} to vary significantly based on differing assumptions. The LCC model in LBSAM2 is used for demand analysis in this paper. For each OD pair, the switching percentage ($R_{sub2sup}$) in (II.3) is computed as the ratio of subsonic premium passengers satisfying Eq. (3) to the total number of subsonic premium passengers.

An indirect approach to using the value proposition in Eq. (3) is developed in Ref. [7]. Instead of using a direct estimation of C_{sup}, ($C_{sup} - C_{sub}$) is approximated by a constant fare difference for a route. The fare difference is derived from the great circle distance for an OD pair, a reference airline yield per nmi, and a scaling factor. The scaling factor depends on the fuel burn for the route, seating capacity, and annual flight hours of the supersonic aircraft. For an OD pair, $R_{sub2sup}$ is calculated using Eq. (3) with ($C_{sup} - C_{sub}$) replaced by the fare difference, the 2014 hourly income data in the US, and an empirical formula for the estimated number of flights per year by an individual based on the hourly income (that is used as VOT).

Equation (3) as a passenger preference model is far from realistic. There are many other factors that might influence the passenger preferences, such as timing and seating comfort for the trip [4,11]. A market share factor of 50% is used in Ref. [8] to assume that other factors make half of the subsonic premium passengers satisfying Eq. (3) remain subsonic passengers. Further studies are required to determine what passenger preference model is appropriate for the market demand analysis of supersonic transports. A first step in this direction is the following value proposition for a subsonic premium passenger to take a supersonic flight, proposed in Ref. [6].

$$C_{sup} - C_{sub} \leq \mathrm{VOT} \cdot (T_{sub} - T_{sup}) - (\text{Value of Passenger Comfort Loss}) \tag{4}$$

Equation (4) was developed after discovering a missing element in Eq. (3) when integrating the market demand analysis with conceptual design of supersonic transport aircraft. During conceptual design, an aircraft designer needs to carefully consider fuselage size to maximize the vehicle performance while also trying to maximize seating capacity for better fleet productivity. When not accounting for seating comfort, market demand analysis results indicate that reducing the seat pitch is always beneficial. As a result, the more spacious seat pitch of 48 inches for two 40-PAX low-boom supersonic transports in Refs. [18,19] was reduced to 38 inches (similar to current premium economy seat pitch) so that the seating capacity could be increased to 50 in the subsequent work (see Table 1 in Ref. [6] or Sec. III.F in this paper). This led to the value proposition of Eq. (4) with the seat pitch as an input.

An appropriate seat pitch is difficult to determine for market demand analysis of a given supersonic transport aircraft. A data-driven regression model of value of comfort (VOC) for each OD pair was derived in Ref. [6] from several sources of air travel data. This enables the use of Eq. (4) and the subsonic premium ticket price distribution to predict the number of supersonic passengers for each OD pair [6], with a seat pitch input that determines the value of passenger comfort loss for the trip. The value of passenger comfort loss is calculated using the VOC, reference subsonic seat pitches, and specified supersonic seat pitch (see Eq. (2) in Ref. [6]). This provides an interdependency model for the aircraft cabin design and the market demand analysis, which is more desirable than an arbitrary decision on the seating capacity for market demand analysis.

When R_{sub2sup} is derived using a passenger preference model, (II.4) is typically computed using Eq. (2) with $F_{\text{scale}} = 1$. The data in (II.4), (II.5), and (II.6) are interdependent in modeling the supersonic market dynamics. Specifically, (II.5) and (II.6) depend on (II.4). However, an increase in the number of aircraft needed generally reduces the aircraft unit cost [25,26], which will reduce the supersonic ticket prices (with a fixed airline profit margin) and induce a higher demand for supersonic flight. Moreover, for the same number of aircraft in the airline fleet, a better set of fleet productivity parameters in (II.5) will also reduce the supersonic ticket prices and induce a higher demand for supersonic flight. So, (II.4) also depends on (II.5) and (II.6).

The airline fleet productivity parameters in (II.5) are typically assumed to avoid solving an integer programming problem for the fleet assignment. With the assumed fleet productivity parameters, the number of aircraft needed to serve the supersonic passengers in the future year can be estimated with an algebraic approximation formula (e.g., see Eq. (16) in Ref. [7] or Eq. (7) in Ref. [8]). However, a fleet assignment model is used in LBSAM2 to compute (II.5) and (II.6) simultaneously from (II.4). LBSAM2 uses a multidisciplinary optimization (MDO) method with (II.5) and (II.6) as coupling variables to model the interdependencies between the analysis variables in (II.4)-(II.6) consistently. See Ref. [27] for a formal definition of consistency of coupling variables for MDO methods.

B. Consistent Coupling of Market Demand Analysis, Mission Analysis, and Low-Boom Constraint

LBSAM2 has consistency not only in modeling the interdependencies of all the analysis variables in (II.4)-(II.6) but also in coupling of the market demand analysis, mission analysis, and low-boom constraint. This subsection briefly describes how the fuel burn data from a detailed mission analysis is used consistently for the supersonic market demand analysis. A numerical example is given to illustrate the accuracy of the approximation formulas used by LBSAM2 when estimating the fuel burn and travel time for the great circle route between an OD pair. The estimates generated by these formulas are approximately identical to those obtained from a mission analysis using different flight modes for various overland and overwater segments of the route, which ensures a consistent coupling between the detailed mission analysis and low-boom constraint [Eq. (1a)]. This level of fidelity for a consistent coupling of the market demand analysis, mission analysis, and low-boom constraint is important for an accurate propagation of the aircraft performance and low-boom characteristics to the level of economic metrics such as those in (II.4)-(II.6).

For a route between an OD pair, LBSAM2 partitions the route into overland and overwater segments. Here, the overland and overwater qualifiers mean that the supersonic aircraft flies each overwater segment unrestricted using an overwater supersonic cruise Mach and each overland segment in a low-boom or no-boom cruise mode. Let τ denote the ratio of the total distance of overland segments to the total distance of the route. A route is eligible for supersonic flight service if its range is no more than $\tau \cdot R_{\text{max,land}} + (1-\tau) \cdot R_{\text{max,water}}$. Here, $R_{\text{max,land}}$ and $R_{\text{max,water}}$ are the maximum overland and overwater ranges of a supersonic transport. For an eligible route, the fuel burn and travel time for the route can be estimated with the following approximation formulas.

$$FW_{\text{sup}} \approx \tau \cdot FW_{\text{overland}} + (1-\tau) \cdot FW_{\text{overwater}} \tag{5}$$
$$T_{\text{sup}} \approx \tau \cdot T_{\text{overland}} + (1-\tau) \cdot T_{\text{overwater}} \tag{6}$$

The values of FW_{overland} and T_{overland} are determined by flying the total route distance with a fixed-AoA cruise at the overland cruise Mach, whereas $FW_{\text{overwater}}$ and $T_{\text{overwater}}$ are determined by flying the total route distance at the overwater cruise Mach and cruise conditions for minimum fuel burn. The Flight Optimization System (FLOPS) [28] is used for all the mission analyses in this paper.

Limited verification runs indicate that Eqs. (5) and (6) provide very accurate approximations of FW_{sup} and T_{sup} for the great circle routes used by the low-boom aircraft concepts in this paper. Here, we use one example to illustrate the accuracy of Eqs. (5) and (6). For a wind-optimal great circle route [8] from Dubai to Glasgow, the FLOPS mission analyses are performed for the Mach 1.7/1.8 concept in Ref. [6] (see also Sec. III.F) using three different mission profiles: a mixed mission with low-boom cruise at Mach 1.7 for overland segments and unrestricted cruise at Mach 1.8 for overwater segments, a low-boom mission with a constant cruise Mach of 1.7, and an unrestricted mission with a constant cruise Mach of 1.8. This route has five overwater/overland segments with the corresponding ranges (in nmi) of (i) 419, (ii) 725, (iii) 567, (iv) 1015, and (v) 405. The overwater segments of (i), (iii), and (v) are over the Persian Gulf, Black Sea, and North Sea, respectively. The route has a total distance of 3131 nmi. The value of τ for this route is 55.6%. All the transitions between the overwater and overland segments are modeled in the FLOPS analysis of the mixed mission (see Fig. 1). The aircraft completes a transition from unrestricted overwater cruise to low-boom overland cruise at the end of the unrestricted overwater cruise. This involves changing altitude to the low-boom cruise altitude and decelerating to the overland cruise Mach, which enables the aircraft to fly the overland segment at the designed AoA for low-boom cruise. An acceleration to the overwater cruise Mach with minimum fuel burn is used to start an overwater segment after an overland segment. So, the mission analysis is consistent with the low-boom constraint [Eq. (1a)].

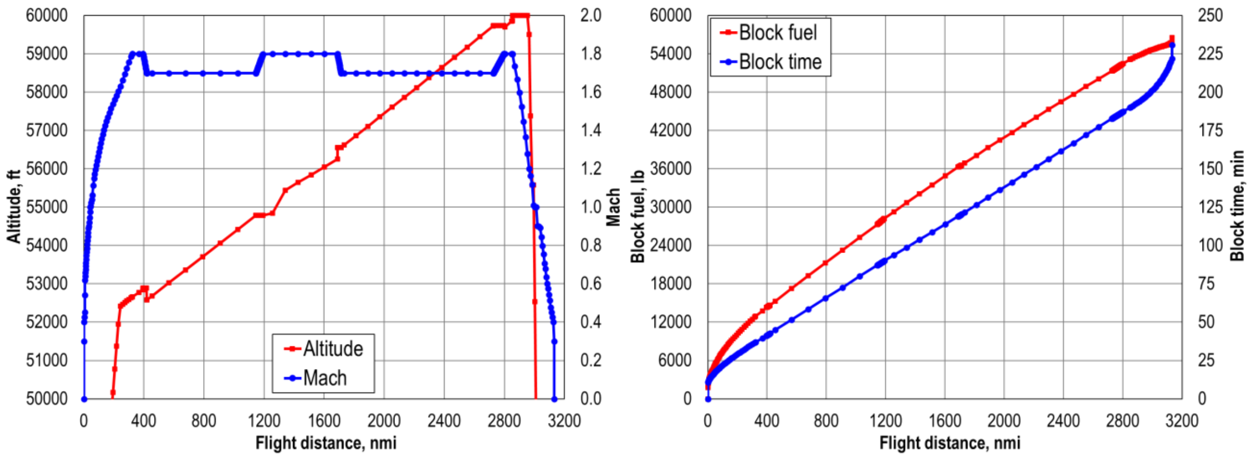

Fig. 1 Altitude, Mach, block fuel, and block time for the mixed mission from Dubai to Glasgow.

Table 1 compares the fuel burn and travel time for the three different mission profiles. The fuel and time differences refer to the differences between the corresponding mission and the mixed mission. A negative fuel difference means that the corresponding mission requires less fuel than the mixed mission. All three missions have approximately the same values for fuel burn and travel time. Equations (5) and (6) for this route yield the approximation values of 56,304 lb and 229.4 min for FW_{sup} and T_{sup}, respectively. So, when the low-boom overland and unrestricted overwater cruise Mach numbers differ by only 0.1 as in this case, Eqs. (5) and (6) provide an approximately consistent coupling for the market demand analysis, FLOPS mission analysis, and low-boom constraint [Eq. (1a)].

Table 1 Comparison of three mission profiles of the Mach 1.7/1.8 concept from Dubai to Glasgow

Operating mode	Mixed	Low-boom overland Mach 1.7	Unrestricted overwater Mach 1.8
Low-boom cruise Mach	1.7	1.7	—
Unrestricted cruise Mach	1.8	—	1.8
Low boom overland	Yes	Yes	No
Total fuel burn, lb	56,564	56,064	56,605
Fuel difference, lb	0	-500	41
Relative fuel difference	0.0%	-0.88%	0.07%
Total travel time, min	230.8	233.4	224.4
Time difference, min	0	2.6	-6.4
Relative time difference	0.0%	1.13%	-2.77%

The mission analysis for a low-boom aircraft to fly a route with a mixture of overland and overwater segments is a non-trivial numerical analysis problem: the aircraft must fly with the designed low-boom AoA for the altitude at the end of an overwater segment (EOWS). The altitude at EOWS is modified iteratively until the AoA of the aircraft at EOWS is the same as that used to satisfy Eq. (1a). This iterative process is performed for an estimated value of the required TOGW for the mixed mission. Then an optimization process is performed to ensure that the estimated TOGW can complete the mixed mission with the minimum amount of leftover fuel to satisfy all the fuel reserve requirements, such as enough leftover fuel to fly to an alternate airport in an emergency. For the mixed mission from Dubai to Glasgow, it takes hours to obtain the mission analysis results in Fig. 1 using FLOPS. There are hundreds of OD pairs with different combinations of overland and overwater segments for the market demand analysis. So, the mixed mission analysis needs to be performed hundreds of times for all selected OD pairs in (II.1), which means days or weeks for the calculation of the values of T_{sup} and FW_{sup} directly using FLOPS. Equations (5) and (6) are useful approximation formulas that significantly reduce the computational time for the operational cost estimation of a low-boom supersonic transport aircraft.

Although Eqs. (5) and (6) are accurate approximation formulas for a consistent coupling of the FLOPS mission analysis and estimation of supersonic ticket price for an OD pair, T_{overland} and $T_{\text{overwater}}$ generated by FLOPS do not account for atmospheric wind effects. For example, the travel time from LAX to JFK[2] is noticeably shorter than that from JFK to LAX for a subsonic transport aircraft using the same cruise Mach. LBSAM2 uses a wind-optimal great circle route between an OD pair for a low-boom aircraft to take advantage of the wind effects. The values of T_{overland} and $T_{\text{overwater}}$ including wind effects are calculated in LBSAM2 using the following approximation formulas (see Sec. V in Ref. [8] for the details of the wind analysis model).

$$T_{\text{overland}} \approx \text{(route distance)/(overland cruise speed + wind speed adjustment) + (non-cruise time adjustment)} \tag{7a}$$

$$T_{\text{overwater}} \approx \text{(route distance)/(overwater cruise speed + wind speed adjustment) + (non-cruise time adjustment)} \tag{7b}$$

A wind vector field is used to adjust the travel speed along the flight path to compute the wind-optimal travel time for the cruise speed. The non-cruise time adjustment accounts for climb, descent, and taxi times based on the FLOPS data. The adjustments of T_{overland} and $T_{\text{overwater}}$ for wind effects approximately achieve a consistent coupling of travel time analysis and atmospheric wind data.

Another detail worth noting is that the partition of overland and overwater segments of a route in LBSAM2 is determined using the geographic location of a point on the flight path. A higher fidelity approach [7,29-32] is to partition a route using the physics of sonic boom propagation. That is, a location on the flight path is considered as a point on an overwater segment if the sonic boom from unrestricted supersonic cruise at this location will not reach land. Such a physics-based partition method for overland and overwater segments is more time consuming than the geography-based partition method of LBSAM2. For the Mach 1.7/1.8 concept and the route from Dubai to Glasgow, the two partition methods might yield different partitions of overland and overwater segments; however, Table 1 indicates that they will have approximately the same values for T_{sup} and FW_{sup} due to a small difference of 0.1 between the two cruise Mach numbers. The different effects on T_{sup} and FW_{sup} of the geographic and physics-based partitions of overland and overwater segments for sonic-boom-restricted missions have not been quantified in the literature.

III. Low-Boom Supersonic Aircraft Trade Methods and Concept Aircraft

In this section, Eq. (1a) is incorporated in the trade methods for conceptual low-boom aircraft design using FLOPS. For a given supersonic configuration satisfying Eq. (1a), the first four subsections show how to perform trades between MTOGW, aircraft range, PAX, and PLdB($A_{\text{e,r}}^{\text{target}}$) for $A_{\text{e,r}}^{\text{target}}$ satisfying Eq. (1a). Moreover, the low-boom MDO process in Ref. [19] is implemented for weight estimation and mission analysis of any supersonic aircraft with geometry modeled in OpenVSP [33]. This MDO process leverages CFD-based low-boom design tools and generates the required data for analysis in LBSAM2 (see Sec. III.E). The last subsection shows how we leveraged these trade methods to generate four 50-PAX low-boom concepts for market demand analysis.

A. Trade Between PAX and Aircraft Range Capability with Fixed MTOGW

The trade between PAX and aircraft range is the standard trade for reducing the total fuel weight by the amount of increased payload so that MTOGW of the aircraft remains the same. Increasing the number of passengers (with fixed OML) is accomplished by reducing seat pitch. This increase in the number of passengers leads to a reduction of the maximum design range. Moreover, because MTOGW is fixed, the weight of the aircraft at SOC remains the same. Because the original concept and the concept with a different PAX have the same OML, they also have the same AoA at SOC. So, the two concepts have the same undertrack $A_{\text{e,r}}$ at SOC and satisfy Eq. (1a) with the same $A_{\text{e,r}}^{\text{target}}$. This trade does not change PLdB($A_{\text{e,r}}^{\text{target}}$) for $A_{\text{e,r}}^{\text{target}}$ satisfying Eq. (1a). An example is a trade from 40 to 46 PAX for a low-boom concept in Sec. III of Ref. [23].

B. Trade Between Overland Range and Target PLdB with Fixed MTOGW

For a fixed MTOGW, one could trade PLdB($A_{\text{e,r}}^{\text{target}}$) for $A_{\text{e,r}}^{\text{target}}$ satisfying Eq. (1a) with TOGW for the overland mission. When TOGW for the overland mission is changed to a value not exceeding MTOGW, the weight at SOC changes too. There are two methods to achieve consistency between the weight and AoA at SOC. The simplest approach is to maintain the altitude at SOC and change the AoA. But an AoA different from that used for Eq. (1a) will change the undertrack pressure distribution used to compute $A_{\text{e,r}}$ and invalidate Eq. (1a). Another approach is to

[2] This paper uses the International Air Transportation Association airport codes for OD pairs.

change the altitude at SOC so that the required AoA at SOC is identical to that used to compute $A_{\mathrm{e,r}}$ in Eq. (1a). As shown in Ref. [23], the undertrack off-body nondimensional overpressure at three body lengths below the aircraft (used to calculate $A_{\mathrm{e,r}}$) is approximately invariant with respect to altitude change. So, with an appropriate altitude at SOC, the $A_{\mathrm{e,r}}$ for the supersonic concept at the new altitude is about the same as that in Eq. (1a) for the same AoA. This indicates that, for any TOGW for the overland mission, one could find an altitude for SOC so that Eq. (1a) holds.

A hypothesis used in this paper is that PLdB($A_{\mathrm{e,r}}^{\mathrm{target}}$) is lower when the altitude for sonic boom propagation is higher. There is no mathematical verification of this hypothesis, but numerical results [23] do agree with this assumption. Intuitively, the hypothesis means that an increased propagation distance reduces the loudness of the noise from the same acoustic source. So, by reducing the overland range or TOGW for the overland mission, one could reduce PLdB($A_{\mathrm{e,r}}^{\mathrm{target}}$) for $A_{\mathrm{e,r}}^{\mathrm{target}}$ satisfying Eq. (1a), with an appropriate increment of the altitude for SOC. This trade method is mainly for increasing the overwater range without increasing PLdB($A_{\mathrm{e,r}}^{\mathrm{target}}$) for $A_{\mathrm{e,r}}^{\mathrm{target}}$ satisfying Eq. (1a), as shown in Sec. III.D. It is also the analysis method used to compute the fuel burn for low-boom overland missions with different range requirements, because the aircraft has to use the same AoA as that used in Eq. (1a) for low-boom overland cruise.

C. Trade Between Overland Range and Target PLdB with Variable MTOGW

This method uses the same computational method as in the previous subsection. For any given MTOGW, one can find an appropriate altitude at SOC so that Eq. (1a) is satisfied. An increased MTOGW will increase the overland range and the weight at SOC. To match the increased weight at SOC, the altitude at SOC must be reduced for a higher air density and the same AoA (or lift coefficient) as that used in Eq. (1a). By the hypothesis in Sec. III.B, the corresponding PLdB($A_{\mathrm{e,r}}^{\mathrm{target}}$) is higher due to the reduced propagation distance to the ground. So, an increase of MTOGW will increase the overland range and the corresponding PLdB($A_{\mathrm{e,r}}^{\mathrm{target}}$) for $A_{\mathrm{e,r}}^{\mathrm{target}}$ satisfying Eq. (1a), with a reduction of the altitude at SOC.

D. Trade Between MTOGW and Overwater Range

The trade between MTOGW and overwater range is a standard trade for conceptual design of supersonic aircraft. However, there are two options for the overland performance of the aircraft with an increased MTOGW. The first is to use the method in Sec. III.B to reduce the overland range while keeping PLdB($A_{\mathrm{e,r}}^{\mathrm{target}}$) for $A_{\mathrm{e,r}}^{\mathrm{target}}$ satisfying Eq. (1a) unchanged. The other is to use the method in Sec. III.C to increase PLdB($A_{\mathrm{e,r}}^{\mathrm{target}}$) for $A_{\mathrm{e,r}}^{\mathrm{target}}$ satisfying Eq. (1a) for the maximum overland range.

E. Consistent Mission Analysis of a CFD-Based Low-Boom Design

If a supersonic configuration is designed using CFD for either a shaped undertrack ground signature or a reduction of sonic boom ground noise level, one could use the method in this subsection to generate the corresponding mission performance data and $A_{\mathrm{e,r}}^{\mathrm{target}}$ such that Eq. (1a) is satisfied and coupled consistently with the mission performance data. The method in this subsection is directly applicable using the integrated analysis process in Ref. [19], where OpenVSP is used to model the aircraft and FLOPS is used to compute the weights and mission performance data.

The flight condition (Mach, altitude, and AoA) for the CFD-based low-boom design is used as the flight condition at SOC for the mission analysis. The $A_{\mathrm{e,r}}$ for the undertrack direction at SOC is computed. The optimal $A_{\mathrm{e,r}}^{\mathrm{target}}$ for $A_{\mathrm{e,r}}$ under the constraint of Eq. (1a) is generated using the method in Ref. [23]. So, the supersonic configuration satisfies Eqs. (1a) and (1b) with an appropriate value of σ_{max}. Then the OpenVSP geometry for the aircraft is imported into the low-boom MDO process in Ref. [19]. The calibrated lift method in Ref. [19] is used to calibrate the low-fidelity aerodynamic analysis so that the total lift at SOC is the same as the total CFD lift for the sonic boom analysis at SOC. This leads to a mission analysis of the CFD-based low-boom design using the calibrated low-fidelity aerodynamics analysis that is consistent with the CFD-based sonic boom analysis for the lift calculation at SOC. Then, MTOGW is used as a design variable for the aircraft configuration. Its value is adjusted so that the weight of the resulting configuration at SOC matches the total CFD lift for sonic boom analysis at SOC. This configuration with the adjusted MTOGW still satisfies Eq. (1a) at SOC.

F. Concept Aircraft

For the remainder of this paper, we explore the market potential of the four low-boom supersonic aircraft concepts that are described in Table 2. Each of these aircraft is a 50-PAX variant of the 40-PAX aircraft described in Refs. [18,19] that maintains the same overall cabin dimensions.[3] These aircraft were generated with the trade methods described above and have various ranges, PLdB values, and cruise speeds.

[3] Seat pitch for the 50-PAX concepts is reduced to 38 in from the 48 in seat pitch of the 40-PAX aircraft.

Table 2 Comparison of four low-boom concepts of 50 passengers with seat pitch of 38 inches

Label for concept	Mach 1.8/1.8	Mach 1.7/1.8	M1.7 HiPLdB	M1.7 EqPLdB
MTOGW, lb	167,250	147,600	163,413	163,413
Operating empty weight	67,433	66,518	67,703	67,703
Sea-level engine thrust per engine, lbf	36,000	34,000	34,000	34,000
Fuel weight for overwater mission, lb	79,339	61,665	75,536	75,536
Fuel weight for overland mission, lb	79,339	61,665	75,536	60,862
Overwater range at cruise Mach 1.8, nmi	4329	3572	4200	4200
Overland range, nmi	4163	3622	4234	3540
Overland cruise Mach	1.8	1.7	1.7	1.7
Low-boom cruise altitude at SOC, ft	47,836	52,100	49,674	52,100
Target PLdB for low-boom constraint	69.9	69.9	70.6	69.9

The first aircraft we study in this paper is the 50-PAX Mach 1.8/1.8 concept described in Ref. [6]. This aircraft was obtained after successive applications of the trade methods in Secs. III.E, III.A, and III.C. Specifically, we began with the CFD-based low-boom design in Ref. [18] labeled as $\hat{\boldsymbol{D}}_{\mathrm{LoT}}$, which satisfies Eq. (1a) with PLdB($A_{\mathrm{e,r}}^{\mathrm{target}}$) = 69.9 PLdB (see Fig. 13 in Ref. [18]). Using the method in Sec. III.E, we conducted a consistent mission analysis of this aircraft along with the weight estimation. Then PAX was increased from the Ref. [18] value of 40 to 50 using the method in Sec. III.A. The method in Ref. [23] was used to find $A_{\mathrm{e,r}}^{\mathrm{target}}$ satisfying Eq. (1a) with the lowest value of PLdB($A_{\mathrm{e,r}}^{\mathrm{target}}$). The method in Sec. III.C was used to adjust the overland range while PLdB($A_{\mathrm{e,r}}^{\mathrm{target}}$) remained at 69.9 PLdB. The altitude for SOC was reduced from 50,700 ft to 47,836 ft for the Mach 1.8/1.8 concept. Some specifications for the Mach 1.8/1.8 concept are listed in Table 2. The Mach 1.8/1.8 concept has the same OML as that of $\hat{\boldsymbol{D}}_{\mathrm{LoT}}$ in Ref. [18].

The second concept aircraft that we consider in this paper is a low-boom concept with an overland cruise speed of Mach 1.7 and an overwater cruise speed of Mach 1.8 that is described in Ref. [6]. The trade method in Sec. III.A was used to generate a 50-PAX version of the 40-PAX low-boom concept in Ref. [19]. Then the method in Ref. [23] was used to find $A_{\mathrm{e,r}}^{\mathrm{target}}$ satisfying Eq. (1a) with the lowest value of PLdB($A_{\mathrm{e,r}}^{\mathrm{target}}$). The method in Sec. III.C was used to increase MTOGW in such a way that the increased PLdB($A_{\mathrm{e,r}}^{\mathrm{target}}$) value is 69.9 PLdB. The resulting supersonic aircraft concept is called the Mach 1.7/1.8 concept.

The third and fourth concept aircraft were generated with an increased overwater range compared to the Mach 1.7/1.8 concept. As shown later in Sec. IV, the range of a low-boom supersonic transport is an important driver for its economic viability. We performed an in-depth assessment of the market demand analysis results at the OD level for the Mach 1.8/1.8 concept aircraft and found that a range of 4200 nmi would provide a good coverage of the market. So, we increased the overwater range of the Mach 1.7/1.8 concept to 4200 nmi following the method in Sec. III.D. Then, leveraging the two options for the overland mission performance in Secs. III.B and III.C, we generated two trade concept aircraft, called the M1.7 EqPLdB and M1.7 HiPLdB concepts, which have PLdB values equivalent to and higher than the Mach 1.7/1.8 concept, respectively. Some specifications for these two trade concepts are listed in Table 2.

Compared to the original concepts in Refs. [18,19], the Mach 1.8/1.8 and 1.7/1.8 concepts have slightly increased overwater ranges due to the trade between PLdB and MTOGW with an $A_{\mathrm{e,r}}^{\mathrm{target}}$ having a lower PLdB($A_{\mathrm{e,r}}^{\mathrm{target}}$) than the target PLdB used for the original low-boom inverse design optimization and significantly increased overland ranges mainly due to the use of a fixed-AoA overland cruise [23] instead of an overland cruise with a fixed low-boom altitude [18,19]. All the concepts with an overland cruise Mach of 1.7 share the same OML as the low-boom concept in Ref. [19].

All four low-boom concepts in Table 2 have a payload of 10,450 lb, an overwater cruise Mach of 1.8, and two engines on the sides of the fuselage. Figure 2 shows the top view of the pressure contours for the Mach 1.7/1.8 and Mach 1.8/1.8 concepts at the corresponding SOC conditions. A detailed shape comparison of the Mach 1.7/1.8 and Mach 1.8/1.8 concepts can be found in Figs. 14 and 15 of Ref. [19]. The M1.7 HiPLdB and M1.7 EqPLdB concepts have the same pressure distribution at SOC as the Mach 1.7/1.8 concept in Fig. 2. The target PLdB differences among the three low-boom concepts with an overland cruise Mach of 1.7 are due to the altitude differences at SOC.

The travel time and fuel burn curves (piecewise linear interpolations of the discrete data generated by FLOPS for a finite set of flight ranges) for these four aircraft concepts are plotted in Fig. 3. These curves determine T_{overland},

FW_{overland}, $T_{\text{overwater}}$, and $FW_{\text{overwater}}$, which are used to compute T_{sup} and FW_{sup} by Eqs. (5) and (6) for a consistent coupling of the supersonic market demand analysis, mission analysis, and low-boom constraint.

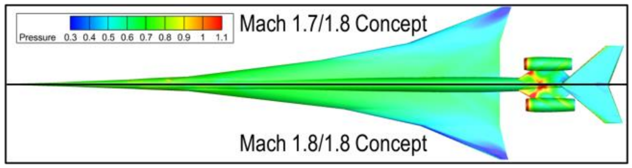

Fig. 2 Top view of pressure contours for the Mach 1.7/1.8 and Mach 1.8/1.8 concepts.

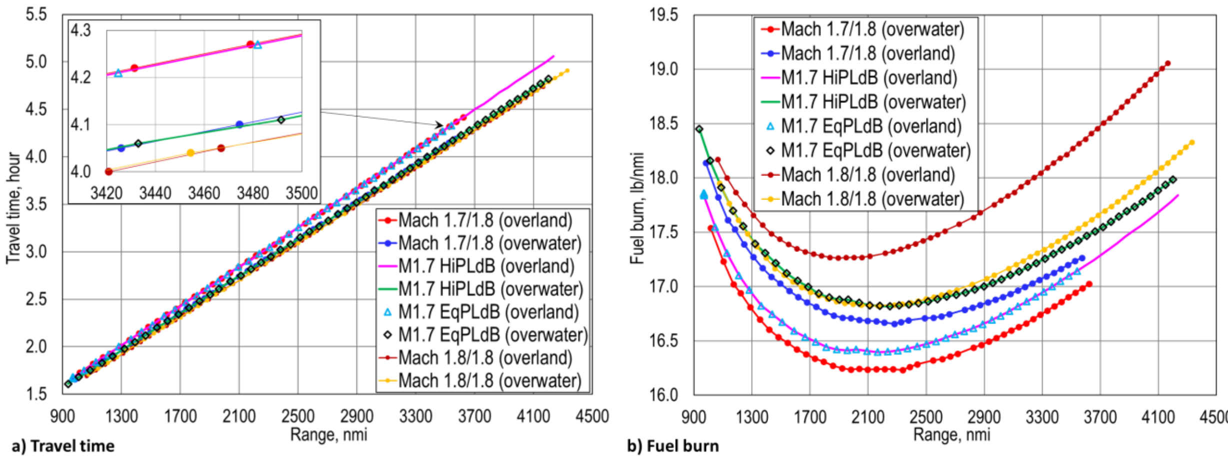

Fig. 3 Travel time and fuel burn curves for the four low-boom supersonic transport concepts.

IV. Market Demand Analysis for Conceptual Low-Boom Design

This section addresses the primary focus of this paper: leveraging LBSAM2 to guide conceptual design of low-boom supersonic transports. The background information is given in Sec. IV.A, which explains why the overland range cannot be set as a requirement for conceptual design of low-boom supersonic aircraft. The basics of market demand analysis using LBSAM2 are discussed in Sec. IV.B, including the assumed input values for LBSAM2, the analysis results for the four low-boom concepts, and convergence errors for the four coupling variables in the multidisciplinary feasible (MDF) [27] optimization method used by LBSAM2. The detailed analysis results are compared to understand which performance metric makes one aircraft concept better than another in terms of market demand capture. Section IV.C shows that the range, cruise speed, and MTOGW could each become the dominant driver for higher demand under some unique condition.

A. Necessity of Using Market Demand Analysis for Conceptual Low-Boom Design

The possibility of simultaneous achievement of low-boom and mission performance goals was demonstrated in Refs. [18,19] using a block coordinate optimization method for minimizing the aircraft weight while satisfying Eqs. (1a) and (1b) with $\sigma_{\max} = 70$ PLdB. A set of minimum performance requirements was imposed, with the overland range and MTOGW as the analysis outputs instead of the design requirements. The overwater design range was required to be at least 3600 nmi to enable a nonstop flight between JFK and LHR. If a desired overland range is set as a design requirement with Eqs. (1a) and (1b) as the low-boom constraint, the formulated conceptual design problem might be infeasible. So, without a priori knowledge of the feasibility of the design requirements, the low-boom optimization problem formulation must use the overland range as an unconstrained output in order to always produce feasible designs. If the overland range of the generated low-boom concept is less than desirable, the option is to either reduce the overland cruise Mach for a longer overland range solution with the same low-boom constraint or increase the sonic boom ground noise limit ($\sigma_{\max}$) in Eq. (1b) using the trade method in Sec. III.C for a longer overland range. This could result in a variety of low-boom concepts that are not comparable using the typical metrics such as MTOGW, cruise Mach, fuel burn, and range. LBSAM2 has the capability of propagating these metrics to the level of economic metrics, with a consistent coupling of the involved disciplinary analyses, to guide favorable system-level trades for economic viability.

B. LBSAM2 Market Demand Analysis

Table 3 lists some of the required input data used by LBSAM2, which indirectly shows the level of modeling detail in LBSAM2. For example, the demand prediction assumes that there are seven flight test vehicles for certification of a novel supersonic aircraft bookkept as a part of the aircraft development cost paid by the airline; this number is based on the Concorde, which required six flight test vehicles to be produced prior to certification, and the assumption that it will be somewhat more complicated to certify a low-boom aircraft. The operating empty weight, maximum cruise speed, number of engines, engine thrust, and number of aircraft produced (based on the supersonic fleet assignment) are inputs for the RAND aircraft cost model [25,26] to compute the aircraft cost in 1998 dollars, which is converted to a 2016 base year dollar value based on the Consumer Price Index (CPI). The number of engines and engine thrust are inputs to compute the engine maintenance cost. The target PLdB in Table 3 provides an explicit relationship between the conceptual-level low-boom constraint for a supersonic transport aircraft and the assumed noise regulation for supersonic overland flight in the market demand analysis. In conceptual design, the limit ($\sigma_{\max}$) for PLdB($A_{\mathrm{e,r}}^{\mathrm{target}}$) in Eq. (1b) ultimately drives much of the OML design of the supersonic transport concept satisfying Eq. (1a). In general, a lower PLdB($A_{\mathrm{e,r}}^{\mathrm{target}}$) for $A_{\mathrm{e,r}}^{\mathrm{target}}$ satisfying Eq. (1a) makes the supersonic concept more easily able to achieve an acceptable sonic boom ground noise level as minor modifications of the OML are made in the preliminary design phase (see Ref. [19]). Computationally, the PLdB($A_{\mathrm{e,r}}^{\mathrm{target}}$) value is not used in LBSAM2, but it quantifies the low-boom constraint under which the supersonic overland flight is allowed in the market demand analysis.

Table 3 Some required input data for LBSAM2 analysis

Category	Data
Aircraft design data	MTOGW, operating empty weight, seating capacity, seat pitch, number of engines, engine thrust, and target PLdB value for undertrack ground noise level in the conceptual-level low-boom constraint at SOC
Aircraft performance data	Two sets of (cruise Mach, maximum range, fuel burn curve, and travel time curve) for overland and overwater flights, respectively
LCC model	Number of years in life cycle cost model for aircraft, interest rate for amortization cost of aircraft, and resale value of aircraft, dollar in the base year as unit of money
Supersonic ticket prices	Sonic-boom-restricted missions (for any supersonic transport), wind-optimal great circle routes (for a low-boom supersonic transport), fuel cost, VOT, profit margin for the aircraft manufacturer, and profit margin for the airline
Demand prediction	The base year of 2016, a future year, percentage of the required supersonic aircraft as spares for maintenance, and seven aircraft as flight test vehicles for certification

Table 4 Input values that define a future scenario for LBSAM2 analysis

Variable	Value	Variable	Value
Future year for equilibrium analysis	2045	VOT in 2045, 2016 $ per hour	200
Annual interest rate for 20-year aircraft loan	7%	Runway length for airport, ft	≥ 8300
Number of years in LCC model for aircraft	20	Min distance between OD pair, sm	1000
Resale value of aircraft vs aircraft cost	10%	Min number of flights per day for OD pair	1
Profit margin for the aircraft manufacturer	15%	Number of days for flights per week	5
Profit margin for the airline	15%	Number of days for aircraft to return to base	2
Fuel price in 2045, 2016 $ per gallon	3.5	Spare aircraft for maintenance operations	10%

The assumed input values for the LBSAM2 analysis of the four low-boom aircraft concepts in Table 2 are listed in Table 4. These values define a future economic and operational scenario for the equilibrium analysis of the supersonic transportation market. The fuel price of 3.50 (2016 $)/gal in 2045 is a conservative assumption compared to the US Energy Information Administration's predicted value of 2.92 (2022 $)/gal for jet fuel in 2045 [34], which is about 2.39 (2016 $)/gal (when converted based on the CPI). The VOT value in Table 4 is based on Ref. [6], which surveyed multiple published VOT studies. The required runway length for the low-boom concepts in Refs. [18,19] is approximately 8300 ft, which is used as a minimum value for LBSAM2 to select the candidate OD pairs. The choice of 1000 statute miles as the minimum distance for the selected OD pairs ensures that there will be sufficient cruise segment length for supersonic flight to provide some time saving benefits to passengers (see Eq. (4)). The five flight days per week assumes that most supersonic passengers travel for business and only fly on weekdays.[4] The other assumed values for the inputs in Table 4 have significant effects on the market demand prediction but marginal effects on the relative economic advantages among the four low-boom aircraft concepts.

LBSAM2 uses four coupling variables (N_{aircraft}, annual revenue and non-revenue flight hours per aircraft, and load factor per flight) [6] for an MDF optimization method [27] to consistently model the interdependencies between the key variables in the LCC analysis of the aircraft, the revenue and profit analysis, the analysis of demand for supersonic flight, and the optimal fleet assignment for airline fleet productivity. The iteration steps of the MDF optimization method in LBSAM2 can be summarized as follows:

(IV.1) A set of values for the four coupling variables is initially assumed for the first iteration; in subsequent iterations, these values are updated with the predicted values from the previous iteration.
(IV.2) C_{flight} and C_{hour} are independent of OD pairs and computed by the aircraft LCC model using the values of the four coupling variables, aircraft data in Table 3, and input values in Table 4.
(IV.3) For each OD pair, C_{sup} = [1 + (airline profit margin)] · [1 + (US excise tax rate)] · [C_{flight} + $C_{\text{hour}} \cdot T_{\text{sup}}$ + FW_{sup} · (fuel price in $/lb)] / [(load factor for the future year) · (seating capacity of aircraft)].
(IV.4) N_{sup} for each OD pair is determined by C_{sup}, the subsonic premium ticket price distribution for the OD pair, the passenger preference model [Eq. (4)], and the air travel demand prediction model.
(IV.5) The values of the four coupling variables are predicted by LBSAM2 after solving a fleet assignment problem of transporting the supersonic passengers for all the OD pairs as efficiently as possible, with constraints for curfew time, operating time for any intermediate stop, and each aircraft returning to its base airport within two days [6].
(IV.6) If the assumed and predicted values of each coupling variable differ by less than a specified error tolerance, terminate the iteration; otherwise, go back to (IV.1).

In this paper, the coupling consistency of the LBSAM2 analysis results is ensured by the following two convergence criteria:

- The assumed and predicted values of N_{aircraft} are identical.
- The relative errors between the assumed and predicted values of the coupling variables are less than 0.1%.

Multiple runs of LBSAM2 with different sets of initial values for the four coupling variables are required to obtain an LBSAM2 solution satisfying the relatively strict convergence criteria specified above, starting with a relatively high initial guess of N_{aircraft}. Figure 4 shows the relative iteration errors of the four coupling variables for the final LBSAM2 solutions for the four concept aircraft.

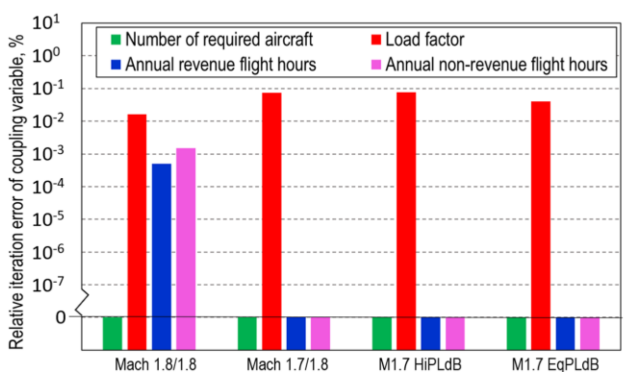

Fig. 4 Relative LBSAM2 iteration errors of the four coupling variables for the four low-boom concepts.

[4] Having five instead of seven days for flights per week technically increases the daily passenger demand because LBSAM2 computes the number of passengers per year and then divides this yearly number by the number of operational days to determine the daily passenger demand. The assumption of five days per week for flights affects the demand prediction, but it provides a common point of comparison among the four low-boom concepts in terms of their economic merits.

It is worth noting that LBSAM2 achieves coupling consistency among the involved disciplinary analyses—specifically the supersonic market demand analysis, FLOPS mission analysis, and conceptual-level low-boom constraint [Eq. (1a)]—with a mixture of MDO architectures. The coupling consistency within the market demand analysis is achieved by the MDF method outlined in (IV.1)-(IV.6). The coupling consistency between the FLOPS mission analysis and low-boom constraint [Eq. (1a)] is enforced by the individual discipline feasible (IDF) [27] architecture using two sets of travel time and fuel burn data for purely overland and overwater routes, with the travel time adjusted for wind effects. For a route with mixed overland and overwater segments, the coupling consistency between the FLOPS mission analysis and low-boom constraint is approximately achieved with the IDF architecture using the approximation formulas in Eqs. (5) and (6). The coupling consistency between the market demand analysis and FLOPS mission analysis is approximately ensured using the IDF architecture, in which (IV.3) is used to compute C_{sup}, and Eqs. (5), (7a), (7b), and (6) are used to compute FW_{sup} and T_{sup} for each OD pair. So, the approximately accurate data for the mixed mission analysis are used to compute the supersonic ticket price.

C. Analysis Results and Discussion

Table 5 lists the market demand analysis results for the four concept aircraft introduced earlier. Ultimately, the Mach 1.8/1.8 aircraft is predicted to have the highest load factor and the largest number of aircraft, OD pairs served, and flight hours, as well as the lowest unit aircraft price and average ticket cost per nmi. In the following discussion, we analyze these results and show that range, cruise speed, and MTOGW could each become the dominant driver for higher demand under some unique condition.

Table 5 Market demand analysis results in 2045 for four low-boom concepts

Label for concept	Mach 1.8/1.8	Mach 1.7/1.8	M1.7 HiPLdB	M1.7 EqPLdB
Number of required aircraft (including spares)	560	376	432	355
Number of OD pairs with at least one daily flight	646	402	458	368
Number of supersonic passengers per year, million	18.6	11.0	12.4	10.2
Aircraft unit price, 2016 $ in million	169.3	186.4	179.8	191.4
Weighted average ticket cost per nmi, 2016 $/nmi	0.9382	1.0333	1.0111	1.0436
Annual revenue flights hours per aircraft	2602.2	2397.9	2399.5	2348.7
Annual non-revenue flights hours per aircraft	108.1	114.5	133.3	118.0
Load factor for 2045	0.816	0.801	0.799	0.805

Prior to conducting the analysis, we did not expect that the Mach 1.8/1.8 concept would have 69% more passenger demand than the Mach 1.7/1.8 concept. Compared with the Mach 1.8/1.8 concept, the Mach 1.7/1.8 concept has a lower MTOGW and decreased fuel burn (see Table 2 and Fig. 3b). So, the economic advantage of the Mach 1.8/1.8 concept must come from its range capability and/or overland cruise speed.

To help understand why the Mach 1.8/1.8 aircraft captured a greater number of passengers than the Mach 1.7/1.8 aircraft, we analyzed results at the OD level. Selected OD-level analysis results are listed in Table 6. These results include OD pairs between which one aircraft must make a refueling stop, neither aircraft must make a refueling stop, or both aircraft must make a refueling stop.[5] The range value listed in the table represents the full distance traveled from origin to destination including any additional distance required for a refueling stop. The refueling stop airport is indicated by its airport identifier, and dash (—) in the table indicates no refueling stop was required. The VOT_{OD} parameter combines the time saved by flying supersonically (compared to a subsonic flight) and the value of time for passengers on the flight to yield a monetary value for the value of time saved.[6]

Although curfew constraints and a 60-minute time gap at each intermediate stop are modeled for each aircraft in LBSAM2 (see Sec. IV of Ref. [6]), no other air traffic control or airport-related constraints are included, which can lead to some unrealistic results. For example, Table 6 shows 38 flights in two days from JFK to LHR for the Mach 1.8/1.8 concept, which equates to approximately 1 to 2 flights per hour. Such frequent flights are not likely feasible due in part to airport throughput limits, and these results indicate that a supersonic transport with a larger seating capacity than 50 would be beneficial for this route. Despite these limitations, the relative comparisons among the

[5] LBSAM2 analysis allows OD pairs with up to one refueling stop and models this stop as a 60-minute time penalty without boarding or debarkation [6].
[6] VOT_{OD} is defined by the right-hand side of Eq. (3).

various concept aircraft are helpful to discern what design characteristics are likely beneficial in low-boom supersonic aircraft.

Table 6 OD-level analysis results for the Mach 1.7/1.8 and Mach 1.8/1.8 concepts

OD pair	SYD to HKG		JFK to LAX		JFK to LHR		LHR to LAX	
Short concept label	1.8/1.8	1.7/1.8	1.8/1.8	1.7/1.8	1.8/1.8	1.7/1.8	1.8/1.8	1.7/1.8
Refueling stop	—	DPS	—	—	—	—	BDA	BDA
Range, nmi	3952	4292	2120	2120	2981	2981	5602	5602
τ, %	44	44	100	100	20	20	53	53
N_{flight}	6	0	10	8	38	28	5	4
N_{sup}	23,700	3,000	60,300	47,100	258,000	191,400	32,700	27,000
N_{pax}	23,700	0	60,300	47,100	258,000	191,400	32,550	27,000
N_{sub}	312,444	312,444	471,168	471,168	781,486	781,486	317,769	317,769
T_{sub}, hr	9.43	9.43	6.29	6.29	7.03	7.03	11.27	11.27
T_{sup}, hr	4.81	7.14	3.02	3.14	3.76	3.79	8.19	8.40
VOT_{OD}, 2016 $	922.39	457.61	654.24	629.09	655.04	648.41	615.54	573.09
FW_{sup}, lb	72,315	70,950	36,644	34,430	51,397	50,106	97,226	93,061
C_{nmi}, 2016 $/nmi	0.8648	1.0011	0.9118	1.0091	0.8696	0.9550	0.8781	0.9658
C_{sup}, 2016 $	3417.30	4296.53	1933.24	2139.71	2591.80	2846.50	4920.16	5411.00

1. Range Matters

A careful inspection of the selected OD-level analysis results in Table 6 provides some insight into the economic characteristics of these two aircraft concepts. In comparison with the Mach 1.8/1.8 concept, the most obvious demand loss for the Mach 1.7/1.8 concept is the SYD-to-HKG OD pair. The Mach 1.7/1.8 aircraft cannot complete a nonstop flight between these airports, and the refueling stop requires a significant detour; these factors eliminate the passenger demand on this route for this aircraft. This demand loss reduces the number of required aircraft and increases C_{nmi}.

This knowledge from the OD-level analysis results led us to develop the M1.7 HiPLdB concept, which eliminates the range disadvantage of the Mach 1.7/1.8 concept compared to the Mach 1.8/1.8 concept. The longest range OD pair for which the Mach 1.8/1.8 concept has a nonstop flight is from LHR to IAH with a range of 4195 nmi. So, the M1.7 HiPLdB concept was generated with an overwater range of 4200 nmi. Even though the M1.7 HiPLdB concept is heavier and less fuel efficient than the Mach 1.7/1.8 concept (see Table 2 and Fig. 3b), its supersonic passenger demand is 12.4 million in 2045, 12.7% higher than that of the Mach 1.7/1.8 concept as shown in Table 5. So, range matters.

2. Speed Matters

The M1.7 HiPLdB concept has a nonstop flight for every OD pair that the Mach 1.8/1.8 concept has a nonstop flight. Moreover, the M1.7 HiPLdB concept is better than the Mach 1.8/1.8 concept in terms of MTOGW and fuel burn (see Table 2 and Fig. 3b). So, it is unexpected to see in Table 5 that the Mach 1.8/1.8 concept has a 50% higher passenger demand than the M1.7 HiPLdB concept (18.6 million passengers compared to 12.4 million passengers for 2045).

To help determine the reasons for this large difference in passenger demand, Table 7 shows results for four representative OD pairs predicted to be flown by the M1.7 HiPLdB and Mach 1.8/1.8 aircraft. First, compare the values of VOT_{OD}, which is a typical means of assessing the value of increased cruise speed. There are relatively small differences between the two aircraft concepts across all OD pairs; the Mach 1.8/1.8 concept has only approximately 1% to 4% larger VOT_{OD} for these OD pairs. However, the number of passengers predicted to be served by the Mach 1.8/1.8 concept is increased by approximately 10% to 31% over the M1.7 HiPLdB concept. This indicates that the effect of speed cannot adequately be measured by considering the VOT_{OD} parameter alone, as is often done.

Another important economic driver that cruise speed affects is fleet productivity. The revenue passenger miles (RPM) per year for one aircraft in the fleet provide one metric for fleet productivity. RPM are the product of the number of passengers carried and the distance traveled by those passengers. The seating capacities, average cruise speeds (instead of average travel speeds), and the fleet productivity parameters in Table 5 (annual revenue flight hours and load factor) can be used for a rough estimate of the ratio between the RPM per aircraft in 2045 for these two concepts.[7]

[7] The average cruise speed ratio is used instead of the average travel speed ratio to estimate the RPM ratio. From LBSAM2's analysis data, the M1.7 HiPLdB fleet's average cruise Mach for all flights in 2045 is about 1.78.

$$\frac{\text{RPM in 2045 for one Mach 1.8/1.8 aircraft}}{\text{RPM in 2045 for one M1.7 HiPLdB aircraft}} \approx \frac{1.8 \cdot (\text{speed of sound}) \cdot 2602.2 \cdot 0.816 \cdot 50}{1.78 \cdot (\text{speed of sound}) \cdot 2399.5 \cdot 0.799 \cdot 50} = 1.12$$

Table 7 OD-level analysis results for the Mach 1.8/1.8 and M1.7 HiPLdB concepts

OD pair	ATL to MEX		JFK to LAX		JFK to LHR		LHR to SEA	
Short concept label	1.8/1.8	HiPLdB	1.8/1.8	HiPLdB	1.8/1.8	HiPLdB	1.8/1.8	HiPLdB
Refueling stop	—	—	—	—	—	—	—	—
Range, nmi	1145	1145	2120	2120	2981	2981	4173	4173
τ, %	33	33	100	100	20	20	61	61
N_{flight}	2	2	10	8	38	28	4	4
N_{sup}	12,900	11,700	60,300	48,600	258,000	197,400	21,600	18,600
N_{pax}	12,900	11,700	60,300	48,600	258,000	197,400	21,600	18,600
N_{sub}	44,507	44,507	471,168	471,168	781,486	781,486	101,251	101,251
T_{sub}, hr	3.65	3.65	6.29	6.29	7.03	7.03	9.90	9.90
T_{sup}, hr	2.11	2.13	3.02	3.14	3.76	3.79	4.87	5.01
$\mathrm{VOT_{OD}}$, 2016 $	309.24	304.8	654.24	629.09	655.04	648.41	1006.45	978.55
FW_{sup}, lb	20,401	20,136	36,644	34,779	51,397	50,605	78,053	74,459
C_{nmi}, 2016 $/nmi	1.0441	1.1457	0.9118	0.9952	0.8696	0.9428	0.8692	0.9360
C_{sup}, 2016 $	1195.09	1311.33	1933.24	2110.17	2591.80	2810.17	3627.22	3906.12

The ultimate passenger demand increase from a cruise-speed-induced fleet productivity improvement of 12% depends on the geographic distribution of supersonic passengers and scheduling constraints. Furthermore, changes to other model inputs or assumptions, such as a passenger preference model change, a different base year, inclusion of additional scheduling constraints, or a passenger demand reduction with a fuel price increase, could make the fleet productivity advantage for overland cruise Mach 1.8 much less dramatic (see Sec. V.C). Regardless, these results indicate that aircraft cruise speed can have significant impacts on the fleet productivity that can, in turn, create large impacts on passenger demand.

To indirectly verify the impact of speed on the fleet productivity, the overland cruise Mach of the M1.7 HiPLdB concept was artificially replaced with Mach 1.8 without changing other weight and performance data. The resulting concept is called the fake Mach 1.8 concept. The LBSAM2 analysis results of the fake Mach 1.8 concept show that the fake Mach 1.8 concept has 590 aircraft and 19.5 million passengers for the equilibrium state in 2045, with annual revenue flight hours of 2612 and a load factor of 0.819. The RPM for one fake Mach 1.8 aircraft are slightly higher than the RPM for one Mach 1.8/1.8 aircraft, due to demand increase. Compared with the Mach 1.8/1.8 concept, the better fuel efficiency and decreased weight of the fake Mach 1.8 concept lead to a 4.8% increase in passenger demand. The assumed overland cruise Mach increase of 0.1 for the fake Mach 1.8 concept results in a 57.3% passenger demand increase compared to the M1.7 HiPLdB concept. Even though this is a made-up example, it confirms the drastic impact of a 0.1 increase in overland cruise Mach on the fleet productivity and passenger demand. So, speed matters.

3. *Weight Matters*

To see whether an increased overwater range alone is economically beneficial when the overland range is reduced to satisfy the same low-boom constraint, we compare the analysis results for the M1.7 EqPLdB concept to those for the Mach 1.7/1.8 concept. The results in Table 5 show a reduction of 7.3% in the passenger demand for the M1.7 EqPLdB aircraft when compared with the Mach 1.7/1.8 concept. Table 8 has some OD-level analysis results for these two concepts. If an OD pair requires a refueling stop for the Mach 1.7/1.8 concept but has a nonstop flight for the M1.7 EqPLdB concept, such as the SEA-to-NRT route in Table 8, C_{sup} for the M1.7 EqPLdB concept is cheaper than that for the Mach 1.7/1.8 concept; so, the M1.7 EqPLdB concept has a higher passenger demand than the Mach 1.7/1.8 concept for such an OD pair. On the other hand, for OD pairs that the M1.7 EqPLdB concept has no range advantage (such as the other three OD pairs in Table 8), the higher MTOGW of the M1.7 EqPLdB concept makes C_{nmi} higher, due to the detrimental effects of weight growth on fuel burn and aircraft unit cost.

In general, there is no a priori answer to which concept is economically better for a trade between weight and overwater range. The winning concept depends on the passenger demand distribution and the weight penalty for extending the overwater range. LBSAM2 provides a means of assessing the economic differences between the two low-boom concepts based on the specified economic scenario, assumptions, and existing air travel data. In this case, for the higher weight M1.7 EqPLdB concept, the passenger demand reduction for the OD pairs that have nonstop flights for the Mach 1.7/1.8 concept cannot be offset by the passenger demand increase from extending overwater range to 4200 nmi. This is an example showing that the weight matters.

Table 8 OD-level analysis results for the Mach 1.7/1.8 and M1.7 EqPLdB concepts

OD pair	SEA to NRT		JFK to LAX		JFK to LHR		LHR to LAX	
Short concept label	1.7/1.8	EqPLdB	1.7/1.8	EqPLdB	1.7/1.8	EqPLdB	1.7/1.8	EqPLdB
Refueling stop	ANC	—	—	—	—	—	BDA	BDA
Range, nmi	4179	4128	2120	2120	2981	2981	5602	5602
τ, %	3	3	100	100	20	20	53	53
N_{flight}	0	2	8	8	28	26	4	4
N_{sup}	6900	13,500	47,100	44,700	191,400	188,100	27,000	26,400
N_{pax}	0	13,500	47,100	44,700	191,400	188,100	27,000	26,400
N_{sub}	52,353	52,353	471,168	471,168	781,486	781,486	317,769	317,769
T_{sub}, hr	10.37	10.37	6.29	6.29	7.03	7.03	11.27	11.27
T_{sup}, hr	6.92	4.85	3.14	3.14	3.79	3.79	8.40	8.36
VOT_{OD}, 2016 $	690.68	1103.90	629.09	629.09	648.41	648.41	573.09	581.04
FW_{sup}, lb	71,124	73,868	34,430	34,779	50,106	50,605	93,061	94,043
C_{nmi}, 2016 $/nmi	1.0081	0.9386	1.0091	1.0393	0.9550	0.9604	0.9658	0.9809
C_{sup}, 2016 $	4212.56	3875.01	2139.71	2203.58	2846.50	2862.60	5411.00	5495.25

V. Discussion of Uncertainties in Market Demand Analysis

This section provides a brief discussion of uncertainties associated with the market demand analysis results from LBSAM2. The typology for uncertainty in decision support described in Ref. [35] is used. This typology defines each uncertainty with the three attributes of location, nature, and level, which aim for a precise technical description of the uncertainty. We discuss the uncertainties in terms of their locations in the analysis process in the following four subsections, focusing on their impacts on the relative economic advantages among low-boom aircraft concepts. Two analysis examples are included in Secs. V.A and V.C to illustrate potential decision support applications of LBSAM2.

A. Context Uncertainty

Any market demand analysis will have context uncertainty that involves the inability to predict the impacts of external economic, environmental, political, social, and technological changes on the demand. Table 9 contains several scenarios and their associated contexts as an example of what future unknown events could impact a supersonic transport market demand analysis. These context uncertainties are defined at the level of total ignorance [35], meaning none of them could be accurately quantified a priori using either a mathematical or statistical model. However, the listed context uncertainties in Table 9 can be used to determine plausible values of uncertain inputs for scenario analysis.

Table 9 Uncertain future scenarios affecting supersonic market demand

Scenario	Context
Subsonic premium passengers resist supersonic travel due to environmental concerns.	environmental, social
Subsonic airlines cut premium ticket prices to defend the existing market shares.	economic
Regulation is made to impose punitive fees for commercial supersonic overland flight.	political, economic
Government provides substantial incentives for developing commercial supersonic transports to strengthen the nation's technology infrastructure in aviation.	political, economic
Trade disputes or wars completely change how an airline serves the global market.	political, social

Technology development could also have a significant influence on the economic viability of a supersonic transport. Some plausible technologies include laminar flow for drag reduction, engine technology for cruise efficiency and airport noise reduction, and low-boom design technology. Any breakthrough in these technologies will profoundly affect the performance of a future low-boom supersonic aircraft and its economic viability. If a technology is well understood, it is possible to use a model-based analysis method for assessment of the economic impact of the technology. For example, based on the natural laminar flow assessment of a low-boom supersonic concept in Ref. [36], it may be plausible to increase the laminar flow on the upper surface of the wing of the Mach 1.8/1.8 concept by 40%. For a quick technology benefit analysis, it is assumed that the wing is unchanged, but the 40% increase in natural laminar flow on the upper surface of the wing is applied using an empirical skin friction model [37] to derive an improved Mach 1.8 concept. The MTOGW of the improved Mach 1.8 concept is reduced to 166,790 lb so that the improved Mach 1.8 concept has the same weight as the Mach 1.8/1.8 concept at SOC. These two concepts have the same OML and cruise weight at SOC; so, they have the same AoA at SOC and satisfy the same low-boom constraint [Eq. (1a)]. The improved Mach 1.8 concept has an overland range of 4332 nmi and an overwater range of 4536 nmi. The low-fidelity *L*/*D* ratio at SOC is increased from 7.06 for the Mach 1.8/1.8 concept to 7.38 for the improved Mach 1.8 concept. The predicted market demand for the improved Mach 1.8 concept in 2045 using LBSAM2 shows a total of 610 aircraft serving 20.6 million passengers. So, laminar flow technology could potentially increase the passenger demand in 2045 by 10.7% for the Mach 1.8/1.8 concept. This example demonstrates the capability of LBSAM2 to quantify the market sensitivity for a particular technology development.

B. Model Uncertainty

There are two types of model uncertainty: model structure uncertain and model technical uncertainty. Model structure uncertainty is due to "a lack of sufficient understanding of the system (past, present, or future) … including the behavior of the system and the interrelationships among its elements." [35] The model structure uncertainty for LBSAM2 is mainly Eq. (4) as a passenger preference model. Some factors influencing passenger choices, such as timing of flight and vying for prestige [4,11], are not modeled in Eq. (4).

Model technical uncertainty is "generated by software or hardware errors." [35] The model technical uncertainty for LBSAM2 is in the *passenger willingness-to-pay* curve, VOC model, passenger demand forecast model, and software errors. The passenger willingness-to-pay curve in LBSAM2 is based on the subsonic premium ticket price distribution in 2016. The implicit assumption is that the future passenger willingness-to-pay curve is approximately the same as that of the base year. The uncertainty in this assumption depends on the airline business model and passenger behavior in the future year. This model uncertainty increases with the time lapse between the base and future years. For decision support analysis, the most recent air travel data should be used to generate the ticket price data for the base year. The passenger demand forecast model is sensitive to disruptive events such as the COVID-19 pandemic. So, it is important to use the past dataset which is most representative of the expected future.

C. Input Uncertainty

Any input to the LBSAM2 model is subject to some level of uncertainty. The fuel price, VOT, interest rate, and profit margins are examples of uncertain inputs related to future economic, social, and financial conditions. As another example, an acceptable PLdB value for future regulation on supersonic overland flight depends on human behavior and political dynamics.

A standard method for understanding the impact of input uncertainty is scenario analysis, which involves varying the values of uncertain inputs to define potential future scenarios. These scenarios enable sensitivity analysis of model outputs by using plausible values of the uncertain inputs, typically by creating themes around which multiple input parameters can be jointly adjusted in coordination with one another.

To provide a simple example, one future scenario of interest could involve a mandate that all supersonic aircraft flying in 2045 must be fueled by some form of sustainable aviation fuel (SAF). If we assume that the fuel source for the supersonic concept aircraft is e-Kerosene, a zero-carbon SAF, the fuel price is likely to increase considerably from the baseline assumption. Assuming a future fuel price of 4.3 (2016 $)/gal[8] and that the aircraft's performance characteristics do not change when jet fuel is replaced with e-Kerosene, LBSAM2's demand prediction for the Mach 1.8/1.8 and M1.7 HiPLdB concepts is 217 and 200 aircraft[9] serving 5.9 and 5.1 million PAX in 2045, respectively. Compared with the M1.7 HiPLdB concept, the Mach 1.8/1.8 concept has only a 15.7% passenger demand increase with a fuel price of 4.3 (2016 $)/gal, instead of the 50% passenger demand increase with a fuel price of 3.5 (2016 $)/gal as we discussed previously. These results indicate that these two low-boom concepts are still potentially economically viable in a scenario with mandated SAF usage in 2045, even though the fuel cost increase of 22.9% leads to 68.3% and 58.9% passenger demand reductions compared to the baseline case for the Mach 1.8/1.8 and M1.7 HiPLdB concepts, respectively.[10]

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[8] A SAF price of 4.3 (2016 $)/gal in 2045 is a conservative assumption compared to the forecasted 2045 average e-Kerosene price of approximately 4.7 (2023 $)/gal in the US predicted by Ref. [38] (see Fig. 2 therein), which is about 3.7 (2016 $)/gal (when converted based on the CPI).

[9] Note that the number of aircraft is just for the equilibrium state in 2045, not for the full production life cycle of the aircraft.

[10] In this case, due to the passenger demand reductions, the Mach 1.8/1.8 and M1.7 HiPLdB fleets have annual revenue flight hours of 2441.2 and 2219.5 hr per aircraft and load factors of 0.772 and 0.773, respectively. The RPM for the Mach 1.8/1.8 fleet are 4.3% higher than the RPM for the M1.7 HiPLdB fleet.

D. Parameter Uncertainty

Parameters are constants in the model, supposedly invariant within the chosen context and scenario [35]. The most obvious parameter uncertainty is in the regression parameters of the aircraft cost model [25,26]. The parameter uncertainty has a significant effect on the predicted market demand, but it has a marginal effect on the relative economic advantages between the low-boom concepts in this paper because these low-boom concepts have no significant differences in weights and engine thrust, which are the drivers for the aircraft cost.

VI. Summary and Conclusions

A mathematical framework is introduced to illustrate that the interdependencies among key variables in the equilibrium analysis of the supersonic transportation market are consistently modeled by LBSAM2. Numerical examples are used to demonstrate that LBSAM2 can be integrated with conceptual low-boom design for a consistent coupling of market demand analysis, detailed mission analysis, and a low-boom constraint. Such an integration enables the propagation of performance characteristics for a supersonic transport to the level of economic metrics, such as the number of aircraft needed to meet the market demand and the size of the market. This allows an unbiased comparison of the economic advantages of various supersonic transport concepts, enables a scientific (in addition to knowledge-based) down-select process for a future supersonic transport aircraft when multiple low-boom supersonic transport concepts are developed, and provides model-based analysis of the economic impact of a future regulation for acceptable sonic boom ground noise level to support regulatory decisions on supersonic overland flight.

Multiple useful conclusions can be drawn from the LBSAM2 analysis results in this paper. First, it might be beneficial to use the overland range instead of weight as the primary performance objective in MDO for achieving both low-boom and mission performance goals. An increased overland range can enable a low-boom supersonic aircraft to serve additional OD pairs with nonstop flights compared to a lighter, shorter-range version. Our LBSAM2 analysis indicates that the higher per-mile operational costs of the heavier aircraft can be overcome, at least in some cases, by the increased addressable market size and economies of scale enabled from opening up additional OD pairs with nonstop flights. Low-boom supersonic aircraft designers are encouraged to explore the design space to determine advantageous break points in the tradeoff between gross weight and overland range with the guidance of market analysis; our study indicates that results may be non-intuitive.

Second, the overland cruise speed of a low-boom supersonic transport aircraft may have significant impacts on the fleet productivity and the resulting passenger demand. Further study is required to understand how significant these impacts would be for differing cruise speeds, potential OD pairs, and other modeling assumptions. However, we have demonstrated that the typical means of assessing the benefits of cruise speed among aircraft concepts with the VOT_{OD} metric is insufficient to predict market demand impacts under at least some set of reasonable inputs and assumptions. Close coordination among supersonic aircraft manufacturers, airlines, and other stakeholders may be required to adequately model the fleet productivity gains that can be practically realized from increased overland cruise speed to determine specific design cruise speed values for low-boom supersonic transport aircraft.

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