

Updating High-Order Aeroservoelastic Models from Low-Order System Identification Results

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Estimating aircraft models from test data requires several simplifying assumptions that introduce biases into the parameter estimates. In this paper, these biases are defined and a method for estimating the biases is discussed. Having an estimate of the bias allows the parameters estimated from test to be integrated into a high-order model. The Integrated Adaptive Wing Technology Maturation (IAWTM) wind tunnel model is discussed and the bias is demonstrated for one of the testing configurations. The methodology was able to estimate these biases and apply corrections to high-order aeroelastic models to improve the fit to test data. The consideration of the biases allows more meaningful comparisons and avoids the erroneous differences between pretest predictions and the fitted model.

Nomenclature

$\mathbf{B}$	input matrix
$\mathbf{B}^0$	steady generalized aerodynamic input coefficients
$\bar{c}$	mean aerodynamic chord
$\mathbf{C}$	damping matrix
$\mathbf{D}$	feed-forward matrix
$\mathbf{H}(\omega)$	aeroelastic frequency response function
$\mathbf{K}$	stiffness matrix
k	reduced frequency
$\mathbf{K}_{bias}^0$	steady aerodynamic coefficient bias corrections
$\mathbf{K}_{corr}^0$	steady aerodynamic coefficient correction factors
$\mathbf{K}_{bias}^1$	quasi-steady aerodynamic coefficient bias corrections
$\mathbf{K}_{corr}^1$	quasi-steady aerodynamic coefficient correction factors
$\tilde{k}$	reference reduced frequency
$\mathbf{M}$	mass matrix
$\mathbf{q}$	generalized modal coordinate
$\mathbf{Q}(k)$	generalized aerodynamic forces
$\mathbf{Q}^0$	steady generalized aerodynamic coefficients
$\mathbf{Q}^1$	quasi-steady generalized aerodynamic coefficients
$\mathbf{Q}^2$	added mass generalized aerodynamic coefficients
$\mathbf{Q}^j$	aerodynamic lag generalized aerodynamic coefficients
$\bar{q}$	dynamic pressure
$\mathbf{u}$	input signals
V_T	true airspeed
$\mathbf{z}$	output signals
γ_j	aerodynamic lag poles
Φ	mode shape matrix
ω	frequency
$\tilde{\omega}$	reference frequency
<i>Subscripts</i>	
d	deleted modes
r	retained modes

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Symbols

$\triangleq$	equal by definition
$\oslash$	Hadamard (elementwise) division
$\odot$	Hadamard (elementwise) multiplication
imag	imaginary part of the complex number
$\hat{}$	quasi-steady estimates
$\bar{}$	reduced order matrix
real	real part of the complex number

I. Introduction

THE verification of airworthiness criteria for flight control laws requires consideration of the coupling between the commands from the control system and the vehicle structure which leads to the field of aeroservoelasticity (ASE). The presence of flexibility in the structure of an aircraft represents an important limitation in the design of aircraft flight controls. The airworthiness criteria, such as gain and phase margins, are typically verified using computational models [1]. These computational models typically use relatively low-fidelity methods [2, 3] and can vary significantly from test results. The ability to update computational models can significantly improve the efficiency and the likelihood of a successful test [4]. Additionally, actively controlling the structure, in contrast to showing passive stability as in rigid-body control, can increase damping or reduce loads from maneuvers and gusts [5, 6].

The computational models used in ASE analysis [7–10] are very high-order, and the models either have frequency dependent aerodynamic coefficients or additional aerodynamic lag states due to unsteady aerodynamics. In contrast, creating models from test data, system identification, requires relatively simple low-order models.

System identification methods [11, 12] have been developed that are very effective at creating dynamic models from test data. These methods are commonly used in updating the aerodynamic parameters associated with rigid aircraft. However, such identification methods always require approximations and compromises in the model fidelity and the number of parameters estimated. For rigid aircraft models, the system identification analysis typically considers only the short period mode or the roll and Dutch roll modes. These identification methods typically neglect the lower frequency phugoid and spiral modes. Depending on the vehicle, such as a statically unstable aircraft, the omission of these lower frequency modes can introduce significant bias in the estimated parameters. The problem is even more significant when trying to extend the system identification to an aircraft model that considers the flexibility [13]. The higher order flexible models have thousands of parameters, which cannot be consistently estimated from test data. Successful attempts to identify models from test data [14–16] have generally used low-order models, with few structural degrees of freedom. Grauer and Boucher [15] showed that there can be a significant bias in the coefficients due to omitted modes. To avoid the bias due to omitted modes, Danowski et al. [16] performed the updating on a model with additional higher order modes that were not updated. However, these models were still of a lower order than typically used in ASE analysis.

Additionally, the successful application of system identification to model updating has used quasi-steady aerodynamics such that the coefficients are constant with the frequency of the inputs. In defining the aerodynamic lag states, Roger [7] suggests that the aerodynamic lags are not critical for a single flight condition. A quasi-steady model can be effective at a single flight condition used in the system identification. However, in order for coefficients to be used for flutter and ASE analyses at a different airspeeds, consideration of the aerodynamic lags is essential.

A method is needed to integrate results from these low-order, quasi-steady system identification models into the higher order analysis models with flexibility and low-frequency rigid body modes. First, Section II provides the general mathematical definition of the problem. Then Section III derives the relationship between the identified models and the analysis models and the means of relating them. In Section IV, the Integrated Adaptive Wing Technology Maturation (IAWTM) wind tunnel model and testing is described. In Section V, the method is used to update IAWTM models. Finally, Section VI discusses the current proposed method.

II. Problem Definition

The general form of the second-order equations of motion for an aeroelastic system in the nondimensional frequency domain is shown in Eq. (1a) [17]. When using the actuator modeling method of [8, 18], the actuators are additional modes and the input matrix, $\mathbf{B}$, is a constant. Equation (1b) represents the sensor model.

$$[\mathbf{H}(\omega)] \{\mathbf{q}\} = [\mathbf{B}] \{\mathbf{u}\} \quad (1a)$$

$$\{\mathbf{z}\} = [\Phi] \{\mathbf{q}\} \quad (1b)$$

where,

$$[\mathbf{H}(\omega)] \triangleq -[\mathbf{M}] \omega^2 + [\mathbf{C}] i\omega + [\mathbf{K} - \bar{q}\mathbf{Q}(k)] \quad (2a)$$

$$k \triangleq \frac{\omega \bar{c}}{2V_T} \quad (2b)$$

When considering only symmetric or anti-symmetric motions of a rigid aircraft, there are typically about two modes and about ten aerodynamic coefficients. In contrast, the models used in aeroelastic and aeroservoelastic analyses typically include on the order of tens of modes and on the order of thousands of coefficients. Practically speaking, it is impossible to identify that many coefficients. Furthermore, the unsteady aerodynamic forces, $\mathbf{Q}(k)$, are a function of reduced frequency. High-order state-space models, such as [7, 8, 19], require some rational function approximation (RFA), such as Eq. 3, for the unsteady aerodynamics.

$$[\mathbf{Q}(k)] \approx [\mathbf{Q}^0] + ik[\mathbf{Q}^1] - k^2[\mathbf{Q}^2] + \sum_j \frac{ik}{ik - \gamma_j} [\mathbf{Q}^j] \quad (3)$$

At best, capturing the unsteadiness will further increase the number of parameters. Even worse, the aerodynamic lag states are often unobservable and not identifiable from a test point at a fixed airspeed. System identification such as in [15, 16] has typically used a quasi-steady approximation, Eq. (4), for the generalized aerodynamic forces (GAF). The matrices, $\mathbf{Q}^0$ and $\mathbf{Q}^1$, are independent of reduced frequency.

$$[\mathbf{Q}(k)] \approx [\mathbf{Q}^0] + ik[\mathbf{Q}^1] \quad (4)$$

Under the quasi-steady and reduced order approximations, the second-order equations of motion used in system identification of aeroelastic systems are typically of a form equivalent to:

$$\left(-[\mathbf{M}_{rr}] \omega^2 + \left[\mathbf{C}_{rr} - \bar{q} \frac{\bar{c}}{2V_T} \widehat{\mathbf{Q}}^1_{rr} \right] i\omega + [\mathbf{K}_{rr} - \bar{q} \widehat{\mathbf{Q}}^0_{rr}] \right) \{\mathbf{q}_r\} = [\widehat{\mathbf{B}}_r] \{\mathbf{u}\} \quad (5)$$

The quasi-steady approximation is typical for rigid-body analysis, but can significantly limit the accuracy of an aeroelastic model when extending to other flight conditions.

III. Methodology

The primary goal of the present work is to create a relationship between the full-order model of Eq. (1a) to the identified model in Eq. (5). To achieve this, first the dynamics, Eq. (6), are separated into the “retained” modes that are common between the two models, Eq. (7a), and the deleted modes, Eq. (7b). The coupling between retained and deleted modes is shown in Eqs. (7c) and (7d),

$$[\mathbf{H}(\omega)] \triangleq \begin{bmatrix} \mathbf{H}_{rr}(\omega) & \mathbf{H}_{rd}(\omega) \\ \mathbf{H}_{dr}(\omega) & \mathbf{H}_{dd}(\omega) \end{bmatrix} \quad (6)$$

where,

$$[\mathbf{H}_{rr}(\omega)] = -[\mathbf{M}_{rr}] \omega^2 + [\mathbf{C}_{rr}] i\omega + [\mathbf{K}_{rr} - \bar{q}\mathbf{Q}_{rr}^h(k)] \quad (7a)$$

$$[\mathbf{H}_{dd}(\omega)] = -[\mathbf{M}_{dd}] \omega^2 + [\mathbf{C}_{dd}] i\omega + [\mathbf{K}_{dd} - \bar{q}\mathbf{Q}_{dd}^h(k)] \quad (7b)$$

$$[\mathbf{H}_{rd}(\omega)] = -[\mathbf{M}_{rd}] \omega^2 + [\mathbf{C}_{rd}] i\omega + [\mathbf{K}_{rd} - \bar{q}\mathbf{Q}_{rd}^h(k)] \quad (7c)$$

$$[\mathbf{H}_{dr}(\omega)] = -[\mathbf{M}_{dr}] \omega^2 + [\mathbf{C}_{dr}] i\omega + [\mathbf{K}_{dr} - \bar{q}\mathbf{Q}_{dr}^h(k)] \quad (7d)$$

A reduced-order system can be produced by deleting (truncating) such that the frequency response is then approximated by Eq. (7a). However, the method of truncation does not account for the effects of the deleted modes on the retained modes and does not match the full-order system as well. A method is needed to retain the contribution of the deleted modes without the additional degrees of freedom.

For unforced systems without damping, Paz created the method of dynamic condensation [20] to remove modes while matching the frequency response of full-order model at some reference frequency, $\tilde{\omega}$. At other frequencies, the match will not be exact, but with an appropriate selection of modes to retain/identify, the frequency response will be close. In the case that the reference frequency tends to zero (i.e., steady state), this method becomes known as static condensation or residualization. To extend the dynamic condensation to more general systems with forcing, the Eqs. (1a) and (6) are solved for the deleted states, $\mathbf{q}_d$, to give an estimate of these deleted states in Eq. (8).

$$\{\mathbf{q}_d\} \approx -[\mathbf{H}_{dd}^{-1}(\tilde{\omega}) \mathbf{H}_{dr}(\tilde{\omega})] \{\mathbf{q}_r\} + [\mathbf{H}_{dd}^{-1}(\tilde{\omega}) \mathbf{B}_d] \{\mathbf{u}\} \quad (8)$$

The estimate of the deleted states, $\mathbf{q}_d$, in Eq. (9a) is used to remove the deleted states from Eqs. (1a) and (6). The resulting reduced order equations are of the form in Eqs. (9a) and (9b).

$$[\bar{\mathbf{H}}(\omega)] \{\mathbf{q}_r\} = [\bar{\mathbf{B}}(\omega)] \{\mathbf{u}\} \quad (9a)$$

$$\{\mathbf{z}\} = [\bar{\Phi}] \{\mathbf{q}\} + [\bar{\mathbf{D}}] \{\mathbf{u}\} \quad (9b)$$

$$\bar{\mathbf{H}}(\omega) \triangleq \mathbf{H}_{rr}(\omega) - \mathbf{H}_{rd}(\omega) \mathbf{H}_{dd}^{-1}(\tilde{\omega}) \mathbf{H}_{dr}(\tilde{\omega}) \quad (9c)$$

$$\bar{\mathbf{B}}(\omega) \triangleq \mathbf{B}_r - \mathbf{H}_{rd}(\omega) \mathbf{H}_{dd}^{-1}(\tilde{\omega}) \mathbf{B}_d \quad (9d)$$

$$\bar{\Phi} \triangleq \Phi_r - \Phi_d \mathbf{H}_{dd}^{-1}(\tilde{\omega}) \mathbf{H}_{dr}(\tilde{\omega}) \quad (9e)$$

$$\bar{\mathbf{D}} \triangleq -\Phi_d \mathbf{H}_{dd}^{-1}(\tilde{\omega}) \mathbf{B}_d \quad (9f)$$

The term $\mathbf{H}_{rr}$ in Eq. (9c) corresponds to the system response if the higher modes were truncated without considering their contribution to the dynamics. The remaining terms capture the contributions of the higher order modes. Similarly, the reduced input matrix, Eq. (9d), has an additional term to capture the contribution of the deleted modes. The strain modes, Eq. (9e), are also modified in the condensed model. Additionally, the model reduction introduces a feed-forward matrix, Eq. (9f).

Because the matrices in Eq. (7a) are a linear combination, the dynamic condensation can be applied separately to the GAF for the modes and control surfaces Eqs. (10),

$$\bar{\mathbf{Q}}(k) = \mathbf{Q}_{rr}(k) - \mathbf{Q}_{rd}(k) \mathbf{H}_{dd}^{-1}(\tilde{\omega}) \mathbf{H}_{dr}(\tilde{\omega}) \quad (10)$$

To achieve a quasi-steady model, the typical generalized aerodynamic coefficients are approximated by Eq. (11a). The $\bar{\mathbf{Q}}^0$ term represents the steady aerodynamic coefficients, and $\bar{\mathbf{Q}}^1$ represents the quasi-steady (rate dependent) coefficients. The input matrix is also approximated by a steady matrix, Eq. (11b).

$$\bar{\mathbf{Q}}(k) \approx \bar{\mathbf{Q}}^0 + \bar{\mathbf{Q}}^1 ik \quad (11a)$$

$$\bar{\mathbf{B}}(\omega) \approx \bar{\mathbf{B}}^0 \quad (11b)$$

The quasi-steady and the modal reduction are combined to give Eqs (12a) and (12b),

$$\bar{\mathbf{Q}}^0 = \text{real} [\bar{\mathbf{Q}}(\tilde{k})] \quad (12a)$$

$$\bar{\mathbf{Q}}^1 = \frac{1}{\tilde{k}} \text{imag} [\bar{\mathbf{Q}}(\tilde{k})] \quad (12b)$$

$$\bar{\mathbf{B}}^0 = \text{real} [\bar{\mathbf{B}}(\tilde{\omega})] \quad (12c)$$

Although the term $\frac{1}{\tilde{k}}$ has the potential to cause problems as $\tilde{k} \rightarrow 0$, the imaginary part of Eq. (10) also goes to zero.

These quasi-steady reduced order models, Eqs (12a), (12b) and (12c), are assumed to be equivalent to the low-order model from system identification, i.e., $\bar{\mathbf{Q}} \approx \hat{\mathbf{Q}}$. The final step before the system identification results can be integrated into the full-order model is to define a similarity between the reduced order model, Eqs (12a) and (12b), and the model estimated from test data, Eq. (5). Initially a pseudoinverse of Eq. (9a) was considered. It was thought that the pseudoinverse would be better to reverse the interaction between modes shown by the additional terms in Eq. (10). However, attempts to apply the pseudoinverse to test data show unreliable and often nonsensical results. The problem

with using a pseudoinverse is that it cannot accurately recover the original coefficients even when applied to ideal data. Therefore, a simpler ratio between the two models has been used, Eqs. (13a) and (13b),

$$\mathbf{K}_{bias}^0 = \mathbf{Q}_{rr}^0 \oslash \bar{\mathbf{Q}}^0 \quad (13a)$$

$$\mathbf{K}_{bias}^1 = \mathbf{Q}_{rr}^1 \oslash \bar{\mathbf{Q}}^1 \quad (13b)$$

Alternatively, scaling corrections, $\mathbf{K}_{corr}^0$ and $\mathbf{K}_{corr}^1$, can be calculated directly, Eqs. (14a) and (14b).

$$\mathbf{K}_{corr}^0 = \mathbf{Q}_{fit}^0 \oslash \bar{\mathbf{Q}}^0 \quad (14a)$$

$$\mathbf{K}_{corr}^1 = \mathbf{Q}_{fit}^1 \oslash \bar{\mathbf{Q}}^1 \quad (14b)$$

The corrections are applied directly to the RFA in Eq. (15).

$$[\mathbf{Q}_{update}(k)] \approx [\mathbf{K}_{bias}^0 \odot \mathbf{Q}^0] + ik [\mathbf{K}_{bias}^1 \odot \mathbf{Q}^1] - k^2 [\mathbf{Q}^2] + \sum_j \frac{ik}{ik - \gamma_j} [\mathbf{K}_{bias}^1 \odot \mathbf{Q}^j] \quad (15)$$

IV. Experimental Description

The IAWTM model, shown in Fig. 1, was tested in the Transonic Dynamics Tunnel (TDT) at the NASA Langley Research Center (LaRC) in Hampton, VA in July and August of 2024. IAWTM was developed to study active control of the aircraft structure to reduce loads and improve aerodynamic efficiency [21]. The initial tunnel entry was focused on characterization of the model dynamics. The model was tested up to a Mach of 0.8 and dynamic pressures of 200 psf.



Fig. 1 IAWTM model mounted in the TDT (credit: NASA / Mark Knopp).

IAWTM has 10 control surfaces, but for the present analysis only the outboard Aileron C, which is shown in Fig. 2, was considered. Because it is the outboard surface it will tend to give the largest excitation of the structure. Furthermore, this surface showed the largest errors when compared to the pretest predictions. During this test point, Aileron C was excited by a logarithmic frequency sweep from 0.1 to 20 Hz. Data from these points were analyzed to demonstrate the method described in this paper. There were ten strain gauges laid out along the wing spar as shown in Figure 2. The strain gauges were used for the initial system identification results in this report because they generally had higher signal-to-noise ratios than the accelerometers.

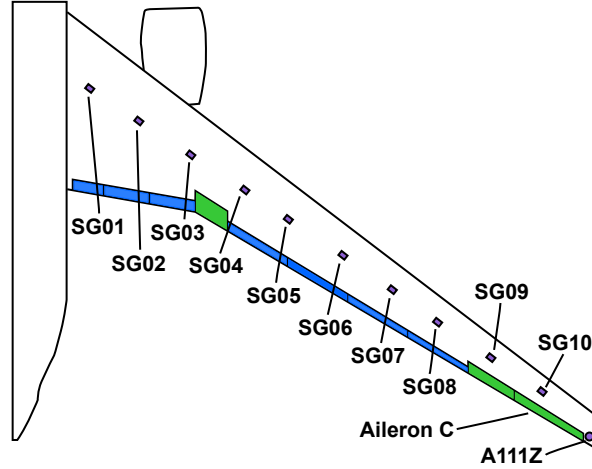


Fig. 2 IAWTM instrumentation and control surface used in the present analysis.

Multiple configurations were tested, but for the current work only a shims-on configuration is considered. The shims are used to bypass or lock out the balance during dynamic testing in order to protect the balance from excessive loads. The shims-on configuration is being considered because there is more excitation data available for this configuration. A description and abbreviation for the first eight modes of the model, in the shims-on configuration, is shown in Table 1. The system identification model was fit at a dynamic pressure of 110 psf and Mach 0.7. An additional test condition at a dynamic pressure of 160 psf and Mach 0.7 was used for validation.

Table 1 IAWTM shims-on structural modes.

Mode	Frequency, Hz	Damping Ratio, %	Update	Abbreviation	Description
1	3.61	0.25	✓	W1VB	Wing first vertical bending
2	8.08	1.21		W1FA	Wing first fore-aft bending
3	9.22	1.12	✓	VP	Vehicle pitch
4	11.63	0.66	✓	W2VB	Wing second vertical bending
5	14.48	1.57		W2FA	Wing second fore-aft bending
6	15.48	0.92		NCL-P1	Wing torsion and nacelle pitch
7	18.05	2.00	✓	VP-NCL-P	Vehicle and nacelle pitch
8	19.48	1.09	✓	NCL-S1	Nacelle Side Bending

V. Results

The results are broken into two sections. First a brief summary of the results from the updates to the reduced order model is discussed. Understanding of the updates provides context to the origin of these parameters and to establish expectations on how well they may work. The second section is the primary focus of this work. These updated parameters are then corrected and applied to the full-order model.

For the present work the reference frequency was selected to be $\bar{\omega} = 0$ Hz. Other frequencies were examined, but for the IAWTM model the selection of this frequency was less critical. For IAWTM, changing the reference frequency increased the bias correction. Therefore, the selection of the steady state is likely to underestimate the bias and be a more conservative test for IAWTM.

A. Updates to Reduced Order Model

The pretest prediction models are based on doublet lattice aerodynamics [2] using the approach of Ouellette and Valdez [19]. Overall these models were well correlated to the test data up to Mach 0.8. Therefore, the corrections are expected to be relatively small. It was found that five modes, indicated by the update column in Table 1, were sufficient for a reduced order model to match for the frequency range that was excited. The importance of these modes is a function of the inputs and outputs being used and would change for different test points. The resulting reduced order model is shown in Tables 3 and 5 in the Appendix.

A preliminary system identification model was generated by updating the 10 parameters shown in Table 2. The $Q_{1,1}^0$, $Q_{3,3}^0$, and $Q_{4,4}^0$ parameters represent the aerodynamic stiffness. The $Q_{3,3}^1$ and $Q_{4,4}^1$ represent aerodynamic damping. Finally, the B^0 terms represent the control power or how much the aileron can deflect each mode.

It was assumed that the validation of the FEM mode shapes in the ground vibration tests was sufficient and that further updates would be unnecessary. This assumption is probably very good for modes one, three and four. However, the ground vibration tests did identify potential problems with modes seven and eight.

Table 2 IAWTM parameters updated.

Parameter	Description
$Q_{1,1}^0$	Aerodynamic force on W1VB due to W1VB displacement
$Q_{3,3}^0$	Aerodynamic force on VP due to VP displacement
$Q_{4,4}^0$	Aerodynamic force on W2VB due to W2VB displacement
$Q_{3,3}^1$	Aerodynamic force on VP due to VP rate
$Q_{4,4}^1$	Aerodynamic force on W2VB due to W2VB rate
$B_{1,1}^0$	Aerodynamic force on W1VB due to Aileron C
$B_{3,1}^0$	Aerodynamic force on VP due to Aileron C
$B_{4,1}^0$	Aerodynamic force on W2VB due to Aileron C
$B_{7,1}^0$	Aerodynamic force on VP-NCL-P due to Aileron C
$B_{8,1}^0$	Aerodynamic force on NCL-S1 due to Aileron C

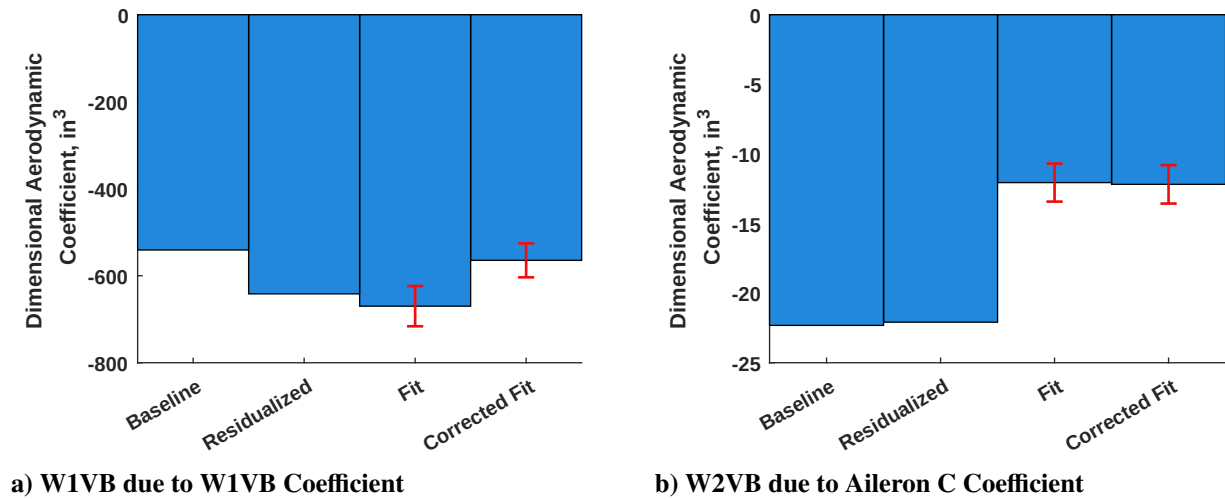


Fig. 3 Updates to aerodynamic coefficients.

Once the parameters were selected, the frequency domain quasi-maximum likelihood approach [10] was used to

adjust these 10 aerodynamic parameters to improve the fit.

The accuracy of the model reduction and fitting are shown in Fig. 4. The baseline model is the high-order state space model using an RFA. The reduced model is shown at discrete frequencies because it is calculated directly from the frequency domain doublet lattice output without an RFA. The reduced model matches very well to the baseline model at all of these frequencies. The fit model is the updated low-order model. Overall the fit model matches well to the test data. At Mach 0.7 the wing was sensitive to the turbulence resulting in a relatively low signal-to-noise ratio and the low coherence in Fig. 4.

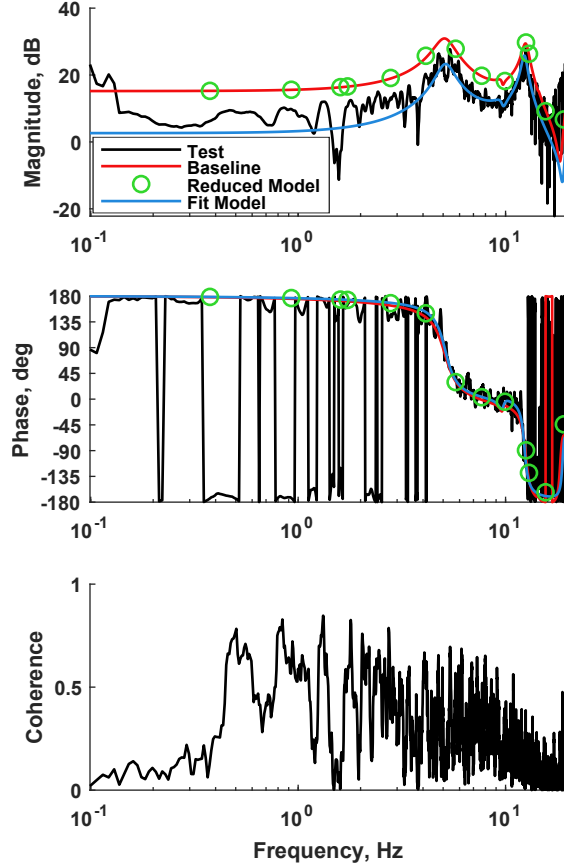


Fig. 4 Frequency response of Aileron C position to Strain Gage 03 showing goodness of model fit at fitting condition (Mach 0.7 and $\bar{q} = 110$ psf).

B. Applying Updates to Full-Order Model

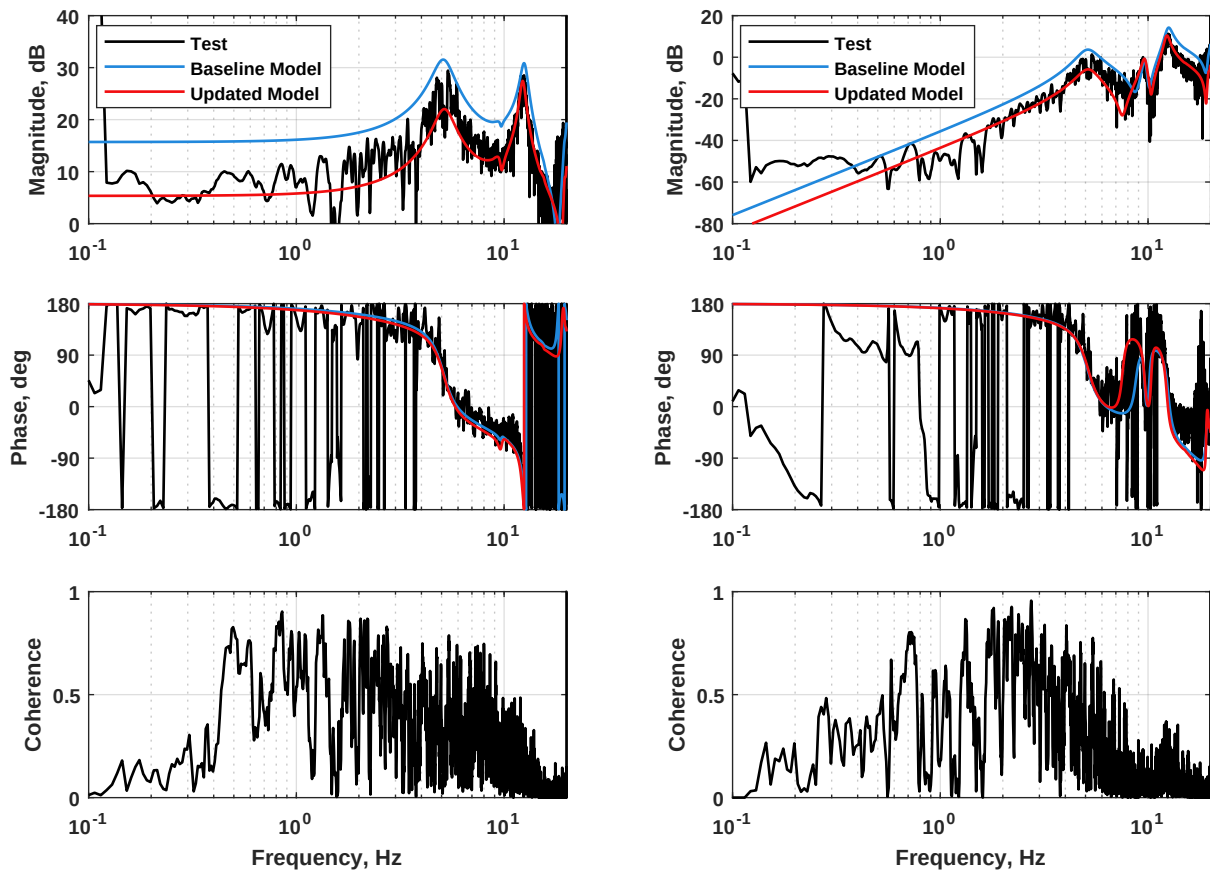
Figure 3 shows the changes in two of the aerodynamic coefficients. The baseline coefficient is the coefficient taken directly from doublet lattice, Eq. (3). The residualized coefficient is after correcting for the deleted modes, Eq. (10). The fit coefficient is the value of the parameter after the model update, as discussed in Section V.A. The error bars represent 95% uncertainty from the Cramér-Rao bounds. Finally, the corrected fit coefficients are the value after correcting the update for the bias, Eq. (15), and are ready to update the high-order model.

With the modes and the test condition being used for updating, the selected parameters in Table 2 do not show a large bias in the parameters due to these deleted modes. However, the aerodynamic stiffness in Fig. 3a, was one of the larger changes due to residualization. In Fig. 3a the change in the parameter, comparing residualized to fit and baseline to corrected, is small relative to the uncertainty. However, for this coefficient the bias from the extra modes is more significant than the error in the coefficient. If the fit data were used without correction, it would actually be

increasing the error more than making no correction. This bias correction can also be used for direct comparison of the coefficients. Based on the results in Fig. 3a, this coefficient may not be worth updating. However, for the remaining results the change in the parameter were kept.

The control effectiveness parameter in Fig. 3b is representative of the other terms in the $\bar{\mathbf{B}}^0$ matrix and shows almost no bias due to the residualization. However, this coefficient does show a much more statistically significant difference between the baseline and the corrected fit.

The accuracy of the update after being applied to the full-order model at the condition used in the fit is shown in Fig. 2. The baseline model matches reasonably well to the test data, but there is a noticeable error in the gain. After updating the model matches much better to the test. The strain gauge in Fig. 5a is one of the sensors used in the fitting. The accelerometer in Fig. 5b is not used to fit the updated model, but the update still shows a good match to the test data. These results show that the corrected aerodynamic coefficients were able to be integrated into the full-order model to produce a more accurate fit to the test data.

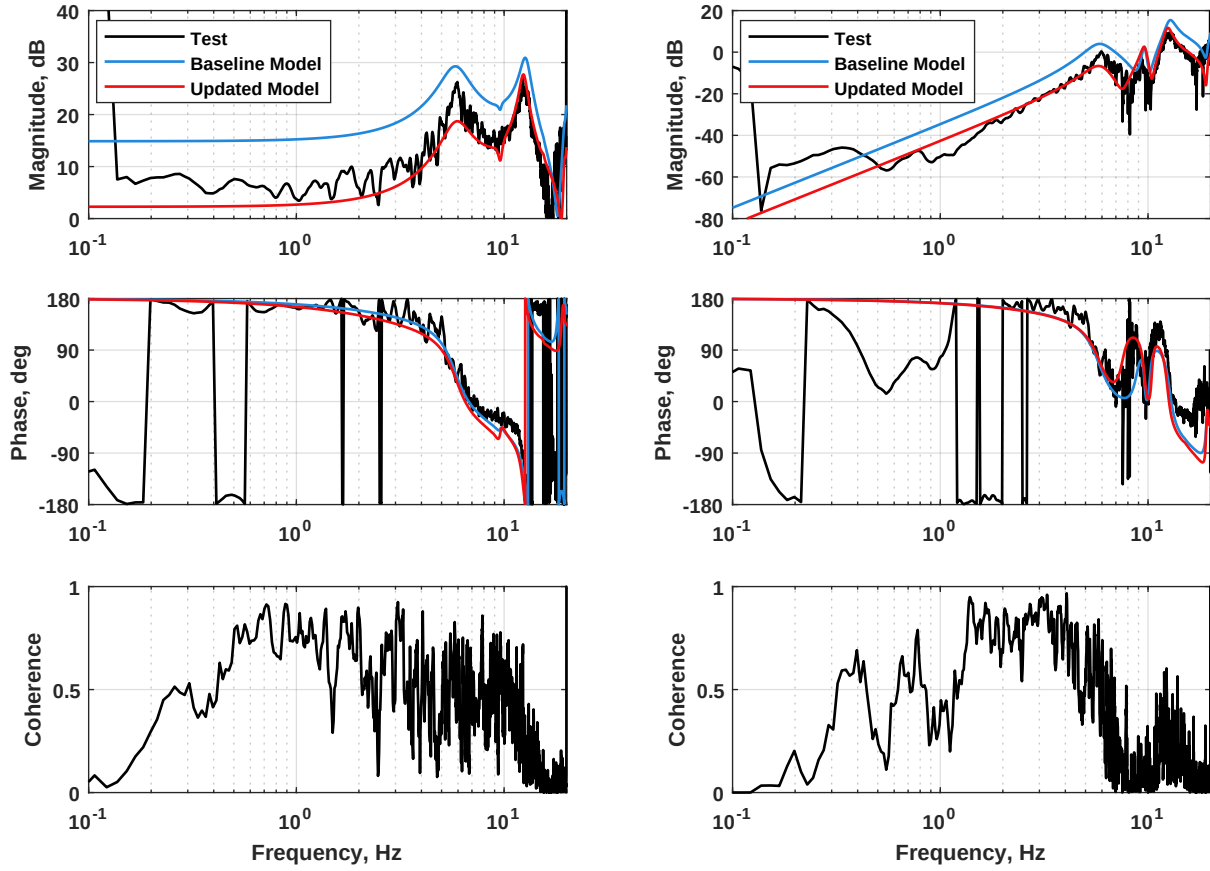


a) Aileron C command to Strain Gage 03

b) Aileron C command to wingtip accelerometer (A111Z)

Fig. 5 Comparison of frequency response of full-order pretest and updated models at fitting condition (Mach 0.7 and $\bar{q} = 110$ psf).

An additional test of the updated models is shown in Fig. 6 at a higher dynamic pressure than was used in the fitting. The updated model does not match as well at this validation condition. However, the update is still improving the fit. The accuracy of this updated model is also limited by the quality of the fit model. The parameter updates were very limited to only a few aerodynamic coefficients. An improved system identification result would also improve the fit at this validation condition.



a) Aileron C command to Strain Gage 03

b) Aileron C command to wingtip accelerometer (A111Z)

Fig. 6 Comparison of frequency response full-order pretest and updated models at validation condition (Mach 0.7 and $\bar{q} = 160$ psf).

VI. Conclusion

The bias due to low-order quasi-steady models used in system identification has been discussed. A dynamic condensation method was discussed that uses a test dependent reference frequency for quantification of these estimation biases due to using a reduced order model.

A preliminary system identification for the IAWTM wind tunnel model was used to demonstrate the method of estimating and correcting for these estimation biases. Overall the estimation bias for IAWTM was small, but neglecting the bias was shown to result in inaccurate conclusions about the accuracy of pretest coefficient predictions. The updated and corrected coefficients were then integrated into a high-order model. The updated high-order model was able to more accurately represent the test data, both at the conditions used for fitting as well as at higher dynamic pressures not used in the modeling.

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Appendix A: Aerodynamic Coefficients

The aerodynamic coefficients from the reduced model are shown in Table 3.

Table 3 IAWTM state space reduced model at Mach 0.7 and $\bar{q} = 110$ psf.

$\bar{\mathbf{Q}}^0$ Matrix (5×5)					$\bar{\mathbf{B}}^0$ Matrix (5×1)
-6.3570E+02	6.2937E+02	-5.8758E+02	7.0901E+02	-4.1666E+02	-3.0304E+01
-3.5873E+01	7.3259E+01	2.7330E+02	-2.7984E+01	-3.2818E+01	6.3846E+00
6.9361E+01	-7.0112E+01	-5.7600E+02	3.3344E+02	-3.6791E+01	-2.2094E+01
-5.2341E+01	3.1566E+01	-8.3067E-01	-9.8517E+01	1.9468E+01	8.9736E+00
2.6782E+01	-2.9240E+01	-8.3135E+00	-1.3004E+01	7.9413E+01	-2.1773E+00
$\bar{\mathbf{Q}}^1$ Matrix (5×5)					
-8.1305E+00	-1.8275E+00	-1.0201E+00	-1.2708E+00	1.4751E+00	
4.4842E-01	-1.2956E+00	2.0199E+00	-1.7081E-01	3.2287E-01	
-9.5634E-01	2.1467E+00	-5.9161E+00	2.2052E+00	-8.8685E-01	
-3.4101E-01	-5.3353E-01	1.3182E+00	-1.9621E+00	6.7015E-01	
1.7527E-01	2.4674E-01	-3.5220E-01	4.8919E-01	-3.7301E-01	

The coefficients after fitting the reduced order model are shown in Table 4. The updated coefficients are shown in bold.

Table 4 IAWTM state space fit model at Mach 0.7 and $\bar{q} = 110$ psf.

$\bar{\mathbf{Q}}^0$ Matrix (5×5)					$\bar{\mathbf{B}}^0$ Matrix (5×1)
-6.6650E+02	6.2937E+02	-5.8758E+02	7.0901E+02	-4.1666E+02	-1.2011E+01
-3.5873E+01	8.8659E+01	2.7330E+02	-2.7984E+01	-3.2818E+01	8.2652E+00
6.9361E+01	-7.0112E+01	-2.9218E+02	3.3344E+02	-3.6791E+01	-1.1813E+01
-5.2341E+01	3.1566E+01	-8.3067E-01	-9.8517E+01	1.9468E+01	3.5983E+00
2.6782E+01	-2.9240E+01	-8.3135E+00	-1.3004E+01	7.9413E+01	-1.8125E+00
$\bar{\mathbf{Q}}^1$ Matrix (5×5)					
-8.1305E+00	-1.8275E+00	-1.0201E+00	-1.2708E+00	1.4751E+00	
4.4842E-01	-3.3172E+00	2.0199E+00	-1.7081E-01	3.2287E-01	
-9.5634E-01	2.1467E+00	-4.0519E+00	2.2052E+00	-8.8685E-01	
-3.4101E-01	-5.3353E-01	1.3182E+00	-1.9621E+00	6.7015E-01	
1.7527E-01	2.4674E-01	-3.5220E-01	4.8919E-01	-3.7301E-01	

The output matrices of the reduced model are shown in Table 5.

Table 5 IAWTM reduced model output matrices at Mach 0.7 and $\bar{q} = 110$ psf.

	$\bar{\Phi}$ Matrix (10×5)					$\bar{D}$ Matrix (10×1)
SG01	1.2528E+02	1.6225E+02	-4.5866E+02	-1.0542E+02	2.0022E+02	-1.2141E+01
SG02	2.2881E+02	1.0737E+02	-7.7292E+02	-1.0641E+01	9.0854E+02	-1.8682E+01
SG03	4.1733E+02	8.2691E+01	-9.1041E+02	8.7601E+02	-2.8371E+02	-2.7861E+01
SG04	5.0998E+02	-4.5185E+01	-5.2117E+02	1.2880E+03	-4.1818E+02	-8.8213E-01
SG05	5.1426E+02	-1.4751E+02	6.7678E+00	1.2160E+03	-4.2213E+02	2.8042E+01
SG06	4.9731E+02	-2.8765E+02	9.5759E+02	5.7317E+02	-2.7055E+02	5.5318E+01
SG07	4.6232E+02	-3.7665E+02	1.7628E+03	-4.3308E+02	2.0417E+01	3.8002E+01
SG08	4.0582E+02	-4.3236E+02	2.4678E+03	-1.7739E+03	4.5013E+02	-2.4407E+01
SG09	3.0899E+02	-4.1026E+02	2.6620E+03	-2.8514E+03	8.3999E+02	-9.2734E+01
SG10	1.8064E+02	-2.7153E+02	1.8373E+03	-2.4077E+03	7.5572E+02	-8.6409E+01

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