

Desensitized Aerocapture Guidance

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Mission design flexibility can be increased using an aerocapture maneuver to capture into an orbit around a planet from a hyperbolic trajectory. In past work, bang-bang control has been shown as an optimal guidance solution that minimizes the ΔV required to get into a desired orbit using bank modulation. This work revisits aerocapture using a bank modulation problem to search for an optimal solution using a direct collocation method. Previous studies have shown that aerocapture performance is sensitive to atmospheric density. In the literature, several studies have used a desensitized control approach to make the optimal guidance more robust to uncertainties in the model. To this end, this work aims to develop a desensitized aerocapture guidance robust to uncertainty in atmospheric density. Furthermore, this work compares the robustness of the open-loop desensitized aerocapture guidance with optimal aerocapture guidance with an aerocapture application at Earth.

I. Nomenclature

ρ	=	atmospheric density
r	=	radial distance from the center of the planet
V	=	planet relative velocity
V_I	=	inertial velocity
γ	=	planet relative flight-path angle
γ_I	=	inertial flight-path angle
ψ	=	planet relative azimuth angle
ϕ	=	latitude
θ	=	longitude
σ	=	bank angle
$\mathbf{x}$	=	state vector
$\mathbf{p}$	=	parameters
Λ	=	sensitivity terms
B	=	scale height
h	=	altitude
t	=	time
s	=	sensitivity matrix
I	=	identity matrix
μ	=	gravitational parameter
a	=	semi-major axis
H	=	Hamiltonian function
λ	=	Lagrange multipliers
ρ_0	=	exponential atmospheric model constant
$\hat{\rho}$	=	magnitude of sine signal used to perturb the density profile
$\mathcal{F}$	=	frequency of sine signal used to perturb the density profile
Φ	=	phase of the sine signal used to perturb the density profile

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a = apoapsis
 p = periapsis
 f = final
 I = inertial
 Superscripts
 T = Transpose
 * = optimal values

II. Introduction

AEROCAPTURE is a maneuver in which a spacecraft enters a planetary atmosphere from a hyperbolic trajectory and dissipates excess energy using aerodynamic forces to capture into an orbit. There are several control mechanisms for aerocapture, such as modulating the drag or lift forces. Within lift modulation, one can change the lift force by varying the angle of attack and the side slip angle individually, known as direct force control, or roll the vehicle for control, known as bank angle modulation. This work focuses on bank angle modulation to perform an aerocapture. Figure 1 shows an aerocapture mission's typical concept of operations (CONOPS). Initially, the vehicle will approach the planet on a hyperbolic trajectory. Using the planet's atmosphere, the vehicle dissipates enough energy to skip the planet's atmosphere and get into an orbit around the planet. The vehicle's post-aerocapture orbit may not accurately target the desired apoapsis target altitude because of several uncertainties, such as the uncertainty in the vehicle states, atmosphere, and aerodynamics. Also, the vehicle must raise its periapsis to avoid passing through the planet's atmosphere again. To this end, the vehicle must perform two maneuvers using the onboard propulsion system to raise the periapsis and to correct the apoapsis. Typically, using aerocapture, these maneuvers are much less expensive than using a complete propulsion design to get capture into a final desired orbit. This work will mainly consider the aerocapture during the sensible atmosphere, which is the red trajectory shown in Figure 1.

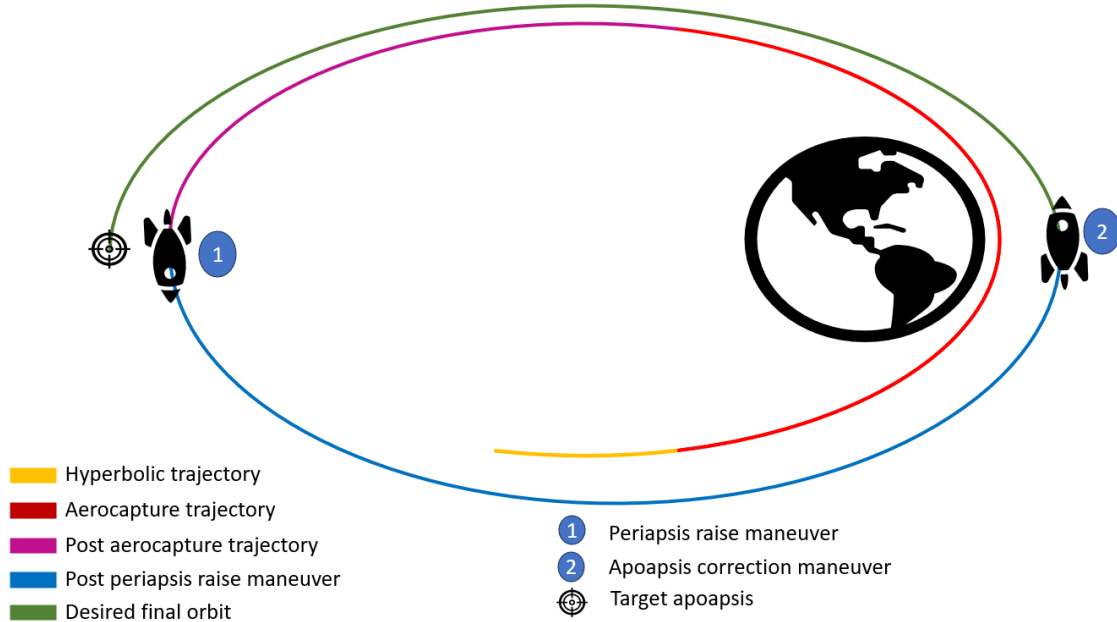


Fig. 1 Typical aerocapture CONOPS

Ignoring the planet's rotation and geo-potential perturbations, one can decouple the longitudinal dynamics. Using these decoupled dynamics, it is well-studied that an optimal aerocapture guidance that minimizes the Δv required to capture into a target orbit post-aerocapture is a bang-bang control [1] with a single switch. That is, the vehicle comes in with full lift up, and at some optimal switching point, the vehicle will start flying with full lift down. Here, the full lift up is at a 0 deg bank, and the full lift down is at a 180 deg bank. This work revisits this problem and determines the optimal aerocapture guidance using the direct collocation technique [2]. Here, longitudinal and lateral dynamics are considered to determine the optimal aerocapture guidance without neglecting the planet's rotation and J2 terms.

The techniques to determine the optimal trajectories have been studied significantly in the last few decades. These techniques can be broadly classified into direct and indirect methods. Indirect optimization techniques employ calculus of variations to derive the necessary analytic conditions of optimality [3] and then construct the corresponding two-point boundary value problem, which is solved numerically [4, 5]. More recently, with the advancements in computing capabilities, direct methods [6, 7] are gaining popularity. These methods transcribe the differential equations governing the vehicle's motion into algebraic equations, thereby formulating a nonlinear parameter optimization problem involving the discretized states, controls, and timescales. Over the years, several nonlinear programming solvers such as Sparse Nonlinear OPTimizer (SNOPT), Interior Point OPTimizer (IPOPT), and Nonlinear Interior point Trust Region Optimization (KNITRO) (the "K" is silent) have been developed to solve these large-scale problems [8]. The direct optimization techniques generally display better convergence characteristics compared to indirect methods. The convergence of the direct and indirect methods depends on the initial guess. Providing a good initial guess for the costates (Lagrange multipliers) for the indirect methods is not intuitive. In addition, multiple direct collocation methods (Pseudospectral collocation methods and local discretization methods such as trapezoidal and Simpson quadrature methods) that use a set of interpolation points to approximate the solution of the differential equation, resulting in a system of algebraic equations, are well-studied. This work focuses on aerocapture trajectory optimization using the direct collocation technique and uses the direct collocation method solution to determine whether the necessary optimality conditions are satisfied. This investigation will improve one's confidence in the optimal solution obtained by the collocation method.

Aerocapture performance can be significantly affected by the uncertainties in the atmospheric density. [9–11]. In the literature, several studies have solved a Desensitized Optimal Control (DOC) problem by augmenting the objective function with the sensitivity of quantities of interest to uncertainties in the model [12–15]. By minimizing these sensitivities, one can determine a more robust optimal solution. To this end, this work focuses on defining a DOC for aerocapture, which will be more robust to the uncertainties in atmospheric density. This work solves the aerocapture DOC problem using the direct collocation method. This work will focus on the aerocapture trajectory during the sensible atmosphere rather than the entire trajectory discussed in the aerocapture CONOPS.

The rest of the paper is organized as follows: Section III describes the optimal aerocapture problem and aerocapture using the DOC problem. Section IV investigates if the optimal solution for the problems described in Section III satisfies the optimality conditions. Section V simulates aerocapture using the optimal bank profile obtained by solving the optimal control problem, and compares the robustness of the DOC solution with respect to the atmospheric density with the optimal aerocapture solution using a Monte Carlo analysis. Finally, the paper's conclusions are presented in Section VI.

III. Problem Formulation

This section discusses the problem setup for a typical aerocapture optimization (No-DOC) problem and a DOC aerocapture problem.

A. No-DOC problem

This section describes the optimization problem one must solve to determine the optimal bank profile to perform aerocapture using bank modulation. The aerocapture optimization problem considered in this work is defined below:

$$\begin{aligned}
\min_{\sigma} \quad & \Delta v = \sqrt{2\mu} \left(\sqrt{\frac{1}{r_a^{Target}} - \frac{1}{r_a^{Target} + r_p^{Target}}} - \sqrt{\frac{1}{r_a^{Target}} - \frac{1}{2a}} \right) \\
\text{s.t.} \quad & \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \sigma, \mathbf{p}) \\
& |r_a - r_a^{Target}| \leq \epsilon \\
& r_0 - r_{exit} = 0 \\
& \sigma \geq \sigma_{min} \\
& \sigma \leq \sigma_{max} \\
& \mathbf{x}(0) = \mathbf{x}_0
\end{aligned} \tag{1}$$

where r_a^{Target} and r_p^{Target} are the target apoapsis and periapsis radius of the target orbit, a is the semi-major axis, and σ represents the bank angle. The vector $\mathbf{x} = \{r, \theta, \phi, v, \gamma, \psi\}$ corresponds to the states of the spacecraft, where r is

the radial distance of the vehicle from the center of the planet, θ is the longitude, ϕ is the geocentric latitude, v is the planet's relative velocity, γ is the planet relative flight path angle, and ψ is the planet relative azimuth angle. Here, subscript 0 describes the initial state vector. $\mathbf{p}$ contains parameters such as the lift and drag coefficients, reference area, vehicle mass, planet's rotation rate, planet's radius, J2 coefficients, and atmospheric scale height. ϵ is a tolerance on the apoapsis target constraint. The radius of apoapsis (r_a) as a function of the spacecraft states at the end of the aerocapture is defined below:

$$r_a = a \left(1 + \sqrt{1 - \frac{V_{exit}^2 r^2 \cos^2 \gamma_{exit}^2}{\mu a}} \right) \quad (2)$$

where μ is the gravitational parameter of the planet, the semi-major axis of the captured orbit post aerocapture (a) is determined as shown below:

$$a = \frac{\mu}{2\mu/r - V_{exit}^2} \quad (3)$$

Here, exit states are the states at the final time for the optimization problem; therefore, the optimization problem is in Mayer's form [16]. Also, the final time for the DOC problem has not been fixed. The optimization problem should also satisfy the differential equations shown in Appendix-I VII that govern the equations of motion of the entry vehicle, described as $f(\mathbf{x}, \sigma)$ in the problem formulation [1]. σ is the bank angle and is the only control variable. The spacecraft is assumed to have constant lift and drag coefficients, and an exponential function models the atmospheric density as shown below:

$$\rho = \rho_0 e^{-B(h-h_0)} \quad (4)$$

where ρ_0 is a constant, h is the altitude, h_0 is the reference altitude, and B is the inverse of scale height. The optimal bank (σ^*) and states ($\mathbf{x}^*$) are obtained from the solution of the No-DOC optimization problem.

B. DOC problem

As discussed earlier, this work also considers DOC to perform aerocapture, which can be robust to the uncertainties in atmospheric density. A DOC problem considers ρ_0 as an additional state and arguments in the objective function of the No-DOC problem along with terms that describe the sensitivity of v_{exit} and γ_{exit} with respect to ρ_0 , which are shown below:

$$\begin{aligned} \min_{\sigma} \quad & \Delta v = \sqrt{2\mu} \left(\sqrt{\frac{1}{r_a^{Target}} - \frac{1}{r_a^{Target} + r_p^{Target}}} - \sqrt{\frac{1}{r_a^{Target}} - \frac{1}{2a}} \right) + w_1 J_1 \\ \text{s.t.} \quad & J_1 = \int_0^{t_f} \left(\Lambda_{v,\rho_0}^2 + \Lambda_{\gamma,\rho_0}^2 \right) \\ & \dot{\mathbf{x}} = f(\mathbf{x}, \sigma, \mathbf{p}) \\ & \dot{\rho}_0 = 0 \\ & \dot{\Lambda} = -(\Lambda) \left(\frac{\partial f(\mathbf{x}, \sigma, \mathbf{p})}{\partial \mathbf{x}} \right) \\ & \left| r_a - r_a^{Target} \right| \leq \epsilon \\ & r_0 - r_{exit} = 0 \\ & \sigma \geq \sigma_{min} \\ & \sigma \leq \sigma_{max} \\ & \mathbf{x}(0) = \mathbf{x}_0 \\ & \Lambda(t_f) = I_{14 \times 14} \end{aligned} \quad (5)$$

All the definitions in the above optimization problem are identical to the No-DOC problem. The augmented objective function J_1 for the DOC problem considers elements of $\Lambda = s(t_f)s(t)^{-1}$ matrix, which carries the sensitivity information of final states at time t_f with respect to the states at time t , where $t \in [0, t_f]$ [17]. Here, s is the sensitivity matrix. Please see Ref [17] for a detailed discussion on the sensitivity matrix for DOC problems. DOC optimization problem considers additional constraints, such as the differential equations governing the elements of the Λ matrix. The Λ matrix for the No-DOC problem considered in this work will have 49 elements. However, this work only considers the 14 elements of the Λ matrix enough to compute the sensitivities of final velocity and flight path angle with respect to the ρ_0 along the trajectory, which are of interest as they are the only elements of the Λ matrix included in the objective

function. The differential equations that govern the matrix Λ are shown in Appendix-I VII. w_1 is the scalar weight considered for the running cost for the DOC problem that minimizes the sensitivity of final velocity and flight path angle to the uncertainties in ρ_0 along the trajectory. If one chooses the w_1 to be zero, the solution of the DOC problem should be identical to the No-DOC problem. Similar to the No-DOC problem, the optimal bank (σ^*) and states (x^*) are obtained from the solution of the DOC optimization problem, which is also a free final time problem.

IV. Optimal Solution Verification

This work uses the direct collocation method to solve the No-DOC and DOC problems discussed in the previous section. This section studies the solution's optimality obtained by solving the optimization problems described in Equation (1) and Equation (5). These problems are solved using the open-source direct pseudospectral optimization software package, *PSOPT* [2]. Using the Hermite-Simpson method, the differential equations are presented as algebraic equations, and the optimal control problem is solved as a parameter optimization problem. *PSOPT* uses the optimization algorithm of the Interior Point OPTimizer (IPOPT) library and Automatic Differentiation by OverLoading in C++ (ADOL-C) library to compute the gradients. *PSOPT* also provides the estimates of the costates or Lagrange multipliers profiles as part of the optimal solution, along with the optimal states and control profiles.

Considering Earth as the central body, No-DOC and DOC problems are solved in *PSOPT*. For the No-DOC problem, there are seven state variables and one control variable. For the DOC problem, there are 21 state variables and one control variable. The additional 14 state variables belong to the sensitivity terms. The DOC and No-DOC problems consider 300 nodes for the direct collocation method. The initial states at the beginning of the sensible atmosphere are the same for the No-DOC and DOC problems and are shown in Table 1.

Table 1 Initial states for the No-DOC and DOC optimal control problems

State	Value
Radius	6499900 m
Longitude	4.2368 rad
Latitude	-0.8145 rad
Velocity	11055 m/sec
Flight path angle	-0.1031 rad
Azimuth angle	0 rad
ρ_0	1.2092 kg/m ³

Table 2 Parameters used for the No-DOC and DOC optimal control problems

Parameters	Value
Gravitational parameter	3.9860121e14 m ³ /sec ²
Cd	1.37
Cl	0.37
Mass	8983 kg
J2	0.00108263
Planet's radius	6378000 m
Planet's rotation rate	7.292115851e-5 rad/sec
Atmospheric reference height	0 m
Atmospheric scale height	8500 m
Gravitational acceleration	9.81 m/sec ²

The No-DOC and DOC problem σ is constrained between 0 deg and 180 deg. The target apoapsis and the target periapsis altitudes are 200 km. The sensitivity states do not have initial conditions, but the final conditions are prescribed as shown in Equation 5. The radius at the final time is constrained to be the initial radius, and the final targeted apoapsis

has a 150 m tolerance (ϵ). Furthermore, several parameters describe the No-DOC and DOC optimization problems. These parameters are shown in Table 2.

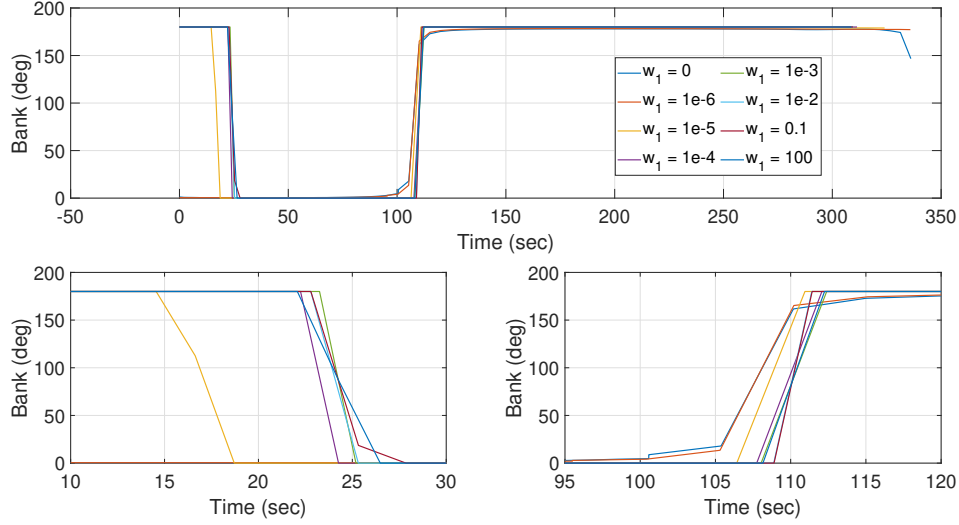


Fig. 2 Optimal solution for No-DOC problem from PSOPT for different values of w_1

For the DOC problem, multiple values for the w_1 are investigated. For this study, only 100 nodes were considered to solve each optimization problem. Figure 2 shows the bank solutions for each optimization problem solved with different w_1 values. If the w_1 is zero or very small, like in the case $w_1 = 1e - 6$, the solution of DOC and No-DOC was identical with a single switch. The first switch of the optimal bank profile for the DOC problem with $w_1 = 1e - 5$ and $w_1 = 1e - 4$ happens sooner than cases with larger w_1 values. For values of w_1 greater than $1e - 3$, the optimal bank solutions converge to a particular switching time. In this work, the value of the w_1 is considered to be $1e - 3$ for the DOC problems. Quantifying the solution's optimality obtained from the direct methods is challenging. In order to study the solution's optimality, this section will consider the Hamiltonian function of the No-DOC and DOC problem, which is discussed in detail in the following sections.

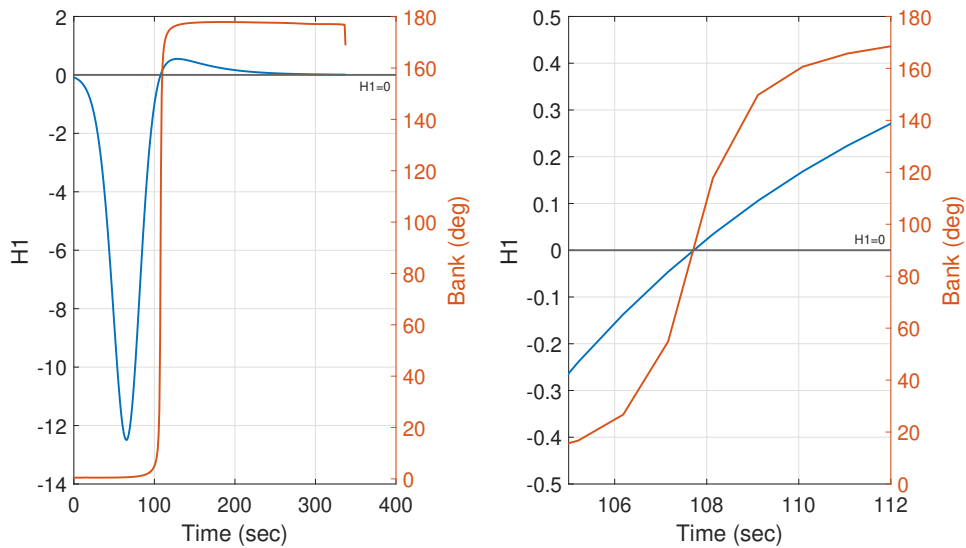


Fig. 3 Optimal solution for No-DOC problem from PSOPT switching with the sign of H_1

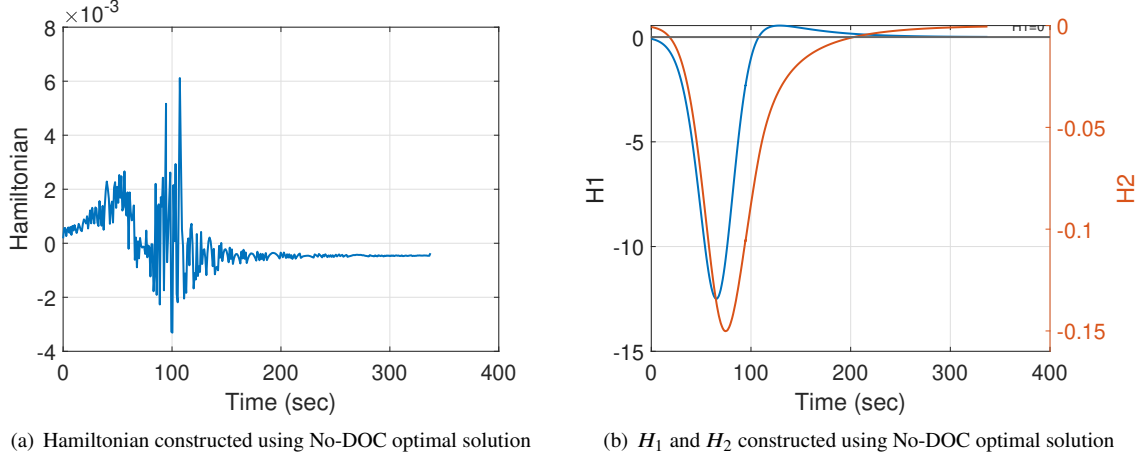


Fig. 4 Hamiltonian and its components constructed using the optimal solution from PSOPT for No-DOC problem

A. No-DOC Problem

This section considers the optimal solution for the No-DOC problem from PSOPT and constructs the Hamiltonian using the optimal states, optimal control, parameters, and costates estimates as shown below:

$$\mathcal{H}(\mathbf{x}, \sigma, \boldsymbol{\lambda}, \mathbf{p}, t) = \boldsymbol{\lambda}^T (f(\mathbf{x}, \sigma, \mathbf{p})) \quad (6)$$

where $\boldsymbol{\lambda}$ is 7×1 vector of Lagrange multipliers, one for each state derivative constraint (equations of motion constraints). The control variable is constrained as the bank angle (σ) can only vary between σ_{min} and σ_{max} . Therefore, Pontryagin's minimum principle says that the optimal control should minimize the Hamiltonian function for all time in comparison to all possible control profiles, as shown below:

$$\mathcal{H}(\mathbf{x}^*, \sigma^*, \boldsymbol{\lambda}^*, \mathbf{p}, t) \leq \mathcal{H}(\mathbf{x}^*, \sigma, \boldsymbol{\lambda}^*, \mathbf{p}, t) \quad \forall t \in [0, t_f] \quad (7)$$

for the No-DOC problem, one can group the terms that include the control terms as shown below:

$$\mathcal{H}(\mathbf{x}, \sigma, \boldsymbol{\lambda}, \mathbf{p}, t) = H_0(\mathbf{x}, \boldsymbol{\lambda}, \mathbf{p}, t) + H_1(\mathbf{x}, \boldsymbol{\lambda}, \mathbf{p}, t) \cos \sigma + H_2(\mathbf{x}, \boldsymbol{\lambda}, \mathbf{p}, t) \sin \sigma \quad (8)$$

Let us consider the case where the bank is allowed to vary between 0 and 180 deg. As one who is not interested in any lateral control during aerocapture, this is the physical control space an aerocapture problem will have. Now, without the $H_2(\mathbf{x}, \sigma, \boldsymbol{\lambda}, t)$ and $\sin \sigma$ terms in the Hamiltonian, as the $\cos \sigma$ monotonically increases between 0 and 180 deg, the optimal bank profile will be a bang-bang profile that switches with the sign of the $H_1(\mathbf{x}, \sigma, \boldsymbol{\lambda}, t)$. However, the $\sin \sigma$ terms in the Hamiltonian make the solution more complex. The optimal solution from the PSOPT is a bang-bang solution, as shown in Figure 3, and shows that it switches with the sign of H_1 . Figure 4 compares the magnitude of H_1 and H_2 constructed using the optimal solution from PSOPT. As one can see, the magnitude of H_2 is much smaller than H_1 . Therefore, it makes sense that the optimal control maximizes the negative value of the magnitude of H_1 by switching the optimal bank sign accordingly, which then minimizes the Hamiltonian. As the cosine of the bank maximizes, the sine of the bank approaches zero. Given the smaller magnitude of H_2 , it is appropriate for the optimal control to optimize the cosine of the bank by riding on the boundaries, which are zero and 180 deg. Also, as this is a free final time problem and the dynamics and terminal constraints are not explicit time functions, the Hamiltonian for the optimal solution should be zero along the trajectory as shown in Figure 4. This analysis shows that the optimal PSOPT solution satisfies the No-DOC problem's optimality conditions.

B. DOC Problem

This section will consider the Hamiltonian for the DOC problem discussed in Equation 5. The Hamiltonian for the DOC problem can be constructed as shown below:

$$\mathcal{H}(\mathbf{x}, \sigma, \boldsymbol{\lambda}, \boldsymbol{\lambda}_\Lambda, \boldsymbol{\Lambda}, \mathbf{p}, t) = w_1 (\Lambda_{v,\rho_0}^2 + \Lambda_{\gamma,\rho_0}^2) + \boldsymbol{\lambda}^T (f(\mathbf{x}, \sigma, \mathbf{p})) + \boldsymbol{\lambda}_\Lambda^T \left(\frac{\partial f(\mathbf{x}, \sigma, \mathbf{p})}{\partial \mathbf{x}} \delta \sigma \right), \quad (9)$$

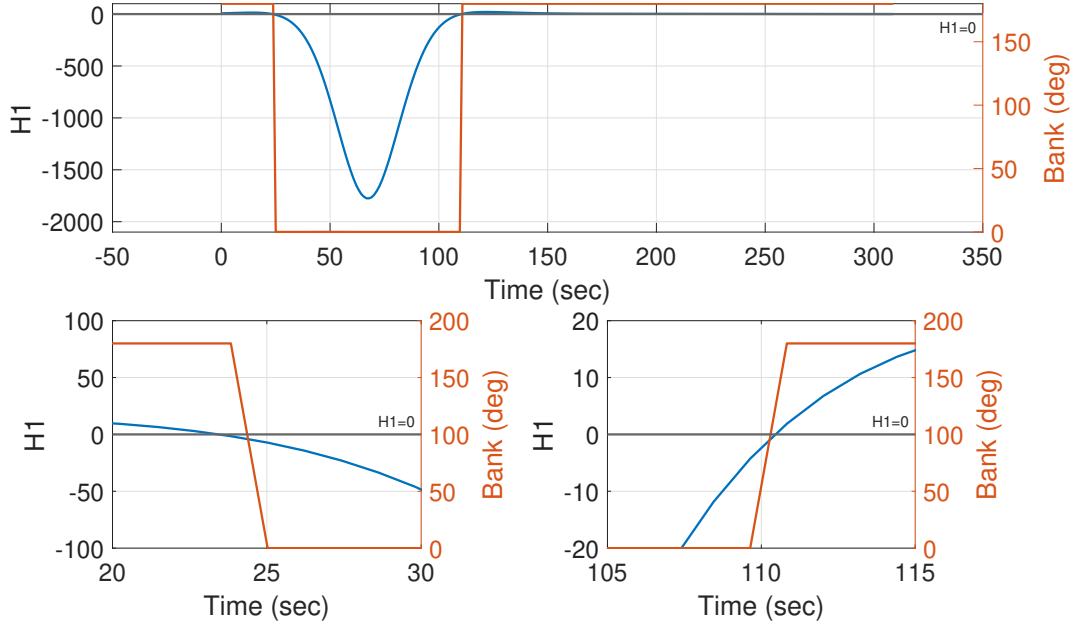


Fig. 5 Optimal solution for DOC problem from PSOPT switching with the sign of H_1

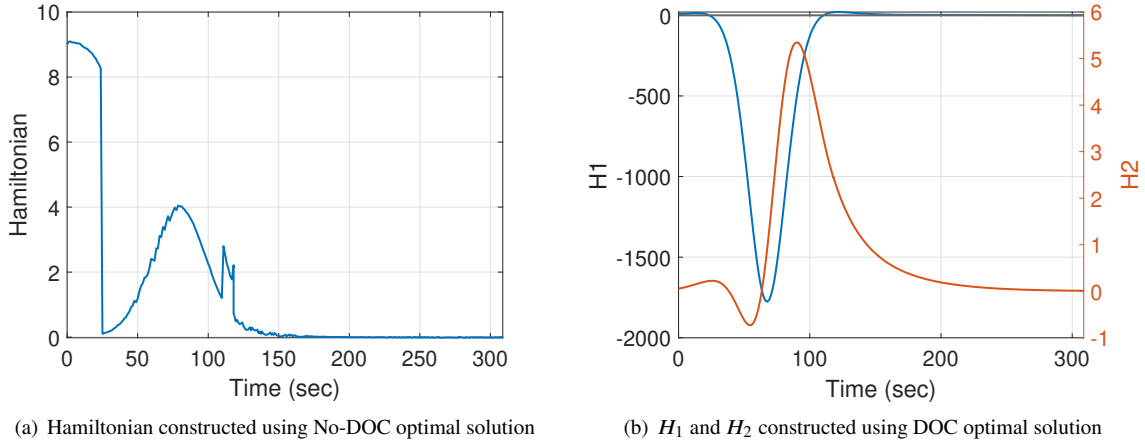


Fig. 6 Hamiltonian and its components constructed using the optimal solution from PSOPT for DOC problem

For the DOC problem, consider the sensitivity terms within the integral of the optimization problem's objective function. In addition, the Hamiltonian for the DOC problem also includes the constraints that dictate the rate of change of the sensitivity terms Λ and their corresponding Lagrange multipliers λ_Λ of size 14×1 . Similar to the No-DOC problem, as the control is constrained, Pontryagin's minimum principle says that the optimal control should minimize the Hamiltonian function for all time compared to all possible control profiles, as shown below:

$$\mathcal{H}(\mathbf{x}^*, \sigma^*, \lambda^*, \lambda_\Lambda^*, \Lambda^*, \mathbf{p}, t) \leq \mathcal{H}(\mathbf{x}^*, \sigma, \lambda^*, \lambda_\Lambda^*, \Lambda^*, \mathbf{p}, t) \forall t \in [0, t_f] \quad (10)$$

Similar to the DOC problem, one can group the No-DOC's Hamiltonian terms that include the control terms, as illustrated below:

$$\mathcal{H}(\mathbf{x}, \sigma, \lambda, \lambda_\Lambda, \Lambda, \mathbf{p}, t) = H_0(\mathbf{x}, \lambda, \lambda_\Lambda, \Lambda, \mathbf{p}, t) + H_1(\mathbf{x}, \lambda, \lambda_\Lambda, \Lambda, \mathbf{p}, t) \cos \sigma + H_2(\mathbf{x}, \lambda, \lambda_\Lambda, \Lambda, \mathbf{p}, t) \sin \sigma \quad (11)$$

Synthesizing the optimal control that minimizes the Hamiltonian is not intuitive because the control variable has sine and cosine terms in the Hamiltonian. This work studies the Hamiltonian function using the optimal solution obtained from the PSOPT. The optimal solution for the bank profile is a bang-bang solution with two switches, as shown in Figure 5. As one can see from Figure 5, the optimal bank switches with the sign of the H_1 , similar to the DOC problem. Figure 6 shows the magnitude of the H_1 and H_2 constructed using the optimal solution from the PSOPT. The magnitude of H_1 is significantly larger than that of H_2 for the DOC problem. Therefore, the optimal bank changes the sign with H_1 to minimize the Hamiltonian. When the optimal bank is riding at the boundaries of 0 deg or 180 deg, the sine approaches zero. However, given the small magnitude of H_2 compared to H_1 , maximizing the negative value of the magnitude of H_1 is more effective in minimizing the Hamiltonian. Similar to the No-DOC problem, as the DOC problem has dynamics and terminal constraints which are not explicit time functions, the Hamiltonian should be close to zero, as shown in Figure 6. The Hamiltonian is much closer to zero than the magnitude of H_1 . However, as this work only uses costate estimates from the PSOPT but not the costates obtained as part of the two-point boundary value problem, the Hamiltonian is not entirely zero. This analysis improves confidence in the optimal solution for the No-DOC problem from PSOPT.

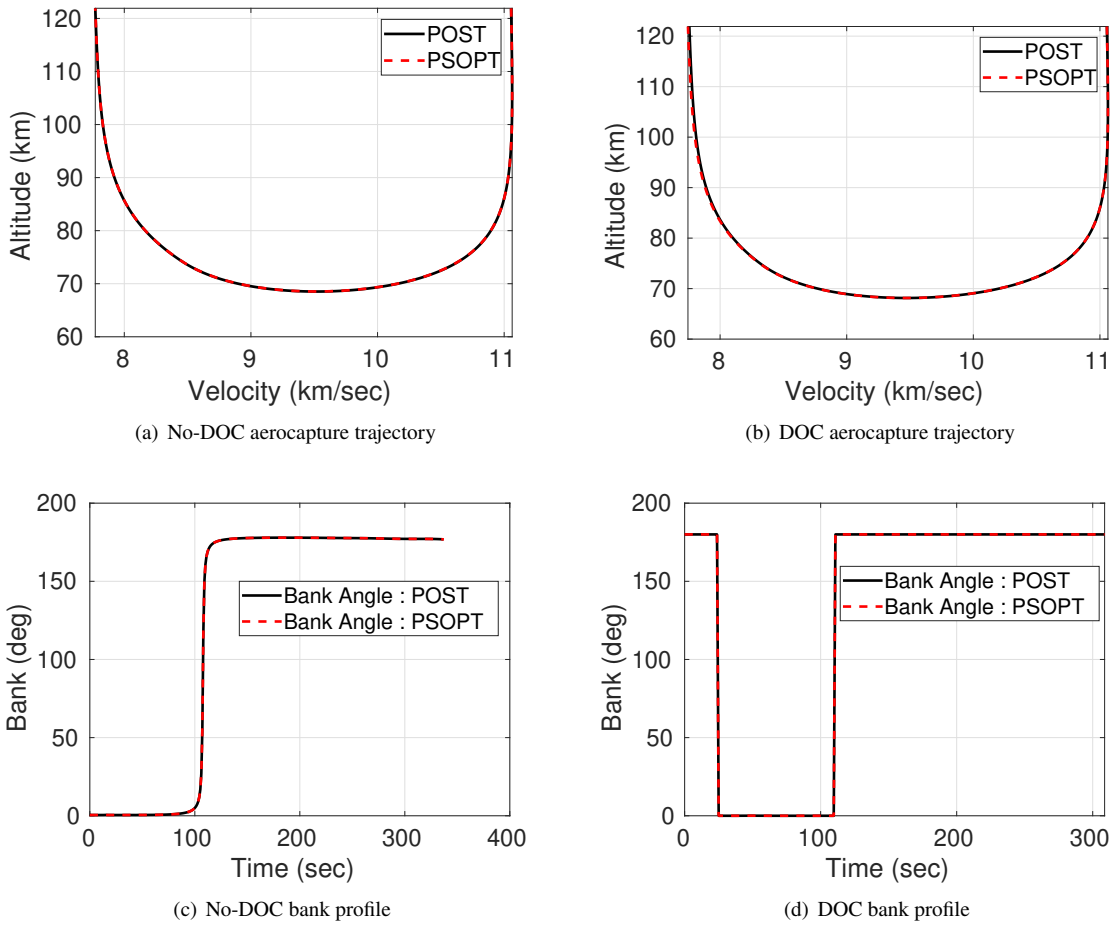


Fig. 7 Comparisons of a nominal solution from PSOPT with the simulated trajectories in POST2.

V. Monte Carlo Analysis

This section will use the optimal solution for the DOC and No-DOC solution from PSOPT and simulate the trajectory in Program to Optimize Simulated Trajectories II (POST2). Using the optimal solution from PSOPT as the reference, tables are generated for reference bank angle. In POST2, the initial states, aerodynamics, gravity, and atmosphere are modeled identically to the optimization problem setup used in PSOPT. Figure 7 compares the optimal

solution from the PSOPT with the POST2 trajectory simulated using the reference bank commands from the lookup table as a function of time. The POST2 trajectory matches closely with the PSOPT solution. It improves confidence in the optimization problem setup, especially with the discretization where PSOPT converts the differential equations that define the equations of motion into algebraic constraints. Figure 8 compares the No-DOC trajectory with the DOC trajectory simulated in POST2. Figure 8 shows that the dynamic pressure is the same at the region where the optimal bank profile varies from the No-DOC and DOC problem and is not negligible.

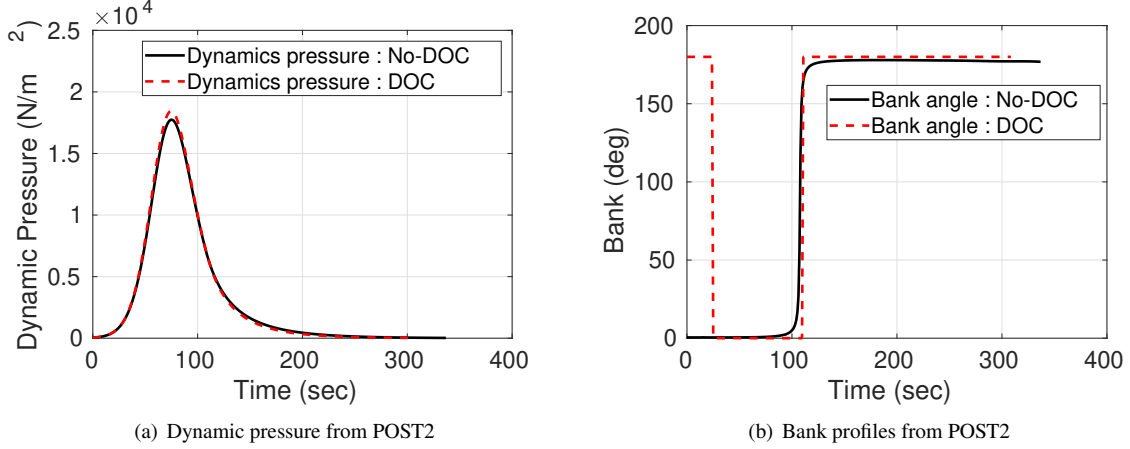


Fig. 8 Comparisons of nominal No-DOC and DOC trajectory simulated in POST2.

This work considered Monte Carlo analysis by dispersing the density profiles used to model the atmosphere to study the robustness of the DOC solution. To disperse the density profiles, this work has added a sine signal with magnitude $\hat{\rho}$ and a phase Φ to the ρ_0 [10]. For each run in the Monte Carlo analysis, the ρ_0 is dispersed as shown below:

$$\rho_{0Disp}(t) = \rho_0 + \hat{\rho} \sin(2\pi\mathcal{F}t + 2\pi\Phi) \quad (12)$$

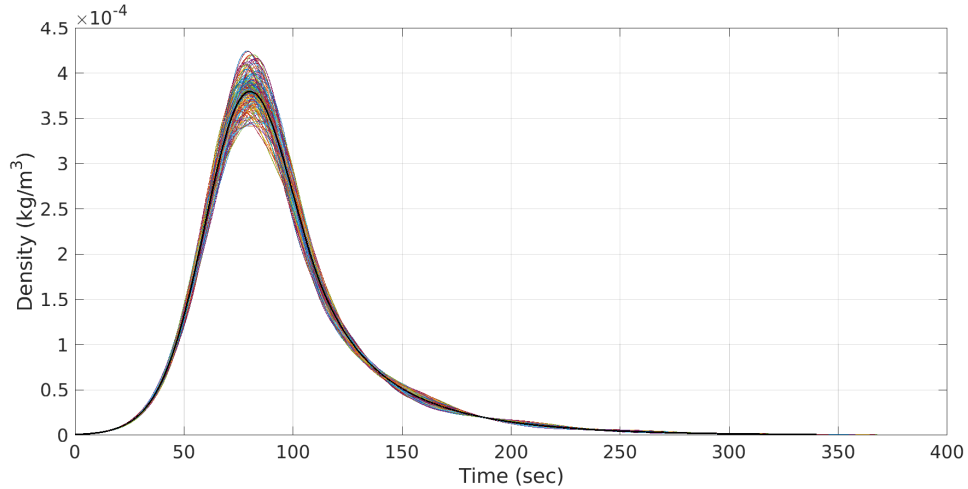


Fig. 9 The ρ_{0Disp} profiles from the Monte Carlo simulation with $\hat{\rho} \pm 10\% \rho_0$ dispersion

where ρ_{0Disp} is the dispersed ρ_0 as a function of time (t), ρ_0 is the nominal value used to compute the DOC and No-DOC guidance, and $\mathcal{F}$ is the frequency of the sine signal. The magnitude $\hat{\rho}$ is randomly chosen for each run in the Monte Carlo analysis uniformly between $\pm$ user-defined percent of the ρ_0 . The phase Φ of the sine signal is randomly chosen for each run in the Monte Carlo analysis uniformly between zero and 2π . The frequency of the sine signal ($\mathcal{F}$) is considered to be 0.02. That is, the sine signal has a period of 50 sec. Figure 9 shows the perturbed density profiles

used for the Monte Carlo analysis with amplitude of the sine signal chosen $\pm 10\%$ percent of the ρ_0 . The black line in Figure 9 corresponds to the nominal density profile used to compute the optimal guidance for the DOC and No-DOC problems. The perturbed density profiles oscillate around the nominal density profile. Having a density profile biased on one side will be a more stressful case for the open loop guidance with constant switching time and, therefore, is not considered for the comparisons in this work.

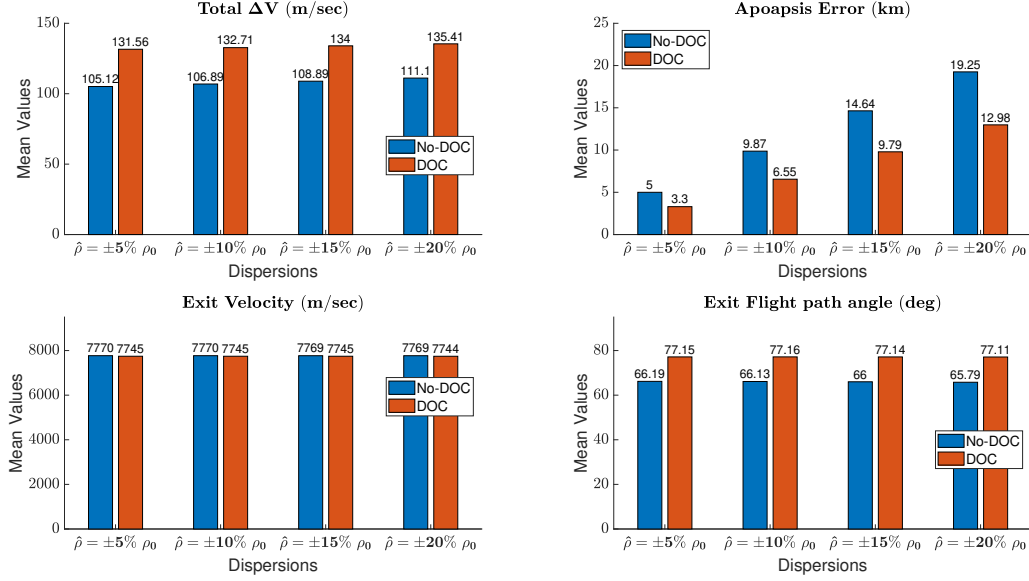


Fig. 10 Comparison of mean values from 8001 case Monte Carlo analysis for No-DOC and DOC considering different ρ_{0Disp} profiles with open loop guidance.

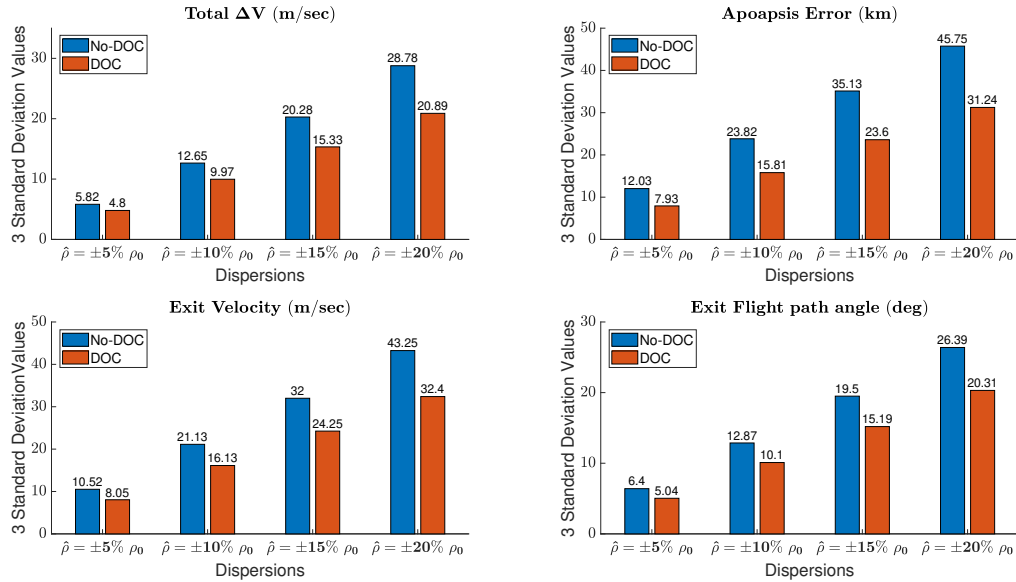


Fig. 11 Comparison of three standard deviation from 8001 case Monte Carlo analysis for No-DOC and DOC considering different ρ_{0Disp} profiles with open loop guidance

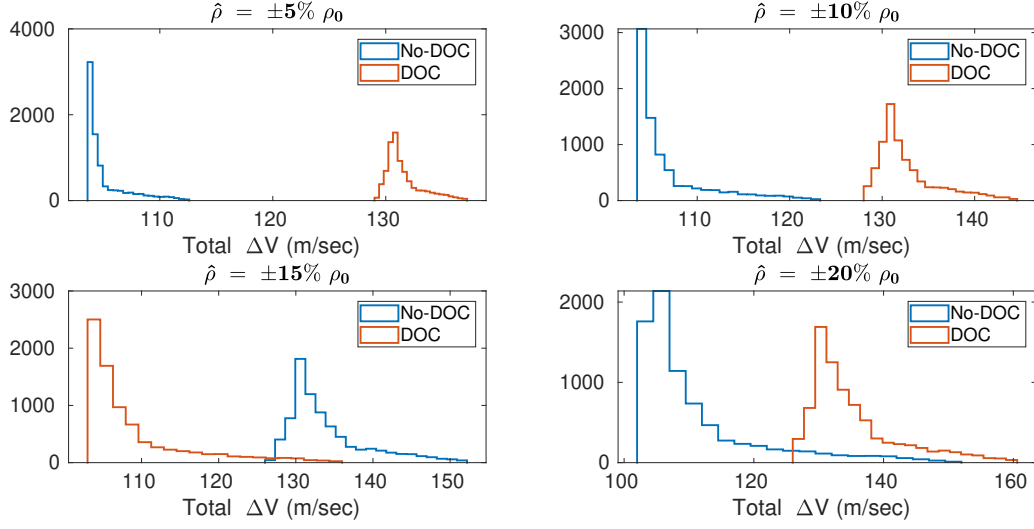


Fig. 12 Total ΔV distribution comparison between No-DOC and DOC using 8001 case Monte Carlo analysis for different ρ_{0Disp} profiles.

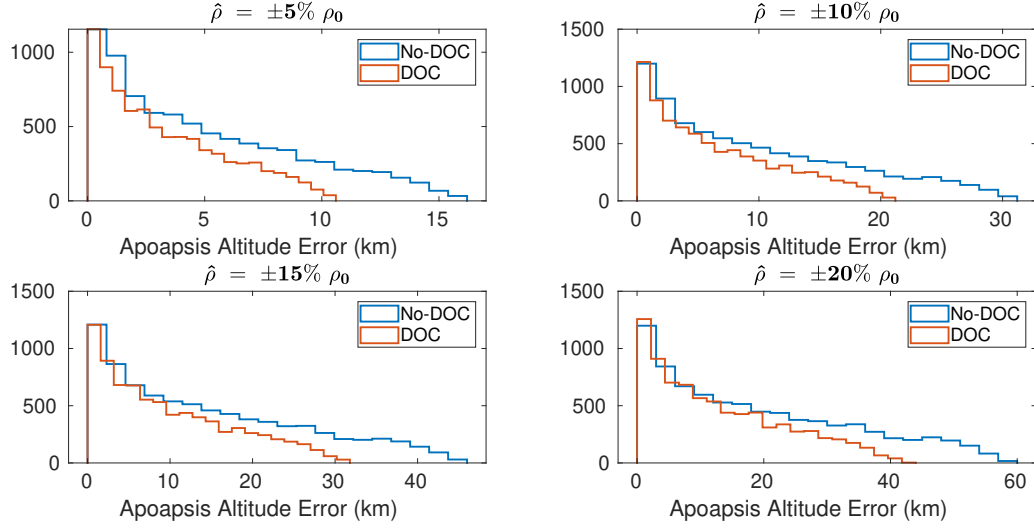


Fig. 13 Target apoapsis error distribution comparison between No-DOC and DOC using 8001 case Monte Carlo analysis for different ρ_{0Disp} profiles.

This work considered four different levels of atmospheric dispersions. The $\hat{\rho}$ is uniformly dispersed to be within $\pm 5\%$, $\pm 10\%$, $\pm 15\%$, and $\pm 20\%$ of the nominal ρ_0 . Additionally, a Monte Carlo analysis of 8001 cases is performed. Figure 10 shows the mean values for total Δv , which is the sum of the Δv required for the periapsis raise maneuver and the apoapsis correction maneuver after the aerocapture. In addition, Figure 10 also shows the error in apoapsis of the post-aerocapture orbit compared to the target orbit's apoapsis, along with the velocity and the flight path angle at the exit. Figure 10 compares these parameters for No-DOC and DOC problems from different 8001 case Monte Carlo runs, which considered different atmospheric density profiles. The total Δv values for the DOC problem are higher compared to the No-DOC problem. This is expected as the DOC problem tries to minimize the sensitivity of the exit velocity and flight path angle with respect to the ρ_0 for the DOC problem along with the total Δv . The exit velocity for the DOC problem is slightly less than the exit velocity of the No-DOC problem, as the vehicle flies initially lift down for the DOC problem. This additional dissipated energy results in an expensive periapsis raise maneuver. The DOC problem has a

higher total Δv as the Δv required for the periapsis raise maneuver is more for the DOC problem. As the DOC problem reduces the sensitivity of the ρ_0 on the final velocity and the final flight path angle, the mean of the error in the apoapsis post-aerocapture from Monte Carlo analysis is smaller for the DOC problem than the No-DOC problem. Figure 11 shows the three standard deviation values of the total Δv , error in the post aerocapture orbit's apoapsis, exit velocity, and exit flight path angle from the 8001 case Monte Carlo analysis performed considering different density dispersions. The three standard deviation values for all the parameters are smaller for the DOC problem, showing more robustness than the No-DOC problem.

Figure 12 shows the plots that compare the total Δv distribution of DOC and No-DOC problems from the 8001 case Monte Carlo analysis for different density dispersions. As the density dispersions get larger, the overlap of the total Δv distributions of the DOC and No-DOC problems keep increasing. The tails of the No-DOC problem are longer compared to the DOC problem, which shows the robustness of the DOC problem to the atmospheric density dispersions. As the DOC problem loses more velocity than the No-DOC solution, the Δv for the periapsis raise maneuver required for the DOC problem is larger, which shows in the total Δv distributions. Figure 13 shows the error in the post-aerocapture orbit's apoapsis compared to the target apoapsis for DOC and No-DOC problems for different density dispersions. Similar to the total Δv , the error in the apoapsis has longer tails for the No-DOC problem than the DOC problem. As the apoapsis is a function of exit velocity and exit flight path angle, which have smaller standard deviation values, it leads to a smaller spread of the error in the apoapsis, showing more robustness to the atmospheric dispersions. With open-loop guidance from the PSOPT for the DOC and No-DOC problems, the DOC problem shows more robustness to the uncertainty in the ρ_0 . In addition, no lander or hyperbolic cases are among all the Monte Carlo analyses considered in this work.

VI. Conclusion

This work revisits the aerocapture (No-DOC) using the bank modulation problem and determines the optimal solution using the direct collocation method within the PSOPT. In addition to considering the optimization problem to determine the optimal guidance for aerocapture that minimizes the total Δv required to perform the periapsis raise maneuver and the apoapsis correction maneuver post aerocapture, this work also considers the desensitized aerocapture guidance. Specifically, this work studies the aerocapture guidance (DOC) desensitized with respect to the uncertainty in the ρ_0 within the exponential atmospheric density model, which is also solved using the direct collocation method within PSOPT. To verify the solution's optimality, this work constructs the Hamiltonian for the DOC and No-DOC problem using optimal states, control, and the estimates of the costates from the PSOPT. This work shows that the optimal bank solutions for the DOC and No-DOC problems are bang-bang profiles with switches happening at optimal times to satisfy Pontryagin's minimum principle. The optimal solutions from the PSOPT are used as guidance through lookup tables within the POST2 simulation of the aerocapture. Monte Carlo analysis using open-loop guidance with different density dispersions is performed in POST2 for the DOC and No-DOC problems. This analysis shows that the DOC solution loses more velocity at the end of the aerocapture than the No-DOC problem, which results in more Δv for the periapsis raise maneuver and thereby more total Δv . The DOC problem has smaller apoapsis errors, with tighter exit velocity and exit flight path angle dispersions. The spread of the total Δv and error in the apoapsis post-aerocapture is smaller for the DOC problem than the No-DOC problem, showing more robustness to the uncertainty in the ρ_0 within the exponential atmospheric density model.

VII. Appendix -I

The differential equations that govern the motion of the aerocapture vehicle are given below in spherical coordinates [1]:

$$\begin{aligned}\dot{r} &= v \sin(\gamma) \\ \dot{\theta} &= \frac{v \cos(\gamma) \sin(\psi)}{r} \\ \dot{\phi} &= \frac{v \cos(\gamma) \sin(\psi)}{r}\end{aligned}$$

$$\begin{aligned}
\dot{v} &= \frac{\mu \sin(\gamma) \left(\frac{J_2 \text{Re}^2 \left(\frac{9 \sin(\phi)^2}{2} - \frac{3}{2} \right)}{r^2} - 1 \right)}{r^2} + \Omega^2 r \cos(\phi) (\cos(\phi) \sin(\gamma) - \cos(\gamma) \cos(\psi) \sin(\phi)) \\
&\quad - \frac{\text{Cd } S_{\text{ref}} \rho_0 v^2 e^{B(\text{Re}+h_0-r)}}{2m} - \frac{3 J_2 \text{Re}^2 \mu \cos(\phi) \sin(\phi) \cos(\gamma \cos(\psi))}{r^4} \\
v \dot{\gamma} &= \cos(\gamma) \left(\frac{v^2}{r} + \frac{\mu \left(\frac{J_2 \text{Re}^2 \left(\frac{9 \sin(\phi)^2}{2} - \frac{3}{2} \right)}{r^2} - 1 \right)}{r^2} \right) + \Omega^2 r \cos(\phi) (\cos(\gamma) \cos(\phi) + \cos(\psi) \sin(\gamma) \sin(\phi)) \\
&\quad + 2 \Omega v \cos(\phi) \sin(\psi) + \frac{\text{Cl } S_{\text{ref}} \rho_0 v^2 e^{B(\text{Re}+h_0-r)} \cos(\sigma)}{2m} + \frac{3 J_2 \text{Re}^2 \mu \cos(\phi) \cos(\psi) \sin(\gamma) \sin(\phi)}{r^4} \\
v \dot{\psi} &= 2 \Omega v (\sin(\phi) - \cos(\phi) \cos(\psi) \tan(\gamma)) + \frac{v^2 \cos(\gamma) \sin(\psi) \tan(\phi)}{r} + \frac{\Omega^2 r \cos(\phi) \sin(\phi) \sin(\psi)}{\cos(\gamma)} \\
&\quad + \frac{3 J_2 \text{Re}^2 \mu \cos(\phi) \sin(\phi) \sin(\psi)}{r^4 \cos(\gamma)} + \frac{\text{Cl } S_{\text{ref}} \rho_0 v^2 e^{B(\text{Re}+h_0-r)} \sin(\sigma)}{2m \cos(\gamma)} \\
\dot{\rho}_0 &= 0
\end{aligned}$$

The differential equations that govern the elements of the Λ matrix are shown below:

$$\begin{aligned}
\dot{\Lambda}_{v,r} &= -\Lambda_{v,r} \left(\frac{\partial \dot{r}}{\partial r} \right) - \Lambda_{v,\theta} \left(\frac{\partial \dot{\theta}}{\partial r} \right) - \Lambda_{v,\phi} \left(\frac{\partial \dot{\phi}}{\partial r} \right) - \Lambda_{v,v} \left(\frac{\partial \dot{v}}{\partial r} \right) - \Lambda_{v,\gamma} \left(\frac{\partial \dot{\gamma}}{\partial r} \right) - \Lambda_{v,\psi} \left(\frac{\partial \dot{\psi}}{\partial r} \right) - \Lambda_{v,\rho_0} \left(\frac{\partial \dot{\rho}_0}{\partial r} \right) \\
\dot{\Lambda}_{v,\theta} &= -\Lambda_{v,r} \left(\frac{\partial \dot{r}}{\partial \theta} \right) - \Lambda_{v,\theta} \left(\frac{\partial \dot{\theta}}{\partial \theta} \right) - \Lambda_{v,\phi} \left(\frac{\partial \dot{\phi}}{\partial \theta} \right) - \Lambda_{v,v} \left(\frac{\partial \dot{v}}{\partial \theta} \right) - \Lambda_{v,\gamma} \left(\frac{\partial \dot{\gamma}}{\partial \theta} \right) - \Lambda_{v,\psi} \left(\frac{\partial \dot{\psi}}{\partial \theta} \right) - \Lambda_{v,\rho_0} \left(\frac{\partial \dot{\rho}_0}{\partial \theta} \right) \\
\dot{\Lambda}_{v,\phi} &= -\Lambda_{v,r} \left(\frac{\partial \dot{r}}{\partial \phi} \right) - \Lambda_{v,\theta} \left(\frac{\partial \dot{\theta}}{\partial \phi} \right) - \Lambda_{v,\phi} \left(\frac{\partial \dot{\phi}}{\partial \phi} \right) - \Lambda_{v,v} \left(\frac{\partial \dot{v}}{\partial \phi} \right) - \Lambda_{v,\gamma} \left(\frac{\partial \dot{\gamma}}{\partial \phi} \right) - \Lambda_{v,\psi} \left(\frac{\partial \dot{\psi}}{\partial \phi} \right) - \Lambda_{v,\rho_0} \left(\frac{\partial \dot{\rho}_0}{\partial \phi} \right) \\
\dot{\Lambda}_{v,v} &= -\Lambda_{v,r} \left(\frac{\partial \dot{r}}{\partial v} \right) - \Lambda_{v,\theta} \left(\frac{\partial \dot{\theta}}{\partial v} \right) - \Lambda_{v,\phi} \left(\frac{\partial \dot{\phi}}{\partial v} \right) - \Lambda_{v,v} \left(\frac{\partial \dot{v}}{\partial v} \right) - \Lambda_{v,\gamma} \left(\frac{\partial \dot{\gamma}}{\partial v} \right) - \Lambda_{v,\psi} \left(\frac{\partial \dot{\psi}}{\partial v} \right) - \Lambda_{v,\rho_0} \left(\frac{\partial \dot{\rho}_0}{\partial v} \right) \\
\dot{\Lambda}_{v,\gamma} &= -\Lambda_{v,r} \left(\frac{\partial \dot{r}}{\partial \gamma} \right) - \Lambda_{v,\theta} \left(\frac{\partial \dot{\theta}}{\partial \gamma} \right) - \Lambda_{v,\phi} \left(\frac{\partial \dot{\phi}}{\partial \gamma} \right) - \Lambda_{v,v} \left(\frac{\partial \dot{v}}{\partial \gamma} \right) - \Lambda_{v,\gamma} \left(\frac{\partial \dot{\gamma}}{\partial \gamma} \right) - \Lambda_{v,\psi} \left(\frac{\partial \dot{\psi}}{\partial \gamma} \right) - \Lambda_{v,\rho_0} \left(\frac{\partial \dot{\rho}_0}{\partial \gamma} \right) \\
\dot{\Lambda}_{v,\psi} &= -\Lambda_{v,r} \left(\frac{\partial \dot{r}}{\partial \psi} \right) - \Lambda_{v,\theta} \left(\frac{\partial \dot{\theta}}{\partial \psi} \right) - \Lambda_{v,\phi} \left(\frac{\partial \dot{\phi}}{\partial \psi} \right) - \Lambda_{v,v} \left(\frac{\partial \dot{v}}{\partial \psi} \right) - \Lambda_{v,\gamma} \left(\frac{\partial \dot{\gamma}}{\partial \psi} \right) - \Lambda_{v,\psi} \left(\frac{\partial \dot{\psi}}{\partial \psi} \right) - \Lambda_{v,\rho_0} \left(\frac{\partial \dot{\rho}_0}{\partial \psi} \right) \\
\dot{\Lambda}_{v,\rho_0} &= -\Lambda_{v,r} \left(\frac{\partial \dot{r}}{\partial \rho_0} \right) - \Lambda_{v,\theta} \left(\frac{\partial \dot{\theta}}{\partial \rho_0} \right) - \Lambda_{v,\phi} \left(\frac{\partial \dot{\phi}}{\partial \rho_0} \right) - \Lambda_{v,v} \left(\frac{\partial \dot{v}}{\partial \rho_0} \right) - \Lambda_{v,\gamma} \left(\frac{\partial \dot{\gamma}}{\partial \rho_0} \right) - \Lambda_{v,\psi} \left(\frac{\partial \dot{\psi}}{\partial \rho_0} \right) - \Lambda_{v,\rho_0} \left(\frac{\partial \dot{\rho}_0}{\partial \rho_0} \right) \\
\dot{\Lambda}_{\gamma,r} &= -\Lambda_{\gamma,r} \left(\frac{\partial \dot{r}}{\partial r} \right) - \Lambda_{\gamma,\theta} \left(\frac{\partial \dot{\theta}}{\partial r} \right) - \Lambda_{\gamma,\phi} \left(\frac{\partial \dot{\phi}}{\partial r} \right) - \Lambda_{\gamma,v} \left(\frac{\partial \dot{v}}{\partial r} \right) - \Lambda_{\gamma,\gamma} \left(\frac{\partial \dot{\gamma}}{\partial r} \right) - \Lambda_{\gamma,\psi} \left(\frac{\partial \dot{\psi}}{\partial r} \right) - \Lambda_{\gamma,\rho_0} \left(\frac{\partial \dot{\rho}_0}{\partial r} \right) \\
\dot{\Lambda}_{\gamma,\theta} &= -\Lambda_{\gamma,r} \left(\frac{\partial \dot{r}}{\partial \theta} \right) - \Lambda_{\gamma,\theta} \left(\frac{\partial \dot{\theta}}{\partial \theta} \right) - \Lambda_{\gamma,\phi} \left(\frac{\partial \dot{\phi}}{\partial \theta} \right) - \Lambda_{\gamma,v} \left(\frac{\partial \dot{v}}{\partial \theta} \right) - \Lambda_{\gamma,\gamma} \left(\frac{\partial \dot{\gamma}}{\partial \theta} \right) - \Lambda_{\gamma,\psi} \left(\frac{\partial \dot{\psi}}{\partial \theta} \right) - \Lambda_{\gamma,\rho_0} \left(\frac{\partial \dot{\rho}_0}{\partial \theta} \right) \\
\dot{\Lambda}_{\gamma,\phi} &= -\Lambda_{\gamma,r} \left(\frac{\partial \dot{r}}{\partial \phi} \right) - \Lambda_{\gamma,\theta} \left(\frac{\partial \dot{\theta}}{\partial \phi} \right) - \Lambda_{\gamma,\phi} \left(\frac{\partial \dot{\phi}}{\partial \phi} \right) - \Lambda_{\gamma,v} \left(\frac{\partial \dot{v}}{\partial \phi} \right) - \Lambda_{\gamma,\gamma} \left(\frac{\partial \dot{\gamma}}{\partial \phi} \right) - \Lambda_{\gamma,\psi} \left(\frac{\partial \dot{\psi}}{\partial \phi} \right) - \Lambda_{\gamma,\rho_0} \left(\frac{\partial \dot{\rho}_0}{\partial \phi} \right) \\
\dot{\Lambda}_{\gamma,v} &= -\Lambda_{\gamma,r} \left(\frac{\partial \dot{r}}{\partial v} \right) - \Lambda_{\gamma,\theta} \left(\frac{\partial \dot{\theta}}{\partial v} \right) - \Lambda_{\gamma,\phi} \left(\frac{\partial \dot{\phi}}{\partial v} \right) - \Lambda_{\gamma,v} \left(\frac{\partial \dot{v}}{\partial v} \right) - \Lambda_{\gamma,\gamma} \left(\frac{\partial \dot{\gamma}}{\partial v} \right) - \Lambda_{\gamma,\psi} \left(\frac{\partial \dot{\psi}}{\partial v} \right) - \Lambda_{\gamma,\rho_0} \left(\frac{\partial \dot{\rho}_0}{\partial v} \right) \\
\dot{\Lambda}_{\gamma,\gamma} &= -\Lambda_{\gamma,r} \left(\frac{\partial \dot{r}}{\partial \gamma} \right) - \Lambda_{\gamma,\theta} \left(\frac{\partial \dot{\theta}}{\partial \gamma} \right) - \Lambda_{\gamma,\phi} \left(\frac{\partial \dot{\phi}}{\partial \gamma} \right) - \Lambda_{\gamma,v} \left(\frac{\partial \dot{v}}{\partial \gamma} \right) - \Lambda_{\gamma,\gamma} \left(\frac{\partial \dot{\gamma}}{\partial \gamma} \right) - \Lambda_{\gamma,\psi} \left(\frac{\partial \dot{\psi}}{\partial \gamma} \right) - \Lambda_{\gamma,\rho_0} \left(\frac{\partial \dot{\rho}_0}{\partial \gamma} \right)
\end{aligned}$$

$$\begin{aligned}\dot{\Lambda}_{\gamma,\psi} &= -\Lambda_{\gamma,r} \left(\frac{\partial \dot{r}}{\partial \psi} \right) - \Lambda_{\gamma,\theta} \left(\frac{\partial \dot{\theta}}{\partial \psi} \right) - \Lambda_{\gamma,\phi} \left(\frac{\partial \dot{\phi}}{\partial \psi} \right) - \Lambda_{\gamma,v} \left(\frac{\partial \dot{v}}{\partial \psi} \right) - \Lambda_{\gamma,\gamma} \left(\frac{\partial \dot{\gamma}}{\partial \psi} \right) - \Lambda_{\gamma,\psi} \left(\frac{\partial \dot{\psi}}{\partial \psi} \right) - \Lambda_{\gamma,\rho_0} \left(\frac{\partial \dot{\rho}_0}{\partial \psi} \right) \\ \dot{\Lambda}_{\gamma,\rho_0} &= -\Lambda_{\gamma,r} \left(\frac{\partial \dot{r}}{\partial \rho_0} \right) - \Lambda_{\gamma,\theta} \left(\frac{\partial \dot{\theta}}{\partial \rho_0} \right) - \Lambda_{\gamma,\phi} \left(\frac{\partial \dot{\phi}}{\partial \rho_0} \right) - \Lambda_{\gamma,v} \left(\frac{\partial \dot{v}}{\partial \rho_0} \right) - \Lambda_{\gamma,\gamma} \left(\frac{\partial \dot{\gamma}}{\partial \rho_0} \right) - \Lambda_{\gamma,\psi} \left(\frac{\partial \dot{\psi}}{\partial \rho_0} \right) - \Lambda_{\gamma,\rho_0} \left(\frac{\partial \dot{\rho}_0}{\partial \rho_0} \right)\end{aligned}$$

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