

Considerations for Optimal Sensor Placement for Higher Accuracy Object Localization for Urban Air Mobility

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Previous research into object localization has shown that sensor placement and alignment plays an important role in achieving higher accuracy levels of the estimated location of a tracked Urban Air Mobility Vehicle. In general, a near-orthogonal intersection between the ground node observation vectors results in the highest accuracy due to a smaller overlapping uncertainty region between both. This applies to triangulation by means of ground node camera angle observations as well as trilateration by means of ground node distance measurements. However, this simple concept is not easily fulfilled with a network of a limited number of static ground nodes and a moving object to be localized. This case study performs sensitivity analyses and explores practical ways on how to achieve higher estimate accuracy levels in this context.

I. Introduction

RECENTLY, there have been numerous developments in the area of advanced air taxi operations. Combined with other novel transportation concepts such as autonomous aerial cargo delivery etc., they form the Advanced Air Mobility (AAM) ecosystem that will help to meet the challenging mobility needs of an increasing population density in metropolitan areas, taking into account the limited capacity of the currently available transportation infrastructure. To realize the full vision of AAM, technology will be needed to allow a few operators to operate many vehicles (m:N). As most m:N implementations will require highly autonomous vehicles, the current state-of-the-art will have to advance for on-board autonomous functionality. One of the key areas requiring further technology development is sensor-based perception techniques. For this perception, techniques are being developed to address problems such as GPS-free Alternative Positioning, Navigation and Timing (APNT) [1]. Advanced sensing and tracking algorithms will be necessary from the ground in an urban environment, where traditional traffic tracking infrastructure such as air traffic radars will be useless. This infrastructure will be needed for traffic monitoring, traffic guidance, as well as spacing and separation in order to increase overall capacity and efficiency in the airspace. In this context, ground-based perception systems, capable of APNT, are critical to safe autonomous operations. This APNT relates to all moving objects in the sky, which can be cooperative (aircraft, AAM vehicles, etc.) as well as noncooperative (smaller drones, birds, ...). The distinction between both lies in the fact that there is some form of data exchange (ADS-B or similar) with the former category in order to coordinate path planning and separation. This exchange of information is nonexistent for the latter category, which means that other sensors will be required to detect and track these objects independently from the ground. Ground-based object detection and tracking are two essential prerequisites for the purpose of traffic monitoring, and also to provide some off-board redundancy for detect and avoid (DAA). For multi-target tracking, there is an additional challenge of observation-to-track association.

Previous research has shown preliminary results for localization of a flying target in different contexts, namely:

- from another flying vehicle, evaluated in a simulation environment, [2]
- from a distributed network of ground nodes (cameras and radars), evaluated in a simulation environment [3]
- from a distributed network of ground nodes (cameras and RF antennas), evaluated in a simulation environment and field tests [4]

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This research fits in the larger project context of Perception and Distributed Sensing as elaborated in the following references. [5] elaborates on distributed sensing and advanced perception technologies to enable Advanced Air Mobility (AAM). [6] describes the simulation environment that was used to evaluate the sensor fusion approach that was developed in [3]. [7] and [8] discuss the field test set up and the sensor payload and its flight test configuration that were relevant to the work presented in [4]. [9] elaborates on the image processing of a ground based vision tracker, where its output is used as raw sensor input data by the sensor fusion approach that was discussed in [4].

Some major observations in the aforementioned research were the need for proper sensor calibration and accurate sensor boresight alignment as well as the need for optimal sensor placement with sufficient overlapping coverage as well as a well-defined geometry as defined by the ground node locations and the localized object. The ultimate goal is to achieve equivalent or better accuracy than GPS (1 to 3 meter level), which is not yet achieved in the aforementioned research. The work in this publication aims to increase these accuracy levels. In cruise, 1 to 3 meter precision levels are expected to be adequate, but higher accuracy (1m and lower) may be needed for specific phases of flight, such as takeoff and landing. Especially for higher density operations around the vertiports, where GPS signals degrade in an urban environment, APNT techniques have the potential to become enabling technologies.

Fig. 1 shows typical fitting error value levels for the simulation based evaluations (Fig. 1(a)) as well as for the field tests (Fig. 1(b)), which are respectively around 2 – 10 m for the synthetic scenarios and 10 – 40 m in real life (without compensation for lens distortion).

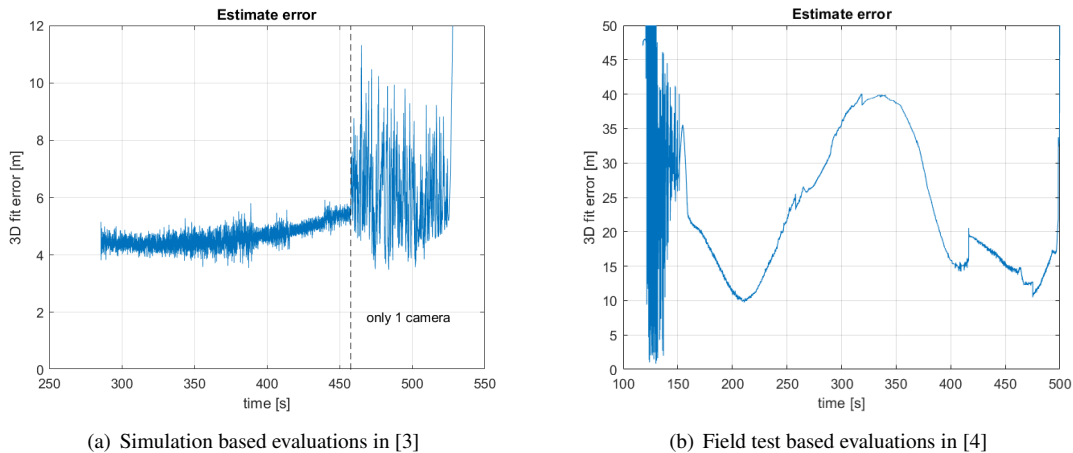


Fig. 1 Typical fitting error value levels for previous research results

These error values are clearly not yet at the intended accuracy level. The trends in these error values show correlations with the number of ground nodes that are observing the target at that time instant, as well as with the topology of the ground nodes, i.e. the relative angles between the lines connecting the ground nodes with the tracked object. The latter help to distinguish between well-posed and ill-posed geometries, are related to optimal sensor placement, and will be an important topic of this paper.

Other research into optimal sensor placement for object tracking in the context of surveillance and monitoring can be found in the following references. [10] studies optimal sensor placement and motion coordination strategies for mobile sensor networks. For a target-tracking application with range sensors, this reference investigates the determinant of the Fisher Information Matrix and computes it in the 2D and 3D cases. They also propose motion coordination algorithms in this context. The research presented here, however, assumes (at least some) static ground nodes. [11] analytically characterizes optimal sensor placement for target localization and tracking in 2D and 3D. Three types of sensors are considered: bearing-only, range-only, and received-signal-strength. The optimal placement problems of the three sensor types are formulated as an identical parameter optimization problem and consequently analyzed in a unified framework. [12] gives a closed-form formula for the optimal range sensor placement problem, by minimizing the errors both in the position and velocity estimates when there are two sensors. Using the formula, the effects of errors in a prior position and velocity estimates of a target and tracking filter parameters on the optimal sensor placement are analyzed. [13] develops sensor selection strategies for 3D wireless positioning based on time of arrival (TOA) and

received signal strength (RSS) measurements to handle two distinct scenarios: (i) known approximated target location, for which dynamic sensor selection is conducted to minimize the positioning error; and (ii) unknown approximated target location, in which the worst-case positioning error is minimized via robust sensor selection. Expressions are derived for the Cramér-Rao lower bound (CRLB) as a performance metric to quantify the positioning accuracy resulted from selected sensors.

Four areas in surveillance and navigation were identified where optimal sensor placement will contribute to higher accuracy:

- Vertiport airspace surveillance (immediate airspace near the vertiport, including vertiport surface movement surveillance)
- Approach and departure corridor airspace surveillance (the approach/departure paths into/out-of vertiports)
- Regional airspace surveillance (including enroute corridors)
- Precision navigation (including enroute, approach and landing)

The distinction between surveillance and navigation is primarily that surveillance is ground-based and mostly independent from the localized object to account for non-cooperative objects. Navigation on the other hand is vehicle centric and should be seen in the wider context of the GNC challenges of the vehicle in a GPS-denied or degraded environment.

II. Optimality criteria for sensor placement

Object localization relies on the concepts of triangulation and trilateration as previously elaborated in general in [4]. However, the geometry of the localization problem has a major influence on the accuracy of the position estimate as will be illustrated next.

A. Observation vector distances and intersection angles

Fig. 2 highlights the contrast between an ill-posed and a well-posed geometry for triangulation of camera observations, which are based on intersecting angles. Fig. 2(a) illustrates that a dual camera angle either close to 0 or 180 deg between the vectors connecting the target with the two ground nodes results in a significantly larger area of overlap between the two observation cones (the dashed line being the true bearing $\psi_\bullet$ and the blue cone is the uncertainty region due to among others sensor noise). However, an intersection angle of around 90 deg between these two vectors results in an optimal smallest area of overlap of these cones, as shown in Fig. 2(b). In addition, a shorter distance between the ground nodes and the target results in a shorter cross range uncertainty distance, since these uncertainties are based on angular specifications for noise and alignment errors.

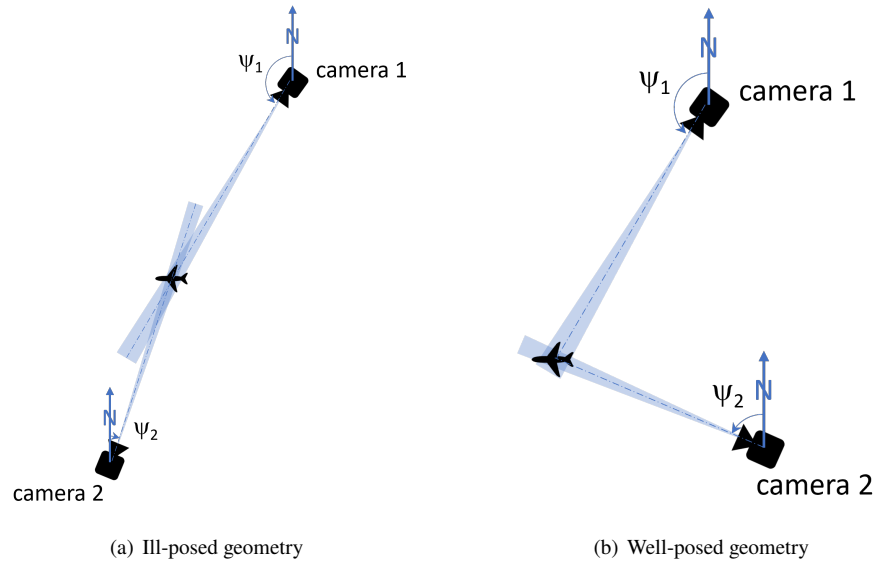


Fig. 2 Geometries for triangulation between ground node angle measurements

Similarly, Fig. 3 highlights the contrast between an ill-posed and a well-posed geometry for trilateration of distance measurements, which are based on intersecting spheres. Fig. 3(a) illustrates that a dual transmitter angle close to either 0 or 180 deg between the vectors connecting the target with the two ground nodes results in a significantly larger area of overlap between the two observation spheres (the dashed line being the true distance R_0 and the blue thicker sphere is the uncertainty region due to among others sensor noise). However, an intersection angle of around 90 deg between these two spheres results in an optimal smallest area of overlap of these spheres, as shown in Fig. 3(b). Please note that a third observation is still needed here for this two dimensional localization problem, in order to eliminate the secondary ‘empty’ location candidate as illustrated on the right side of Fig. 3(b). Three dimensional localization problems require four simultaneous observations to eliminate ‘false’ detected locations. Also here, a shorter distance between the ground nodes and the target typically results in a narrower uncertainty band, since magnitude of these uncertainties depends on either the time-of-flight or the distance to noise ratio of the signal [4].

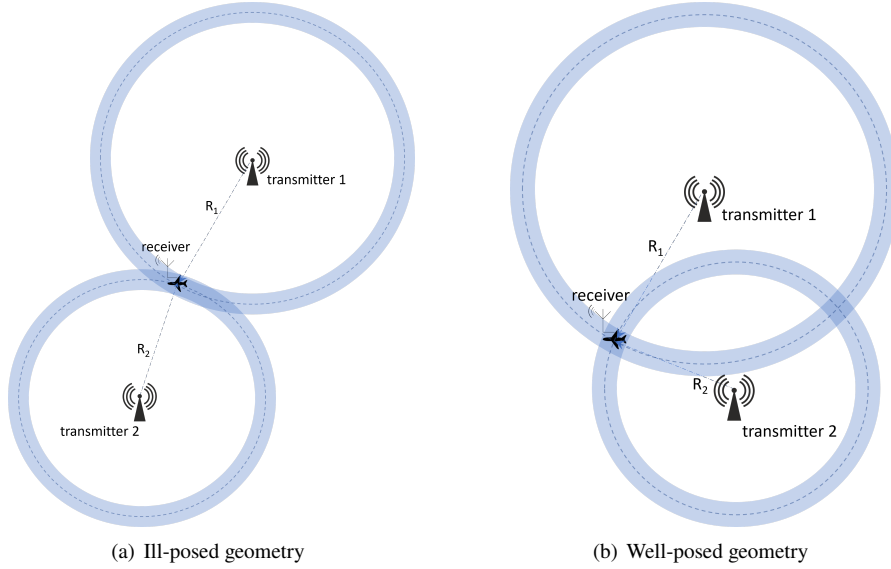


Fig. 3 Geometries for trilateration between ground node distance measurements

Based on the aforementioned findings, one can state that an intersection angle between observation vectors of around 90 deg results in the highest accuracy due to a smallest area of overlap. This means that for any two ground nodes, the circle that encompasses both locations of the ground nodes shows all the locations where highest accuracy triangulation or trilateration can be achieved. This observation relies on the theorem that the angle inscribed in a semicircle is always a right angle, as illustrated in Fig. 4. Moreover, narrower observation vectors also result in a smaller area of overlap, indicating an advantage for shorter observation distances. In the remainder of this paper, this will be the central crux in the context of considerations for optimal sensor placement.

Based on these observations, the distance between tracked object and the ground nodes, together with the angle defined by their respective observation vectors, are the main variables for analysis as well as optimization purposes. The inner vector product serves as an efficient evaluation metric for this purpose, since it considers the distance of each of two vectors, together with the angle between them. For an existing track and ground node geometry, it permits to analyze if the geometry is ill- or well-posed. And otherwise for a given track, it can serve as an optimality criterion for optimal sensor placement.

B. Sensitivity analysis

Define $\mathbf{T}$ as the vector from the origin to the tracked object, and vectors $\mathbf{A}$ and $\mathbf{B}$ define the locations of the ground nodes. Then the vectors from tracked object towards the ground nodes are defined by:

$$\mathbf{TA} = \mathbf{A} - \mathbf{T} \quad (1)$$

$$\mathbf{TB} = \mathbf{B} - \mathbf{T} \quad (2)$$

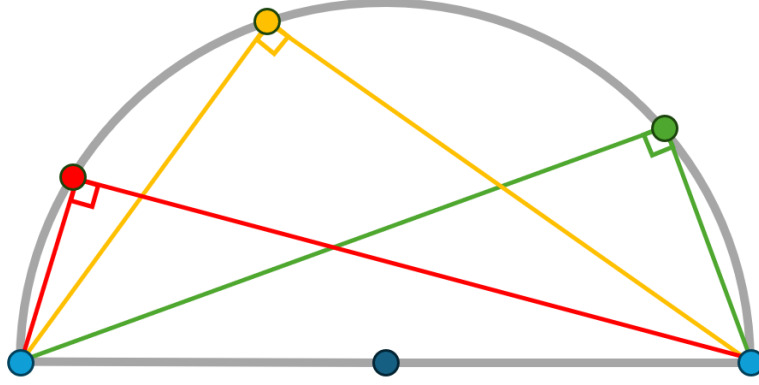


Fig. 4 The angle inscribed in a semicircle is always a right angle

and $\psi_{\hat{\mathbf{A}}\hat{\mathbf{T}}\hat{\mathbf{B}}}$ represents the angle defined by the two vectors, connecting the tracked object with the ground nodes.

This set up allows to perform a sensitivity analysis that indicates the magnitude of the expected estimate errors based on observation range and intersection angle.

Finding the worst case expected estimate error corresponds to calculating half the length of the longest diagonal of a parallelogram with variable base length and variable interior angles. Two separate cases are considered for completeness: equidistant cases from the tracked object to the ground nodes as a best case scenario, where the parallelogram becomes a rhombus, and otherwise non-equidistant cases where the other side of the parallelogram is the longest possible, reflecting the worst case scenario.

The base of the parallelogram depends on the distance between ground node and tracked object $\|\mathbf{TA}\|$, and the standard deviation for the angular measurements at the ground node σ_A as follows:

$$\Delta_{\mathbf{TA}} = 2 \times \|\mathbf{TA}\| \tan \sigma_A \quad (3)$$

The length of the other side of the parallelogram depends on which one of the two aforementioned cases is considered:

- 1) equidistant case from tracked object to ground nodes (minimum length scenario): overlap area becomes rhombus

$$\Delta_{\mathbf{TB}} = \Delta_{\mathbf{TA}} \quad (4)$$

- 2) non-equidistant case from tracked object to ground nodes (maximum length scenario): overlap area similar to parallelogram:

$$\Delta_{\mathbf{TB}} = \min \left(\frac{2 \times R_{\max} \tan \sigma_A}{|\cos(\frac{\pi}{2} - \psi_{\hat{\mathbf{A}}\hat{\mathbf{T}}\hat{\mathbf{B}}})|}, R_{\max} \right) \quad (5)$$

where $R_{\max}$ is the maximum detection range of the sensor. This is also the maximum length of this side of the parallelogram.

One can calculate both the long and short diagonals of the parallelogram by means of the cosine rule:

$$\text{diag}_1 = \sqrt{\Delta_{\mathbf{TA}}^2 + \Delta_{\mathbf{TB}}^2 - 2\Delta_{\mathbf{TA}}\Delta_{\mathbf{TB}} \cos(\pi - \psi_{\hat{\mathbf{A}}\hat{\mathbf{T}}\hat{\mathbf{B}}})} \quad (6)$$

$$\text{diag}_2 = \sqrt{\Delta_{\mathbf{TA}}^2 + \Delta_{\mathbf{TB}}^2 - 2\Delta_{\mathbf{TA}}\Delta_{\mathbf{TB}} \cos(\psi_{\hat{\mathbf{A}}\hat{\mathbf{T}}\hat{\mathbf{B}}})} \quad (7)$$

The worst case expected estimate error is then half the length of the longest diagonal, but cannot be longer than the longest possible detection range $R_{\max}$:

$$\Delta_{\mathbf{R}_{\max}} = \min \left(\frac{\max(\text{diag}_1, \text{diag}_2)}{2}, R_{\max} \right) \quad (8)$$

The actual worst case estimate error is expected to be somewhere in the range between the equidistant and non-equidistant cases as described above.

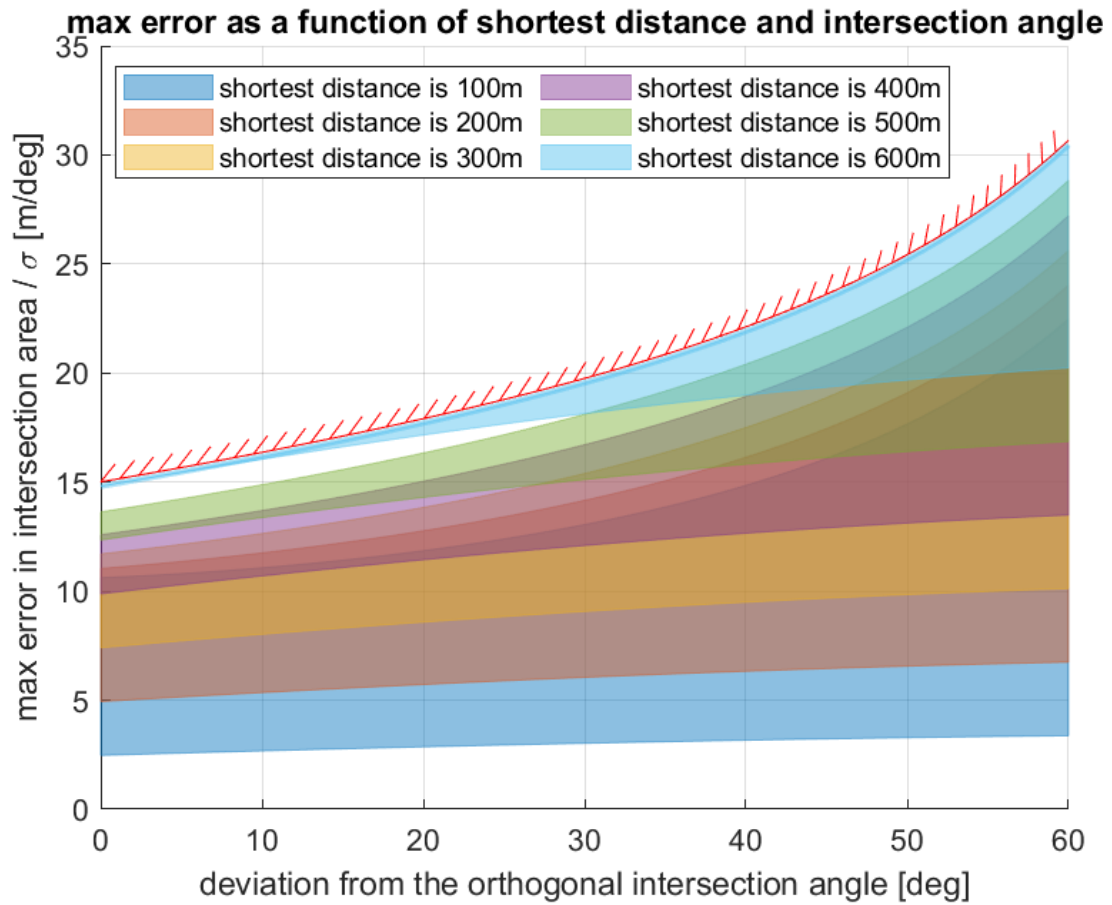


Fig. 5 Sensitivity analysis of the worst case expected estimate error as a function of the shortest distance to the closest ground node and the deviation from the ideal orthogonal intersection angle

The results of this sensitivity analysis are shown in Fig. 5. This graph shows the worst case expected estimate error as a function of the shortest distance to the closest ground node and the deviation from the ideal orthogonal intersection angle. Differences between equal (rhombus) and non-equal (parallelogram) distances between tracked target and ground nodes causes the wider range for larger deviation angles. The red hatched line on top of the graph shows the boundary imposed by sensing limit range of 600m. The worst case error is normalized by the standard deviation for scaling purposes.

Based on the results presented in Fig. 5, by proper scaling one can assess the order of magnitude of the worst case error for various errors, such as the expected noise levels in the ground node angular measurements and the ground node viewing angle alignment errors. For the former category, one needs to multiply the error values from Fig. 5 by the magnitude of σ_A . The standard deviation for the angular measurements at the ground node was chosen as $\sigma_A = 4.4 \times 10^{-3}$ deg, based on field test results in [4]. This shows that when the shortest distance to the closest ground node is 100m, then the best case max estimate error (equidistant to the ground nodes) is 1cm and worst case error is 5cm (second ground node at max range) for orthogonal intersection angles and barely more best case but up to 10cm worst case when the intersection angle deviates 60 deg from the optimal orthogonal intersection. However, at 600m distance from the closest ground node, this estimate error grows up to 6.5cm for orthogonal intersection angles and between 9cm and 13cm when the intersection angles deviates 60 deg from orthogonality. At a shorter distance of 400m, this estimate error at 60 deg deviation angle can vary between 6cm (equidistant) and 12cm (non-equidistant). These errors are fairly small and acceptable for the aforementioned required levels of estimate accuracy (1 to 3m). However, viewing angle alignment errors are usually of another order of magnitude. An error of 1 deg in estimated azimuth and/or elevation in the ground node viewing angle is not unusual. This will result in estimate errors between 2.5m and 11m (orthogonal intersection) and even between 3m and 22m (60 deg from orthogonality) at a shortest distance of 100m. At 600m shortest distance, these estimate errors will scale up to 15m (orthogonal intersection) and even between 20m and 30m when the intersection angles deviates 60 deg from orthogonality. At a shorter distance of 400m, this estimate error at 60 deg deviation angle can vary between 13m (equidistant) and 27m (non-equidistant) when the intersection angle deviates 60 deg from orthogonality. As a last observation from Fig. 5, one can state that, if the estimated position has to be accurate within 1m of the tracked target in worst case conditions (here 600m distance at 60 deg deviation from an orthogonal intersection angle), then the ground node viewing angle estimates have to be precise up to 0.03 deg. This is a very strict requirement, given the available equipment. Significantly better performance is achieved at shorter range and closer to orthogonal intersection angles. This is more likely to happen in a higher density environment of ground nodes, as will be illustrated in the scenario.

III. Examples

A. Calculating location based worst case errors for a pair of ground nodes

One can calculate the worst case errors for a grid of locations, given an outline of a pair of ground nodes as shown in Fig. 6.

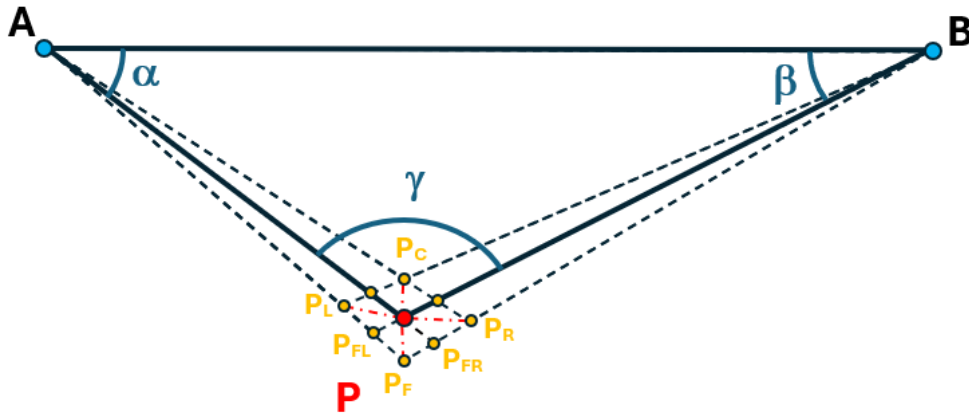


Fig. 6 Outline of locations and angles for an uncertain gridpoint and two ground nodes

The position vectors of the two ground nodes are defined as $\mathbf{A}$ and $\mathbf{B}$. The grid point is in location $\mathbf{P}$. The inscribed angles inside the triangle defined by the three aforementioned locations are respectively α , β and γ . However, due to noise, alignment errors etc., one can expect deviations from the true angles α and β , with their extreme values defined as α^+ , α^- , β^+ and β^- :

$$\alpha^+ = \alpha + \delta \quad (9)$$

$$\alpha^- = \alpha - \delta \quad (10)$$

$$\beta^+ = \beta + \delta \quad (11)$$

$$\beta^- = \beta - \delta \quad (12)$$

Due to these deviations, the measured grid point can be situated anywhere inside the generic quadrangle defined by the corner points $\mathbf{P}_C$ (close), $\mathbf{P}_F$ (far), $\mathbf{P}_L$ (left) and $\mathbf{P}_R$ (right). The maximum deviation is then defined as:

$$\max \Delta \mathbf{P} = \max\{\|\mathbf{P} - \mathbf{P}_C\|, \|\mathbf{P} - \mathbf{P}_F\|, \|\mathbf{P} - \mathbf{P}_L\|, \|\mathbf{P} - \mathbf{P}_R\|\} \quad (13)$$

One can calculate the length of each aforementioned segment in Eq. (13) by means of the cosine rule, for example for the left corner point $\mathbf{P}_L$:

$$\|\mathbf{P} - \mathbf{P}_L\| = \sqrt{\|\mathbf{P}_{FL} - \mathbf{P}\|^2 + \|\mathbf{P}_{FL} - \mathbf{P}_L\|^2 - 2\|\mathbf{P}_{FL} - \mathbf{P}\|\|\mathbf{P}_{FL} - \mathbf{P}_L\| \cos \gamma_{FL}} \quad (14)$$

One can calculate the length of each aforementioned subsegment in Eq. (14) by subtracting the relevant sides of the corresponding triangles, for example for the subsegment $\|\mathbf{P}_{FL} - \mathbf{P}_L\|$:

$$\|\mathbf{P}_{FL} - \mathbf{P}_L\| = \|\mathbf{P}_{FL} - \mathbf{A}\| - \|\mathbf{P}_L - \mathbf{A}\| \quad (15)$$

The magnitude of the angle γ_{FL} in Eq. (14) is calculated by means of the corresponding triangle:

$$\gamma_{FL} = 180^\circ - \alpha^+ - \beta \quad (16)$$

Each length in Eq. (15) is calculated by means of the sine rule:

$$\|\mathbf{P}_{FL} - \mathbf{A}\| = \|\mathbf{B} - \mathbf{A}\| \cdot \frac{\sin \beta}{\sin \gamma_{FL}} \quad (17)$$

$$\|\mathbf{P}_L - \mathbf{A}\| = \|\mathbf{B} - \mathbf{A}\| \cdot \frac{\sin \beta^-}{\sin \gamma_L} \quad (18)$$

with $\gamma_L = 180^\circ - \alpha^+ - \beta^-$. This procedure gives for any location in a predefined grid the worst case localization fit error due to inaccuracies in the observed angles in the ground nodes. These errors are not only dependent on the magnitude of the inaccuracies in the observed angles, but also in the first place on the magnitude of the inscribed angle γ and also on the distances from the respective ground nodes (GNs). Two examples will illustrate this.

As an illustrating example, the above equations were applied to a horizontal ground level as well as a tilted surface situation where the ground nodes are located in the points $\mathbf{A} = (150; 120; 0)$ and $\mathbf{B} = (-150; -120; 0)$. The to-be-localized object can be anywhere in the box $(-500; -500) - (500; 500)$ at a sloped altitude linearly varying between 30 and 170m. This is the setup for the 3D scenario. For the simplified 2D scenario, all z values are set equal to zero. The corresponding altitude profiles are shown in Fig. 7.

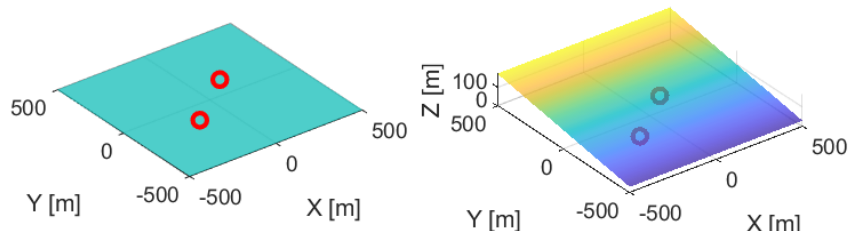


Fig. 7 Altitude profiles for the 2D and 3D scenarios

1. Horizontal ground level surface scenario

Fig. 8 shows the uncertainty range (worst case fitting error) for the area around a pair of GNs as described above. The surface area under investigation is at ground level, just like the two GNs, as shown in the left in Fig. 7. For this simplified 2D scenario, the uncertainty range gets larger with distance from the GNs, but also varies with the intersection angle of the observation vectors from the GNs. The smallest errors of around 8m occur at the optimal intersection angle of 90 deg, which results in a light purple circle centered in the halfway point between the two ground nodes that encompasses the locations of both ground nodes, based on the aforementioned theorem. In the darker purple area around this circle, one can observe the area with slightly increasing errors but still within 1m max deviation from the smallest error for 1 deg deviation of the observation angles from the GNs. Please note that these graphs do not take into account limitations from field of view. GN locations are represented by the small red circles. The maximum observation range of 600m is taken into account. However, near the straight line connecting the two ground nodes, one observes significantly increased errors. These locations correspond to an intersection angle of either 0 deg (top right or bottom left) or 180 deg (in between the two ground nodes), which will result in a localization error up to the maximum range of the sensors. This is illustrated in the sketch in Fig. 2(a). Finally, Fig. 8 can also serve to derive sensor alignment specifications to achieve a certain level of required accuracy. For example, sufficiently far off the straight line connecting the ground nodes and off the edges of the detection range, the uncertainty range goes up to around 20m for alignment errors around 1 deg. If one would require a minimum accuracy of 1m, that would translate into a requirement for GN angular alignment errors of max 0.05 deg for this specific set up.

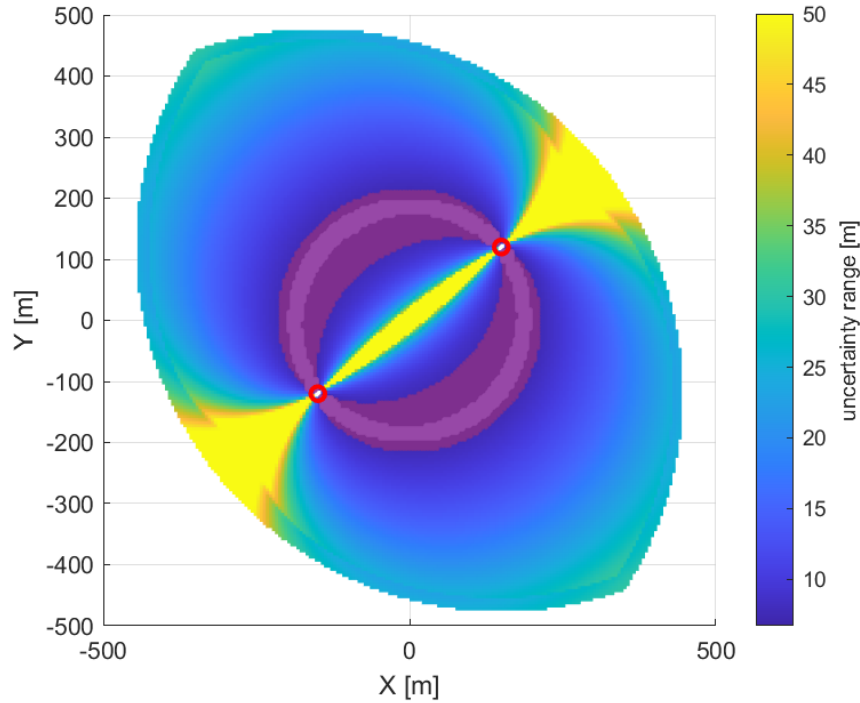


Fig. 8 Optimal localization zones for a 2D scenario

2. Tilted surface scenario

Fig. 9 shows the uncertainty range (worst case fitting error) for the tilted surface scenario with the sloped flight path, with similar observations as before: the optimal angle of 90 deg results in a circle encompassing the two ground nodes, with 8m error. In the nearby area around this circle, one can observe the slightly increasing error up to 1m extra. Please note that these graphs do not take into account limitations from field of view. GN locations are represented by the red circles. The maximum observation range of 600m is taken into account. Thanks to the third dimension, there are fewer 'singularity' locations with an intersection angle of either 0 deg or 180 deg. Only the lower left and upper right corners

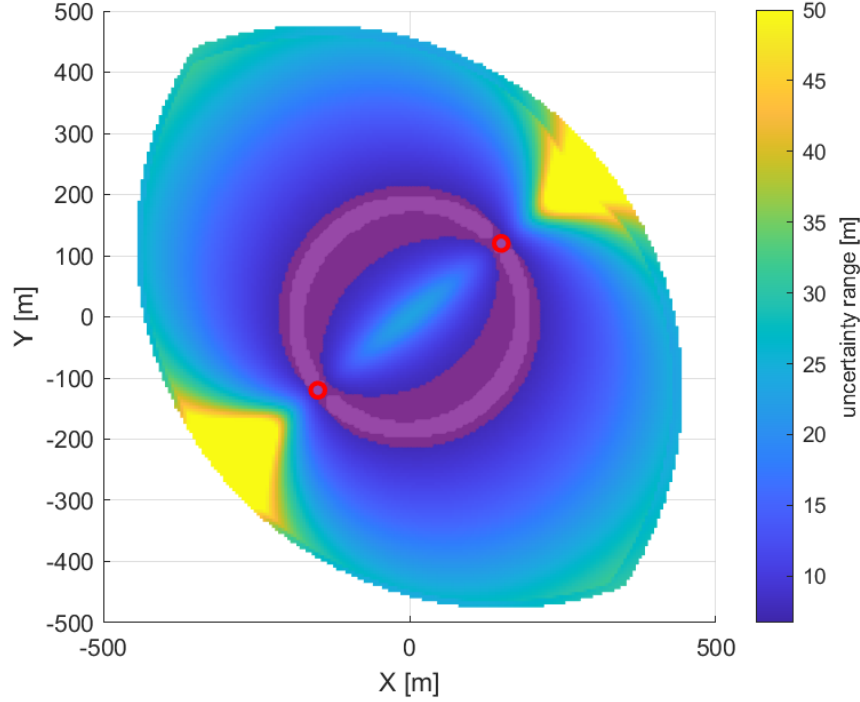


Fig. 9 Optimal localization zones for a 3D scenario

show an increase. The increase in uncertainty is very minor in between the GNs. Comparing Fig. 9 with Fig. 8 shows that the third dimension eliminates a large portion of the region with large errors.

IV. Modification of Kalman Filter to prevent overcorrection in the correction step

Since the Kalman Filter uses a gradient based optimization approach, it has the tendency to overcompensate significantly during the initial convergence phase, even when the initial condition for the estimated state is chosen in the vicinity of the starting point. Due to this overcompensating behavior, the estimate takes a long time to finally converge. Therefore, the Kalman gain matrix $\mathbf{W}$ is rescaled so that its most extreme matrix entry value doesn't exceed a certain threshold $\varepsilon_{\text{limit}}$ while preserving sign and relative differences between the matrix values so that the gradient direction is maintained. The Kalman Filter procedure was described in detail in previous publications [3, 4] and is included, with rescaling modification, in the Appendix. For this specific application, the threshold was defined as: $\varepsilon_{\text{limit}} = 6 \times 10^3$, based on closer inspection of overcompensating behavior in some scenarios.

V. Sensor Characteristics, Measurement Processing and Data Calibration

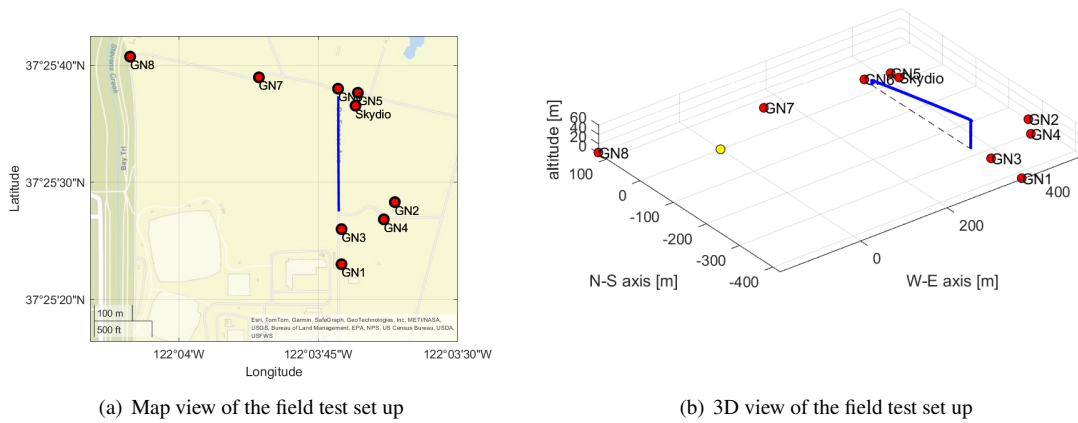
Only one type of sensor is considered in this study, namely electro-optical cameras. Radars, wireless RF equipment and distance measuring equipment (DME) were considered in previous work [3, 4] and can be modeled in the Kalman Filter observation models, but were not incorporated for this study. Cameras provide azimuth and elevation only and have specific sensor characteristics, such as limitations on field of view and useful range, noise, latency, update rate and resolution. Some characteristic values from the literature of state-of-the-art sensor hardware [14] are given in Table 1. [15] provides more details on sensor hardware. Specific details of the sensor orientations and measurement readings for the specific scenario considered in this study are given in Sec. VI for the field tests.

Table 1 Sensor characteristics

sensor	EO camera
sensed variables	azimuth [deg] elevation [deg]
model / type (hardware) [15]	GoPro Hero 10
limits on field of view (16/9)	az: 110deg el: 62.88deg
max useful range	700m
min useful range	0m
noise sigma	$9.1 \cdot 10^{-3}$ deg
latency	0.1s
update rate	30fps
resolution	$1.3 \cdot 10^{-3}$ deg

VI. Application scenario

The main scenario in this publication is at Ames Research Center, between the gantry of the Outdoor Aerodynamic Research Facility (OARF) and the Ames's Disaster Assistance and Rescue Team (DART) training area, with strategically placed cameras throughout the area. It is a similar set up as discussed in [4], but for a shorter flight segment with a sloped angle, representing a descent or climb depending on the direction flown, and with a larger number of ground nodes. This scenario also includes one airborne observer. More details about the design and implementation of the scenarios are given in [16]. The current study discussed in this paper is using synthetically generated ground node measurement data instead of the real life ones, as raw data image processing is currently still ongoing. Results based on real life measurements will be shown in future publications.

**Fig. 10 Map and 3D view of the set up of the scenario with location of observers and trajectory.**

Each ground node as well as the airborne observer are equipped with a camera that is facing in a specific direction horizontally as well as vertically, and tracking the moving object flying along the track, in this specific case an Alta X multicopter drone, shown in Fig. 11(a). The airborne observer is a Skydio drone, shown in Fig. 11(b).



(a) Multicopter drone Alta X, used as tracked vehicle



(b) Skydio drone, used as airborne observer

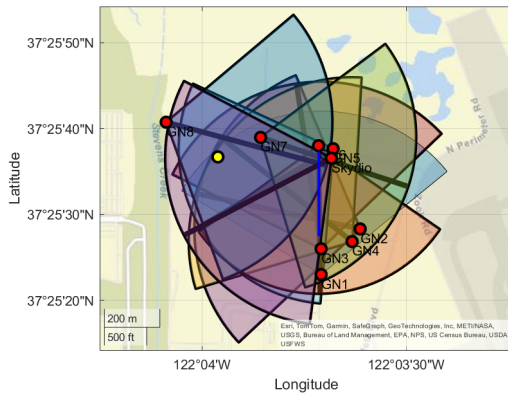
Fig. 11 Pictures of multicopter drone and airborne observer drone Skydio used for the test

Tab. 2 shows accurate location and viewing angle and range information for each individual ground node. Experiments have shown that the accuracy of the estimates depends to a large extent on the precision of this ground node information.

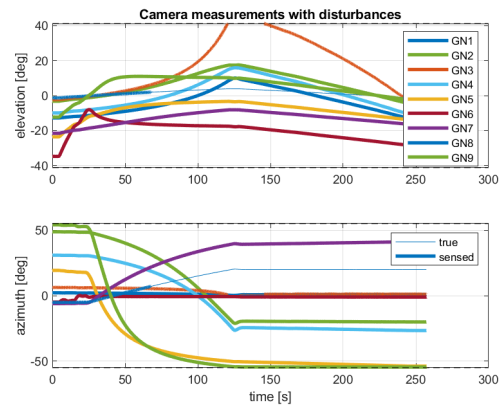
Table 2 Distributed sensors locations and angles around DART site for field test

Observer	Lat ($^{\circ}$)	Long ($^{\circ}$)	Abs. Alt. (m)	Heading ($^{\circ}$)	Elev. ($^{\circ}$)	Range (m)
GN 1	37.4230556	-122.061803	1.27	-4.14	12.05	600
GN 2	37.4245310	-122.060209	1.51	-77.87	3.75	600
GN 3	37.4238889	-122.061803	0.83	-8.54	1.63	600
GN 4	37.4241220	-122.060538	1.51	-51.70	10.60	600
GN 5	37.4271260	-122.061314	1.52	-120.25	21.74	600
GN 6	37.4272230	-122.061907	1.75	179.40	27.67	600
GN 7	37.4274930	-122.064273	1.54	108.33	20.58	600
GN 8	37.4279850	-122.068125	1.71	105.02	0.57	600
Skydio	37.4268210	-122.061385	10.00	-114.75	0.00	600

The observation cones and synthetic measurements are shown in Fig. 12. The measurements as shown here reflect the sensor characteristics which were discussed in Sec. V.



(a) Map view of the field test set up with sensor cones



(b) synthetic measurements

Fig. 12 Orientation of sensor cones and synthetic measurements.

VII. Results

A. Field coverage analysis of observer geometry

Relying on the field test outline, as sketched in Fig. 10(a), one can perform a sensitivity analysis similar as was described in Sec. III.A that shows the uncertainty in the estimated position. This is done for each possible pair of observers out of the total set of 8 GNs and the airborne Skydio drone. This results in a total of 36 possible combinations. The overall uncertainty analysis is achieved by finding the minimum uncertainty for all 36 combinations in each gridpoint for the tilted location grid. The evaluation grid is tilted over an angle of 3 deg, as was shown in Fig. 7, so that it is consistent with the vehicle trajectory as shown in Fig. 10(b). The result is shown in Fig. 13. The circular pattern of higher precision, visible in Fig. 8 and 9, also appears here between each pair of GNs. Moreover, shorter distances make the circular pattern more pronounced (such as between GN2-GN4, GN5-GN6 and GN5-Skydio) compared to longer distances (such as between GN1-GN7, GN2-GN7, GN3-GN7 and GN4-GN7). The actual flightpath is marked with the white dash dotted line and the white squares at takeoff and landing location. This path crosses 3 regions of varying precision levels: highest precision near GN5-GN6-Skydio below 2m uncertainty, lowest precision for the en route part of the path around 6m uncertainty, and medium precision near GN3 around 4m uncertainty. Also here, Fig. 13 can serve to derive sensor alignment specifications to achieve a certain level of required accuracy. For example, along the flight path, the uncertainty range goes up to around 6m for alignment errors around 1 deg. If one would require a minimum accuracy of 1m, that would translate into a requirement for GN angular alignment errors of max 0.15 deg for this specific outline.

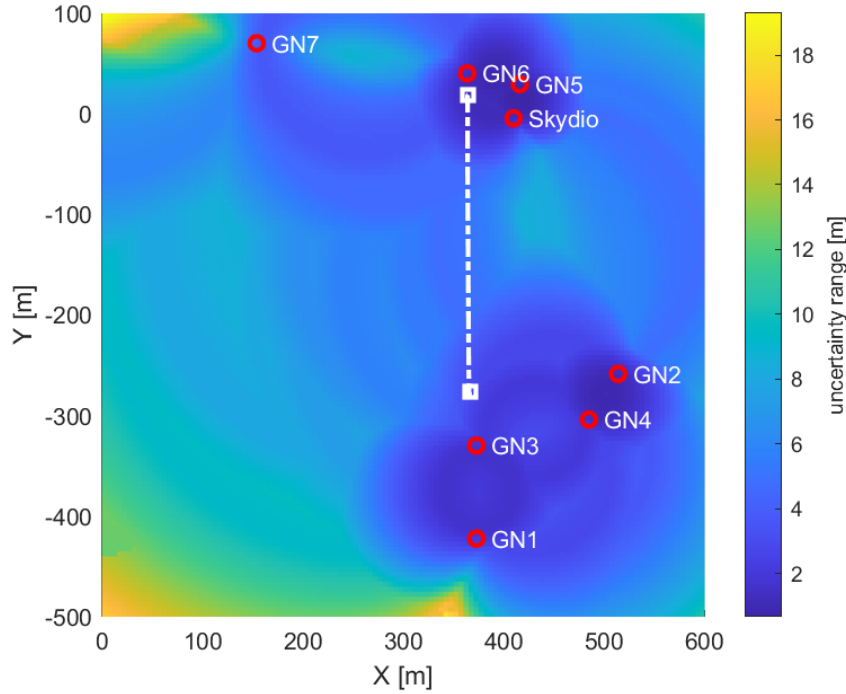


Fig. 13 Optimal localization zones for the field test outline

B. Evaluation metric applied on actual flight path

One can also apply the evaluation metric for the precision on the actual flight path of the scenario as sketched in Fig. 10. Calculating the maximum fit error for each pair of observers (36 possible combinations) and displaying them for each location along the track results in Fig. 14. Many errors are small, thanks to small distances and intersection angles that are sufficiently close to orthogonal. But some errors are significantly larger because of larger distances. The error values that start with a high value and trend lower during flight are pairwise related to the GNs near the landing zone: GN1-GN4. The error values with opposite trends are linked to GNs near the take-off location: GN5-8 and Skydio.

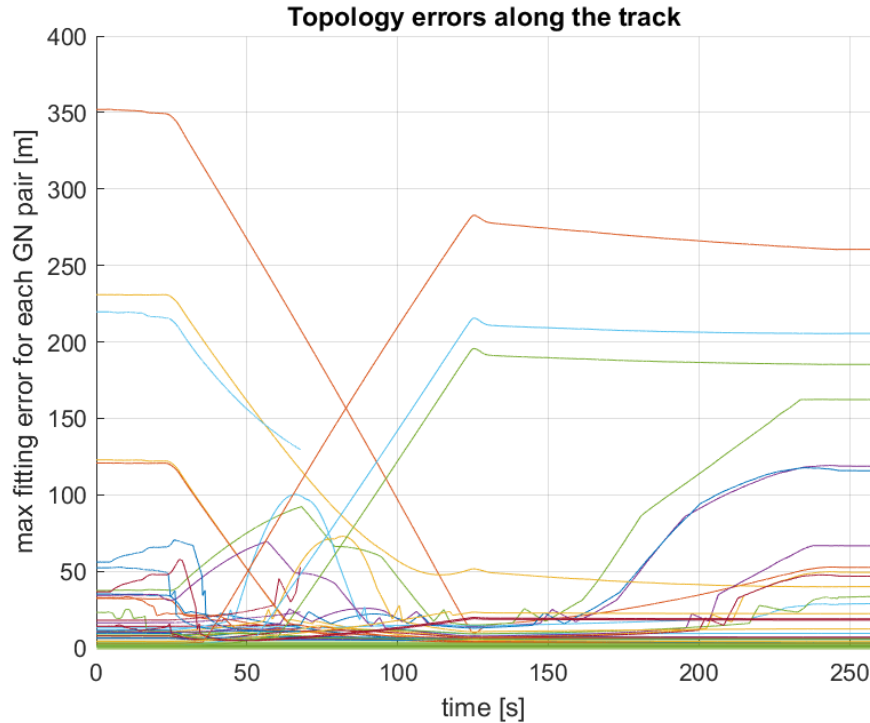


Fig. 15 shows a deeper analysis of the fitting errors shown in Fig. 14. The upper part of Fig. 15 shows the total number of fitting errors (in a total of 36) that are within a certain margin. For this specific scenario with a 1 deg bearing error in each GN, there is no error smaller than 1m, 3 errors within 4m in the vicinity of the GNs and at least 7 errors within 7m near the GNs. Halfway the trajectory in between the GNs 7m precision is achieved by only 3 GNs. The lower part of Fig. 15 shows the 3 smallest fitting errors along the track. The smallest one starts slightly above 1m, increases to around 5m halfway the flightpath in between the GNs and lowers again to around 3m in the landing zone area. These error levels correspond well with the values that were found in Fig. 13.

Fig. 16 shows the number and identification numbers of the GNs that constitute the 3 smallest fitting errors as shown in Fig. 15. Fig. 16(a) displays the selection of the best GN's along the track, which are selected based on respective distances and intersection angles. GN's in the vicinity of the vehicle are usually the better ones. Around the takeoff zone, these are GN5, GN6 and Skydio and to a lesser extent GN7. Towards the landing area, the GN's in that region start to outperform the other ones. These are GN1-4. GN8 is too far out to play a role of any importance during any segment of the flight. The number of ground nodes in this down selected group starts at 3 (GN5, GN6 and Skydio), briefly jumps to 5 when GN7 and GN2 join but drops back to 4 when GN6 drops out of the selection. After that the total number remains 4.

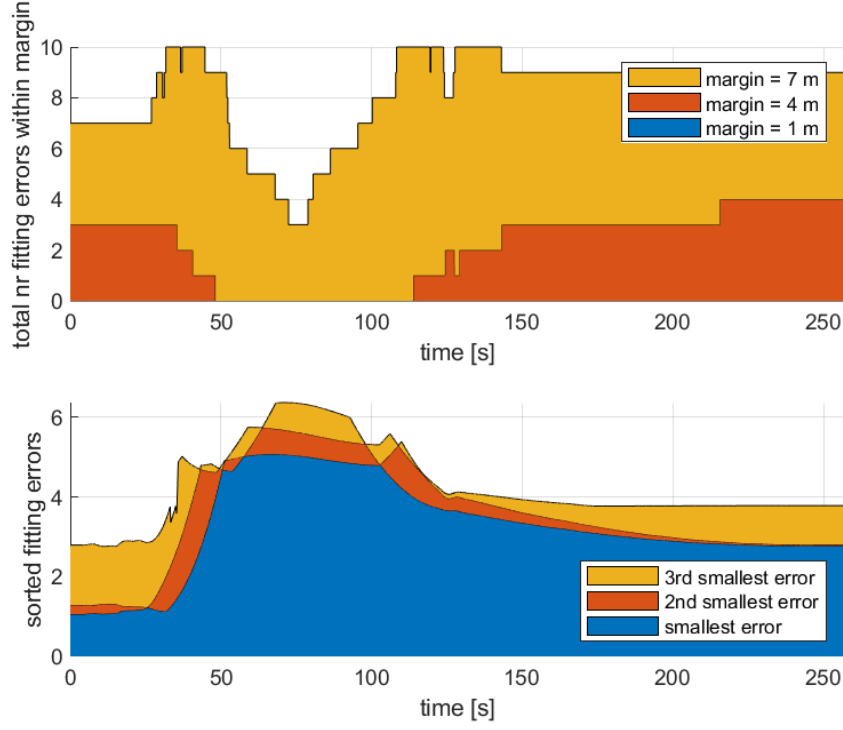
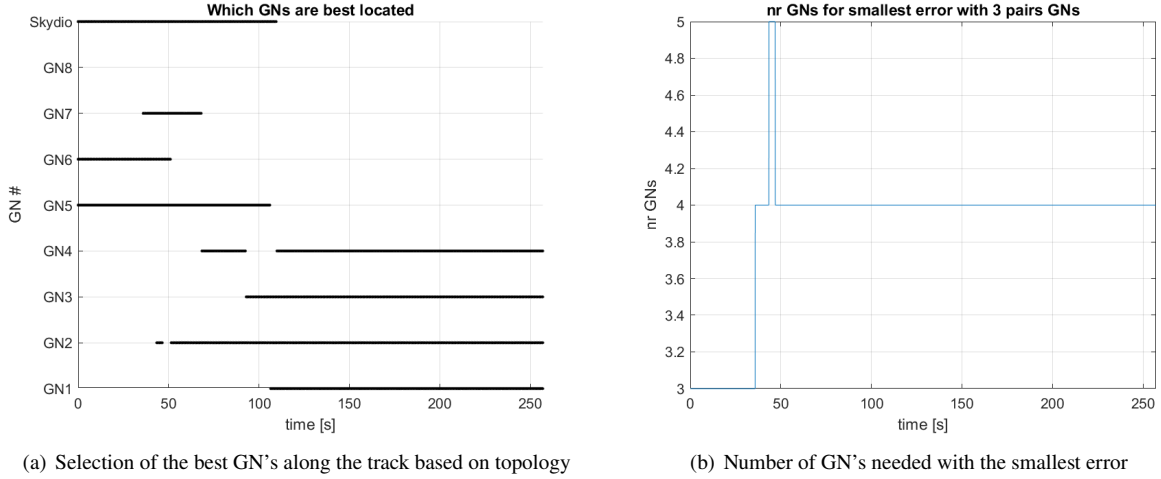


Fig. 15 Nr of errors within margin and sorted error values along track



(a) Selection of the best GN's along the track based on topology

(b) Number of GN's needed with the smallest error

Fig. 16 Number and identification numbers of the best GN's along the track based on topology

C. Localization errors due to sensor noise

This section focuses on the magnitude of the localization errors that one can expect due to realistic levels of sensor noise and other sensor characteristics for these kind of EO cameras. Sensor characteristics for these cameras were discussed in detail in Sec. V. This error magnitude analysis is performed and compared for a smaller set of GNs and a larger one. Sec. VII.C.1 focuses on coverage provided by 3 ground nodes only. Sec. VII.C.2 investigates the magnitude of the localization errors for the entire set of 8 GNs and the airborne observer.

1. Coverage with 3 ground nodes

The reduced coverage scenario relies solely on GNs 4, 5 and 8. Their coverage is shown in Fig. 17(a). GNs 4 and 5 cover the entire trajectory, but GN8 covers only the starting segment of the flight path. The corresponding camera measurements including disturbances are shown in Fig. 17(b). The first 70s of the flight are covered by 3 GNs but coverage is reduced to 2 GNs afterwards since the vehicle leaves the maximum sensing range of GN8.

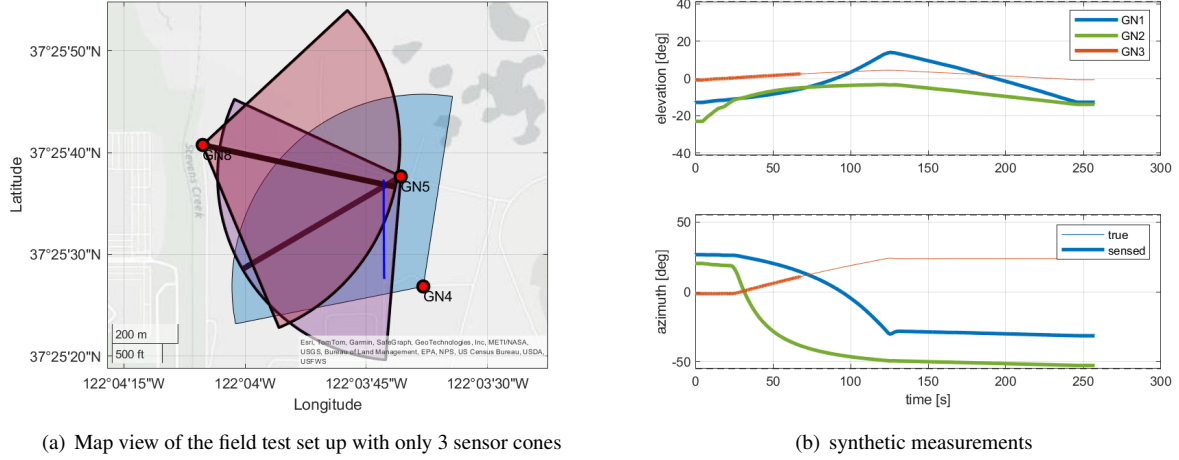


Fig. 17 Orientation of sensor cones and synthetic measurements for 3 GNs with sensor noise

Fig. 18 compares the estimated path and the true path. Fig. 18(a) shows the map view with the GN locations. The estimated locations are color coded red for low fidelity (high condition number for the observability matrix, greater than 10^{10}) and green for high fidelity (condition number smaller than 10^{10}). The values of the condition number over time are shown in Fig. 20(b). Fig. 18(b) shows the estimated 3D path in the East-North-Up reference frame, comparing the true path with predicted and corrected estimates from the KF. All results appear fairly accurate on this scale throughout the maneuver.

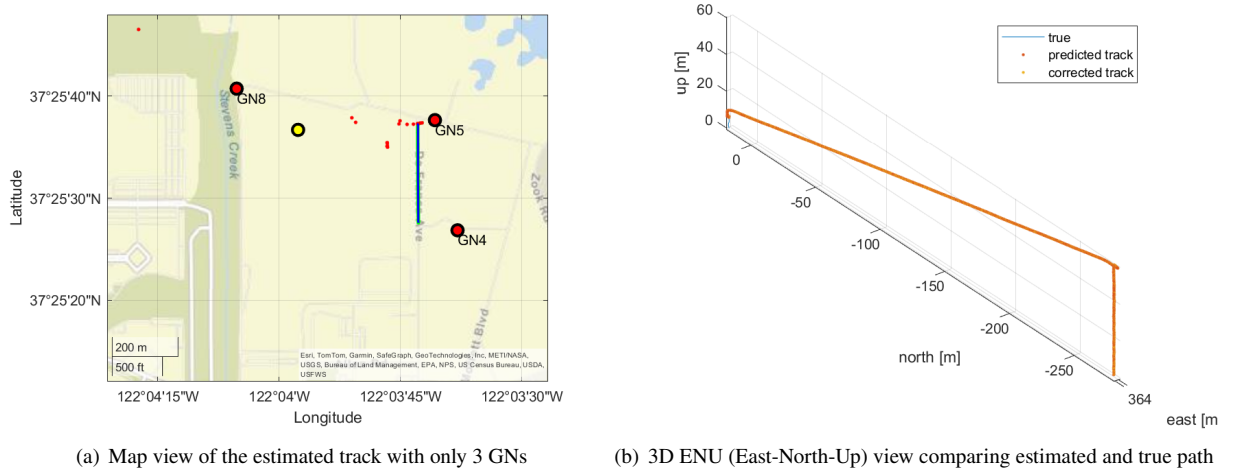


Fig. 18 Map and 3D view comparing estimate track and true path

The East-North-Up (ENU) errors as components per axis direction are shown in the left column of Fig. 19(a). The East component of the estimate fit error is dominated by sensor noise, but this does not apply for the North and Up

components. Here, the sensor time delay causes a small but significant fit error. The figures in the right column of Fig. 19(a) show the ENU velocity components, and the red lines in the left column show the values of these velocity components multiplied by the time delay. This shows that the largest share of the fit error is caused by the sensor time delay. The magnitude and direction of fit errors North and Up are consistent with a 3 deg climbing flight up to 120s and a vertically descending flight segment afterwards. Fig. 19(b) shows the standard deviations of the estimates per axis and confirms the accuracy of the results.

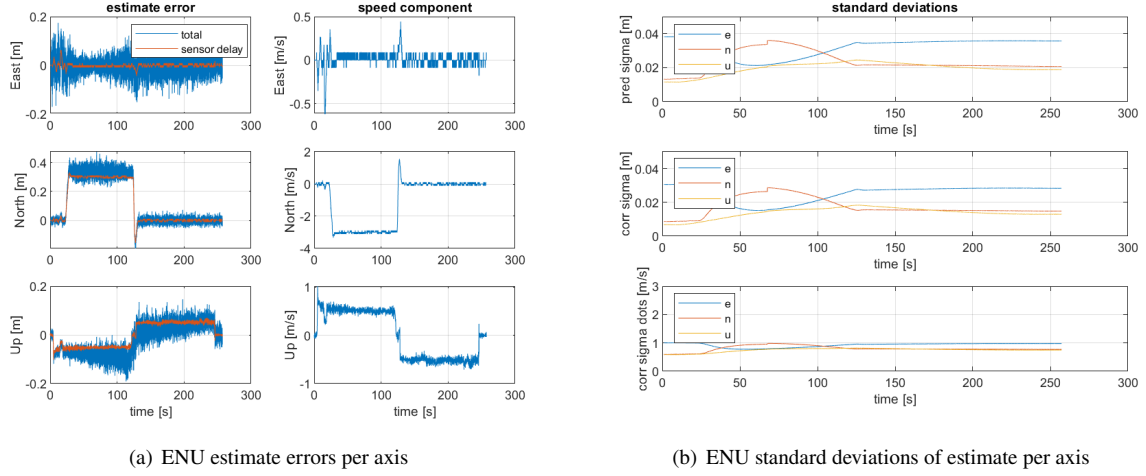


Fig. 19 ENU errors and standard deviations per axis

The full rank and the low value of the condition number of the observability matrix, both shown in Fig. 20, confirm that the estimation problem is well posed in this scenario. In general, these two metrics of rank and condition number of the observability matrix show how well the position and speed components of the tracked object (KF states) can be inferred from knowledge of the sensor measurements (KF measurements).

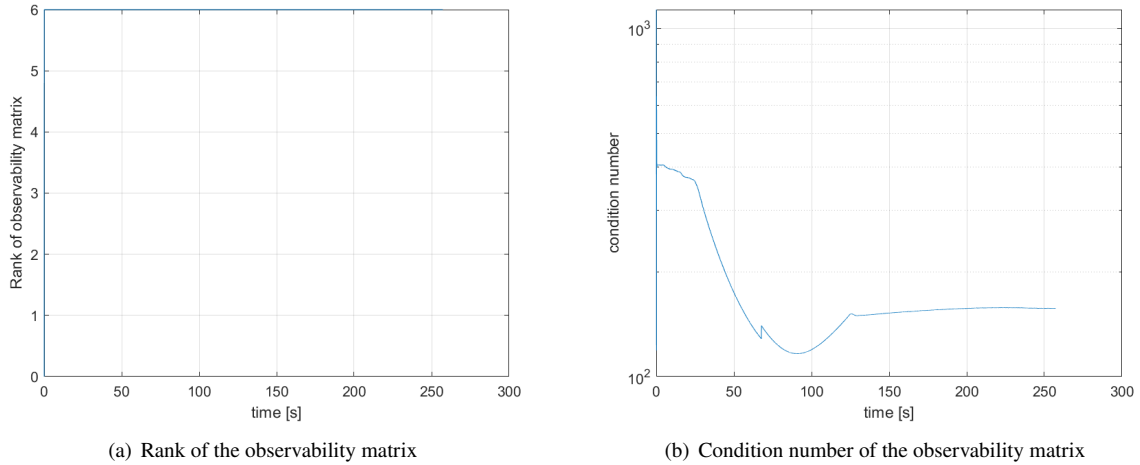
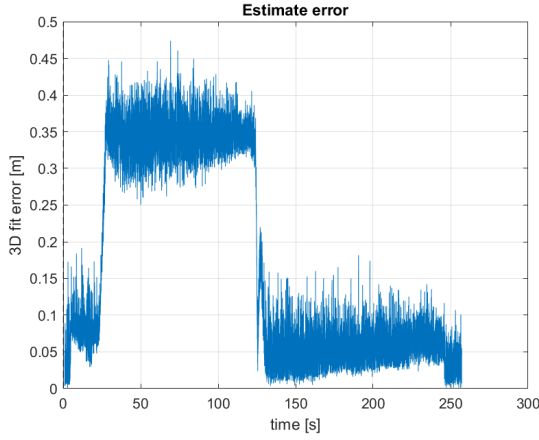
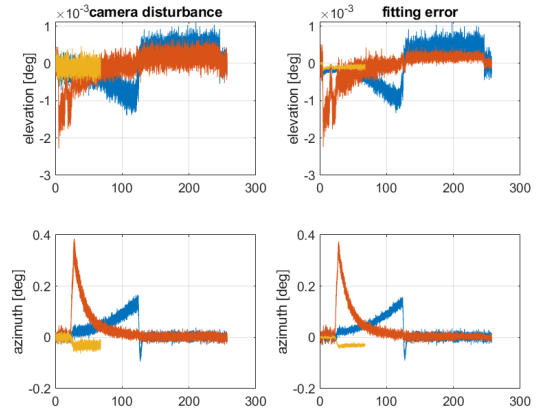


Fig. 20 Rank and condition number of the observability matrix

The 3D fit error over time between estimate and true path in Fig. 21(a) shows that the average error is around 0.35m in the higher speed 3 deg climbing flight segment and between 0.05 and 0.1m in the vertically descending segment, well within the required precision of 1m to 3m. Finally, Fig. 21(b) compares the actual camera disturbances on elevation and



(a) 3D ENU fit error



(b) Comparison between sensor disturbance and fit errors per axis

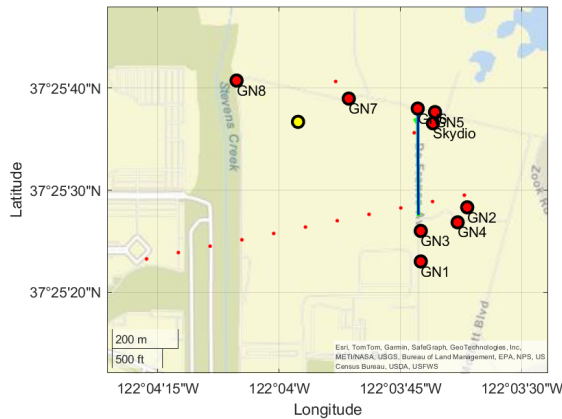
Fig. 21 3D ENU fit error and comparison between sensor disturbance and fit errors

azimuth in the left column with the fit error induced deviations in the reconstructed sensed variables in the right column. Comparing left and right column in Fig. 21(b) shows the beneficial effect of the KF in removing sensor characteristics.

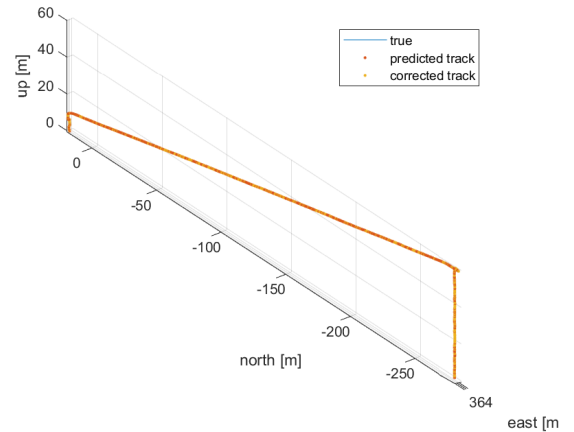
2. Coverage with 8 ground nodes and an airborne observer

This section repeats the same scenario as before, but this time with the full sensor set as described in Sec. VI. Comparing these results with Sec. VII.C.1 shows the effect of a denser network of ground nodes on the precision of the estimated position in the presence of sensor noise, time delay and other sensor characteristics.

Fig. 22 compares the estimated path and the true path. Fig. 22(a) shows the map view with all the GN locations, compared to the reduced set in the previous section. The estimated locations are color coded red for low fidelity and green for high fidelity in a similar fashion as before. The corresponding values of the condition number over time are shown in Fig. 24(b). Fig. 22(b) shows the estimated 3D path in the East-North-Up reference frame, comparing the true path with predicted and corrected estimates from the KF. All results appear fairly accurate on this scale throughout the maneuver.



(a) Map view of the estimated track with 8GNs and the airborne observer



(b) 3D ENU view comparing estimated and true path

Fig. 22 Map and 3D view comparing estimate track and true path

The East-North-Up errors as components per axis direction are shown in the left column of Fig. 23(a). As before, the East component of the estimate fit error is dominated by sensor noise, and the magnitude of the fit error is significantly smaller compared to the 3 GN setup that was shown in Fig. 19(a). However, no significant difference shows for the North and Up components of the fit error, because they are dominated by the sensor time delay which is not compensated for by a denser network of ground nodes. The remainder of the figures in Fig. 23(a) are presented in a similar way as was done in Sec. VII.C.1. Fig. 23(b) shows the standard deviations of the estimates per axis and confirms the accuracy of the results. Comparing this figure with Fig. 19(b) confirms that a denser and well positioned network of GNs helps to reduce the standard deviations for the predicted and corrected estimates (values reduced by more than 50% in this case). The standard deviations for the speed components do not show a similar reduction.

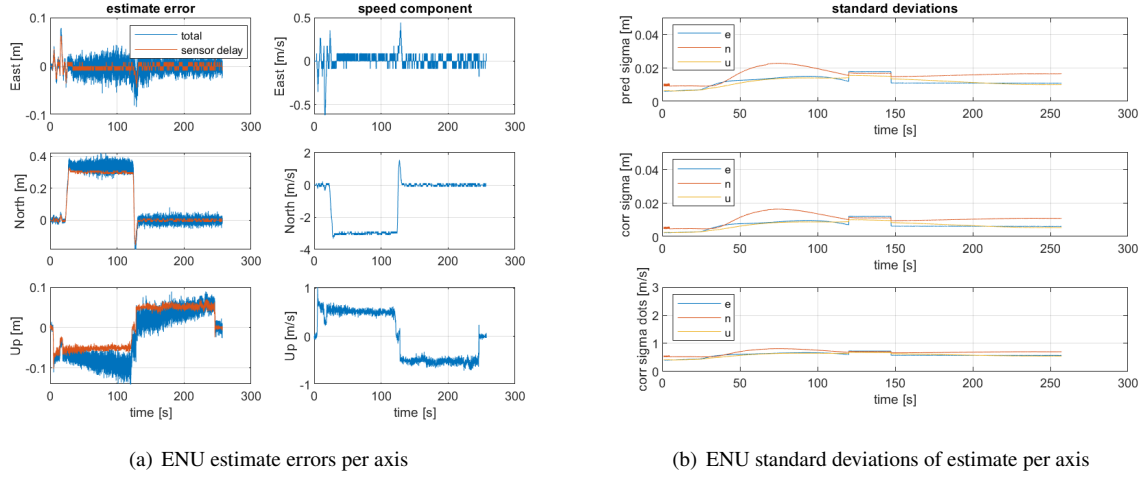


Fig. 23 ENU errors and standard deviations per axis

The full rank and the low value of the condition number of the observability matrix, both shown in Fig. 24, confirm that the estimation problem is well posed here as well. Comparing the condition numbers for the different ground coverage setups between Fig. 24(b) and 20(b) shows that the larger number of ground nodes significantly reduces the condition number of the observability matrix. This is another indicator of a better posed localization problem.

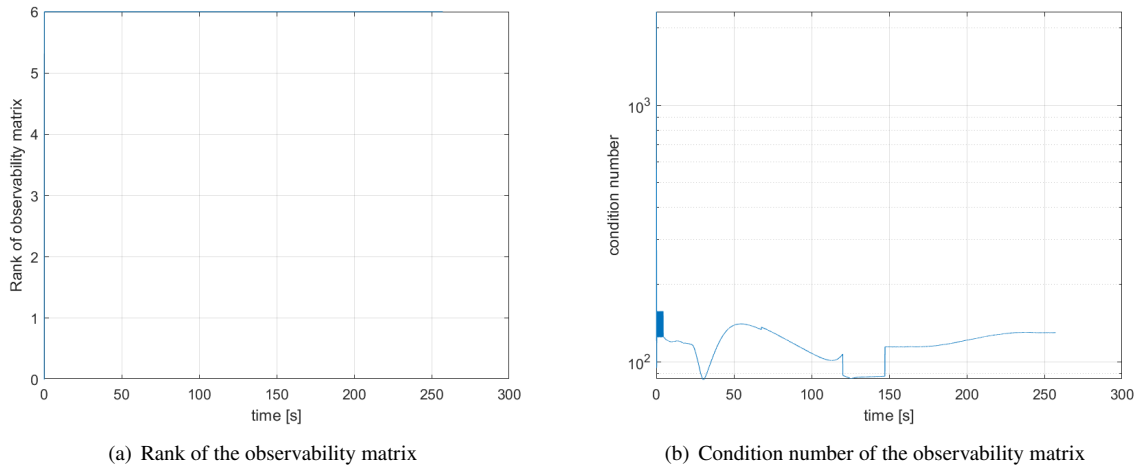


Fig. 24 Rank and condition number of the observability matrix

The 3D fit error and the comparison between sensor disturbance and fit error do not show any significant differences compared to the previous section and have been omitted for brevity.

D. Localization errors due to sensor alignment offset

This section investigates the localization errors due to a misalignment in the viewing angles of the ground nodes. Previous field test results as presented in [4] were clearly affected by these misalignments due to the lack of accurate viewing angle information. For this specific case study, the bearing and elevation angles of each ground node, as provided in Tab. 2, were perturbed by one degree for bearing and half a degree for elevation angle. The sensor characteristics as discussed in Sec. V are also incorporated in addition to the aforementioned misalignments. Just like for the sensor noise, this error magnitude analysis is also performed and compared for a smaller set of GNs and a larger one. Sec. VII.D.1 focuses on coverage provided by 3 ground nodes only. Sec. VII.D.2 investigates the magnitude of the localization errors for the entire set of 8 GNs and the airborne observer.

1. Coverage with 3 ground nodes

The orientation of the sensor cones and the synthetic measurements for this scenario with only 3 GNs is similar as the outline given in Fig. 17 and was omitted for brevity. The only difference is in the synthetic measurement offset (one degree for azimuth and half a degree for elevation) due to the misalignment of the viewing angles, but this is indistinguishable on the scale as presented here.

Fig. 25 compares the estimated path and the true path. Fig. 25(a) shows the map view with the GN locations. The estimated locations are color coded red for low fidelity and green for high fidelity in a similar fashion as for the sensor noise analysis. The values of the condition number over time are shown in Fig. 27(b). Fig. 25(b) shows the estimated 3D path in the East-North-Up reference frame, comparing the true path with predicted and corrected estimates from the KF. The results here achieve a lower precision level than for the sensor noise, which is especially visible in the 3D trajectory in Fig. 25(b).

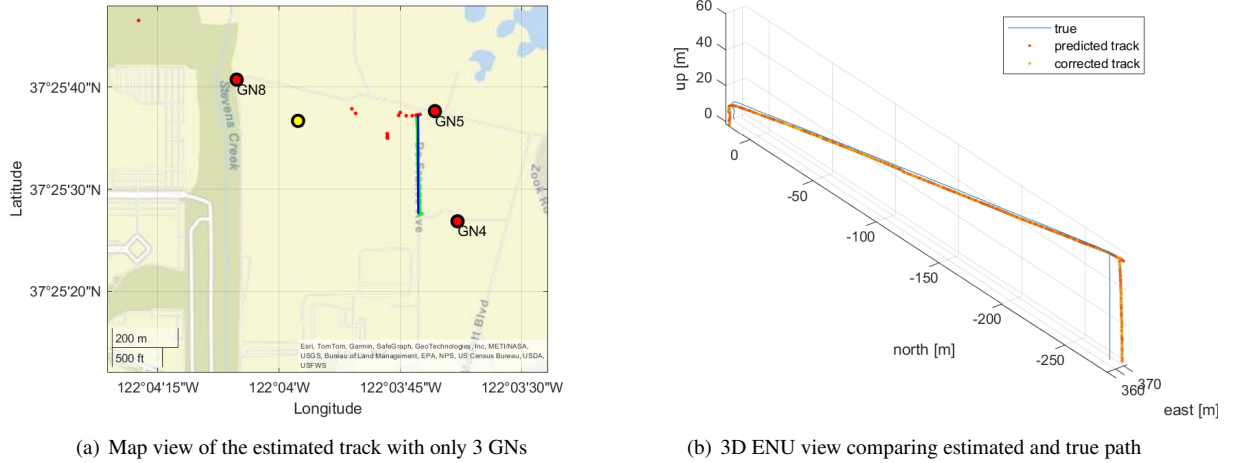
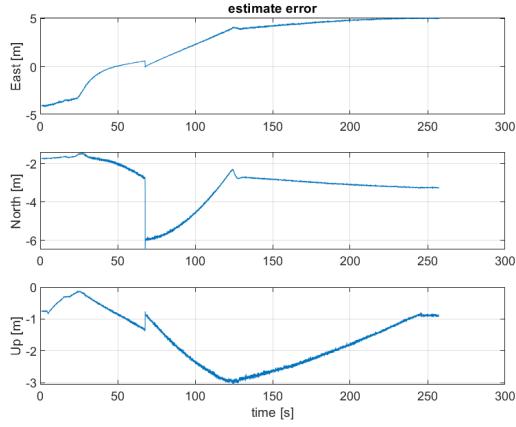
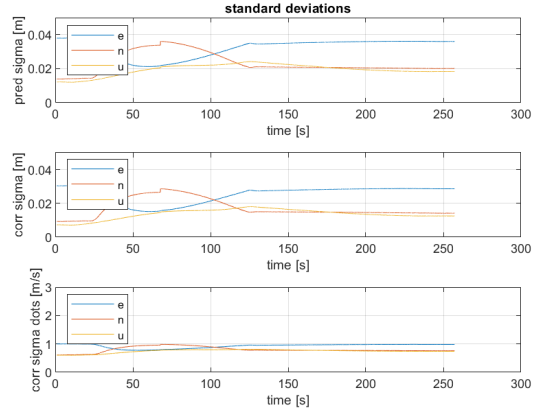


Fig. 25 Map and 3D view comparing estimate track and true path

The East-North-Up errors as components per axis direction are shown in Fig. 26(a). The alignment error dominates the previously discussed sensor time delay offset by far, resulting in a fit error of multiple meters which is more than between one and three meters acceptable error. Even a minor misalignment at the GNs of one degree results in an offset of several meters at these longer detection ranges between the tracked object and the GN. Especially in a sparser GN network, the tracking ranges become longer which makes these errors worse. Moreover, the transition of 3 GN coverage to only 2 GN coverage at around 70 s also causes jumps in the fit error. Fig. 26(b) shows the standard deviations of the estimates per axis and comparing them with Fig. 19(b) shows an increase in the values for the standard deviations of predicted and corrected position compared to the scenarios with sensor noise only.



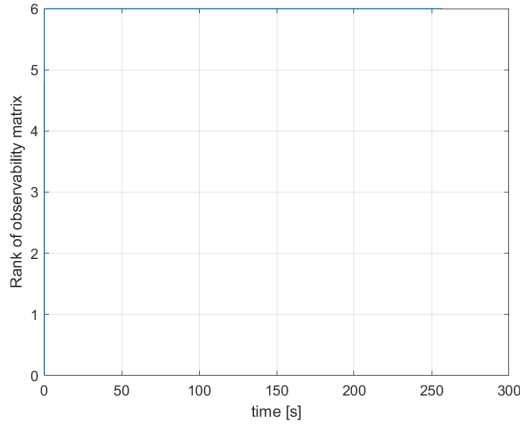
(a) ENU estimate errors per axis



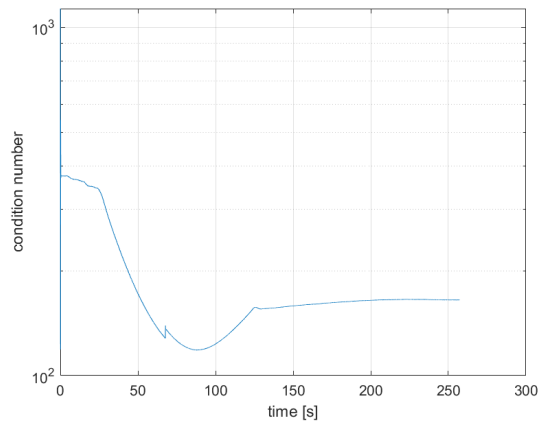
(b) ENU standard deviations of estimate per axis

Fig. 26 ENU errors and standard deviations per axis

Fig. 27 shows both full rank and a relatively low value of the condition number for the observability matrix in this case. However, comparing these values for the condition number with the values in Fig. 20 confirms that this set up is not as well posed as with sensor noise only.



(a) Rank of the observability matrix



(b) Condition number of the observability matrix

Fig. 27 Rank and condition number of the observability matrix

Fig. 28 shows that the 3D ENU fit error gets as large as 6m which is beyond the acceptable range (between 1m and 3m for similar precision levels as GPS). One can also observe an increase in fit error when the coverage reduces from 3 GNs to only 2, after 70s.

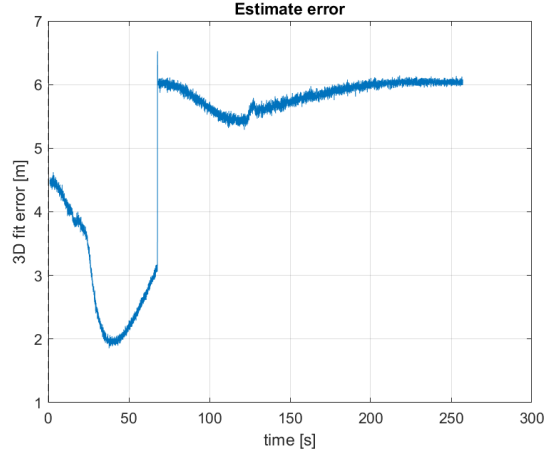
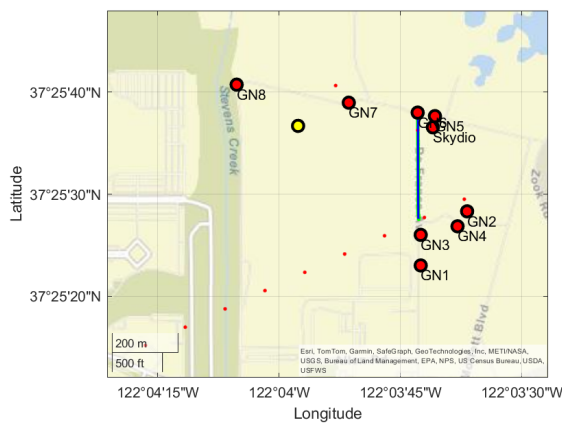


Fig. 28 3D ENU fit error

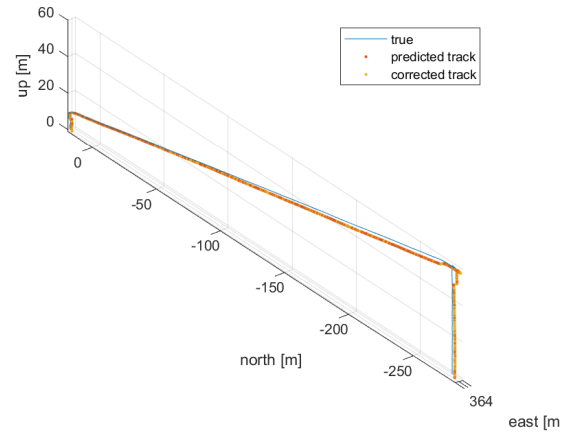
2. Coverage with 8 ground nodes and an airborne observer

This section repeats the same scenario as before, but this time with the full sensor set as described in Sec. VI. Comparing these results with Sec. VII.D.1 shows the effect of a denser network of ground nodes on the precision of the estimated position in the presence of sensor alignment errors, in addition to the other usual sensor characteristics.

Fig. 29 compares the estimated path and the true path. Fig. 29(a) shows the map view with all the GN locations, compared to the reduced set in the previous section. The estimated locations are color coded red for low fidelity and green for high fidelity in a similar fashion as before. The corresponding values of the condition number over time are shown in Fig. 31(b). Fig. 29(b) shows the estimated 3D path in the East-North-Up reference frame, comparing the true path with predicted and corrected estimates from the KF. Especially in Fig. 29(b), the results appear more accurate with the larger set of observers compared to Fig. 25(b), but still lower than with sensor noise only. This shows that a denser network of observers can reduce the effect of sensor misalignments, but it cannot eliminate the effect entirely.



(a) Map view of the estimated track with 8 GNs and the airborne observer



(b) 3D ENU view comparing estimated and true path

Fig. 29 Map and 3D view comparing estimate track and true path

The East-North-Up errors as components per axis direction are shown in Fig. 30(a). The magnitude of the fit error is smaller compared to the 3 GN setup that was shown in Fig. 26(a), but still covers multiple meters which is more than between one and three meters acceptable error. Even in a denser GN network, the tracking ranges are still too long such that even a minor misalignment at the GNs of one degree results in an offset of several meters at these shorter detection

ranges between the tracked object and the GN. Fig. 30(b) shows the standard deviations of the estimates per axis and confirms the accuracy of the results. Comparing this figure with Fig. 26(b) confirms that a denser and well positioned network of GNs helps to reduce the standard deviations for the predicted and corrected estimates (values reduced by more than 50% in this case). The standard deviations for the speed components do not show a similar reduction. Both ENU estimate errors and standard deviations show sudden jumps up in their values when the coverage by the number of GNs drops from 8 to 7 because the tracked object exceeds the vertical field of view of GN3, as shown in Fig. 12(b).

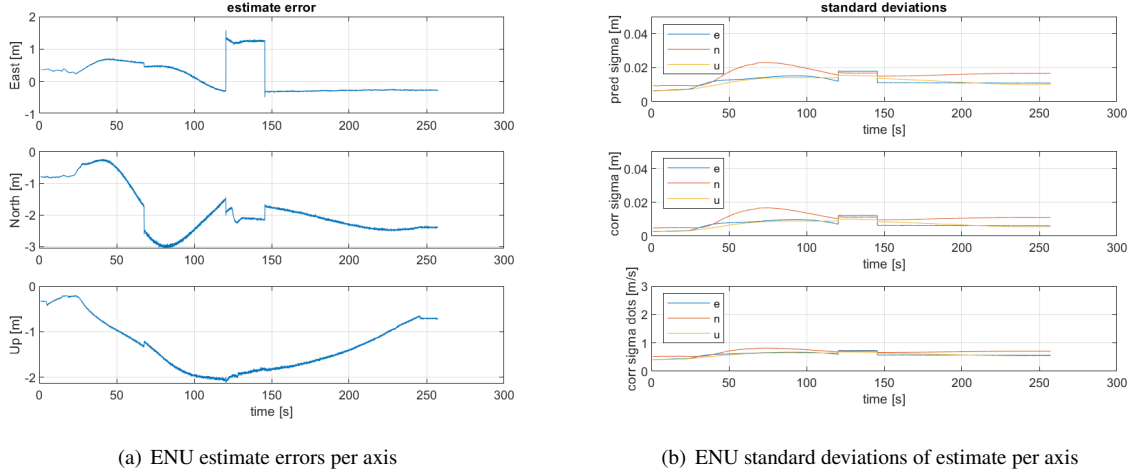


Fig. 30 ENU errors and standard deviations per axis

The full rank and the low value of the condition number of the observability matrix, both shown in Fig. 31, confirm that the estimation problem is well posed here as well. Comparing the condition numbers for the different ground coverage setups between Fig. 31(b) and 27(b) shows that the larger number of ground nodes significantly reduces the condition number of the observability matrix. This is another indicator of a better posed localization problem.

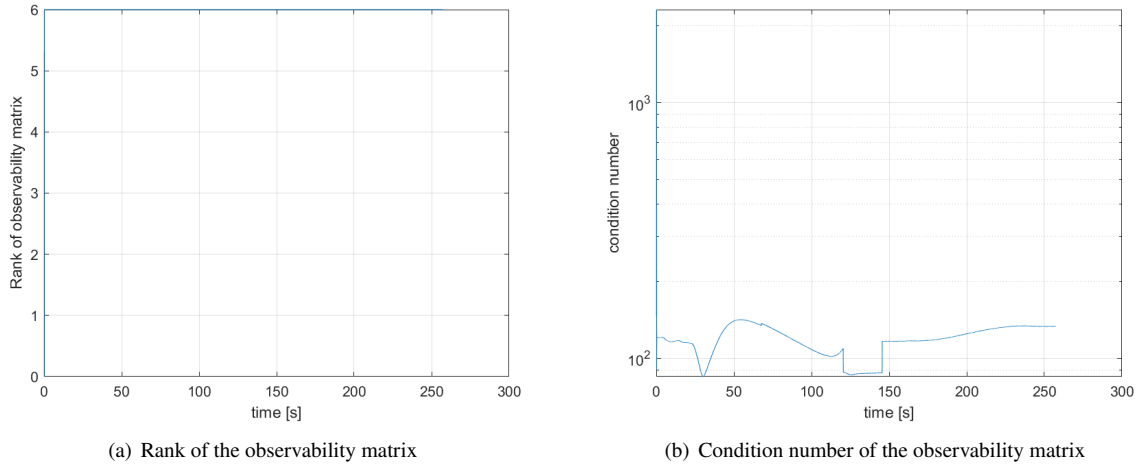


Fig. 31 Rank and condition number of the observability matrix

Fig. 32 shows that the 3D ENU fit error gets as large as 3.5m which is still beyond the acceptable range (between 1m and 3m for similar precision levels as GPS) but nearly half as much as for the network with 3 GNs as was shown in Fig. 28. One can also observe an increase in fit error when the coverage reduces from 9 GNs to 8, after 70s, and also during

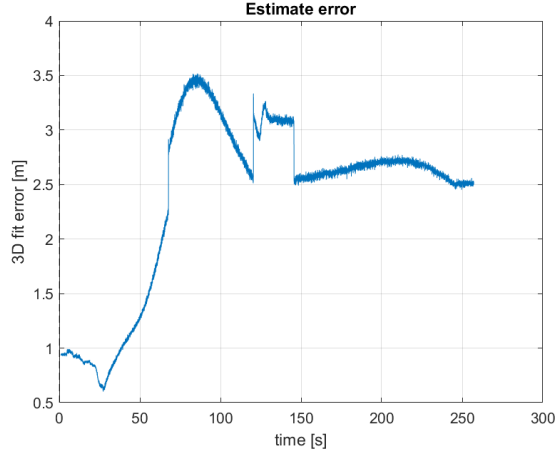


Fig. 32 3D ENU fit error

the momentary drop of coverage by GN3 between 120 and 150s.

The trends as shown by Fig. 28 and 32 provide an indicator of the impact of the maximum spacing between GNs, and also for the maximum allowable sensor alignment error given the current setups: with only 3 GNs the maximum allowable sensor alignment error is only 0.15 deg. With the full coverage of 8 GNs and the airborne observer, this requirement is relaxed to 0.30 deg.

VIII. Observations, Conclusions and Future Work

This section focuses on some general observations and conclusions that can be drawn after the aforementioned case studies and analyses of variations in independent variables.

The required accuracy of object localization for Urban Air Mobility with APNT techniques is between 1m and 3m if one aims to achieve similar precision levels as traditional GPS methods. In cruise, these precision levels are expected to be adequate, but higher accuracy (1m and lower) may be needed for specific phases of flight, such as takeoff and landing. Especially for higher density operations around the vertiports, where GPS signals degrade in an urban environment, APNT techniques have the potential to become enabling technologies.

Based on the aforementioned case studies, the most important factors that influence the precision of the localization results are the following:

- **Precision of the location and alignment of the ground nodes:** In WGS-84 coordinates, lat and lon values should be defined down to tenths of arcseconds, vertical precision down to centimeter level, and the viewing angles need a precision down to less than a tenth of a degree. Table 2 shows the values and the corresponding precision levels that were used in this study. Ref. [17] shows the potential of bundle adjustment for this purpose.
- **Field of view of the sensors:** Aligning the viewing angles of the sensors to maximize coverage, while taking into account their field of view limitations, is also important. One ground node lost the target outside its field of view for a short time and the impact was immediately visible in the magnitude of fitting error and standard deviation, as was shown in Fig. 30 and 32.
- **Sensor noise:** The sensor noise levels based on state of the art sensor characteristics (specified in Table 1) and observed noise levels after image processing of field test imaging do not prevent achieving the aforementioned precision levels. An important observation here is that images at closer ranges tend to become more noisy because the image processing software spreads the target detection over more available pixels. More advanced target detection techniques have the potential to bring benefits here.
- **Sensor delay:** Typical sensor delay values based on state of the art sensor characteristics (specified in Table 1) have a more significant impact on precision levels than noise, but do not prevent achieving the aforementioned precision levels. However, image processing and network time delays are not yet taken into account in these numbers. Ref. [18] investigates the impact of communication delays in the context of distributed sensing. The impact on the precision is always a combination of the total time delay and the speed of the vehicle. Higher accuracy positioning in takeoff and landing areas will take place at lower speeds and shorter distances compared

to cruise flight conditions, which will help improving the precision levels.

- **Density and positioning of the ground nodes and airborne observers:** Distances between ground nodes and airborne observers on one hand and the tracked object on the other hand, together with the intersection angles of the observation vectors have a major impact on the precision levels, as previously shown in Fig. 8 and 9. One can perform a sensitivity analysis for local worst case errors in the field test outline, as shown in Fig. 13, and discussed in more detail in Fig. 15 and 16. A more generic plot to show the impact of distances and intersection angles on worst case estimate error was shown in Fig. 5. Finally, the error traces as were shown in Fig. 28 and 32 for 3 GNs (shown in Fig. 17(a)) and the full sensor network (shown in Fig. 10(a)) respectively are superimposed on the aforementioned Fig. 5 with the result shown in Fig. 33. This figure illustrates how a denser and well-positioned network of ground nodes and airborne observers improves precision by means of shorter observation distances as well as closer to orthogonal intersection angles between the observation vectors. The results shown here are for the aforementioned one degree alignment error cases.

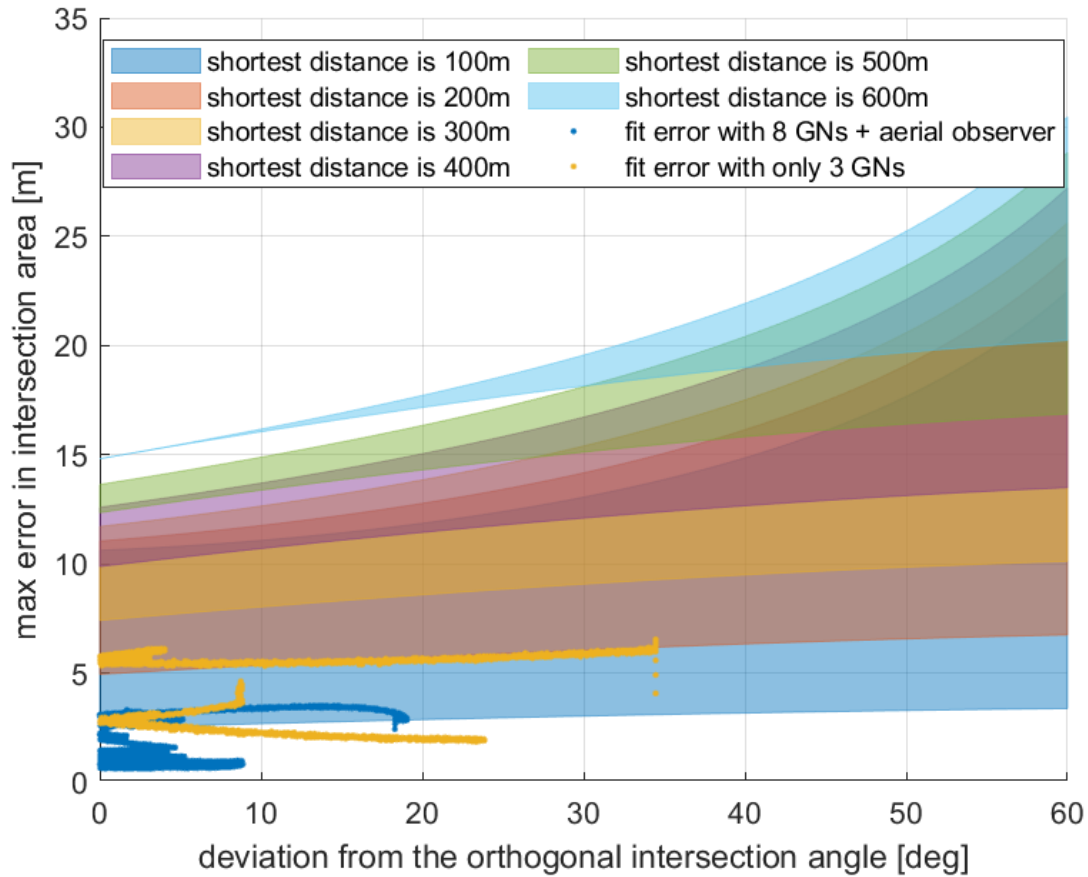


Fig. 33 Sensor alignment error traces superimposed on areas of worst case expected estimate errors as a function of the shortest distance to the closest ground node and the deviation from the ideal orthogonal intersection angle

As a final remark, tactical positioning of the ground nodes based on the anticipated path of the tracked object is key. Comparing Fig. 13, 15 and 32 shows that the most significant gain in precision and fit error reduction can be achieved by adding GNs to improve coverage in the middle segment of the flight and also near the landing zone. One can interpret this work in two ways. For a given scenario outline and sensor characteristics, the worst case errors along the trajectory can be deduced. However, one can also determine the required minimum sensor characteristics for achieving a certain level of precision in the estimated trajectory. This could support the selection of adequate sensor equipment and its calibration. For this specific scenario, corresponding numbers are given in Tab. 3 and 4. The precision values in Tab. 4 depend on groundspeed and time delay, and are emphasized when the precision requirements are not satisfied.

Table 3 Sensor characteristics versus precision in this case study

3 GNs	
given: 1 deg misalignment	required: 1m / 3m precision
max 6 m fit error	max 0.16 deg misalignment
up to 35 deg deviation from $\perp$	max 0.48 deg misalignment
8 GNs + airborne observer	
given: 1 deg misalignment	required: 1m / 3m precision
max 3.5 m fit error	max 0.28 deg misalignment
up to 18 deg deviation from $\perp$	max 0.84 deg misalignment

Table 4 Delay versus precision

Fit error [m]	groundspeed [m/s]				
time delay [s]	1	2	3	4	5
0.1	0.1	0.2	0.3	0.4	0.5
0.3	0.3	0.6	0.9	1.2	1.5
0.5	0.5	1.0	1.5	2.0	2.5
0.8	0.8	1.6	2.4	3.2	4.0
1.0	1.0	2.0	3.0	4.0	5.0

Future work will explore this topic further, especially in combination with real life measurement information from image processing of the field test videos.

Appendix: Kalman Filter based Object Tracker

The object tracker is a further development which is based on a conventional Kalman Filter as elaborated in detail in Ref. [2–4, 19]. This filter estimates 3D position and velocity components in ENU coordinates (East–North–Up) and assumes constant speed in the prediction step, and updates the estimate in the correction step based on the available measurements. The filter gain calculation is modified in order to incorporate valid flags for the different sensor measurements, which makes this method adaptive and robust against invalid data, loss of signal between the different distributed sensors, outliers and lower sampling rates. The filter gain is also rescaled to prevent overcompensation in this new implementation. This modular architecture is fairly straightforward to extend so that other sensors and sensor locations can be incorporated as well, such as radar and others. Fig. 34 illustrates the general overview of how the Kalman Filter is set up, including the possibility to add radar. The prediction step assumes constant velocity for calculating the predicted state $\hat{\mathbf{x}}_k$ based on the previous state $\mathbf{x}_{k-1}$. The state consists of six elements, namely the three position coordinates and the three velocity coordinates, all in ENU with respect to a predefined fixed referenced point. Predicted measurements $\hat{\mathbf{z}}_{k,i}$ are calculated for each sensor location i , which are a function of the predicted state $\mathbf{h}_i(\hat{\mathbf{x}}_k)$ and the sensor location coordinates. These predicted measurements consist of the measured variables that are provided by the relevant sensors for each location, namely range, azimuth and elevation from the radar (when available) as well as azimuth and elevation from the EO camera. By comparing the actual measurement $\mathbf{z}_{k,i}$ with the predicted measurement $\hat{\mathbf{z}}_{k,i}$ for each sensor location i , one can calculate the innovation $\varepsilon_i = \mathbf{z}_{k,i} - \hat{\mathbf{z}}_{k,i} = \mathbf{z}_{k,i} - \mathbf{h}_i(\hat{\mathbf{x}}_k)$. Main drivers for the innovation are errors in the predicted state (due to the constant speed assumption) and sensor disturbances. Next, the information from all individual sensor locations and sensors is stacked together in one large innovation array ε . As last but not one, the Kalman gain matrix $\mathbf{W}$ is calculated by means of the predicted covariance $\hat{\mathbf{P}}_k$, measurement matrix $\mathbf{H} = \frac{d\mathbf{h}(\mathbf{x})}{d\mathbf{x}}$, and the innovation covariance matrix $\mathbf{S}$. This Kalman gain matrix $\mathbf{W}$ is weighted with valid flags to deal appropriately with the sensor measurements as explained previously. The corrected state $\mathbf{x}_k$ is calculated by means of predicted state $\hat{\mathbf{x}}_k$, the innovation ε and the Kalman gain matrix $\mathbf{W}$ via the following equation: $\mathbf{x}_k = \hat{\mathbf{x}}_k + \mathbf{W} \cdot \varepsilon$. Finally, the corrected covariance $\mathbf{P}_k$ is calculated. The complete procedure is described in more detail below.

A. State Prediction $\hat{\mathbf{x}}_k$

The one step ahead prediction of the state vector is split up as follows:

$$\hat{\mathbf{x}}_k = \begin{bmatrix} \hat{\mathbf{x}}_{k1} \\ \hat{\mathbf{x}}_{k2} \end{bmatrix} \quad (19)$$

With the position in the upper half of the vector:

$$\hat{\mathbf{x}}_{k1} = \begin{bmatrix} \hat{E} \\ \hat{N} \\ \hat{U} \end{bmatrix}_k = \begin{bmatrix} E \\ N \\ U \end{bmatrix}_{k-1} + \Delta t \cdot \begin{bmatrix} \dot{E} \\ \dot{N} \\ \dot{U} \end{bmatrix}_{0:k-1} \quad (20)$$

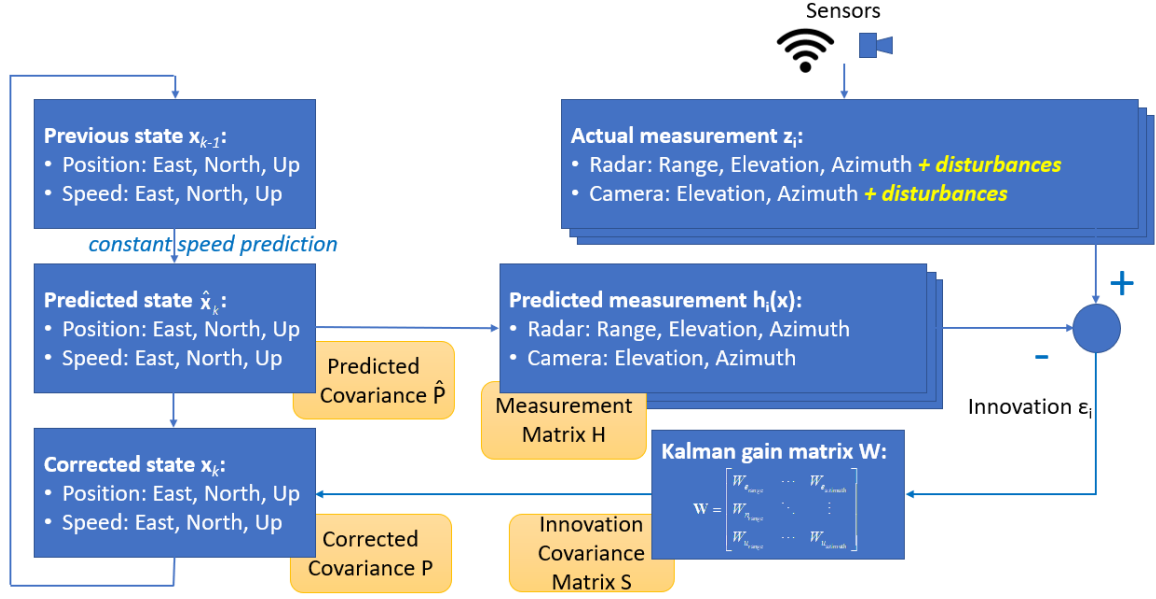


Fig. 34 General overview of the Distributed Sensor Kalman Filter setup

And the velocity in the lower half of the vector:

$$\hat{\mathbf{x}}_{k_2} = \begin{bmatrix} \dot{E} \\ \dot{N} \\ \dot{U} \end{bmatrix}_k = \begin{bmatrix} \dot{E} \\ \dot{N} \\ \dot{U} \end{bmatrix}_{0_{k-1}} \quad (21)$$

Note that the coordinates are East–North–Up with respect to a predefined fixed reference point and that the velocity is assumed to be close to constant. Moreover:

$$\begin{bmatrix} \dot{E} \\ \dot{N} \\ \dot{U} \end{bmatrix}_{0_{k-1}} = \mathbf{L}^\top \cdot \begin{bmatrix} \dot{E} \\ \dot{N} \\ \dot{U} \end{bmatrix}_{T_{k-1}} \quad (22)$$

where $\mathbf{L}^\top$ is the rotation matrix from the target ENU-frame to the reference point ENU-frame, as elaborated in the Appendix. Next step is to rotate from ENU to ECEF reference frame:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{0_{\text{ECEF}}} = \mathbf{T} \begin{bmatrix} E \\ N \\ U \end{bmatrix}_{0_{k-1}} \quad (23)$$

where $\mathbf{T}$ is the rotation matrix from the ENU reference frame to the ECEF reference frame, as defined in the Appendix. The predicted target position is converted straight from ECEF to WGS-84 (i.e. latitude, longitude and altitude) via the transformation elaborated in the next appendix, resulting in the predicted target position coordinates $\begin{bmatrix} \phi_t & \lambda_t & h_t \end{bmatrix}_{\text{pred}}^\top$.

B. State Error Covariance Prediction $\hat{\mathbf{P}}_k$

$$\hat{\mathbf{P}}_k = \mathbf{F}\mathbf{P}_{k-1}\mathbf{F}^\top + \mathbf{Q} \quad (24)$$

with:

$$\mathbf{F} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t \\ \hline & & \mathbf{0}_{3 \times 3} & 1 & 0 & 0 \\ & & & 0 & 1 & 0 \\ & & & 0 & 0 & 1 \end{array} \right] \quad \mathbf{Q} = \mathbf{G}\mathbf{F}\mathbf{G}^\top \quad (25)$$

and:

$$\mathbf{G} = \left[\begin{array}{ccc|ccc} 0.5\Delta t^2 & 0 & 0 & & & \\ 0 & 0.5\Delta t^2 & 0 & & & \\ 0 & 0 & 0.5\Delta t^2 & & & \\ \hline \Delta t & 0 & 0 & & & \\ 0 & \Delta t & 0 & & & \\ 0 & 0 & \Delta t & & & \end{array} \right] \quad \mathbf{\Gamma} = \begin{bmatrix} \sigma_e^2 & 0 & 0 \\ 0 & \sigma_n^2 & 0 \\ 0 & 0 & \sigma_u^2 \end{bmatrix} \quad (26)$$

C. Predicted Measurement Calculation $\hat{\mathbf{z}}_k$

The total predicted measurement vector $\hat{\mathbf{z}}_k$ consists of the predicted measurement vectors $\hat{\mathbf{z}}_{k,i}$ of each individual sensor location i . The predicted measurement vectors contain the relevant equipment-dependent information per individual sensor location and include only EO camera information in this specific study: elevation α and azimuth γ (no range due to the lack of stereo view and depth perception). The predicted measurement vector per sensor location is described as follows with only the hardware-relevant parts present in order to reduce vector size and computational load:

$$\hat{\mathbf{z}}_{k,i} = \hat{\mathbf{z}}_{k,i\text{EOcam}} \quad (27)$$

$$\hat{\mathbf{z}}_{k,i\text{EOcam}} = \begin{bmatrix} \hat{\alpha}_i & \hat{\gamma}_i \end{bmatrix}^\top \quad (28)$$

The individual measured variables are defined as follows:

$$\hat{\alpha}_i = \arctan \frac{-\hat{Z}_{S_i}}{\hat{\rho}_{i\text{hor}}} \quad (29)$$

$$\hat{\gamma}_i = \arctan \frac{\hat{Y}_{S_i}}{\hat{X}_{S_i}} \quad (30)$$

where:

$$\hat{\rho}_{i\text{hor}} = \sqrt{\hat{X}_{S_i}^2 + \hat{Y}_{S_i}^2} \quad (31)$$

and:

$$\begin{bmatrix} \hat{X}_{S_i} \\ \hat{Y}_{S_i} \\ \hat{Z}_{S_i} \end{bmatrix} = \mathbf{M}_i^\top \cdot \begin{bmatrix} \hat{E}_i \\ \hat{N}_i \\ \hat{U}_i \end{bmatrix}_k \quad (32)$$

where $\mathbf{M}_i^\top$ is the rotation matrix from the i^{th} sensor-based ENU reference frame to the sensor fixed reference frame of the i^{th} sensor, as defined in the Appendix. It rotates the East-North-Up coordinates for each sensor location towards the sensor orientation (viewing direction of camera (and radar if applicable)). These local ENU _{i} coordinates are calculated from the aforementioned predicted state in WGS-84 coordinates $\begin{bmatrix} \phi_t & \lambda_t & h_t \end{bmatrix}_{\text{pred}}^\top$ via the transformation elaborated in the appendix, resulting in the predicted target position coordinates in ECEF, and a subsequent rotation from ECEF to the local sensor's ENU as follows:

$$\begin{bmatrix} \hat{E}_i \\ \hat{N}_i \\ \hat{U}_i \end{bmatrix}_k = \mathbf{T}_i^\top \begin{bmatrix} \hat{X} \\ \hat{Y} \\ \hat{Z} \end{bmatrix}_{0\text{ECEF}} \quad (33)$$

where $\mathbf{T}_i^\top$ is the rotation matrix from the ECEF reference frame to the local sensor ENU reference frame, as defined in the Appendix.

D. Kalman Filter Gain $\mathbf{W}$

The following steps are performed in order to calculate the gain of the filter:

- 1) Calculate measurement matrix $\mathbf{H}$
- 2) Calculate innovation covariance matrix $\mathbf{S}$
- 3) Calculate gain matrix $\mathbf{W}$

Each step will be elaborated in detail next:

1. Measurement matrix $\mathbf{H}$

The measurement matrix $\mathbf{H}$ is basically the Jacobian of the predicted measurement vector:

$$\hat{\mathbf{z}}_k = \begin{bmatrix} \hat{\mathbf{z}}_{k,1} & \dots & \hat{\mathbf{z}}_{k,i} & \dots & \hat{\mathbf{z}}_{k,N} \end{bmatrix}^\top$$

with N the number of sensor locations and each individual sensor location based component defined as follows with only the hardware-relevant parts present in order to reduce matrix size and computational load:

$$\hat{\mathbf{z}}_{k,i} = \hat{\mathbf{z}}_{k,i_{\text{EOcam}}} = \hat{\mathbf{h}}_{\text{EOcam}_i}(\hat{\mathbf{x}}_k)$$

with respect to the state vector $\hat{\mathbf{x}}_k = [\hat{\mathbf{x}}_{k_1} \quad \hat{\mathbf{x}}_{k_2}]^\top$, with $\mathbf{x}_{k_1}$ being the position components and $\mathbf{x}_{k_2}$ the velocity components, both in the ENU reference frame. The measurement matrix $\mathbf{H}$ basically consists of the blocks $\mathbf{H} = [\mathbf{H}_1 \quad \dots \quad \mathbf{H}_i \quad \dots \quad \mathbf{H}_N]^\top$ with N the number of sensors and decomposing each $\mathbf{H}_i$ in 8 blocks results in the following structure:

$$\mathbf{H}_i(\mathbf{x}_k) = \frac{d\hat{\mathbf{h}}_i(\hat{\mathbf{x}}_k)}{d\hat{\mathbf{x}}_k} = \begin{bmatrix} \frac{d\hat{\mathbf{h}}_{\text{EOcam}_i}(\hat{\mathbf{x}}_{k_1})}{d\hat{\mathbf{x}}_{k_1}} & \frac{d\hat{\mathbf{h}}_{\text{EOcam}_i}(\hat{\mathbf{x}}_{k_2})}{d\hat{\mathbf{x}}_{k_2}} = \mathbf{0}_{2 \times 3} \end{bmatrix} \quad (34)$$

The measurements depend only on the position components and thus $\mathbf{x}_{k_1}$. All nonzero blocks in Eq. (34) are elaborated next:

$$\frac{d\hat{\mathbf{h}}_{\text{EOcam}_i}(\hat{\mathbf{x}}_{k_1})}{d\hat{\mathbf{x}}_{k_1}} = \begin{bmatrix} \frac{d\hat{e}_i}{dE_i} & \frac{d\hat{n}_i}{dN_i} & \frac{d\hat{u}_i}{dU_i} \end{bmatrix} = \begin{bmatrix} \frac{d\hat{x}_i}{dX_{S_i}} & \frac{d\hat{y}_i}{dY_{S_i}} & \frac{d\hat{z}_i}{dZ_{S_i}} \end{bmatrix} \cdot \mathbf{M}^\top = \begin{bmatrix} \frac{\dot{X}_{S_i} \cdot \dot{Z}_{S_i}}{\hat{\rho}_{\text{hor}_0} \hat{\rho}_0^2} & \frac{\dot{Y}_{S_i} \cdot \dot{Z}_{S_i}}{\hat{\rho}_{\text{hor}_0} \hat{\rho}_0^2} & \frac{-\hat{\rho}_{i_{\text{hor}}}}{\hat{\rho}_0^2} \\ \frac{-\dot{Y}_{S_i}}{\hat{\rho}_{\text{hor}_0}^2} & \frac{\dot{X}_{S_i}}{\hat{\rho}_{\text{hor}_0}^2} & 0 \end{bmatrix} \cdot \mathbf{M}^\top \quad (35)$$

The denominators $\hat{\rho}_0$ and $\hat{\rho}_{\text{hor}_0}$ in Eq. (35) are singularity protected as follows:

$$\hat{\rho}_0 = \max(\hat{\rho}_i, \varepsilon_\rho) \quad (36)$$

$$\hat{\rho}_{\text{hor}_0} = \max(\hat{\rho}_{i_{\text{hor}}}, \varepsilon_{\rho_{\text{hor}}}) \quad (37)$$

where ε_ρ and $\varepsilon_{\rho_{\text{hor}}}$ are fairly small positive values to prevent singularities.

2. Innovation covariance matrix $\mathbf{S}$

The innovation covariance matrix is calculated as follows:

$$\mathbf{S} = \mathbf{H} \hat{\mathbf{P}}_k \mathbf{H}^\top + \mathbf{R}_{\text{sensors}} \quad (38)$$

where: $\mathbf{R}_{\text{sensors}} = \text{diag}[\mathbf{R}_{\text{sensor}_1} \dots \mathbf{R}_{\text{sensor}_i} \dots \mathbf{R}_{\text{sensor}_N}]$ with the diagonal elements per sensor location defined as follows:

$$\mathbf{R}_{\text{sensor}_i} = \text{diag} \left[\sigma_{\alpha_{\text{EOcam}_i}}^2 \quad \sigma_{\gamma_{\text{EOcam}_i}}^2 \right]$$

where only the hardware relevant elements per sensor location are kept. A diagonal matrix of sensor valid flags is defined similarly, $\mathbf{M}_{\text{sensors}_{\text{valid}}} = \text{diag}[\mathbf{M}_{\text{sensors}_{1_{\text{valid}}}} \dots \mathbf{M}_{\text{sensors}_{i_{\text{valid}}}} \dots \mathbf{M}_{\text{sensors}_{N_{\text{valid}}}}]$ with:

$$\mathbf{M}_{\text{sensors}_{i_{\text{valid}}}} = \text{diag}[\mu_{\text{EOcam}_i} \quad \mu_{\text{EOcam}_i}]$$

These valid flags $\mu_{\bullet_i}$ are one for valid sensor readings or zero for invalid ones. These flags allow to take into account lower sampling rates for specific sensors, signal dropouts (e.g. object is outside field of view), sensor malfunctions, invalid signals, etc. This setup assumes that there is a monitoring algorithm in place that drives the valid flag values accordingly. This valid flag matrix is needed for calculating the inverse of the innovation covariance matrix:

$$\mathbf{S}_{\text{valid}}^{-1} = \mathbf{M}_{\text{sensors}_{\text{valid}}} \cdot \mathbf{S}^{-1} \cdot \mathbf{M}_{\text{sensors}_{\text{valid}}} \quad (39)$$

3. Gain matrix $\mathbf{W}$

The Kalman filter gain matrix is calculated with all the aforementioned information as follows:

$$\mathbf{W} = \hat{\mathbf{P}}_k \mathbf{H}^T \mathbf{S}_{\text{valid}}^{-1} \quad (40)$$

$$\mathbf{W}_{\text{valid}} = \mathbf{W} \mathbf{M}_{\text{sensors}_{\text{valid}}} \quad (41)$$

Invalid flags in $\mathbf{M}_{\text{sensors}_{\text{valid}}}$ will result in zero columns in $\mathbf{W}_{\text{valid}}$. As a consequence, that sensor information will not be used for the correction step. This means that the prediction step will have a higher relative importance for the states that are the main drivers of these invalid measurements, except when there are other valid sensor measurements that contain significant information of these states. When valid measurements are sparse, the Kalman Filter will extrapolate a trajectory that will gradually diverge from reality. This growing uncertainty is indicated by the rank and/or condition number of the observability matrix, as well as by the standard deviation of the estimates.

To prevent overcompensation, the gain matrix $\mathbf{W}_{\text{valid}}$ is rescaled in case one of the matrix entries exceeds a certain upper threshold $\varepsilon_{\text{limit}}$: If $\mathbf{W}(i, j)_{\text{valid}} > \varepsilon_{\text{limit}}$ for any i and j , then define the scaling factor $\sigma_{\text{scale}} = \frac{\varepsilon_{\text{limit}}}{\max \mathbf{W}(i, j)_{\text{valid}}}$ and rescale the weighting matrix as follows: $\mathbf{W}(i, j)_{\text{valid}_{\text{scaled}}} = \sigma_{\text{scale}} \mathbf{W}(i, j)_{\text{valid}}$. Similarly, if $\mathbf{W}(i, j)_{\text{valid}} < -\varepsilon_{\text{limit}}$, define the scaling factor $\sigma_{\text{scale}} = \frac{-\varepsilon_{\text{limit}}}{\max \mathbf{W}(i, j)_{\text{valid}}}$ and rescale the weighting matrix as follows: $\mathbf{W}(i, j)_{\text{valid}_{\text{scaled}}} = \sigma_{\text{scale}} \mathbf{W}(i, j)_{\text{valid}}$.

E. State Update $\mathbf{x}_k$

The state update step relies on the total sensor measurements vector covering all sensor locations, which will give only measurement values for the locations where the tracked object is in the field of view.

The innovation is calculated first, which is the difference between the predicted measurements $\hat{\mathbf{z}}_k$ and the actual sensor measurements $\mathbf{z}_k^*$, including sensor limitations and perturbations:

$$\varepsilon = \mathbf{z}_k - \hat{\mathbf{z}}_k = \mathbf{z}_k - \mathbf{h}(\hat{\mathbf{x}}_k) \quad (42)$$

This innovation ε is then used to calculate the state update by means of the previously computed Kalman Gain matrix:

$$\mathbf{x}_k = \hat{\mathbf{x}}_k + \mathbf{W}_{\text{valid}} \cdot \varepsilon \quad (43)$$

F. Geodetic Coordinates Calculation

Next step is calculating the absolute position of the target in WGS-84 coordinates. This is achieved by combining the known sensor position and the relative position of the target with respect to the sensor position, which is the first half of the corrected state vector $\mathbf{x}_{k1}$. Since both are defined in different ENU reference frames, they are first converted to the ECEF (earth centered earth fixed) reference frame, then added together and converted back to WGS-84.

The sensor position is converted straight from WGS-84 (i.e. latitude, longitude and altitude) to ECEF via the transformation elaborated in the appendix, resulting in the position coordinates $[X \ Y \ Z]_{\text{ECEF}}^T$. The relative position of the target with respect to the sensor position in ECEF coordinates is calculated as follows:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{(S \rightarrow T)_{\text{ECEF}}} = \mathbf{T} \begin{bmatrix} E \\ N \\ U \end{bmatrix}_k = \mathbf{T} \mathbf{x}_{k1} \quad (44)$$

where $\mathbf{T}$ is the rotation matrix from the ENU reference frame to the ECEF reference frame, as defined in the Appendix.

The absolute position of the target in ECEF coordinates is then calculated as follows:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{T_{\text{ECEF}}} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{\text{ECEF}} + \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{(S \rightarrow T)_{\text{ECEF}}} \quad (45)$$

As the last step, this absolute target position is converted back to WGS-84 (i.e. latitude, longitude and altitude) via the reverse transformation elaborated in the Appendix.

*In the numerical calculations, an 'empty' measurement scalar in $\mathbf{z}_k$ is usually represented by a 'NaN'. In order to prevent numerical issues these have to be replaced by zeros together with their respective valid flag being set to zero in $\mathbf{W}_{\text{valid}}$ so that they are ignored.

G. Velocity Update

The velocity components of the target, which are the second half of the corrected state vector $\mathbf{x}_{k_2}$ are defined with respect to the sensor ENU reference frame, these are transformed to the target ENU reference frame as follows:

$$\begin{bmatrix} \dot{E} \\ \dot{N} \\ \dot{U} \end{bmatrix}_{k_T} = \mathbf{L}^T \begin{bmatrix} \dot{E} \\ \dot{N} \\ \dot{U} \end{bmatrix}_k = \mathbf{L}^T \mathbf{x}_{k_2} \quad (46)$$

H. State Error Covariance Update $\mathbf{P}_k$

By first defining:

$$\mathbf{W}_H = \mathbf{I}_{6 \times 6} - \mathbf{W}_{\text{valid}} \mathbf{H} \quad (47)$$

where $\mathbf{I}_{6 \times 6}$ is a 6-by-6 identity matrix, the state error covariance matrix is then updated in the reference point reference frame:

$$\mathbf{P}_k = \mathbf{W}_H \hat{\mathbf{P}}_k \mathbf{W}_H^T + \mathbf{W}_{\text{valid}} \mathbf{R}_{\text{sensors}} \mathbf{W}_{\text{valid}}^T \quad (48)$$

which is finally converted to the target reference frame via the aforementioned rotation matrix $\mathbf{L}$:

$$\mathbf{P}_{k_T} = \begin{bmatrix} \mathbf{L} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{L} \end{bmatrix} \mathbf{P}_k \quad (49)$$

Appendix: Reference Frame Transformations for Kalman Filter

Throughout this paper, five different reference frames are used: geodetic (WGS-84), Earth-Centered Earth-Fixed (ECEF), East–North–Up (ENU), body fixed and sensor spherical reference frames. They are all illustrated in Fig. 35.

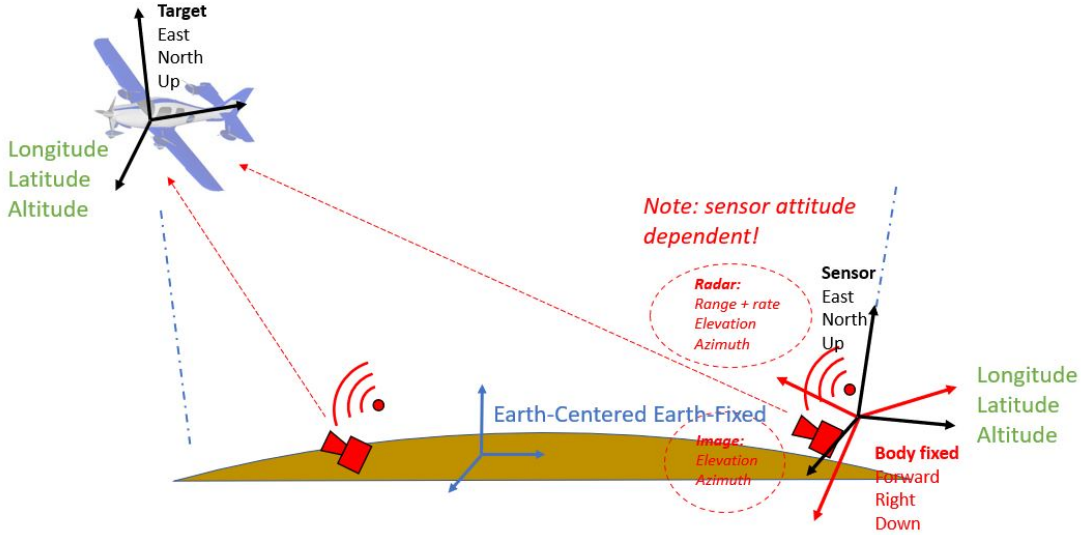


Fig. 35 Reference frames used by Kalman Filter

The geodetic reference frame expresses the spherical coordinates as longitude λ_s with respect to the Greenwich meridian, latitude ϕ_s with respect to the equator and altitude h_s above the average sea-level. The ECEF reference frame is Cartesian with the origin in the Earth center and its coordinates are as follows: the Z_{ECEF} axis extends through true North, X_{ECEF} points through the earth surface where the equator and Greenwich meridian intersect, and Y_{ECEF} is finally perpendicular on the two previous axes. The ENU Cartesian reference frame is sensor centered with the axes pointing East–North–Up. As illustrated in Fig. 35, the ‘up’ direction is location dependent, as well as the others. As a consequence, the ENU frames for different sensors and target are all differently oriented. This difference increases for

increasing distances between all of them. This must be reflected in the calculations. Next is the body fixed reference frame, which is Cartesian and centered at the sensor's center of gravity. The X_B axis points forward, Y_B axis points to the right and the Z_B axis is downwards. Finally, the sensor centered spherical reference frame expresses the coordinates as range ρ (distance from center of gravity), elevation α (vertical angle with respect to the $X_B - Y_B$ -plane) and azimuth γ (horizontal angle with respect to the forward axis). The sensor measurements from radar and camera are expressed in this spherical reference frame. Note that both latter sensor fixed reference frames are sensor attitude dependent, i.e. the viewing direction.

The transformations between the different aforementioned reference frames are illustrated in Fig. 36. The transformations between body-fixed, ENU, ECEF as well as between different ENU's of different sensors and target at different locations are pure angular rotations by means of the respective rotation matrices $\mathbf{M}(\phi, \theta, \psi)$ over the three Euler angles between body fixed and ENU, $\mathbf{T}(\lambda_s, \phi_s)$ over longitude and latitude between ENU and ECEF and $\mathbf{L}(\lambda_s, \phi_s, \lambda_t, \phi_t)$ over longitude and latitude of both locations between different ENU's at different locations. The mappings between body fixed and sensor spherical as well as between geodetic and ECEF are more complex nonlinear transformations. Each of them is elaborated next.

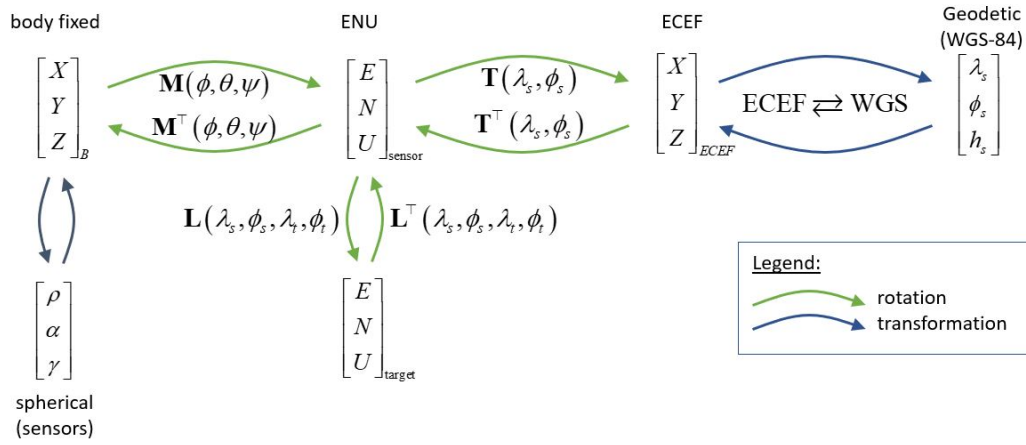


Fig. 36 Transformations between the different reference frames

Transformation from body-fixed to spherical and backwards

The sensor spherical coordinates are defined as a function of the body-fixed coordinates as follows:

$$\rho = \sqrt{X_B^2 + Y_B^2 + Z_B^2} \quad (50)$$

$$\alpha = \arctan \frac{-Z_B}{\rho_{\text{hor}}} \quad (51)$$

$$\gamma = \arctan \frac{Y_B}{X_B} \quad (52)$$

$$(53)$$

where:

$$\rho_{\text{hor}} = \sqrt{X_B^2 + Y_B^2} \quad (54)$$

And the other way around:

$$X_B = \rho \cos \alpha \cos \gamma \quad (55)$$

$$Y_B = \rho \cos \alpha \sin \gamma \quad (56)$$

$$Z_B = \rho \sin \alpha \quad (57)$$

Rotation from body-fixed to ENU and backwards

From body-fixed to ENU is a sequential rotation over the three Euler angles roll ϕ , pitch attitude θ and yaw ψ [20]:

$$\mathbf{M}(\phi, \theta, \psi) = \begin{bmatrix} \sin \psi \cos \theta & \cos \phi \cos \psi + \sin \phi \sin \psi \sin \theta & -\sin \phi \cos \psi + \cos \phi \sin \psi \sin \theta \\ \cos \psi \cos \theta & -\cos \phi \sin \psi + \sin \phi \cos \psi \sin \theta & \sin \phi \sin \psi + \cos \phi \cos \psi \sin \theta \\ \sin \theta & -\sin \phi \cos \theta & -\cos \phi \cos \theta \end{bmatrix} \quad (58)$$

The transpose is used for the backwards direction.

Rotation from ENU to ECEF and backwards

From the vehicle centered ENU to the earth centered ECEF is a sequential rotation over the longitude λ_s and latitude ϕ_s of the current position[20]:

$$\mathbf{T}(\lambda_s, \phi_s) = \begin{bmatrix} -\sin \lambda_s & -\cos \lambda_s \sin \phi_s & \cos \lambda_s \cos \phi_s \\ \cos \lambda_s & -\sin \lambda_s \sin \phi_s & \sin \lambda_s \cos \phi_s \\ 0 & \cos \phi_s & \sin \phi_s \end{bmatrix} \quad (59)$$

The transpose is used for the other way around.

Rotation between ENU's

A rotation between ENU's is basically a rotation from the first ENU to ECEF and subsequently from ECEF back to the second ENU:

$$\mathbf{L}(\lambda_s, \phi_s, \lambda_t, \phi_t) = \mathbf{T}^T(\lambda_t, \phi_t) \cdot \mathbf{T}(\lambda_s, \phi_s) \quad (60)$$

Substituting Eq. (59) results in:

$$\begin{aligned} \mathbf{L}(\lambda_s, \phi_s, \lambda_t, \phi_t) &= \\ &= \mathbf{T}^T(\lambda_t, \phi_t) \cdot \mathbf{T}(\lambda_s, \phi_s) \\ &= \begin{bmatrix} -\sin \lambda_t & -\cos \lambda_t \sin \phi_t & \cos \lambda_t \cos \phi_t \\ \cos \lambda_t & -\sin \lambda_t \sin \phi_t & \sin \lambda_t \cos \phi_t \\ 0 & \cos \phi_t & \sin \phi_t \end{bmatrix}^T \begin{bmatrix} -\sin \lambda_s & -\cos \lambda_s \sin \phi_s & \cos \lambda_s \cos \phi_s \\ \cos \lambda_s & -\sin \lambda_s \sin \phi_s & \sin \lambda_s \cos \phi_s \\ 0 & \cos \phi_s & \sin \phi_s \end{bmatrix} \\ &= \begin{bmatrix} \cos(\lambda_t - \lambda_s) & \sin(\lambda_t - \lambda_s) \sin \phi_s & -\sin(\lambda_t - \lambda_s) \cos \phi_s \\ -\sin(\lambda_t - \lambda_s) \sin \phi_t & \cos \phi_t \cos \phi_s + \sin \phi_t \sin \phi_s \cos(\lambda_t - \lambda_s) & \cos \phi_t \sin \phi_s - \sin \phi_t \cos \phi_s \cos(\lambda_t - \lambda_s) \\ \sin(\lambda_t - \lambda_s) \cos \phi_t & \sin \phi_t \cos \phi_s - \cos \phi_t \sin \phi_s \cos(\lambda_t - \lambda_s) & \sin \phi_t \sin \phi_s + \cos \phi_t \cos \phi_s \cos(\lambda_t - \lambda_s) \end{bmatrix} \end{aligned} \quad (61)$$

Transformation from ECEF to geodetic and backwards

ECEF $\Rightarrow$ geodetic

The target position in WGS-84 geodetic coordinates is obtained following the 15-step process presented below (extracted from RTCA DO-317A)[19]:

$$r_{\text{ecef}} = \sqrt{X_t^2 + Y_t^2} \quad (62)$$

$$E^2 = a^2 - b^2 \quad (63)$$

$$F = 54b^2 Z_t^2 \quad (64)$$

$$G = r_{\text{ecef}}^2 + (1 - \varepsilon^2) Z_t^2 - \varepsilon^2 E^2 \quad (65)$$

$$C = \frac{\varepsilon^4 F r_{\text{ecef}}^2}{G^3} \quad (66)$$

$$S = \sqrt[3]{1 + C + \sqrt{C^2 + 2C}} \quad (67)$$

$$P = \frac{F}{3 \left(S + \frac{1}{S} + 1 \right)^2 G^2} \quad (68)$$

$$Q = \sqrt{1 + 2\varepsilon^4 P} \quad (69)$$

$$r_o = \frac{-P\varepsilon^2 r_{\text{ecef}}}{1 + Q} + \sqrt{\frac{a^2}{2} \left(1 + \frac{1}{Q} \right) - \frac{P(1 - \varepsilon^2) Z_t^2}{Q(1 + Q)} - \frac{P r_{\text{ecef}}^2}{2}} \quad (70)$$

$$U = \sqrt{(r_{\text{ecef}} - \varepsilon^2 r_o)^2 + Z_t^2} \quad (71)$$

$$V = \sqrt{(r_{\text{ecef}} - \varepsilon^2 r_o)^2 + (1 - \varepsilon^2) Z_t^2} \quad (72)$$

$$Z_o = \frac{b^2 Z_t}{aV} \quad (73)$$

$$\phi_t = \tan^{-1} \left(\frac{Z_t + \varepsilon'^2 Z_o}{r_{\text{ecef}}} \right) \quad (74)$$

$$\lambda_t = \tan^{-1} \left(\frac{Y_t}{X_t} \right) \quad (75)$$

$$h_t = U \left(1 - \frac{b^2}{aV} \right) \quad (76)$$

where:

$a = 6.378137 \cdot 10^6 m$, semimajor axis of the Earth

$b = 6.3567523142 \cdot 10^6 m$, semiminor axis of the Earth

$\varepsilon = 0.081819190842622$, first eccentricity of the Earth

$\varepsilon' = 0.082094437935831$, second eccentricity of the Earth

ϕ_t = target latitude [rad]

λ_t = target longitude [rad]

h_t = target geodetic altitude [m]

geodetic $\Rightarrow$ *ECEF*

The ownship's Earth-Centered, Earth-Fixed (ECEF) position is calculated by[19]:

$$\begin{bmatrix} X_s \\ Y_s \\ Z_s \end{bmatrix} = \begin{bmatrix} (N_s + h_s) \cos \phi_s \cos \lambda_s \\ (N_s + h_s) \cos \phi_s \sin \lambda_s \\ (N_s (1 - \varepsilon^2) + h_s) \sin \phi_s \end{bmatrix} \quad (77)$$

where:

ϕ_s = ownship geodetic latitude [rad]

λ_s = ownship geodetic longitude [rad]

h_s = ownship geodetic altitude [m]

$$N_s = \frac{a}{\sqrt{1 - \varepsilon^2 \sin^2 \phi_s}}$$

$a = 6.378137 \cdot 10^6 m$, semimajor axis of the Earth

$\varepsilon = 0.081819190842622$, first eccentricity of the Earth

If the altitude of the ownship is measured as height above Mean Sea Level (MSL) from a barometric sensor, it

should be converted to WGS-84 Height Above Ellipsoid (HAE) using the following steps:

$$\xi(\phi_s) = g_{0eq} \frac{1 + k \sin^2 \phi_s}{\sqrt{1 - \varepsilon^2 \sin^2 \phi_s}} \quad (78)$$

$$R(\phi_s) = \frac{a}{1 + f + m_r - 2f \sin^2 \phi_s} \quad (79)$$

$$h_s = \frac{R(\phi_s)^h}{\frac{R(\phi_s)\xi(\phi_s)}{\xi(\frac{\pi}{4})} - h} \quad (80)$$

where:

ϕ_s = ownship geodetic latitude [rad]

$\varepsilon = 0.081819190842622$, first eccentricity of the Earth

$g_{0eq} = 9.7803253359m/s^2$, theoretical gravity at the equator

$g_{0pl} = 9.8321849378m/s^2$, theoretical gravity at the poles

$$k = \frac{b \cdot g_{0pl}}{a \cdot g_{0eq}} - 1$$

$\xi(\phi_s)$ = normal gravity on the surface of an ellipsoid at ϕ_s [m/s^2]

$a = 6.378137 \cdot 10^6m$, semimajor axis of the Earth

$b = 6.3567523142 \cdot 10^6m$, semiminor axis of the Earth

$f = \frac{a-b}{a}$, flattening of the Earth

$m_r = 0.003449787$, gravity ratio

$R(\phi_s)$ = radius of the Earth at ϕ_s [m]

h = measured ownship height above MSL

h_s = ownship geodetic altitude [m]

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References

- [1] Aubuchon, V. V., Hashemi, K. E., Shively, R. J., and Wishart, J. M., "Multi-Vehicle (m:N) Operations in the NAS - NASA's Research Plans," *AIAA AVIATION 2022 Forum*, American Institute of Aeronautics and Astronautics, 2022. doi:10.2514/6.2022-3758.
- [2] Lombaerts, T., Shish, K., Keller, G., Stepanyan, V., Cramer, N., and Ippolito, C., "Adaptive Multi-Sensor Fusion Based Object Tracking for Autonomous Urban Air Mobility Operations," *AIAA SciTech Forum*, 2022. doi:10.2514/6.2022-0362.
- [3] Lombaerts, T., Kannan, K., Kawamura, E., Dolph, C., Stepanyan, V., Gorospe, G. E., and Ippolito, C. A., "Distributed Ground Sensor Fusion Based Object Tracking for Autonomous Advanced Air Mobility Operations," *AIAA SCITECH 2023 Forum*, American Institute of Aeronautics and Astronautics, 2023. doi:10.2514/6.2023-0896.
- [4] Lombaerts, T., Kawamura, E., Kannan, K., Dolph, C., Stepanyan, V., Gorospe, G. E., and Ippolito, C. A., "Development and Field Test Results of Distributed Ground-Based Sensor Fusion Object Tracking," *AIAA SCITECH 2024 Forum*, American Institute of Aeronautics and Astronautics, 2024. doi:10.2514/6.2024-1389.
- [5] Ippolito, C. A., Martin, R. A., Kawamura, E., Gorospe, G. E., Holforthy, W., Kannan, K., Stepanyan, V., Lombaerts, T., Brown, N., and Dolph, C., "Enabling Smart Urban Airspaces through Distributed Sensing Technologies," *AIAA SCITECH 2025 Forum*, American Institute of Aeronautics and Astronautics, 2025.
- [6] Kannan, K., Baculi, J. E., Lombaerts, T., Kawamura, E., Gorospe, G. E., Holforthy, W., Ippolito, C. A., Stepanyan, V., Dolph, C., and Brown, N., "A Simulation Architecture for Air Traffic Over Urban Environments Supporting Autonomy Research in Advanced Air Mobility," *AIAA SCITECH 2023 Forum*, American Institute of Aeronautics and Astronautics, 2023. doi:10.2514/6.2023-0895.

- [7] Stepanyan, V., Ippolito, C. A., Kannan, K., Kawamura, E., Lombaerts, T., Holforty, W., and Dolph, C., “Flight Test Design and Implementation for Independent Surveillance of an Airspace Through a Distributed Ground Sensing Network,” *AIAA SCITECH 2024 Forum*, American Institute of Aeronautics and Astronautics, 2024. doi:10.2514/6.2024-1387.
- [8] Kannan, K., Ippolito, C. A., and Holforty, W., “An Indoor Flight Test Setup with Sensor Payload and Distributed Ground Nodes for Distributed Sensing Concepts,” *AIAA SCITECH 2024 Forum*, American Institute of Aeronautics and Astronautics, 2024. doi:10.2514/6.2024-2007.
- [9] Kawamura, E., Kannan, K., Lombaerts, T., Stepanyan, V., Dolph, C., and Ippolito, C. A., “Ground-Based Vision Tracker for Advanced Air Mobility and Urban Air Mobility,” *AIAA SCITECH 2024 Forum*, American Institute of Aeronautics and Astronautics, 2024. doi:10.2514/6.2024-2010.
- [10] Aranda, S., Martinez, S., and Bullo, F., *On Optimal Sensor Placement and Motion Coordination for Target Tracking*, 2004. doi:10.21236/ada460179.
- [11] Zhao, S., “Optimal sensor placement for target localisation and tracking in 2D and 3D,” *International Journal of Control*, Vol. 86, No. 10, 2013, pp. 1687–1704. doi:10.1080/00207179.2013.792606.
- [12] Yoo, K., and Chun, J., “Analysis of Optimal Range Sensor Placement for Tracking a Moving Target,” *IEEE Communications Letters*, Vol. 24, No. 8, 2020, pp. 1700–1704. doi:10.1109/lcomm.2020.2994448.
- [13] Oh, M. S., Hosseinipour, S., Kim, T., Love, D. J., Krogmeier, J. V., and Brinton, C. G., “Dynamic and Robust Sensor Selection Strategies for Wireless Positioning With TOA/RSS Measurement,” *IEEE Transactions on Vehicular Technology*, 2023, pp. 1–16. doi:10.1109/tvt.2023.3279833.
- [14] Shish, K. H., Cramer, N. B., Gorospe, G., Lombaerts, T., Stepanyan, V., and Kannan, K., “Survey of Capabilities and Gaps in External Perception Sensors for Autonomous Urban Air Mobility Applications,” *AIAA Scitech 2021 Forum*, American Institute of Aeronautics and Astronautics, 2021. doi:10.2514/6.2021-1114.
- [15] Kannan, K., Ippolito, C., and Holforty, W., “Flight Test Configuration of the Sensor Payload and the Ground Nodes in Distributed Sensing Frameworks,” *AIAA SciTech 2024 Forum*, 2024. doi:10.2514/6.2024-2007.
- [16] Stepanyan, V., Ippolito, C., Kannan, K., Kawamura, E., Lombaerts, T., Holforty, W., and Dolph, C., “Flight Test Design and Implementation for Airspace Independent Surveillance through a Distributed Ground Based Sensor Network,” *AIAA SciTech 2024 Forum*, 2024. doi:10.2514/6.2024-1387.
- [17] Kawamura, E., Kannan, K., Lombaerts, T., Stepanyan, V., Dolph, C., Brown, N., and Ippolito, C. A., “Vision-Based Distributed Sensing at Vertiports for Advanced Air Mobility and Urban Air Mobility Approach and Landing,” *AIAA SCITECH 2025 Forum*, American Institute of Aeronautics and Astronautics, 2025.
- [18] Stepanyan, V., Ippolito, C. A., Kannan, K., Kawamura, E., and Lombaerts, T., “Optimal Communication Topology Determination and Sensor Selection for Independent Airspace Surveillance,” *AIAA SCITECH 2025 Forum*, American Institute of Aeronautics and Astronautics, 2025.
- [19] *Minimum Operational Performance Standards for Detect and Avoid Systems*, DO-365, RTCA Inc., 2019, techreport Appendix F: Example Functional Description For Airborne Surveillance Data Processor (ASDP) Subsystem. URL <https://standards.globalspec.com/std/14281907/rtca-do-365>.
- [20] Mohinder, S., Grewal, P., Lawrence, R., Weill, A. P., and Andrews, “Appendix C: Coordinate Transformations,” *Global Positioning Systems, Inertial Navigation, and Integration*, John Wiley & Sons, Inc., 2006, pp. 456–501. doi:10.1002/9780470099728.app3.