Gas Phase Effects on Slosh Dynamics

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Gas phase effects on slosh dynamics were quantified using computational fluid dynamics (CFD) simulation for a range of propellant and gas combinations. Historical slosh modeling using potential flow solutions typically neglects gas phase effects. Regardless, the results have been shown to compare well with slosh ground tests typically performed with water and air at standard temperature and pressure. CFD analysis reveals that as the liquid-to-gas density ratio decreases, slosh dynamics change due to the relative increase in gas inertia and thus influence on liquid motion. The result is a profound impact on slosh dynamics over certain parameter spaces particularly for liquid hydrogen. Gas phase effects on slosh dynamics should be considered in slosh models especially for liquid hydrogen propellant tanks.

I. Introduction

The Fluid Dynamics Branch at the NASA Marshall Space Flight Center supports slosh model development for 1st and 2nd stage propellant tanks of the Space Launch System (SLS) rocket among other customers. Historical modeling approaches based on potential flow theory [1][2] are used for many slosh model parameters which are supplemented by recent theoretical, computational, and experimental work for damping quantification [3]. CFD simulation of liquid hydrogen (LH2) propellant tanks at flight-like conditions resulted in differences from both the previously described historical modeling approaches and the breadth of water-and-air ground test data even when corrected for liquid properties [1][4]. A study was begun to identify the cause of the observed differences in slosh model parameters.

Parametric study of slosh dynamics for a generic large propellant tank reveals that liquid-to-gas density ratio is the primary reason for the observed differences. CFD simulation was shown to compare very well with potential flow solutions of non-damping slosh model parameters for high liquid-to-gas density ratios. For fluid properties like water and air ground test setups or typical liquid oxygen (LOX) propellant tanks, liquid dynamics dominate the bulk slosh motion. Gas inertia and thus dynamics become important as fluid properties move towards those for high pressure liquid methane (LCH4) and LH2 propellant tanks. Slosh model parameter dependencies on liquid-to-gas density ratio are provided here to help inform the aerospace community of the need to account for this effect in future modeling efforts.

II. Background

The analysis documented herein is applicable to lateral liquid slosh which is the oscillation of a liquid mass along a single axis in a partially filled container. This type of liquid slosh can only exist when the storage tank is subjected to an appreciable acceleration, due either to gravitational fields or vehicle accelerations. The direction of oscillation

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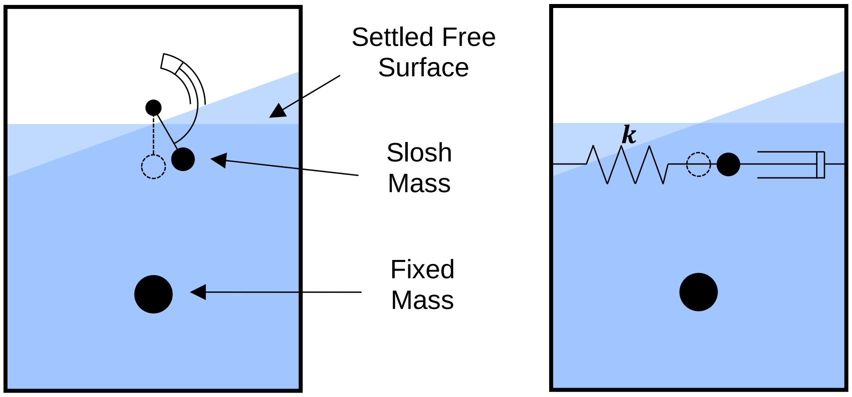
is perpendicular to the primary acceleration and is onset by a force in the direction of oscillation. An example of this motion is shown in Fig. 1.



a) Simple Pendulum Model   b) Spring-Mass-Damper Model

Fig. 1 Illustration of lateral liquid slosh and common mechanical models of slosh.

Lateral slosh can be represented by an equivalent mechanical model, two of which are shown in Fig. 1. In Fig. 1a, a simple pendulum is used to represent the oscillation of the liquid center of mass, while in Fig. 1b the liquid center of mass is represented by a mass attached to a spring. Both models result in the same net forces and torques on a tank resulting from liquid slosh. Both models show that a horizontal or lateral motion of the tank causes the liquid to slosh, i.e., causes the mass of the model to oscillate relative to the tank. The models do not capture vertical slosh motion like Faraday waves but that motion is not expected during high acceleration like powered flight nor would it significantly impact vehicle stability for which the models are primarily used. The linearized ordinary differential equations (ODE) for these mechanical models are shown below in Equations 1 and 2. The model variables are described in Table 1. Additional consideration is needed to place the oscillating mass in the correct position to recreate both slosh forces and moments, i.e., pendulum hinge location or slosh mass location. Also, a fixed mass and corresponding location are typically modeled to preserve the total liquid mass and fully settled inertia in force and moment balance equations.

Linearized Simple Pendulum ODE:

$$m\frac{d^2\theta}{dt^2} + 2\gamma\omega_n m\frac{d\theta}{dt} + \frac{mg}{L}\theta = \frac{ma}{L} \tag{1}$$

Spring-Mass-Damper ODE:

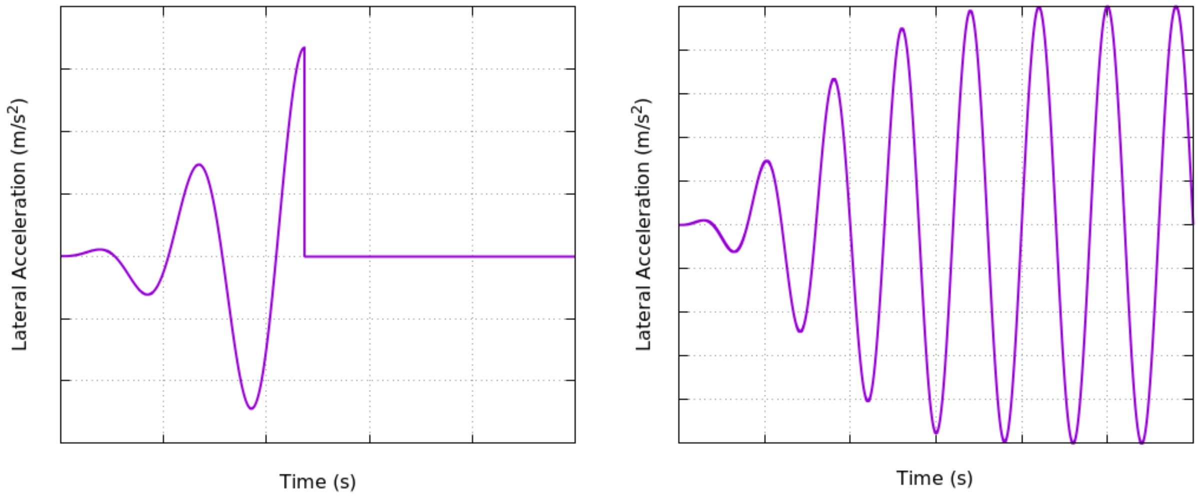
$$m\frac{d^2x}{dt^2} + 2\gamma\omega_n m\frac{dx}{dt} + kx = ma \tag{2}$$

Table 1 Legend for slosh model ODE's.

Parameter	Description
θ	Pendulum Angle (°)
x	Slosh Mass Displacement (m)
t	Time (s)
m	Slosh Mass (kg)
ω_n	Natural Frequency (rad/s)
γ	Critical Damping Ratio (%)
L	Pendulum Length (m) ($L = g/\omega_n^2$)
k	Spring Constant (kg/s^2) ($k = m\omega_n^2$)
g	Primary Acceleration (m/s^2)
a	External Lateral Acceleration (m/s^2)
L	Pendulum Length (m)

The model equations can be further reduced given the relationship between certain parameters to yield a minimum number of model parameters that need to be calculated. The minimum set of model parameters requiring calculation from a flow field solution, like CFD analysis results, include natural frequency, slosh mass, slosh mass location, and critical damping ratio. All other parameters may be calculated from the established model parameters and tank geometry given typical slosh modeling assumptions.

Natural frequency, slosh mass location, and slosh damping were all calculated from CFD simulation results of freely decaying slosh waves. A single forcing function was used to excite slosh motion before entering free decay for each analyzed fill level and liquid-to-gas density ratio. The forcing function was designed to allow gradual wave amplitude growth minimizing nonlinear features that may develop under impulsive forcing, and thereby reducing any initial transients during free decay. A typical forcing function of this type is shown in Fig. 2a.



a) Excitation to free decay

b) Forced periodicity

Fig. 2 Typical forcing functions.

Forcing was applied at natural frequencies predicted by the Southwest Research Institute (SwRI) SLOSHXL code [1][2], a potential flow solver commonly used for slosh model parameter quantification, for the subject tank and fill level and for water and air at standard temperature and pressure. Acceleration was abruptly stopped upon reaching a peak value prior to reaching large wave amplitudes where nonlinear features like wave crests and breakup may occur. As a result, slosh dynamics are expected to have a negligible dependence on wave amplitude over the analyzed design space. This also makes the ODE parameters linear and eases model solution. External forcing was applied by placing the computational domain in a non-inertial reference frame that moved according to the specified acceleration vector.

Instantaneous natural frequency was calculated from peak displacements of liquid center of mass every half slosh period that resulted from the simulation, specifically using Equation 3. Here, f is the natural frequency and t_i is the time at peak i of the total liquid mass center displacement history. An illustration of a typical total liquid mass center displacement history is shown in Fig. 3 along with the peak values used to calculate the natural frequency.

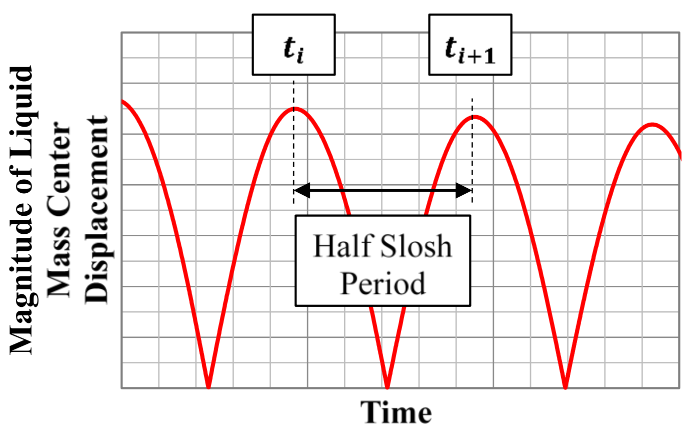


Fig. 3 Typical total liquid mass center displacement history for free-decay simulations.

$$f = \frac{2}{t_{i+1} - t_i} \tag{3}$$

Slosh mass location, H, lies along the primary acceleration vector and is relative to the tank bottom. It is the moment arm of the lateral slosh force calculated at peak values of force as is shown in Equation 4. Here, F_i is the lateral slosh force on the tank at peak i from the force transient and M_i is the corresponding moment.

$$H = \frac{\mathrm{M_i}}{\mathrm{F_i}} \tag{4}$$

Critical damping ratio is the energy dissipation over a slosh period and was calculated using logarithmic decay every half period, see Equation 5. Here, $x_{T,i}$ is the total liquid mass center displacement at peak i from the mass center displacement transient.

$$\gamma = \ln\left(\frac{|x_{T,i}|}{|x_{T,i+1}|}\right)/\pi \tag{5}$$

Averaging of the instantaneous natural frequency, slosh mass location, and critical damping ratio was performed over at least 5 slosh periods for the reported results.

Slosh mass was calculated to ensure consistency of the proposed slosh model with the predicted lateral slosh force. Specifically, the particular solution to the linear spring-mass model, shown in Equation 6, with input from CFD simulation results was used to calculate the slosh mass. In Equation 6, m_t is the total liquid mass, ω_f is the external frequency (in rad/s), A_0 is the amplitude of the applied periodic sinusoidal acceleration, and $x_{t,0}$ is the corresponding total liquid mass displacement amplitude during periodic motion. Parameter dimensions are in standard SI units.

$$\frac{m}{m_t} = \frac{x_{t,0}}{A_0}\left(\omega_n^2 - \omega_f^2\right) \tag{6}$$

The procedure does not include slosh damping since the calculated slosh mass is only significantly affected by high slosh damping. For example, the actual slosh mass is only decreased by 1% given a slosh damping of 7.5% and a driving frequency that is half of the natural frequency. For any common tank design, 7.5% is a very large damping value and orders of magnitude greater than the linear regime damping expected in a large-scale bare tank like that of the present study. The particular solution of the spring-mass ODE was used to calculate the slosh mass to eliminate dependence upon the initial condition of a slosh wave. This calculation method facilitates repeatability and calculation of the slosh mass from CFD results.

Simulation of a periodic slosh wave oscillation was achieved in a minimum amount of simulation time by driving a fully settled liquid at a particular fill level using the forcing function shown in Fig. 2b. The driving frequency was set to 0.25 Hz, about half of the natural frequency over the analyzed fill level range, to avoid resonance which often precludes periodicity. Additionally, driving at frequencies above the natural frequency but still off resonance can excite higher order slosh modes. These higher order slosh modes are unintended and not considered in the slosh modeling method presented herein. The acceleration amplitude was increased over several driving periods to avoid applying an impulse to the liquid as was done in the free decay simulations. Typical simulation results used to calculate slosh mass are shown in Fig. 4. The average of the absolute value of $x_{t,0}$ over at least 5 slosh periods was used in Equation 6.

The final background topic is that of slosh behavior dependence upon the liquid-to-gas density ratio, i.e., the subject of this work. Traditionally, analytical calculations of lateral slosh dynamics have only accounted for liquid masses and corresponding dynamics neglecting gas effects on slosh, see the models in Fig. 1. [1][2][4] Slosh experiments have primarily been conducted with water and air at standard temperature and pressure. Neglecting the gas entirely yields an infinite gas-to-liquid density ratio while water and air experiments were conducted with a density ratio of nearly 1000. Other published experiments have been conducted with different liquid properties through either additives or a different chemical makeup entirely. However, density ratios have nearly always remained high. Analysis results have been shown to compare well with experimental data where applicable. As a result, little incentive existed to prompt engineers to study the effects of gas mass on slosh dynamics.

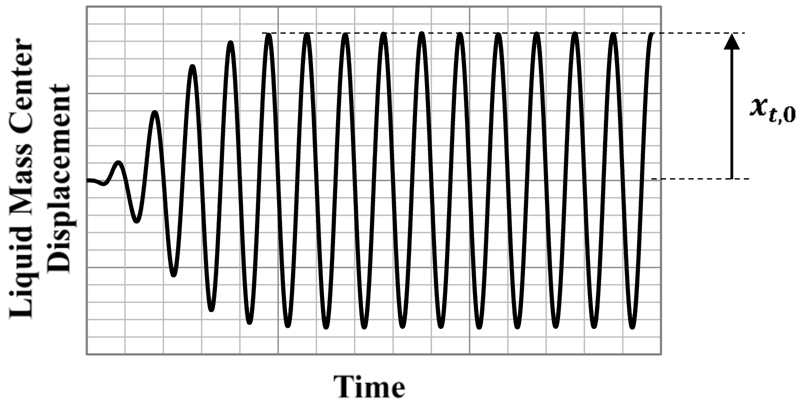


Fig. 4 Typical total liquid mass center displacement history for forced periodicity simulations.

Some slosh experiments have been conducted with pressurized LCH4 [5] and LH2 [6] which yield significantly lower density ratios. However, those experiments were not conducted with the purpose of measuring slosh dynamics but rather the effects of slosh on pressurant mass usage and liquid temperature. Still other experiments have been conducted with LCH4 and LH2 that included slosh, but the results are either unpublished or insufficient data has been published to draw conclusions about slosh dependency on density ratio. As a result, the present work includes analysis only, but validation opportunities are being sought.

In the limit of a density ratio that approaches 1, it is easy to see that a liquid slosh mass would have to push a comparable gas mass in order to move, similar to the illustration in Fig. 5 of an added pendulum in the gas phase. Due to the low speed of typical slosh dynamics and the corresponding incompressibility of the gas phase, a rigid connection between the slosh masses is likely needed. While some development on a mechanical model that includes the gas phase has been done by the authors, the work is immature and is an area that can be expanded upon.

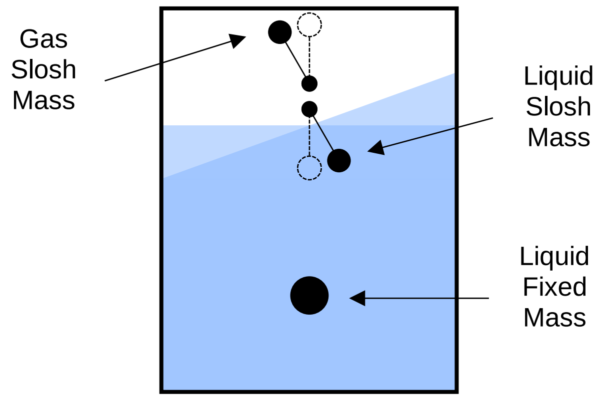


Fig. 5 Concept of a possible slosh model including gas phase effects.

III. Computational Methodology

CFD simulations were conducted using a cryogenic propellant tank representative of a large-scale main tank for an orbital rocket. The radius of the tank is 2 meters, the total height from end to end is 8 meters, the barrel section height is 4 meters, and the forward and aft dome sections are 2 meter radius hemispheres. An illustration of the tank, along with coordinate axes and liquid fill level increments of 10% by volume, is shown in Fig. 6. The origin of the coordinate system is at the bottom of the tank along the barrel section axis of symmetry, but the coordinate axes are labeled outside of the tank for visibility. No tank internal geometry was analyzed in the present study.

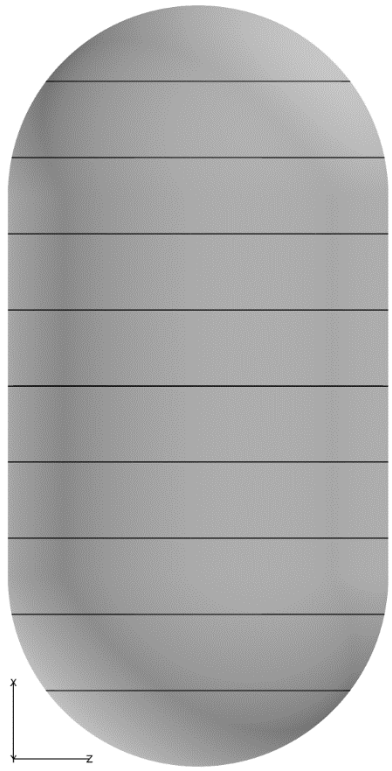


Fig. 6 Analyzed tank geometry with liquid fill levels at 10% by volume increments shown.

The extent of gas phase effects on slosh dynamics was demonstrated through a parametric study of liquid level and liquid-to-gas density ratio variation for the tank configuration mentioned. Most density ratios were obtained by manipulating the gas molecular mass to achieve the desired ratio. This enabled a smooth transition between realistic conditions for common cryogenic propellant tanks which were also directly analyzed.

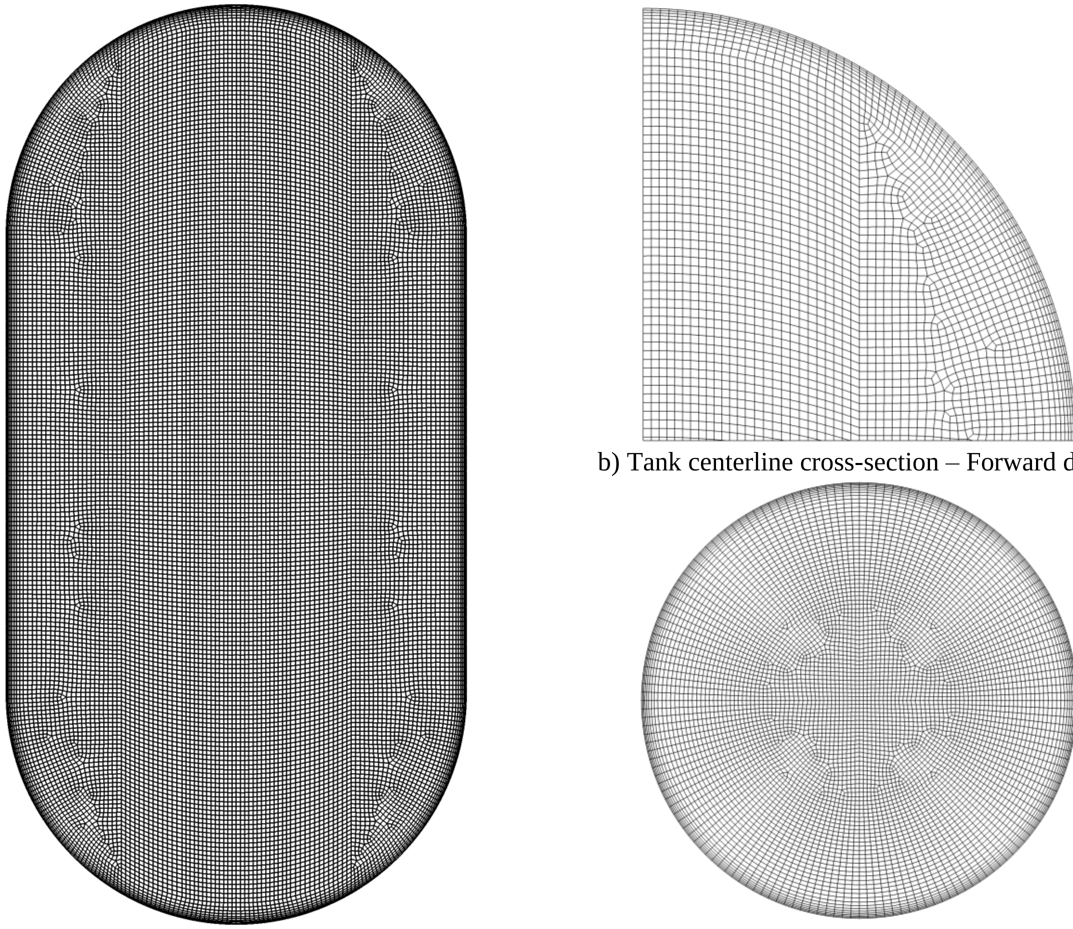
A. Computational Tools

Loci/STREAM, a pressure-based solver, was used in the preparation of these results. Loci/STREAM [7][8] is an all-speed CFD code for generalized grids in the rule-based programming framework called Loci [9]. The Loci framework allows the solver to integrate new multidisciplinary physics using a modular manner and automatic handling of massively parallel computing. The Loci/STREAM code has been applied and validated over a wide range of problems, including incompressible laminar flows to compressible turbulent flows with heat transfer. Loci/STREAM has been shown to also scale very well on significant problems [7][8]. Loci/STREAM is a pressure-based solver with SIMPLE, SIMPLEC, and PISO algorithms available for pressure-momentum coupling. Also available are various turbulence models that can be executed in Reynolds-Averaged Navier-Stokes (RANS) or Large Eddy Simulation (LES) mode. It has support for reacting flows and flamelet models for turbulent combustion. Loci/STREAM supports first and second-order discretization for inviscid fluxes as well as first and second-order temporal discretization. The code has been in use at NASA MSFC for large-scale simulations of low-speed flows, turbulent combustion, cavitation, and other problems.

The code employs multiple methodologies to represent distinct fluid phases and their shared interface. The volume of fluid (VoF) method distinctly represents a liquid and gaseous phase in the same simulation by tracking the volume of a particular fluid and the geometric liquid interface in each cell of the discretized computational domain. This simulation technique can capture gas-liquid interface deformation and breakup and has been applied extensively to propellant tank dynamics and cryogenic fluid management applications. The VoF method was used in the present analysis.

B. Computational Mesh

The tank volume was discretized with unstructured hexahedral cells as is shown in Fig. 7. Stretching normal from the tank wall was done to better capture shear flow along the wall. A mesh study was performed to demonstrate spatial resolution convergence for slosh model parameter quantification.



a) Tank centerline cross-section – Full tank

b) Tank centerline cross-section – Forward dome

c) Barrel cross-section

Fig. 7 Nominal computational mesh.

C. Simulation Inputs

Unsteady laminar Navier-Stokes equations with first order spatial and temporal accuracy were solved for every simulation. Rigid adiabatic walls were used to represent the closed system of the tank. The maximum time step was set to 0.01 s and limited by the maximum Courant-Fredericks-Lewy number in gas-liquid interface cells of 0.2. Standard Earth gravity, 9.81 m/s^2, was specified for the axial acceleration as a body force acting on the simulated fluids and along the negative x-axis. Lateral tank acceleration profiles were applied in the form of a non-inertial reference frame. Air and incompressible water were used as the gaseous and liquid phases in the simulation. Water properties at 300 K and 14.7 psia [10] were used which included a density of 996.56 kg/m^3 and dynamic viscosity of 8.5374x10^{-4} Pa*s. The ideal gas law was used to quantify gas density throughout a simulation, but variation was predictably negligible. The molecular mass for air was changed artificially to isolate the liquid-to-gas density ratio across simulations as is shown in Table 2. Air dynamic viscosity was set to 1.8460x10^{-5} Pa*s. [11] The free surface of every simulation was initialized to the fully settled condition at the desired fill level. Quiescent, isothermal initial conditions were used where gas pressure was 14.7 psia and temperature was uniformly 300 K. Neither natural convection or phase change were expected or modeled due to the specified isothermal conditions.

Table 2 Molecular mass for air specified to yield liquid-to-gas density ratio variation.

Modeled Gas Molecular Mass (g/mol)	Gas Density (kg/m^3)	Liquid Density (kg/m^3)	Liquid-to-Gas Density Ratio
28.89	1.17	996.56	850
288.9	11.7	996.56	85
2889	116	996.56	8.5

IV. Computational Results

A. Observed Dynamics

A typical time history of liquid position in the modeled propellant tank is shown in Fig. 8. The observed smooth contiguous oscillating free surface is indicative of the targeted linear regime slosh dynamics. Maximum wave amplitudes reached about 15 cm (6 inches) but varied with fill level and density ratio. Damped oscillation of liquid mass center displacement, force, and moment were observed as expected. Lateral slosh perpendicular to the driving direction was small, remaining 3 orders of magnitude below that of slosh in the driving direction.

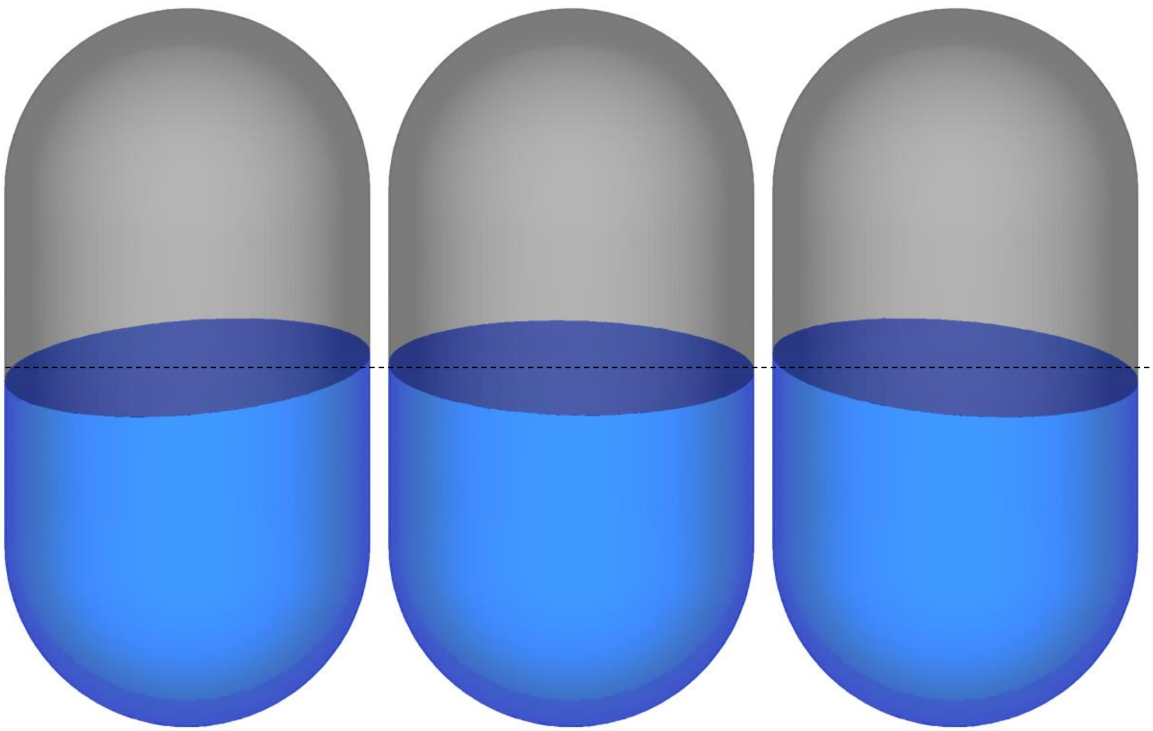


Fig. 8 Time series of CFD analysis results showing typical lateral liquid slosh.

B. Mesh Convergence Study

A spatial resolution study was conducted to demonstrate the corresponding level of simulation convergence for slosh model parameter quantification. Table 3 shows the parameter space studied. For every mesh, cells were formed with near equal cell edge lengths on all sides, but cell edges were stretched at a rate of 1.2 per cell towards the wall to investigate wall shear flow resolution requirements separate from bulk shear flow resolution requirements. Simulations were conducted with water and air at standard temperature and pressure for a realistic air density to best match analytical results and empirical data for slosh model parameters.

Both natural frequency and slosh mass location showed little dependence upon the mesh resolution as is shown in Fig. 9. Natural frequency was within 0.75% of the potential flow solution, from the SwRI SLOSHXL code, for the subject tank for every simulation. Slosh mass location was likewise within 1.5% of the potential flow solution. Slosh mass was not explicitly calculated in the mesh convergence study.

Critical damping ratio did change dramatically with mesh resolution though. Damping as a function of mesh resolution is shown in the rightmost column of Fig. 9. Capturing the correct shear flow to match bare tank linear regime damping requires significant mesh resolution in the bulk flow which drives the computational costs and simulation duration.

For initial assessment of slosh model parameter sensitivity to density ratio, it was decided to use the mesh with a nominal cell edge length of 40 mm and a cell edge length normal to the wall of 9 mm. This choice limited the error in damping while allowing for more computationally efficient simulation and thus a wider parametric study. As a result, damping reported in the density ratio study section of this paper has a known error and should only be used to understand trends.

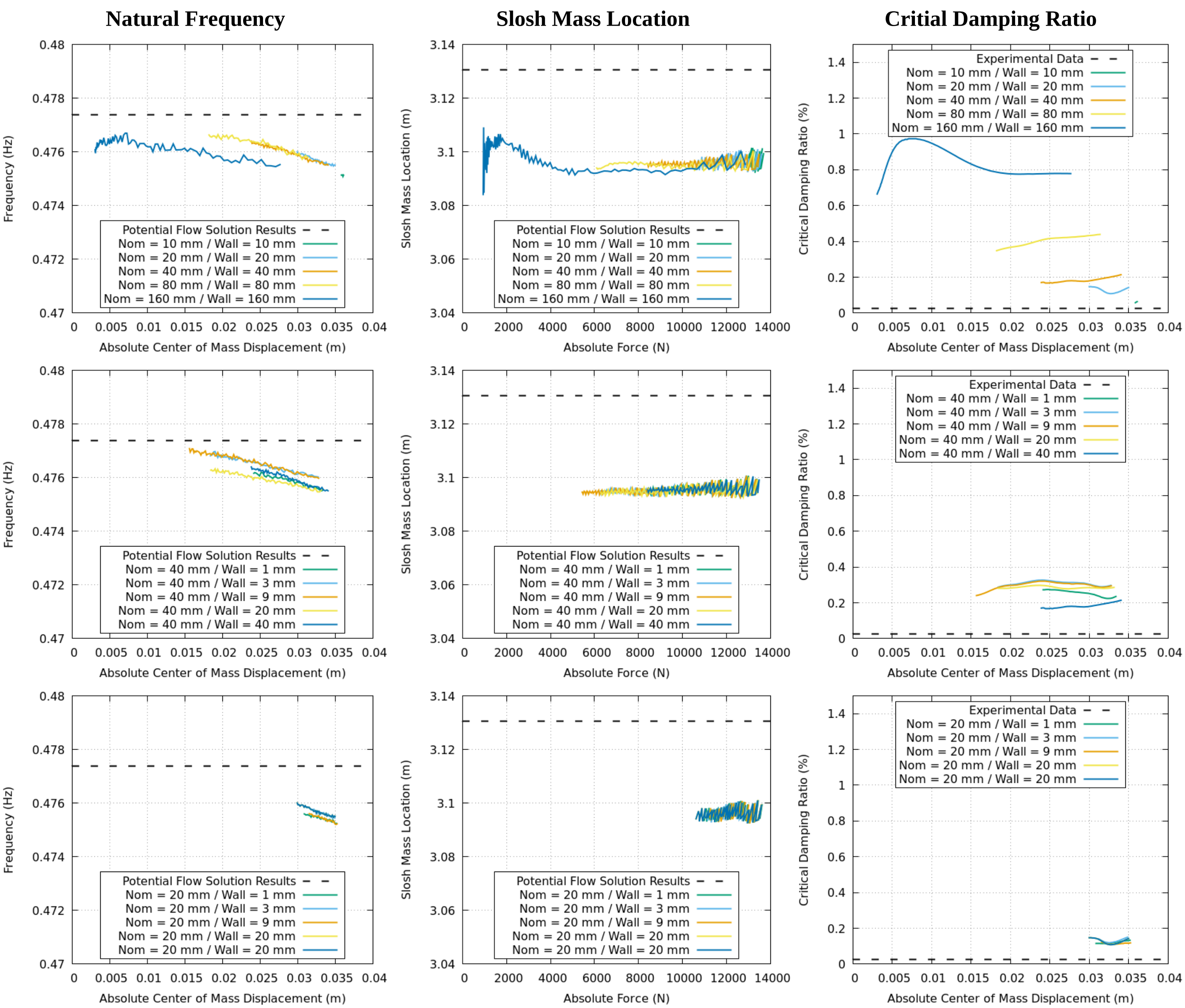


Fig. 9 Slosh model parameter sensitivities to spatial resoultion.

Table 3 Computational mesh parameters analyzed in the mesh convergence study.

Sensitivity Study Name	Nominal Cell Edge Length (mm)	Cell Edge Length Normal to Wall (mm)	# Volume Cells
Nominal Cell Size	10	10	71.0 M
	20	20	9.0 M
	40	40	1.1 M
	80	80	0.14 M
	160	160	0.018 M
Boundary Layer 1	20	1	10.9 M
	20	3	10.0 M
	20	9	9.3 M
	20	20	9.0 M
Boundary Layer 2	40	1	1.7 M
	40	3	1.5 M
	40	9	1.3 M
	40	20	1.2 M
	40	40	1.1 M

C. Density Ratio Study

Low wave amplitude lateral slosh in free decay was simulated at fill levels spanning the tank for numerous liquid-to-gas density ratios. Slosh parameters resulting from the potential flow solution for the subject tank were produced using the SwRI SLOSHXL code. [1][2]

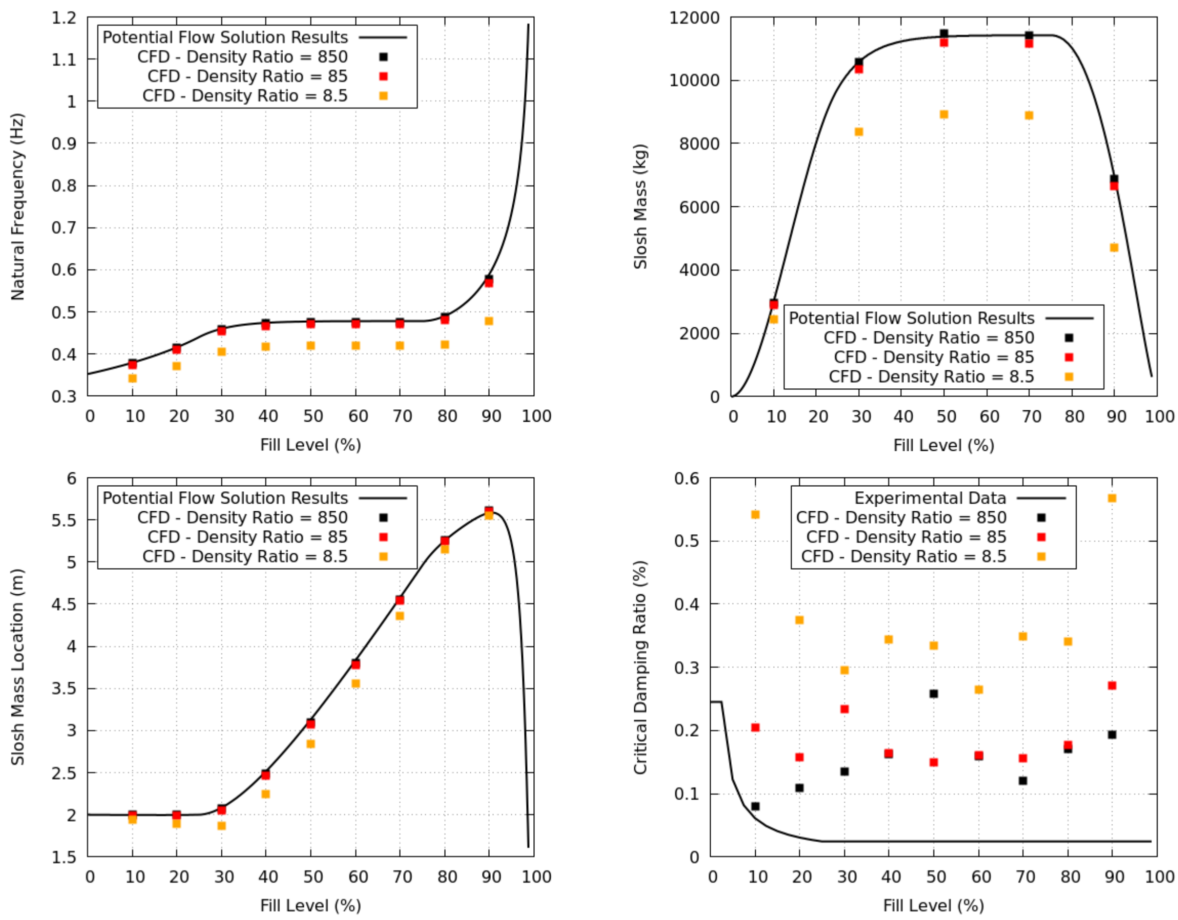


Fig. 10 Slosh model parameter sensitivities to liquid-to-gas density ratio.

Analysis results compare very well with potential flow solutions of non-damping slosh model parameters for high liquid-to-gas density ratios, see Fig. 10. Critical damping ratio is ill resolved as described in the mesh convergence section, and it is more variable owing likely to the lack of resolution and inherent variability of slosh energy decay.

Simulation results show that natural frequency, slosh mass, and slosh mass location decrease with decreasing liquid-to-vapor density ratio, see Fig. 10. Significant variability in these parameters, beyond the typical margins carried by a vehicle program, begins around a density ratio of less than 85. Density ratios during a mission will vary but could be much lower than 100 for cold gases in LH2 and LCH4 tanks, like what might be expected following a large slosh event that cools the gas. The dynamics are like a fluid-structure interaction problem where a vibrating structure submerged in liquid has a reduced natural frequency due to the mass effectively added to the structure, i.e., the structure must move the liquid as well as itself. The gas mass also counteracts the liquid mass since it moves in the opposite lateral direction decreasing net force. Critical damping ratio generally increased with decreasing density ratio.

V. Conclusion

An analysis was conducted to study slosh model parameter dependence on liquid-to-gas density ratio for a generic large propellant tank using CFD simulation results. Analysis results compared very well with potential flow solutions of non-damping slosh model parameters where expected, for high liquid-to-gas density ratios. Spatial resolution sensitivities showed that equivalent comparisons to empirical data for damping calculations could be achieved by further resolving the computational domain.

Slosh model parameters were shown to be affected by gas dynamics if the liquid-to-gas density ratio is sufficiently low. In particular, natural frequency, slosh mass, and slosh mass location all decreased with decreasing density ratio. Critical damping ratio generally increased with decreasing density ratio. For fluid properties in water and air ground tests or typical LOX or storable propellant tanks, liquid dynamics dominate the bulk slosh motion. Gas inertia and dynamics become important as fluid properties move towards those for high pressure LCH4 and LH2 propellant tanks.

More work is needed to identify experimental data that may be used to validate the findings in this paper. Additionally, theoretical work is needed to develop an accurate slosh model that includes gas dynamics for future use. One method of accounting for density ratio effects, if performance margins allow, is to expand uncertainties on non-damping slosh model parameters in Monte Carlo vehicle stability analyses to cover the range of density ratios expected during a mission. Damping quantified for a high-density ratio may be used as-is knowing there will be higher damping during the mission than what is used to design purposes.

Acknowledgments

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References

[1] Dodge, F.T., "The New "Dynamic Behavior of Liquids in Moving Containers"," Southwest Research Institute, San Antonio, TX (2000).

[2] Lomen, D.O., "Liquid Propellant Sloshing in Mobile Tanks of Arbitrary Shape," NASA CR-222.

[3] West, J.S., et.al., "Extension of Miles Equation for Ring Baffle Damping Predictions to Small Sloshing Amplitudes and Large Baffle Widths," JANNAF, 11th MSS / 9th LPS / 8th SPS Joint Subcommittee Meeting, December 2016.

[4] Abramson, H.N., "The Dynamic Behavior of Liquids in Moving Containers," NASA SP-106 (1966).

[5] DeWitt, R.L. and McIntire, T.O., "Pressurant Requirements for Discharge of Liquid Methane from a 1.52-meter- (5-ft-) Diameter Spherical Tank Under Both Static and Slosh Conditions," NASA-TN-D-7638. 1974.

[6] Moran, M.E., McNelis, N.B., Kudlac, M.T., Haberbusch, M.S., and Satomino, G.A., "Experimental Results of Hydrogen Slosh in a 62 Cubic Foot (1750 Liter) Tank," NASA-TM-106625. 1994.

[7] S. Thakur, J. Wright, J., and W. Shyy, "An Algorithm for Chemically Reacting Flows on Generalized Grids Using a Rule-Based Framework." 43rd AIAA Conference 2005. p. 0875.

[8] R. Kamakoti, S. Thakur, J. Wright, W. Shyy, "Validation of a new parallel all-speed CFD code in a rule-based framework for multidisciplinary applications," 36th AIAA Fluid Dynamics Conference and Exhibit, Paper No. AIAA 2006-3063, San Francisco, CA (June 2006).

[9] E. Luke, and T. George, "Loci: "A Rule-Based Framework for Parallel Multidisciplinary Simulation Synthesis," Journal of Functional Programming, Special Issue on Functional Approaches to High-Performance Parallel Programming, Vol. 15, No.3, 2005, pp. 477-502.

[10] Linstrom P, Mallard W, Editors. NIST Chemistry WebBook, NIST Std. Ref. Database Number 69, NIST, 2005, http://webbook.nist.gov.

[11] Dixon, J. C., "The Shock Absorber Handbook," Second Edition, John Wiley & Sons, Ltd. (2007).