Analytical calculation of the energy deposition spectrum and average dose to a spherical target at all impact parameters

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Introduction

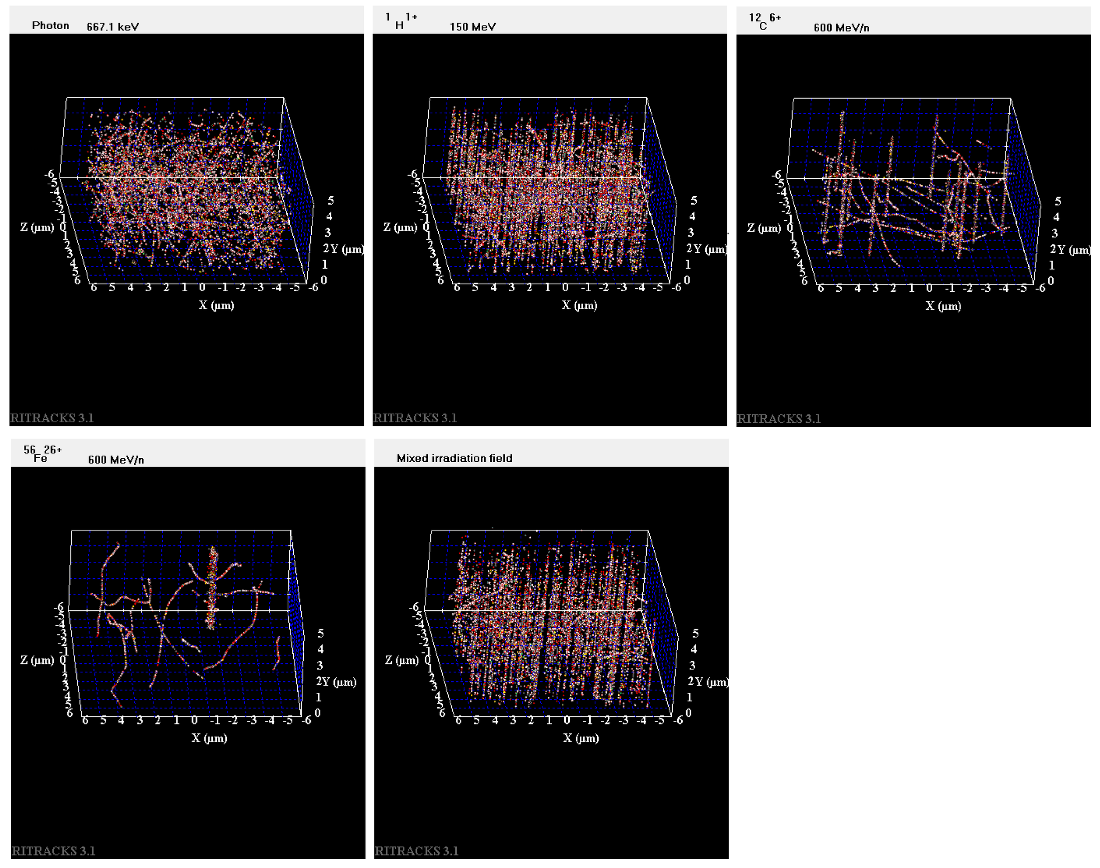
- Energy deposition by ionizing radiation in micrometric targets representative of cells is very important to understand the effect of radiation and the cellular and biological response.
- Microdosimetry is used to estimate quality factors for risk assessment in radiation protection [1] and quantify Relative Biological Effectiveness (RBE) for treatment planning of hadron radiation therapy [2].
- Radiation physics has shown that ionizing radiation deposits energy in a complex manner, referred to as the track structure.
- Consequently, energy deposited in a target is a function of variables such as the ion type, its energy, and the irradiated volume.

Simulation setup: volume irradiation with stochastic tracks

- The code RITRACKS (Relativistic Ion Tracks) [3] simulates detailed stochastic radiation track structures of ions of different types and energies.
- A parallelepiped volume is defined. Its bottom surface is irradiated by n tracks, determined by sampling the Poisson distribution, $p(n) = \lambda^n \exp -\lambda / n!$ where $\lambda = \phi A$ is the average number of tracks, A is the area of the irradiated surface, and ϕ is obtained from a given dose using

$$D(Gy) = 1.6 \times 10^9 \phi(cm^{-2}) \times LET(keV/\mu m)$$

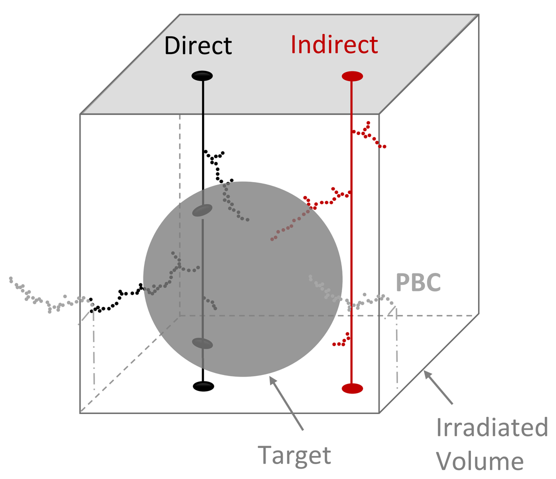
- The LET of an ion is calculated using the Bethe's equation.
- To mimic the contribution of energy deposition of delta-rays from tracks in neighboring volumes, periodic boundary conditions (PBCs) are used. Essentially, all electrons leaving the irradiation volume are put back on the other side of the volume, with the same energy and direction vector.



Example of RITRACK simulations for photons, protons 150 MeV, Carbon 600 MeV/n, Iron 500 MeV/n and mixed irradiation field. The dose is approximately 0.2 Gy in all cases.

Simulation set-up: microtarget

- Wall-less spherical water targets are used and encompassed in the irradiated volume [4].
- The energy deposited is calculated for each individual track.
- The tracks are classified as direct for those which the axis intercepts the target, and indirect for those that do not.
- Electrons are considered to originate from a different track after crossing a boundary.



Definition of direct/indirect contributions

Calculation of the dose to the target at various impact parameters

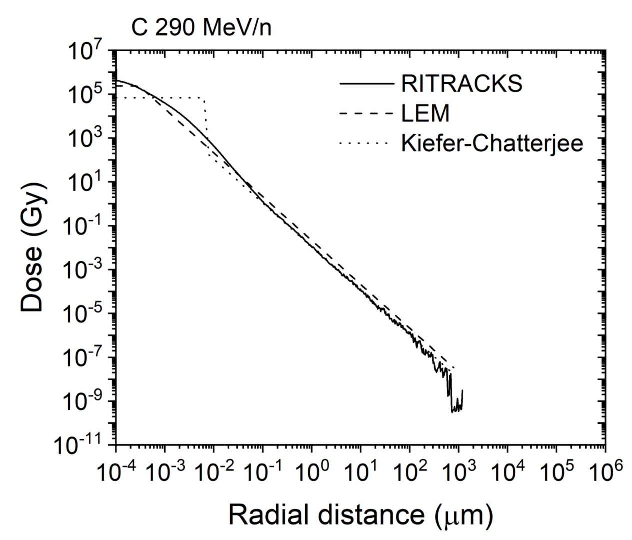
- The radial dose is given in the Kiefer-Chatterjee [5,6] parameterization by

$$D_r(r) = \begin{cases} \frac{\lambda_1}{r_{min}^2} & r \le r_{min} \\ \frac{\lambda_2}{r^2} & r_{min} < r \le r_{max} \\ 0 & r > r_{max} \end{cases} \qquad \lambda_1^{KC} = \left(\frac{LET}{\rho} - 2\pi\lambda_2^{KC} log\left(\frac{r_{max}}{r_{min}}\right)\right). \qquad \lambda_2^{KC} = 1.25 \times 10^{-4}\left(\frac{z^*}{\beta_{ion}}\right)^2.$$

$$r_{min} = 11.6\beta_{ion} \qquad r_{max} = 0.062 \times E^{1.7}$$

E is the energy per nucleon

β_{ion} is the relativistic beta.



Radial dose calculated by the LEM and by the Kiefer-Chatterjee parameterization

- The dose to target contributed by one track can be calculated analytically

Indirect

$$D_{sphere} = \frac{4\pi\lambda LET}{V_{sphere}}\left\{R - \sqrt{b^2 - R^2}\sin^{-1}\left(\frac{R}{b}\right)\right\}$$

b: Impact parameter

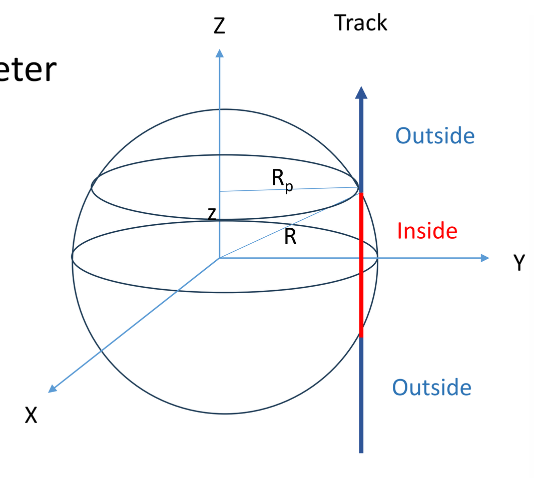
R: target radius

Direct, inside part

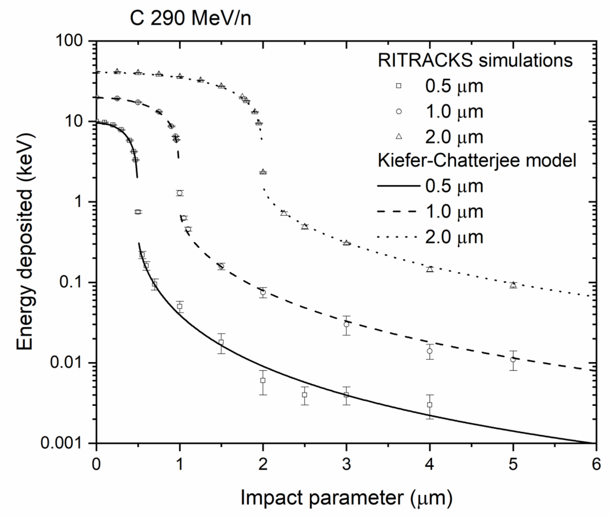
$$D_{sphere}^{in} \approx 2\pi\sqrt{R^2 - b^2}\left[\lambda_2 log\left(4\frac{R^2 - b^2}{r_{min}{}^2}\right) + (\lambda_1 - 2\lambda_2)\right].$$

Direct, outside part

$$D_{sphere}^{out} \approx 2\pi\lambda_2\left[2R + \sqrt{R^2 - b^2}\left(-2 + \log\left(4\frac{(R^2 - b^2)(R - \sqrt{R^2 - b^2})}{b^2(R + \sqrt{R^2 - b^2})}\right)\right)\right].$$



3D view of an ion track and a spherical target.



Energy deposited in spherical targets as a function of impact parameter for one 290 MeV/n carbon track (dots), and the current model prediction with the Kiefer-Chatterjee model for targets of radius 0.5, 1 and 2 μm.

Calculation of the direct and indirect spectra

- Since the target is irradiated uniformly, the number of tracks between the impact parameters b and $b + db$ is given by $\phi 2\pi b db$.
- By the analytical model, each impact parameter is associated with a given dose (or energy deposition).
- The equations cannot be inverted analytically. However, an approximate relationship between b^2 and the energy deposited in the sphere can be derived
- With this approach, an energy deposition spectra can be obtained.

$$P(E)dE = \phi\pi \frac{E}{2\pi^2\left(\lambda_2 log\left(4R^2/r_{min}^2\right) + (\lambda_1 - 2\lambda_2)\right)^2} dE.$$

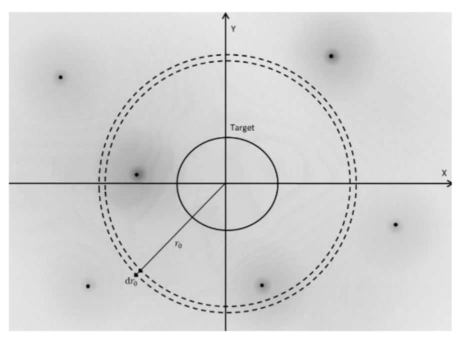
Direct spectra

$$P(E)dE = \phi\pi R^2 \frac{h}{E^2} dE \qquad h = 4\pi\rho\lambda_2 R/3$$

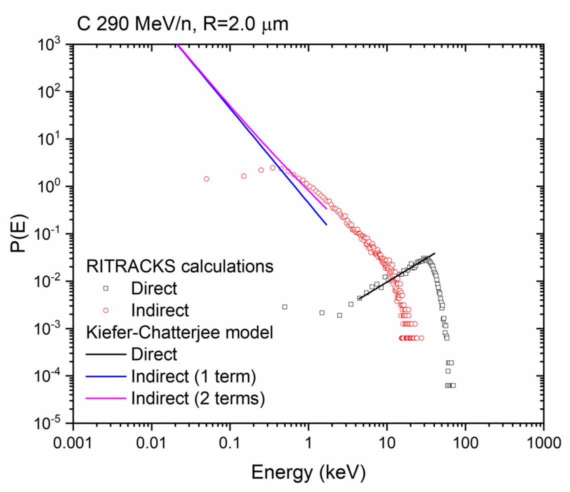
Indirect spectra, 1 term

$$P(E)dE = \phi\pi R^2 \left\{\frac{2}{5E\sqrt{1 + \frac{8E}{5h}}} + \frac{h}{2E^2}\left\{1 + \sqrt{1 + \frac{8E}{5h}}\right\}\right\} dE.$$

Indirect spectra, 2 terms



Dose to target by multiple tracks.



Energy deposition spectra for 290 MeV/n carbon, with a target of radius of 2 μm.

Conclusions

- An analytical model based on the Kiefer-Chatterjee parameterization has been developed to calculate the energy deposition to a target by ion tracks at all impact parameters. An analytical expression was also derived for the energy deposition spectra.
- The calculations were compared with those from the code RITRACKS.
- The energy gap between the indirect and direct spectra is due to the missing calculation between $R - r_{min} < b < r + R_{min}$.

References

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