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Unsteady Aerodynamic Modeling for Atmospheric Entry Vehicles in Subsonic and Incompressible Flow: A Frequency Response Approach

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Engineering Directorate

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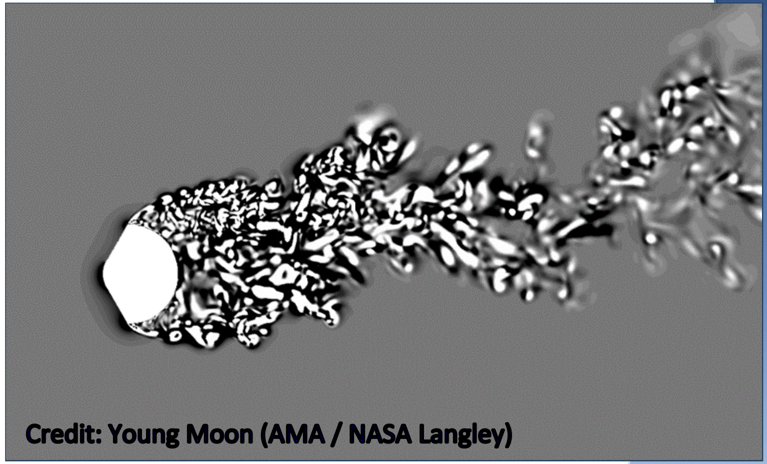
Outline

- Background
- Motivation and Objective
- Classical Aerodynamic Theories
- Frequency Response Approach
- Results from the12 ft Low Speed Tunnel
- Reconstruction from semi-empirical models
- Summary

Motivation

- Planetary missions often require an entry, descent, and landing (EDL) system to successfully deliver a payload to the surface

- *Blunt body entry capsules display complex aerodynamics due to unsteady wake effects*
  - Dynamic instabilities tend to grow with decreasing Mach number
  - Either the vehicle **(i) reaches a stable limit cycle**, or **(ii) the oscillations can grow** to where a stabilizing device may not be safe to deploy, and the vehicle may eventually begin to tumble
  - Flow separation and reattachment shift unpredictably depending on frequency of oscillations, mean angle of attack, and pitch/plunge amplitudes.



- Previous model efforts include:
  - Allen and Tobak, along with Fletcher and Wolhart, laid the foundation for dynamic stability analysis of blunt bodies through early analytical and experimental studies.
  - Schoenenberger and Queen developed differential equations of motion for entry vehicles, emphasizing the dominance of aerodynamic forces over gravitational and centrifugal forces, focusing on the angle of attack's time evolution.
  - Seiff, Whiting, and Ericsson extended Newtonian impact methods to unsteady aerodynamics, refining semi-empirical predictions for slender blunted cones with half-cone angles below 10 degrees and achieving agreement for Mach numbers as low as Mach 3.

Understanding unsteady aerodynamics is an important part of the modeling and simulation of entry vehicle trajectories

[1] Tobak, M., and Allen, H. J., Dynamic stability of vehicles traversing ascending or descending paths through the atmosphere, Vol. 4275, National Advisory Committee for Aeronautics, 1958.
[2] Schoenenberger, Mark, and Eric M. Queen. "Limit cycle analysis applied to the oscillations of decelerating blunt-body entry vehicles." NATO RTO Symposium AVT-152 on Limit-Cycle Oscillations and Other Amplitude-Limited, Self Excited Vibrations. No. RTO-MP-AVT-152. 2008.
[3] Ericsson, L. E., "Generalized unsteady embedded Newtonian flow," Journal of Spacecraft and Rockets, Vol. 12, No. 12, 1975,pp. 718–726. https://doi.org/10.2514/3.27870.

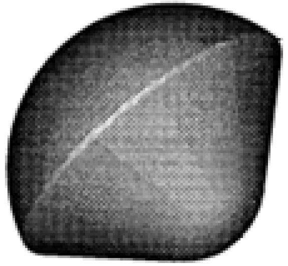
Objectives

Motivations:

- Develop data driven analysis to model unsteady aerodynamics from forced oscillation wind tunnel tests.
- Traditional aerodynamic databases not capturing vehicle frequency effects on aerodynamic response.
- Investigate a new data reduction technique to extract dynamic aero coefficients
- Develop a system identification like approach that characterizes a transfer function for a blunt body entry vehicle should be able to predict lift and moment time histories as a function of oscillatory inputs to inform controller design

Objective: Utilize a data-driven frequency approach to model unsteady aerodynamics from experimental forced oscillation tests on an entry vehicle.

The chosen test article is a 60-degree spherical cone angle with a hemispherical backshell, similar to Mars Microprobe and Varda's Winnebago-1 (W-1)



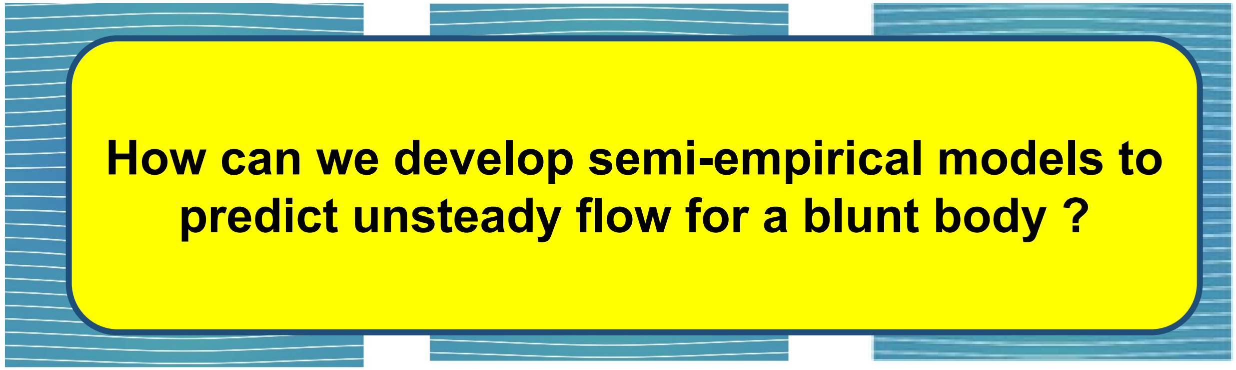
Braun, R. D., Mitcheltree, R. A., and Cheatwood, F. M., "Mars microprobe entry analysis," *1997 IEEE Aerospace Conference*, Vol. 1, IEEE, 1997

Credit: Varda Space Industries

Classical Unsteady Aerodynamic Theories

Joukowski's transform: Conformal mapping from a simple geometric shape (circle) into a complex one (e.g., cambered airfoil, blunt bodies). The solution of velocity potential remains accurate [4].

Kutta Condition : Bounded circulation is conserved between the wake and on the body. Flow leaves the trailing edge smoothly. Lift is proportional to flow circulation [5].



Unsteady aerodynamic models: Theodorsen [6] provides a frequency-dependent solution for the unsteady lift and pitch moment resulting from oscillations. Builds on the steady-lift concepts derived from Joukowski and Kutta condition and extends them to unsteady flow via potential flow theory.

Assumptions: Potential flow theory. I.e., incompressible, irrotational, linear and mostly attached

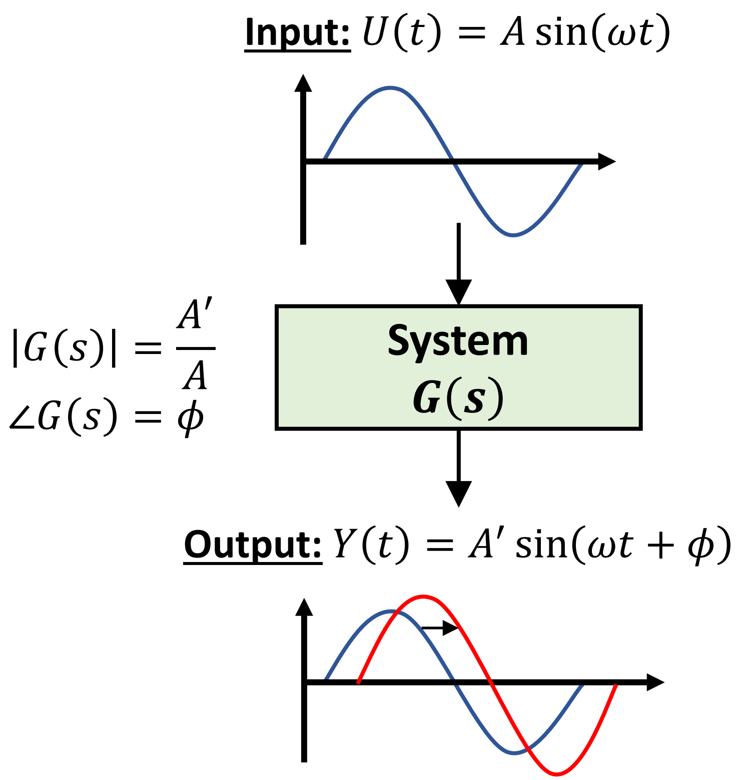
$$C_{L_{\text{Total}}} = \underbrace{\frac{\pi b}{U_\infty^2}\left(U_\infty \dot{\alpha} - ba\ddot{\alpha}\right)}_{C_{L_{\text{Addedmass}}}} + \overbrace{\underbrace{2\pi\left[\alpha + b\left(\frac{1}{2} - a\right)\frac{\dot{\alpha}}{U_\infty}\right]}_{C_{L_{\text{QS}}}} C(k)}^{C_{L_{\text{US}}}}$$

$C(k)$: Approximated transfer function to account for wake effects [7]

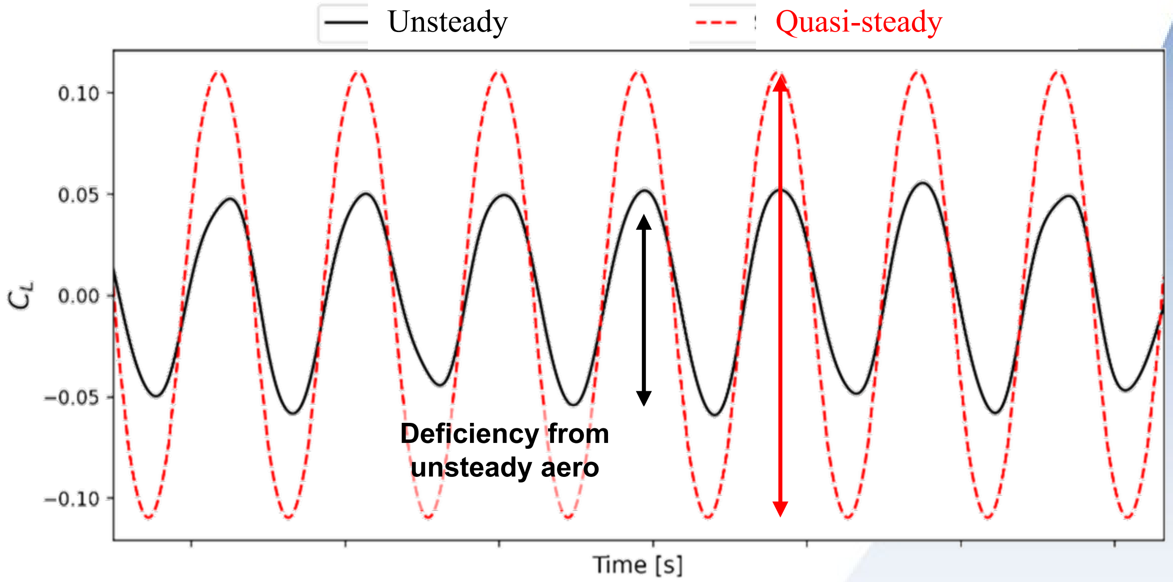
2π: Thin airfoil theory

[4] Joukowsky, N., "Über die konturen der Tragflächen der Drachenflieger," *Zeitschrift für Flugtechnik und Motorluftschiffahrt*, Vol. 1, No. 22, 1910, pp. 281–285.
[5] Kutta, W., "Auftriebskräfte in strömenden Flüssigkeiten," *Illustrierte Aeronautische Mitteilungen*, Vol. 6, No. 133, 1902, pp.133–135.
[6] Theodorsen, T., "General theory of aerodynamic instability and the mechanism of flutter," Tech. Rep. NACA TR-496, National Advisory Committee for Aeronautics, 1949..
[7] Bisplinghoff, Raymond L., Holt Ashley, and Robert L. Halfman. *Aeroelasticity*. Courier Corporation, 2013, pp 232 - 255

Frequency Response Approach



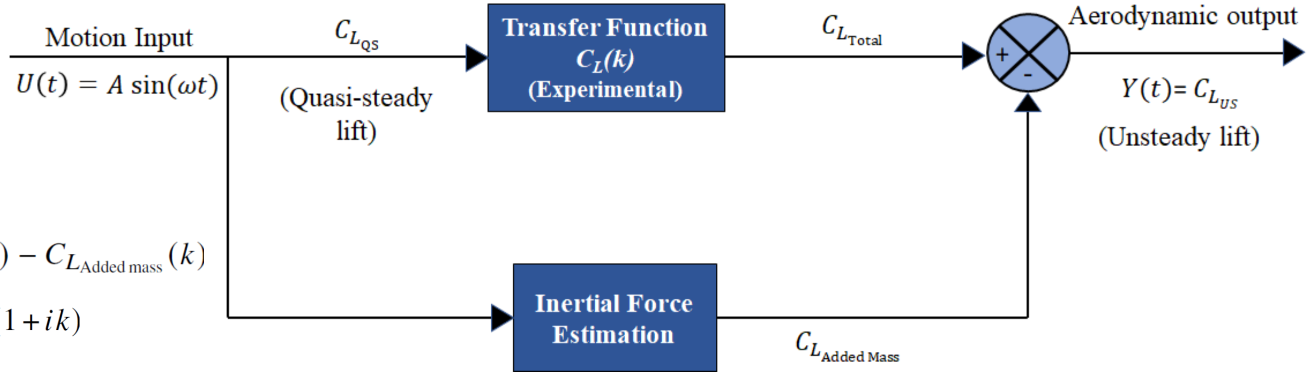
- Model a transfer function, C(k) to describe the system's response (unsteady aerodynamics due to harmonic oscillatory input (quasi-steady aerodynamics)
  - Assumes the system is linear time-invariant (LTI) and that the frequency is preserved
  - Quasi-steady flow follows the motion input (gain of 1 and phase lag of 0°)
  - Manifests as a gain/deficiency, shift in the mean, and/or a phase lag
  - This process can be data-driven and informed by CFD or **dynamic test techniques**



A system identification like approach that **characterizes a transfer function for a blunt body entry vehicle** should be able to **predict lift and moment time histories** as a function of oscillatory inputs.

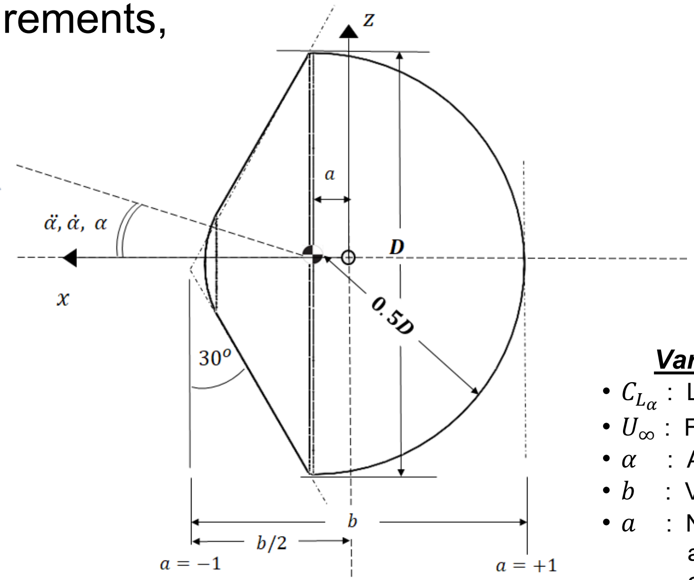
Frequency Response Approach to Unsteady Aerodynamics

An approximate transfer function from forced oscillation wind tunnel test measurements, addressing unsteady aerodynamic analysis from a dynamical systems perspective



$$C_{L_{US}}(k) = C_{L_{Total}}(k) - C_{L_{Added\ mass}}(k)$$

$$\hat{C}_{L_{QS}}(k) = C_{L_\alpha} A_\alpha (1 + ik)$$



Variable Definitions:

- C_{L_α} : Lift-curve slope
- U_∞ : Free-stream velocity
- α : Angle of attack
- b : Vehicle span
- a : Normalized position of the axis of rotation from the center of the body
- $C_L(k)$: Transfer function

The effect of unsteady aerodynamics can be captured with a transfer function (transfer of energy from quasi-steady to unsteady aero):

Lift and moment timeseries can be recreated for any given $C(k)$

$$\int_{-\infty}^{\infty} U(t) e^{i\omega t}\, dt$$

- The transfer function is a reduced frequency dependent complex number:

$$C(k) = F(k) + iG(k)$$

- $C(k)$ is a complex number that contains information on the gain/deficiency and phase lag resulting from dynamic motion

$$C_L(k) = \frac{C_{L_{US}}(k)}{C_{L_{QS}}(k)}.$$

$k = \frac{\omega D}{2V_\infty}$: reduced frequency, a, b : geometric parameters

$$C_L = \underbrace{\frac{C_{L_\alpha} b}{2V_\infty^2}\left[U_\infty \dot{\alpha} - ba\ddot{\alpha}\right]}_{\text{Added Mass Effects}} + \underbrace{C_{L_\alpha}\left[\alpha + b\left(\frac{1}{2} - a\right)\frac{\dot{\alpha}}{U_\infty}\right] C(k)}_{\text{Term modified by Transfer Function}}$$

$$C_m = \frac{C_{L_\alpha} b}{2U_\infty^2}\left[-U_\infty b\left(\frac{1}{2} - a\right)\dot{\alpha} - b^2\left(\frac{1}{8} + a^2\right)\ddot{\alpha}\right] + C_{L_\alpha} b\left(a + \frac{1}{2}\right)\left[\alpha + b\left(\frac{1}{2} - a\right)\frac{\dot{\alpha}}{U_\infty}\right] C_L(k)$$

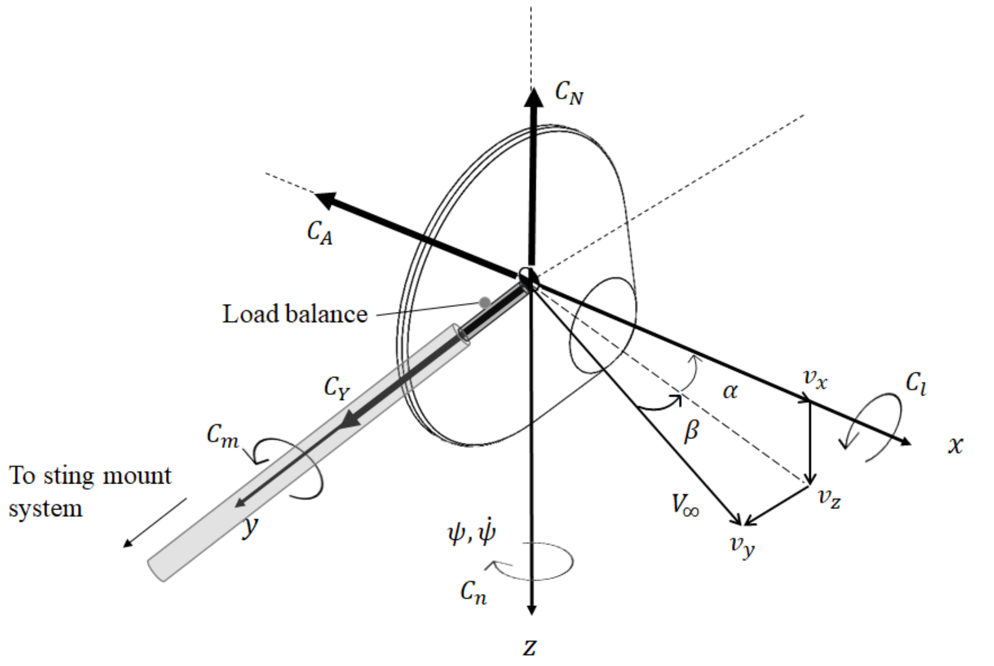
Entry Systems Model - Forced Oscillation Test

Forced oscillation test overview:

- A 60° spherical nose cone and hemispherical backshell vehicle was tested in the *12-Foot Low-Speed Tunnel at NASA Langley*.
- 21 runs were conducted at different reduced frequencies. 250 Hz sampling rate.
- Wind on and Wind off force and moment measurements
- Post-Processing : Cut-off frequency at 4X the driving frequency

Operating conditions

Mean Angle of Attack, α_0	Pitching amplitude, A_α	Reduced frequency, k
0°	10°	0.01 - 0.21



Body coordinate system

Forced oscillation test set up

Entry Systems Model - Forced Oscillation Test

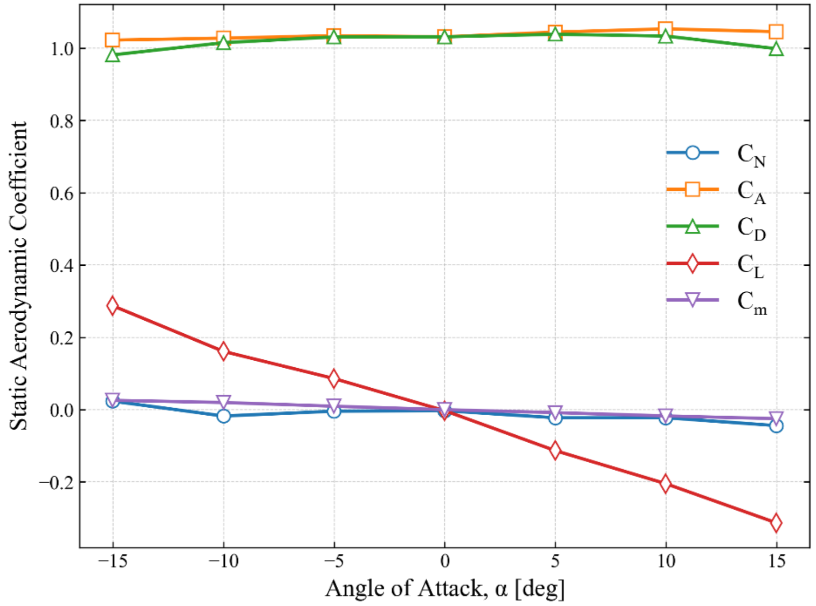
Operating conditions

Mean Angle of Attack, α_0	Pitching amplitude, A_α	Reduced frequency, k
0°	10°	0.01 - 0.21

Results: Static and Unsteady Aerodynamics

Static Aerodynamic data:

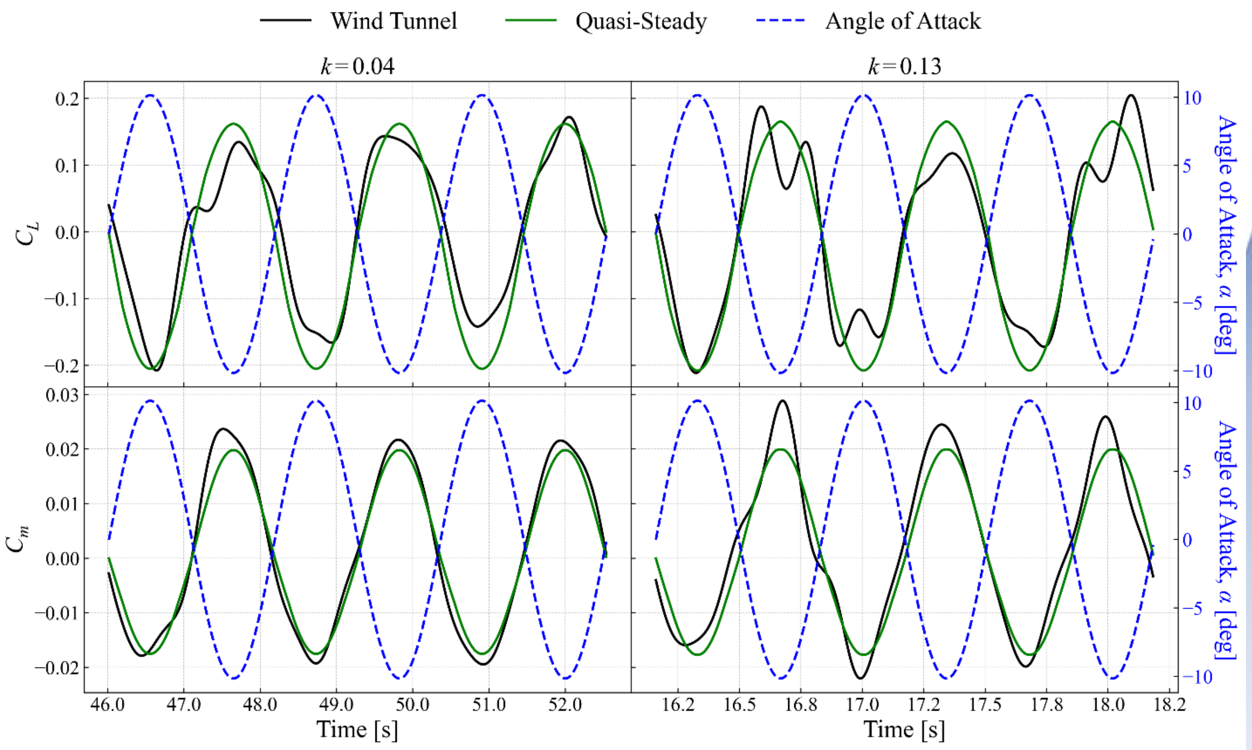
- Static data was also acquired in the 12-Foot tunnel for use in the reconstructions
- C_{L_α} is determined from the static run to generate the quasi-steady lift and moment time-series.



Static aerodynamic data

Unsteady Aerodynamic test data

- Quasi-steady departs from the unsteady aerodynamic at larger k



Unsteady lift and moment time-histories

Results: Transfer Function Database

- Values of the transfer function can be stored in databases and queried to calculate C_L and C_m using oscillatory aerodynamic models [8]
  - These values can be stored and queried from aerodynamic databases
  - Typical aerodatabases do not capture frequency dependent and resonant phenomena. These can be captured using the following transfer function database

k	$C_L(k)$	$C_m(k)$	k	$C_L(k)$	$C_m(k)$
0.01	$1.021-0.035i$	$0.957+0.034i$	0.12	$0.767-0.163i$	$1.122+0.265i$
0.02	$1.019-0.109i$	$0.972+0.101i$	0.13	$0.773-0.154i$	$1.156+0.350i$
0.03	$0.999-0.102i$	$0.984+0.104i$	0.14	$0.768-0.148i$	$1.136+0.287i$
0.04	$0.939-0.106i$	$1.032+0.132i$	0.15	$0.753-0.178i$	$1.167+0.321i$
0.05	$0.940-0.173i$	$1.014+0.186i$	0.16	$0.772-0.169i$	$1.151+0.237i$
0.06	$0.919-0.126i$	$1.039+0.149i$	0.17	$0.734-0.181i$	$1.169+0.342i$
0.07	$0.857-0.185i$	$1.064+0.215i$	0.18	$0.692-0.131i$	$1.212+0.102i$
0.08	$0.882-0.164i$	$1.078+0.263i$	0.19	$0.721-0.116i$	$1.203+0.268i$
0.09	$0.844-0.167i$	$1.076+0.236i$	0.20	$0.711-0.129i$	$1.188+0.241i$
0.10	$0.844-0.232i$	$1.092+0.302i$	0.21	$0.682-0.141i$	$1.226+0.577i$
0.11	$0.824-0.180i$	$1.108+0.272i$			

[8] Rodden, William P., and Joseph P. Giesing. "Application of oscillatory aerodynamic theory to estimation of dynamic stability derivatives." *Journal of Aircraft* 7.3. 1970.

Results: Lift Transfer Function

- Magnitude attenuation and growing phase lag with increasing k
  - Predictions compared with Theodorsen's analytical solution.
- Uncertainty in the phase grows as reduced frequency increases
  - Uncertainties are computed by the Root Sum Square (RSS) of various sources of error: Instrumentation precision, calibration, data interpolation/sampling and data trimming

$$C_L(k) = \mathcal{F}\left\{\frac{L_{wind}(k) - (N/N_{tare})L_{tare}(k)}{qS_{\mathrm{REF}}C_{L_{QS}}(k)}\right\}$$

Note: Tare effects are removed through a frequency domain subtraction.

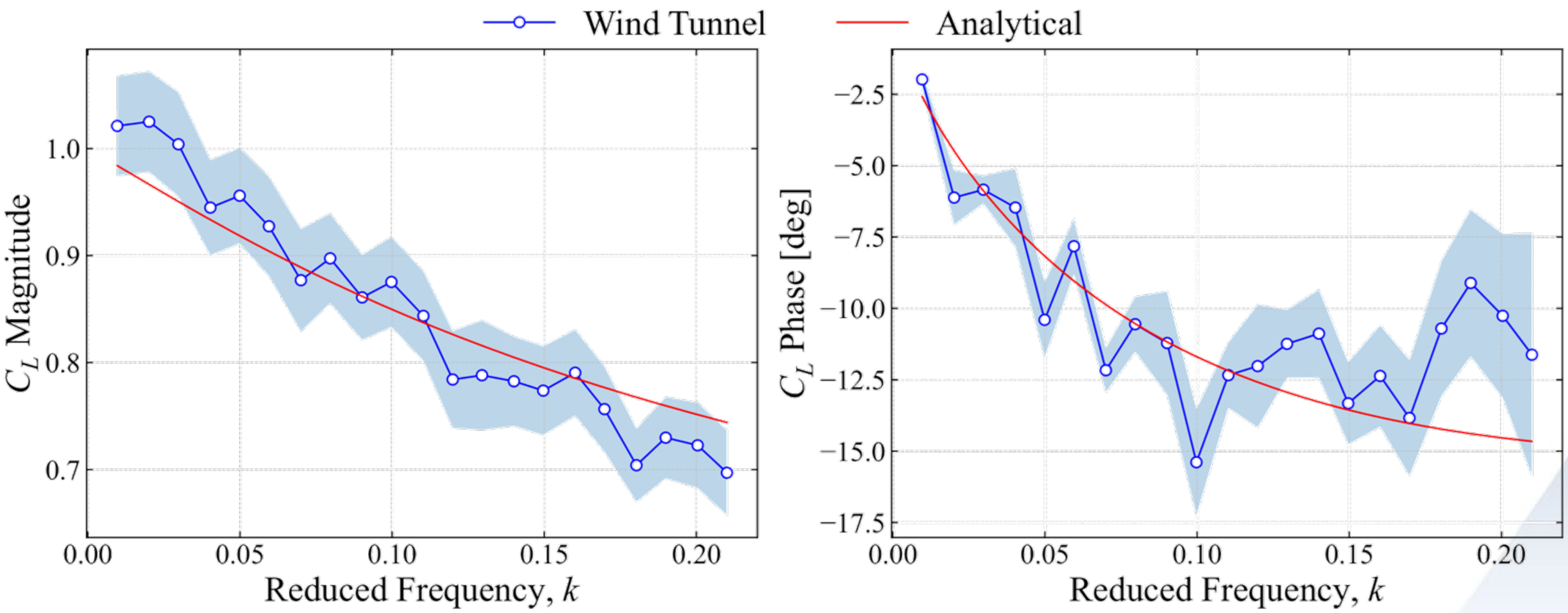


Fig. Bode plot for the lift transfer function and Theodorsen's analytical prediction with 3-sigma uncertainty bands

Results: Pitching Moment Transfer Function

- Magnitude amplification and growing phase shift with increasing k
- Uncertainty in the phase grows significantly as reduced frequency increases
  - Larger uncertainty bounds than were seen in the lift coefficient

$$C_m(k) = \mathcal{F}\left\{\frac{M_{wind}(k) - (N/N_{tare})M_{tare}(k)}{qS_{\mathrm{REF}}DC_{m_{QS}}(k)}\right\}$$

Tare effects are removed through a frequency domain subtraction. * $\mathcal{F}$: Fourier transform

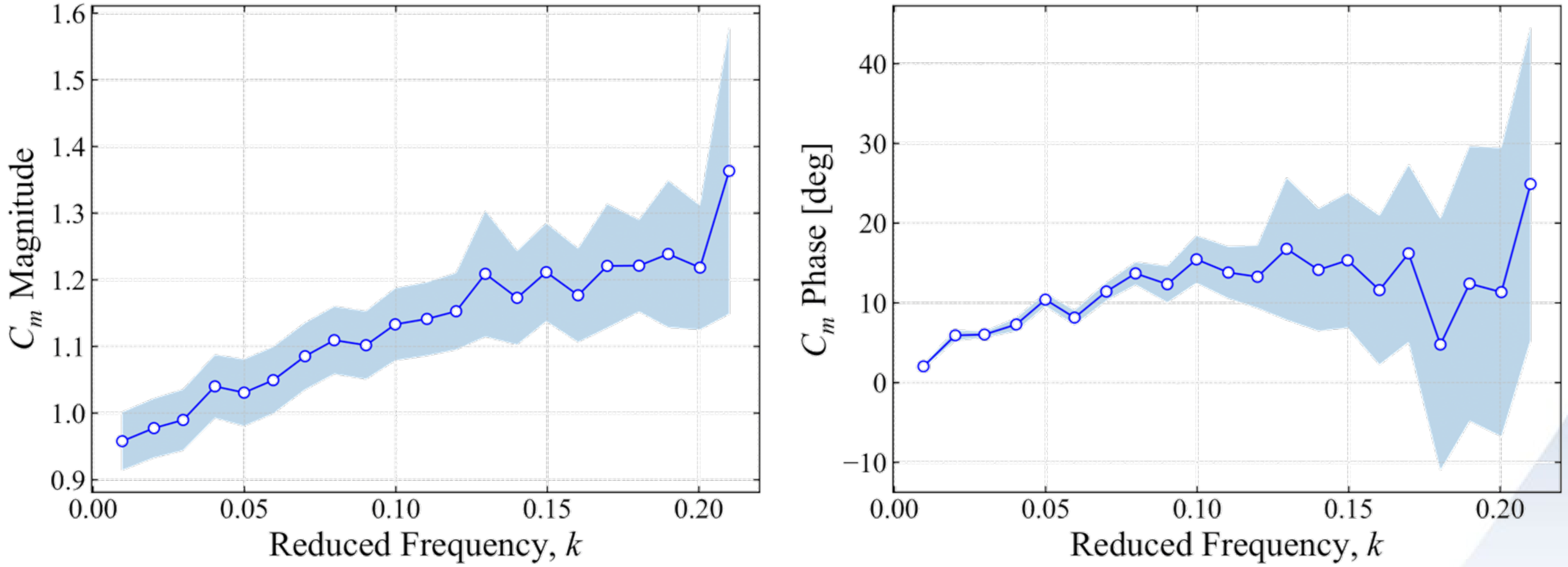


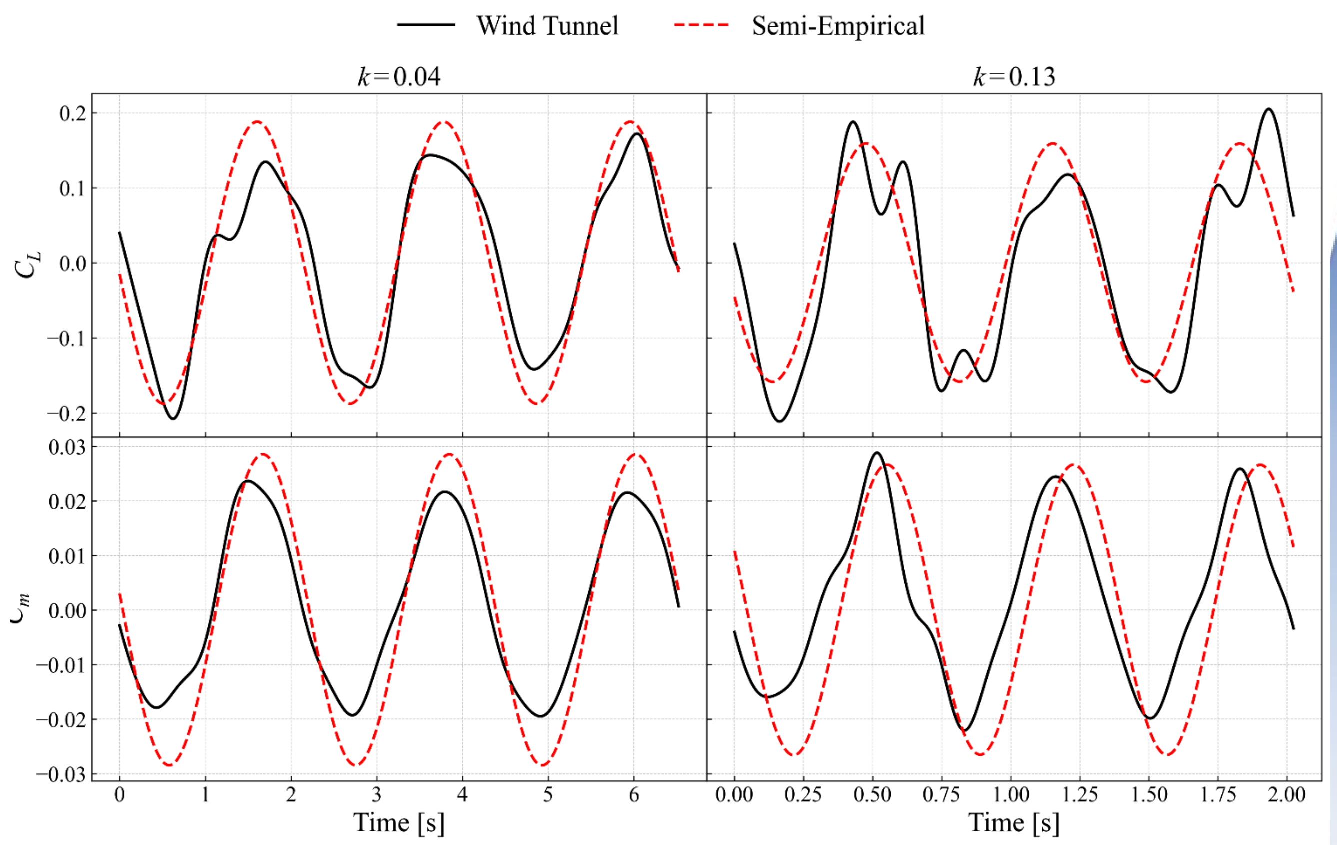
Fig. Bode plot for the pitching moment transfer function where the shaded region represents 3-sigma uncertainty

Results: Time-Varying C_L and C_m Reconstruction

- Lift and moment time histories can be recreated assuming Theodorsen's equations apply here
  - Axis of rotation is assumed to be at the center of mass ($a = -0.329, b = 0.875\text{ft}$)
  - Magnitude and phase discrepancies are observed as expected, since Theodorsen's applies to thin airfoils, not blunt-bodies

$$C_L = \frac{C_{L_\alpha} b}{2V_\infty^2}[U_\infty \dot{\alpha} - ba\ddot{\alpha}] + C_{L_\alpha}\left[\alpha + b\left(\frac{1}{2} - a\right)\frac{\dot{\alpha}}{U_\infty}\right]C_L(k)$$

$$C_m = \frac{C_{L_\alpha} b}{2U_\infty^2}\left[-U_\infty b\left(\frac{1}{2} - a\right)\dot{\alpha} - b^2\left(\frac{1}{8} + a^2\right)\ddot{\alpha}\right] + C_{L_\alpha} b\left(a + \frac{1}{2}\right)\left[\alpha + b\left(\frac{1}{2} - a\right)\frac{\dot{\alpha}}{U_\infty}\right]C_L(k)$$



Reconstruction of the lift and moment time series for two random reduced frequencies.

Results: Time-Varying C_L and C_m Reconstruction w/ Regression Analysis

- Discrepancy can be reduced using a regression approach: a constant can be added to each term that is solved for using a least squares regression
  - All the time series are used together to solve for the constants

$$C_L = \mathbf{C_1} \frac{C_{L_\alpha} b}{2V_\infty^2}\left[U_\infty \dot{\alpha} - b a \ddot{\alpha}\right] + \mathbf{C_2} C_{L_\alpha}\left[\alpha + b\left(\frac{1}{2} - a\right)\frac{\dot{\alpha}}{U_\infty}\right] C_L(k)$$

$$C_m = \mathbf{C_3} \frac{C_{L_\alpha} b}{2U_\infty^2}\left[-U_\infty b\left(\frac{1}{2} - a\right)\dot{\alpha} - b^2\left(\frac{1}{8} + a^2\right)\ddot{\alpha}\right] + \mathbf{C_4} C_{L_\alpha} b\left(a + \frac{1}{2}\right)\left[\alpha + b\left(\frac{1}{2} - a\right)\frac{\dot{\alpha}}{U_\infty}\right] \mathbf{C_m(k)}$$

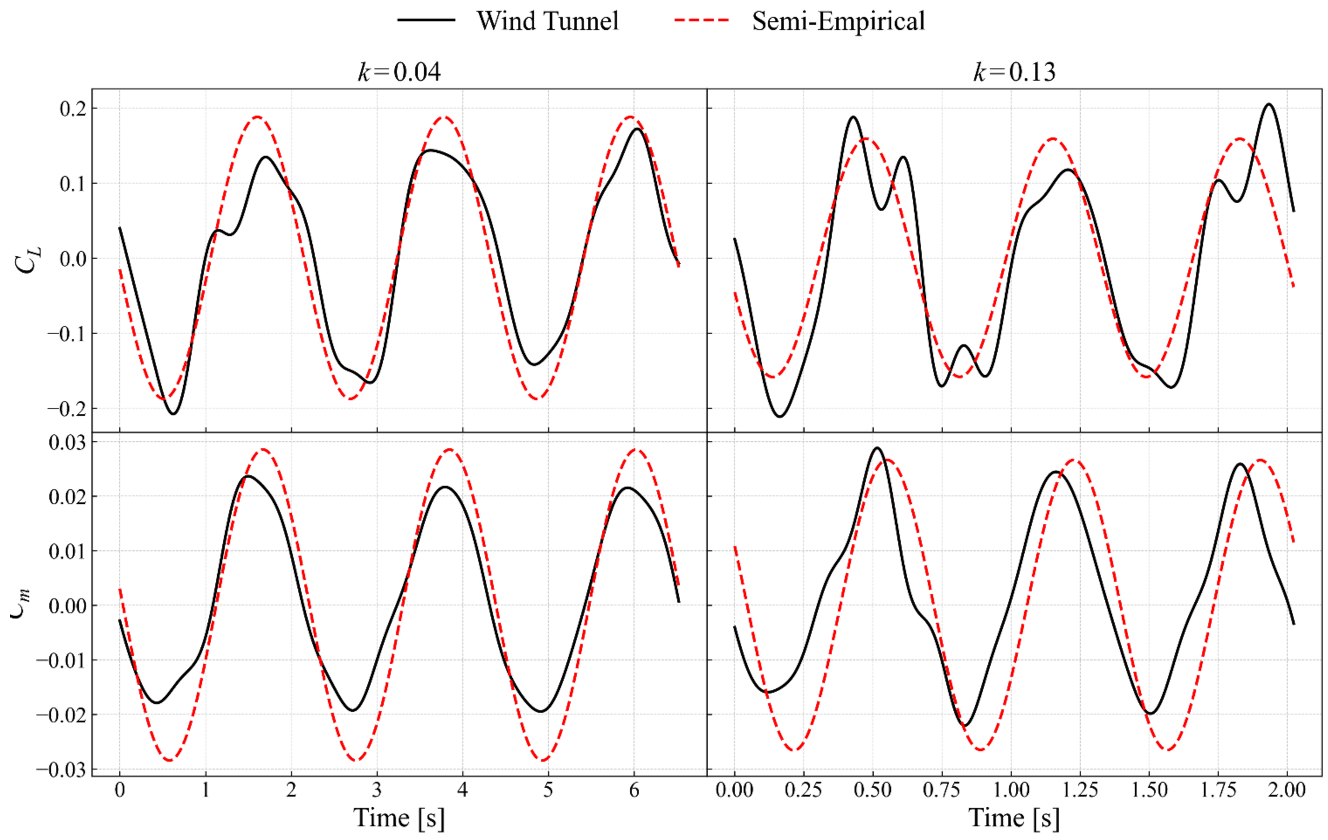


Fig. Reconstruction of the lift and moment time series for two random reduced frequencies. Tare-removed coefficients are shown and only 3 of the 40 cycles are shown.

[1] Bisplinghoff, Raymond L., Holt Ashley, and Robert L. Halfman. *Aeroelasticity*. Courier Corporation, 2013.

Results: Time-Varying C_L and C_m Reconstruction w/ Regression Analysis

- Discrepancy can be reduced using a regression approach: a constant can be added to each term that is solved for using a least squares regression
  - All the time series are used together to solve for the constants
  - Results in better predictions (more detailed error analysis is given in the paper)

$$C_L = \mathbf{C_1} \frac{C_{L_\alpha} b}{2V_\infty^2}\left[U_\infty \dot{\alpha} - ba\ddot{\alpha}\right] + \mathbf{C_2} C_{L_\alpha}\left[\alpha + b\left(\frac{1}{2} - a\right)\frac{\dot{\alpha}}{U_\infty}\right] C_L(k)$$

$$C_m = \mathbf{C_3} \frac{C_{L_\alpha} b}{2U_\infty^2}\left[-U_\infty b\left(\frac{1}{2} - a\right)\dot{\alpha} - b^2\left(\frac{1}{8} + a^2\right)\ddot{\alpha}\right] + \mathbf{C_4} C_{L_\alpha} b\left(a + \frac{1}{2}\right)\left[\alpha + b\left(\frac{1}{2} - a\right)\frac{\dot{\alpha}}{U_\infty}\right] \boxed{C_m(k)}$$

C_1 (C_L Added Mass)	C_2 (C_L Unsteady)	C_3 (C_m Added Mass)	C_4 (C_m Unsteady)
-2.77	0.924	-0.146	0.598

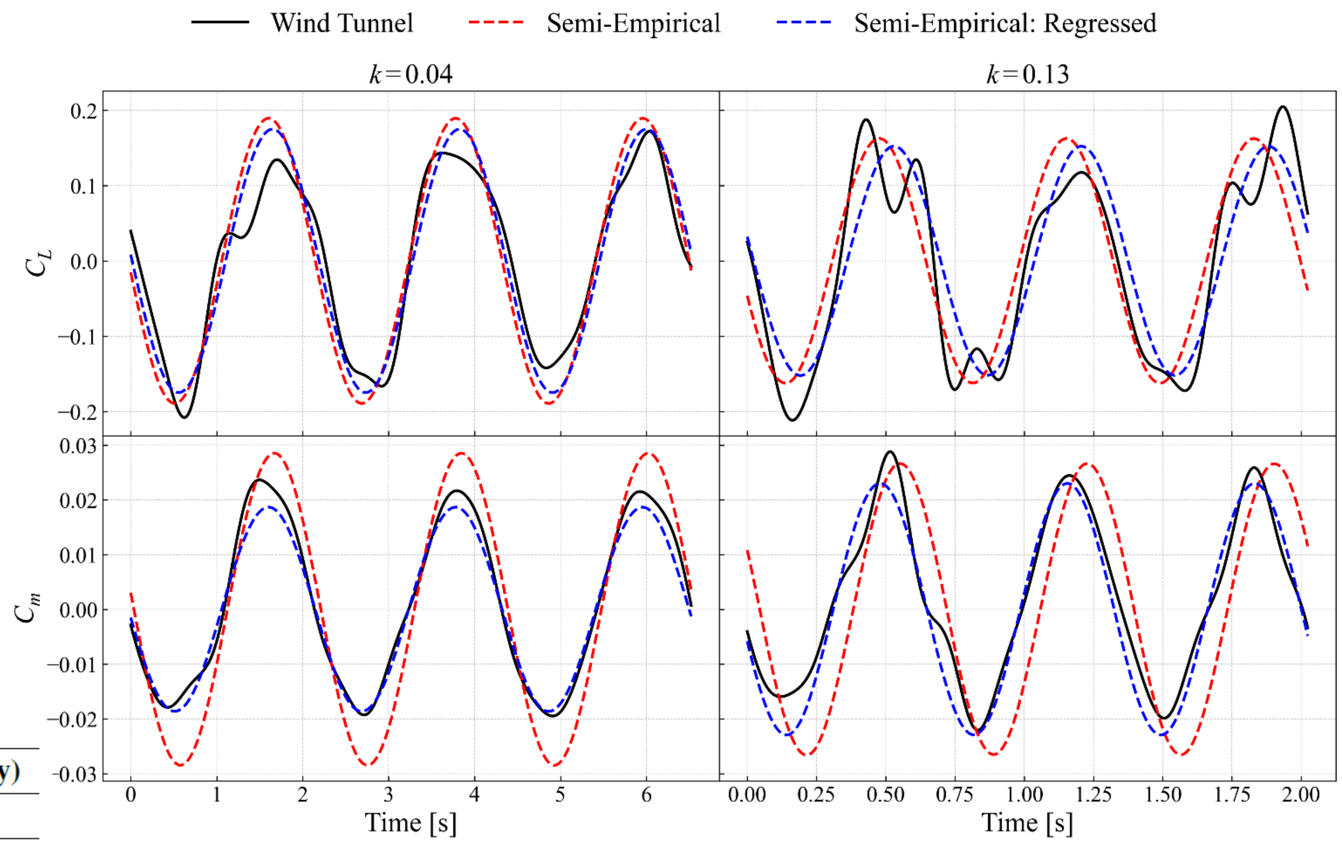
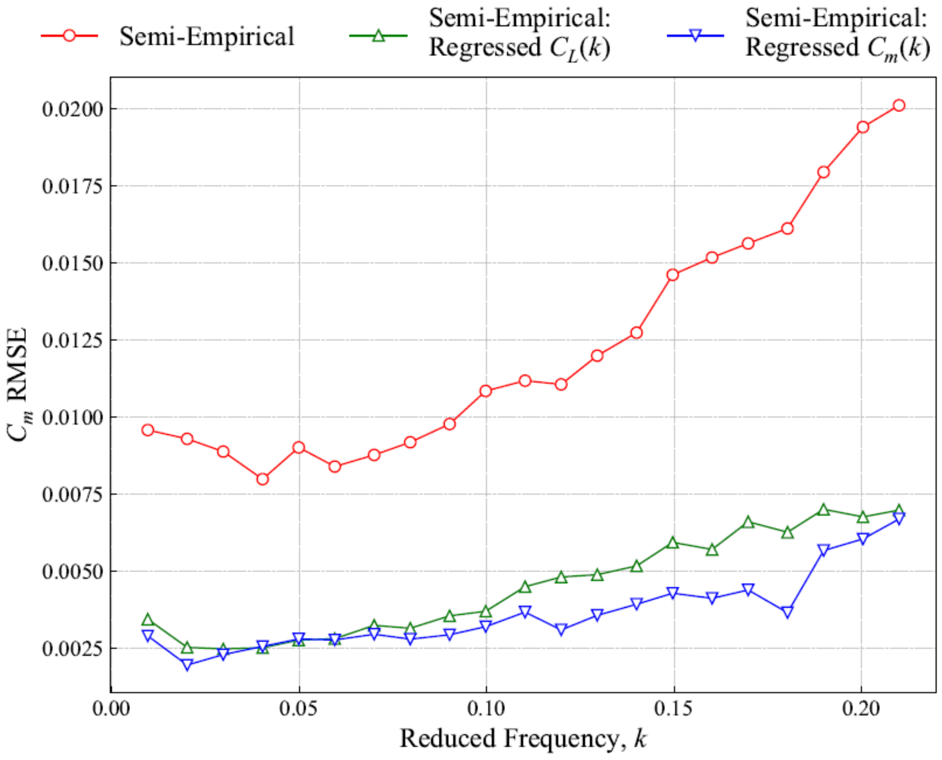


Fig. Improved reconstruction of the lift and moment time series for two random reduced frequencies.

Error Comparisons in Regression Analysis

- Discrepancy can be reduced using a regression approach: a constant can be added to each term that is solved for using a least squares regression
  - The Root mean square error (RMSE) of the moment coefficient reconstruction shows that regressed model provides better predictions to the wind tunnel test data
  - $\boldsymbol{C_m(k)}$ captures the phase shift in flow response reasonably better than $C_L(k)$



Summary

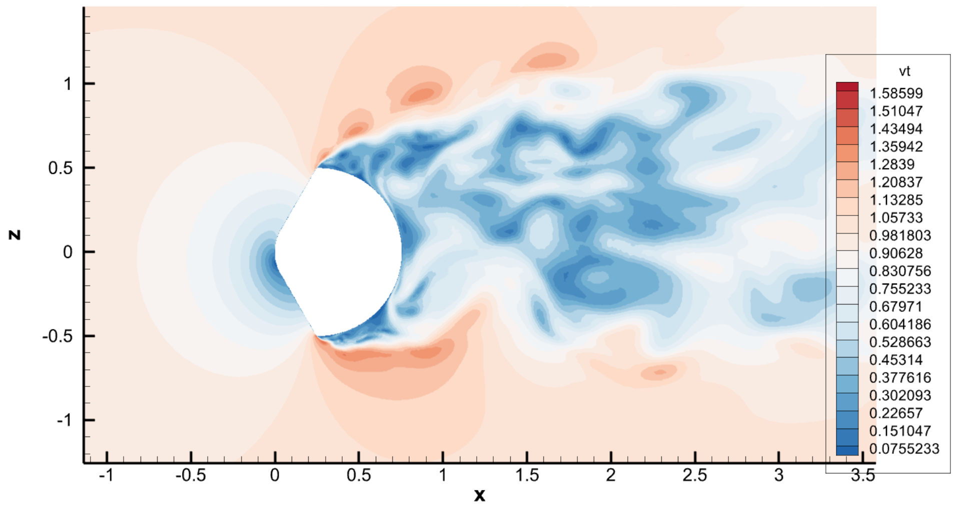
Conclusions

- Determined Transfer functions for a blunt body entry vehicle from forced oscillation tests from NASA Langley's 12 ft Low Speed Tunnel.
- Transfer functions describe the gain and the phase of the aerodynamic response due to oscillatory input motion as a function of frequency.
- Modified classical unsteady aerodynamic theories into semi-empirical formulations tailored to blunt body entry vehicle to recreate lift and moment time histories.
- Applied regression analysis with newly determined regressed coefficients to the existing semi-empirical model to better reconstruct the time series of lift and pitch moment.

Future Work:

- Stability coefficients can be extracted from forced oscillation data using a frequency approach as a new data reduction technique. [9]
- Implement the determined transfer functions to develop of frequency dependent dynamic stability coefficients models in aerodynamic databases.

[9] Dastoor, Jehan and Shehata, Hisham, "*A Frequency Response Approach to Calculating Stability Coefficients from Forced Oscillation Data with an Entry Vehicle Application*". In preparation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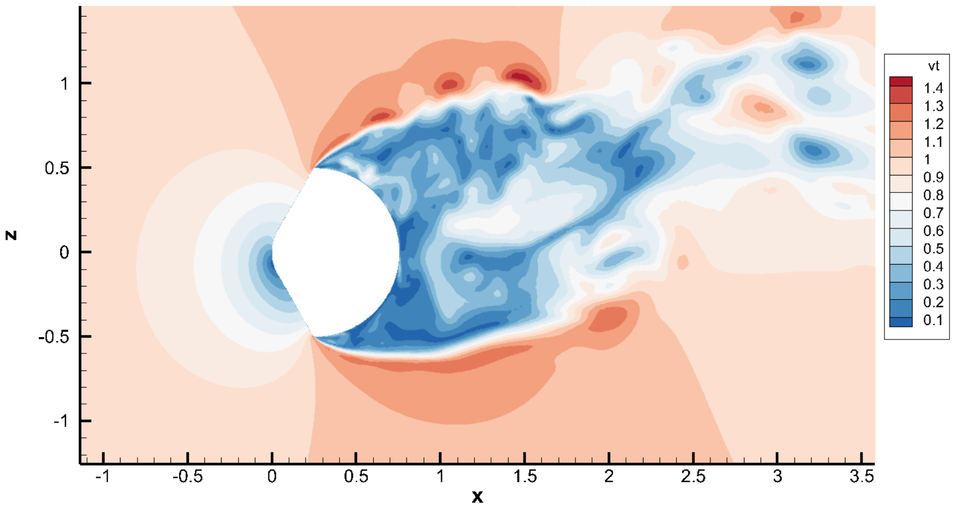
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