Reconstruction of Thermal Protection System Aeroheating Using a Green's Function Approach

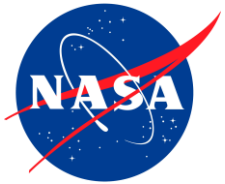


Kenneth McAfee and Oded Rabin

University of Maryland, College Park, MD

Hannah S. Alpert

NASA Ames Research Center, Moffett Field, CA

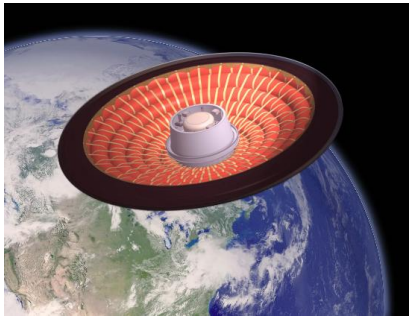


AIAA SciTech Forum, January 10th, 2025

Motivation

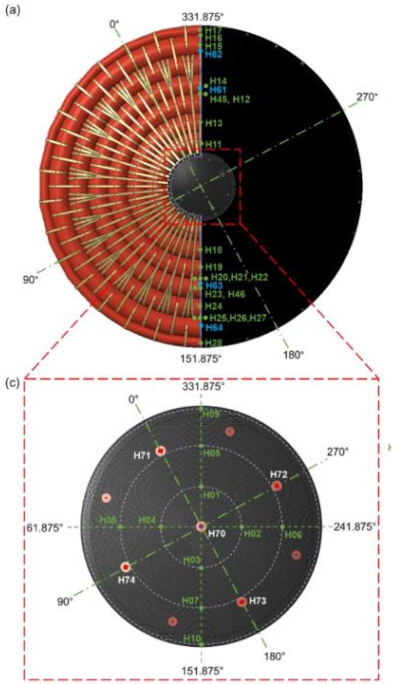


https://mars.nasa.gov/mer/gallery/artwork/entry_br.html



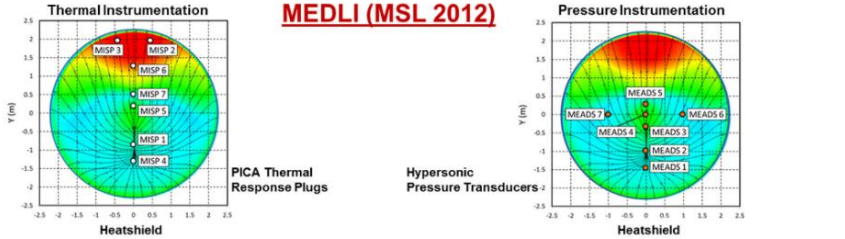
https://www.nasa.gov/general/low-earth-orbit-flight-test-of-an-inflatable-decelerator-loftid-overview/

LOFTID (2022)

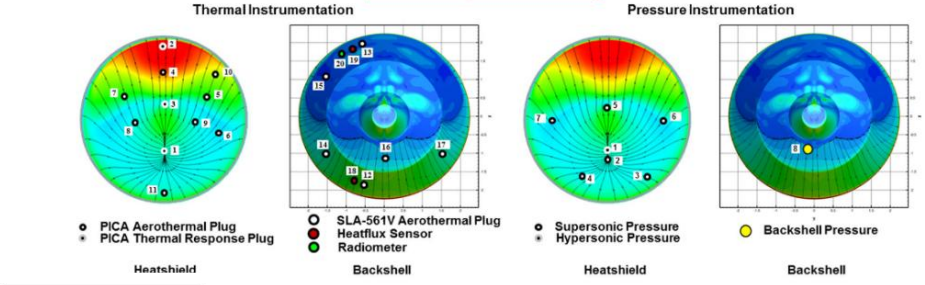


Swanson, G., et al. AIAA SciTech (2024)

MEDLI (MSL 2012)



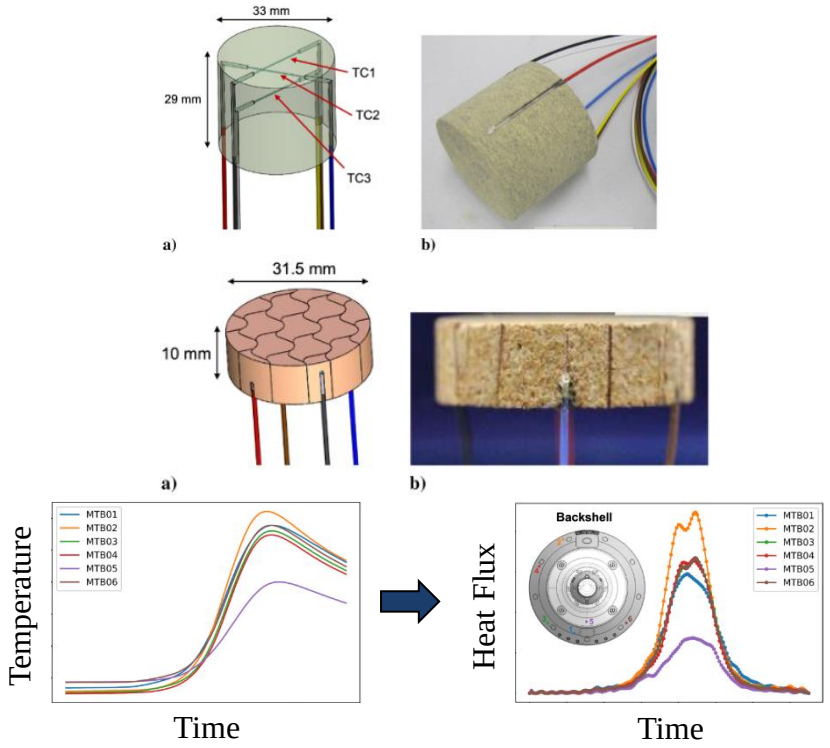
MEDLI2 (Mars 2020)



Hwang, H., et al. AIAA Aviation (2016)

Recovering TPS Surface Heat Flux

- Inverse heat transfer (IHT) methods well-suited for recovering TPS heat flux during atmospheric entry.
- For TPS applications
  - FIAT_Opt [1]
  - INHEAT [2]



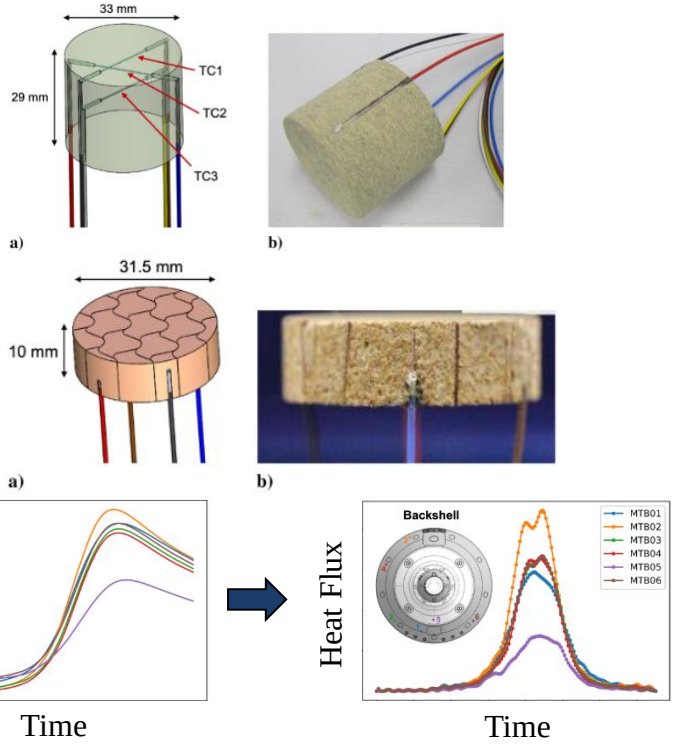
Alpert, H., et al. J Spacecr Rockets (2023)

[1] Alpert, H., et al. J Spacecr Rockets (2023)
[2] Oliver, B., and Amar, A., 46th AIAA Thermophys Conf (2016)

Recovering TPS Surface Heat Flux

- Inverse heat transfer (IHT) methods well-suited for recovering TPS heat flux during atmospheric entry.
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  } Computationally $$$ ~30 mins/reconstruction



Alpert, H., et al. J Spacecr Rockets (2023)

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[2] Oliver, B., and Amar, A., 46th AIAA Thermophys Conf (2016)

Hot-Wall Heat Flux Reconstruction Algorithm

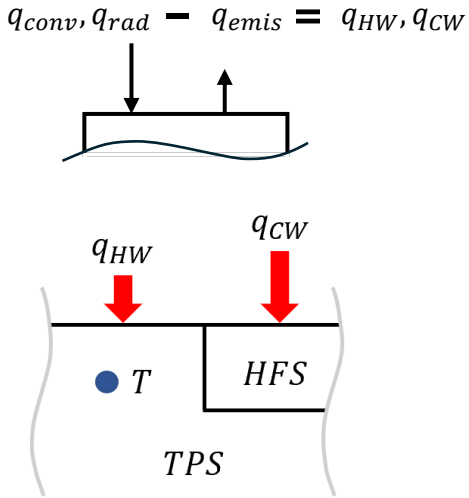
- Prior work
  - **Green's function sensor fusion** approach developed to recover TPS surface heating from collocated heat flux sensor and temperature probe measurements.
    - McAfee, K., et al. AIAA Aviation (2024)
- Objective:
  - Validate the accuracy of the reconstruction approach using experimental measurements.
  - Apply reconstruction algorithm to measurements from the MEDLI2 sensor suite aboard the Mars 2020 entry spacecraft.

Reconstruction Algorithm Overview

- Green's functions used to model the heat conduction within the TPS.

$$T - T_0 = -\int_0^t \frac{\alpha}{k} G(x, t, x_0, \tau) q_{HW}(\tau)\, d\tau$$

$$\{\Delta T\} = -\frac{\alpha}{k}[G]\{q_{HW}\}$$



Reconstruction Algorithm Overview

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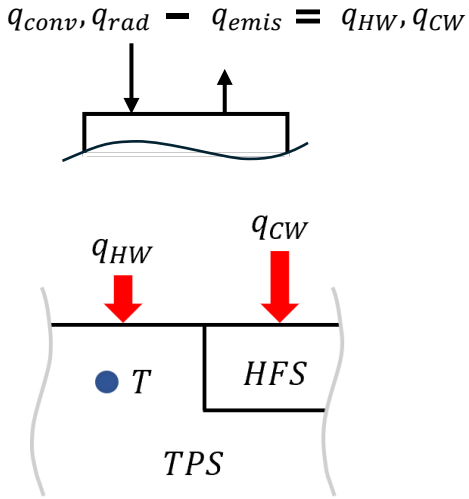
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- Heat flux sensor used to stabilize the inversion of the thermal response.

$$q_{HW} = C(t) q_{CW}$$

$$\{\Delta T\} = -\frac{\alpha}{k}[G][diag(q_{CW})]\{C\}$$



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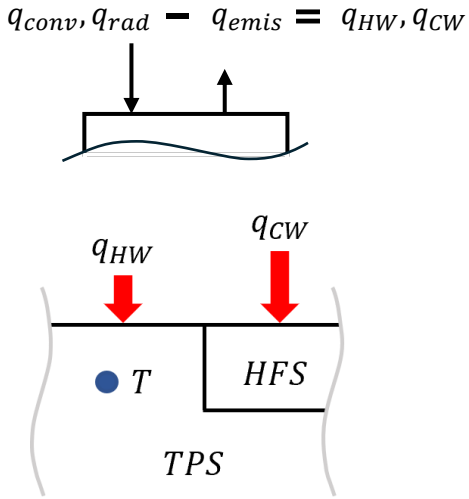
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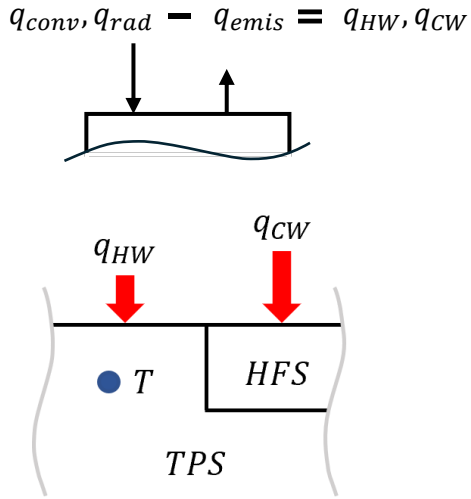
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Omitted details (see full paper):

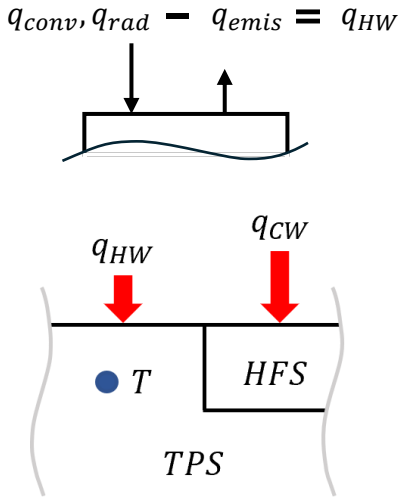
- Temperature-dependent thermophysical properties
- Non-linear solution procedure
- Green's function construction

Recovery of Incident Heating

- Incident heating related to net hot-wall heat flux via surface energy balance.

$$q_{HW} = q_{conv} + \alpha_s q_{rad} - \epsilon_s \sigma_{SB}(T_s^4 - T_\infty^4)$$

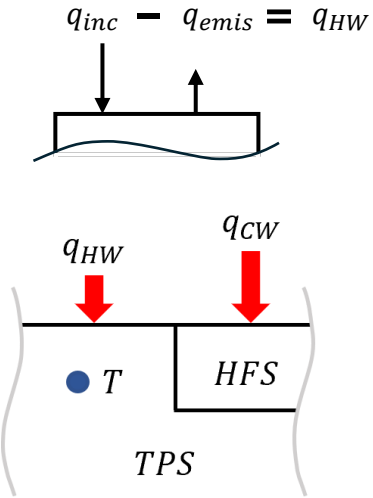
Convection Radiation



Recovery of Incident Heating

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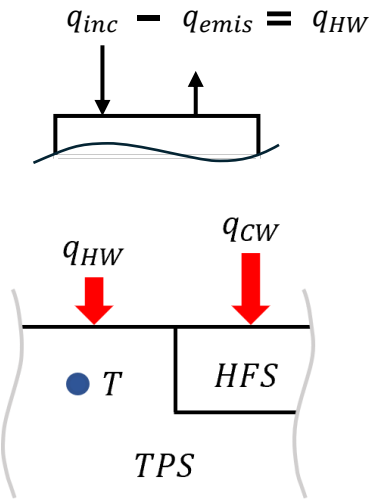


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Recovery of Incident Heating

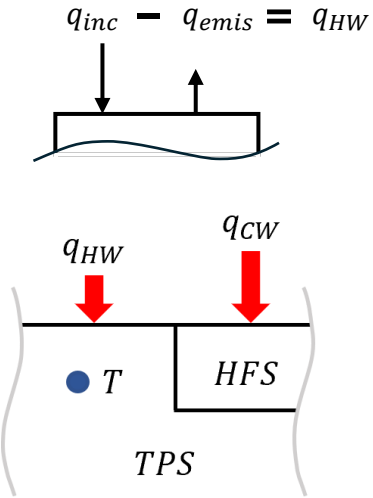
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- Emissivity modeled as a function of surface char fraction χ.

$$\epsilon_s(T, \chi) = \underbrace{[1 - \chi]\frac{\rho_v}{\rho_s}\epsilon_v(T)}_{virgin} + \underbrace{\chi\frac{\rho_c}{\rho_s}\epsilon_c(T)}_{char}$$



Recovery of Incident Heating

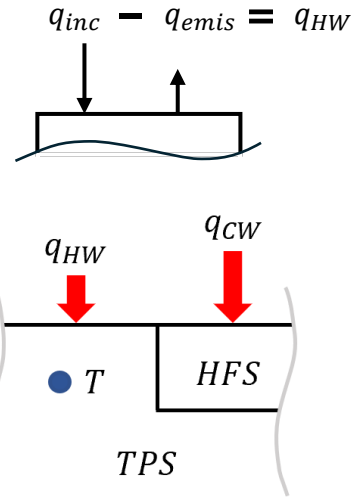
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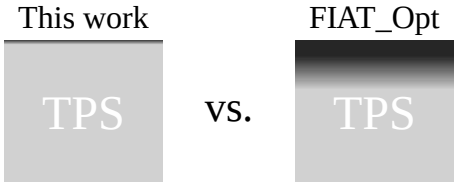
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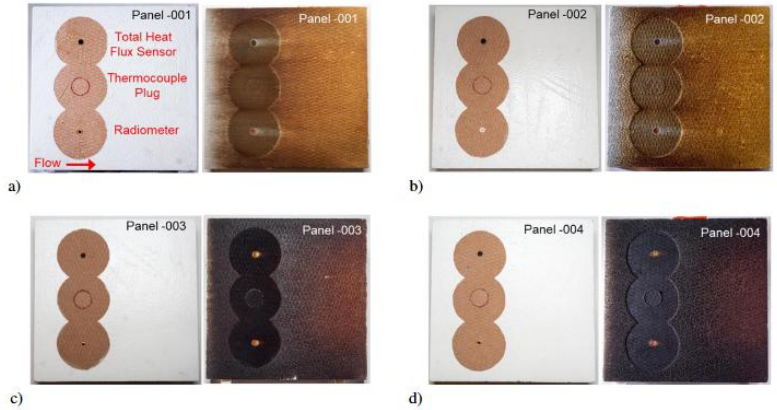




Experimental Reconstruction of TPS Aeroheating

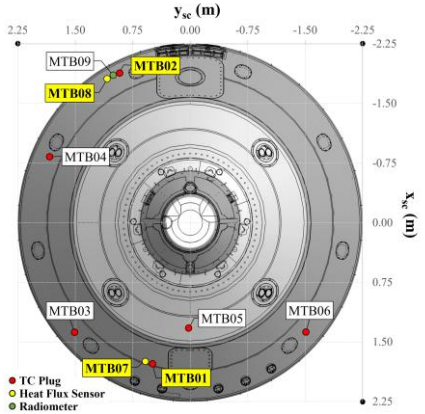
- Panel arc-jet test 162 from NASA Ames PTF.
- Mars 2020 backshell heating & uncertainty quantification.
- Algorithm benchmarked against FIAT_Opt.

PTF162



Miller, R., et al. J Spacecr Rockets (2024)

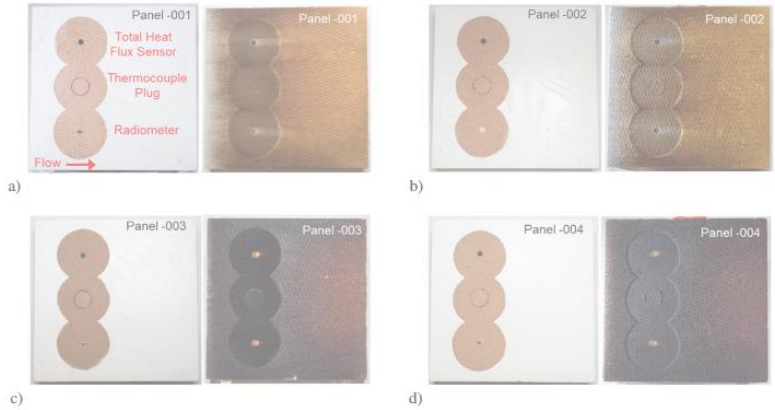
Mars 2020



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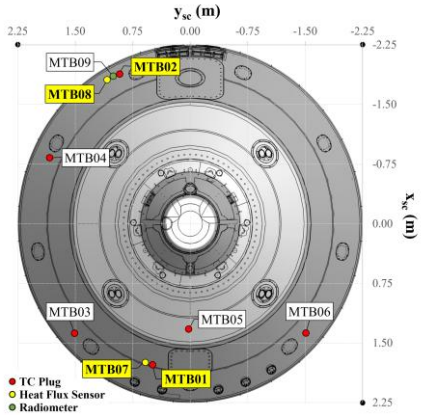
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PTF162

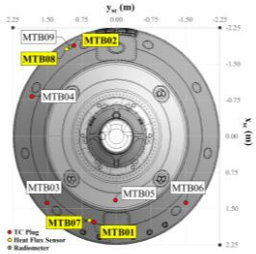


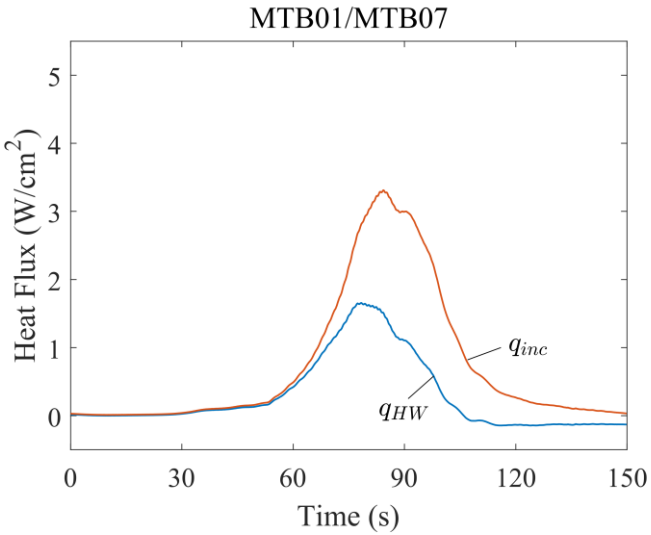
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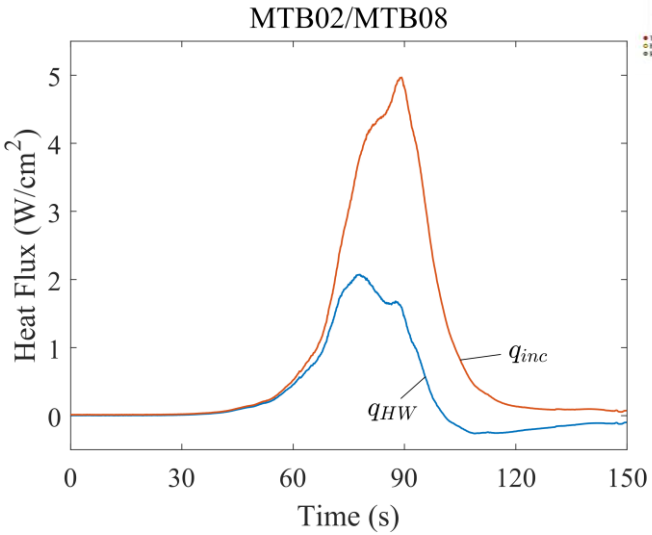
Mars 2020



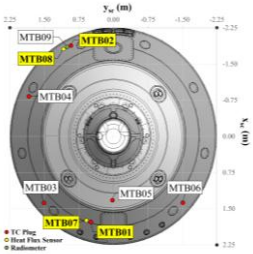
Reconstruction of Mars 2020 Backshell Heating

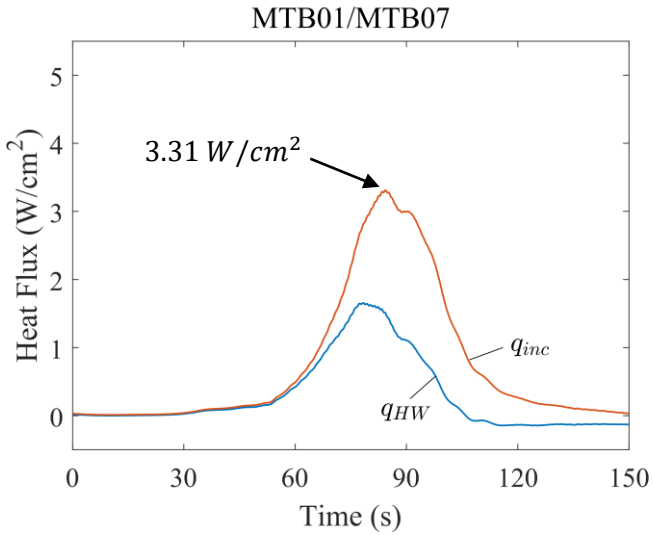


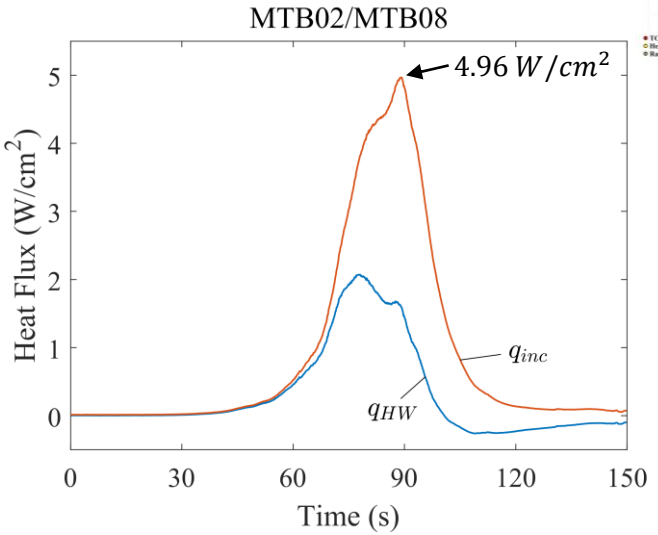




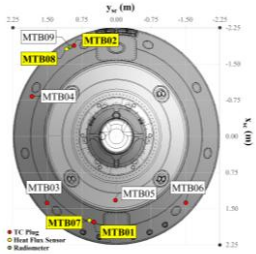
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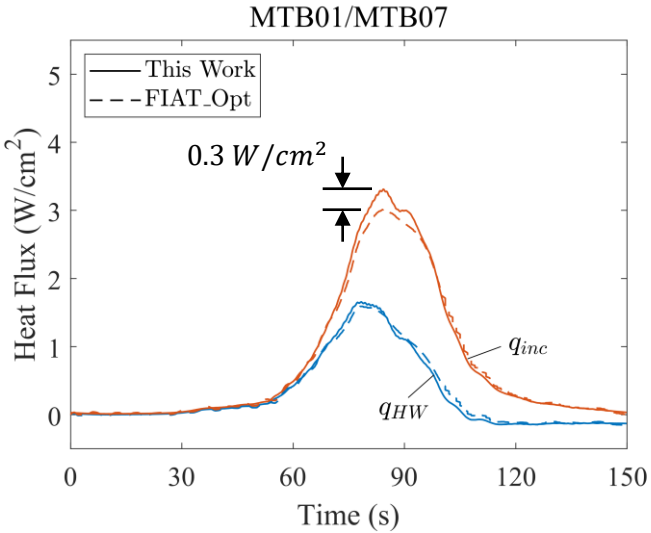


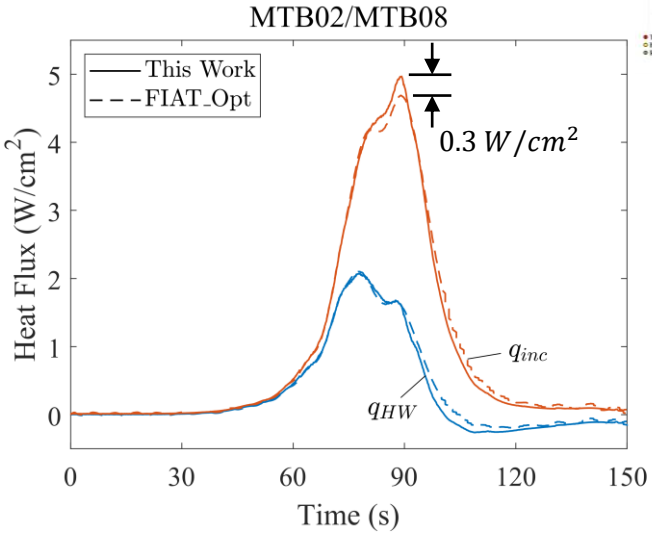




Reconstruction of Mars 2020 Backshell Heating

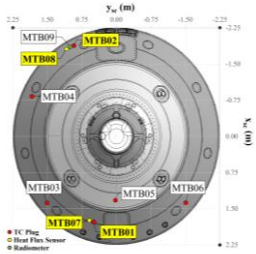


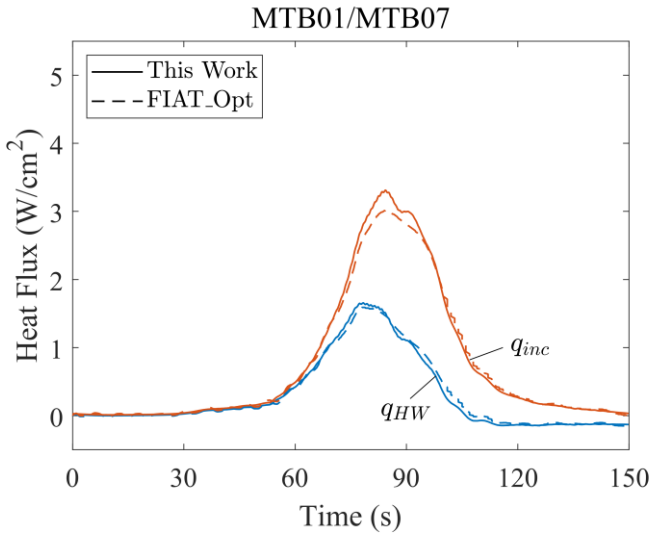


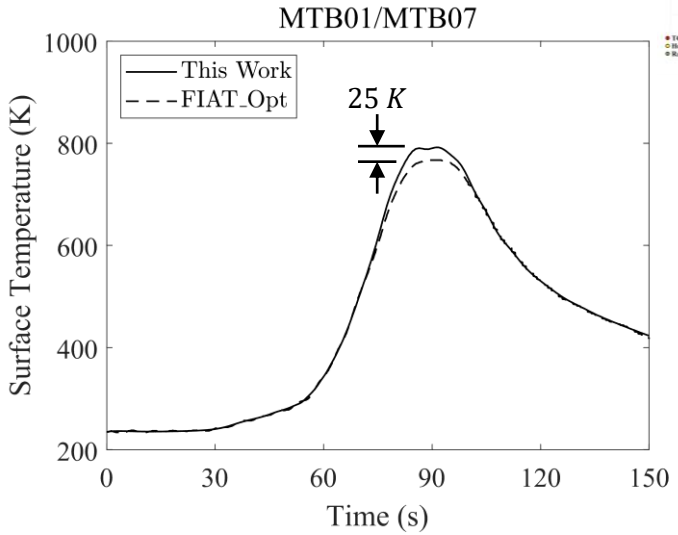


- Max peak heating Δ
  - Net hot-wall heat flux: 4% (MTB01/MTB07)
  - Incident heat flux: 10% (MTB01/MTB07)

Reconstruction of Mars 2020 Backshell Heating

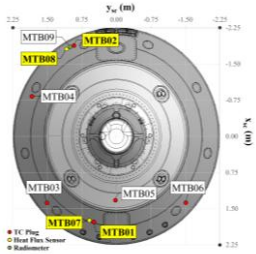


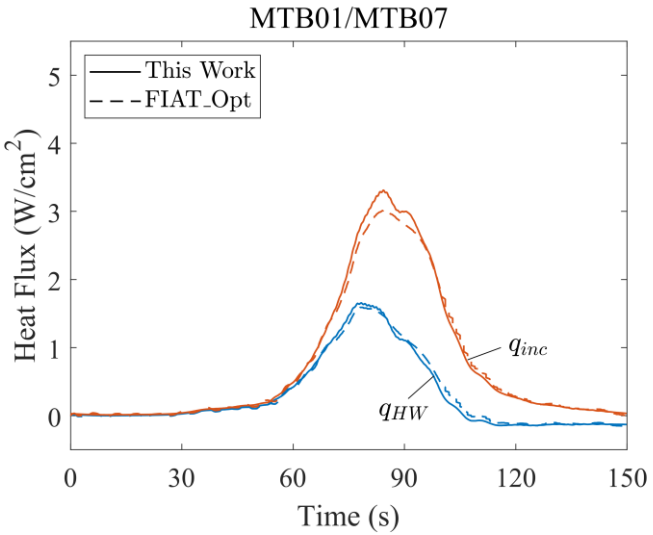


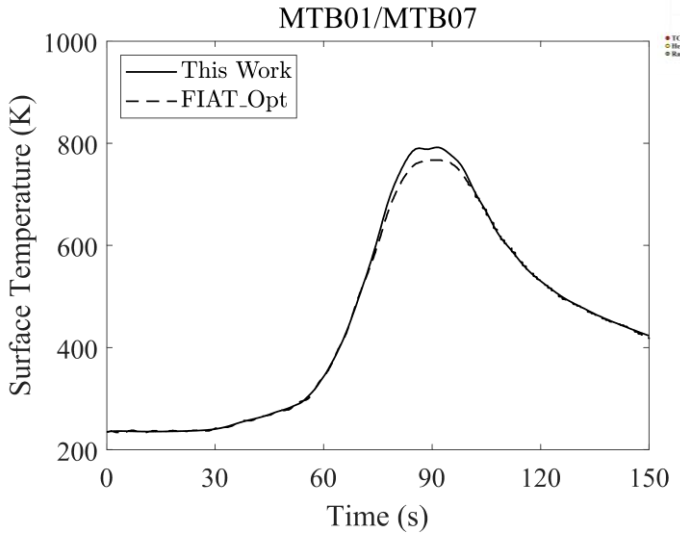


$$q_{inc} = q_{HW} + \epsilon_s \sigma_{SB} (T_s^4 - T_\infty^4)$$

Reconstruction of Mars 2020 Backshell Heating

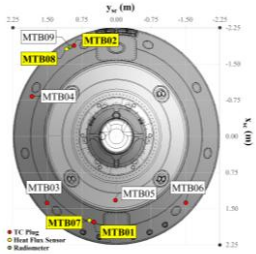




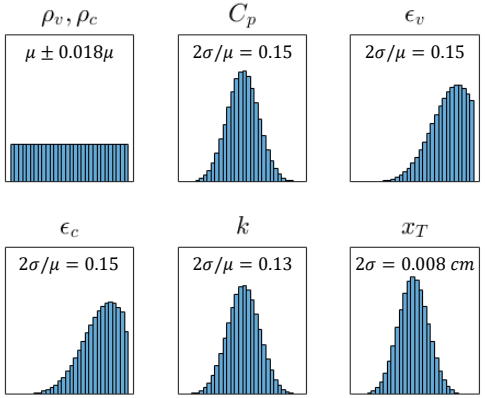


- Total computation time (600 measurement samples)
  - Hot-wall heat flux reconstruction algorithm: **2-3 seconds (laptop)**
  - FIAT_Opt: **30 minutes (cluster)**

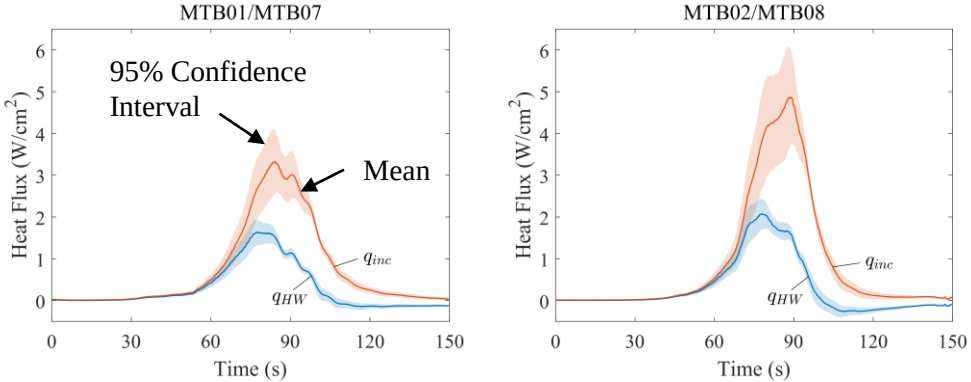
Mars 2020 Backshell Uncertainty Quantification



- Monte Carlo analysis
  - Inputs downsampled by a factor of 2.
  - Latin hypercube sampling.
  - Truncated normal distributions to avoid $\epsilon > 1$.

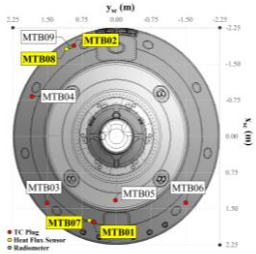


$N_s = 5000$

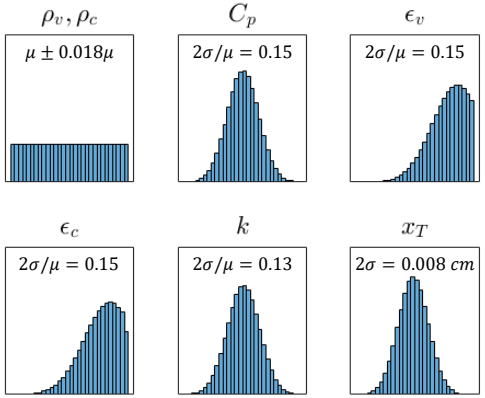


Alpert, H., et al. J Spacecr Rockets (2023)

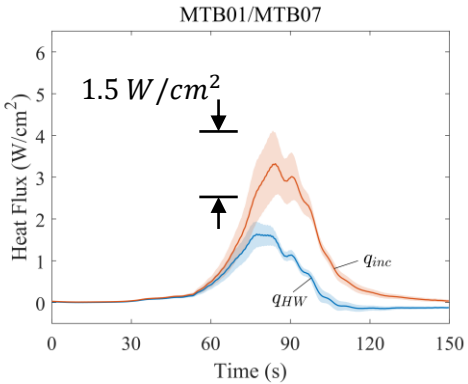
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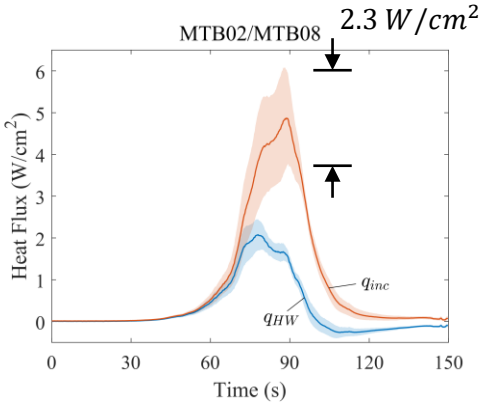


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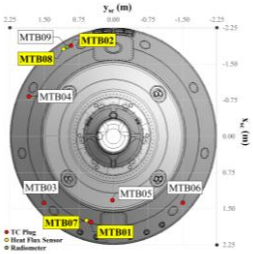
$N_s = 5000$



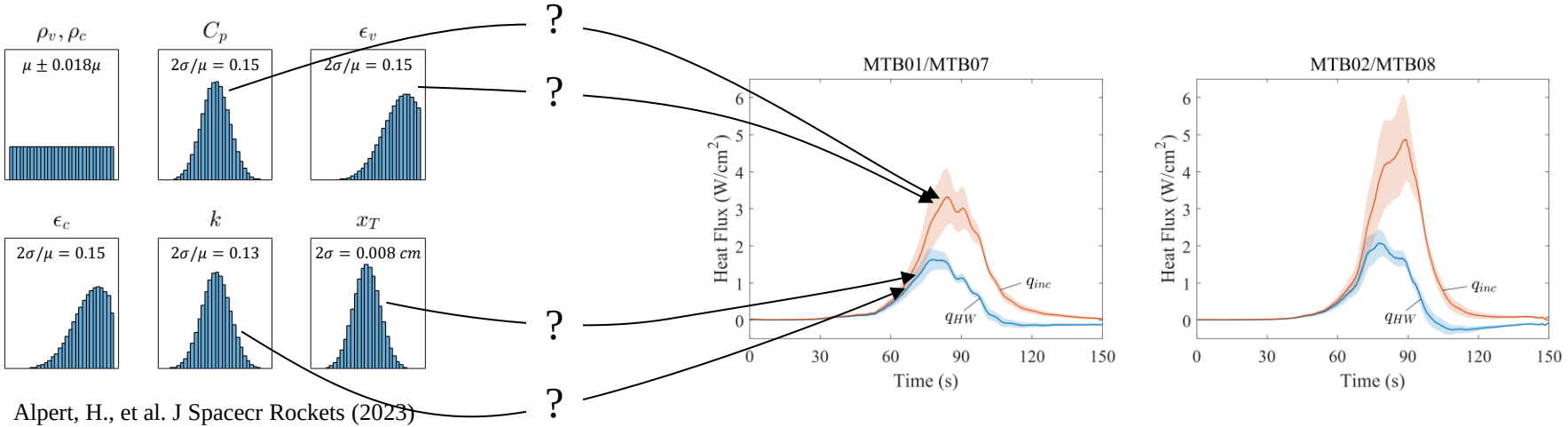


Alpert, H., et al. J Spacecr Rockets (2023)

Mars 2020 Backshell Uncertainty Quantification

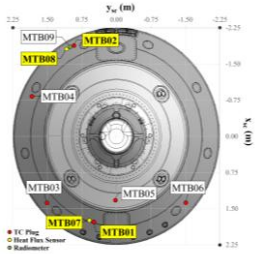


- Monte Carlo analysis
  - Good for overall uncertainty calculation.
  - Not enough to decompose uncertainty contributions from input parameters.



Alpert, H., et al. J Spacecr Rockets (2023)

Variance-Based Sensitivity Analysis

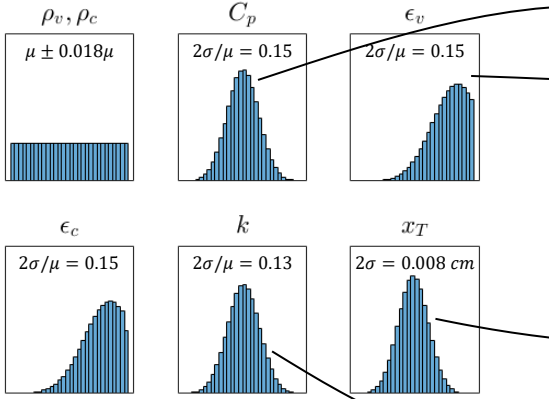


- Total Sobol indices

Saltelli, A., *et al.* Comp Phys Comm (2010)
Jansen, M., Comp Phys Comm (1999)

Sampling matrix

$$S_{T,i} = \frac{1}{2}\frac{\sum_{j=1}^{N_s}\left[q(A)_j - q\left(A_B^{(i)}\right)_j\right]^2}{\sum_{j=1}^{N_s}\left[q(A)_j - q_0\right]^2}$$



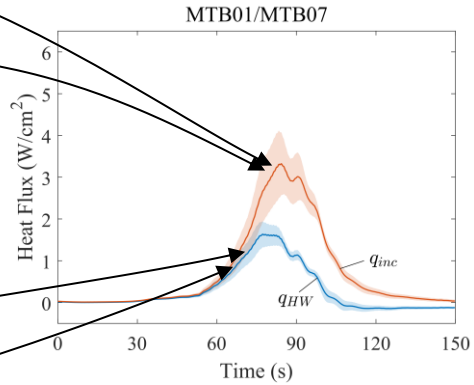
S_{T,C_p}

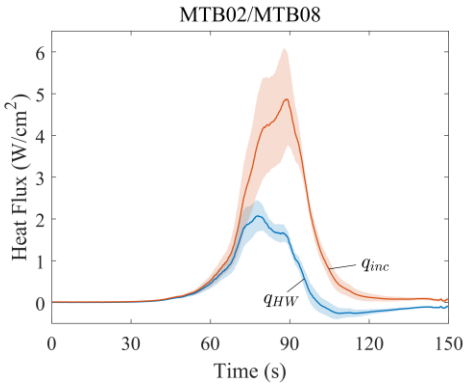
S_{T,ϵ_v}

S_{T,x_T}

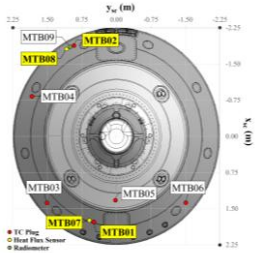
$S_{T,k}$

Alpert, H., et al. J Spacecr Rockets (2023)





Variance-Based Sensitivity Analysis



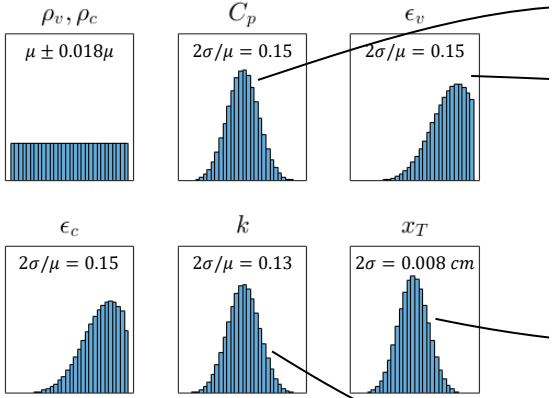
- Total Sobol indices

Saltelli, A., *et al.* Comp Phys Comm (2010)
Jansen, M., Comp Phys Comm (1999)

Sampling matrix

$$S_{T,i} = \frac{1}{2}\frac{\sum_{j=1}^{N_s}\left[q(A)_j - q\left(A_B^{(i)}\right)_j\right]^2}{\sum_{j=1}^{N_s}\left[q(A)_j - q_0\right]^2}$$

Total computations:
$N_s \times (k+1)$
$$$$$



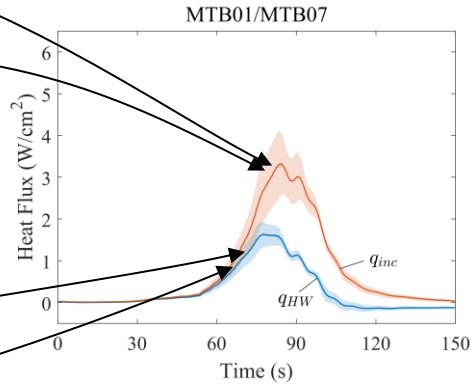
S_{T,C_p}

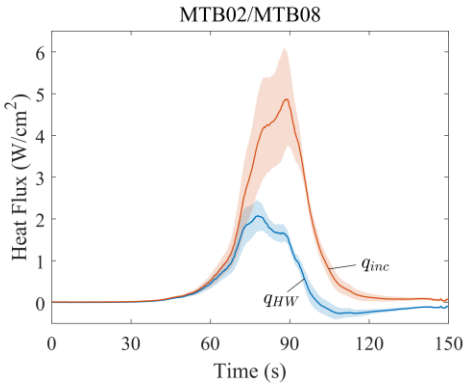
S_{T,ϵ_v}

S_{T,x_T}

$S_{T,k}$

Alpert, H., et al. J Spacecr Rockets (2023)

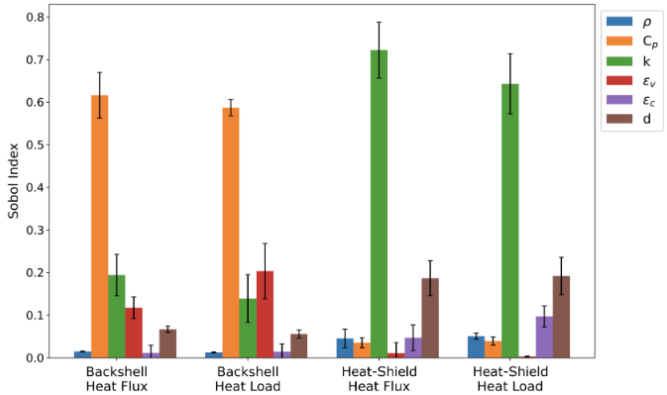




Variance-Based Sensitivity Analysis

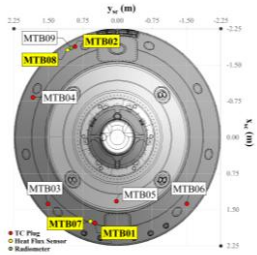
- Prior work
  - Machine learning approach developed to calculate $S_{T,i}$ w/ respect to peak heating and total heat load.

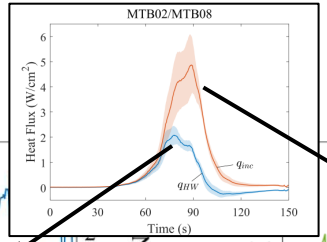
    → Only applicable to peak heating region.
- This work
  - $S_{T,i}$ calculated w/ respect to net hot-wall and incident heat flux for the full entry heat pulse.



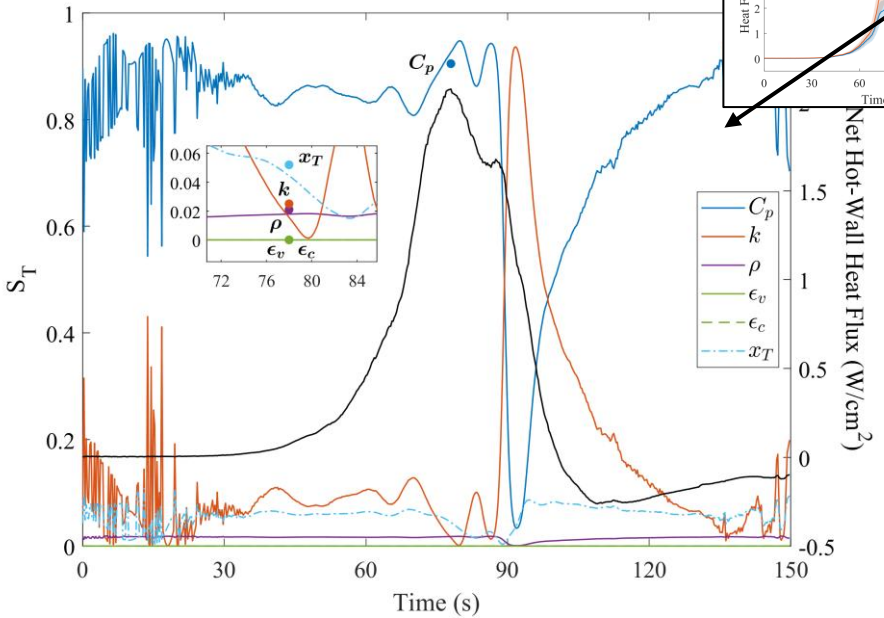
Alpert, H., et al. J Spacecr Rockets (2023)

Total Sobol Indices – MTB02/MTB08

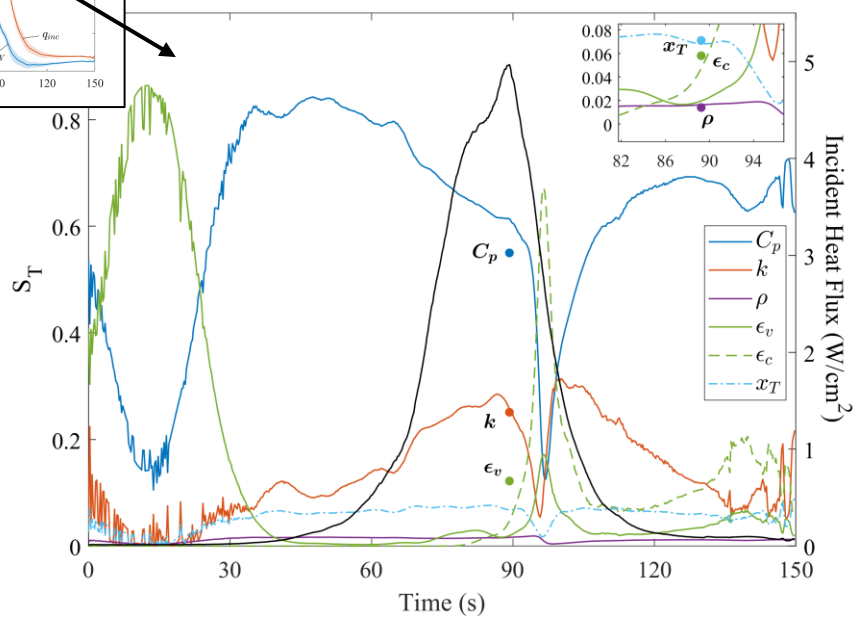




Net Hot-Wall Heat Flux

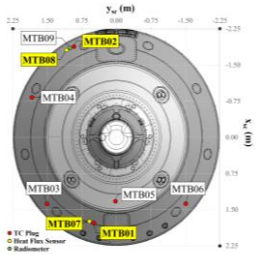


Incident Heat Flux

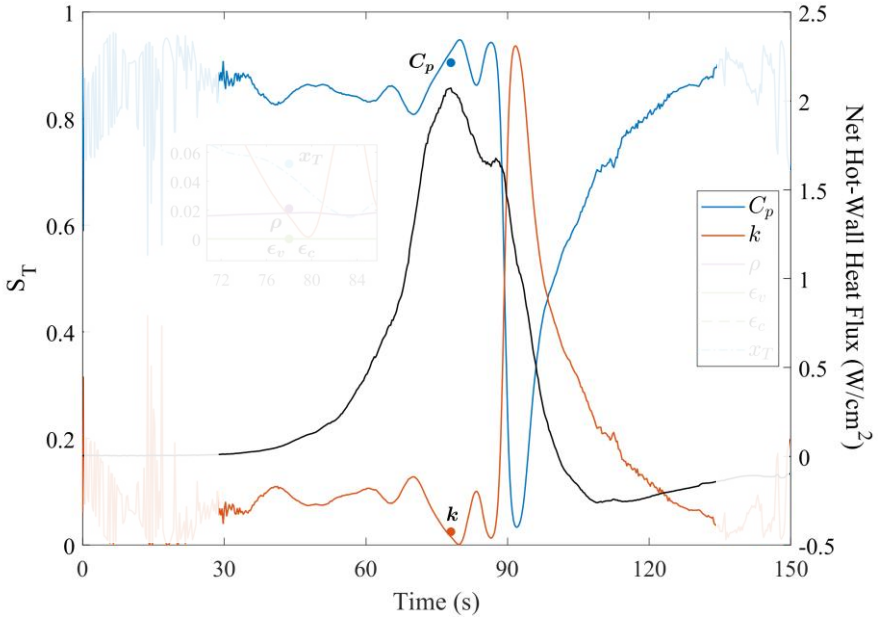


Datapoints from Alpert, H., et al. J Spacecr Rockets (2023)

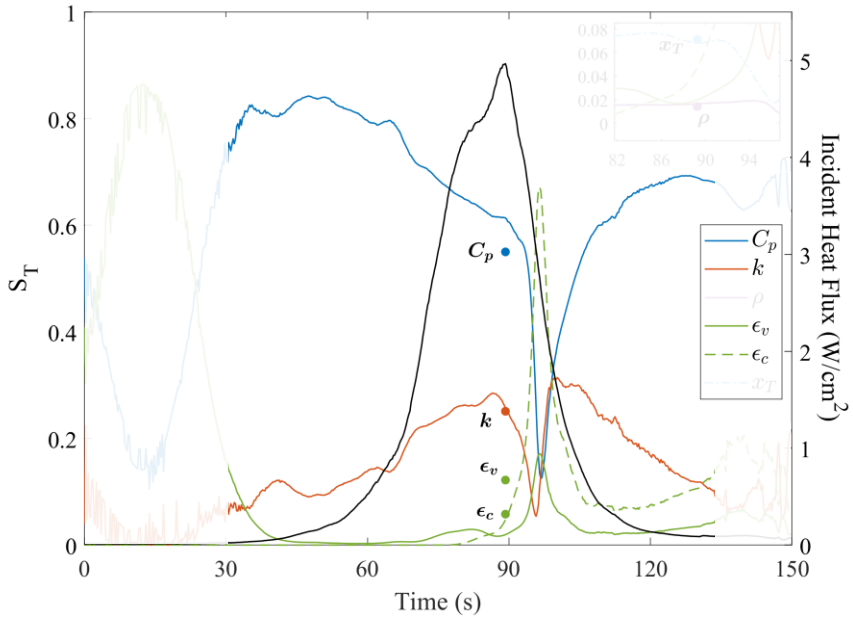
Total Sobol Indices – MTB02/MTB08



Net Hot-Wall Heat Flux

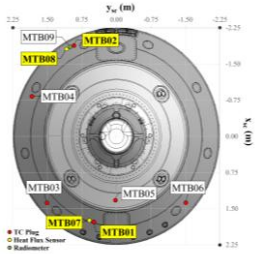


Incident Heat Flux

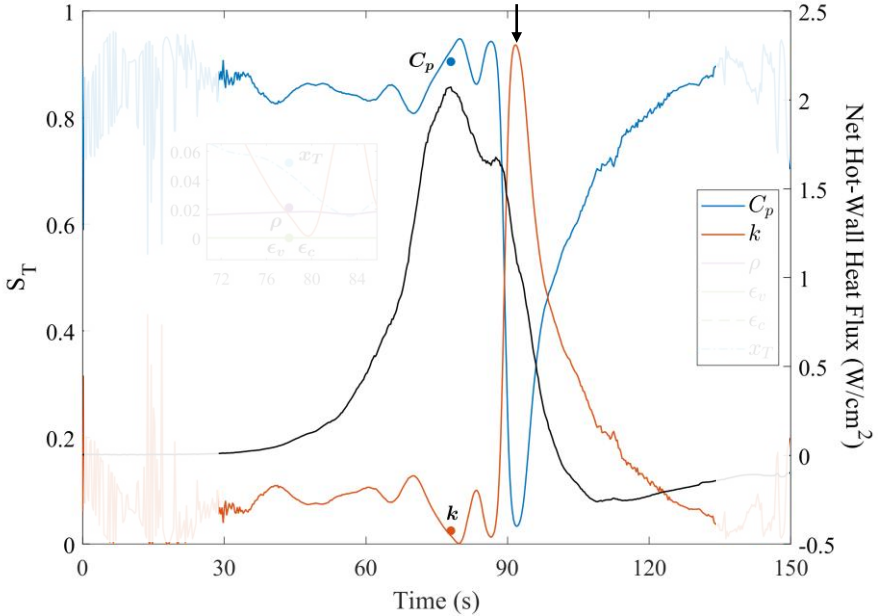


Datapoints from Alpert, H., et al. J Spacecr Rockets (2023)

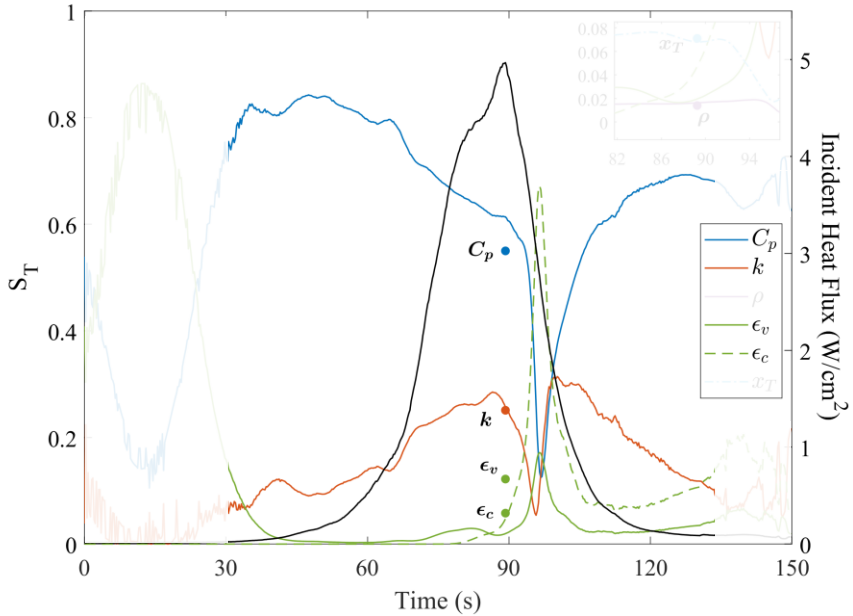
Total Sobol Indices – MTB02/MTB08



Net Hot-Wall Heat Flux

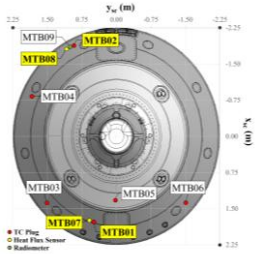


Incident Heat Flux

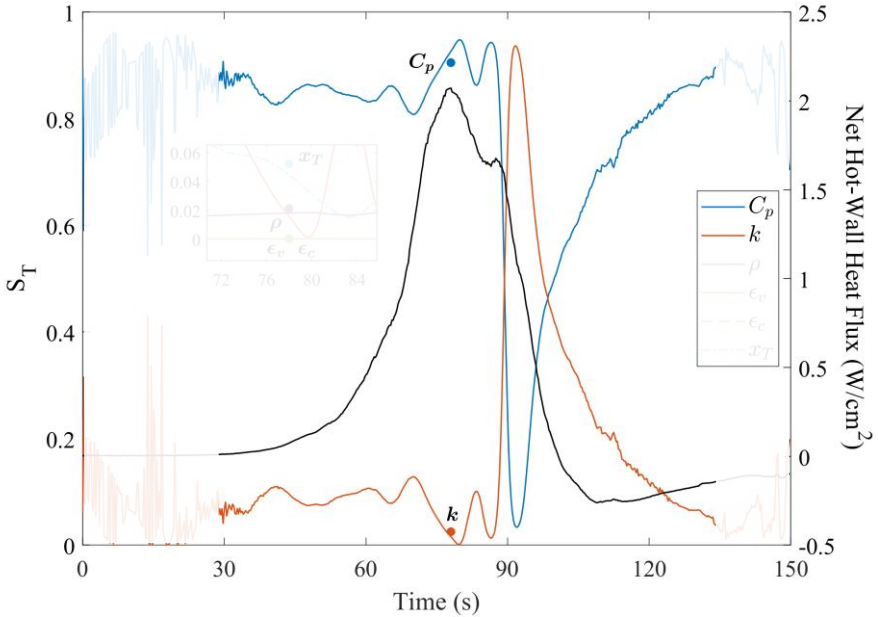


Datapoints from Alpert, H., et al. J Spacecr Rockets (2023)

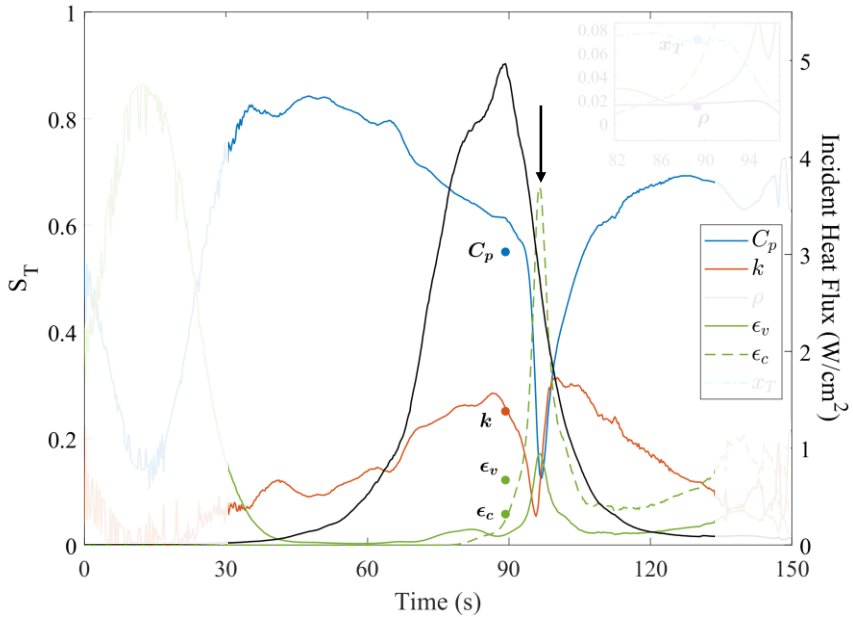
Total Sobol Indices – MTB02/MTB08



Net Hot-Wall Heat Flux

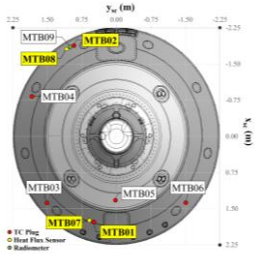


Incident Heat Flux

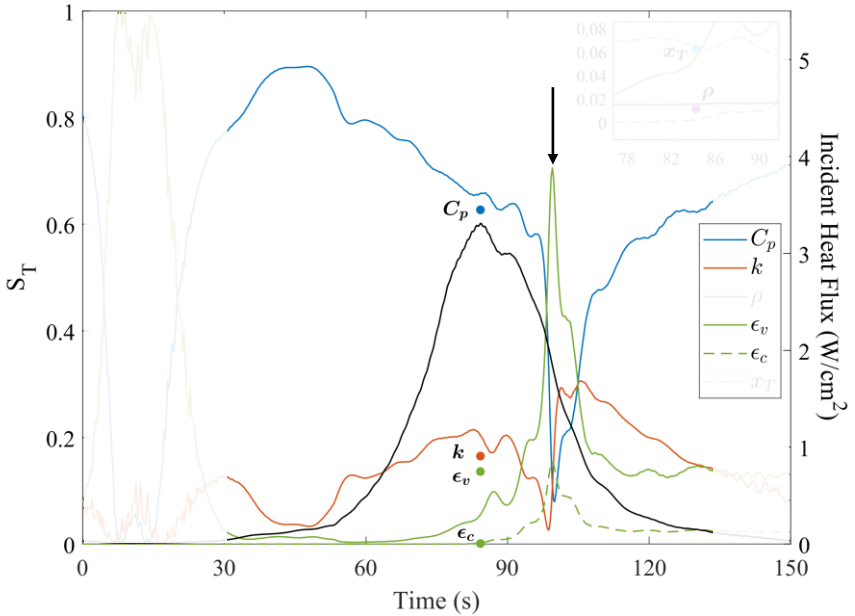


Datapoints from Alpert, H., et al. J Spacecr Rockets (2023)

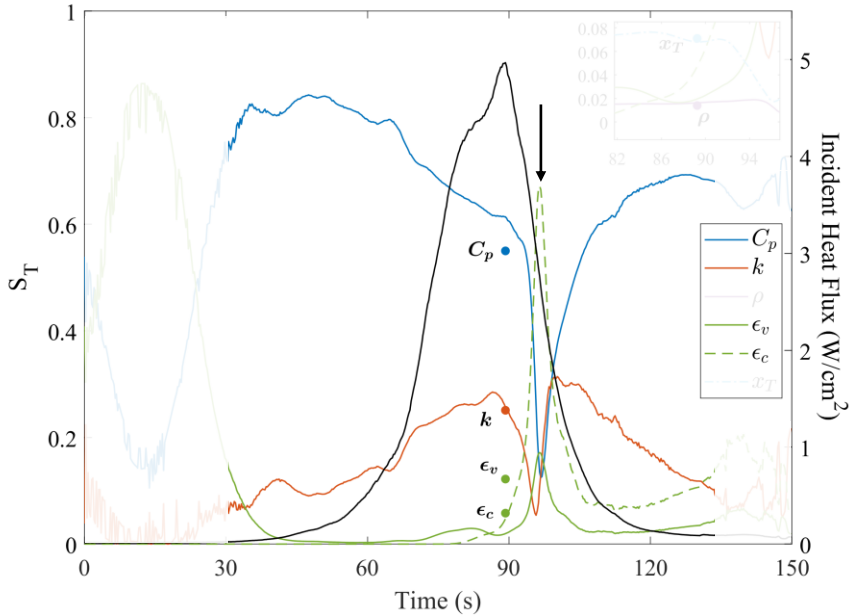
Total Sobol Indices – Emissivity



MTB01/MTB07



MTB02/MTB08



Datapoints from Alpert, H., et al. J Spacecr Rockets (2023)

Conclusions

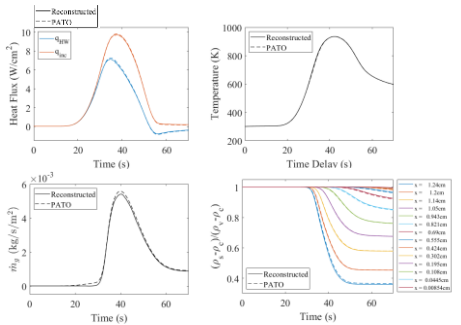
- Green’s function sensor fusion approach used to reconstruct thermal protection system aeroheating in experimental systems.
- Algorithm recovers Mars 2020 backshell heating to within 10% of FIAT_Opt in 3 orders-of-magnitude less time.
- Uncertainty of Mars 2020 backshell heating driven by uncertainties in specific heat, thermal conductivity, and emissivity.

Conclusions

- Green's function sensor fusion approach used to reconstruct thermal protection system aeroheating in experimental systems.
- Algorithm recovers Mars 2020 backshell heating to within 10% of FIAT_Opt in 3 orders-of-magnitude less time.
- Uncertainty of Mars 2020 backshell heating driven by uncertainties in specific heat, thermal conductivity, and emissivity.

Future Work

- Apply fully pyrolyzing TPS formulation to Mars 2020 backshell reconstruction.



Numerical verification on ablative TPS (TACOT)
McAfee, K., et al. 14th Ablation Workshop (2024)

Ackowledgements

- This work was supported by a NASA Space Technology Graduate Research Opportunity (grant 80NSSC23K1201).
- We thank Antonella Alunni, Ruth Miller, Quincy Mckown, Sam Preece, and Prof. Peter Sunderland for their comments and reviews.

Questions?

Appendix

Inverse Heat Transfer (IHT) for TPS

Conventional IHT methods

- E.g., FIAT_Opt, INHEAT

+ Capture relevant heat conduction phenomena in TPS.

- Requires
  - time marching.
  - whole-mesh solution.
  - non-linear optimization/ calculation of sensitivity coefficients.

→ **Computationally expensive**

Green's Functions

- E.g., - Cole, K., et al. CRC Press (2010)
  - Massa, L., and Schetz, J., J Thermophys Heat Trans (2020)
  - Whalen, T., et al. Meas Sci Technol (2021)

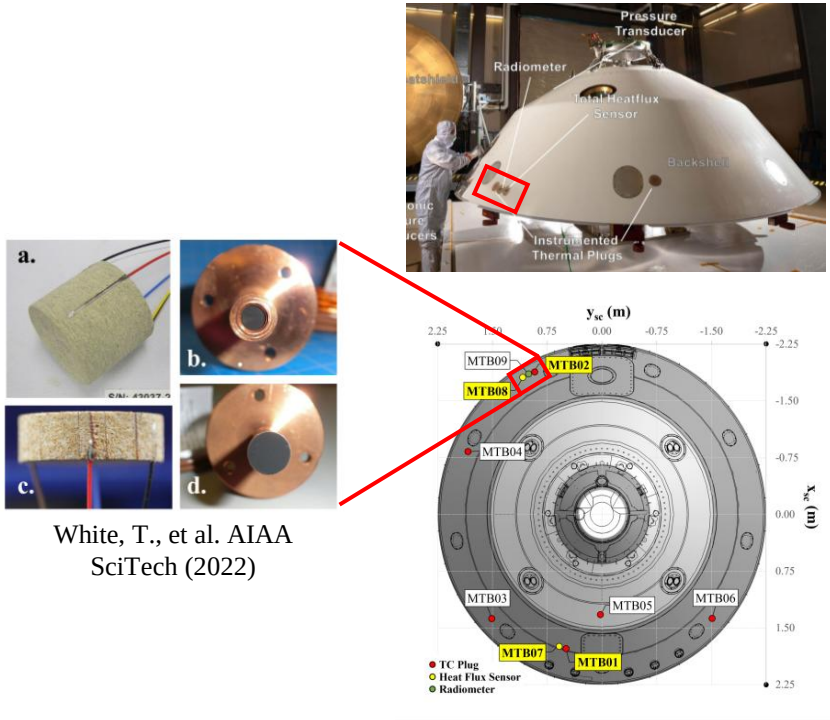
- Traditionally lower fidelity.

+ Only models the material response at a *single* location (i.e., TC location), but for the *entire* time domain at once.

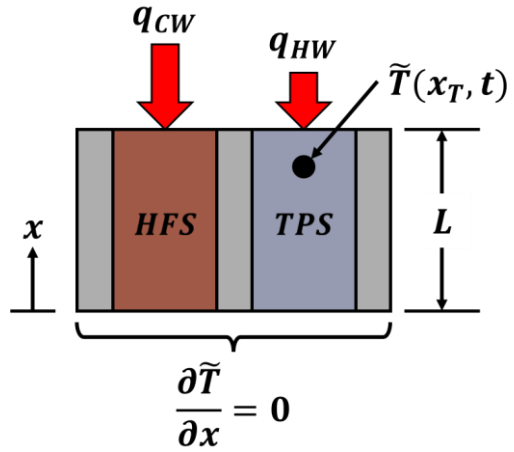
+ Material response captured by linear system → Can be inverted to recover the surface heat flux.

Optimized for IHT

Model System



White, T., et al. AIAA SciTech (2022)



Reconstruction Algorithm Overview (cont.)

- Solution of C is stabilized by penalizing the 2nd derivative

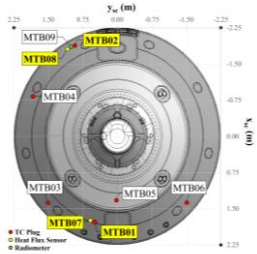
User defined

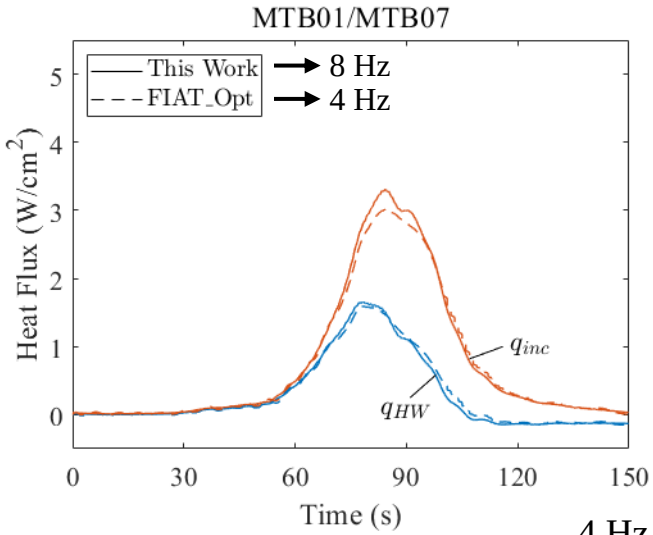
$$\min\left\{\underbrace{\left\|\{\Delta T\}+\frac{\alpha}{k}[G]\,[diag(q_{CW})\,]\{C\}\right\|_2^2}_{\text{Residual}}+\underbrace{\|\lambda_2[L_2]\{C\}\|_2^2}_{\text{2nd derivative penalty term}}\right\}$$

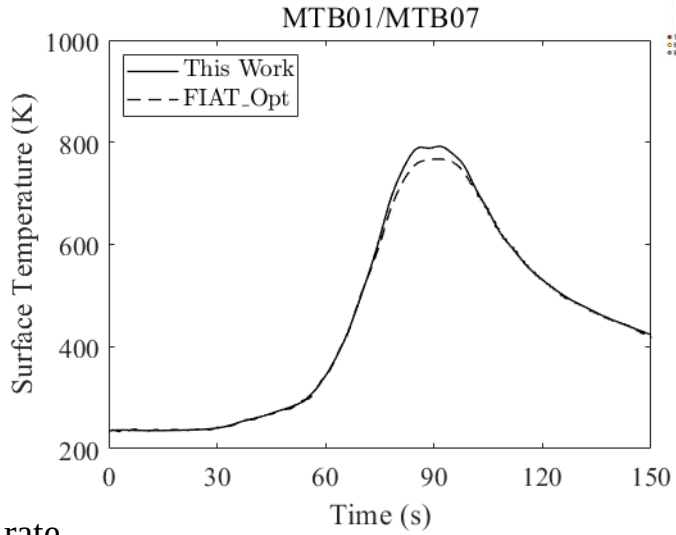
- q_{HW} is recovered via

$$\{q_{HW}\}=[diag(q_{CW})]\{C\}$$

Reconstruction of Mars 2020 Backshell Heating



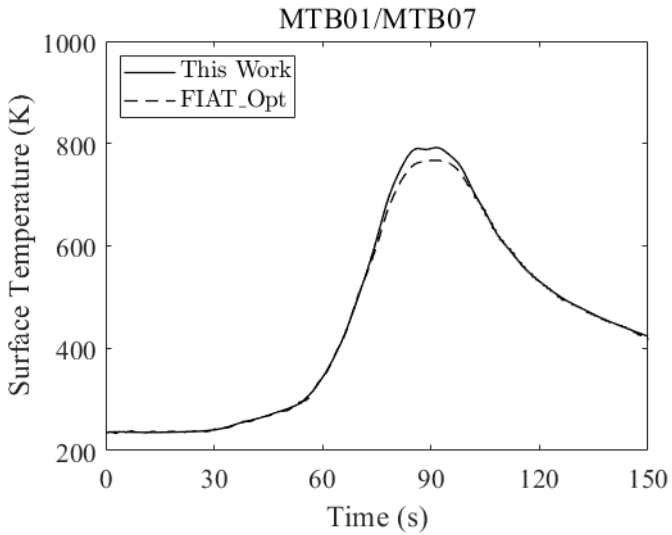


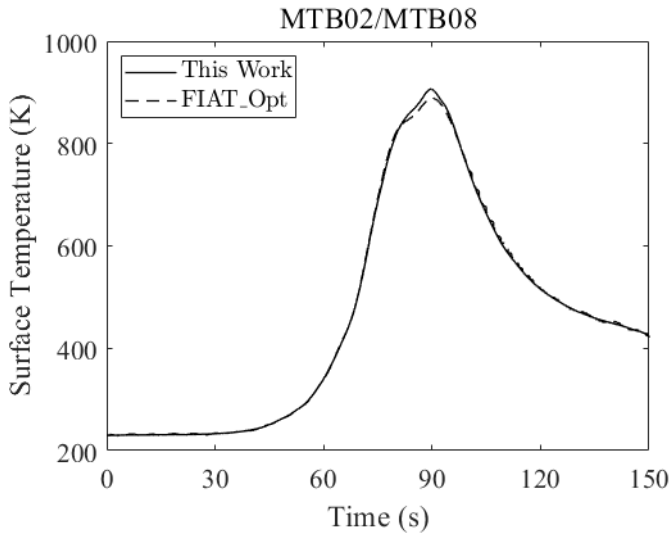


4 Hz sample rate

- Total computation time (600 measurement samples)
  - Hot-wall heat flux reconstruction algorithm: **2-3 seconds (laptop)**
  - FIAT_Opt: **30 minutes (cluster)**

Reconstructed Surface Temperature

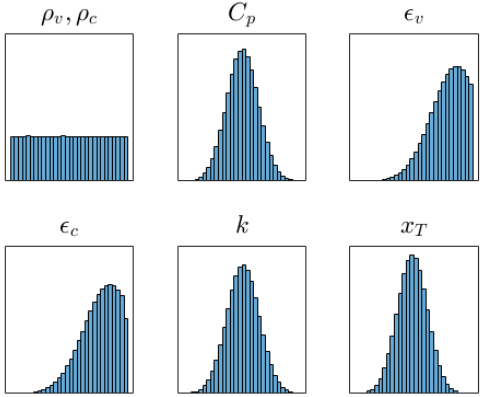




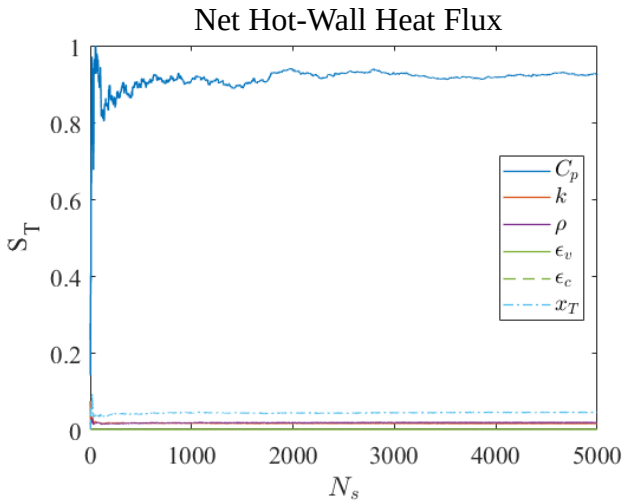
MEDLI2 Flight-Lot Uncertainties

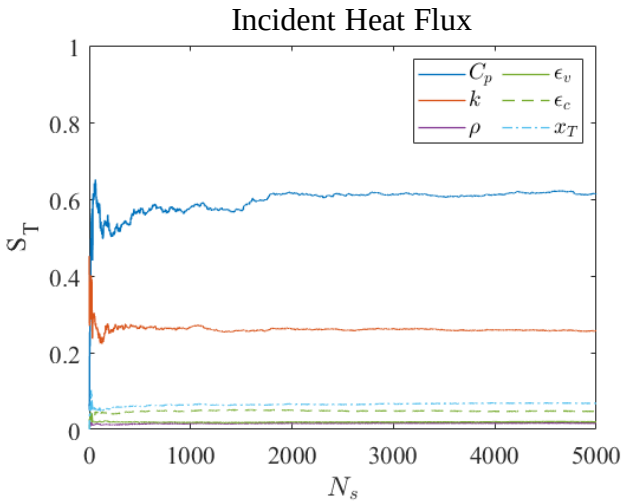
	$\rho_v = \rho_c$	C_p	ϵ_v	ϵ_c	k	x_T
Distribution	U	N	N*	N*	N	N
$2\sigma/\mu$	±0.018†	0.15	0.15	0.15	0.13	0.0076 cm‡

* Truncated normal distribution
† Upper/lower bounds
‡ Not normalized by mean

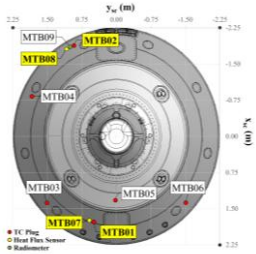


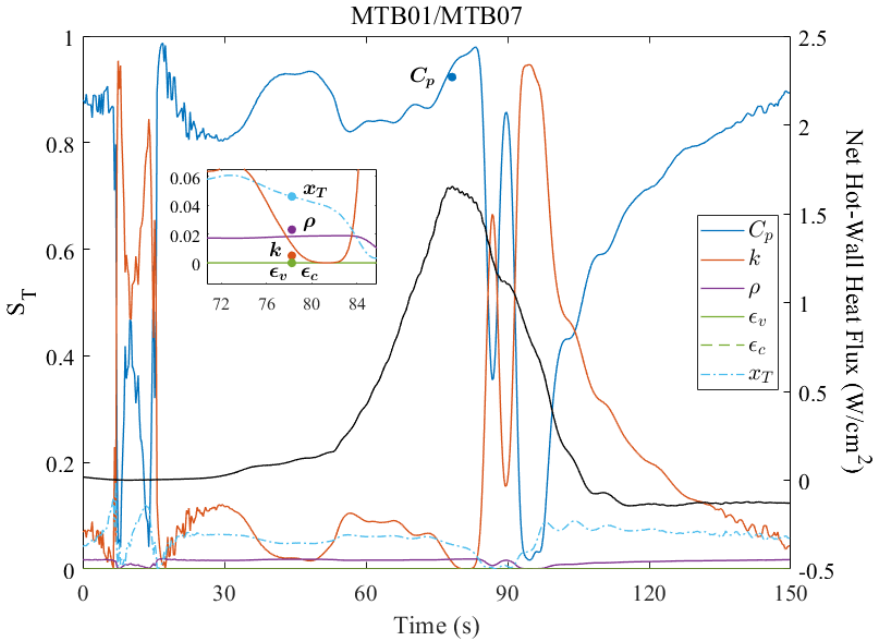
Sobol Indices Convergence (MTB02/MTB08)

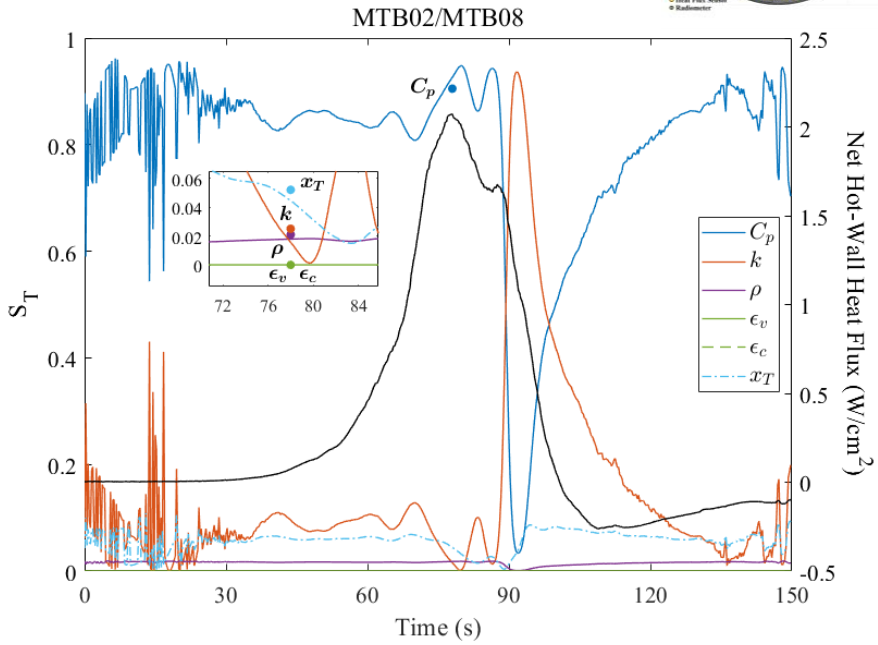




Sobol Indices – Net Hot-Wall Heat Flux

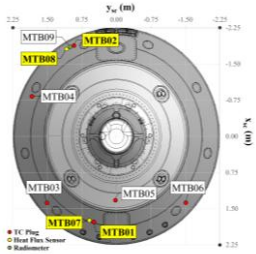


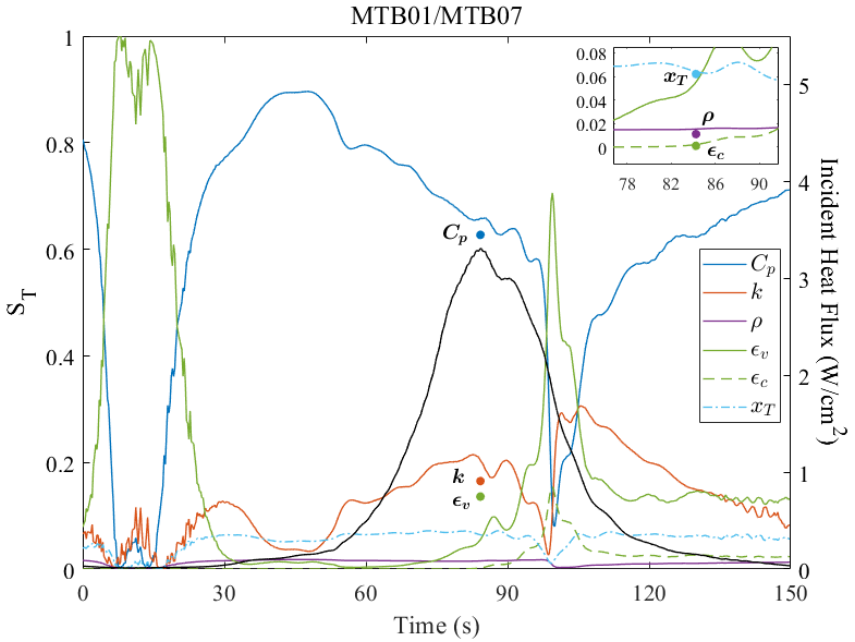


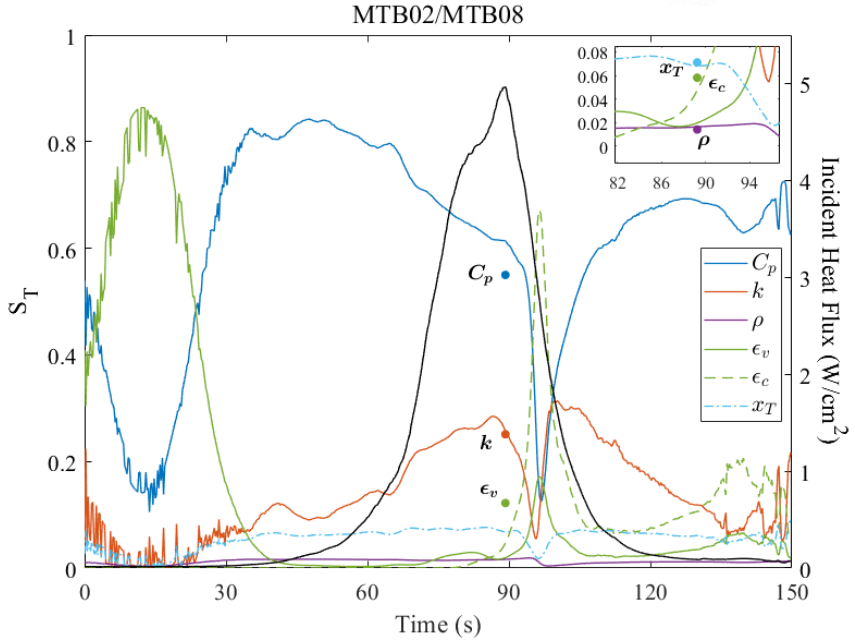


Datapoints from Alpert, H., et al. J Spacecr Rockets (2023)

Sobol Indices – Incident Heat Flux



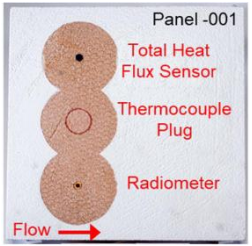


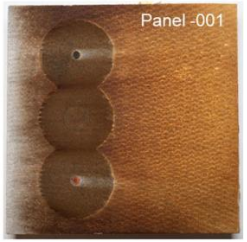


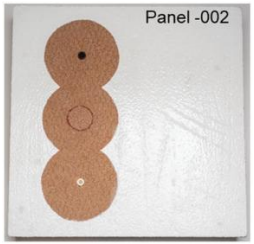
Datapoints from Alpert, H., et al. J Spacecr Rockets (2023)

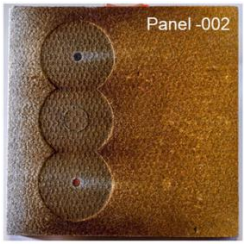
PTF162

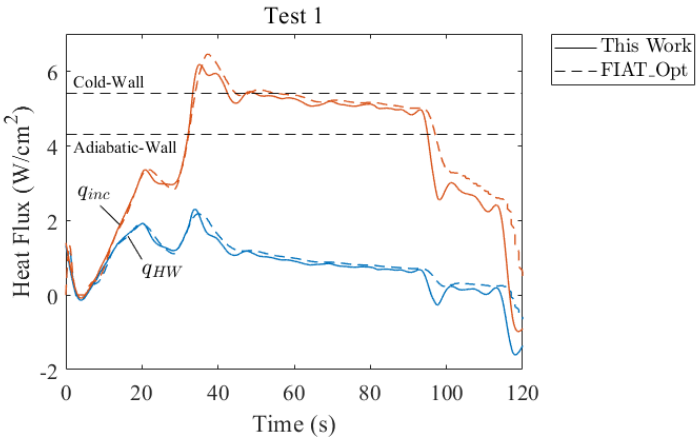
PTF162 Low Heating Scenario

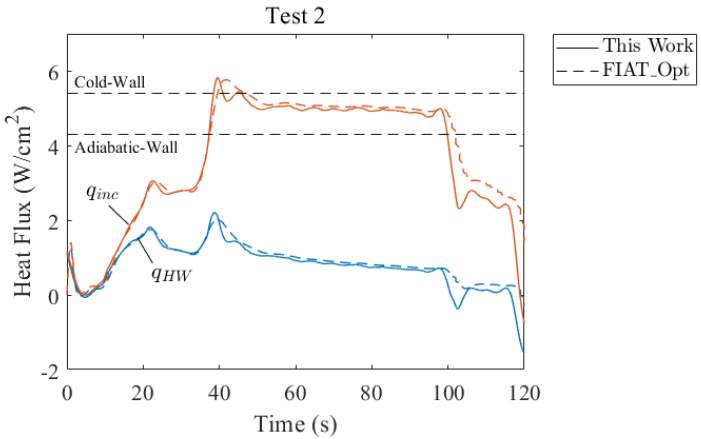












PTF162 High Heating Scenario

