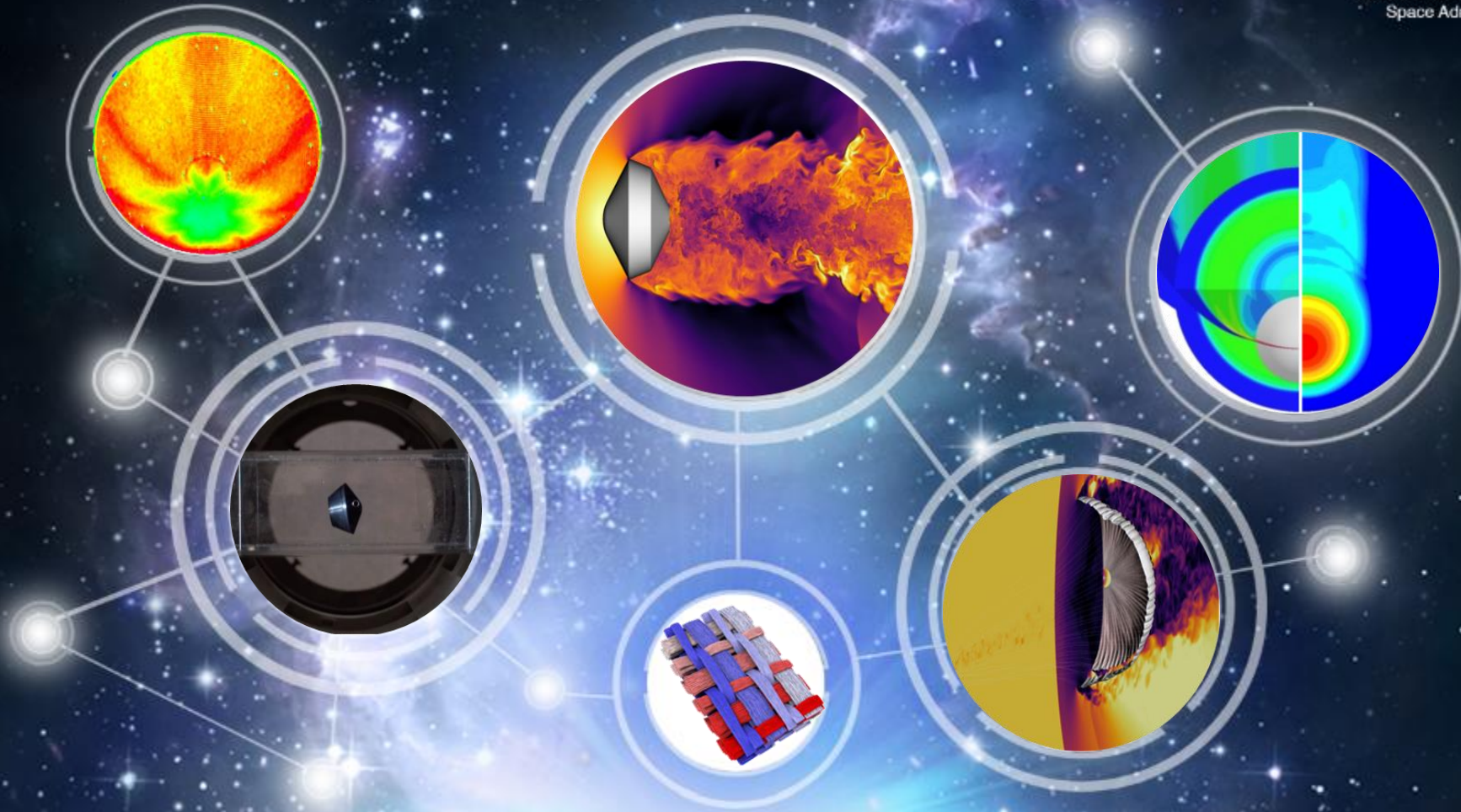




Application of a Dual-Quaternion 6DOF Guidance to Human-Scale Mars Entry, Descent, and Landing

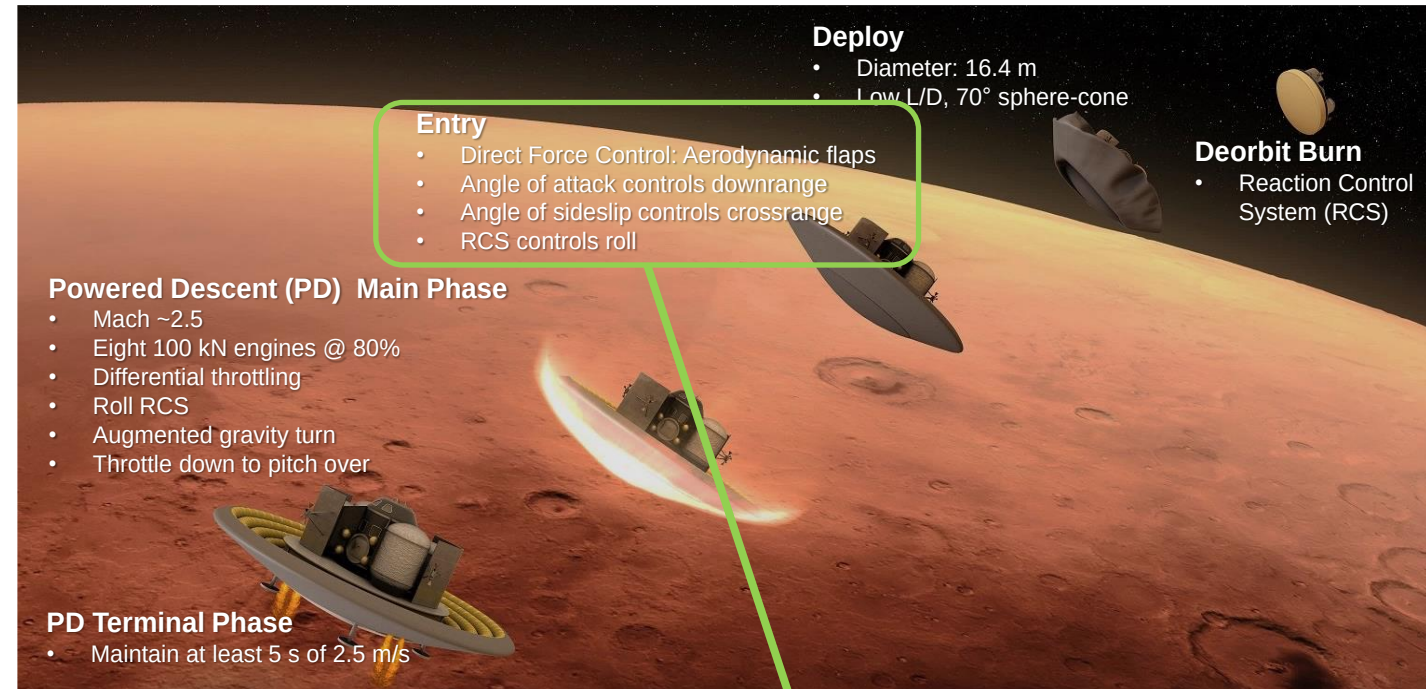
Rafael Lugo (NASA LaRC), Soumyo Dutta (NASA LaRC), & Michael Manwell (AMA/LaRC) | AIAA SciTech Forum & Exposition | January 6th-10th 2025 | Orlando, FL

- New and novel entry guidance algorithms are a key element to safe and precise landing of human-scale elements at Mars
- **2018: Three NASA Early Career Faculty (ECF) awards in the field of advanced EDL guidance algorithms, including to Dr. Ran Dai of Purdue**
 - NASA Entry Systems Modeling (ESM) project through STMD funded efforts to implement guidances in NASA mission and simulation environments
 - Goal: Guidance flyoffs
- **Dr. Dai's dual-quaternion based guidance utilizes equations of motion for translational and rotational motion based on dual-quaternions [1]**

Objective

Apply dual-quaternion based guidance to a NASA reference human-scale Mars entry system [2-5] in a NASA simulation architecture (POST2)

Emphasis of this work is on implementation of guidance and performance results



Today's focus

- 1. POST2 Overview**
- 2. Guidance Overview**
- 3. POST2 Implementation**
- 4. Results**
- 5. Summary**

What is POST2?

Key Capabilities

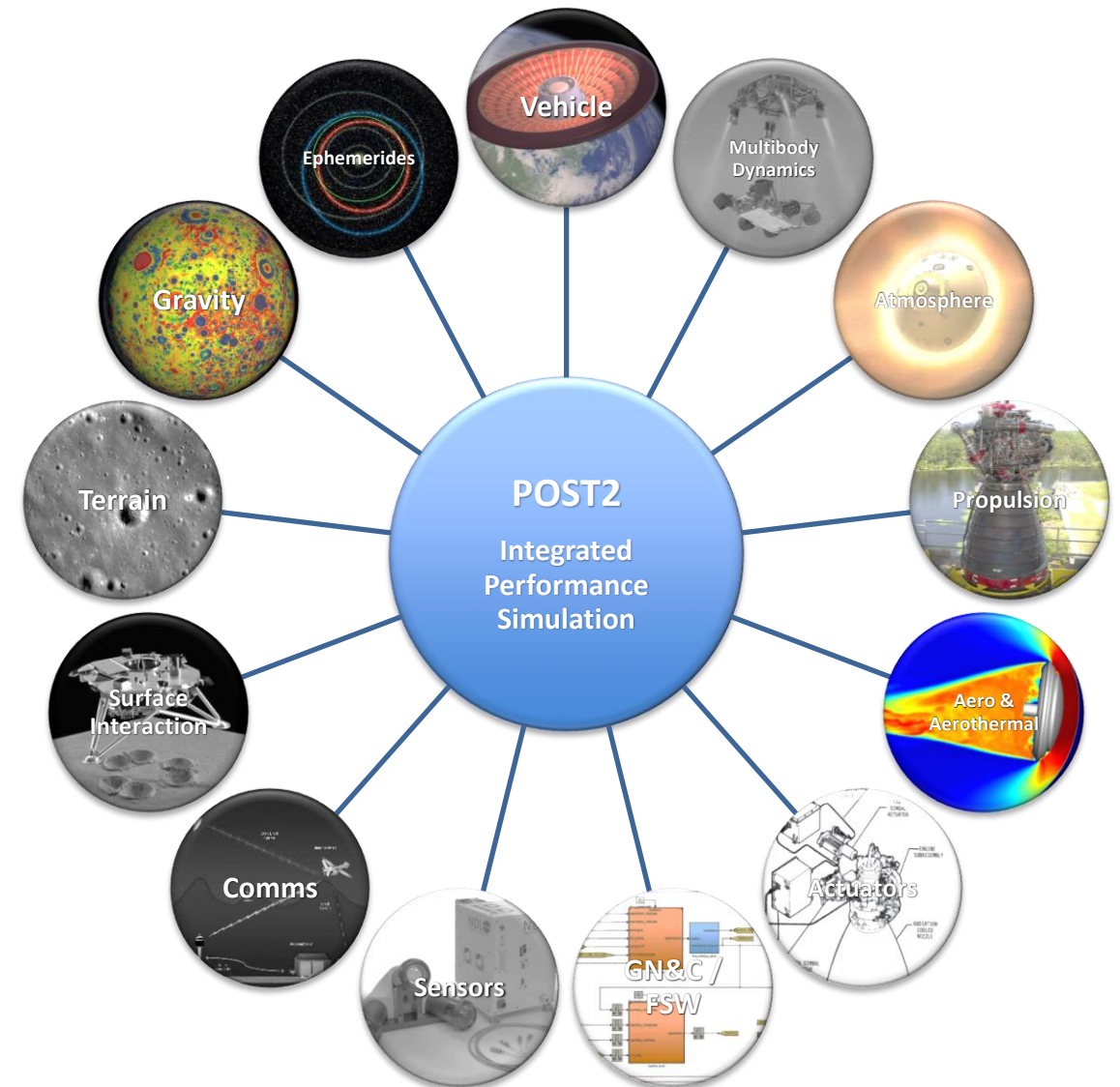
- **3/6/Multi-DOF Flight Dynamics Simulation & Optimization:** powered & unpowered, endo & exoatmospheric, fixed-wing & rotorcraft, single & multi vehicle, etc.
- **Flight Validated:** Used for pre- and post-flight analysis of dozens of Earth & Mars EDL missions and ground/flight tests
- **Scalable Fidelity:** 3DOF cannonballs to 6DOF multi-staged flight systems with integrated GN&C/FSW, flex & slosh, touchdown contact dynamics, etc.
- **User-Driven:** User defines trajectory event sequencing and provides models and data for GN&C/FSW, actuators, aerodynamics, atmosphere, terrain, etc.
- **Continuous Integration:** Automated regression- and unit-testing at core software and project levels
- **Developers-as-Users:** Core development team integrated with D205 users

Key Applications (Pre-Phase A → Phase F)

- Conceptual/research vehicle and trajectory design
- Flight vehicle design & integrated performance assessment (prime & IV&V)
- End-to-end trajectory design & optimization
- Integrated GN&C/FSW design & assessment with subsystem interaction and fault injection
- Post-flight reconciliation

Selected Program Support

- Mars EDL: All NASA Mars entry systems since Pathfinder '97
- Mars Sample Return Campaign: SRL, EES, & MAV
- Human Spaceflight Insight & IV&V: HLS, CCP, SLS
- Numerous architecture studies and conceptual/research efforts



1. Quadratically Constrained Quadratic Problem (QCQP)

- Normalizes inputs received from POST2
- Formulates discretized trajectory into 35 temporal nodes
- Forms linearized equality constraints based on equations of motion
- Forms linearized inequality constraints based on user defined input (e.g. final altitude, attitude, structural loading, & heat flux)
- Upfront computation required (~2.5 minutes)

2. Customized Alternating Direction Method of Multipliers (ADMM)

- Transform into consensus constrained problem
- Formulate Augmented Lagrangian function
- Objective Function
- Consensus Constraint Equality Constraint
- Inequality Constraint

3. Classical ADMM

- Primal (x,y) and Dual Updates (v,μ,λ)
- Update Weighing Factors (ρ1, ρ2, ρ3)
- Check Convergence & Iterate

4. Compute Commands from Converged Nodal Solution

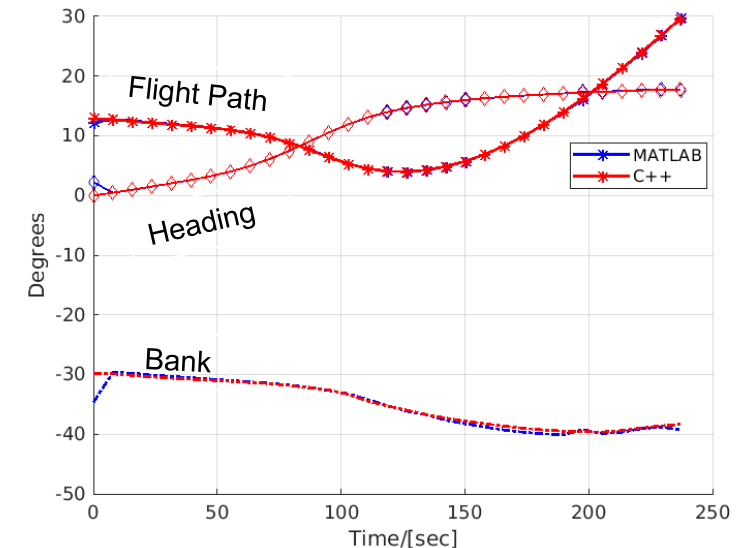
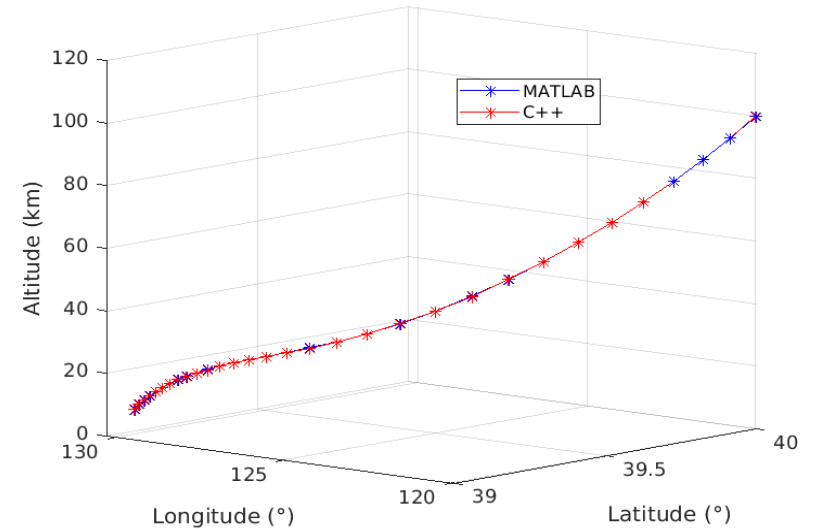
- Unnormalize Solution Vector
- Interpolate & Pass Commands to POST2

$$\begin{aligned}
 &\min_{\mathbf{x} \in \mathbb{R}^n} \mathbf{x}^T \mathbf{Q}_0 \mathbf{x} \\
 &\text{subject to } \mathbf{x}^T \mathbf{Q}_i \mathbf{x} = c_i, i \in \mathcal{E} \\
 &\quad \mathbf{x}^T \mathbf{P}_j \mathbf{x} \leq d_j, j \in \mathcal{I}
 \end{aligned}
 \quad \longrightarrow \quad
 \begin{aligned}
 &\min_{\mathbf{x}, \mathbf{y}} \mathbf{x}^T \mathbf{Q}_0 \mathbf{y} \\
 &\text{subject to } \mathbf{x}^T \mathbf{Q}_i \mathbf{y} = c_i, i \in \mathcal{E} \\
 &\quad \mathbf{x}^T \mathbf{P}_j \mathbf{y} \leq d_j, j \in \mathcal{I} \\
 &\quad \mathbf{x} = \mathbf{y}
 \end{aligned}$$

Consensus Constrained Problem

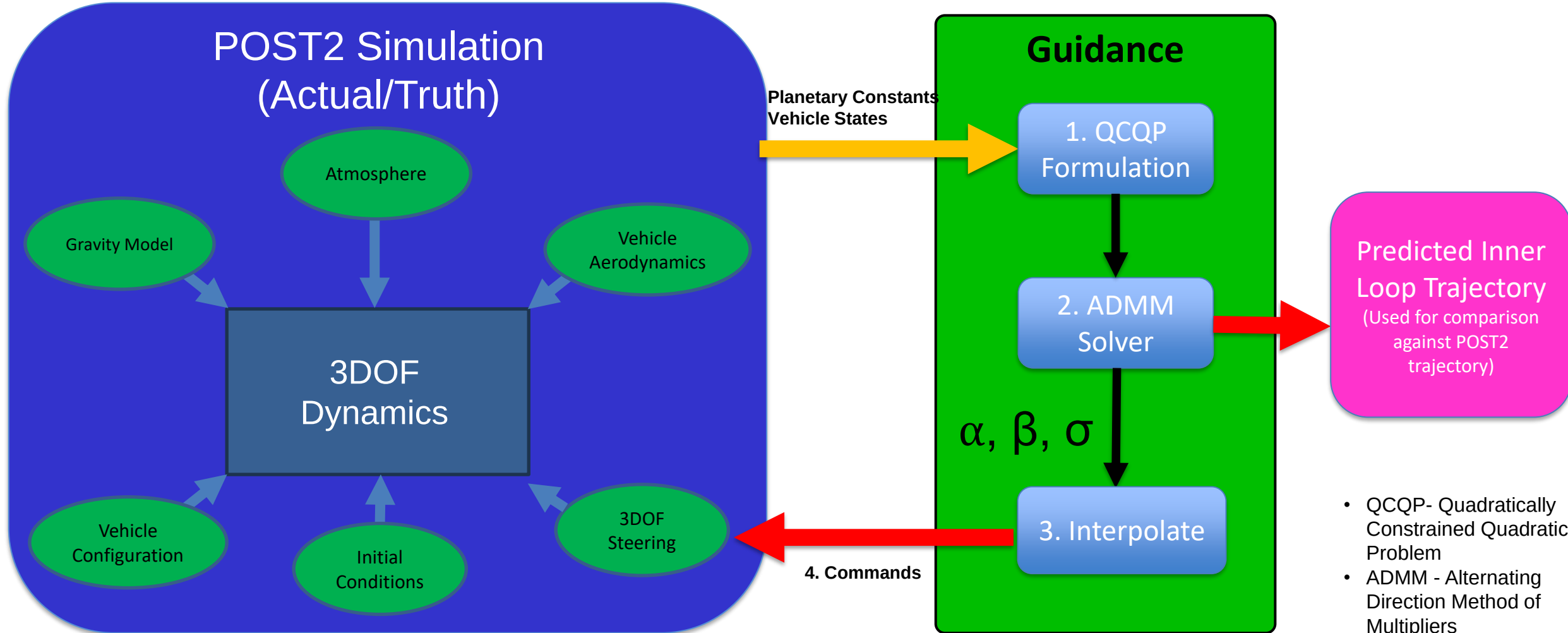
$$\begin{aligned}
 \mathcal{L}_p(\mathbf{x}, \mathbf{y}, \boldsymbol{\theta}) = & \boxed{\mathbf{x}^T \mathbf{Q}_0 \mathbf{y}} \quad \text{Objective Function} \\
 & + \boxed{v(\mathbf{x} - \mathbf{y}) + \frac{\rho_1}{2} \|\mathbf{x} - \mathbf{y}\|^2} \quad \text{Consensus Constraint} \\
 & + \boxed{\sum_{i \in \mathcal{E}} (\mu_i (\mathbf{x}^T \mathbf{Q}_i \mathbf{y} - c_i) + \frac{\rho_2}{2} \|\mathbf{x}^T \mathbf{Q}_i \mathbf{y} - c_i\|^2)} \quad \text{Equality Constraint} \\
 & + \boxed{\sum_{j \in \mathcal{I}} f_{\rho_3}(\lambda_j, \mathbf{x}^T \mathbf{P}_j \mathbf{y} - d_j)} \quad \text{Inequality Constraint}
 \end{aligned}$$

- **Guidance provided as MATLAB scripts**
 - NASA translated to C++ language using Armadillo library to facilitate extensive matrix math
 - Initial translation: Hayden Dean (Georgia Tech)
 - Additional translation and testing: Natalia Marchini (Georgia Tech)
 - Standalone C/C++ implementation run with provided trajectory, used to verify implementation against original MATLAB solutions
 - Did not implement modifications, e.g., to permit different number of nodes
- **Implementation Challenges**
 - Performing matrix operation on very large matrices ($\sim 3000 \times 3000$) in C/C++
 - Debugging code utilizing Armadillo libraries
 - Coordinate frames, commanded angles



Implementation	ADMM Iterations	ADMM Wall Time	Total Wall Time
MATLAB	126	130 s	285 s
POST2 / C++	124	138 s	304 s

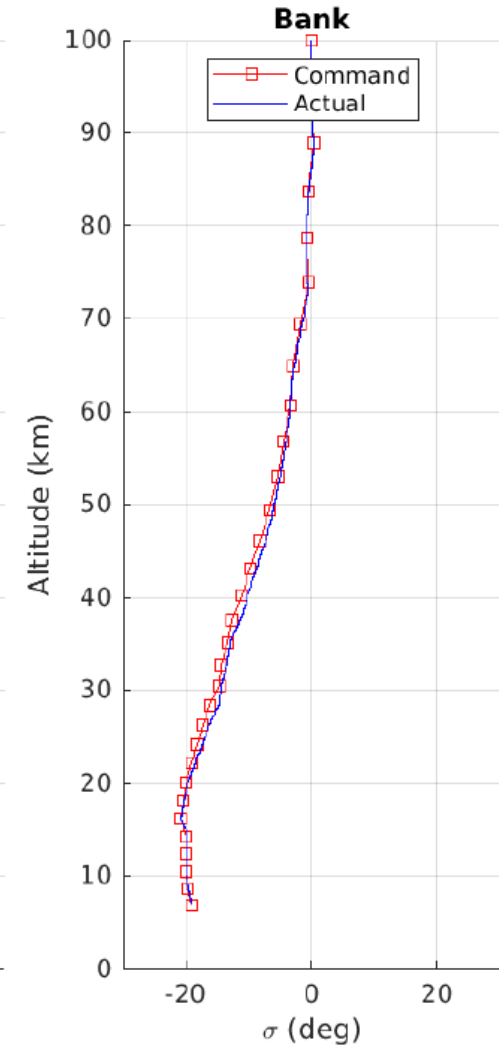
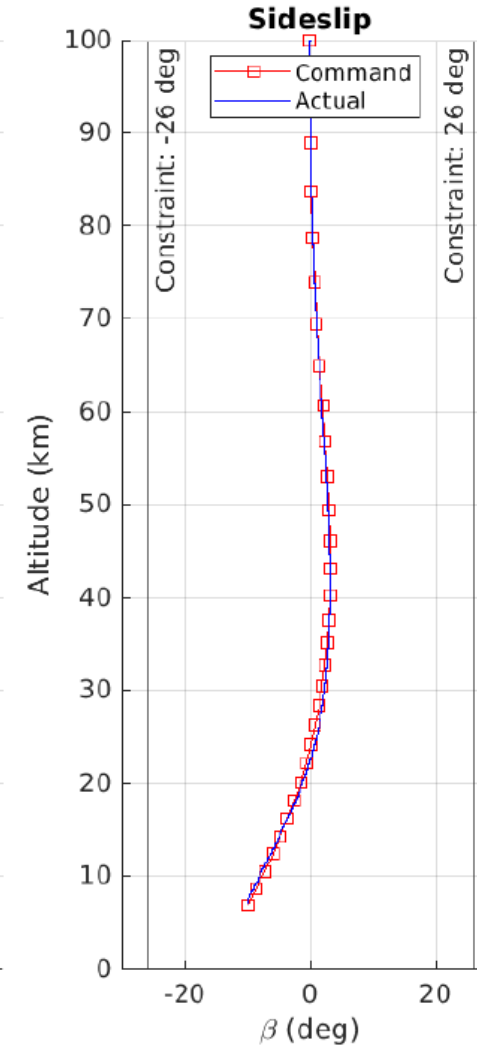
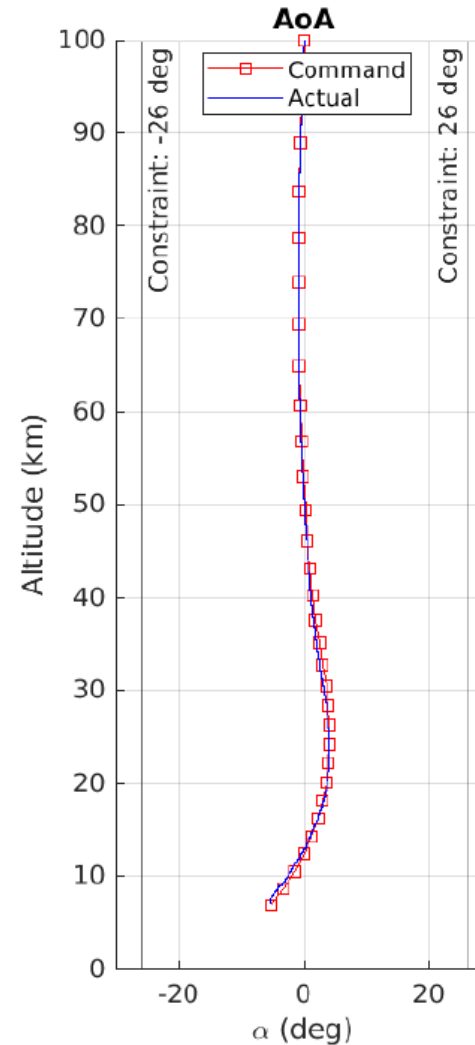
Guidance Flow Diagram



- QCQP- Quadratically Constrained Quadratic Problem
- ADMM - Alternating Direction Method of Multipliers
- α - Angle of Attack
- β - Sideslip Angle
- σ - Bank Angle

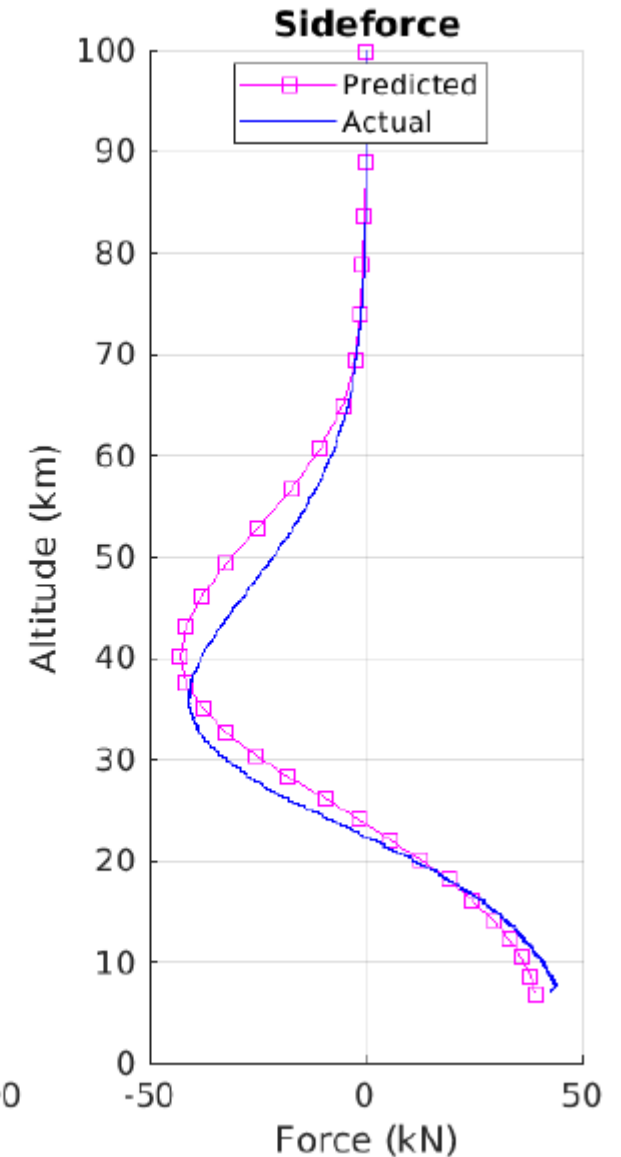
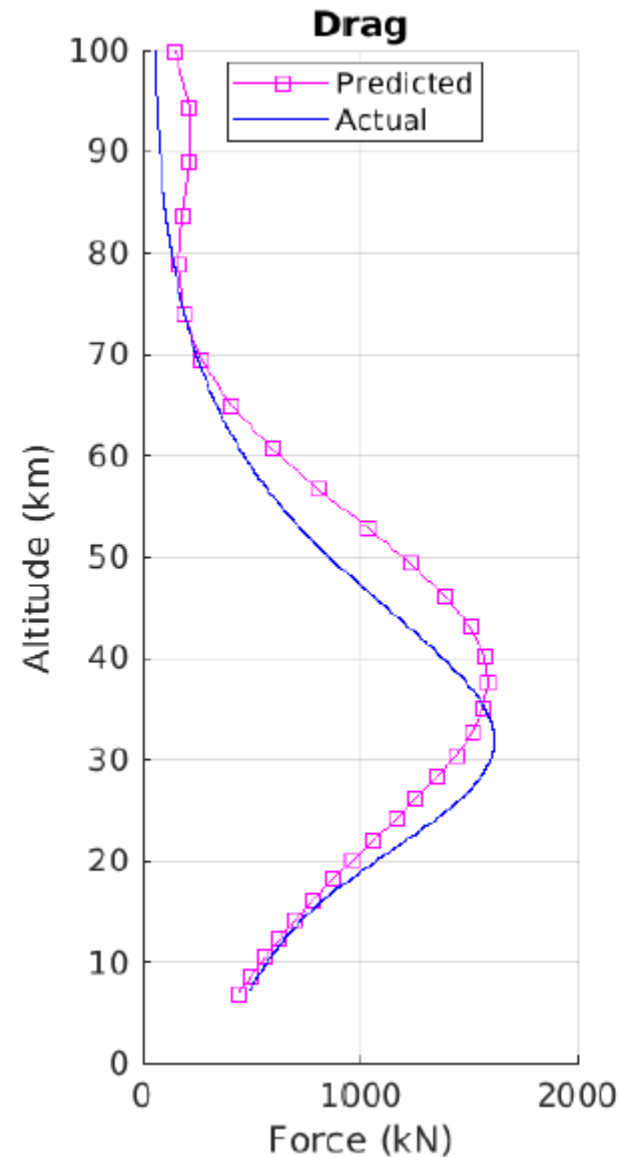
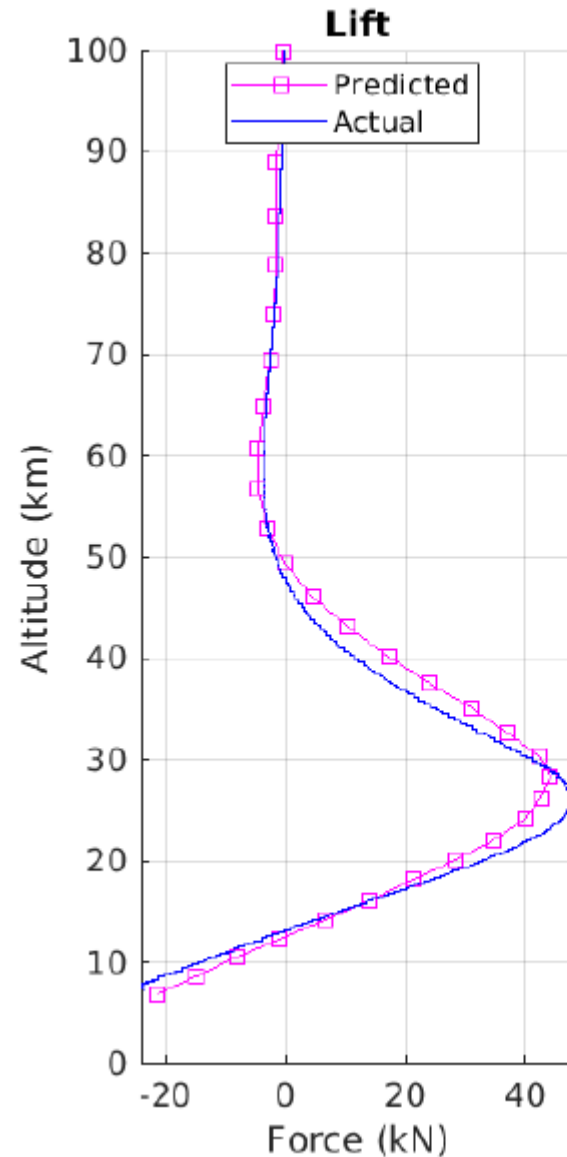
Results: Angles

- **Application: Human-scale Mars EDL with inflatable low L/D entry system**
- **Solution (prediction) from ADMM solver (inner loop) are used as commands to the POST2 steering logic (outer loop)**
 - Solution is generated once
 - Angle of attack (α)
 - Sideslip angle (β)
 - Bank angle (σ)
 - Controls direction of lift about the velocity vector
- **Goals of attitude constraints**
 - Avoid high asymmetric loading that could cause inflatable aeroshell to buckle
 - Avoid flow impingement and radiative heating effects on the payload at high angles of attack

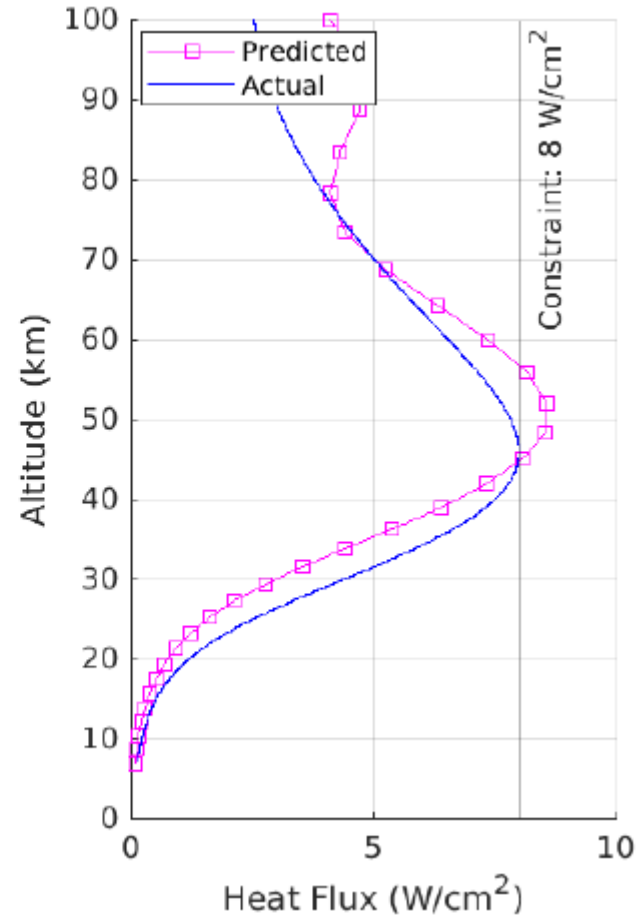
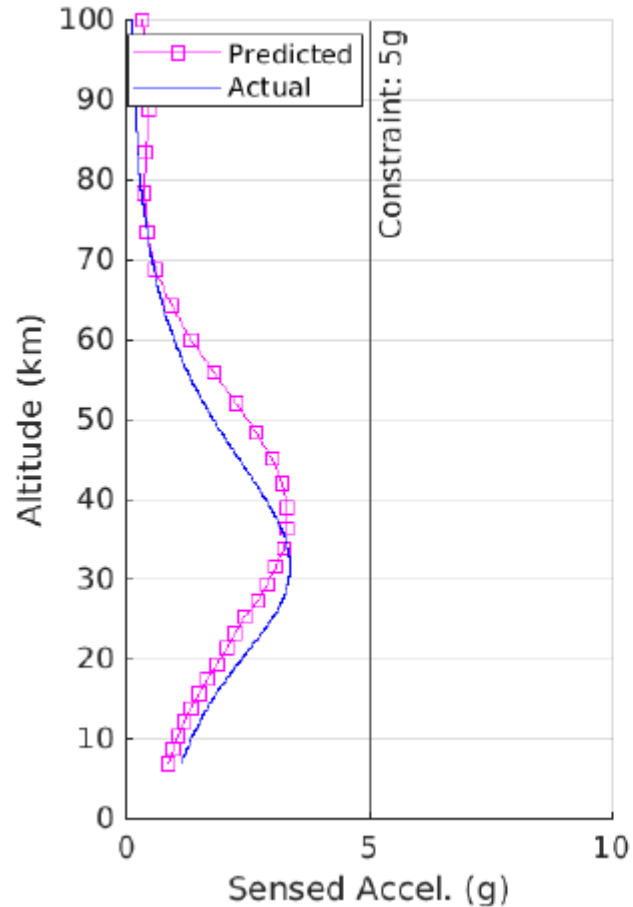


Results: Aerodynamic Forces

- Inner loop model: polynomials
- Outer loop model: Inner loop polynomials converted to lookup tables
- Functions of angle of attack and sideslip angle



Results: Constraints on Structural Loading and Heating

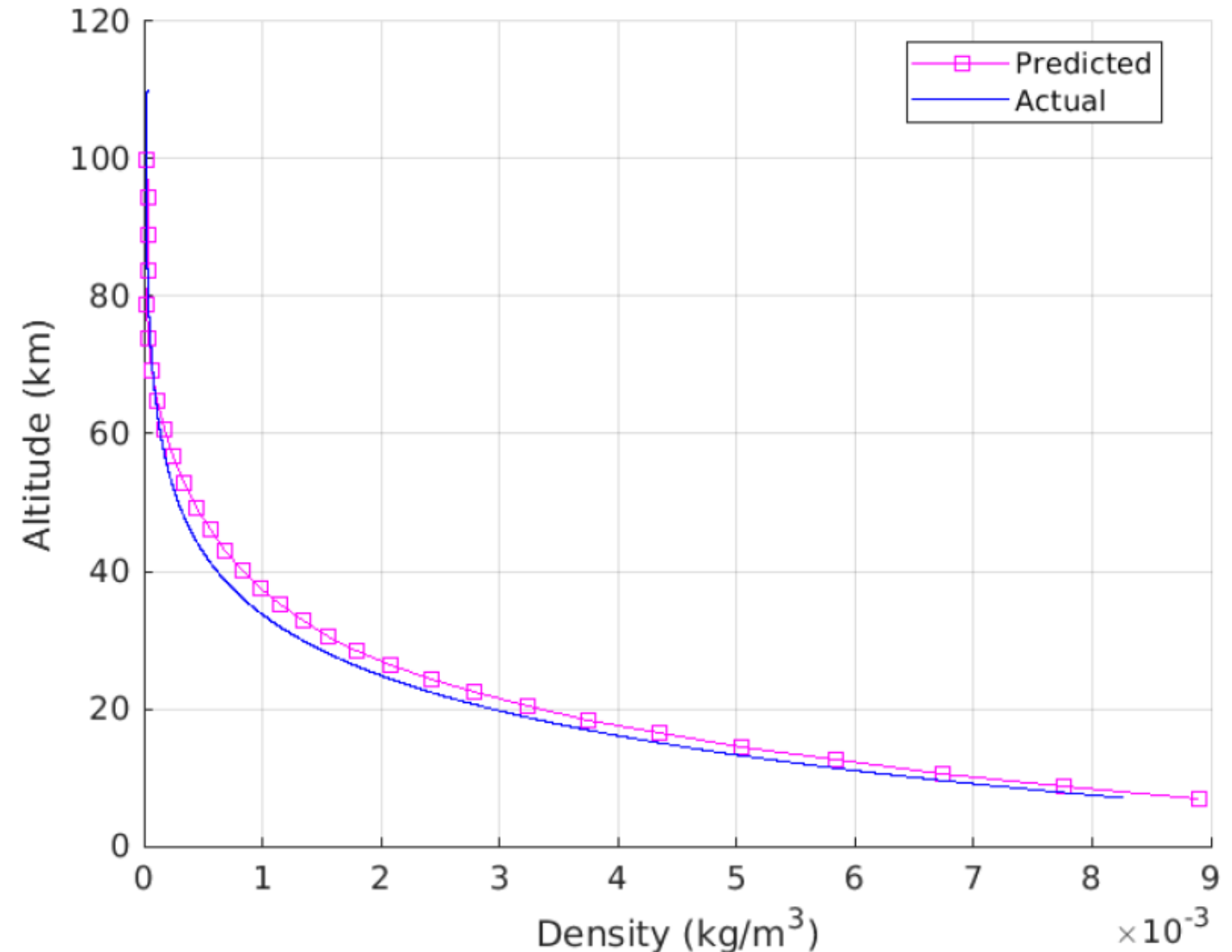


	Peak Value		
	Actual	Predicted	Constraint
Sensed Acceleration	3.37g	3.31g	5g
Heat Flux	7.99 W/cm ²	8.57 W/cm ²	8 W/cm ²

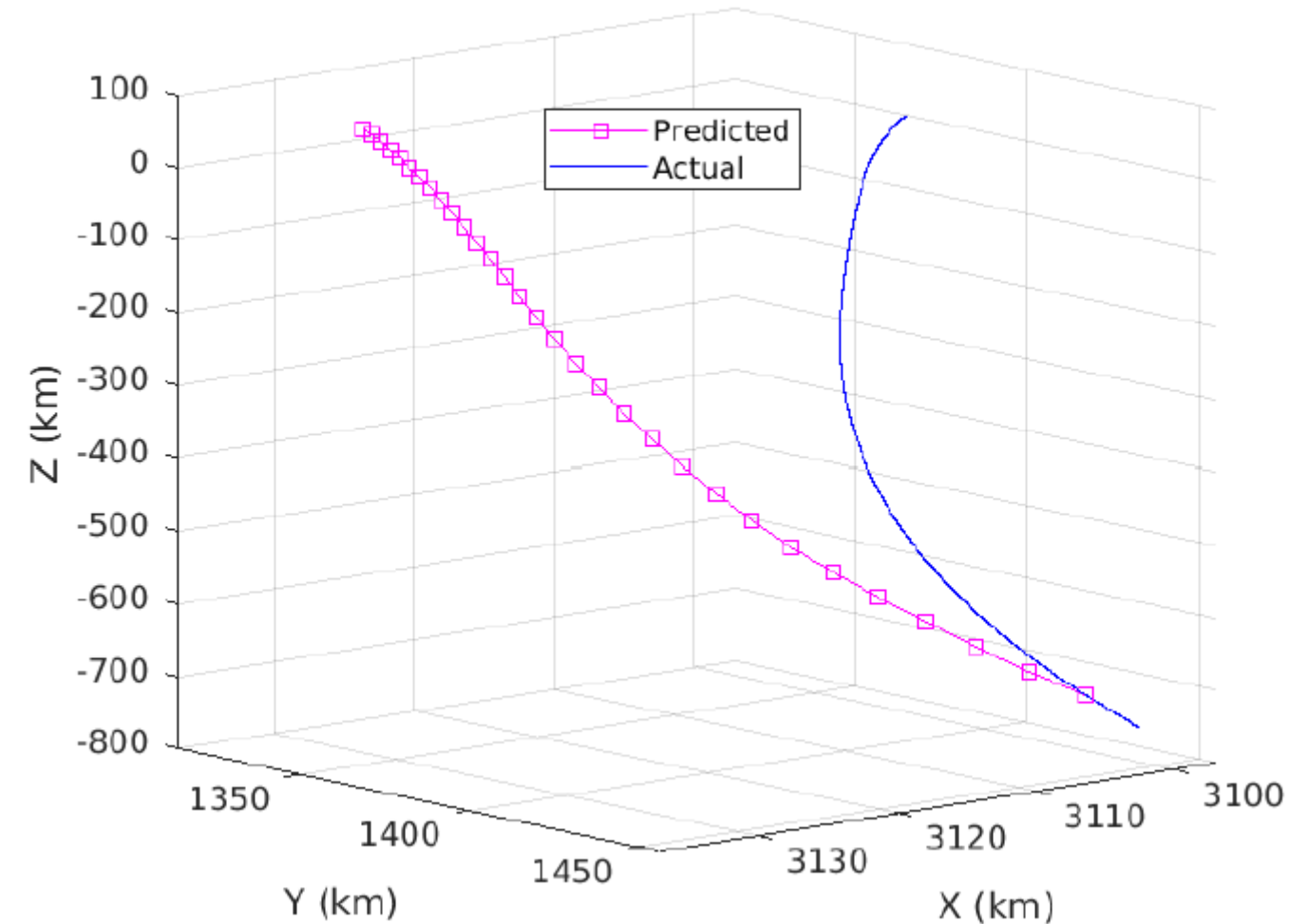
*Tolerance is input as single scalar compared to max percent error of normalized equality, inequality, and consensus constraint.

Results: Density vs. Altitude

- **Density**
 - Inner loop model: 7th order polynomial
 - Outer loop model: Inner loop polynomials converted to lookup tables
- **Variation in density profile**
 - Leading cause of variation in peak g's and heat flux
 - Variation in density profile causing "Predicted" peak forces to occur at higher altitudes
- **Recall: predicted trajectory is computed only once**
 - In-flight, a new trajectory would likely be computed at/near powered descent initiation (i.e., divert)



Planet-Centered Relative



	Absolute Difference at Simulation End (km)
X Position Difference	25.08
Y Position Difference	57.48
Z Position Difference	0.75
Magnitude Difference	62.72

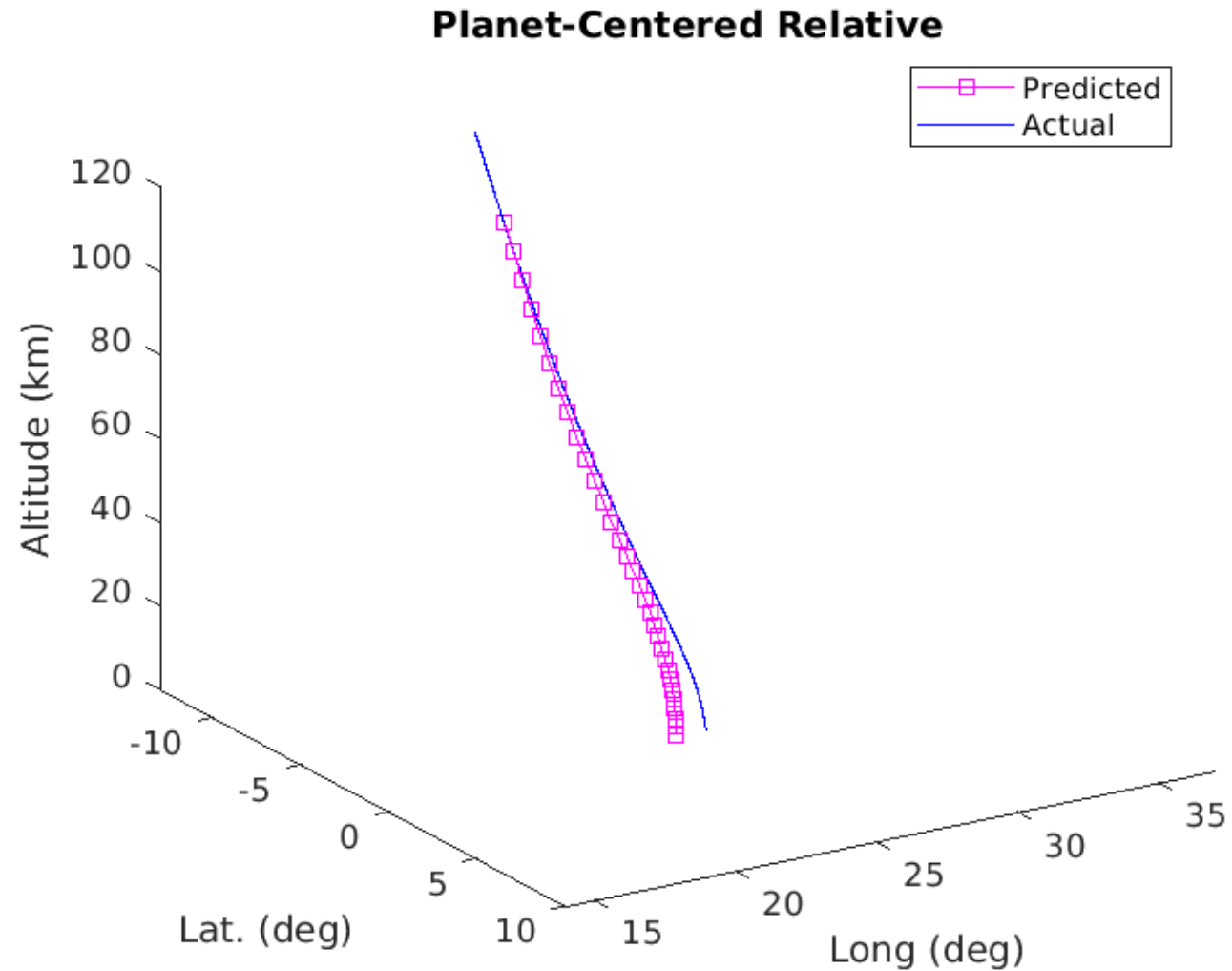
- Recall:
 - Problem formulated with no constraint in final lat/long
 - Final altitude constrained
- Disagreement between predicted and actual trajectory could be due to:
 - Coordinate frame transformations
 - Planet rotation models
 - Different integration schemes

- **Delivered guidance algorithm translated to C++ and implemented in POST2 simulation environment, for application to NASA reference Mars EDL human-scale mission**
 - C++ implementation showed good agreement with original MATLAB code when run with original problem inputs
 - Significant differences observed between as-flown and predicted trajectories when integrated with POST2 and applied to NASA mission
 - Coordinate frame, planet rotation and integration scheme differences
 - Density profile differences
 - ADMM algorithm iterates until solution for all N number of nodes is within user defined tolerance
 - This can lead to a trajectory that is not physically feasible if max number of iterations hit
- **Forward Work**
 - Change outer loop aerodynamics and atmosphere models to MSL aero and GRAM, respectively -- Polynomials help simplify the problem, but may be too oversimplified
 - Switch to 6DOF simulation with navigation errors from simulated navigation sensors [5]

References

- [1] Wan, C., Pei, C., Dai, R., Jing, G., and Rea, J. R., “Six Dimensional Atmosphere Entry Guidance based on Dual Quaternion,” AIAA 2021-050
- [2] Cianciolo, A. D., Dillman, R., Brune, Lugo, R. A., Polsgrove, T. T., Percy T. K., Sutherlin, S., A., Johnson, C., and Cassell, A., “Human Mars Entry, Descent and Landing Architecture Study: Deployable Decelerators,” AIAA Paper 2018.
- [3] Cianciolo, A. D., Polsgrove, T. T., Sostaric, R. R., Edquist, K. T., Korzun, A. M., and Garcia, J. A., "Human Mars Entry, Descent, and Landing Architecture Study: Phase 3 Summary," AIAA 2020-1509. AIAA Scitech 2020 Forum. January 2020.
- [4] Sandoval, S.A., Rafael L.A., Lu, P., Cianciolo, A.S. “Simulation Comparison of Entry and Powered Descent Guidance Algorithms for a Human-Scale Mars Mission”, AAS, Volume 179
- [5] Lugo, R. A., Dwyer-Cianciolo, A. M., Dutta, S., Williams, R. A., Pensado, A. Chen, P., “Integrated Precision Landing Performance Results for a Human-Scale Mars Landing System,” AIAA 2022-0607.

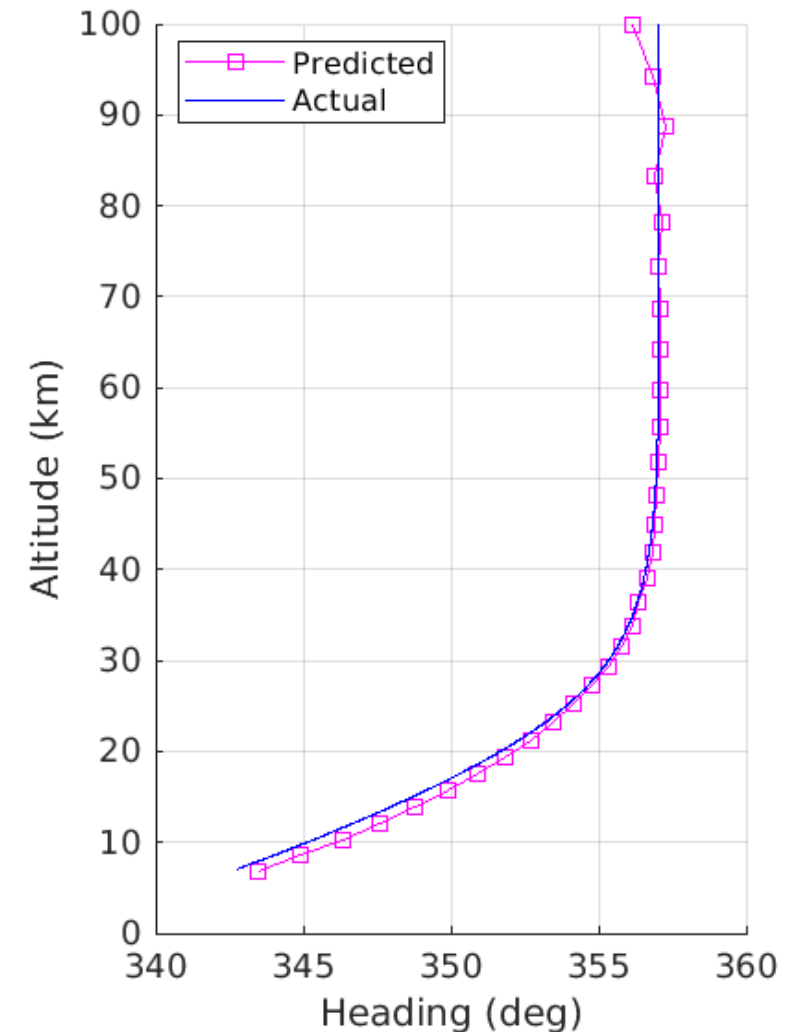
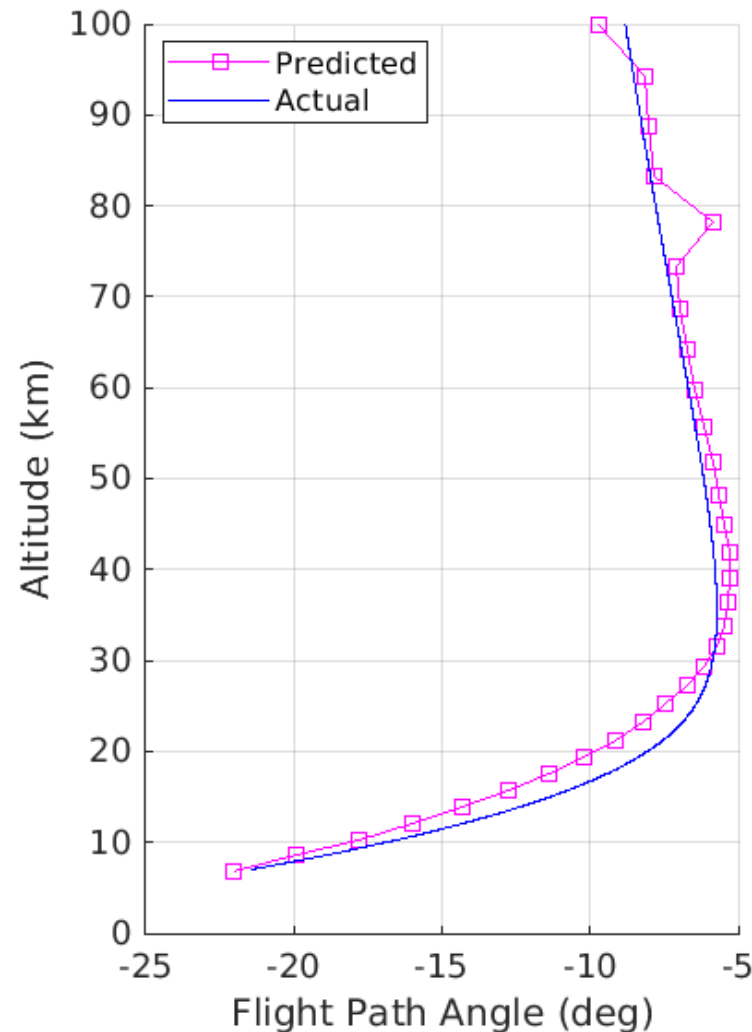
- Plot shows that vehicle is on a descending trajectory traveling North by Northwest (NNW)



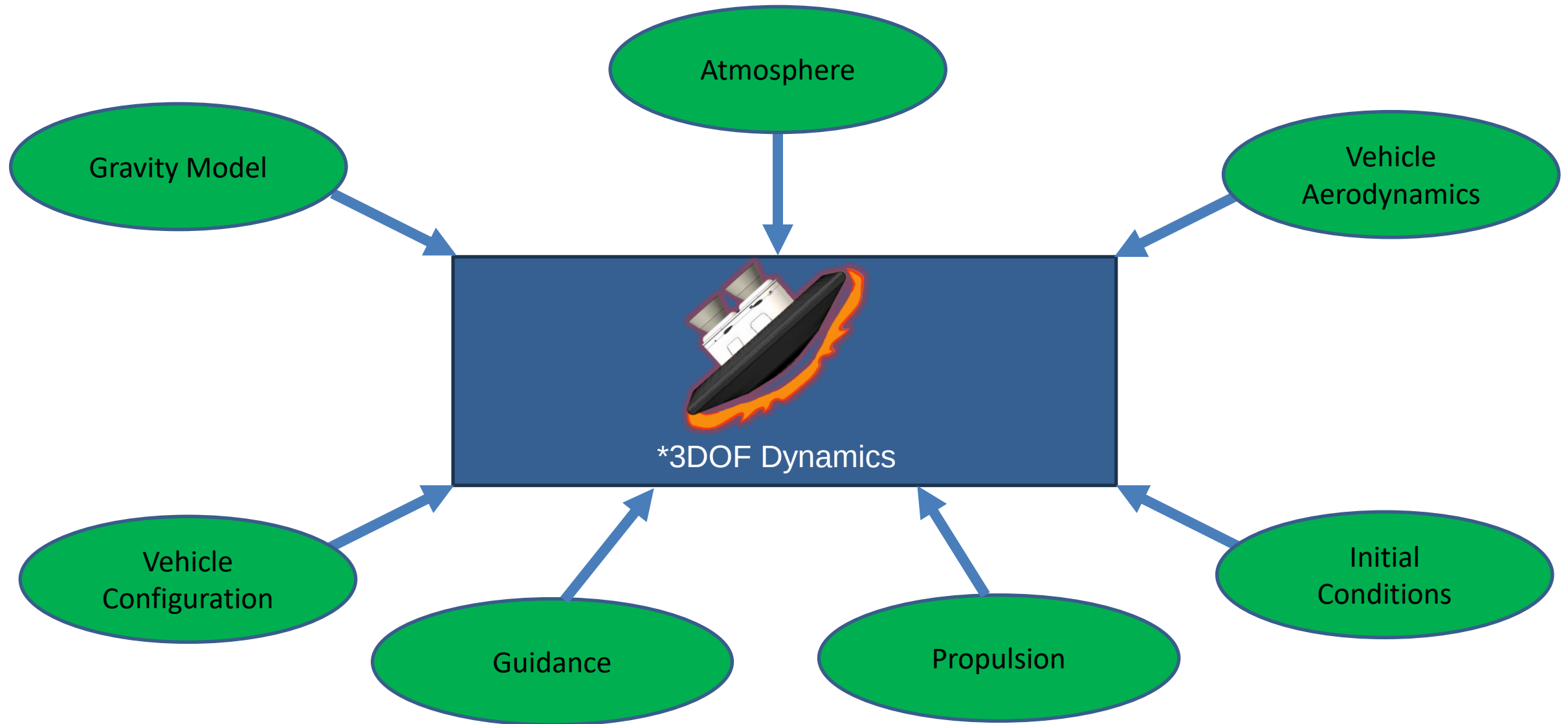
Results

Flight Path Angle & Heading

- Flight path and heading angle plots support trajectory profile shown on previous slide
 - Negative flight path angle profile implies vehicle is on a descending trajectory
 - Heading angle profile implies vehicle is traveling in a NNW direction
- Would expect bigger deviation in heading angle trend lines between actual and predicted trajectory if incorrect bank angle transformation was performed
- Suspect large position difference caused by differences in modeling planet rotation

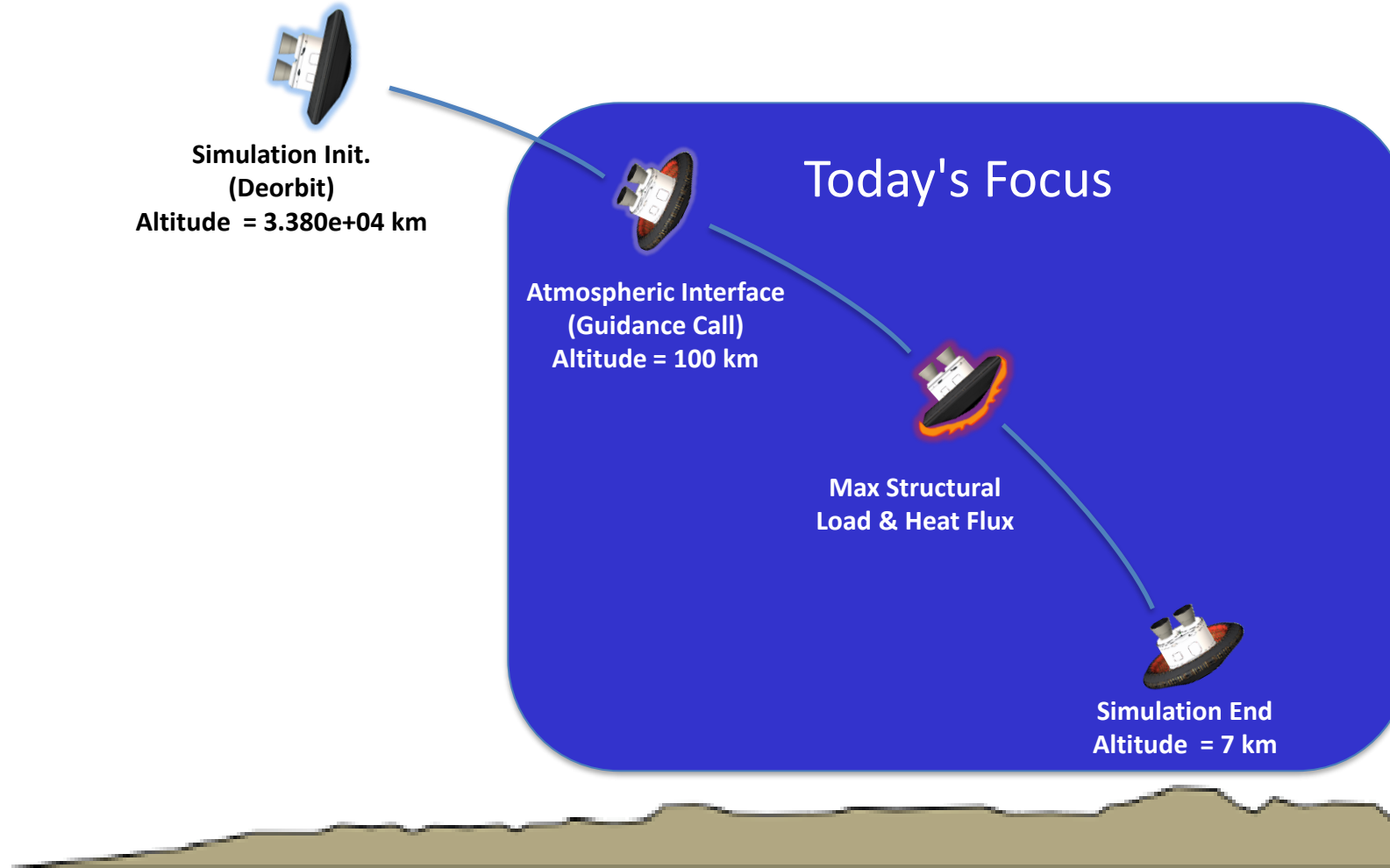


Modeling with Program to Optimize Simulated Trajectories II



CONOPS Overview

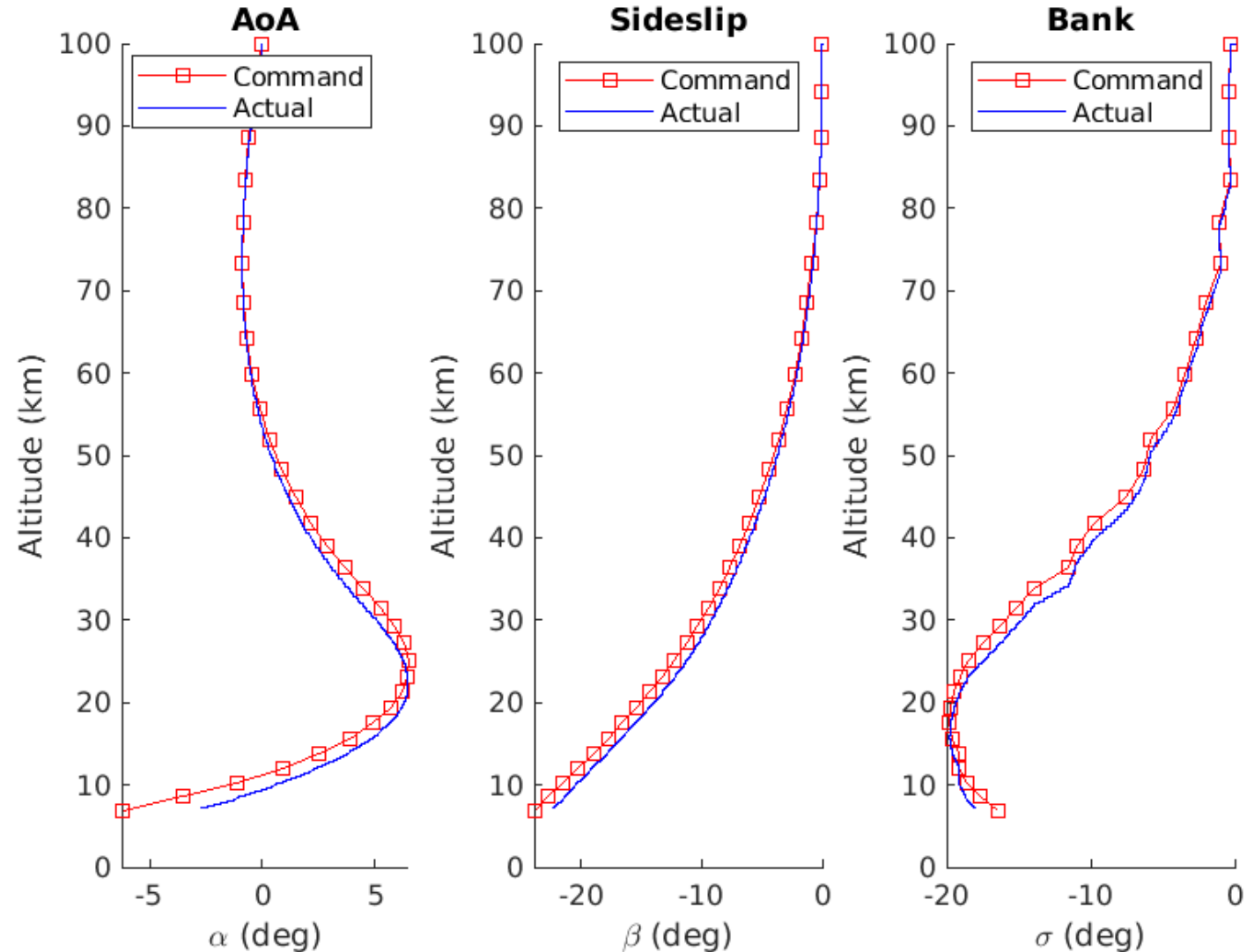
(Not to Scale)



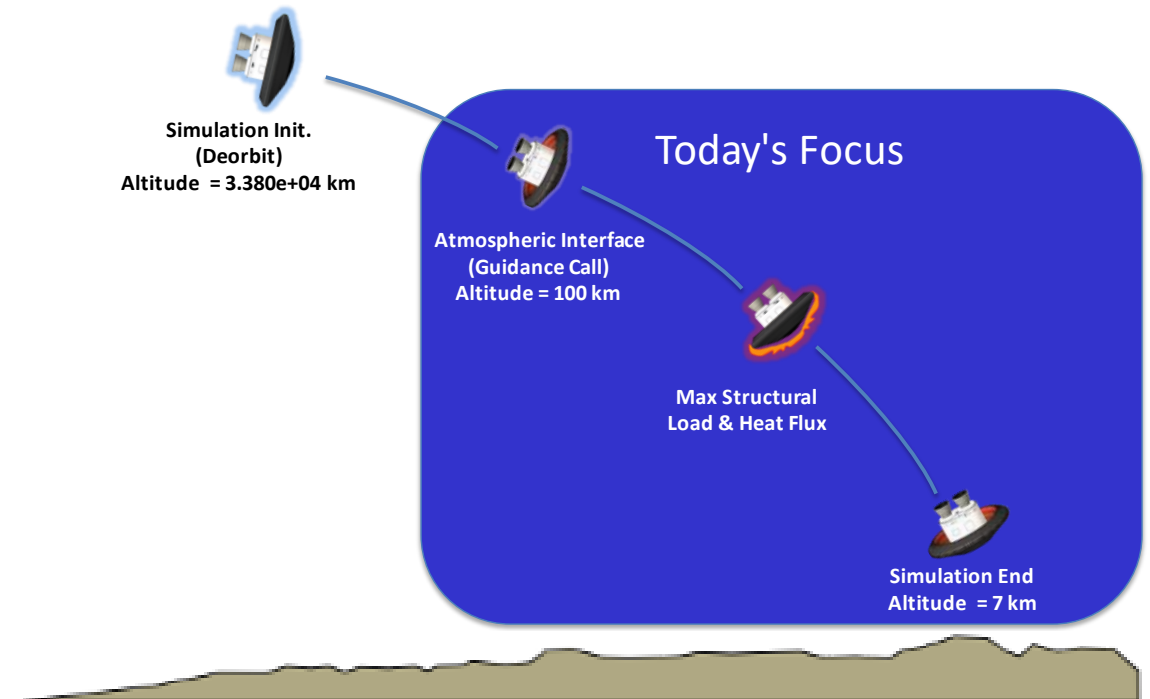
Results

Actuator Commands

- Solution from ADMM solver (inner loop) are used as commands to the POST2 steering logic (outer loop)
 - Angle of Attack (α)
 - Sideslip Angle (β)
 - Bank Angle (σ)
 - Controls direction of the lift around the velocity vector
 - Angle of attack and bank are relatively small
 - Sideslip is large (20+ deg)
 - See next slide



- **2018 Early Career Faculty (ECF) made three awards in field of advanced EDL guidance algorithms, including to Dr. Ran Dai of Purdue**
- **Dual-Quaternion based Guidance**
 - Utilizes equations of motion for translational and rotational motion based on dual-quaternions
 - **Consists of three distinct steps**
 1. **Quadratically Constrained Quadratic Problem (QCQP)**
 - Normalizes inputs received from POST2
 - Formulates discretized trajectory into N temporal nodes
 - Forms linearized equality constraints based on equations of motion
 - Forms linearized inequality constraints based on user defined input (e.g. final altitude, attitude, structural loading, & heat flux)
 - Large upfront computation required (~2.5 minutes)



2. Customized Alternating Direction Method of Multipliers (ADMM) [1]

1. Transform into consensus constrained problem
2. Formulate Augmented Lagrangian function

- Objective Function
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4. Check Convergence & Iterate as Necessary

3. Compute Commands from Converged Nodal Solution

1. Unnormalize Solution Vector
2. Interpolate & Pass Commands to POST2

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 \end{aligned}$$

Consensus Constrained Problem

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 \end{aligned}$$