

# CUMULATIVE DISTRIBUTION OVERLAP TECHNIQUE FOR ARTEMIS MISSION PUBLIC ENTRY RISK ASSESSMENT

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The Artemis missions use a skip-entry profile to accomplish a recovery near the western US coastline, however, the service module debris must still be disposed of safely. For certain contingency return scenarios, the expected dispersed entry profile is biased from the well-analyzed corridor. A novel method is presented for assessing the Cumulative Distribution Function (CDF) of these biased results to ensure they do not contribute additional probabilistic risk to the public. More specifically, the weight of the flight path angle dispersion “tail” of the CDF needs to remain below the well-assessed region. The details of how this method was actually used to screen and plan entry profiles for specific launch windows during the Artemis I mission are shown. This method has potential applicability in other problems where a biased distribution needs to be assessed against a well-defined “core” region.

## INTRODUCTION

The successful Artemis I mission flew in 2022 as an uncrewed lunar test flight for NASA’s Orion Multi-Purpose Crew Vehicle and as a precursor for the Artemis exploration campaign. Re-entry of the vehicle occurs over the Pacific Ocean, using a skip-entry profile to target a nominal crew module landing site near San Diego. The Orion service module is jettisoned shortly before re-entry and breaks up in the atmosphere for a safe disposal in the Pacific Ocean as seen in Figure 1. Extensive pre-flight analysis was performed on the allowable entry interface conditions that would accomplish this breakup safely. More details can be found in McPherson.<sup>1</sup> However, simulation runs in the months just prior to launch showed that the autonomous return system would provide a biased entry condition on certain launch days versus the nominal aim point.

## OFF-NOMINAL PROBLEM

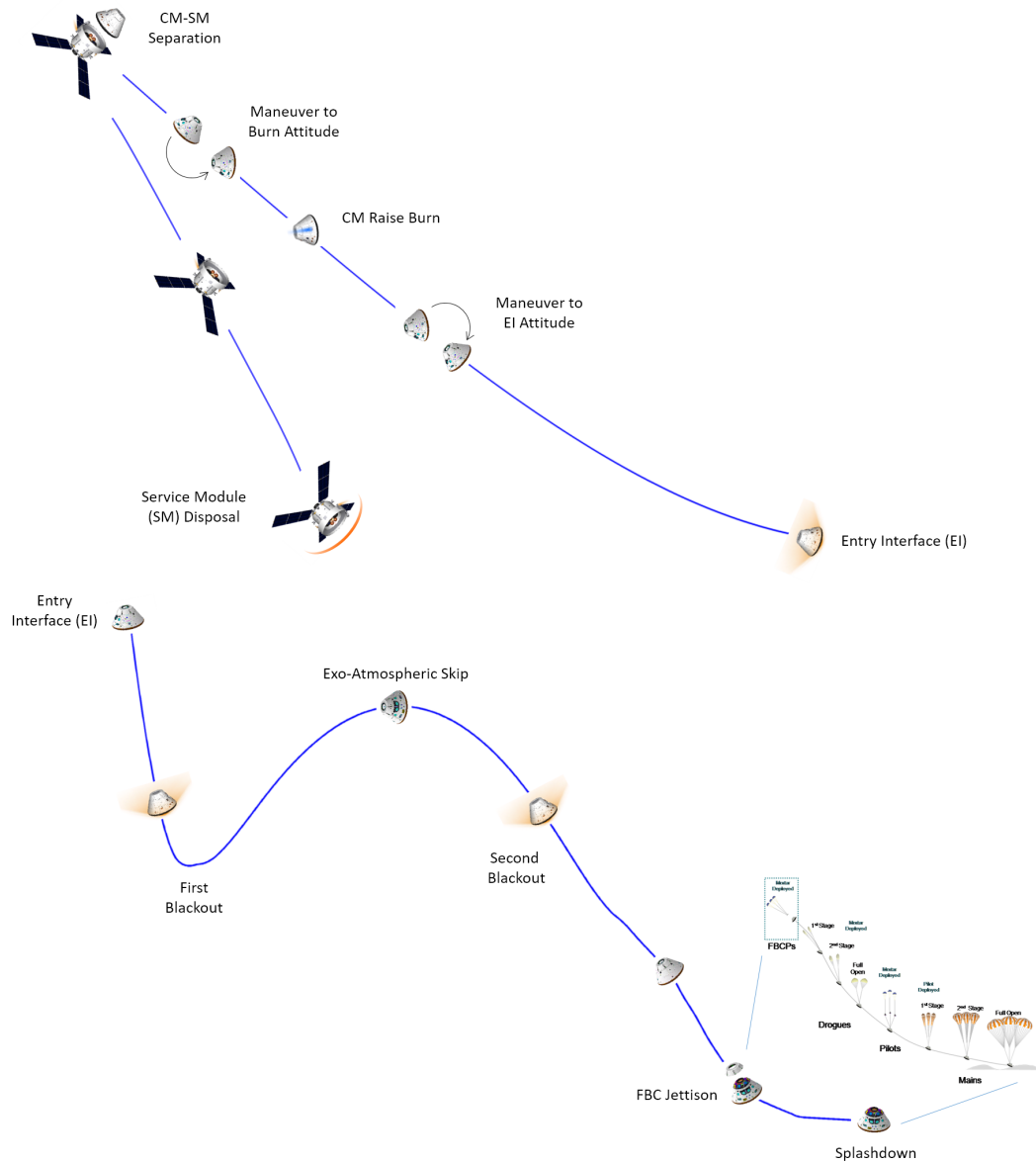
If Orion loses communications with Earth during the Artemis I mission, it is designed to autonomously return using Optical Navigation and onboard guidance/targeting. This, then, was the interesting problem and situation faced by the team preparing for the flight. A large amount of analysis had been performed during early mission planning with a zero-mean distribution, but some of these contingency return scenarios resulted in system performance that was nonzero mean. Redoing the original analysis would have been very time-consuming and costly. The challenge was to evaluate the biased results using the previously assessed and cleared limits from the zero-mean analysis. In this case, the biased distribution was the Entry Interface conditions of the service module, an example of which is seen in Figure 2.

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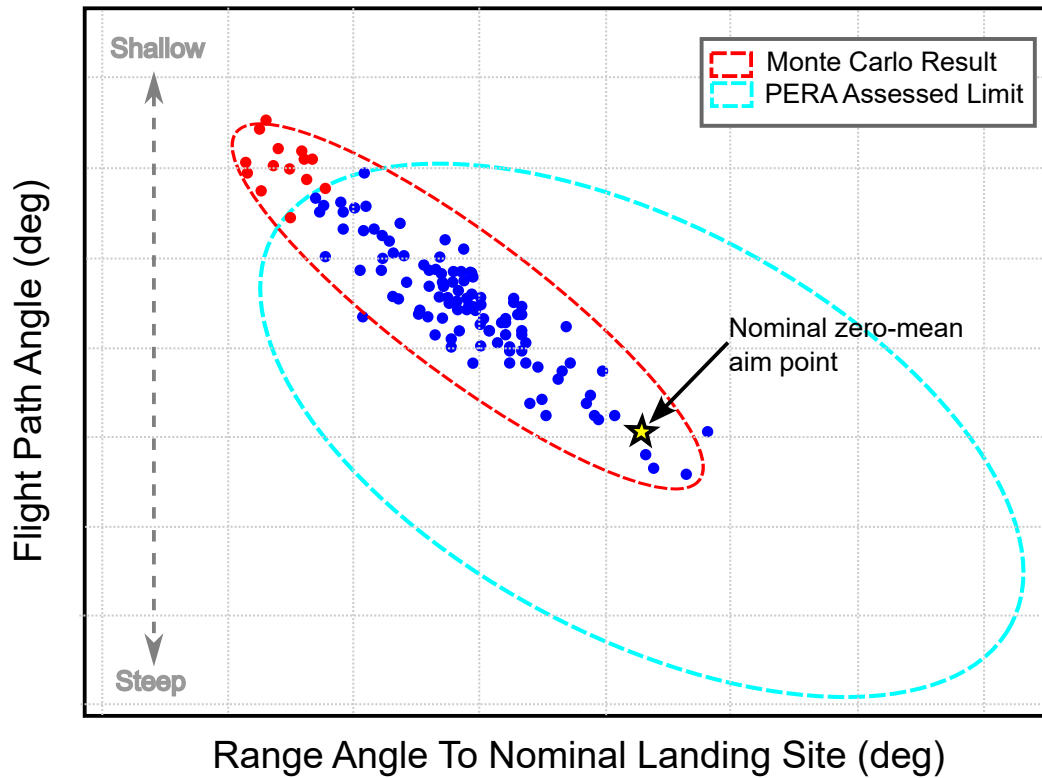
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**Figure 1 Entry Separation Profile**

Preflight Monte Carlo analysis of the contingency loss of communication scenario showed this biased Entry Interface dispersion, while the nominal case did not. However, shifting the aim point would affect both the nominal and contingency cases. Understandably, mission managers did not want to change the nominal Entry Interface aim point solely due to the low-likelihood contingency case. So the flight dynamics and GNC team went to work to understand the unique characteristics of this problem. Again, the chief concern was how to evaluate the risk of a biased dispersion when scored against a zero-mean criteria. The result would either clear the trajectory for use or prescribe the absolute minimum amount of nominal aim shift to protect for the contingency.

For the problem of service module debris, we get a Monte Carlo statistical result at Entry Interface



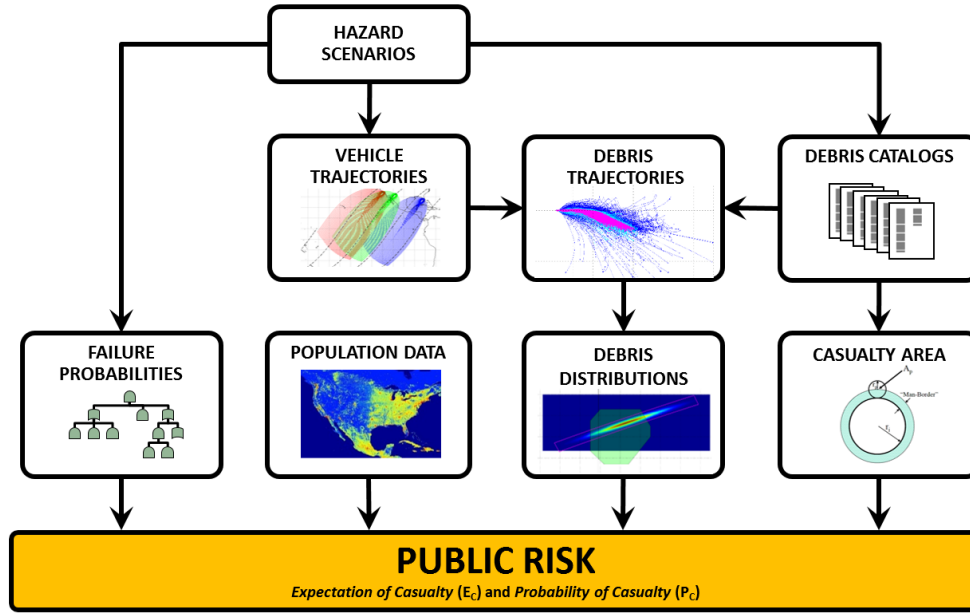
**Figure 2 Example of Biased Monte Carlo Results Against Zero Mean Requirement**

that has some mean bias ( $\mu$ ) and standard deviation ( $\sigma$ ). We compare that against a Public Entry Risk Assessment (PERA) limit based on a zero-mean (unbiased) distribution. Intuitively, we would say that if the Monte Carlo mean is “close enough” to zero, the comparison should be valid, but the difficulty comes in quantifying that intuition. So for this problem, we assume the flight path angle (FPA) is the main driver, and more specifically the shallow “tail” of the FPA distribution that would result in service module debris traveling further downrange into populated regions. This lends itself to a one-dimensional analysis problem focusing on the FPA. Additionally, if the shallow side is most important, the focus will be on the side of the distribution of FPAs less than zero. So the final difficulty then becomes agreeing on a value which constitutes the “tail” (as opposed to the “core” of the distribution). The approach taken was to define this “core” region using the nominal, zero-mean analysis  $3\sigma$  value. The nominal  $1\sigma$  value is 0.04 deg, giving a “core” limit of 0.12 degrees. That  $3\sigma$  value has been used and analyzed extensively, such that any number of points that fall within that region are not concerning. Our task then becomes, for any given Monte Carlo statistical output, to show that we are not introducing any more cumulative probability distribution outside of this “core” region. The PERA limit itself is set at 4 times the nominal for a  $1\sigma$  value of 0.16 degrees.

## STATISTICAL APPROACH

The Public Entry Risk Assessment (PERA) process is an analysis methodology intended to provide a quantifiable estimate of the risk to the public associated with atmospheric entry of a spacecraft. Fundamentally it comprises a number of functional modules that each form a key part of the entire analysis process. Some of these can be performed serially, others in parallel, but all are needed

in order to compute the risk metrics necessary to satisfy NASA Range Flight Safety requirements.<sup>2</sup> The public entry risk assessment process is summarized in Figure 3.



**Figure 3 Public Entry Risk Assessment Process**

The unique circumstances of this problem provide a well-studied, high-confidence core region of the distribution. Additionally, only one side of the parameter space was concerning to “miss”, in our case the shallow side that would result in service module debris traveling further downrange into populated regions. Extensive early analysis was performed on a zero-mean distribution, but certain simulations showed resulting distributions that were nonzero mean. Intuition tells us that Monte Carlo populations with smaller standard deviations but a shifted mean would still have most of their samples fall in the “core” region, with relatively small numbers extending into the “tail” region of concern. But how to quantify when the statistical risk is greater or less than the studied zero-mean case?

Consider a one-dimensional normal (Gaussian) distribution ( $\mathcal{N}(\mu, \sigma^2)$ ) of the form

$$f_G(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2} \frac{(x-\mu)^2}{\sigma^2}} \quad (1)$$

the cumulative distribution function is

$$F_G(x) = \frac{1}{2} \left[ 1 + \operatorname{erf} \left( \frac{x - \mu}{\sigma\sqrt{2}} \right) \right] \quad (2)$$

where the Gauss error function,  $\operatorname{erf}(x)$  is defined as

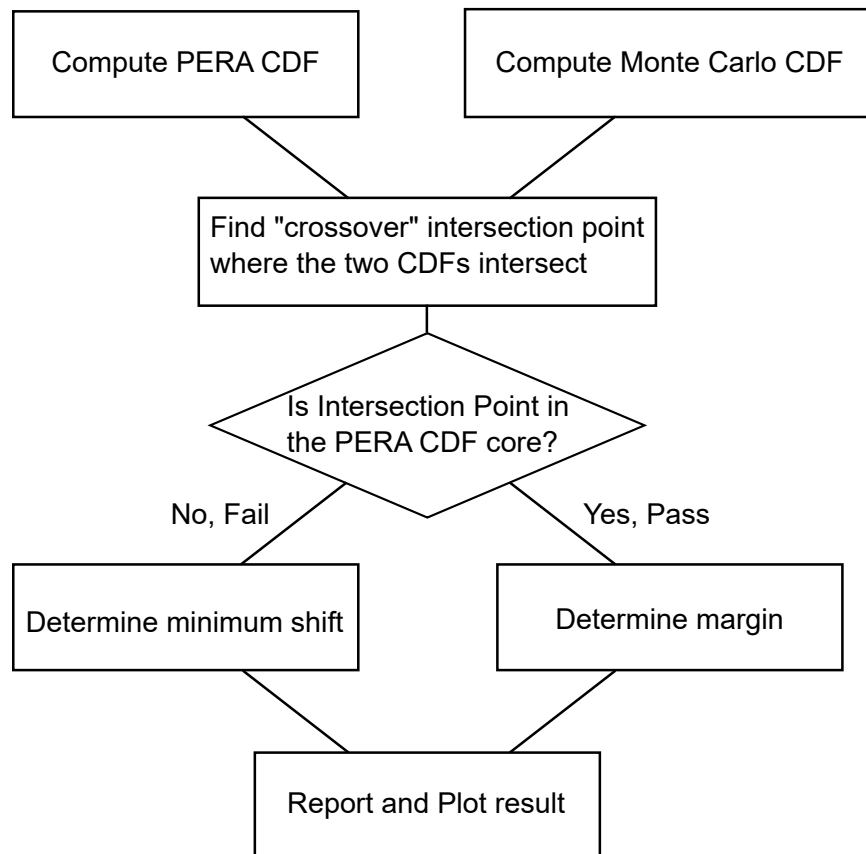
$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \quad (3)$$

If the resulting tail is below the analyzed CDF tail, then that represents margin. Conversely, if the resulting tail is above the analyzed tail, this indicates the smallest amount of bias shift that would be

required to bring the distribution into compliance. This strategy was in place for Artemis I mission in case the as-flown trajectory came back unfavorably, a biased flight path angle target would be commanded to the vehicle to aim steeper and avoid the tail violation.

## EVALUATION METHOD

First, the Monte Carlo results are scored relative to the nominal aim point for mean and standard deviation. Next, the CDF for the zero mean “scoring” distribution (PERA) and the CDF for the “test” (Monte Carlo) distribution are computed. The crossover point is found where the two CDFs intersect. If this intersection point is within the core, not the tail, of the scoring distribution then the test distribution adds no additional risk in this problem and there is margin in the scenario. However, if the intersection point is outside the core, in the tail, of the scoring distribution then the test distribution is adding additional risk and needs to be shifted to at least align the intersection point with the tail limit. This methodology is shown in Figure 4.



**Figure 4 Flowchart of Cumulative Distribution Function Comparison**

## RESULTS

In order to demonstrate this, a sample situation is shown for both a non-passing and passing case. Both cases have the same Monte Carlo sample mean, but one has more spread in the standard deviation. Recall again that the PERA limit is set at 4 times the nominal for a  $1\sigma$  value of 0.16

degrees. The non-passing case example is shown below.

$$\begin{aligned}
\mu_{\gamma_{PERA}} &= 0.0^\circ \\
\sigma_{\gamma_{PERA}} &= 0.16^\circ \\
\mu_{\gamma_{MC_{disp}}} &= -0.07^\circ \\
\sigma_{\gamma_{MC_{disp}}} &= 0.092^\circ
\end{aligned}$$

where  $\mu_{\gamma_{PERA}}, \sigma_{\gamma_{PERA}}$  are the mean and standard deviation associated with the PERA assessed FPA limits and  $\mu_{\gamma_{MC_{disp}}}, \sigma_{\gamma_{MC_{disp}}}$  are associated with the biased Monte Carlo simulator.

The result can be seen in Figure 5. Here the green zone represents allowable distribution intersections inside the 0.12 degree “core”, while the red zone represents the “tail”. Any result where the CDF stays below existing PERA analysis limit in the tail region is considered passing. This is an example of a “FAIL” result, with an intersection which is beyond the -0.12 degree tail limit. The corresponding shift is indicated that would be necessary to bring it back into compliance.

A passing case is demonstrated by another example given below.

$$\begin{aligned}
\mu_{\gamma_{PERA}} &= 0.0^\circ \\
\sigma_{\gamma_{PERA}} &= 0.16^\circ \\
\mu_{\gamma_{MC_{disp}}} &= -0.07^\circ \\
\sigma_{\gamma_{MC_{disp}}} &= 0.04^\circ
\end{aligned}$$

The plot of Figure 6 shows this example and its “PASS” result, having an intersection within the -0.12 degree tail limit. This plot also indicates the margin of the passing result for situational awareness. This result is noteworthy - a Monte Carlo result with the same sample mean is passing because it has a majority of its distribution weight in the “core” region.

## CONCLUSION

The outcome of this problem and study was a useful method for evaluating biased Monte Carlo results against a zero-mean limit under certain conditions. Intuitively, we would say that if a Monte Carlo statistical mean is “close enough” to zero, a comparison with a zero-mean limit should be valid, but the difficulty comes in quantifying that intuition. For this problem, we found that a Monte Carlo result with a majority of its distribution weight in the core region did not present additional risk and could be cleared for flight. This method has potential applicability in other problems where a biased distribution needs to be assessed against a well-defined core region.

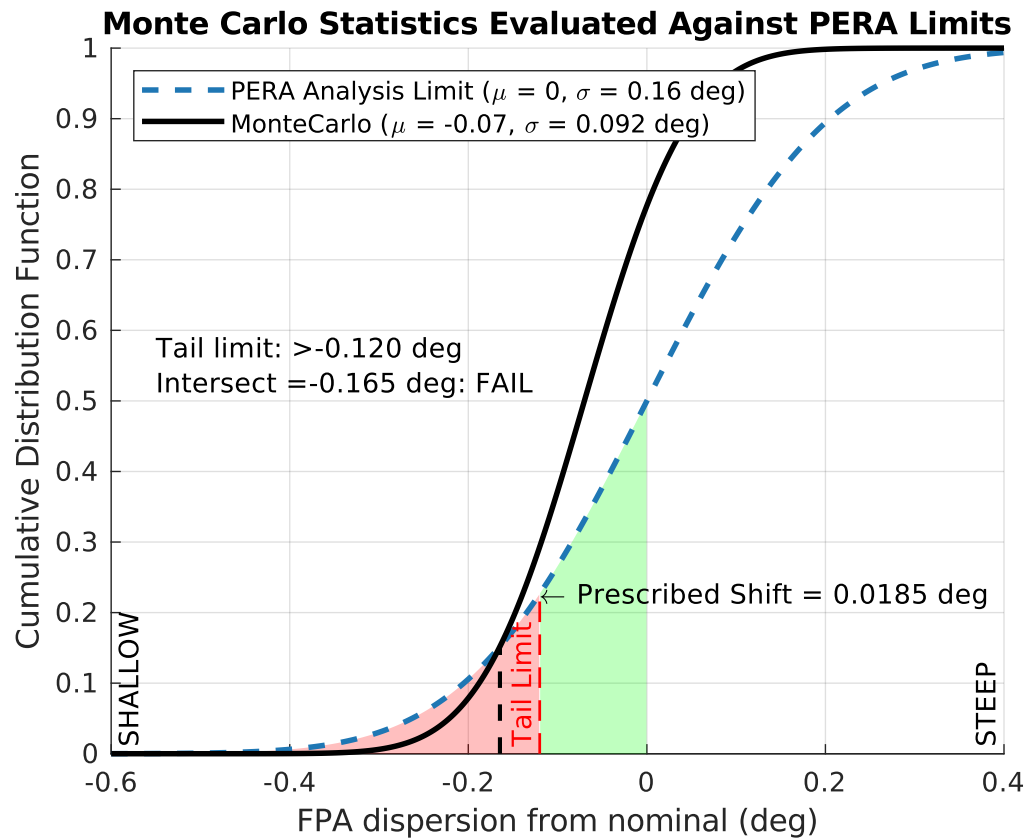


Figure 5 Example Failing Output from Statistical Comparison Tool

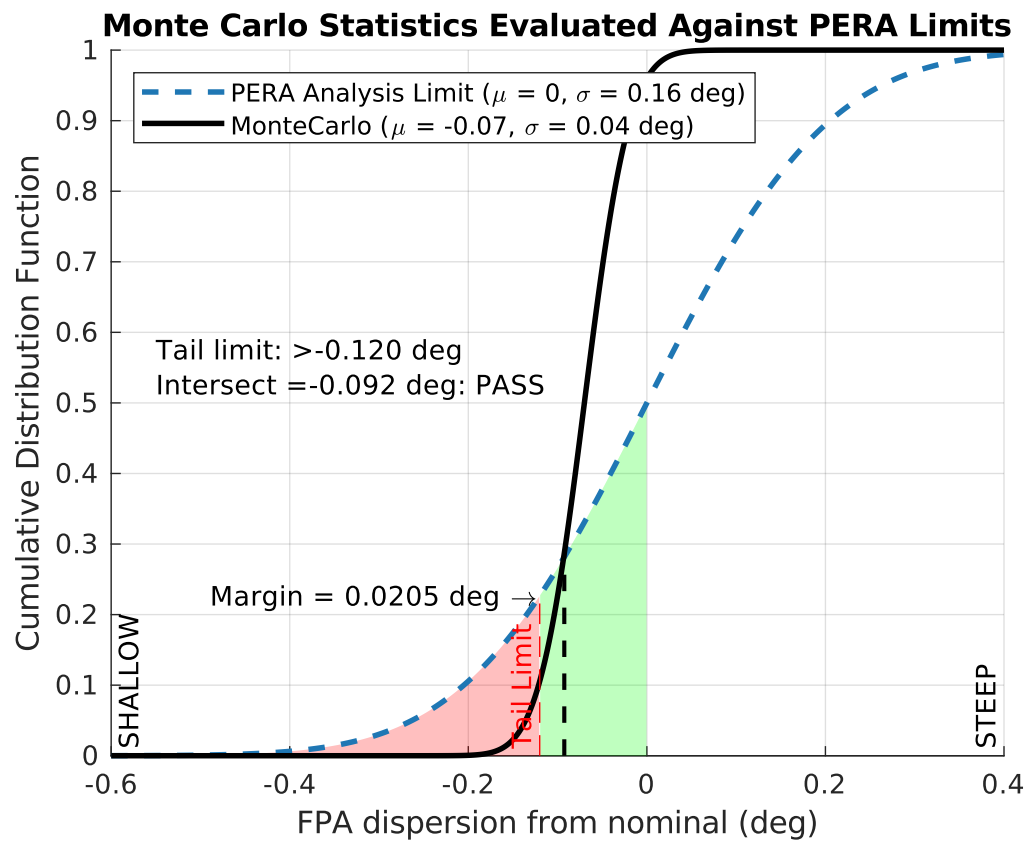


Figure 6 Example Passing Output from Statistical Comparison Tool



## **ACKNOWLEDGMENT**

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## **REFERENCES**

- [1] M. A. McPherson, “Artemis I Mission Entry Risk, Cross-Program Range Safety Public Entry Risk Assessment (PERA) for EM-1,” MPCV 72603-EM1, NASA Johnson Space Center, Houston, TX, 2021.
- [2] “NASA Procedural Requirements (NPR 8715.5A),” NASA Tech Briefs, NASA Johnson Space Center, Houston, TX, June 2010.