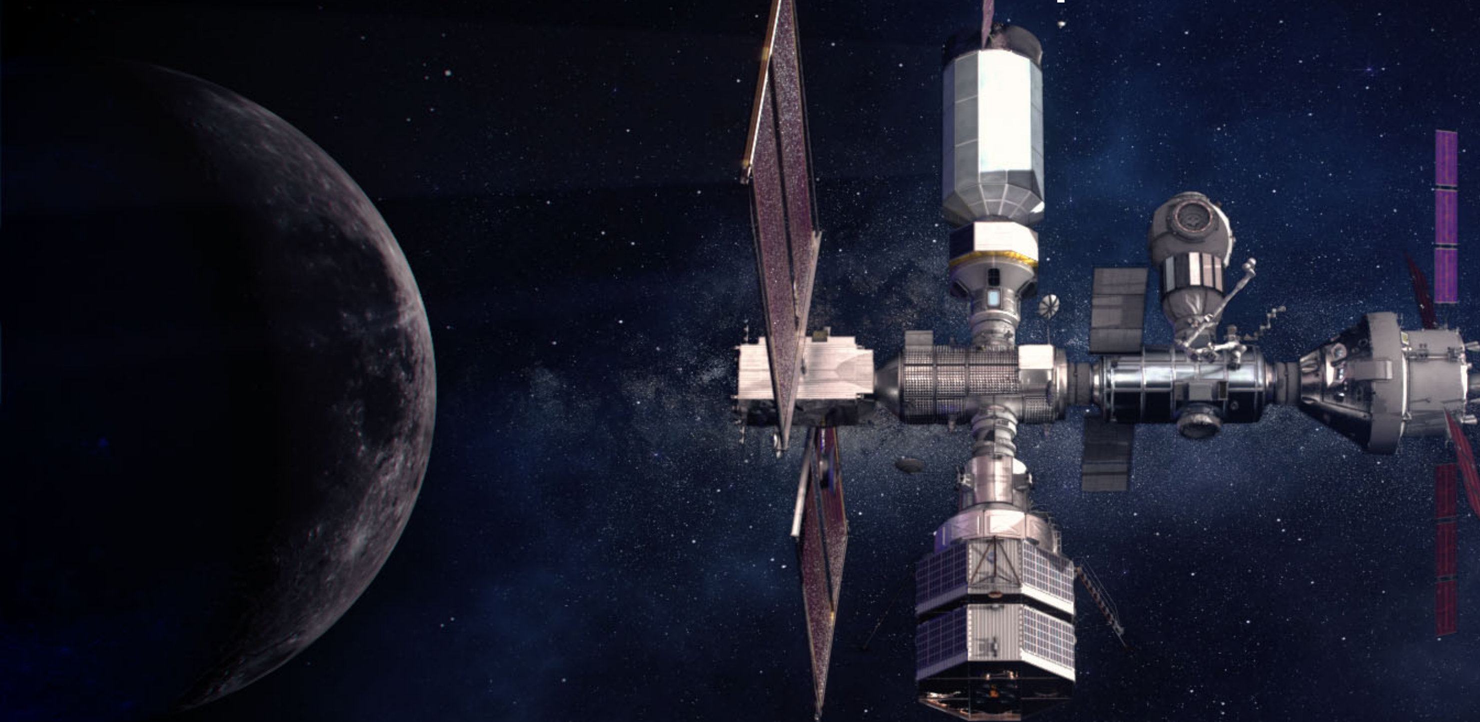
The Effects of the Solar Beta Angle on Low Thrust Spirals For Solar Electric Propulsion Vehicles



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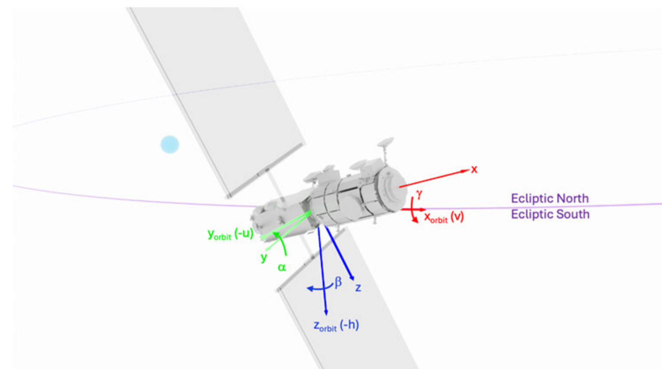
Agenda

- Background
- Attitude Strategies
  - Solar Perpendicular
  - Orbit Normal
  - Gamma Swap
- Methods
  - Time Based Simulation
  - Kinematic Model
- Results
- Conclusion

Background

- **Initial Capability for NASA's Gateway**
  - Co-manifested launch comprised of the Power and Propulsion Element (PPE) and Habitation and Logistics Outpost (HALO), referred to as PPE+HALO
  - PPE+HALO is delivered to a Near Rectilinear Halo Orbit (NRHO) by the low thrust, solar electric propulsion (SEP) system on PPE
  - PPE propels PPE+HALO from an initial, highly elliptic Earth orbit to the NRHO over the course of roughly 1 year
- **Thrusting nearly continuously over the course of many orbits presents unique challenges to the attitude control system to meet various pointing constraints**
  - Thrust vector pointing
    - Typically vehicle +x along velocity vector during a spiral out
  - Solar array pointing
    - Vehicle z-axis perpendicular to the sun allows for arrays to be perpendicular to sun
    - Solar arrays then rotate about the array axis to actually be perpendicular to sun
  - Keeping the sun off of the radiators
    - Typically radiators are on +/- z panels
  - Keeping communication antennas pointed to the ground stations on Earth



- **Maximizing sun incidence on the solar arrays works in tandem with minimizing sun incidence on the radiators**
- **With the thrust vector defining the pitch and yaw of the spacecraft, the roll axis is used to maximize the sun on the solar arrays**

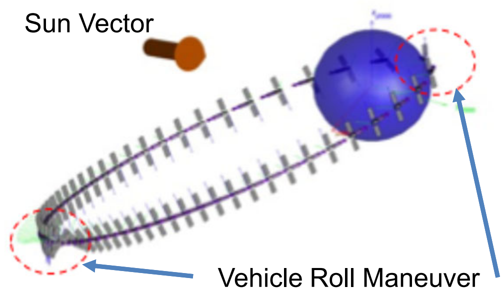
Background

- **Satisfying the two constraints of thrust vector pointing and solar array pointing can lead to a need to roll the vehicle twice per orbit, specifically when the thrust vector is in close alignment with the direction to or away from the Sun.**

- **During the roll maneuvers, there may be transients, where**
  - It may not be possible to maintain adequate pointing of the solar arrays to generate enough power to operate the thrusters at the desired power level during the transients
  - The elevation of the sun above/below the vehicle x-y plane may be too high to operate the thrusters

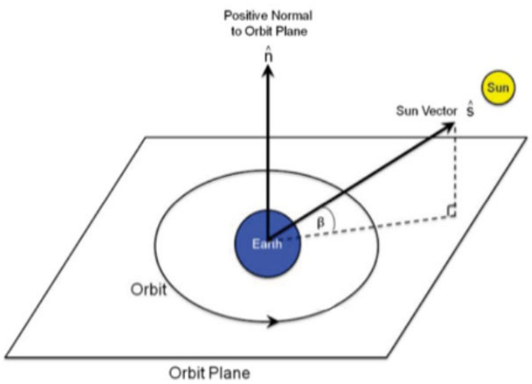
- **This can directly impact the total trip time of the spiral trajectory if the thrusters must be turned off or throttled down twice per orbit**

- **This can also complicate operations of the vehicle to plan for changing thruster power levels throughout the spiral out**



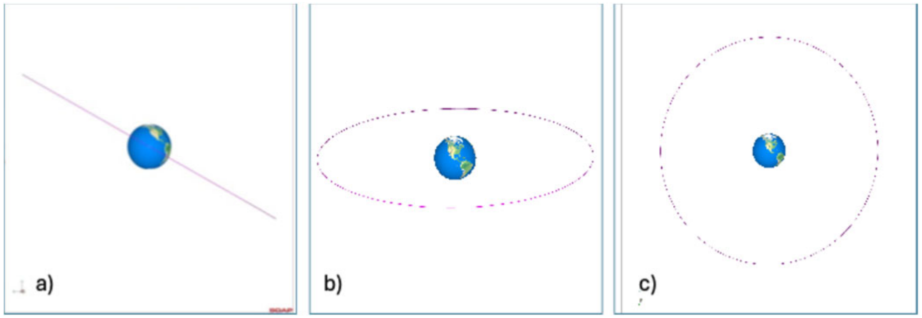
Background, Definition of the Solar Beta Angle

- **One main driver of how much the vehicle may need to roll to maintain the solar arrays pointed to the sun is the solar beta angle, β.**
- **The solar beta angle is defined as the angle between the vehicle orbit plane and the direction to the Sun**



Definition of solar beta angle, β

Examples of solar beta angle for a circular orbit with SMA = 42,166 km
Sun To Earth View



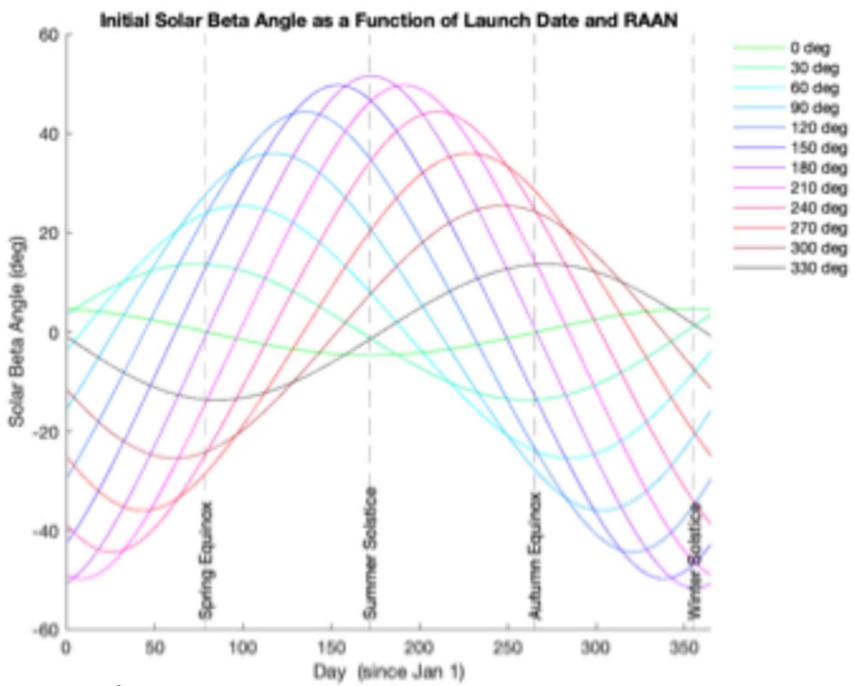
β = 0 deg β = 20 deg β = 90 deg

Background, Initial Solar Beta Angle

- **The solar beta angle can be calculated as**

$$\beta = \sin^{-1}(\cos(\Gamma)\sin(\Omega)\sin(i) - \sin(\Gamma)\cos(\epsilon)\cos(\Omega)\sin(i) + \sin(\Gamma)\sin(\epsilon)\cos(i))$$

  - Γ = Ecliptic solar longitude, provides the seasonal nature of the position of the Earth as it orbits the sun. It is 0 degrees at the vernal equinox, 90 degrees a the summer solstice, 180 degrees at the autumnal equinox, and 270 degrees at the winter solstice
  - Ω = Right ascension of the ascending node, and generally initially dictated by the launch time of day
  - *i* = inclination of the orbit, generally set by the launch latitude, remains relatively constant for the majority of the spiral out when thrusting tangentially
  - ε = obliquity of the ecliptic, the axial tilt of the Earth with respect to the ecliptic plane



- **Generally, for any orbit, the initial solar beta angle could be anywhere between +/-(i + ε).**

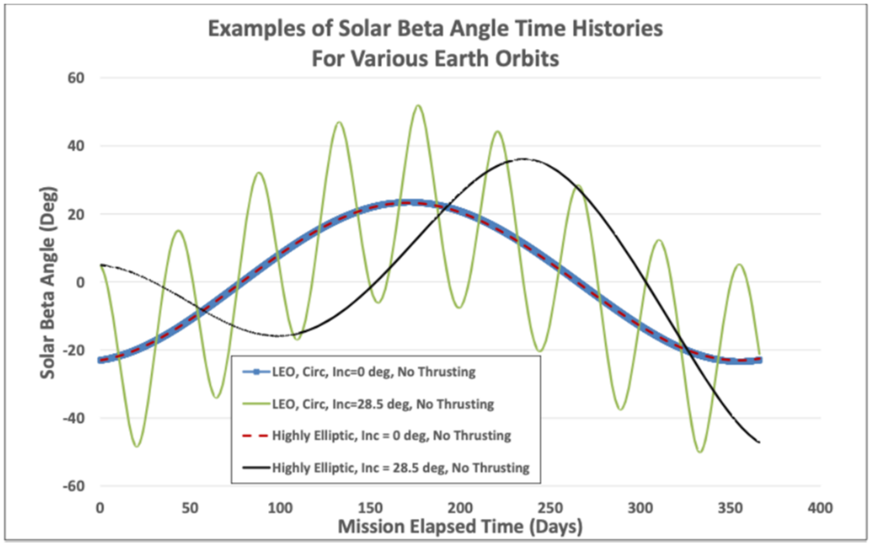
Assuming an Earth orbit, with an inclination of 28.5 degrees, the initial solar beta angle can be anywhere between ~+/- 52 degrees.

Background, How The Solar Beta Angle Changes

- **The solar beta angle is also not constant throughout a spiral out, rather it changes due to**
  - Orbital precession due to Earth oblateness for non-equatorial, non-polar orbits
  - The Earth moving through its orbit around the sun
- **The rate at which Ω changes due to Earth oblateness can be approximated as**

$$\dot{\Omega} = -2.06474 \times 10^{14} a^{-7/2} cos(i)(1 - e^2)^{-2}$$

- **For Earth orbits, the solar beta angle**
  - Varies between +/- 23.5 degrees for equatorial orbits
  - Changes faster for lower energy, non-equatorial, non-polar orbits, and also varies between higher extremes



Time History of the Solar Beta Angle for Various Earth Orbits

These effects are demonstrated because there are a number of spacecraft steering options, or attitude strategies, that can be employed to keep the solar arrays pointed to the sun, and the solar beta angle impacts which strategy to use and when to use it throughout a spiral out.

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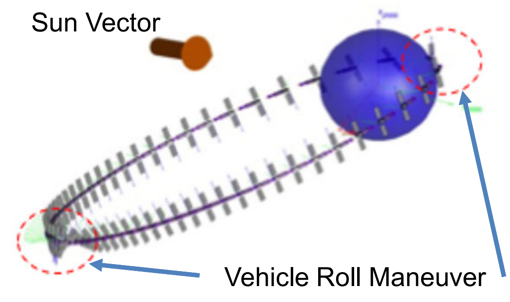
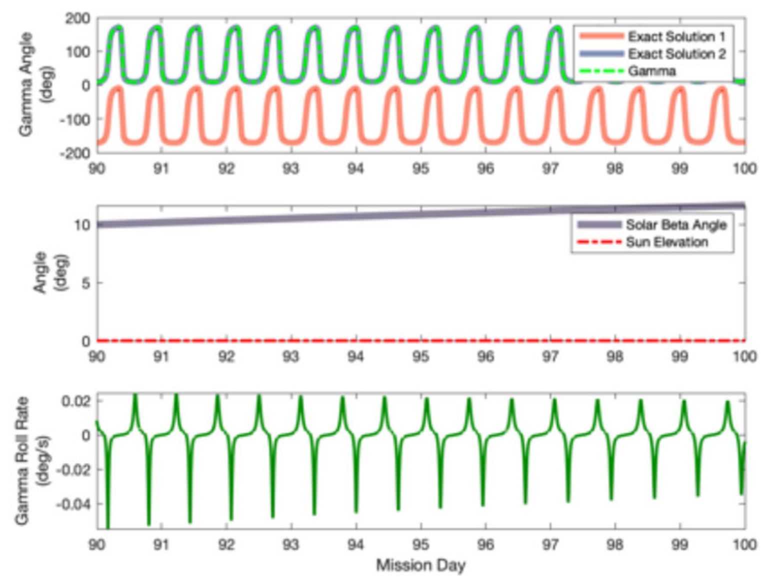
Attitude Strategies, Solar Perpendicular

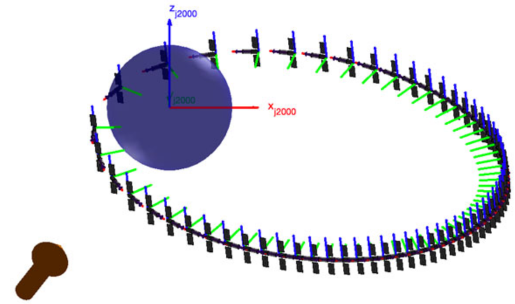
- **The vehicle attempts to maintain <u>perfect</u> pointing of the solar arrays at all times.**
  - Spacecraft rolls about the vehicle x-axis such that the solar array axis (vehicle z-axis) is perpendicular to the sun.
  - It is assumed then that the solar arrays can rotate about the array axis to point to the sun.
- **The roll angle is referred to as the angle gamma, γ.**
- **There are generally two possible solutions to achieve perfect sun pointing, and these two solutions are 180 degrees in opposition from one another.**
- **The vehicle follows 1 of the gamma solutions for the entire orbit**
- **The Sun remains on one side of vehicle**
- **This strategy requires a large roll maneuver twice per orbit when the thrust vector is in close alignment with the direction to or away from the Sun.**
- **Maintains perfect sun pointing and zero sun elevation as long as the required roll rate is within vehicle capabilities.**
- **As the solar beta angle gets smaller, the required roll rate to maintain perfect pointing increases**
  - This results in an increase in required vehicle momentum capacity

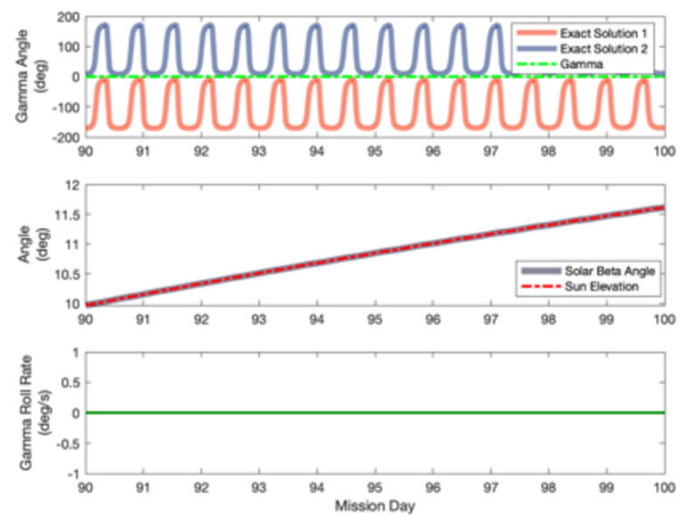




Attitude Strategies, Orbit Normal

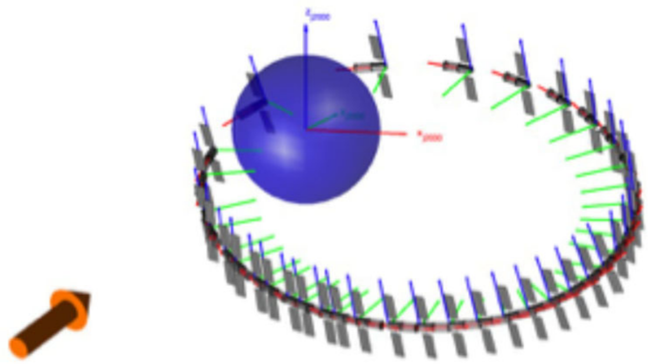
- **The vehicle is aligned such that the solar array axis is normal to the vehicle orbit plane.**
- **Vehicle does not roll, the gamma angle is commanded to 0 or 180 degrees throughout the orbit.**
  - Momentum usage is minimal.
  - The Sun continuously clocks around the vehicle while using the same gamma solution.
  - The same side of the vehicle remains pointed at the Earth while using the same gamma solution.
- **The solar elevation above/below the vehicle x-y plane is equal to the solar beta angle.**
- **The incidence on the solar arrays is equal to the cosine of the solar beta angle.**
- **This can be acceptable during periods of low solar beta angle, but can become prohibitive as the beta angle increases.**

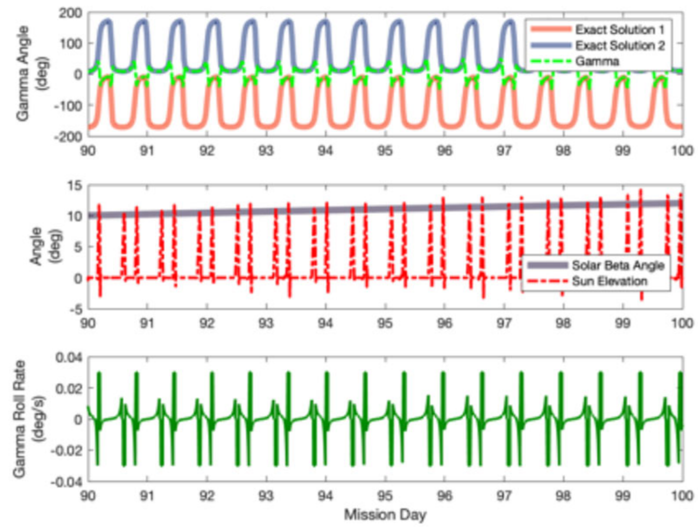




Attitude Strategies, Gamma Swap

- In this strategy, the vehicle swaps back and forth between the 2 gamma solutions.
- The swap from one solution to the other occurs when the thrust vector is in close alignment with the direction to or away from the Sun.
- During the transition from one gamma solution to the other, transients arise where the sun elevation above the vehicle's x-y plane peaks for a short period of time.
- While this strategy does induce transients, it can use less momentum to be within the vehicle's capabilities, compared to the solar perpendicular method, depending on what the value of the solar beta angle is.





Attitude Strategies, Summary

Strategy	Advantages	Disadvantages	When to Use
Solar Perpendicular	Maximizes power generation and minimizes sun elevation above vehicle x-y plane.	Requires snap roll at low beta angles that can introduce transients in power generation and sun elevation.	Non-low beta angles, when vehicle has the required momentum to perform roll maneuver to maintain adequate power generation during transients.
Orbit Normal	Roll angle is constant throughout orbit, momentum usage is minimized.	Cosine loss on solar arrays AND sun elevation on the vehicle is equal to the beta angle.	Low beta angles. When it's acceptable to maintain the same side of the spacecraft pointed to Earth. Small cosine losses for power and sun elevation on the vehicle are acceptable.
Gamma Swap	Can require less momentum and result in better transients then solar perpendicular at low beta angles.	Transients of non-perfect pointing and non-zero sun elevation twice per orbit.	Use as a transition between Orbit Normal strategy and Solar Perpendicular, when beta angle is low but is increasing and the cosine loss in power and sun elevation on vehicle is near violating vehicle requirements.

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Methods

- **Time based simulation**
  - Ingests a previously optimized trajectory and calculates the vehicle performance utilizing the attitude strategies mentioned previously.
  - This method is applicable when an optimized trajectory is available, and provides a higher fidelity analysis then the following approximation method.
  - Will not be discussed in detail in this presentation

- **Kinematic model**
  - Goal of this method is to provide planning values and calculate momentum usage during transients
  - Alleviates the need to calculate where in the orbit the alignment occurs that causes the transients
  - Closed form solution is generated by stacking rotations matrices for the sun azimuth, elevation and vehicle roll angle
    - Provides a symbolic form of the sun vector in the spacecraft frame for a given set of orbital elements
    - Solar perpendicular gamma roll angle is then calculated by using the sun vector's z component, setting it to 0 and symbolically solving for the gamma roll angle
    - Results in a differentiable analytical function that can be used to calculate momentum, inflection points, and sensitivity to orbital elements
  - Used during the tangential thrusting phase of a spiral out trajectory
  - Will be discussed in detail in the following slides

Kinematic Model

- **Derivation of the solar elevation angle**
  - The sun vector in the orbit frame can be expressed using active intrinsic functions

$$p_{sun}^{peri} = R_{azimuth} R_{elevation} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$p_{sun}^{peri} = \begin{bmatrix} cos(\alpha) & -sin(\alpha) & 0 \\ sin(\alpha) & cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos(\beta) & -0 & sin(\beta) \\ 0 & 1 & 0 \\ -sin(\beta) & 0 & cos(\alpha) \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$p_{sun}^{peri} = \begin{bmatrix} cos(\alpha)cos(\beta) \\ cos(\beta)sin(\alpha) \\ -sin(\beta) \end{bmatrix}$$

  - Next, we express the sun vector in the spacecraft body frame which has been rotated by γ about it's roll axis

$$p_{sun}^{sc} = R_{peri}^{sc} p_{sun}^{peri} = \begin{bmatrix} 1 & -0 & 0 \\ 0 & \cos(\gamma) & \sin(\gamma) \\ 0 & -\sin\gamma & \cos(\gamma) \end{bmatrix} \begin{bmatrix} cos(\alpha)cos(\beta) \\ cos(\beta)sin(\alpha) \\ -sin(\beta) \end{bmatrix}$$

$$p_{sun}^{sc} = \begin{bmatrix} cos(\alpha)cos(\beta) \\ \cos(\beta) * \cos(\gamma) * \sin(\alpha) - \sin(\beta) * \sin(\gamma) \\ -\cos(\gamma) * \sin(\beta) - \cos(\beta) * \sin(\alpha) * \sin(\gamma) \end{bmatrix} \quad (1)$$

  - The solar elevation angle in the spacecraft frame is the arcsin of the z-component of Equation 1

$$\theta = \arcsin(-\cos(\gamma) * \sin(\beta) - \cos(\beta) * \sin(\alpha) * \sin(\gamma)) \quad (2)$$

Kinematic Model

- **Calculating the required roll angle and roll rate for the solar perpendicular attitude strategy, perfect sun pointing**
  - For perfect sun pointing (psp), the solar elevation angle is 0 degrees, so we set θ to 0 in Equation 2 and solve for γ to get the roll angle

$$\gamma_{psp,1} = \text{atan2}(-sin(\beta), cos(\beta)sin(\alpha)) \quad (3)$$

$$\gamma_{psp,2} = \text{atan2}(sin(\beta), -cos(\beta)sin(\alpha)) \quad (4)$$

  - There are two solutions because the spacecraft can be rolled 180 degrees to place the sun vector in the vehicle x-y plane
  - The required roll rate can then be calculated by taking the time derivative (both solutions have the same derivative)

$$\dot{\gamma_{psp}} = \frac{\dot{\beta}\sin(\alpha) - \dot{\alpha}\cos(\alpha)\cos(\beta)\sin(\beta)}{\cos^2(\alpha)\cos^2(\beta) - 1}$$

  - Since the duration of the roll maneuvers are very short relative to the rate of change of the solar beta angle, the rate of change of the solar beta angle can be set to 0

$$\dot{\gamma_{psp}} = -\frac{\dot{\alpha}\cos(\alpha)\cos(\beta)\sin(\beta)}{\cos^2(\alpha)\cos^2(\beta) - 1} \quad (5)$$

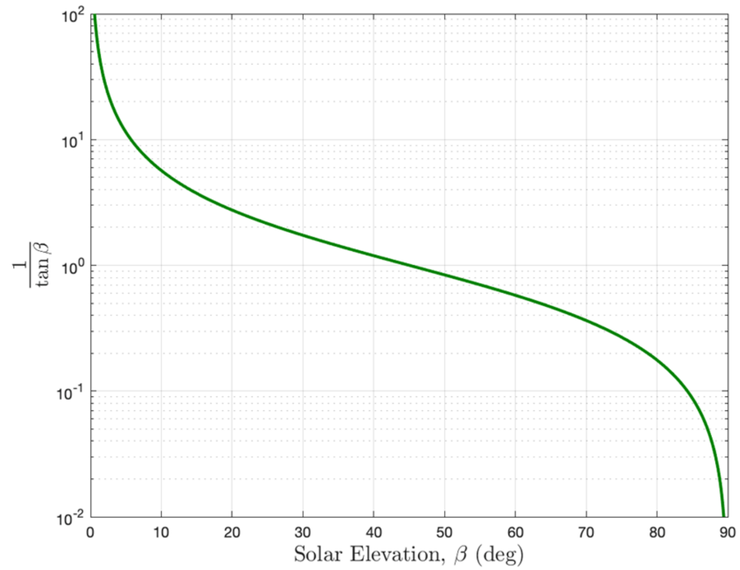
Special Case, Sun in Vehicle x-z Plane

- **When the sun vector is in the vehicle x-z plane, $\alpha = 0$**
- **Using Equations 3, 4 and 5**
  - With $\beta = 0$, the sun is aligned directly off the nose (+x) of the vehicle and the roll angle is undefined because there are infinite solutions. The roll rate is also undefined.
  - With $\beta = \pi/2$, the sun is pointing to the orbit Zenith, the roll angle is either $\pi/2$ or $-\pi/2$, and the roll velocity is 0.
  - For all other values of β, with $\alpha = 0$, Equations 3, 4 and 5 simplify to:

$$\gamma_{psp} = \text{atan2}(-sin(\beta), cos(\beta)sin(0)) = [-\pi/2, \pi/2] \quad (6)$$

$$\dot{\gamma_{psp}} = \frac{\dot{\alpha}}{\tan(\beta)} \quad (7)$$

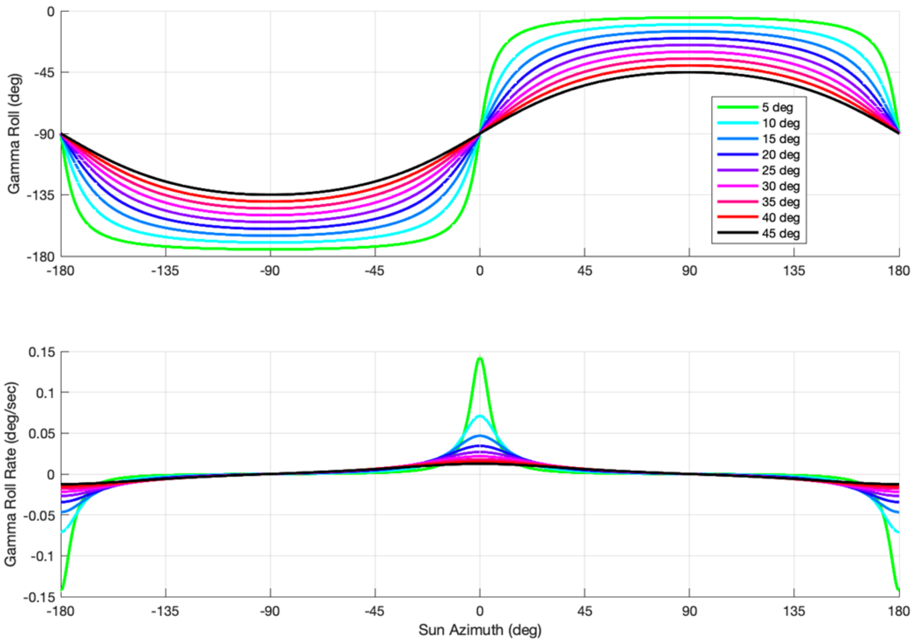
  - This is an important subcase. It represents the sun moving through the spacecraft's x-z plane at any solar beta angle. It illustrates that the roll velocity is a non-linear function of the sun elevation and a linear function of the azimuth rate of the sun.
  - The roll rate becomes maximum as $\tan(\beta)$ goes to 0 which occurs as the solar beta angle approaches 0.



Effect of Solar Beta Angle on Roll Rate at $\alpha = 0$

Required Roll and Roll Rate for Perfect Sun Pointing

- Plotting Equations 4 and 5 for various solar beta angles, over a single orbit (α = -180 deg to 180 deg), and approximating the solar azimuth rate as a circular constant orbit period of about 8 hours
- Lower beta angles require a larger roll maneuver and higher roll rate to maintain perfect sun pointing compared to higher beta angles.
- For non-zero beta angles, a large roll maneuver at a high roll rate is required to maintain perfect sun pointing when the solar azimuth is 0 degrees and 180 degrees, corresponding to when the x-axis of the vehicle is nearly aligned with the direction to the sun.
- If the spacecraft has roll rate limits that are below the peaks, we need to determine the gamma swap angle extent, the amount of time the vehicle is not perfect sun pointing, and minimize that off pointing.



Gamma Angle and Rate for Varying Solar Beta Angle to Maintain Perfect Sun Pointing

Determining the Gamma Swap Parameter

- **To calculate the gamma swap parameter, we want to determine the azimuth when a gamma swap can be completed within the gamma roll rate limit.**
- **We want the vehicle to stay on the perfect sun pointing profile as long as it can and then swap to the other gamma solution only when it needs to, when it will be at the roll rate limit during the swap**
- **If we assume the roll maneuver occurs at the rate limit, from equation 4, we can arrive at :**

$$\frac{\dot{\gamma}\alpha_s}{\dot{\alpha}} + \operatorname{atan2}\left(\sin\left(\beta\right), -\cos\left(\beta\right)\sin\left(\alpha_s\right)\right) = 0 \qquad (8)$$

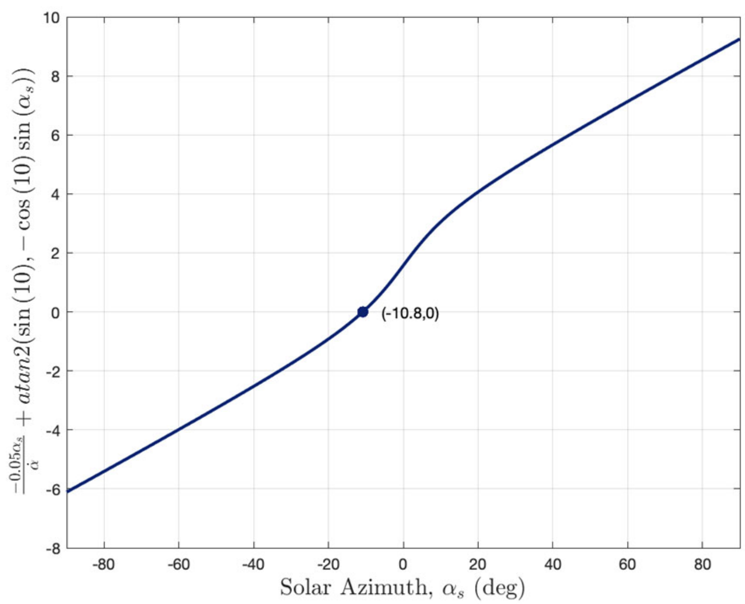
  - There is no closed form solution to Equation 8, but it can be numerically root solved for the α_s
- **As an example, assume the following:**
  - Roll rate limit = 0.05 deg/s
  - Solar azimuth rate = 0.0125 deg/sec (8 hour circular orbit)
  - Beta angle = 10 deg
  - Equation 8 then becomes:

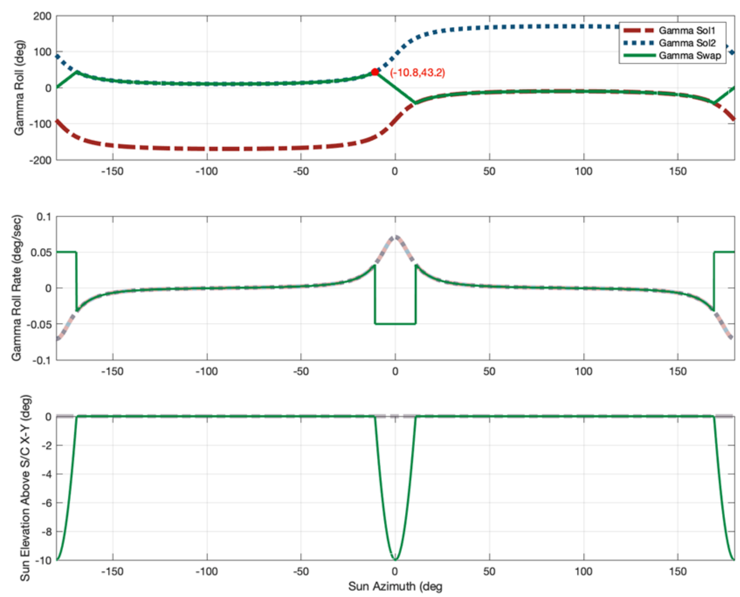
$$\frac{-0.05 * \alpha_s}{0.0125} + \operatorname{atan2}\left(\sin\left(10\right), -\cos\left(10\right)\sin\left(\alpha_s\right)\right) = 0$$

  - We can then numerically root solve for α_s

Determining the Gamma Swap Parameter (Continued)

Numerically root solving for α_s





Gamma Swap Example: 10deg Beta Angle, 8 hour solar azimuth period

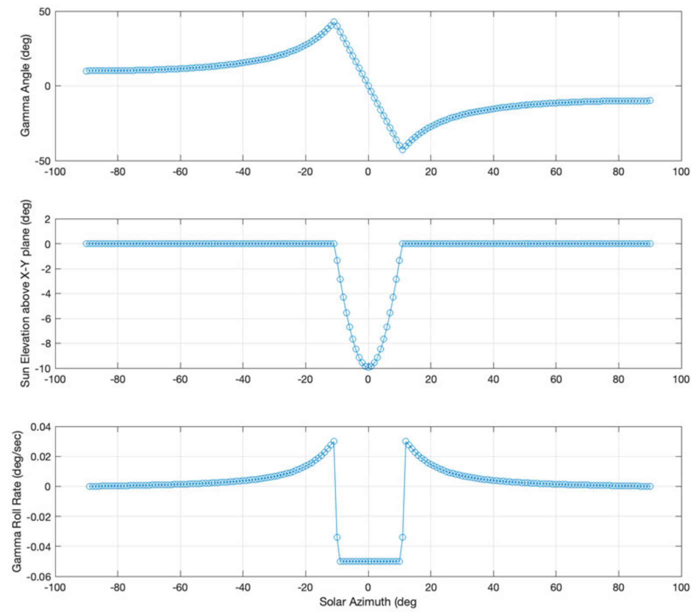
- α_s **is found to be 10.8 degrees.**
- **Using this value in Equation 4 and solving for the gamma swap parameter,** γ_s **is found to be 43.2 degrees.**

Determining the Gamma Swap Parameter (Continued)

- **To estimate the gamma swap parameter, we had previously assumed that the vehicle would follow one gamma solution, and then switch to the other gamma solution at the vehicle roll rate limit.**
- **To check the validity of this assumption, we set up an optimization problem using direct collocation, with the cost function as the time integral of the solar elevation derived in Equation 2**

$$J_{min} = \int_{-t}^{+t} \sin^2(\theta)\, dt = \int_{-t}^{+t} (-\cos(\gamma) * \sin(\beta) - \cos(\beta) * \sin(\alpha) * \sin(\gamma))^2\, dt$$

  - Given the controls: $u = [\gamma(t)]$
  - Fixed Parameters: $p = [\beta, \dot{\alpha}]$
  - Varying Parameters: $\alpha = \dot{\alpha} t$
  - Subject to the constraints that the gamma dot is less than the gamma roll rate limit
  - The time, t, was set such that we consider +/- 90 deg of azimuth
  - The values used for the parameters are the same as those in the previous example
  - The optimization was performed using Matlab's fmincon optimizer
  - While the optimizer solved for the full gamma trajectory, it results in a a gamma swap parameter of 41 deg.



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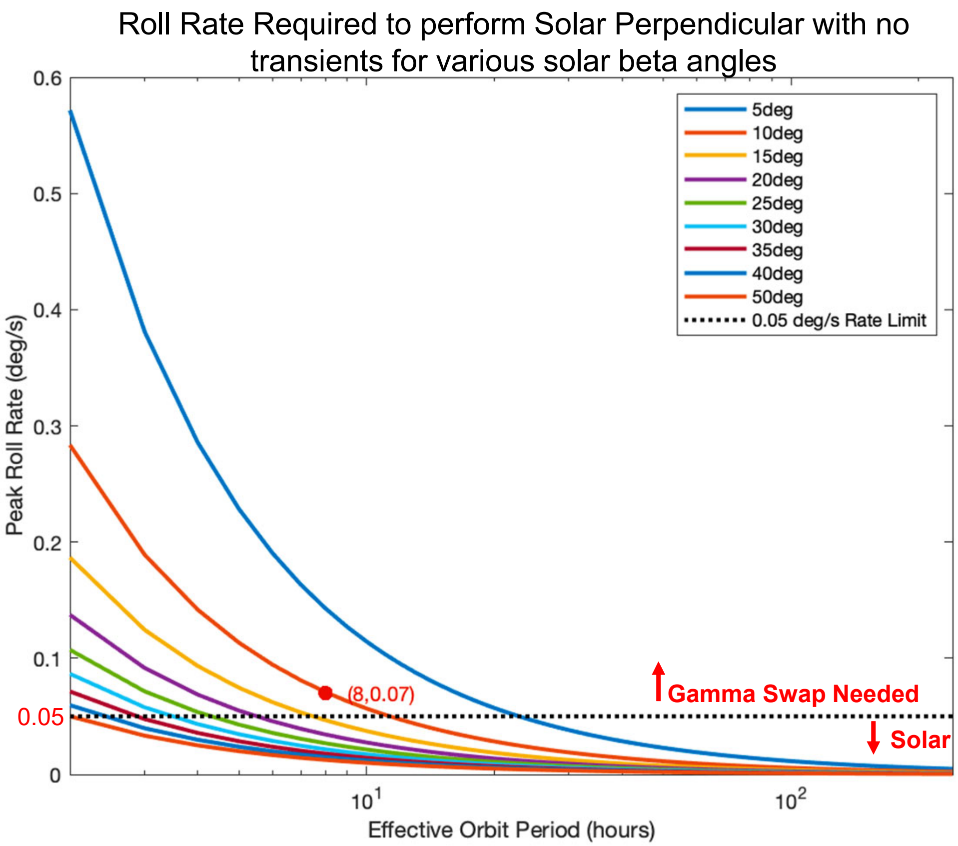
Results

- **Deciding between Solar Perpendicular and Gamma Swapping**



- **A shorter orbit period, especially at low beta angles, drives the need to use the Gamma Swap strategy.**
- **As the orbit period increases, and/or if the solar beta angle increases, solar perpendicular could be used.**

- **For the example:**
  - 8 hour effective orbit period
  - Solar beta angle of 10 degrees
  - Vehicle roll rate limit 0.05 deg/s
  - Gamma swap strategy is needed, since the required roll rate for solar perpendicular is 0.07 deg/s
- **Solar Perpendicular could be employed:**
  - For beta angles > ~ 15 degrees for an 8 hour effective orbit period and roll rate limit of 0.05 deg/s.

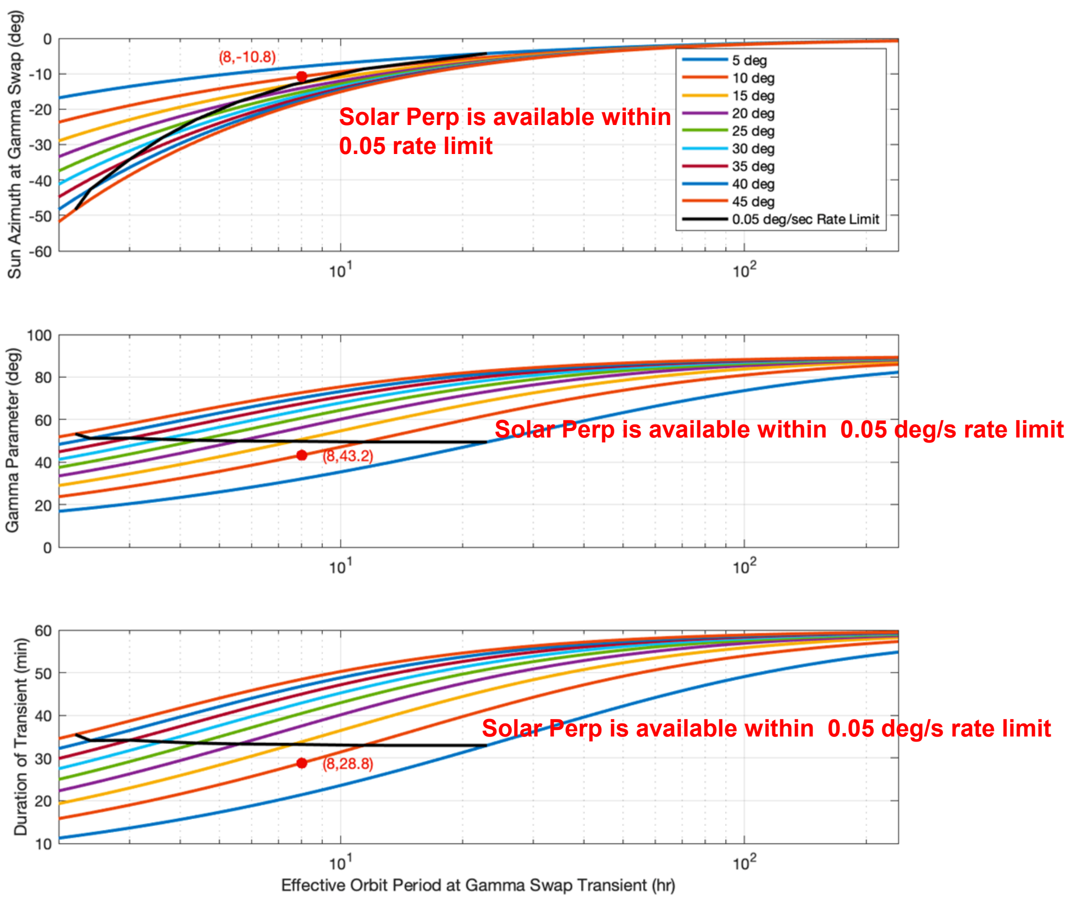
Conclusion

- **Low thrust solar electric propulsion spiral out missions present unique challenges to the vehicle, particularly to the attitude control system to satisfy various pointing constraints when the thrust vector is continually changing.**
- **There are various attitude strategies that can be employed by the vehicle to satisfy pointing constraints simultaneously.**
- **A key parameter that drives which attitude strategy to use and when to use it is the solar beta angle.**
- **The solar beta angle changes throughout a spiral out due to orbit precession if launched to a non-zero inclination, and also due to the Earth moving through its orbit about the sun.**
- **Since PPE+HALO will be launching to an initial highly elliptic, non-zero inclination with the spiral out to the NRHO taking roughly one year, it's likely that all of the attitude strategies discussed here will need to be employed, likely switching between them multiple times throughout the spiral out.**
- **Assessing the time history of the solar beta angle throughout the spiral out will help operators choose which attitude strategy to employ to meet various attitude and other subsystem constraints.**

Back-up Charts

Results

- **Deciding between Solar Perpendicular and Gamma Swapping**



- **Using the kinematic model of gamma swapping, the gamma swap parameters were found for a 0.05 deg/s rate limit**
- **But when does solar perpendicular not only provide better sun elevation performance, but actually need less roll rate?**
  - The black line is the solar beta angle and orbit period at which solar perpendicular has a roll rate of 0.05 deg/s
  - The black line here is the solar perpendicular dashed line from the previous slide mapped onto the gamma swap performance curves
- **As the durations of the transients for gamma swap get larger as orbit period gets longer, you can switch to perfect pointing at and above the black line and get zero transients**

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