National Aeronautics and Space Administration

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New Perspective On Slosh Dynamics in High-Gravity Regimes for Lunar Missions

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Introduction

- Slosh modeling for high-gravity scenarios has predominantly focused on launch vehicles, such as NASA's Space Launch System (SLS), utilizing two mechanical analogies: pendulum and mass spring damper (MSD) models.
- The equations of motion involving slosh and multibody dynamics were often simplified, excluding some of the nonlinear and secondary terms, which is sufficient for heavy launch vehicles with slow dynamics.
- Unlike launch vehicles, lunar landers require higher-fidelity modeling due to quick maneuvers during braking, approach and landing phases.
- Accurate representation demands maintaining the full EOM, including non-linear and multibody dynamics.
- In this study, slosh mechanical model parameters were derived using NASA MSFC's WINSLOSH tool, developed based on NASA's SP-8009 framework [3].

[3] N. Aeronautics and S. Administration, *Propellant Slosh Loads*. Hampton, Virginia: National Aeronautics and Space Administration, 1968.

Mass Spring Damper EOM Derivation

- The derivation of the MSD equations of motion (EOM) is straightforward, as it is obtained through the equation describing differentiated momentum in both rotational and translational forms, as shown below:

$$\dot{\vec{p}} = -\vec{\omega} \times \vec{p} + \vec{f} \qquad \dot{\vec{h}} = -\vec{\omega} \times \vec{h} + \vec{g}$$

- Where $\vec{p}$ is translational momentum, $\vec{f}$ is external force, $\vec{\omega}$ is angular velocity, $\vec{h}$ is angular momentum, $\vec{g}$ is external torque.

Mass Spring Damper EOM Derivation

- The resulting EOM are as follows:
  - Slosh: (Lateral Displacement)

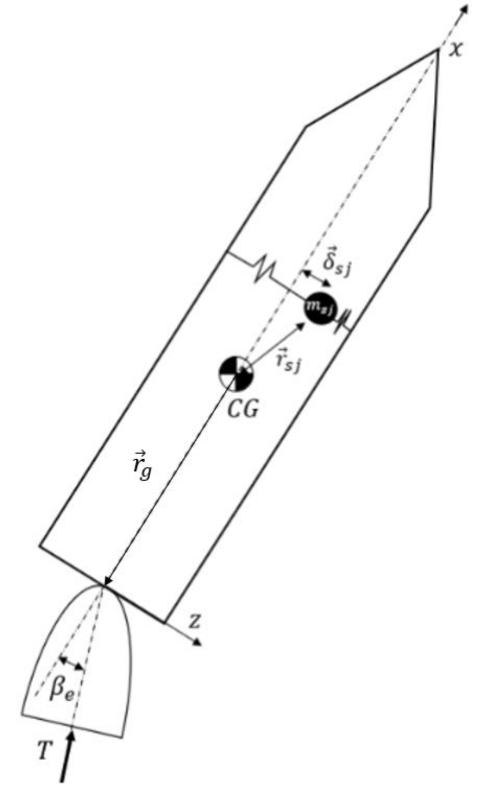
$$\ddot{\vec{\delta}}_{sj} + D\dot{\vec{\delta}}_{sj} + K\vec{\delta}_{sj} = -\vec{a}_b + \vec{r}_{sj} \times \dot{\vec{\omega}} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{sj}) - 2\vec{\omega} \times \dot{\vec{\delta}}_{sj}$$

Where $D = 2\zeta_{sj}\omega_{sj}$ and $K = \omega_{sj}^2$

  - Vehicle Rotational:

$$I_T\dot{\vec{\omega}} = -\vec{\omega} \times I_T\vec{\omega} \boxed{- m_{sj}\vec{r}_{sj} \times \vec{a}_b - m_{sj}\vec{r}_{sj} \times \ddot{\vec{\delta}}_{sj} + m_{sj}\vec{r}_{sj} \times (\vec{r}_{sj} \times \dot{\vec{\omega}}) + m_{sj}\vec{r}_{sj} \times (\vec{\omega} \times (\vec{\omega} \times \vec{r}_{sj})) - 2m_{sj}\vec{r}_{sj} \times (\vec{\omega} \times \dot{\vec{\delta}})} + \boxed{\vec{r}_{sj} \times \vec{T}_e}$$

Slosh Thrust

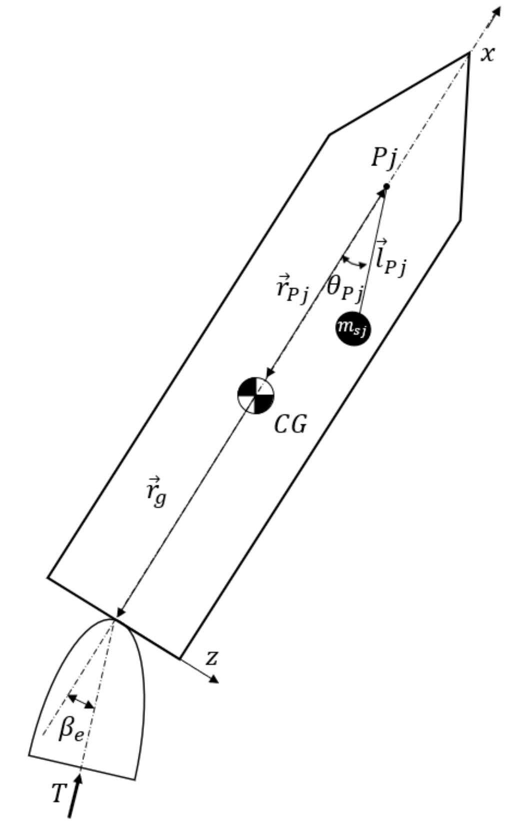


Pendulum EOM Derivation

- Using Newton-Euler method, the resulting EOM are as follows:
  - Slosh: (Pendulum Angle)

$$\ddot{\vec{\theta}}_{Pj} = \frac{\vec{l}_{Pj} \times \left(\frac{\vec{T}}{m_T} - \dot{\vec{\omega}} \times \vec{r}_{Pj} - \vec{\omega} \times (\vec{\omega} \times \vec{r}_{Pj})\right)}{|\vec{l}_{Pj}|^2} - \dot{\vec{\omega}}$$

  - Vehicle Rotational:



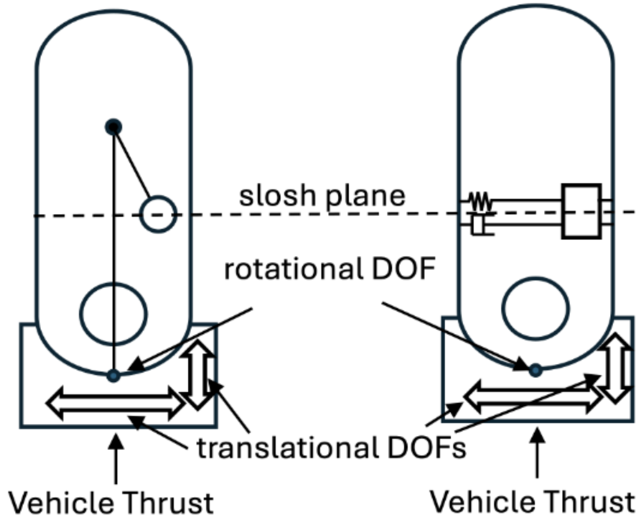
$$I_T\dot{\vec{\omega}} = -\vec{\omega} \times I_T\vec{\omega} + m_{sj}\vec{r}_{Pj} \times \left\{-\ddot{\vec{\theta}}_{Pj} \times \vec{l}_{Pj} - \dot{\vec{\theta}}_{Pj} \times \dot{\vec{\theta}}_{Pj} \times \vec{l}_{Pj} - 2\vec{\omega} \times \left(\dot{\vec{\theta}} \times \vec{l}_{Pj}\right) - \vec{a}_b - \dot{\vec{\omega}} \times \left(\vec{r}_{Pj} + \vec{l}_{Pj}\right) - \vec{\omega} \times \left[\vec{\omega} \times \left(\vec{r}_{Pj} + \vec{l}_{Pj}\right)\right]\right\} + \vec{r}_g \times \vec{T}_e$$

Slosh

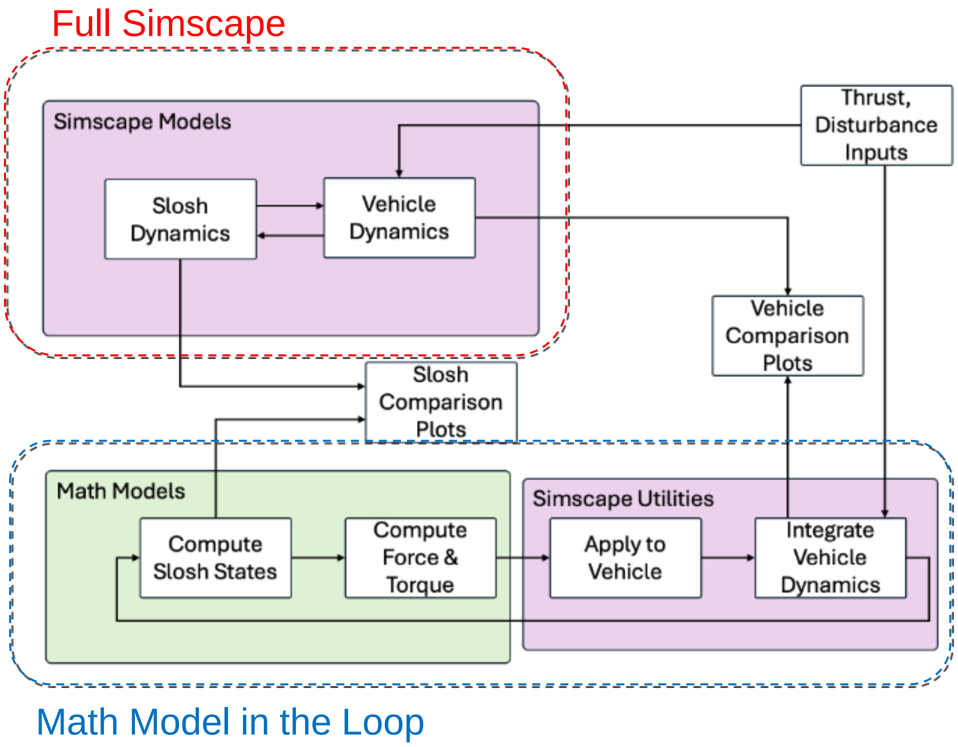
Thrust

EOM Validation

Test Simulation Set-up

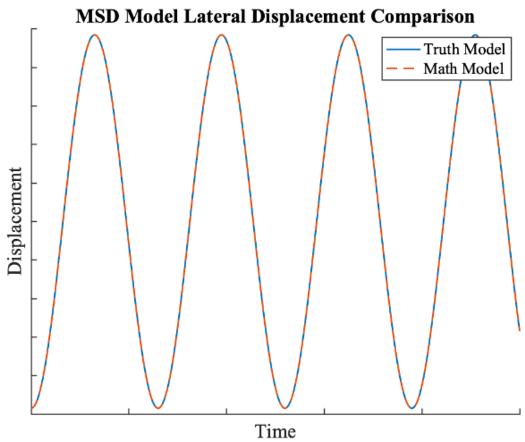


Flow Diagram for Validation

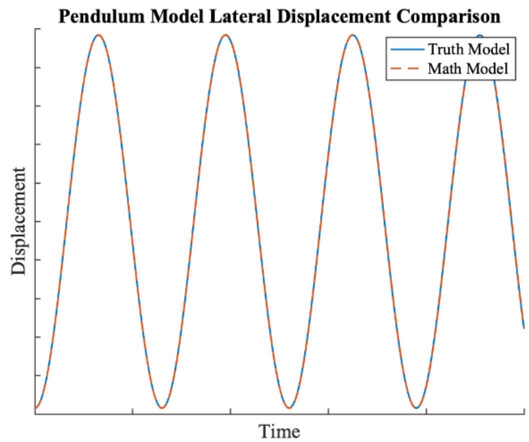


EOM Validation Results

No Damping



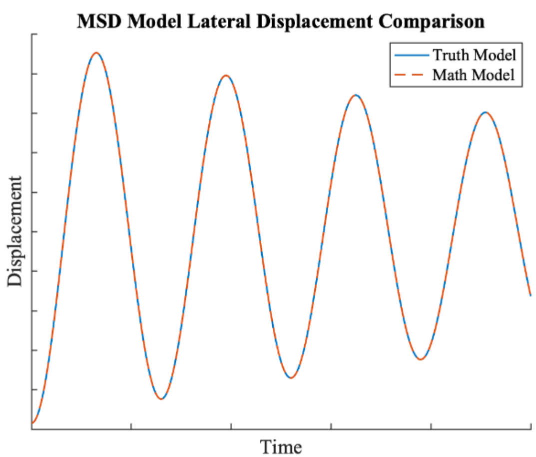
(a) MSD model



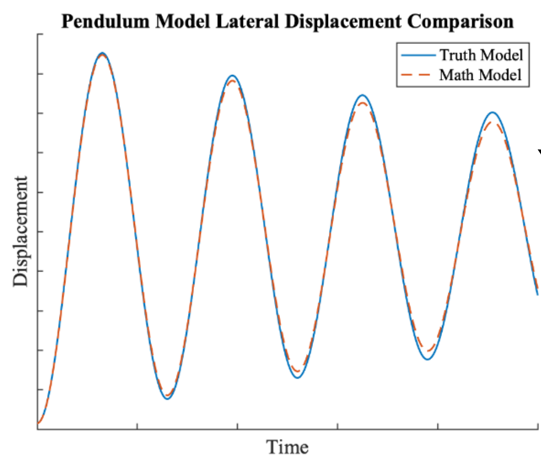
(b) Pendulum model

“Math Model” denotes math model in the loop and “Truth” denotes full Simscape.

Damping Included



(a) MSD model



(b) Pendulum model

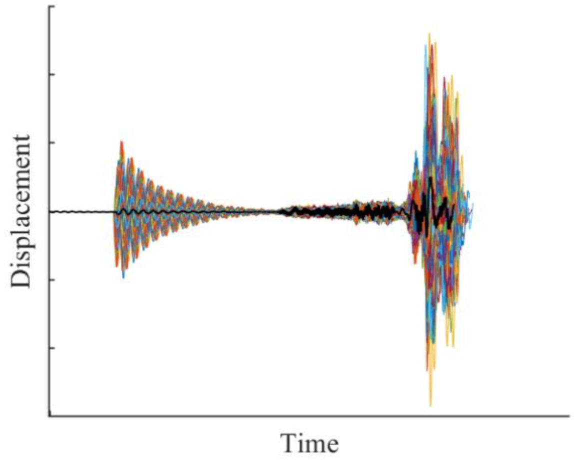
A slight discrepancy is observed in the pendulum model with damping, which will be further investigated in future studies.

GLASS Monte Carlo Results with Math Model

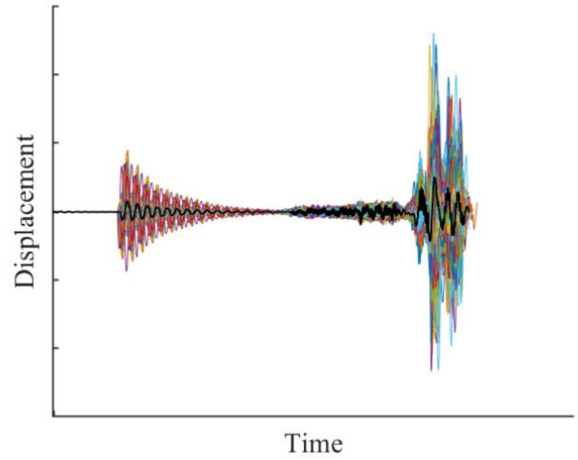
- GLASS (GeneraLized Aerospace Simulation in Simulink) is a 6-DOF flight simulation tool developed at NASA Marshall Space Flight Center for evaluating Guidance, Navigation, and Control (GN&C) performance of the lunar landing and ascent vehicles.
- GLASS's core engine is built on MATLAB Simulink with Mathworks Simscape Multibody dynamics toolbox that is capable of modeling and simulating multibody dynamics.
- GLASS Monte Carlo results is generated with Slosh Math Model in the Loop method, meaning slosh force and moment are derived mathematically and integrated into the simulation that involves Simscape Multibody that handles the overall vehicle dynamics.

GLASS Monte Carlo Results with Math Model

Tank 1 Y axis



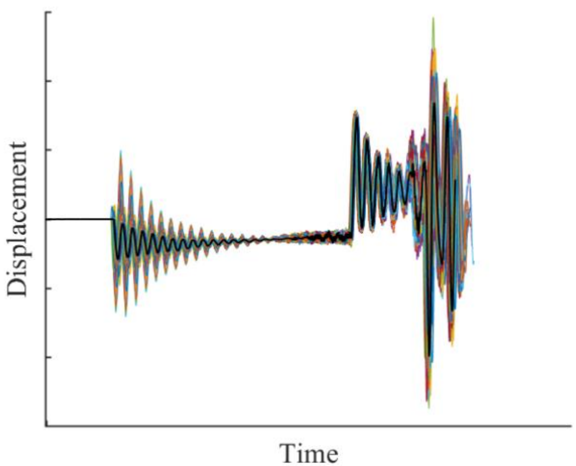
(a) MSD Model



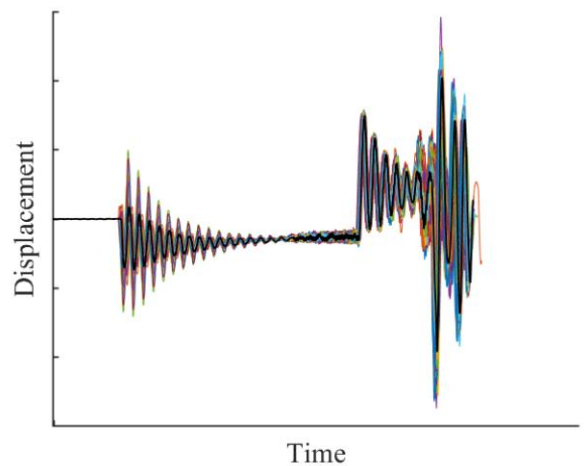
(b) Pendulum Model

Comparable bulk motion characteristics such as damping, excitations and limit cycle oscillations are observed between MSD and Pendulum model results.

Tank 1 Z axis



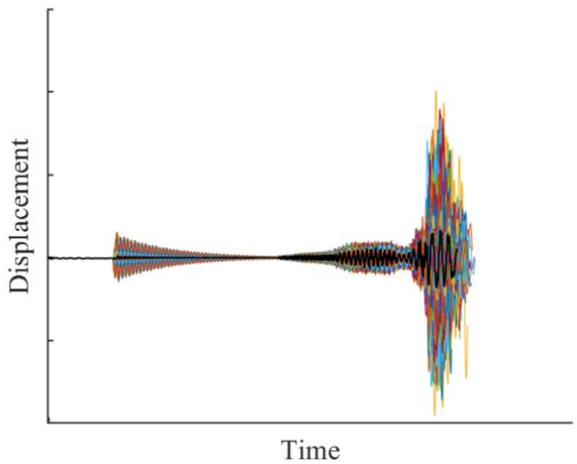
(a) MSD Model



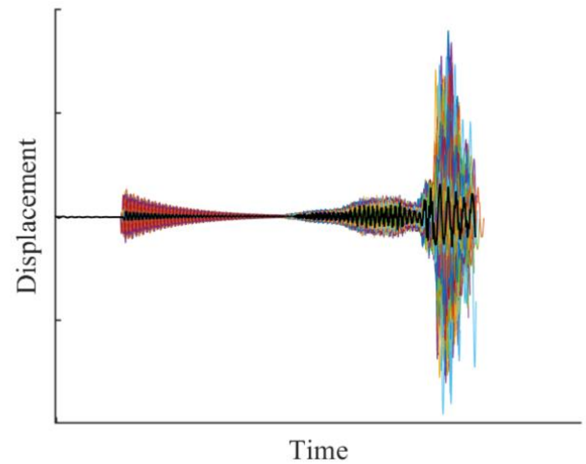
(b) Pendulum Model

GLASS Monte Carlo Results with Math Model

Tank 2 Y axis



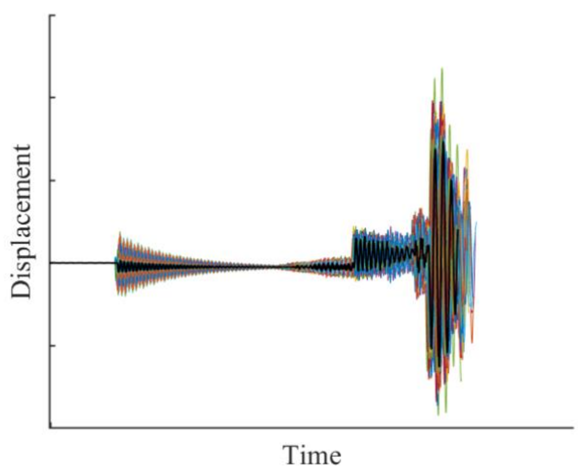
(a) MSD Model



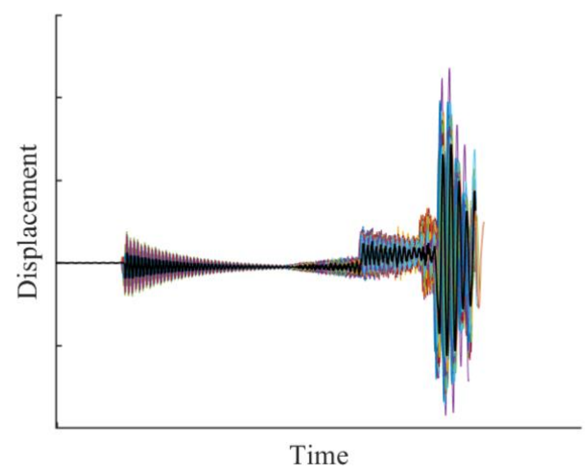
(b) Pendulum Model

Comparable bulk motion characteristics such as damping, excitations and limit cycle oscillations are observed between MSD and Pendulum model results.

Tank 2 Z axis



(a) MSD Model

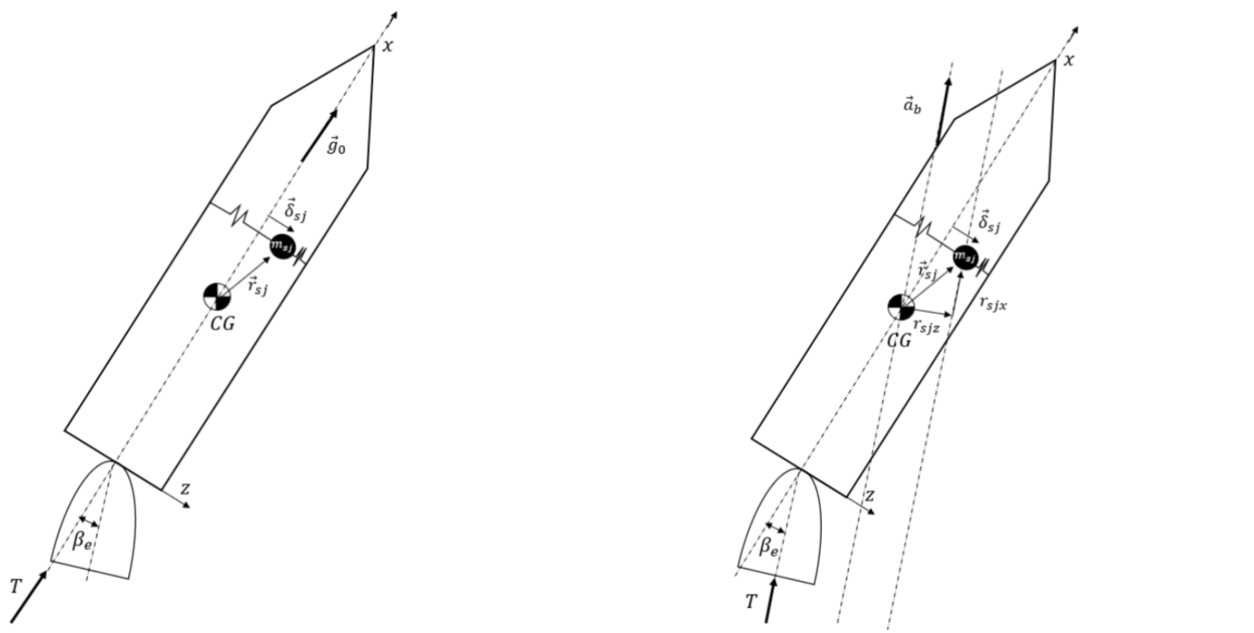


(b) Pendulum Model

Implications to the Proper Axial Acceleration in MSD

- The term $m_{sj}\vec{r}_{sj} \times \vec{a}_b$ not only captures k_3 or $\bar{g}_0$ effect, but also accounts for lateral acceleration effects and the vehicle's tilt from the reference trajectory.

$$I_T\dot{\vec{\omega}} = -\vec{\omega} \times I_T\vec{\omega} \boxed{- m_{sj}\vec{r}_{sj} \times \vec{a}_b} - m_{sj}\vec{r}_{sj} \times \ddot{\vec{\delta}}_{sj} + m_{sj}\vec{r}_{sj} \times (\vec{r}_{sj} \times \dot{\vec{\omega}}) + m_{sj}\vec{r}_{sj} \times (\vec{\omega} \times (\vec{\omega} \times \vec{r}_{sj})) - 2m_{sj}\vec{r}_{sj} \times (\vec{\omega} \times \dot{\vec{\delta}}) + \vec{r}_{sj} \times \vec{T}_e$$

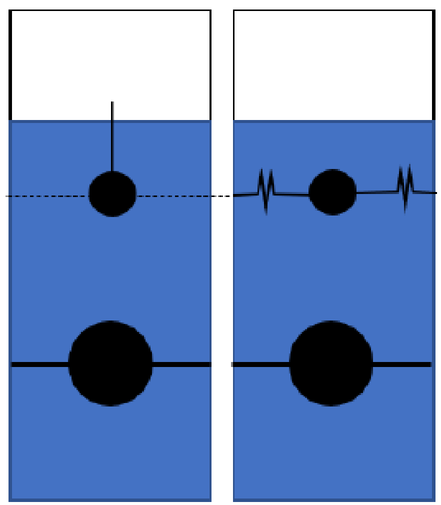


(a) k_3 or $\bar{g}_0$ configuration used in other literature

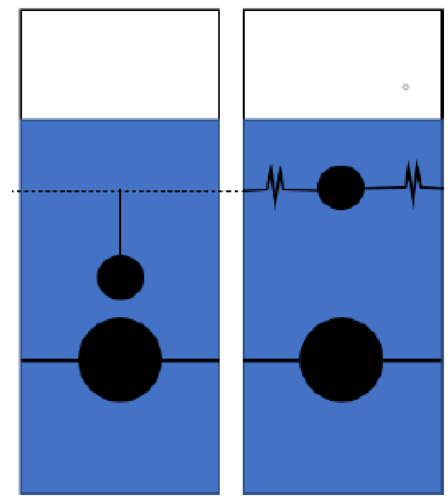
(b) The moment arm effect of $\vec{r}_{sjz}$ due to $\vec{a}_b$.

Case (a) and (b) Comparison

- Case (a)—where pendulum mass coincides with MSD mass location—and case (b)—where pendulum hinge point coincides with MSD mass location—are compared using the simulated test harness to address and clarify common misconceptions.

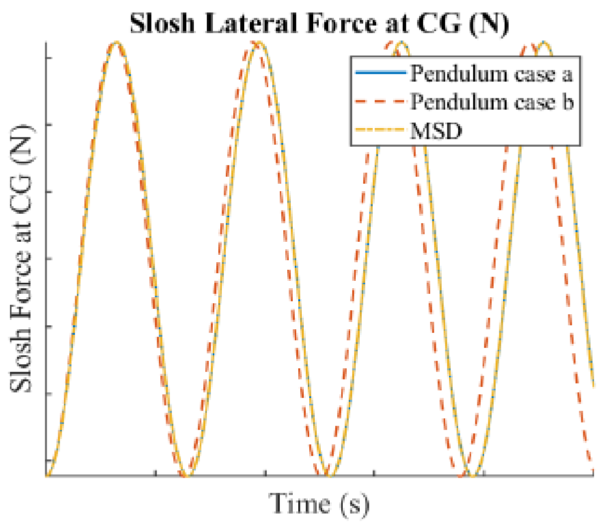


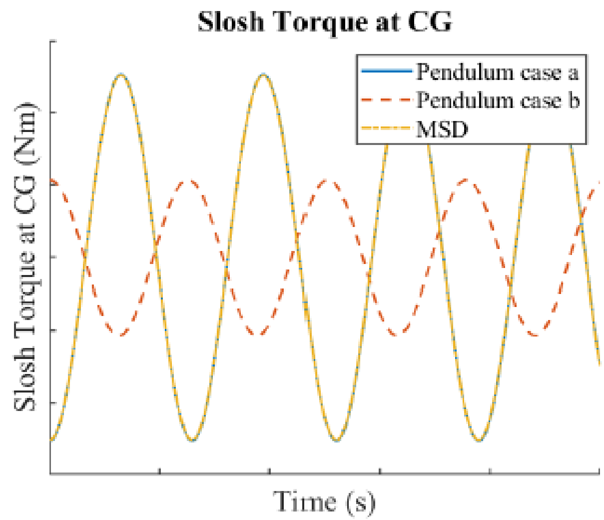
(a) Pendulum mass and mass-spring mass located on the same height

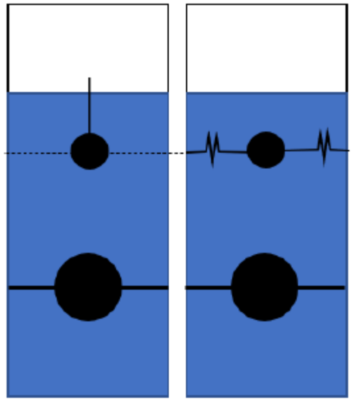


(b) Pendulum hinge point and mass-spring mass located on the same height

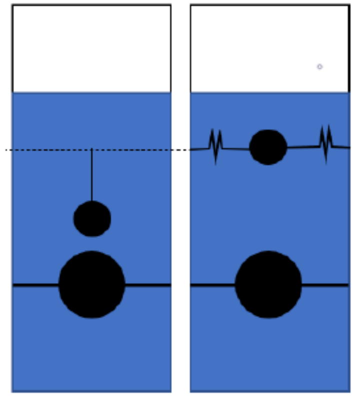
Case (a) and (b) Comparison







(a) Pendulum mass and mass-spring mass located on the same height

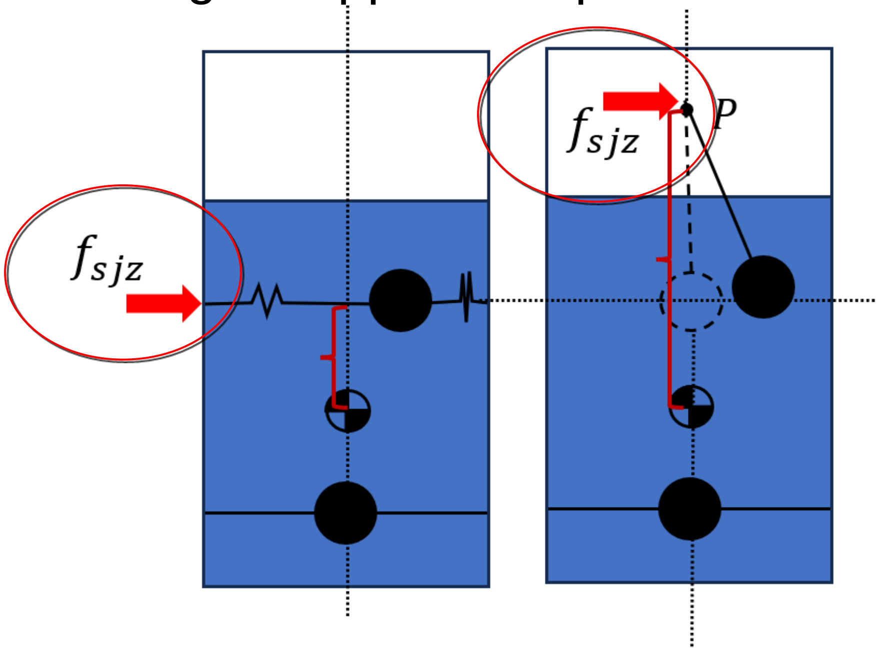


(b) Pendulum hinge point and mass-spring mass located on the same height

Case (a) shows match in both torque and force

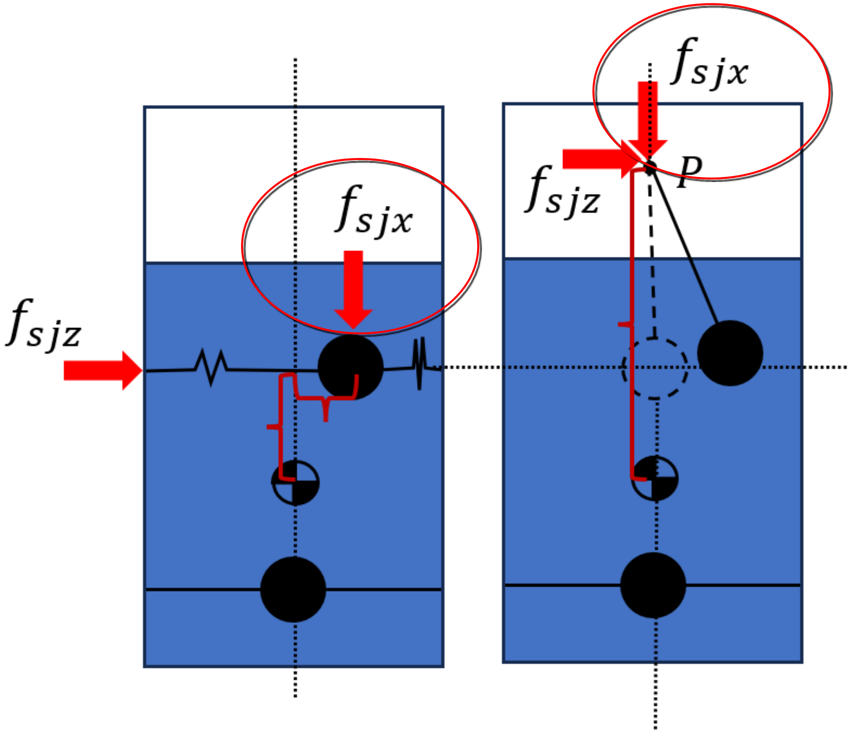
Case (a) and (b) Comparison

- It is well-understood that the lateral forces from the two analogies are equivalent.
- However, these forces are applied at different locations: the MSD mass location for the MSD analogy and the pendulum hinge point location for the pendulum analogy.
- When considering only the lateral force effects, the pendulum model is expected to produce a higher applied torque on the vehicle compared to the MSD model.



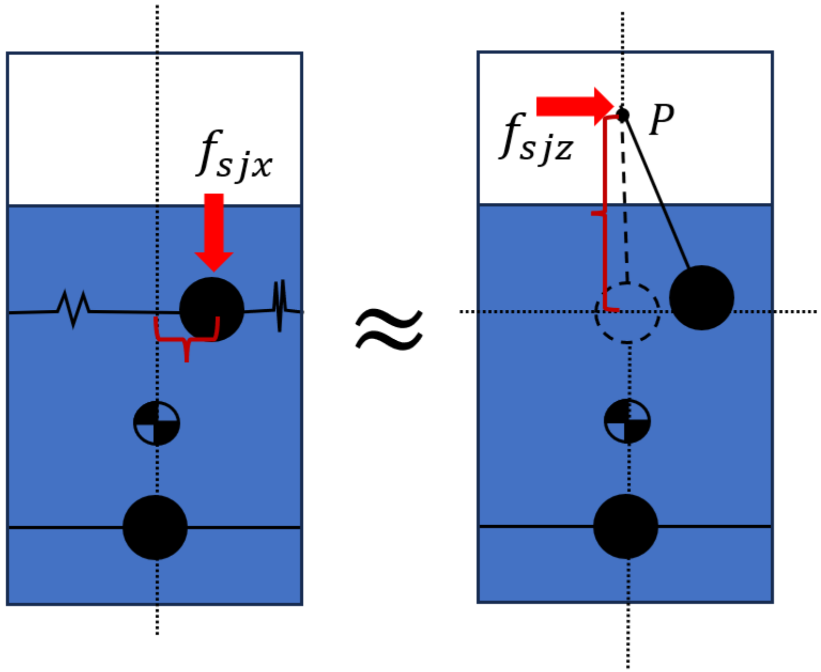
Case (a) and (b) Comparison

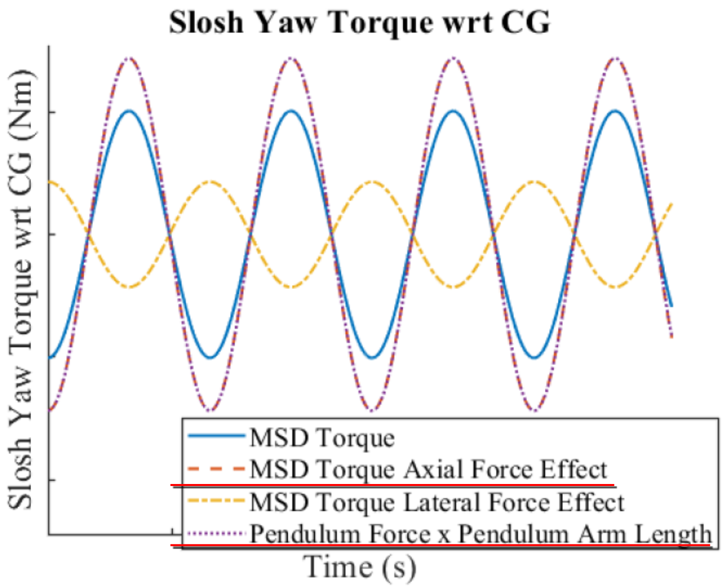
- However, the torque due to axial acceleration effects must also be considered.
- While the MSD model accounts for the existing torque due to the lateral offset of the slosh, the pendulum model shows zero torque because the pendulum hinge point is fixed at the centerline.



Case (a) and (b) Comparison

- MSD torque due to the axial force effect coincides with pendulum force multiplied by the pendulum arm length.
- As a result, Case (a) yields coincident torque despite the pendulum model having a longer vertical moment arm.
- This demonstrates that EOM with pendulum model inherently captures the k_3 effect, or the slosh lateral offset effect.

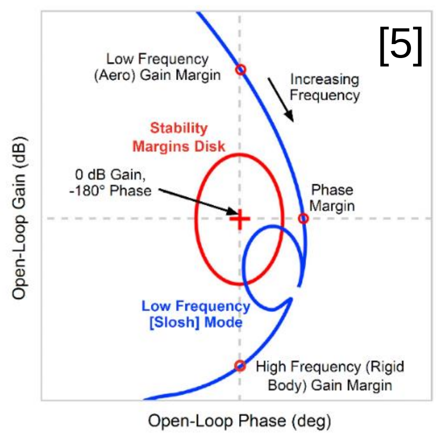




MSD torque due to axial force effect coincides with pendulum force x pendulum arm length

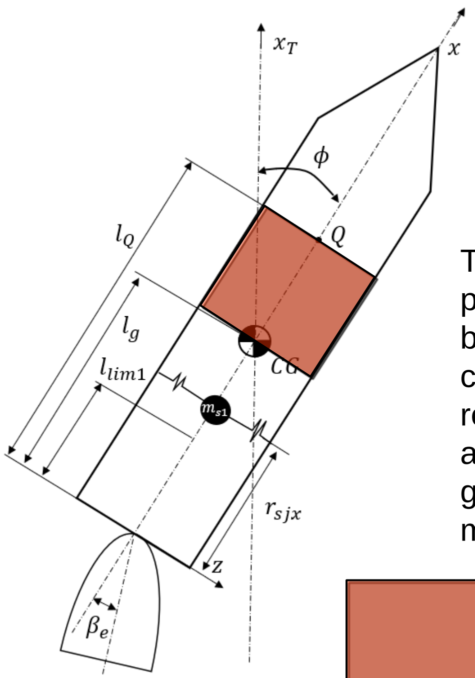
Danger Zone Background I

- During the Saturn Era, a rule-of-thumb known as the slosh danger zone was established.
- As shown in Bauer's paper [9], if the slosh is located between the center of mass of the vehicle and the center of percussion, the slosh becomes unfavorably phased, creeping toward the critical point (0 dB Gain, -180 deg Phase) for instability.



$$0 < r_{sjx} < -\frac{I_T}{m_T l_g}$$

Measured w.r.t. the center of gravity



The center of percussion is defined by the instantaneous center of rotation in response to rotation about the center of gravity and translational movements.

:Slosh Danger Zone

[5] T. VanZwieten, J. Wall, C. Dennehy, D. Dyer, R. Hall, W. Benson, and J. Pei, "Consideration Regarding the Treatment of Launch Vehicle Flight Control Stability Margin Reductions with Emphasis on Slosh Dynamics," AAS 23-151.

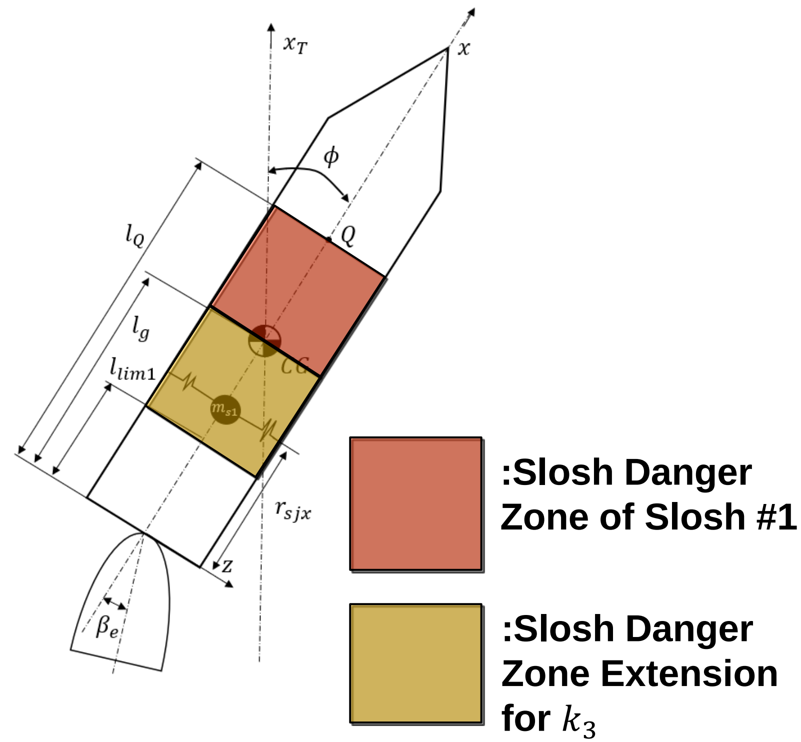
[9] H. F. Bauer, "Stability Boundaries of Liquid-Propelled Space Vehicles with Sloshing," *Journal of Spacecraft*, Vol. 1, No. 7, 1967, pp. 1583–1589.

Danger Zone Background II

- The recent advances replaced lower limit of the danger zone with a new limit defined by $\frac{k_3(m_T-m_{sj})}{m_T\omega_{sj}^2}$ [4].
- The inclusion of the k_3 term expands the slosh danger zone, thereby providing a more accurate representation of the risk instability.

$$-\frac{k_3(m_T-m_{sj})}{m_T\omega_{sj^2}} < r_{sjx} < -\frac{I_T}{m_T l_g}$$

Measured w.r.t. the center of gravity



[4] J. A. Ottander, R. A. Hall, and J. F. Powers, "Practical Methodology for the Inclusion of Nonlinear Slosh Damping in the Stability Analysis of Liquid-propelled Space Vehicles," *AIAA SciTechForum*, No. 2018-2097, 2018.

Danger Zone For Pendulum Model

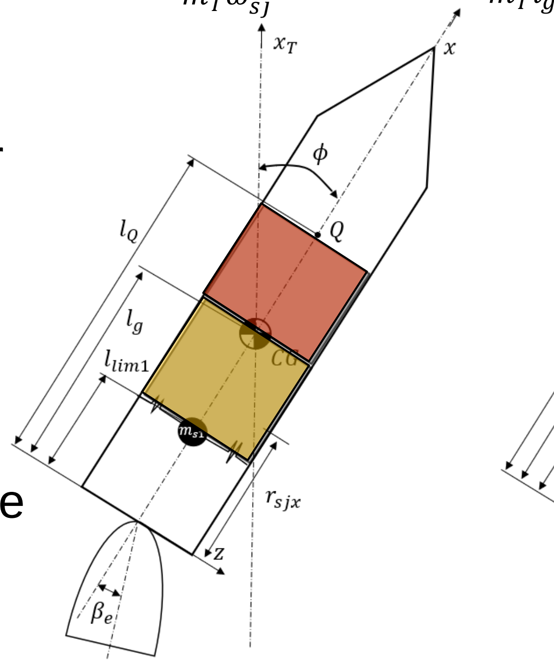
- Assuming $m_T \gg m_{sj}$, it is interesting to observe that aft limit term is equal to the length of the pendulum

$$\frac{k_3}{\omega_{sj}^2} = L_{Pj}$$

- This indicates that using pendulum hinge point as a reference point for the danger zone analysis is valid, provided the k_3 term aft limit extension is not applied.
- The upper boundary $-\frac{I_T}{m_T l_g}$ still relies on slosh mass location for the pendulum case, as shown in [7].
  - To use slosh pendulum hinge point as the analysis reference, simply add pendulum length to the upper boundary.

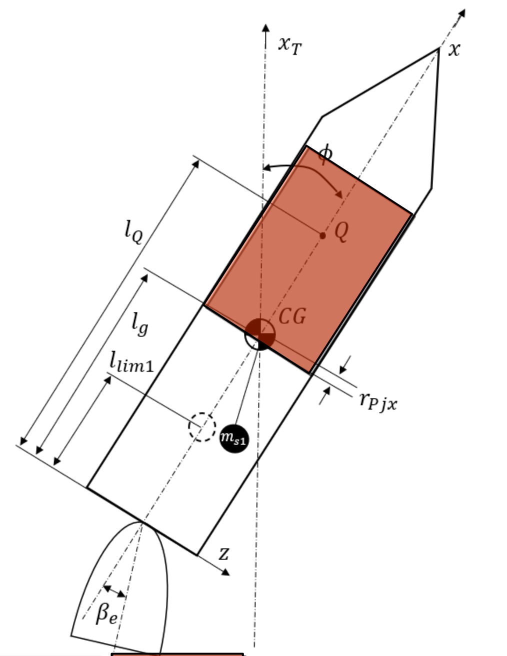
Danger zone for mass spring damper

$$-\frac{k_3(m_T - m_{sj})}{m_T \omega_{sj}^2} < r_{sjx} < -\frac{I_T}{m_T l_g}$$



Danger zone for pendulum

$$0 < r_{Pjx} < -\frac{I_T}{m_T l_g} + \frac{k_3}{\omega_{sj}^2}$$



Appropriate danger zone limits for two analogies

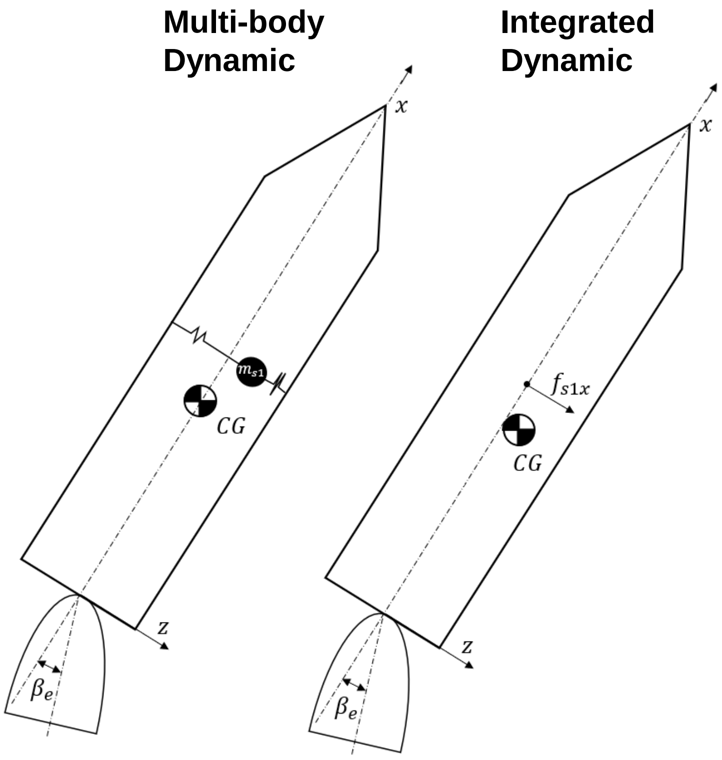
:Slosh Danger Zone of Slosh #1

:Slosh Danger Zone Extension for k_3

[7] J. Pei, "Analytical Investigation of Propellant Slosh Stability Boundary on a Space Vehicle," *Journal of Spacecraft and Rockets*, Vol. 58, No. 5, 2021.

Implications to the Common Modeling Practice

- Integrated dynamics consider the slosh mass and vehicle mass as a single integrated body whereas true multibody dynamic considers these to be separate bodies.
- Integrated dynamics inherently account for the k_3 effect (torque due to the lateral slosh mass offset) because the combined center of the gravity is offset from the center.
- When using the MSD approach only lateral force input, the integrated dynamic captures k_3 effect accurately.
- However, if the lateral force input is used with pendulum approach, it can result in double accounting of the k_3 effect.
- To prevent double bookkeeping in integrated dynamics, applying the case (b) analogy—locating pendulum hinge point at the MSD mass—can provide mitigations.



Conclusions

- **Objective:**
  – Re-derived equations of motion to capture non-linearity and secondary effects in slosh dynamics for two mechanical analogies for lunar landing vehicles and gain deeper insights into modeling gaps.
- **Key Findings:**
  – Pendulum and MSD math models validated via MathWorks® Simscape™ Multibody™ toolbox.
  – Pendulum mass at MSD level accurately represents resultant forces and torques.
  – Torque effects from lateral slosh offsets does not need to be treated in EOM involving pendulum model for multi-body simulation or analysis such as danger zone analysis; however, additional attention needs to be considered when implementing pendulum model for the integrated dynamic.
- **Future Work:**
  – Validate equations for multi-slosh systems and higher DOF constraints.
  – Explore nonlinear damping effects and refine pendulum damping terms.
  – Analyze fast maneuvers and off-centerline multi-tank configurations.

Thank you

Thanks to...
Brock Cazalet, Justin Ganiban, Jing Pei, Jorge Munoz-Burgos, Mark Jackson and John McCullough

Any questions?