

Stabilized mascon solutions through combinations of GRACE and SLR normal equations

M.J. Croteau , B.D. Loomis  and T.J. Sabaka 

Geodesy and Geophysics Laboratory, NASA Goddard Space Flight Center, Greenbelt, MD, 20771, USA. E-mail: michael.j.croteau@nasa.gov

Accepted 2025 January 7. Received 2024 January 6; in original form 2025 October 29

SUMMARY

A new global mascon solution using the Gravity Recovery and Climate Experiment (GRACE) and GRACE Follow-On data is co-estimated with satellite laser ranging (SLR) measurements to seven major geodetic satellites. This combined solution is compared with an otherwise similar GRACE-only solution to determine improvements in the estimate. We find similar performance between both solutions in the recovered mass change, but significant improvements in the associated errors in the combination solution. Errors in recovered basin mass change are 10–20 per cent better for the combination solution across all basin sizes, with the greatest improvements in high-latitude ice sheets. These results lead to our recommendation that all GRACE Level 3 mascon and spherical harmonic user-oriented gridded solutions should include SLR information during the solution inversion. As an ancillary contribution, we also provide validation of the choice of truncated spherical harmonics used in determining GRACE Technical Note 14, the current recommended mechanism for including SLR information with GRACE solutions in post-processing.

Key words: Global change from geodesy; Satellite geodesy; Time variable gravity.

1 INTRODUCTION

The GRACE and GRACE Follow-On missions, hereafter ‘GRACE(-FO)’, have recorded spatiotemporal changes in Earth’s gravity field since 2002 (Tapley *et al.* 2004; Landerer *et al.* 2020). These missions have been instrumental in providing global records of changes in terrestrial water storage, ice sheets and glaciers, solid earth and more, resolving changes on monthly timescales at approximately 300 km spatial resolution (Tapley *et al.* 2019; Landerer *et al.* 2020). However, both missions have shown poor performance recovering changes in Earth’s oblateness, J_2 (or $C_{2,0}$ in spherical harmonics), possibly due to ocean tidal aliasing or β' sun-orbital plane angle effects (Loomis *et al.* 2019b), and since late in the GRACE mission both have likewise had difficulty recovering changes in the ‘pear shape’ of the Earth, J_3 (or $C_{3,0}$) (Landerer *et al.* 2020).

Satellite laser ranging (SLR) has long provided an independent estimate of changes in $C_{2,0}$, with records dating to the launch of Starlette in 1975. As additional SLR satellites have launched (including Stella, Ajisai, LAGEOS-1 and -2, LARETS and LARES), SLR estimates of low-degree/order spherical harmonics have improved, and SLR estimates of $C_{2,0}$ have exceeded the quality of those from GRACE(-FO). For this reason, it has long been recommended that GRACE(-FO) estimates of $C_{2,0}$ be discarded and replaced by those from SLR. More recently, high-quality SLR estimates of $C_{3,0}$ since the launch of LARES in 2012 have been demonstrated to outperform worsening GRACE(-FO) estimates. As a result, it is usually recommended that GRACE(-FO) estimates of $C_{2,0}$ for all months

and of $C_{3,0}$ after 2016 June be discarded and replaced by SLR using a product termed GRACE Technical Note 14 (TN-14; Loomis *et al.* 2019a; Landerer *et al.* 2020). This ‘GRACE and Replace’ method has been widely performed with both Level 2 spherical harmonic solutions and Level 3 mass concentration (mascon) products [e.g. from NASA Goddard Space Flight Center (GSFC), the Jet Propulsion Laboratory (JPL), and the University of Texas at Austin’s Center for Space Research (CSR)]. While this replacement improves the overall quality of all of these time-variable gravity solutions, such replacement is not robust. Errors in the recovered time-variable gravity field associated with recovered $C_{2,0}$ and $C_{3,0}$ including both noise and correlations between these and other recovered terms, are maintained within the resulting gravity field after replacing these terms with SLR, and errors from SLR are likewise not properly mapped into the solution.

In order to robustly include SLR as part of a solution, a combined solution must be estimated incorporating all information from both GRACE and SLR. The inclusion of SLR has a stabilizing effect on the long-wavelength portion of the spectrum, where SLR is most sensitive (Kvas *et al.* 2021). Past studies have demonstrated that combinations of spherical harmonic gravity coefficients from GRACE, SLR, and other missions at the normal equation level further have positive impacts on recovered harmonics across many degrees and orders (e.g. Meyer *et al.* 2019; Kang *et al.* 2022; Weigelt *et al.* 2024). While some available solutions have been designed to combine these data types, including the French National Centre for Space Studies (CNES)/Space Geodesy Research Group (GRGS)

monthly and 10-d solutions and the GOCO06S time-variable gravity model (Lemoine *et al.* 2019; Kvas *et al.* 2021), this has not become common practice and no mascon solution to date has incorporated SLR in a robust way. Here, we develop a first of its kind mascon solution robustly estimated from both GRACE(-FO) and SLR and fully incorporating the information content from each. We then show that this solution, which benefits from both the combination of GRACE(-FO) and SLR and the parametrization and regularization of the gravity field as surface mascons, shows 10–20 per cent improvements in basin mass change recovery for all types of basins across all observable basin sizes. Finally, we recommend all GRACE(-FO) mascon solutions adopt similar GRACE + SLR combination strategies.

2 METHODS AND DATA

In this study, we compute two monthly mascon solutions, one using only information from GRACE(-FO) in the solution inversion (referred to as “GRACE-only” from here) and one combining GRACE(-FO) and SLR information in the solution inversion (referred to as ‘GRACE + SLR’ from here). Both solutions are computed using the fast mascon technique (Croteau *et al.* 2021), as explained in Section 2.1. For each solution after the mascon inversion, we apply the necessary post-processing steps to match each solution to the same standards as the widely available GSFC, JPL and CSR mascon solutions. For the GRACE-only solution, this includes replacing the solution’s $C_{2,0}$ terms for all times and $C_{3,0}$ terms for all times beginning with 2016 July with those from SLR using TN-14 (Loomis *et al.* 2020). For the GRACE + SLR solution, we do not need to do this step since SLR and GRACE spherical harmonics are combined at the normal equation level prior to mascon estimation (as explained in more detail in Section 2.2). For both solutions, we restore the forward modelled ocean bottom pressure signal and subtract the mean atmospheric mass over the ocean, meaning ocean results reflect sea level anomaly.

2.1 Fast mascon estimation

Mascon solutions are determined using the ‘fast mascon’ method (Croteau *et al.* 2021). This method estimates mascons from converged spherical harmonic solutions and their full normal equation information, and is equivalent to robust mascon estimation methods from Level-1B ranging observations as done for the CSR and GSFC mascon solutions (Save *et al.* 2016; Loomis *et al.* 2019a), while also seeing significant gains in simplicity and computational efficiency. The fast mascon method solves the equation:

$$\hat{\mathbf{m}} = \mathbf{P}^{-1} \mathbf{L}^T (\lambda \mathbf{N}^{-1} + \mathbf{L} \mathbf{P}^{-1} \mathbf{L}^T)^{-1} \mathbf{c}, \quad (1)$$

where $\hat{\mathbf{m}}$ is the estimated set of mascons, \mathbf{c} is a converged estimate of spherical harmonics, \mathbf{N} is the normal equation matrix associated with \mathbf{c} , \mathbf{L} is the partial derivatives of the differential spherical harmonics coefficients with respect to the mascon parameters, \mathbf{P} is the mascon regularization matrix and λ is a weighting factor applied to the regularization. A full derivation and computationally stable formulation of this equation is in Croteau *et al.* (2021).

For this study, we follow the formulation and all solution parametrization methods and choices in Croteau *et al.* (2021), using the same set of 41 168 mascons (also used in the GSFC mascon solution), the same mascon regularization and the same method for determining the regularization scaling λ . Core details of that formulation have been reproduced in Section S1 of the Supporting

Information for convenience. We extend the GRACE-only solution from that paper forward in time to now include through 2023 July (previously it only extended through 2021 April) and compare it with a new GRACE + SLR solution also through 2023 July. This new GRACE + SLR solution first combines GRACE and SLR spherical harmonic normal equations (see Section 2.2) and then applies the fast mascon method (using the same parametrization as the GRACE-only solution) to estimate regularized GRACE + SLR mascons. This allows this study to isolate the effects of the GRACE + SLR combination, and we can make direct comparisons between the GRACE-only and GRACE + SLR solutions to evaluate any impacts of the inclusion of SLR.

2.2 Combination of GRACE and SLR spherical harmonic normal equations

We combine the GRACE and SLR solutions at the normal equation level by solving the equation

$$(w_G \mathbf{N}_G + w_S \mathbf{N}_S) \mathbf{c} = w_G \mathbf{n}_G + w_S \mathbf{n}_S \quad (2)$$

for the combined spherical harmonics from the sets of GRACE(-FO) and SLR normal matrices, \mathbf{N}_G and \mathbf{N}_S , and right hand side vectors of these normal equation systems, \mathbf{n}_G and \mathbf{n}_S , as well as their respective data weights, w_G and w_S . This method is well understood and has been shown by previous studies to effectively combine solutions from GRACE, SLR and other missions (Cheng & Ries 2017; Meyer *et al.* 2019; Kang *et al.* 2022; Weigelt *et al.* 2024). To select the GRACE and SLR weighting factors, we set w_G to 1.0 and tested values for w_S from 0.01 to 1000. Ultimately, $w_S = 10$ was selected because this was the smallest weighting that allowed SLR to drive recovery $C_{2,0}$ for all times and $C_{3,0}$ during the GRACE-FO era (after the launch of LARES), which matches the method typically employed using TN-14 where SLR replaces GRACE for these terms. GRACE information is included for all spherical harmonics from degree 2 through 96, while SLR information is included for all spherical harmonics from degree 2 through 10. Note that $C_{2,0}$ and $C_{3,0}$ in TN-14 are estimated as part of a set of spherical harmonics that includes only coefficients for degrees 2 through 5 plus $C_{6,1}$ and $S_{6,1}$. Fig. S2 in the Supporting Information Section S2 depicts this breakdown.

Section S3 of the Supporting Information includes a breakdown of the relative weighting between the GRACE and SLR data types for each coefficient, computed from the weighted variance-covariance matrices for both. Also provided in that portion of the supplement are plots of the resulting relative contributions for GRACE and SLR to each term, following the procedure in Sneeuw *et al.* (2000) and Meyer *et al.* (2019).

GRACE-only solutions contain correlations between estimated spherical harmonic coefficients due to limitations of the observing platform at even the lowest degrees and orders, manifest as clusters of correlated coefficients around pairs of zonal coefficients as shown in Fig. 1 (left). In the SLR-only solution, higher degree/order harmonics are very noisy and introduce correlation issues with lower-degree terms (e.g. between even zonals and their successive tesseral terms) as shown in Fig. 1 (centre). By combining GRACE(-FO) and SLR normal equations, however, we are able to mitigate correlations in each data type while allowing both data types to contribute whatever information they contain to the estimate for all coefficients, as shown in Fig. 1 (right) and Section 3. In fact, we see from Fig. 1 that correlations between individual coefficients are reduced for all coefficients with the exception of the $C_{3,0}/C_{5,0}$ pair, where already

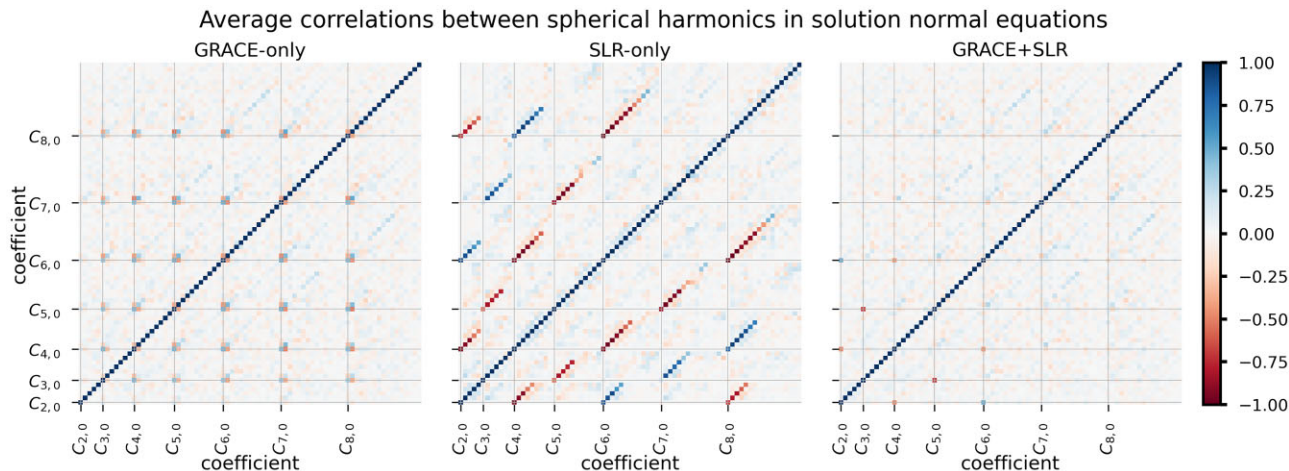


Figure 1. Average correlations between spherical harmonic coefficients in monthly gravity fields from GRACE-only (left), SLR-only (centre) and GRACE + SLR (right) solutions. Correlations between all pairs of coefficient are significantly reduced except for between $C_{3,0}$ and $C_{5,0}$. The x- and y-axes are ordered by degree, then order, then type (i.e. $C_{2,0}$, $C_{2,1}$, $S_{2,1}$, $C_{2,2}$, $S_{2,2}$, $C_{3,0}$, $C_{3,1}$, etc.).

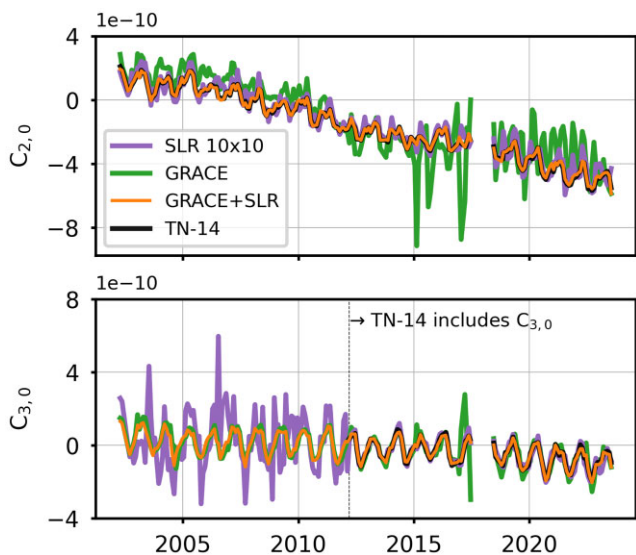


Figure 2. Comparison of recovered $C_{2,0}$ and $C_{3,0}$ terms from GRACE(-FO), SLR 10×10 , TN-14 and combined GRACE + SLR spherical harmonics. The GRACE + SLR solution nearly perfectly recovers the $C_{2,0}$ and $C_{3,0}$ results from TN-14 without needing to limit contributing SLR terms.

large negative correlations from the SLR solution are made slightly worse in the GRACE + SLR solution, and all clusters of correlated coefficients are eliminated for degree 2–10 terms.

Here, we note that our results agree with the results from Cheng & Ries (2017), which shows correlations between $C_{2,0}$ and $C_{4,0}$ for a GRACE + SLR solution are reduced compared to an SLR-only solution. However, our results are ultimately a departure from that paper’s conclusion that such a combination is not helpful. Given the advancements in both GRACE and SLR processing, it makes sense that those conclusions should be re-evaluated, and we show evidence that continuing to replace $C_{2,0}$ from GRACE with that from SLR in post-processing is not optimal today. By examining correlations between many additional terms and by looking at the compounding effects of both GRACE + SLR combination and mascon regularization, we show that a combined GRACE + SLR mascon solution provides added stability in gravity field recovery

(see Section 3). These results agree well with more recent work, for example (Kang *et al.* 2022), and highlight that the combination in Cheng & Ries (2017) did actually produce a better solution even if that was not recognized at the time.

2.3 Data

The fast mascon method requires the use of a converged spherical harmonic solution and its associated normal equations. In this study, the GRACE(-FO) solution used is the ITSG-Grace2018 spherical harmonic solution and full normal equations to degree/order 96 (Mayer-Gürr *et al.* 2018; Kvas *et al.* 2019). We use this solution because it matches the quality of the GRACE Level 2 spherical harmonic solutions released by the GRACE Science Data System team while additionally providing the full normal equation information freely and openly. The SLR normal equations that are used are those from the GSFC SLR solution (to degree/order 10) used in the production of TN-14. These normal equations were provided by GSFC for use in this study, and include SLR tracking observations from the following satellites: Starlette, LAGEOS-1, AJISAI, LAGEOS-2, Stella, Larets (beginning 2003 September), LARES (beginning 2012 February) and LARES-2 (beginning 2022 July). Of note, the TN-14 SLR-only solution is tailored around best recovery of $C_{2,0}$ and $C_{3,0}$ for replacement of the corresponding GRACE terms in post-processing, and that solution only includes degrees 2 through 5 plus $C_{6,1}$ and $S_{6,1}$ in the estimate. This choice of a truncated set of harmonics (relative to our own inclusion of SLR data as a 10×10 field) is a design space in its own right and has evolved over time, and we instead use the full 10×10 gravity field because this provides better combinations with GRACE without needing to find a particular optimal subset of coefficients to use. A short comparison between resulting correlations between coefficients using a truncated field matching that in TN-14 versus the full 10×10 field is included in the Supporting Information (see Section S4). Additionally, comparisons are made to the GSFC RL06v2.0 Global Mascons (Loomis *et al.* 2019a), the JPL RL06.1v03 Global Mascons (Watkins *et al.* 2015; Wiese *et al.* 2018), the CSR RL06.2 Global Mascons (Save *et al.* 2016; Save 2020) and TN-14. As with the fast mascon solutions estimated here, the mean atmospheric mass over the oceans has been subtracted from the GSFC, JPL, and

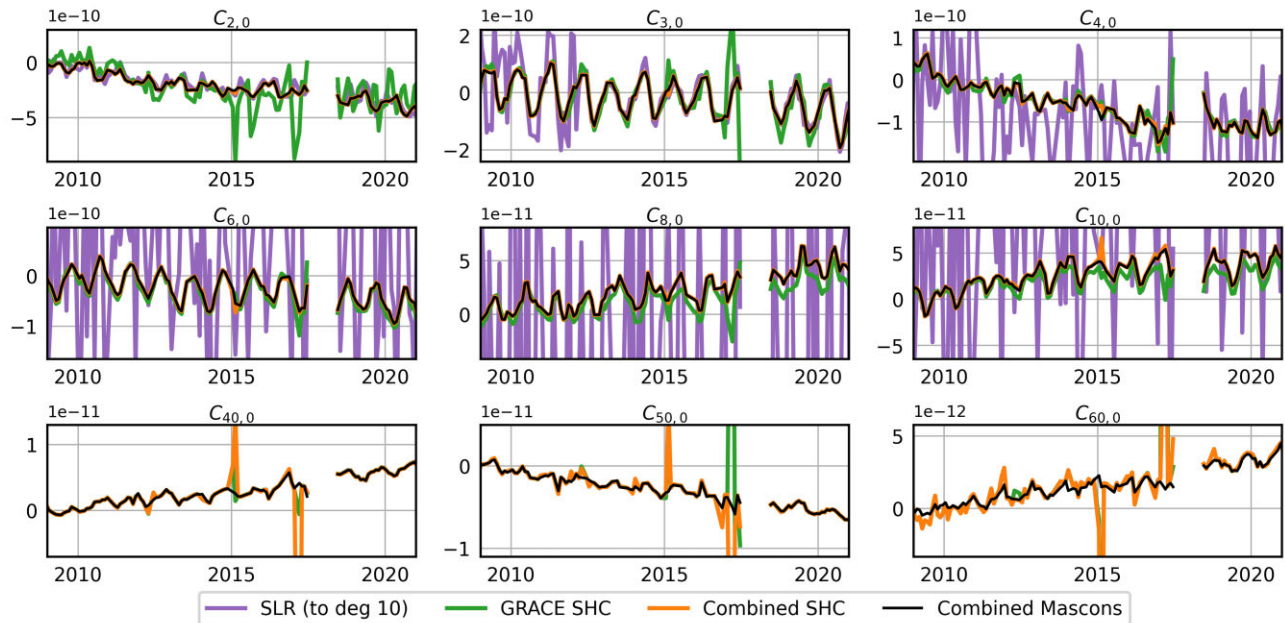


Figure 3. Recovered spherical harmonic time-series from SLR, GRACE, GRACE + SLR combined spherical harmonics and GRACE + SLR combined regularized mascons. The GRACE + SLR combination greatly improves the longer-wavelength portion of the spectrum for all times, while the mascon regularization further improves the shorter-wavelength portion for all times as well as the longer-wavelength portion for certain challenging months in a complimentary manner.

CSR mascon solutions, meaning ocean results for those solutions also reflect sea level anomaly.

3 RESULTS

While this paper primarily focuses on extending GRACE + SLR combinations to mascon solutions, the necessary first step in the fast mascon approach is to combine GRACE(-FO) and SLR spherical normal equations, which allows us to examine the effects of this combination in addition to the effects of applying mascon regularization. Fig. 2 shows a comparison of estimates for $C_{2,0}$ and $C_{3,0}$ from GRACE(-FO), SLR (10×10 spherical harmonic expansion), GRACE + SLR, and TN-14 ($5 \times 5 + C_{6,1}/S_{6,1}$). Interestingly, the combined GRACE + SLR estimate for $C_{2,0}$ matches the estimate for $C_{2,0}$ from TN-14 extremely well, despite these two solutions having significant differences in information content, as well as for $C_{3,0}$ after LARES launches in 2012. The SLR solution in the combination uses a 10×10 expansion that includes significant correlations between multiple harmonics (see Fig. 1, centre) due to observability issues as the harmonic expansion is increased in size, while TN-14 uses a truncated expansion omitting $C_{6,0}$ and all terms higher than $C_{6,1}$ and $S_{6,1}$ to eliminate the harmful effects of these correlations. These differences explain why $C_{2,0}$ and $C_{3,0}$ in the SLR 10×10 are relatively noisier than for TN-14 in Fig. 2. The reduced noise in the GRACE + SLR solution then demonstrates that this combination uses GRACE information to similarly help break correlations between such terms without needing to selectively eliminate certain harmonics from the estimate, and also provides validation for the system employed in estimating the TN-14 SLR-only solution.

After final estimation of our GRACE + SLR mascon solution, we can study the impact of regularization on the gravity field in spherical terms by converting the set of mascons back to spherical harmonics. We found that the regularization further stabilized the

solution, reducing errors across many spherical terms for challenging months and for correlated harmonics, which matches findings by GRACE-only mascon solutions. Fig. 3 shows estimates of nine sample zonal harmonics from $C_{2,0}$ to $C_{60,0}$ from SLR-only, GRACE-only and GRACE + SLR spherical harmonics, as well as GRACE + SLR mascons. Fig. 3 demonstrates that low-degree harmonics are most improved by the spherical combination and not impacted much by the mascon regularization while higher-degree harmonics are most improved by the mascon regularization and only somewhat improved by the spherical combination. We can also see that, in addition to stabilizing $C_{2,0}$ and $C_{3,0}$, the inclusion of SLR has notably changed the trend in other low-degree zonal harmonics such as $C_{8,0}$ and $C_{10,0}$. This is likely not due to the SLR-only estimates of these harmonics themselves, which are not resolved well, but rather is due to the reduction of correlations between coefficients in the GRACE + SLR solution (as discussed in Section 2.2). Similar improvements are seen for non-zonal harmonics, as in other studies.

Finally, basin mass change was computed for 20 basins varying in size and type (including ice covered, land and seas) for both the GRACE-only and GRACE + SLR mascons. Table 1, as well as Fig. S6 in Section S6 of the Supporting Information, shows few substantial changes in the recovered basin trends, annual signals, or interannual signals using the GRACE + SLR solution compared to the GRACE-only solution, though solutions for individual months in Antarctica in particular do show notable improvements. We additionally see very good agreement with both solutions for all basins when comparing to the commonly available mascon solutions from GSFC, JPL and CSR, as had been previously shown for the GRACE-only solution in Croteau *et al.* (2021). However, we importantly find that the GRACE + SLR solution shows substantially reduced month-to-month uncertainties for all basins when compared to the GRACE-only solution. These were computed following the same methods as in the GSFC monthly solution (Loomis

Table 1. Basin mass recovered for GRACE-only (A) and GRACE + SLR (B) mascons, with comparisons to GSFC (G), JPL (J) and CSR (C) mascon solutions. The temporal mean of the total basin uncertainty ($\bar{\epsilon}$) is shown for the GRACE-only and GRACE + SLR solutions, along with the percent reduction in error ($\Delta\%$) due to the GRACE + SLR combination.

	Area (km ²)	Trend (Gt yr ⁻¹)					Annual Amplitude (Gt)					$\bar{\epsilon}$ (Gt)		
		A	B	G	J	C	A	B	G	J	C	A	B	$\Delta\%$
Antarctica	13 043 374	-129	-127	-139	-140	-145	118	105	114	92	151	110.0	87.1	-20.8
Greenland	2451 975	-267	-268	-266	-271	-260	154	153	156	126	132	112.3	93.2	-17.0
Gulf of Alaska	520 064	-69	-70	-68	-66	-66	144	142	147	155	133	41.6	34.9	-16.0
Iceland	161 073	-9	-9	-8	-8	-7	18	18	17	4	11	15.0	12.9	-14.6
Sahara	5528 176	-12	-12	-9	-10	-10	30	26	27	12	22	53.8	47.3	-12.0
Congo	3791 292	16	15	20	19	17	174	175	188	161	176	51.0	43.5	-14.6
Mississippi	3308 286	8	8	8	5	6	189	194	203	193	202	44.0	38.6	-12.2
La Plata	2999 402	-3	-3	-4	-2	-4	151	152	168	162	163	52.2	45.3	-13.2
St. Lawrence	1276 012	12	12	10	11	11	53	55	59	67	64	29.6	25.9	-12.4
Murray	966 794	3	3	2	3	3	12	15	19	16	19	23.8	20.3	-14.6
Japan	619 584	3	3	-5	-1	0	29	28	30	11	13	25.9	22.7	-12.4
California	495 538	-7	-6	-7	-5	-4	48	47	43	39	40	19.4	17.3	-10.8
Lake Balkash	446 146	-2	-2	-3	-2	-3	20	20	20	16	19	14.4	13.4	-7.1
Volta	433 881	4	4	4	4	4	47	47	48	47	45	18.2	15.6	-14.4
Yucatán	260 222	1	1	1	0	1	23	23	22	19	19	14.4	12.7	-11.4
Oder	123 737	-1	-1	-1	-1	-1	7	7	7	6	6	6.2	5.3	-14.3
Hudson Bay	879 243	1	1	-0	2	0	5	7	7	11	14	37.5	32.0	-14.7
Red Sea	532 897	-0	-0	1	3	1	83	83	87	77	73	28.1	24.7	-12.2
Caspian Sea	458 489	-34	-33	-32	-32	-20	55	55	39	53	33	47.0	41.2	-12.3
Black Sea	347 114	-1	-1	-0	0	-1	18	18	17	26	25	28.7	25.6	-10.8

et al. 2019a), which has been summarized in Section S5 of the Supporting Information. Improvements to the regional mean monthly error estimates range from 15–20 per cent for ice covered basins and 10–15 per cent for nearly all hydrologic basins and seas, and this finding holds regardless of the size of the basin or the total trend or annual signal recovered in the basin.

4 DISCUSSION

This study demonstrates the impacts of including SLR information with GRACE(-FO) in estimating Level 3 mascon solutions. Following past studies illustrating positive impacts of similar combinations for Level 2 spherical harmonic solutions, we have demonstrated that significant improvements of 10–20 per cent can be made in the reduction of the overall errors in monthly basin mass change recovery from mascon solutions by allowing the long-wavelength portion of the gravity field to be informed by SLR during the solution inversion. Additional improvements are seen in individual months where GRACE(-FO) solutions are more challenging, as well as in Antarctic mass change recovery. The greatest reductions in basin uncertainties are seen for the major ice sheets, which are located at high latitudes and thus are most impacted by correlations between low-degree zonal harmonics such $C_{2,0}$ and $C_{3,0}$ and other low-degree terms. By reducing correlations between these terms, the combination solution produces a cleaner gravity field estimate with smaller biases, leading to greater uncertainty improvements at higher latitudes. These 15–20 per cent improvements in monthly glacial ice mass change errors are significant in better accounting for ice mass contributions to sea level rise and other climate change processes, though do not have large impacts on the long-term trend in ice mass loss. Additionally, high-latitude glacial regions such as the Greenland and Antarctic ice sheets experience the largest long-term trends in the GRACE record, which maps directly into larger solution bias errors (see the Supporting Information Section S5). Error reductions of 10–15 per cent for all other basins suggest that these improvements will benefit almost all GRACE applications,

regardless of basin size or location. Because of these improvements in mascon solution errors, we recommend all Centres producing GRACE Level 3 mascons or Level 3 spherical harmonic-based gridded solutions similarly incorporate SLR information into their solution inversion.

ACKNOWLEDGMENTS

This work was funded by NASA GRACE Follow-On Science Team Grant NNH23ZDA001N-GRACEFO and NASA Earth Surface And Interior Grant NNH22ZDA001N-ESI. MJC was responsible for conceptualization, data curation, formal analysis, software, visualization and writing the manuscript for this study. BDL assisted with conceptualization, data curation, methodology and review of this manuscript. TJS assisted with conceptualization, formal analysis, methodology and review of this manuscript. Thank you to Kenny Rachlin and Frank Lemoine at NASA GSFC for their expertise and assistance with SLR processing and the inclusion of the GSFC SLR normal equations.

SUPPORTING INFORMATION

Supplementary data are available at *GJIRAS* online.

Figure S1. Expected uncertainties σ 's used to populated the regularization matrix (from Croteau *et al.* 2021).

Figure S2. Spherical harmonic coefficients included in the estimate per source (through degree/order 25). Also depicted are those coefficients included in the TN-14 SLR-only estimate of $C_{2,0}$ and $C_{3,0}$.

Figure S3. Spherical harmonic σ s from the GRACE (red) and SLR (blue) variance-covariance matrices. Smaller values indicate that data type contributes more to the combination, meaning $C_{2,0}$ is primarily resolved by SLR, as is $C_{3,0}$ after the 2012 launch of LARES.

Figure S4. Contributions per spherical harmonic coefficient from GRACE and SLR for portions of the combination time span. Here, a contribution of ‘1’ indicates that the coefficient is driven by GRACE, ‘0’ indicates that the coefficient is driven by SLR, and values in between are driven by GRACE and SLR in proportion to the value.

Figure S5. Average correlations between spherical harmonic coefficients in GRACE-only, SLR-only and GRACE + SLR solutions for SLR solutions to degree/order 10 and truncated to degree/order 5 plus $C_{6,1}$ and $S_{6,1}$.

Figure S6. Basin mass change recovered from GRACE + SLR and GRACE-only solutions, with the GSFC, JPL and CSR mascon solutions shown in grey for comparison. For most months and basins, the inclusion of SLR has only small impacts on the recovered signal but has larger impacts in the reduction of the error of that signal.

Figure S7. Maps of recovered trends from GRACE-only and GRACE + SLR, as well as the difference between the two.

Please note: Oxford University Press is not responsible for the content or functionality of any supporting materials supplied by the authors. Any queries (other than missing material) should be directed to the corresponding author for the paper.

DATA AVAILABILITY

ITSG-Grace2018 normal equations are available from GFZ Data Services and the International Center for Global Earth Models (ICGEM), <https://doi.org/10.5880/ICGEM.2018.003>. The JPL RL06.1 v03 mascon solution and GRACE Technical Note 14 are available from JPL’s Physical Oceanography Distributed Active Archive Center (PO.DAAC), <https://podaac.jpl.nasa.gov>. The GSFC RL06 v2.0 mascon solution is available from GSFC, <https://earth.gsfc.nasa.gov/geo/data/grace-mascons>. The CSR RL06.2 mascon solution is available from CSR, <https://doi.org/10.15781/cgq9-nh24>.

REFERENCES

- Cheng, M. & Ries, J., 2017. The unexpected signal in GRACE estimates of C_{20} , *J. Geod.*, **91**, 897–914.
- Croteau, M.J., Sabaka, T.J. & Loomis, B.D., 2021. GRACE fast mascons from spherical harmonics and a regularization design trade study, *J. geophys. Res. Solid Earth*, **126**, e2021JB022113.
- Kang, Z., Ries, J., Bettadpur, S., Save, H. & Tapley, B., 2022. Combined Gravity Solution from SLR and GRACE/GRACE-FO. in *Geodesy for a Sustainable Earth, International Association of Geodesy Symposia*, Vol. **154**, eds Freymueller, J.T. & Sánchez, L., Springer, Cham.
- Kvas, A., Behzadpour, S., Ellmer, M., Klinger, B., Strasser, S., Zehentner, N. & Mayer-Gürr, T., 2019. ITSG-Grace2018: overview and evaluation of a new GRACE-only gravity field time series, *J. Geophys. Res. Solid Earth*, **124**, 9332–9344.
- Kvas, A., Brockmann, J. M., Krauss, S., Schubert, T., Gruber, T., Meyer, U. & Pail, R., 2021. GOCO06s—a satellite-only global gravity field model, *Earth Syst. Sci. Data Discuss.*, **31**, 99–118.
- Landerer, F.W. *et al.*, 2020. Extending the global mass change data record: GRACE Follow-On instrument and science data performance, *Geophys. Res. Lett.*, **47**, e2020GL088306.
- Lemoine, J.-M., Biancale, R., Reinquin, F., Bourgogne, S. & Gégout, P., 2019. CNES/GRGS RL04 earth gravity field models, from GRACE and SLR data. GFZ Data Services.
- Loomis, B.D., Luthcke, S.B. & Sabaka, T.J., 2019. Regularization and error characterization of GRACE mascons, *J. Geod.*, **93**, 1381–1398.
- Loomis, B.D., Rachlin, K.E. & Luthcke, S.B., 2019. Improved Earth oblateness rate revealed increased ice sheet losses and mass-driven sea level rise, *Geophys. Res. Lett.*, **46**, 6910–6917.
- Loomis, B.D., Rachlin, R.E., Wiese, D.N., Landerer, F.W. & Luthcke, S.B., 2020. Replacing GRACE/GRACE-FO C_{30} with satellite laser ranging: impacts on Antarctic ice sheet mass change, *Geophys. Res. Lett.*, **47**, e2019GL085488.
- Mayer-Gürr, T., Behzadpur, S., Ellmer, M., Kvas, A., Klinger, B., Strasser, S. & Zehentner, N., 2018. ITSG-Grace2018 - monthly, daily and static gravity field solutions from GRACE, GFZ Data Services.
- Meyer, U., Sosnica, K., Arnold, D., Dahle, C., Thaller, D., Dach, R. & Jäggi, A., 2019. SLR, GRACE and swarm gravity field determination and combination, *Remote Sens.*, **11**, 956.
- Save, H., Bettadpur, S. & Tapley, B.D., 2016. High resolution CSR GRACE RL05 mascons, *J. Geophys. Res. Solid Earth*, **121**, 7547–7569.
- Save, H., 2020. *CSR GRACE and GRACE-FO RL06 Mascon Solutions v02*, Center for Space Research, Univ. of Texas at Austin.
- Sneeuw, N., 2000. A semi-analytical approach to gravity field analysis from satellite observations, Deutsche Geodätische Kommission, C 527, München, Germany
- Tapley, B.D., Bettadpur, S., Watkins, M. & Reigber, C., 2004. The gravity recovery and climate experiment: mission overview and early results, *Geophys. Res. Lett.*, **31**, L09607.
- Tapley, B.D. *et al.*, 2019. Contributions of GRACE to understanding climate change, *Nat. Clim. Change*, **9**, 358–369.
- Watkins, M.M., Wiese, D.N., Yuan, D.-Y., Boening, C. & Landerer, F.W., 2015. Improved methods for observing Earth’s time variable mass distribution with GRACE using spherical cap mascons, *J. Geophys. Res. Solid Earth*, **120**, 2648–2671.
- Wiese, D.N., Landerer, F.W. & Watkins, M.M., 2016. Quantifying and reducing leakage errors in the JPL RL05M GRACE mascon solution, *Water Resour. Res.*, **52**, 7490–7502.
- Wiese, D.N., Yuan, D.-N., Boening, C., Landerer, F.W. & Watkins, M.M., 2018. *JPL GRACE Mascon Ocean, Ice, and Hydrology Equivalent Water Height Release 06 Coastal Resolution Improvement (CRI) Filtered Version 1.0. Ver. 1.0*, PO.DAAC, CA, USA.
- Weigelt, M. *et al.*, 2024. Bridging the gap between GRACE and GRACE Follow-On by combining high–low satellite-to-satellite tracking data and satellite laser ranging, *J. Geod.*, **98**, 84.