Camera Calibration and Alignment Metrology at Johnson Space Center's Electro-Optics Laboratory

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What is the bearing from you to your target?

image credit: NASA

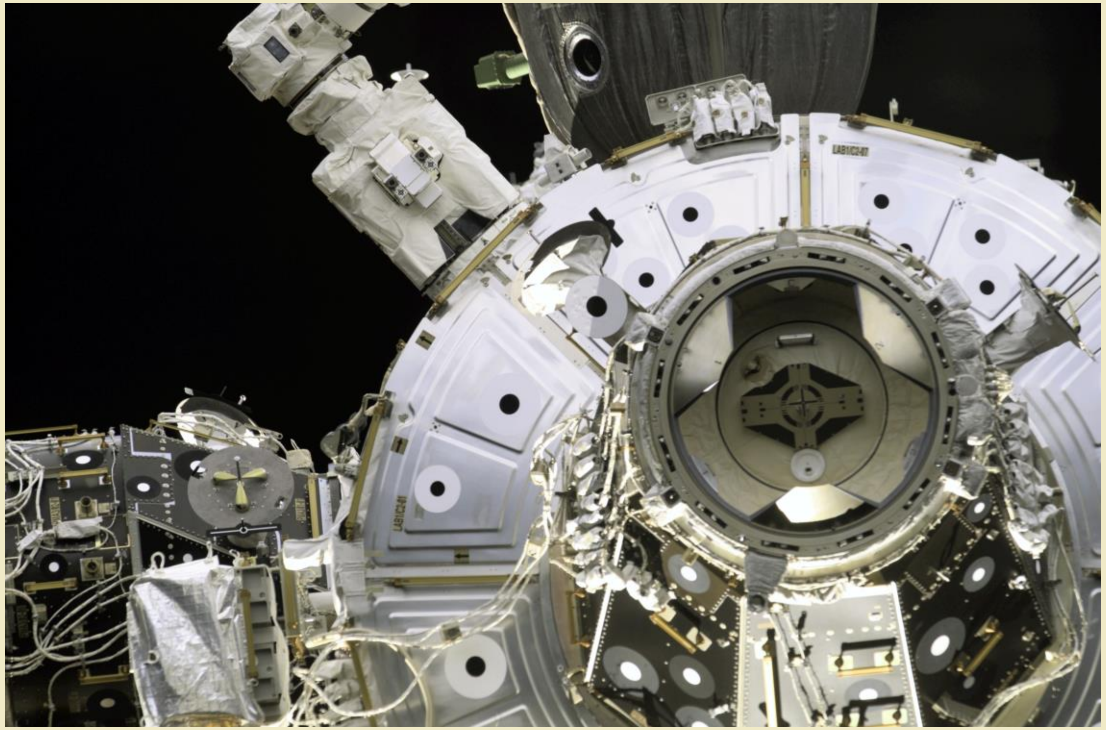

image credit: NASA

- pixels → 3D directions
- camera frame → spacecraft frame

Contents

- **Camera Model**
  - **3D directions to pixels**
  - **pixels to 3D directions**
  - **camera calibration**

- EOL Calibration Hardware
- EOL Calibration Procedure
- COTS Camera Calibration Results

- EOL Alignment Hardware
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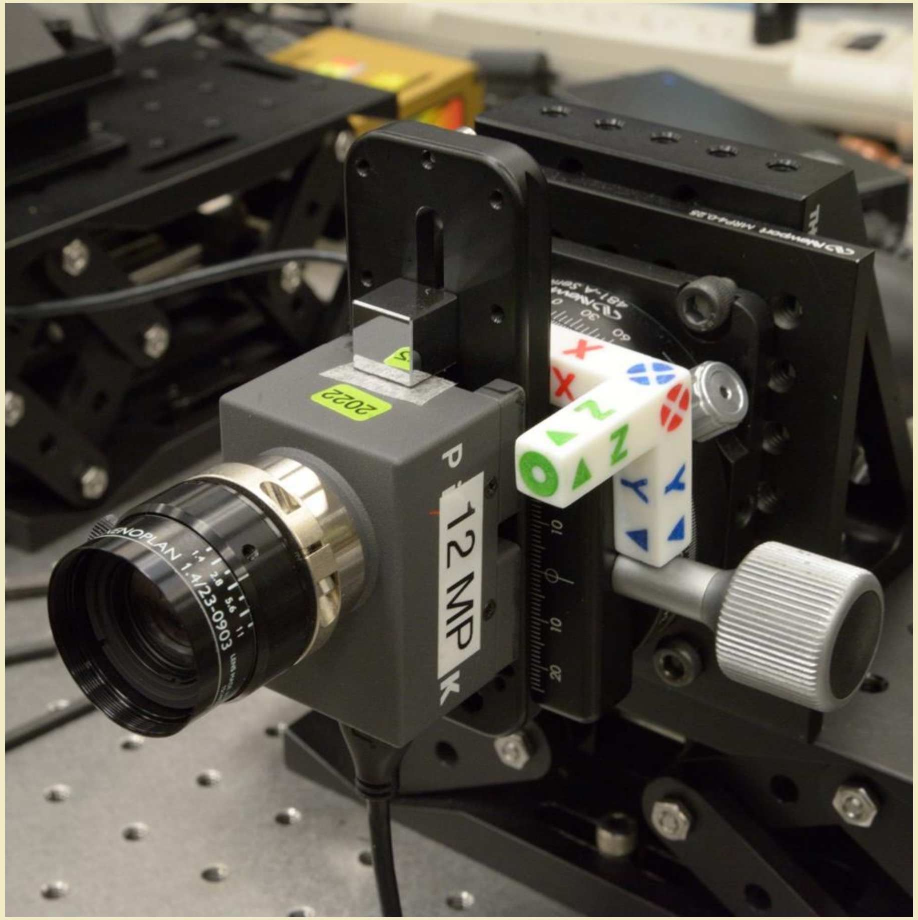
- COTS Camera Alignment Results



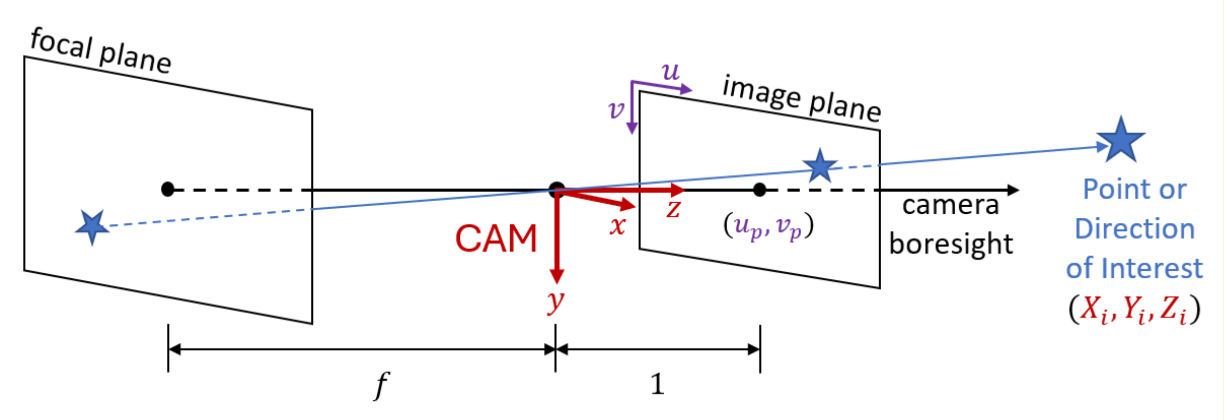
Camera Model: Intrinsic Parameters

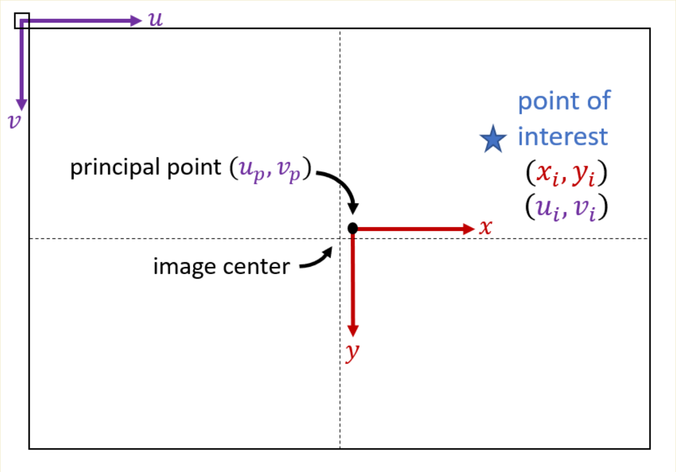
- Pinhole Camera Model

$$x_i = \frac{X_i}{Z_i} \qquad y_i = \frac{Y_i}{Z_i}$$

$$\begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} d_x & \alpha & u_p \\ 0 & d_y & v_p \\ 0 & 0 & 1 \end{bmatrix}}_{K} \begin{bmatrix} x_i' \\ y_i' \\ 1 \end{bmatrix}$$

Parameter	Description
d_x	focal length ÷ pixel width
d_y	focal length ÷ pixel height
α	detector skew
u_p	principal point u-coordinate
v_p	principal point v-coordinate



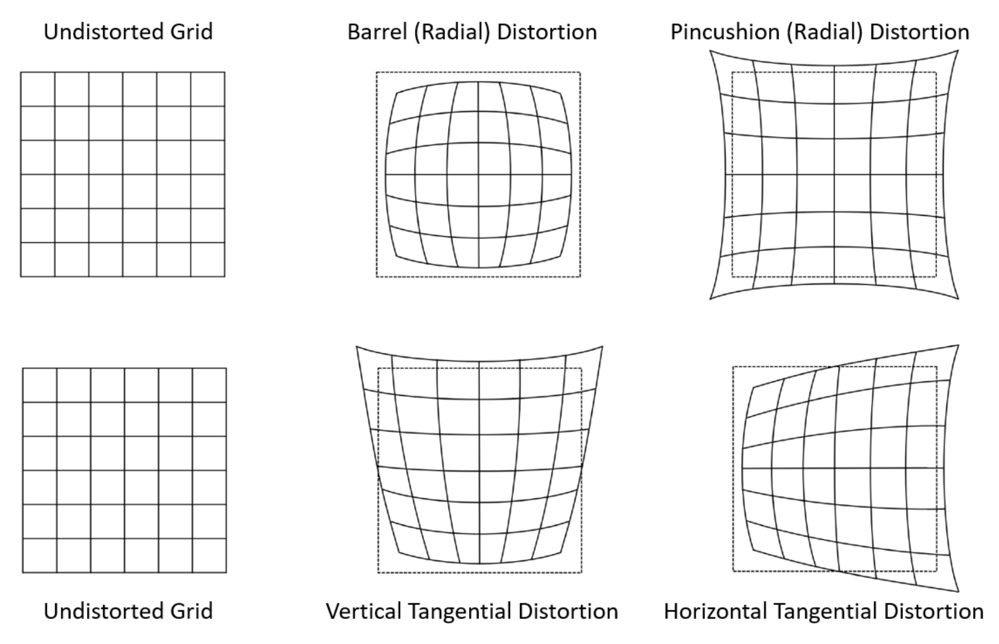


Camera Model: Distortion Parameters

- Brown-Conrady Distortion Model
- We can either un-distort individual points or build an un-distortion map for an entire image

$$\begin{bmatrix} x_i' \\ y_i' \end{bmatrix} = \begin{bmatrix} x_i \\ y_i \end{bmatrix} \left(1 + k_1 r_i^2 + k_2 r_i^4 + k_3 r_i^6\right) + \begin{bmatrix} 2p_1 x_i y_i + p_2(r_i^2 + 2x_i^2) \\ 2p_2 x_i y_i + p_1(r_i^2 + 2y_i^2) \end{bmatrix}$$

Parameter	Description
k_1	radial distortion 2nd order coefficient
k_2	radial distortion 4th order coefficient
k_3	radial distortion 6th order coefficient
p_1	horizontal tangential distortion term
p_2	vertical tangential distortion term



Camera Model: Forward/Reverse Mapping

Forward Mapping Problem

$$(X_i, Y_i, Z_i) \xrightarrow{\text{re-scale}} (x_i, y_i) \xrightarrow{\text{distort}} (x_i', y_i') \xrightarrow{K} (u_i, v_i)$$

Reverse Mapping Problem

$$\hat{e}_i^{CAM} \xleftarrow{\text{normalize}} (x_i, y_i) \xleftarrow{\text{un-distort}} (x_i', y_i') \xleftarrow{K^{-1}} (u_i, v_i)$$

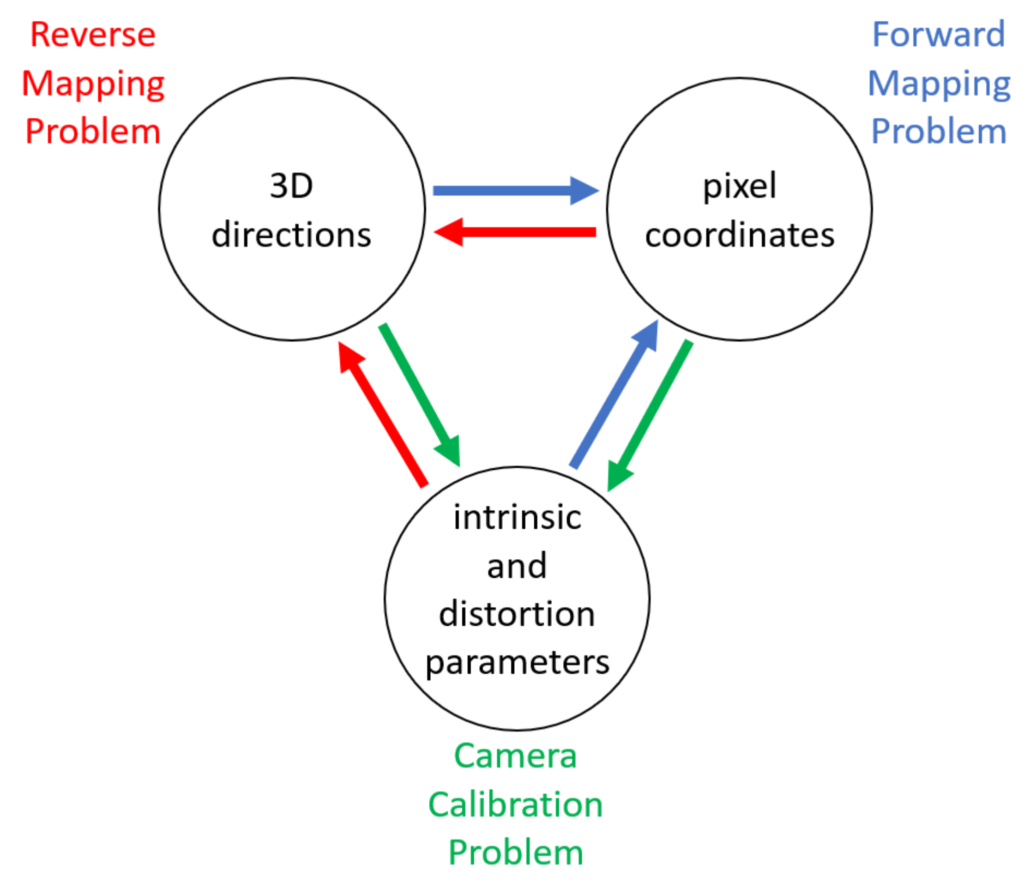
$$\hat{e}_i^{CAM} = \left\langle \begin{bmatrix} x_i & y_i & 1 \end{bmatrix}^T \right\rangle$$

iterate until convergence (usually < 5 iterations)

$$\begin{aligned}
x_{i_0} &= x_i' \\
y_{i_0} &= y_i' \\
r_{i_k}^2 &= x_{i_k}^2 + y_{i_k}^2 \\
x_{i_{k+1}} &= \frac{x_i' - 2p_1 x_{i_k} y_{i_k} - p_2\left(r_{i_k}^2 + 2x_{i_k}^2\right)}{1 + k_1 r_{i_k}^2 + k_2 r_{i_k}^4 + k_3 r_{i_k}^6} \\
y_{i_{k+1}} &= \frac{y_i' - 2p_2 x_{i_k} y_{i_k} - p_1\left(r_{i_k}^2 + 2y_{i_k}^2\right)}{1 + k_1 r_{i_k}^2 + k_2 r_{i_k}^4 + k_3 r_{i_k}^6}
\end{aligned}$$

$$K^{-1} = \begin{bmatrix} \frac{1}{d_x} & -\frac{\alpha}{d_x d_y} & \frac{v_p \alpha - u_p d_y}{d_x d_y} \\ 0 & \frac{1}{d_y} & -\frac{v_p}{d_y} \\ 0 & 0 & 1 \end{bmatrix}$$

Camera Model: Calibration

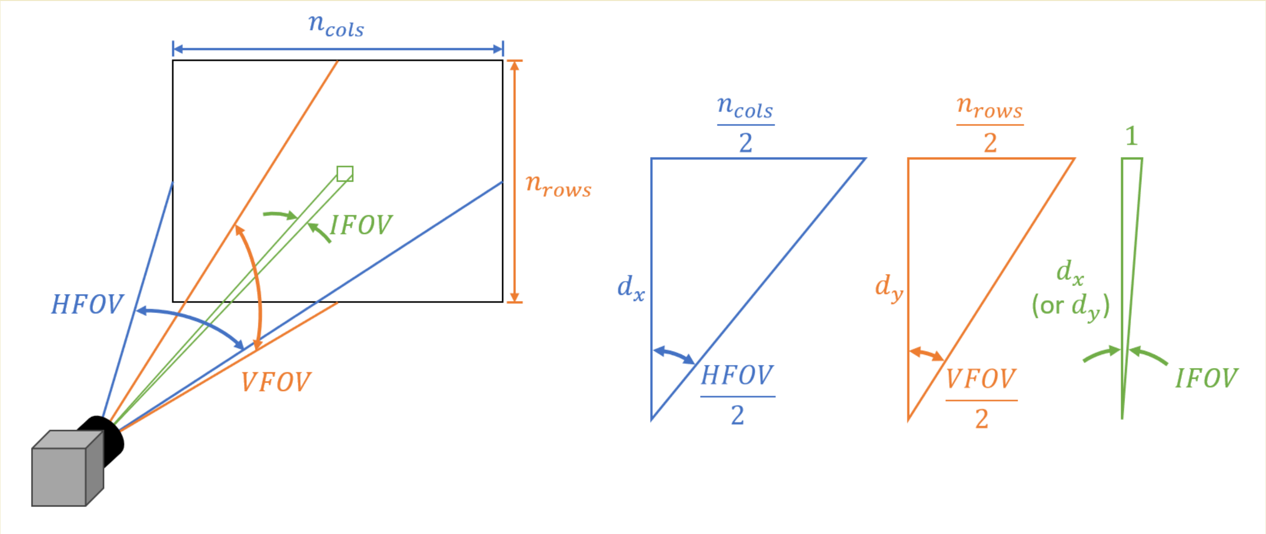


- Starting with some **initial guess** of intrinsic and distortion parameters…
- Project 3D directions into image, compute pixel coordinates
- Compare expected to measured pixel values
- Iterate on parameters to minimize sum of squared reprojection error

- Nonlinear least squares problem
- Solve with Levenberg-Marquardt Algorithm

Camera Model: Calibration – Initial Guess

Initial Guess	Justification
$d_x = \frac{n_{cols}}{2\tan(HFOV/2)}$	(see figure)
$d_y = d_x$	pixels are roughly square
$\alpha = 0$	detector skew is small
$u_p = \frac{n_{cols}+1}{2}$	principal point is near image center
$v_p = \frac{n_{rows}+1}{2}$	principal point is near image center
$k_1 = 0$	radial distortion is small
$k_2 = 0$	radial distortion is small
$k_3 = 0$	radial distortion is small
$p_1 = 0$	tangential distortion is small
$p_2 = 0$	tangential distortion is small



- Alternatively, seed with previous calibration results
- Use lab calibration results to seed on-orbit calibration

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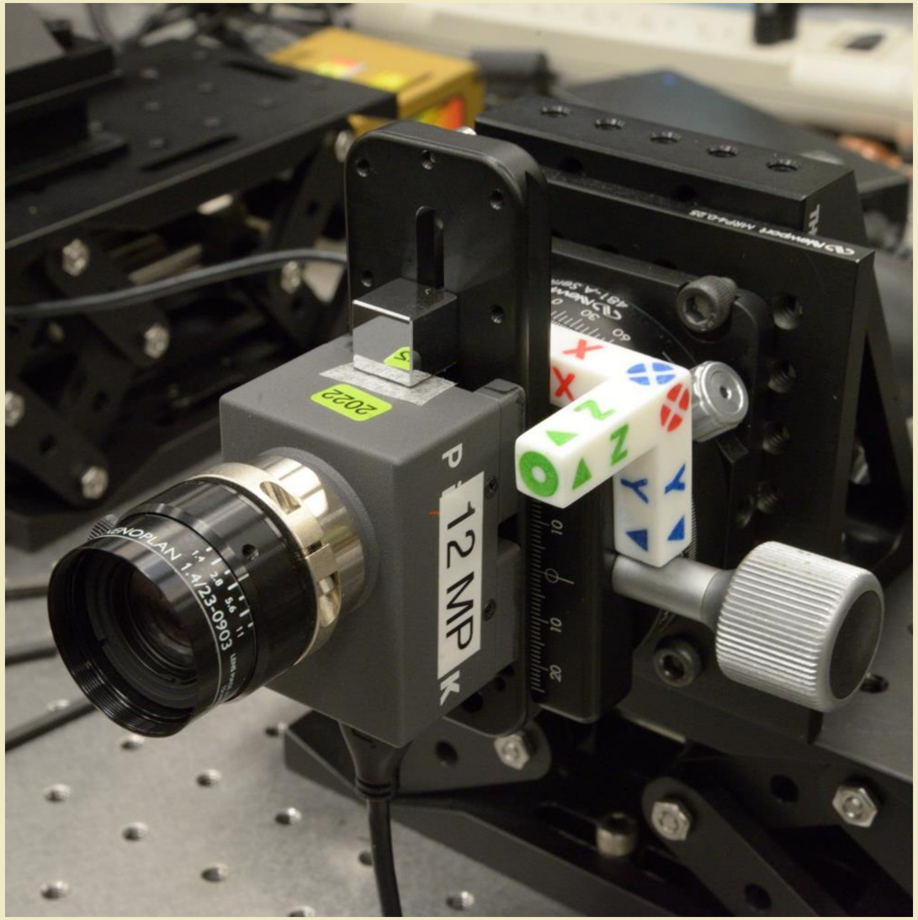
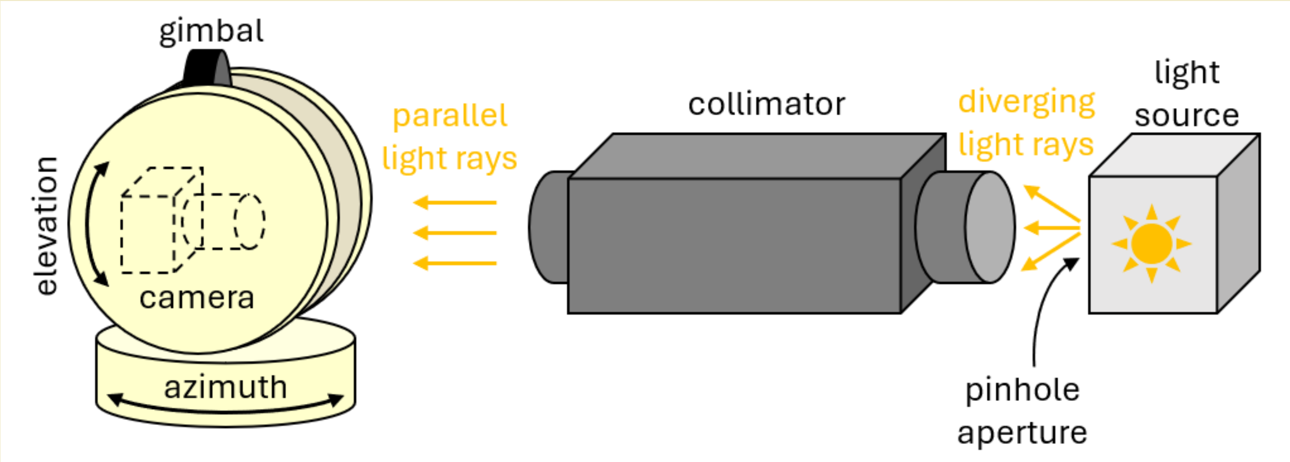
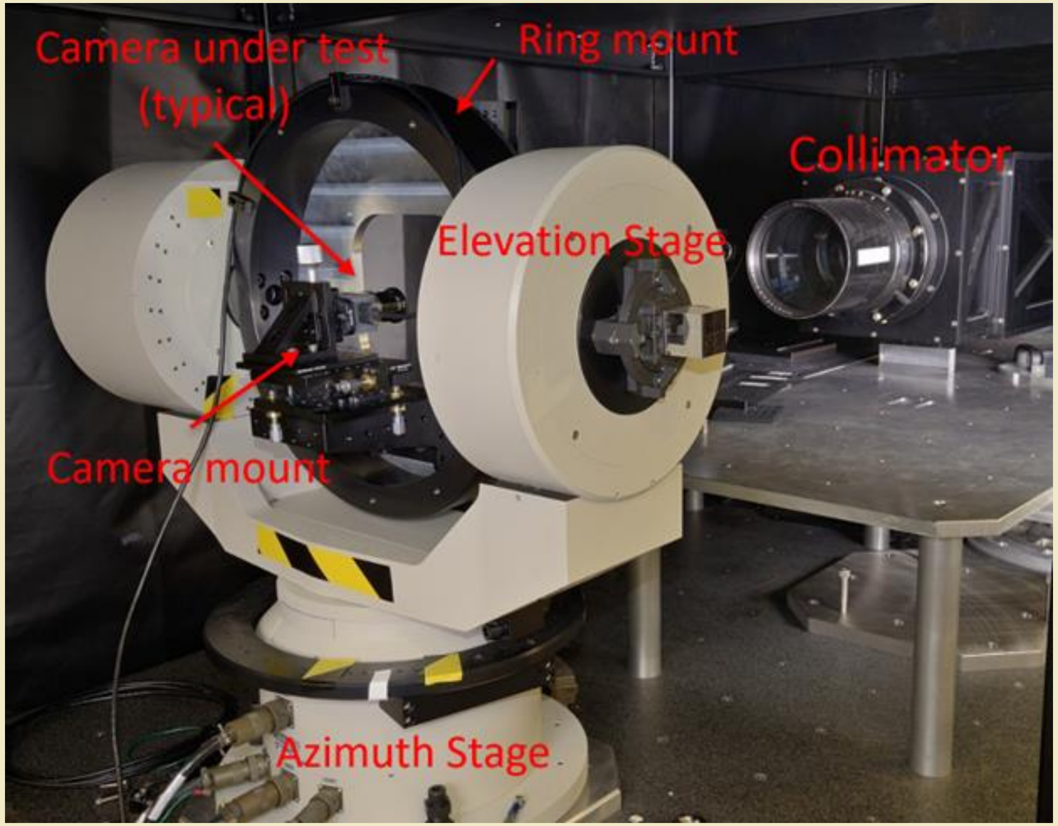


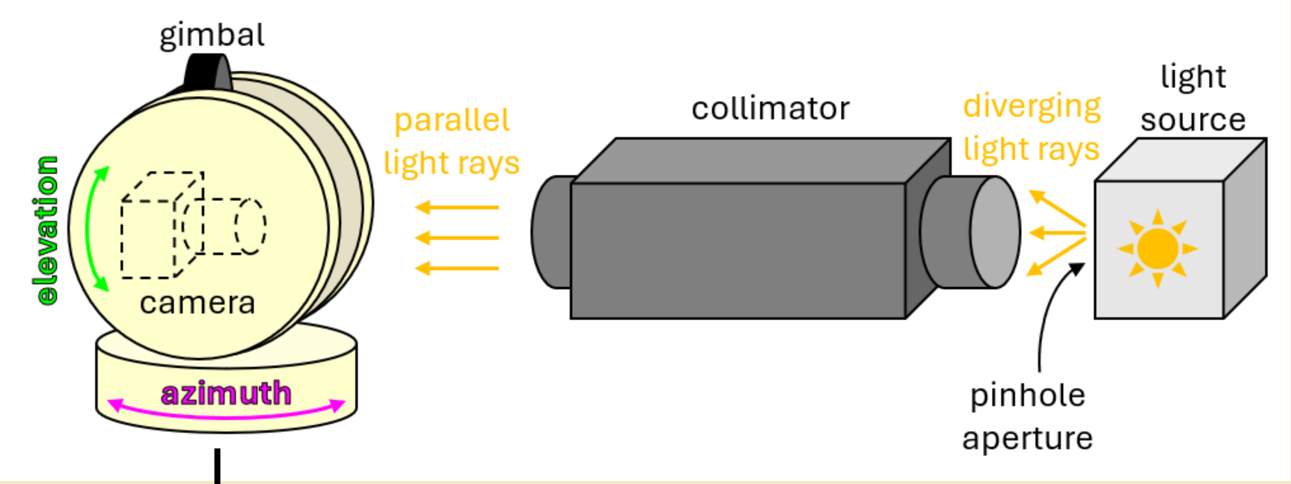
EOL Calibration Hardware

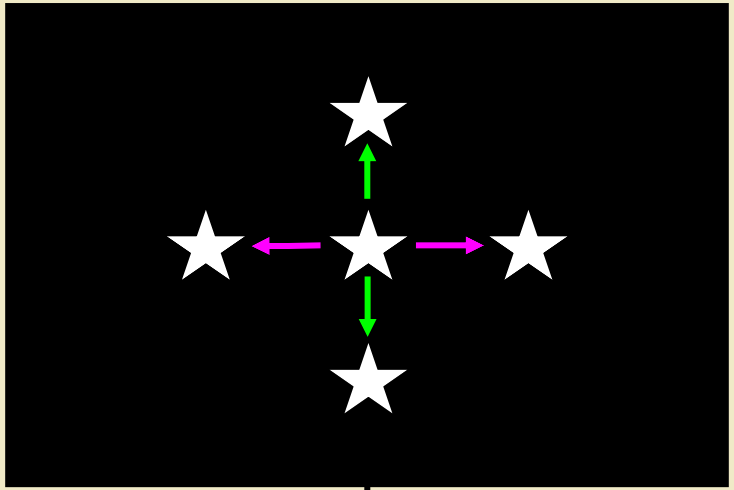




Aerotech AOM360D-400 two-axis gimbal

EOL Calibration Procedure





commanded
gimbal angles

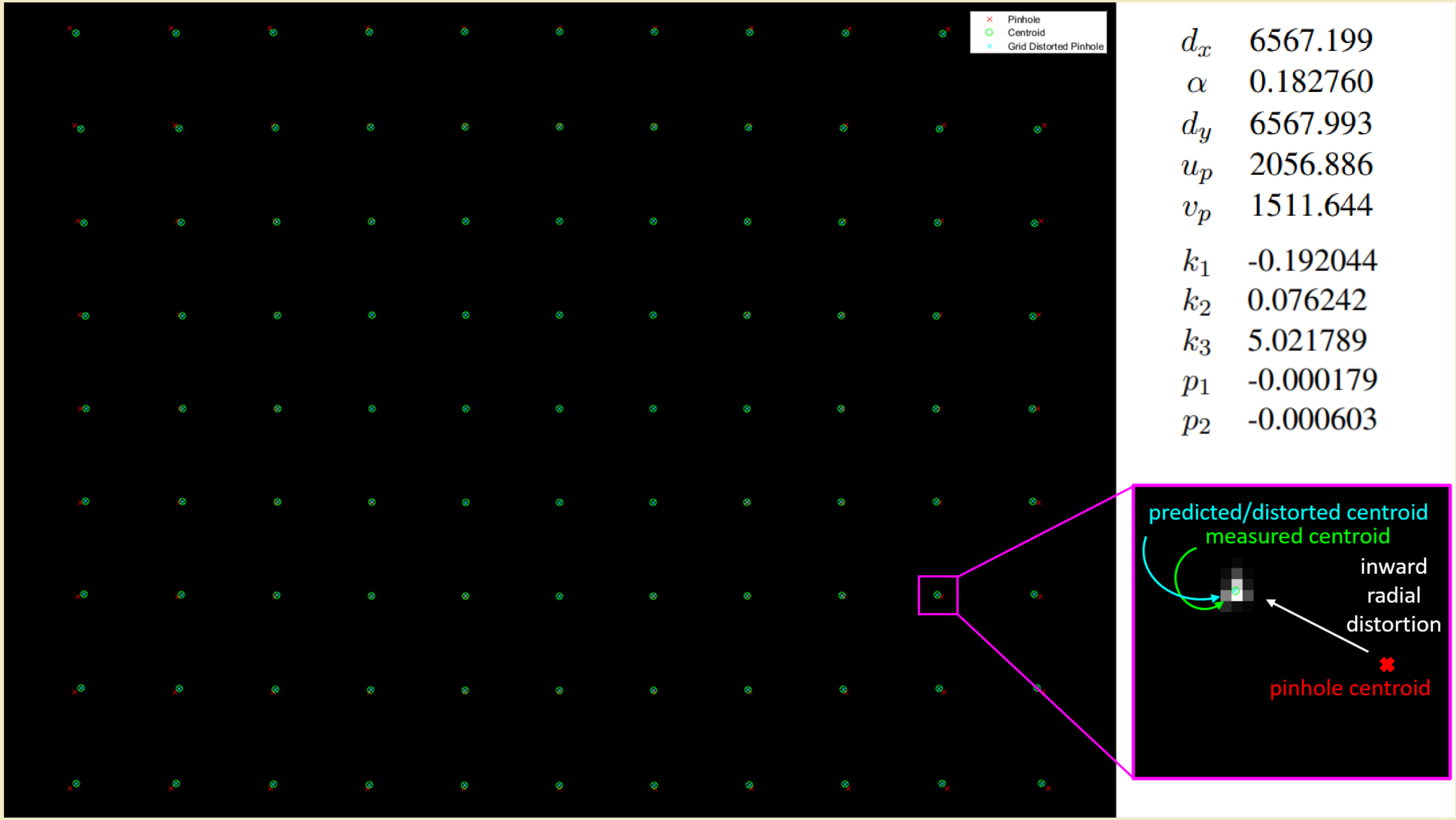
3D directions

camera
calibration

star grid
image

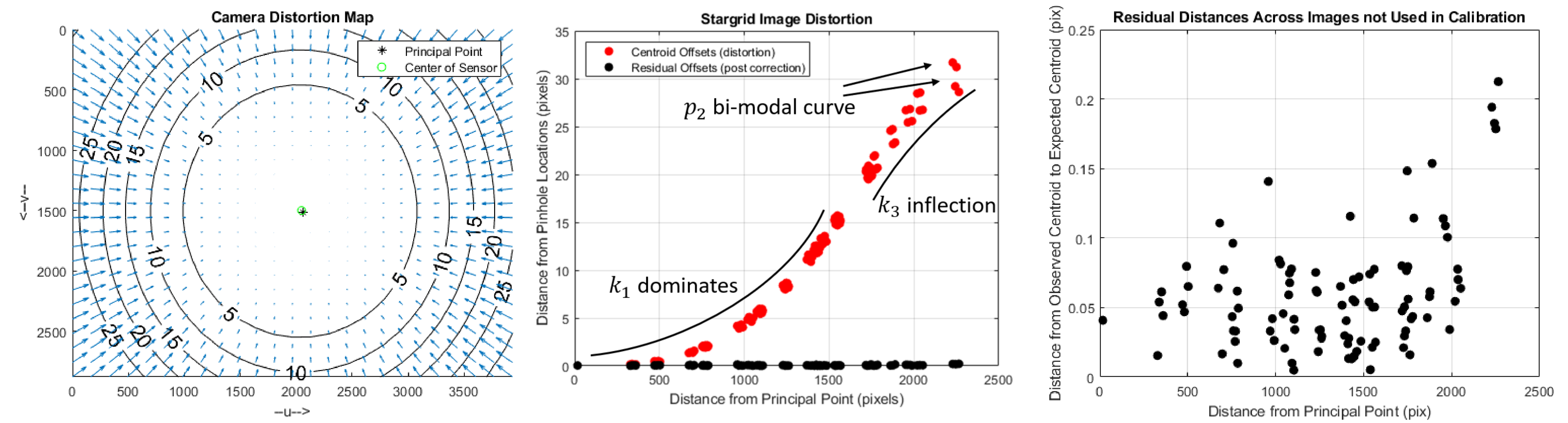
star centroid
pixel coordinates

COTS Camera Calibration Results



d_x	6567.199
α	0.182760
d_y	6567.993
u_p	2056.886
v_p	1511.644
k_1	-0.192044
k_2	0.076242
k_3	5.021789
p_1	-0.000179
p_2	-0.000603

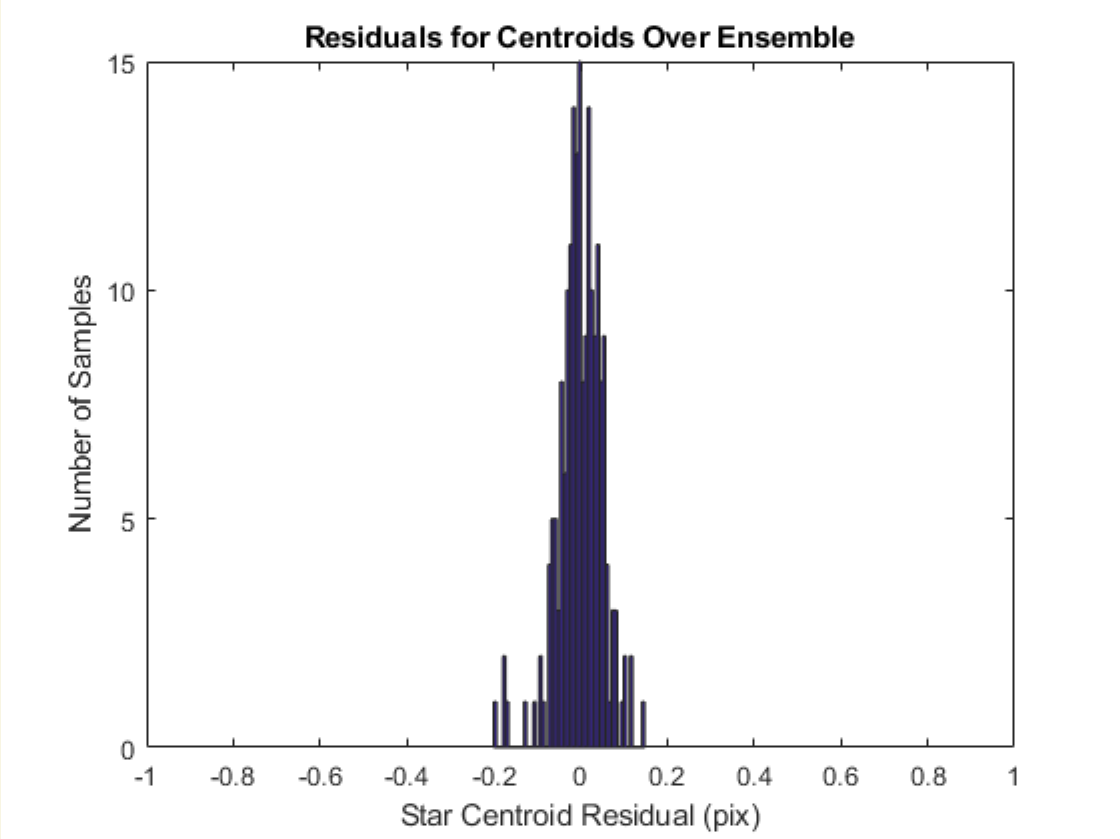
COTS Camera Calibration Results

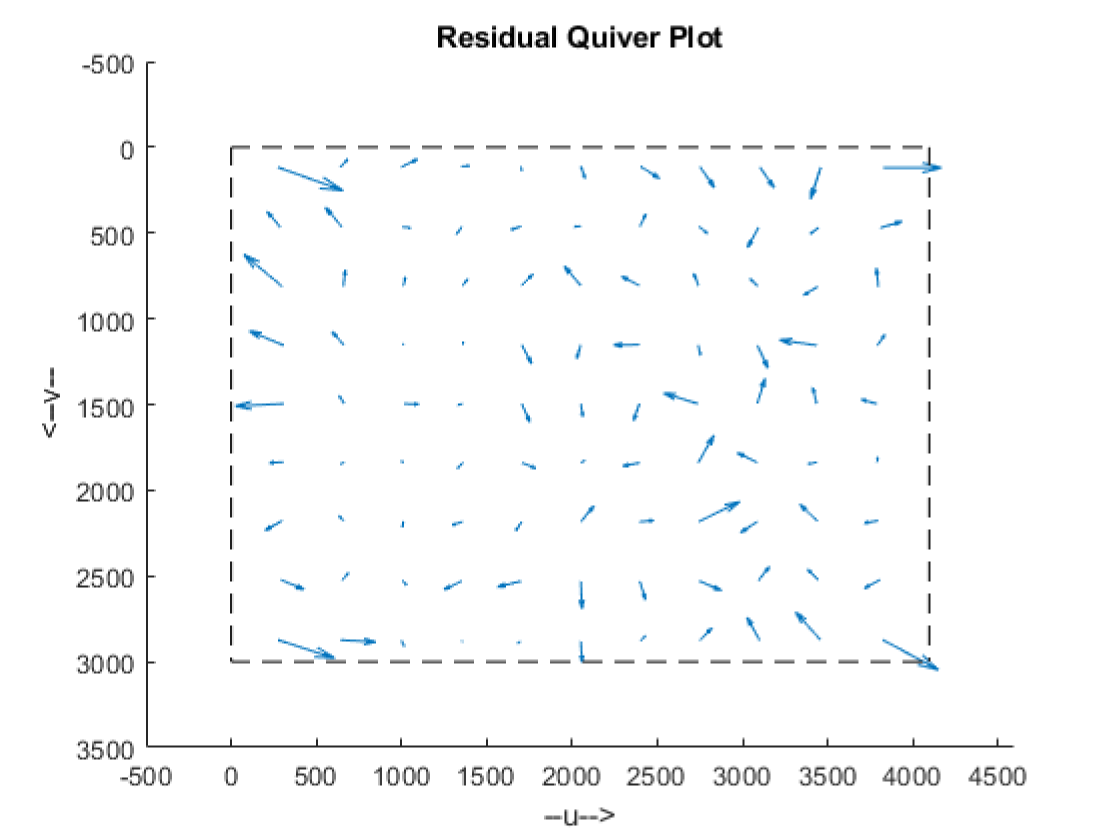


(principal point 13.95 pixels from image center)

$$\begin{bmatrix} x_i' \\ y_i' \end{bmatrix} = \begin{bmatrix} x_i \\ y_i \end{bmatrix} \left(1 + k_1 r_i^2 + k_2 r_i^4 + k_3 r_i^6\right) + \begin{bmatrix} 2p_1 x_i y_i + p_2(r_i^2 + 2x_i^2) \\ 2p_2 x_i y_i + p_1(r_i^2 + 2y_i^2) \end{bmatrix}$$

COTS Camera Calibration Results





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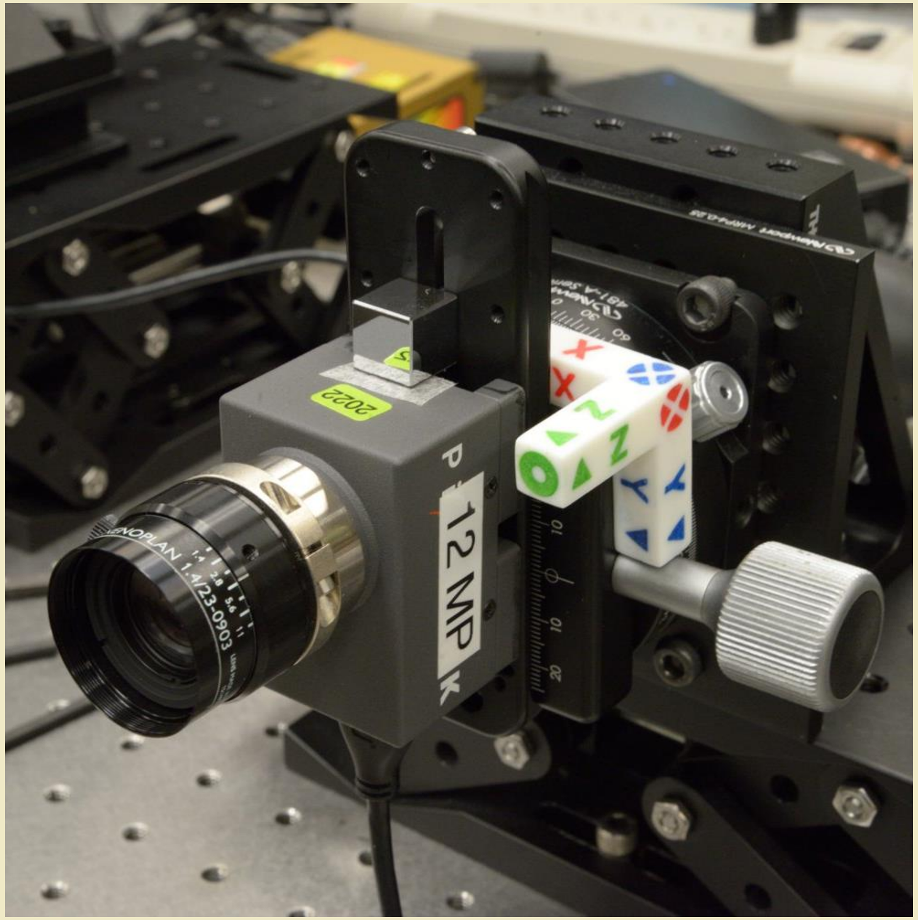
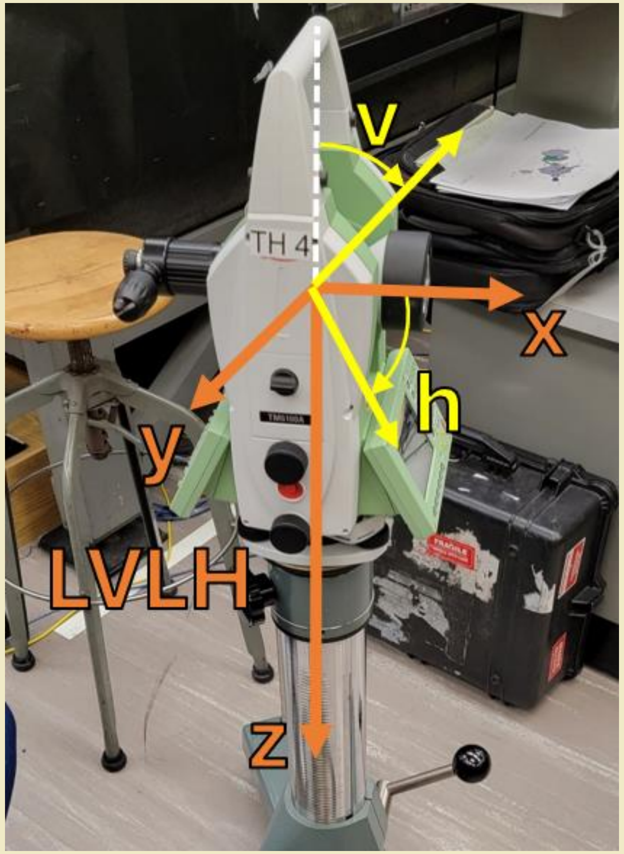
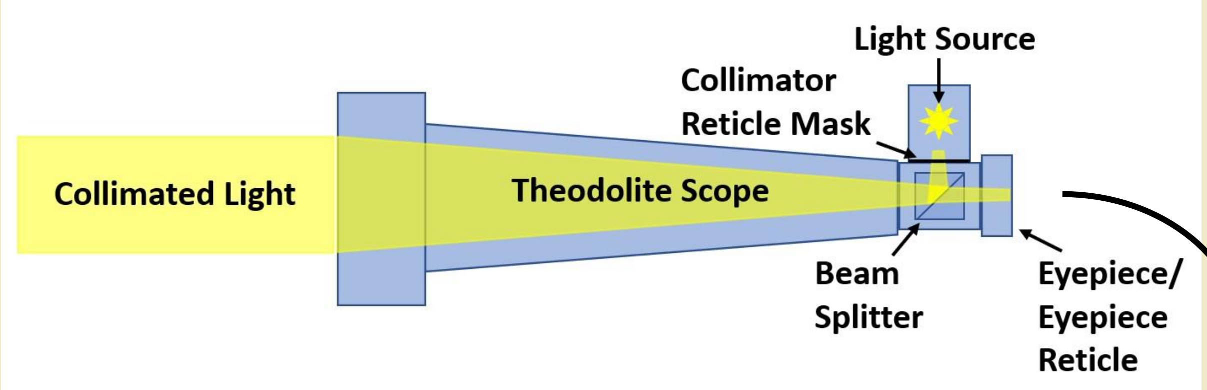


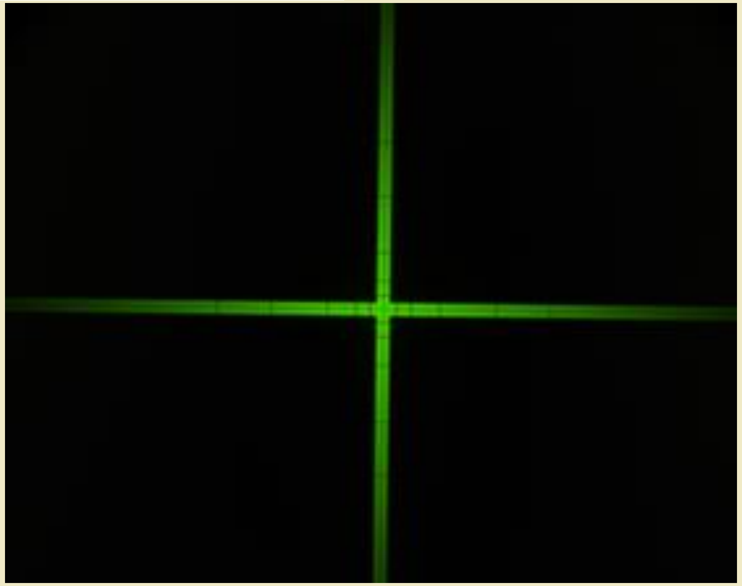
EOL Alignment Hardware



Leica TM6100A theodolite

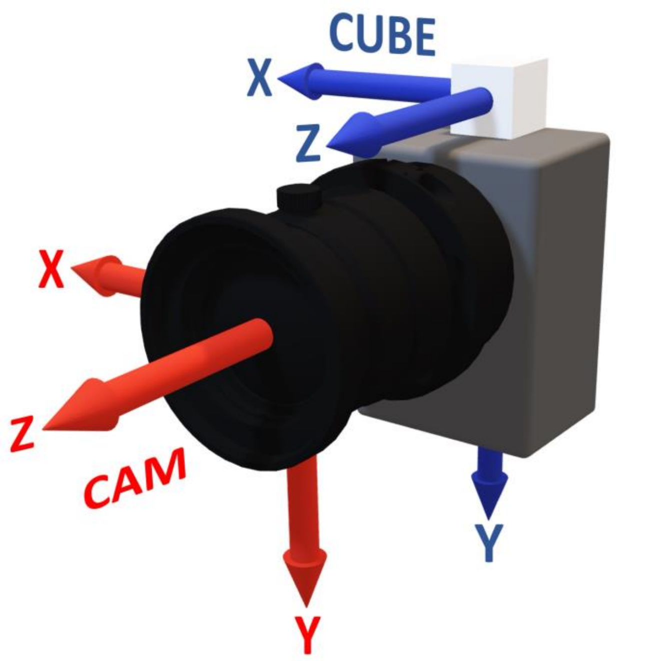


if you look through here you should see...

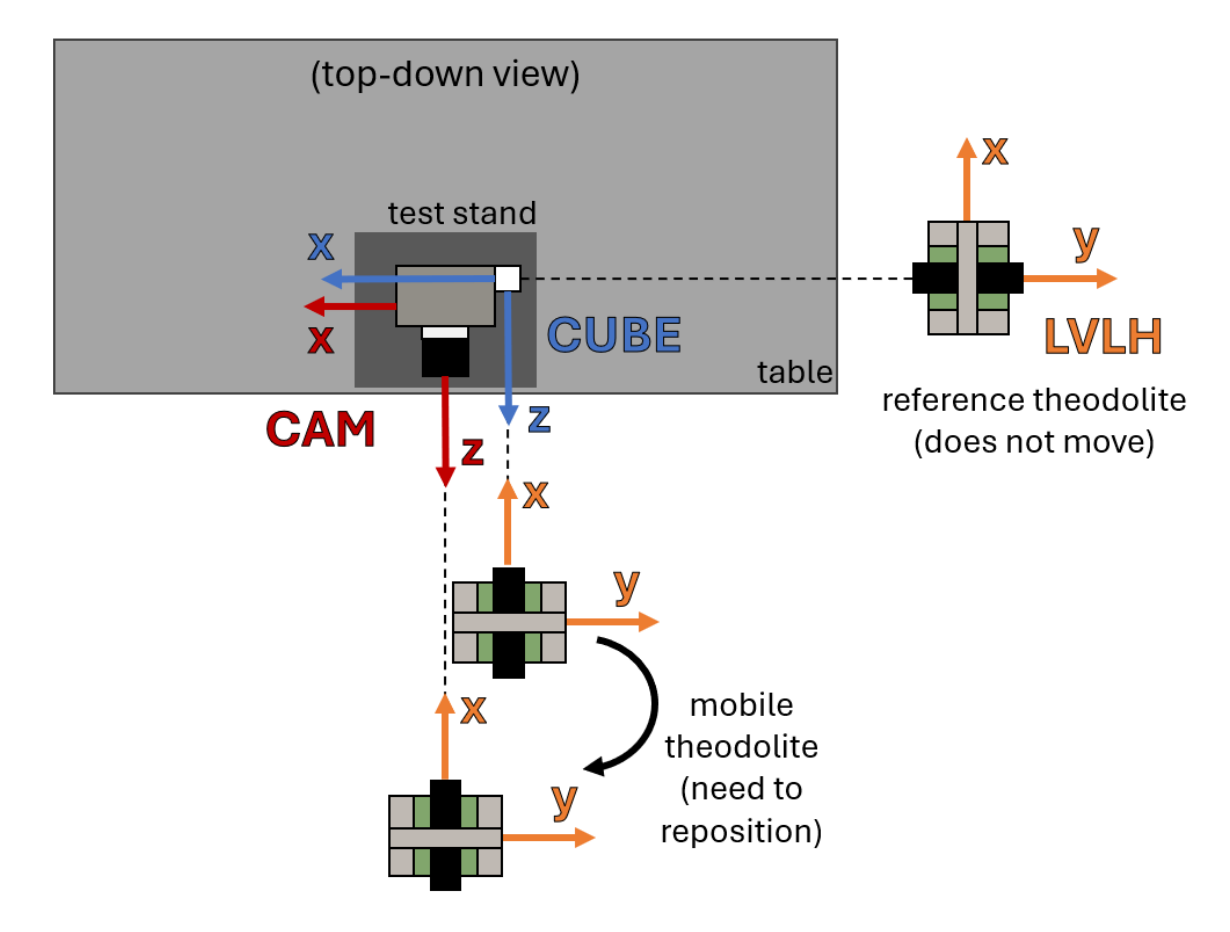


$$\hat{e}_{LVLH} = \begin{bmatrix} \cos(h)\sin(v) \\ \sin(h)\sin(v) \\ -\cos(v) \end{bmatrix}$$

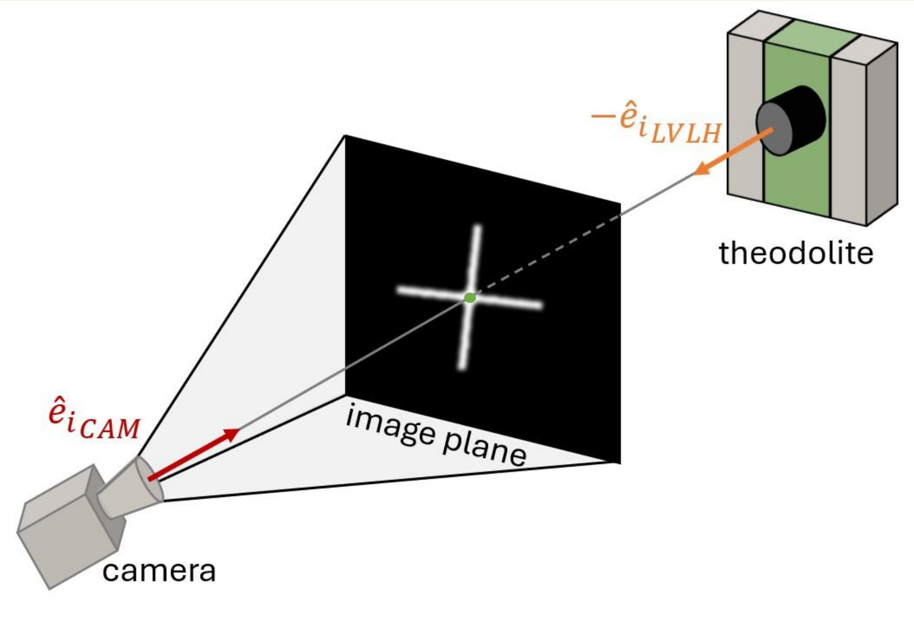
EOL Alignment Procedure

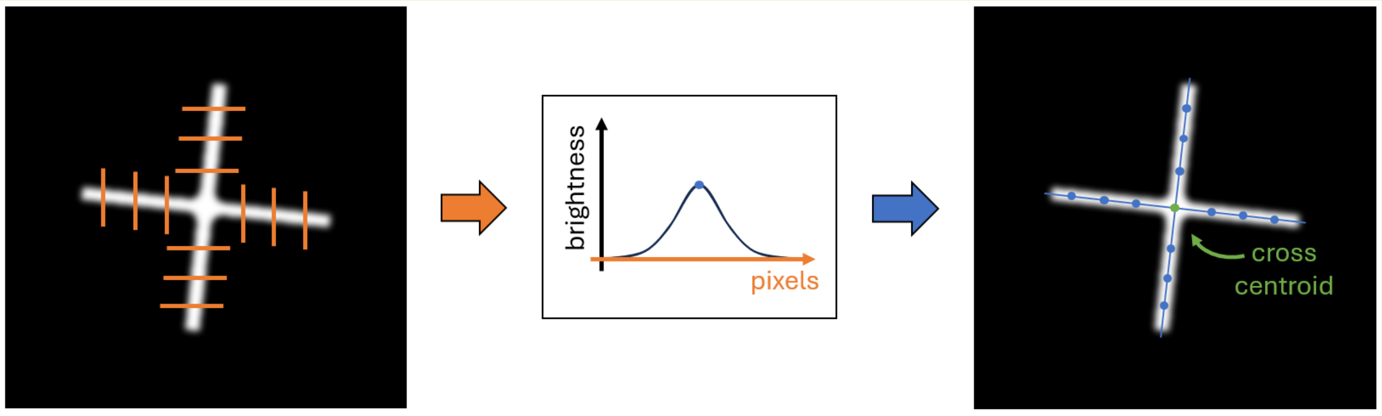


$$\hat{y}_{CUBE} = \hat{z}_{CUBE} \times \hat{x}_{CUBE}$$
$$\hat{x}'_{CUBE} = \hat{y}_{CUBE} \times \hat{z}_{CUBE}$$
$$C_{CUBE \leftarrow LVLH} = \begin{bmatrix} \hat{x}'_{CUBE} & \hat{y}_{CUBE} & \hat{z}_{CUBE} \end{bmatrix}^T$$



EOL Alignment Procedure





$$\underbrace{C_{CAM \leftarrow CUBE}}_{\text{final answer}} = \underbrace{C_{CAM \leftarrow LVLH}}_{\substack{\text{process cross images} \\ \text{solve Wahba's problem}}} \underbrace{C_{LVLH \leftarrow CUBE}}_{\substack{\text{theodolite measurements} \\ \text{of cube (see last slide)}}}$$

COTS Camera Alignment Results

$$C_{LVLH \leftarrow CUBE} = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

- computed from theodolite angles
- simple change of bases
- negligible misalignment

$$C_{CAM \leftarrow LVLH} = \begin{bmatrix} -0.000224 & -0.999872 & 0.015969 \\ -0.000966 & 0.015970 & 0.999872 \\ -0.999999 & 0.000208 & -0.000970 \end{bmatrix}$$

- cross-centroiding + Wahba's problem
- much higher uncertainty
- change of bases and misalignment

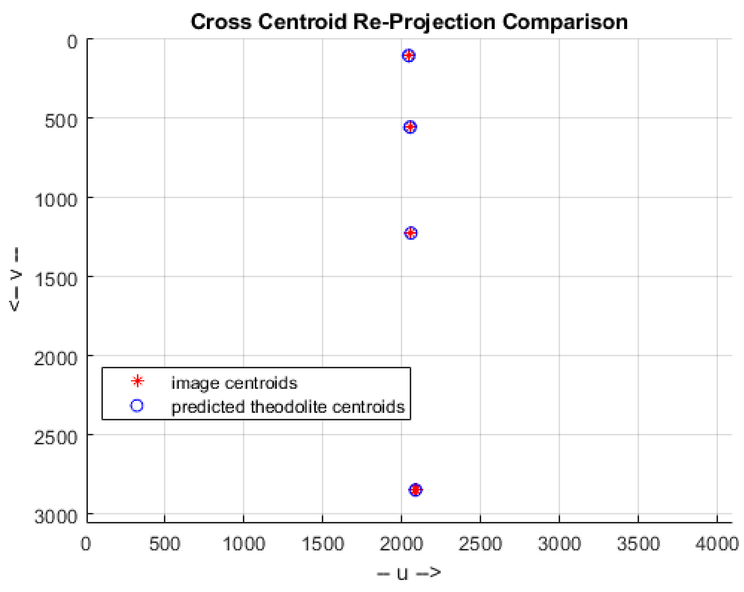
- theodolite measurement error: 5 arcsec
- cross centroiding error: 0.2 pix (6.4 arcsec)

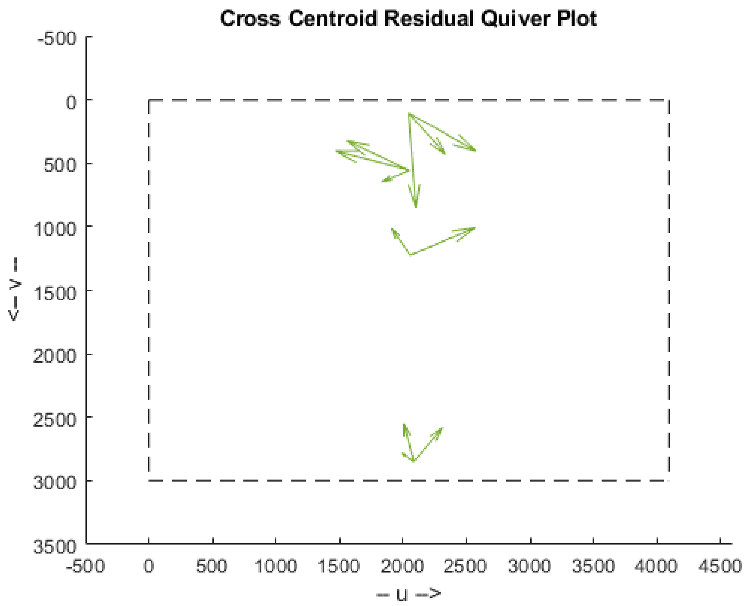
- compute attitude covariance using the Markley approach, collapse to a scalar...

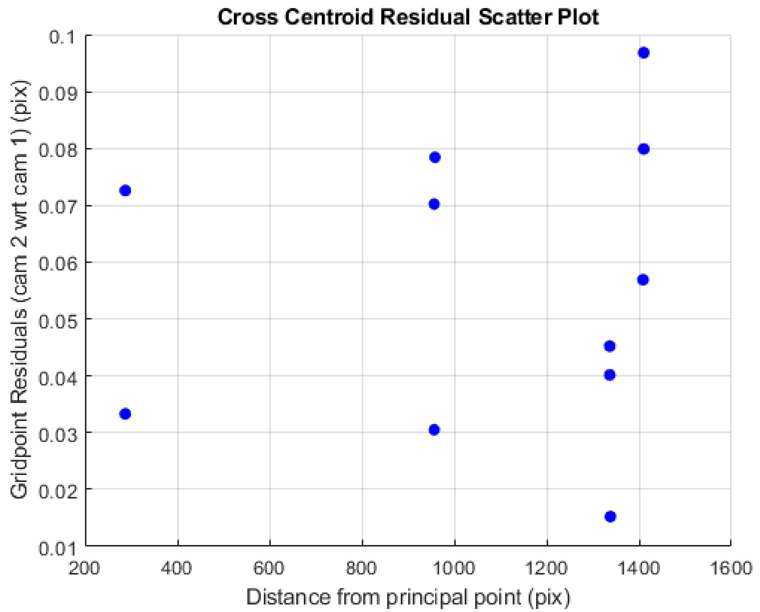
$$P_C = \begin{bmatrix} 24.032351 & 0 & 0 \\ 0 & 0.658363 & 0 \\ 0 & 0 & 0.640808 \end{bmatrix}$$

$$\theta_{uncertainty} = \sqrt{tr(P_C)}$$ = 5.033 arcsec

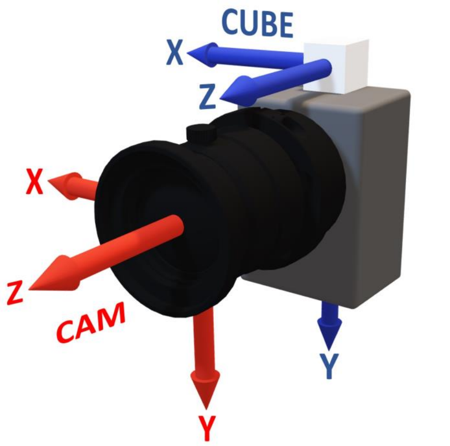
COTS Camera Alignment Results







COTS Camera Alignment Results



$$C_{CAM \leftarrow CUBE} = \begin{bmatrix} 0.999872 & 0.015969 & 0.000224 \\ -0.015970 & 0.999872 & 0.000966 \\ -0.000208 & -0.000970 & 0.999999 \end{bmatrix}$$

$$\theta_{misalignment} = \arccos\left(\frac{tr(C_{CAM \leftarrow CUBE}) - 1}{2}\right)$$ = 0.9168 deg

- Monte Carlo analysis:
- ran whole cube-to-cam analysis 10,000 times
- applied noise to measurements
  - 5 arcsec (3σ) to angles, 0.2 pix (3σ) to centroids

$\theta_{misalignment}$ mean = 0.9168 deg standard deviation = 5.071 arcsec

What is the bearing from you to your target?

image credit: NASA

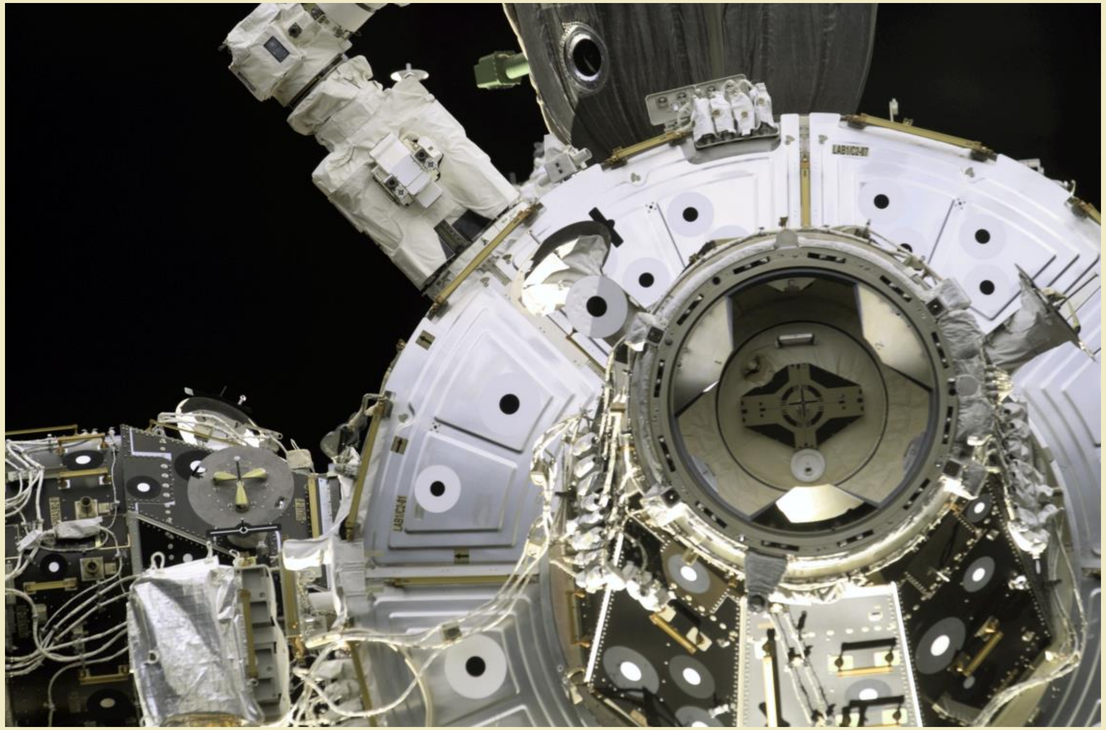
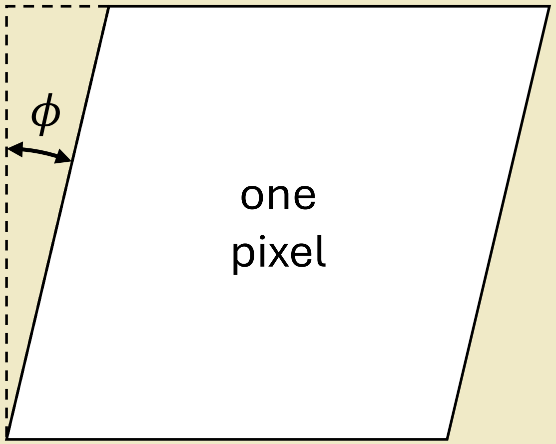


image credit: NASA

- image processing to find points of interest (u_i, v_i)
- solve reverse mapping problem using camera parameters $\hat{e}_i^{CAM} \leftarrow (x_i, y_i) \leftarrow (x_i', y_i') \leftarrow (u_i, v_i)$
- convert to spacecraft body frame $\hat{e}_i^{BODY} = C_{BODY \leftarrow CUBE}\, C_{CUBE \leftarrow CAM}\, \hat{e}_i^{CAM}$
- The EOL regularly does this for OpNav cameras at NASA JSC and for our partners in private industry.

Bonus Slides

Detector Skew



$$\alpha = d_x \tan(\phi)$$

$$u_i = d_x x_i + \alpha y_i + u_p$$
$$v_i = d_y y_i + v_p$$

$$\begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} = \begin{bmatrix} d_x & \alpha & u_p \\ 0 & d_y & v_p \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

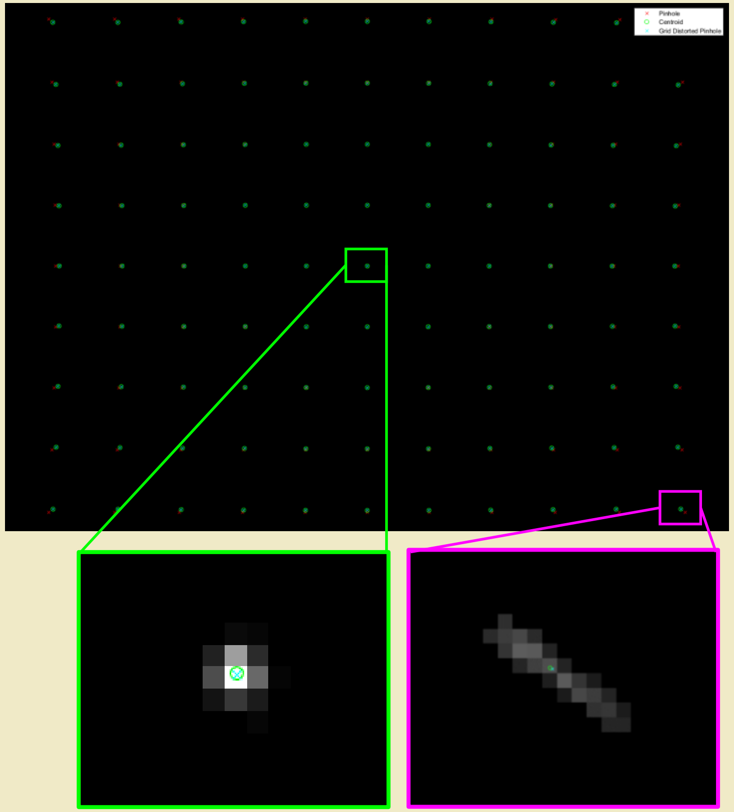
Camera Aberrations (the usual suspects)

blur/ defocus point sources

- Spherical aberration
  - blurs point sources of light
- Chromatic Aberration
  - axial or longitudinal – focal length varies with wavelength
  - lateral – magnification varies with wavelength
- Coma
  - point sources of light near edges of image are smeared
- Astigmatism
  - multiple focal points for incoming light
  - point sources are blurred more along one axis than another
- Curvature of field
  - cannot have in-focus center and in-focus edges of image

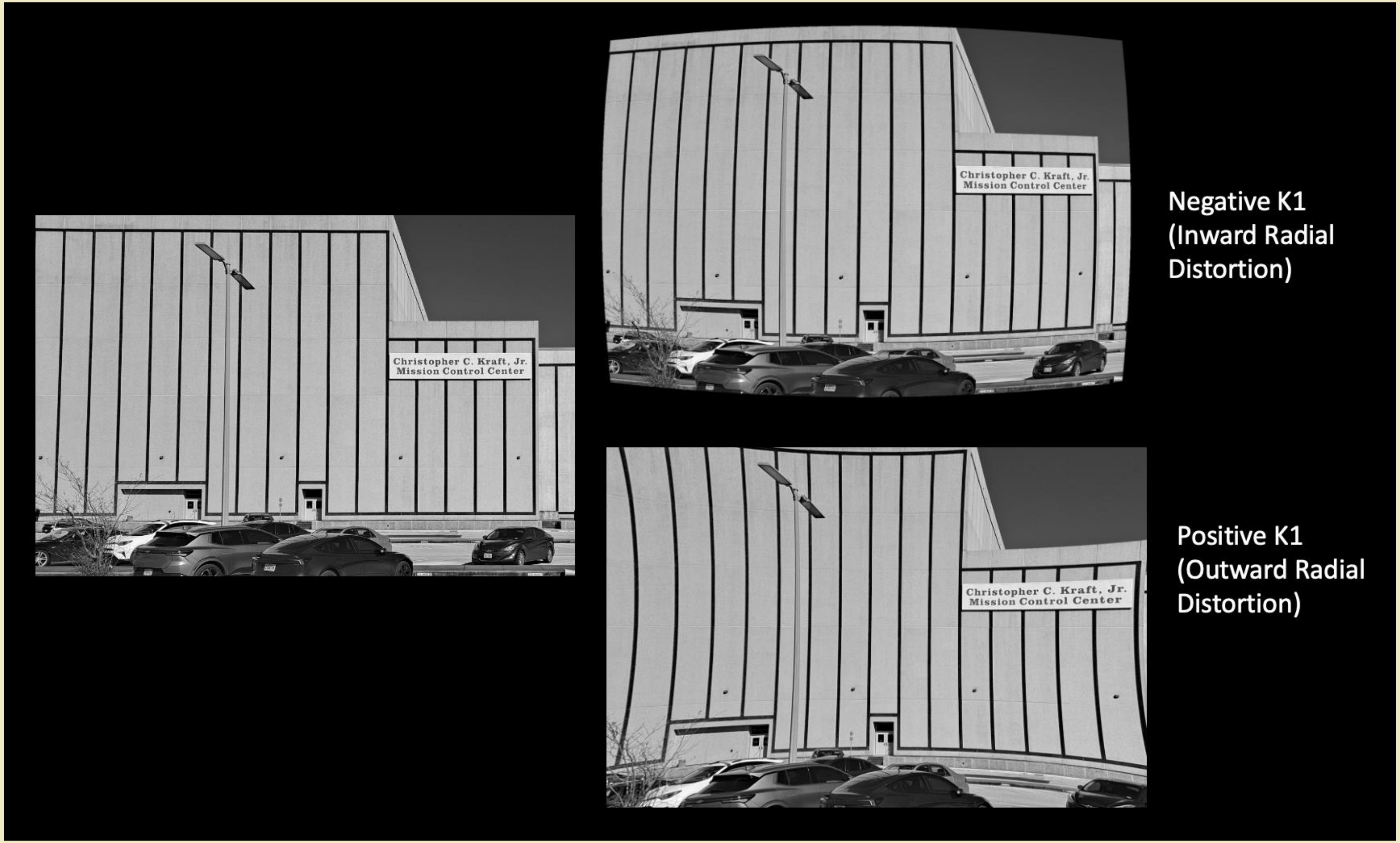
move point sources

- Lens distortion
  - radial and tangential distortion



coma

Lens Distortion



Questions people might ask

- Why not just checkerboard cal?
- Be ready to contrast against cahvor
- What about fish eye?
- Where does theodolite error and centroiding error come from? How are you so sure?