

Progress towards a High-Accuracy 4D Computational Aeroacoustics (CAA) Prediction Tool

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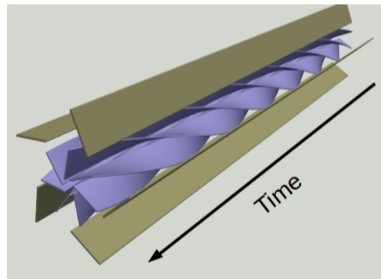
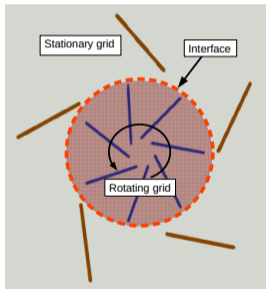
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Acknowledgements

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Continuing The Story...

- The goal: High-accuracy predictions of unsteady flow and noise...
- The idea: Solving the problem in 4D space-time provides new efficiencies for unsteady flow prediction.



A revolutionary CAA prediction tool is being developed at NASA

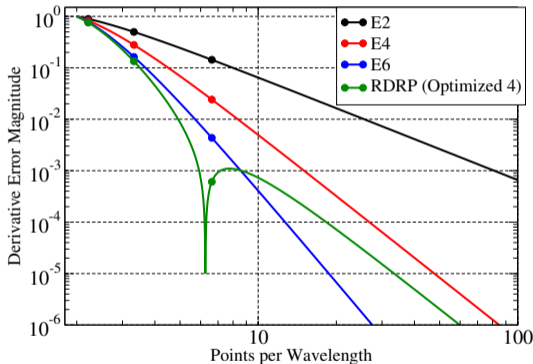
The Advantages of Space-Time

- Some of the practical advantages of the space-time approach are:
 - Can use a *lot* of existing technology:
 - Robust and efficient CAA differencing schemes.
 - Fast CFD steady-flow convergence techniques.
 - Smooth elliptic grid generation methods.
 - This also sidesteps a *lot* of existing problems with time marching:
 - How to resolve a range of time scales?
 - How to determine solution accuracy for an unsteady simulation?
 - How to achieve large-scale parallel efficiency?
 - Gives a pathway to quantum computing...

The space-time method could solve a lot of problems!

Making This Work...

- High-accuracy numerical differencing schemes are necessary for computational efficiency.
- All of these schemes are implemented in the Space-Time Mapping Analysis (STMA) code.



A range of numerical schemes are being tested!

Current Status

- Progress since the Spring 2024 TWG:
 - 2D/3D/4D Inviscid Single Block Solver:
 - Method of Manufactured Solution (MMS) Verification
 - Test Case: Pitching NACA Airfoil at $M = 0.1$
 - 4D 'scaled' LU-SGS method: high order RHS with 'scaled' low-order LHS
 - Shock-capturing artificial dissipation (extension of JST (1981) approach)
 - Accomplishments:
 - MMS used to Verify RHS implementation and order of accuracy.
 - First fully 4D solutions obtained using DRP and sixth order explicit differencing!
 - Stability of multiblock implementation tested (60 'blocks').

High-order 4D solutions have been obtained!

Method of Manufactured Solutions (MMS) Verification

- Given the governing equations:

$$\vec{Q}_t + \vec{E}_x + \vec{F}_y + \vec{G}_z = 0$$

- An analytic 'Manufactured Solution' $\vec{\tilde{Q}}$ is chosen and substituted into the governing equation:

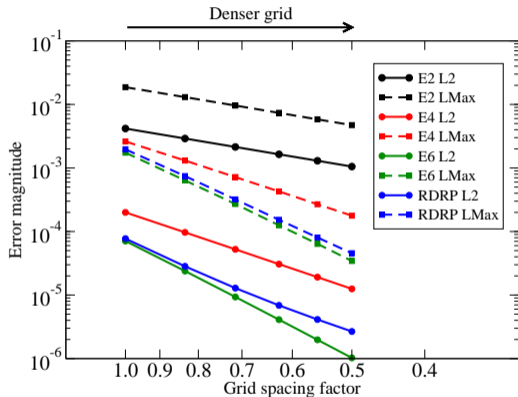
$$\vec{\tilde{Q}}_t + \vec{\tilde{E}}_x + \vec{\tilde{F}}_y + \vec{\tilde{G}}_z = \vec{S}$$

- The code is modified to add the analytic source term and used to determine the actual error and order of accuracy.

MMS provides a method to verify the 4D code!

MMS Error (4D)

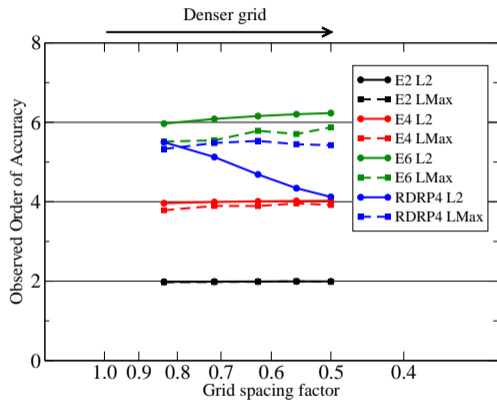
- A series of curvilinear grids with varying grid densities are used...



The high order methods have up to 1000x lower error!

MMS Order of Accuracy (4D)

- The computed error is used to obtain the *actual* order of accuracy...



The 4D STMA code is obtaining design order of accuracy!

Validation Test Case: Pitching Wing of Infinite Span

- The initial validation test case is a NACA 00xx wing of infinite span pitching about the midchord location.
 - Low-speed ($M = 0.1$) inviscid flow test case, periodic in time direction.
 - NACA 00xx airfoil section with a rounded trailing edge (getting ready for fan blades!)
 - Comparing to analytic solution by Theodorsen (1935) for incompressible flow about an infinitely thin flat plate oscillating in pitch and plunge.
 - $181 \times 301 \times 9 \times 9$ single-block O-grid.
 - Thompson 1D characteristic BCs used at wall and inflow/outflow boundaries.
 - Testing showed no need for implicit boundary conditions (!)
 - Test cases were run on a Mac laptop.

First validation of the 4D STMA method against analytic solution!

Theodorsen Solution (1935)

- Given a flat plate that is pitching and plunging as:

$$\alpha = \alpha_0 + \Delta\alpha \cos(\omega t)$$

$$\hat{y} = \frac{\Delta y}{b} \cos(\omega t + \phi)$$

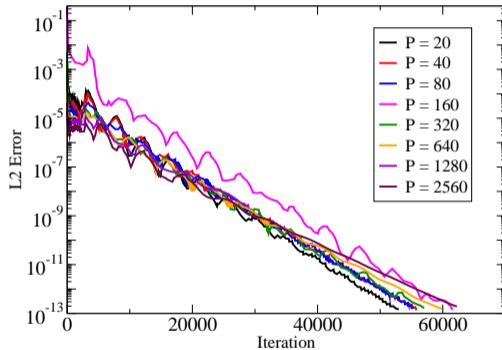
- The unsteady lift coefficient is given as:

$$C_L = 2\pi\alpha_0 + \pi \left(\begin{array}{l} \left(\begin{array}{l} (2(C_R - k(\frac{1}{2} - a)C_I) + ak^2)\Delta\alpha \\ + ((2kC_I + k^2)\cos\phi + 2kC_R\sin\phi)\frac{\Delta y}{b} \end{array} \right) \cos(\omega t) \\ + \left(\begin{array}{l} (2(C_I - k(\frac{1}{2} - a)C_R) + k)\Delta\alpha \\ + (-2kC_R\cos\phi + (2kC_I + k^2)\sin\phi)\frac{\Delta y}{b} \end{array} \right) \sin(\omega t) \end{array} \right)$$

There are a lot of assumptions and approximations needed to obtain this!

4D STMA Convergence History

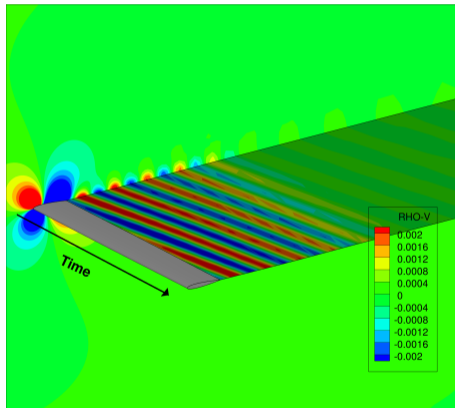
- A range of reduced frequencies were run, using the optimized RDRP differencing scheme and 8th order dissipation...



All test cases converged the L_{max} error to machine precision!

Cool Color Picture

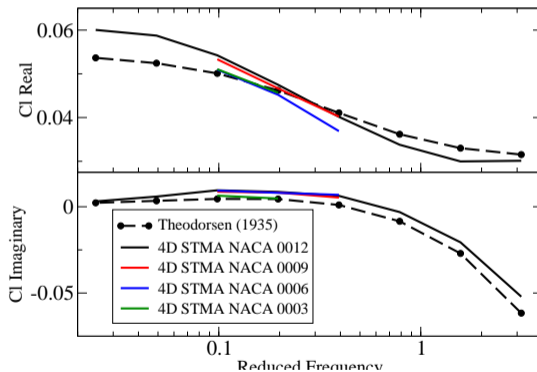
- This is a spanwise slice of the wing, with the time direction shown in the z coordinate direction.



The temporal periodicity speeds convergence!

Comparison with Theory (NACA 00xx sections)

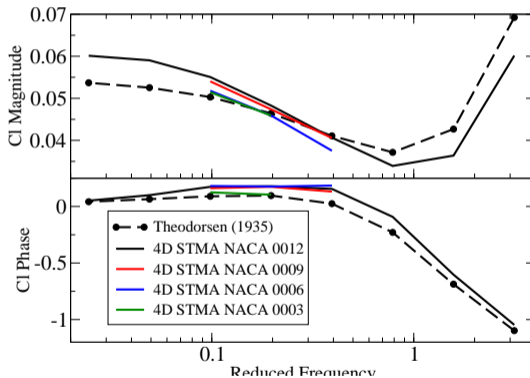
- The midspan plane is used to calculate the lift coefficient.
- The airfoil thickness has a large effect on the unsteady lift!



The STMA method is capturing the effect of the shed vortices!

Comparison with Theory (NACA 00xx sections)

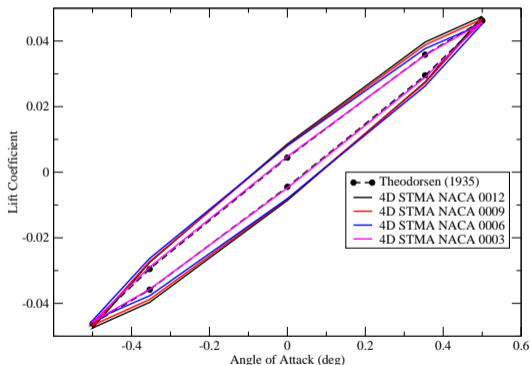
- The magnitude and phase of the unsteady lift coefficient are shown.



The STMA method is capturing the phase variation with 8 points per period!

Effect of Airfoil Thickness (Period = 320)

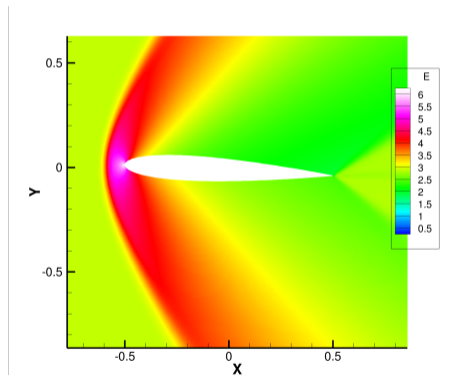
- The Theodorsen theory uses an infinitely thin flat plate airfoil.
- Reducing the thickness of the airfoil improves the comparison.



The STMA method is resolving more flow physics than the theory!

Colorful Aeroacoustics (for now...)

- This is a $M = 1.5$ inviscid flow over a NACA 0012, with optimized RDRP differencing combined with shock-capturing 8th order dissipation (2D steady calculation).



The STMA solution is converged to machine precision!

Coming Attractions

- The next steps in the process:
 - Multigrid and automatic grid sequencing
 - Grid adaption
 - 4D automatic topology generation and improved grid generation
 - Multiblock implementation and testing of 4D flow solver
 - Code parallelization

All objects are coded in 2D, 3D, and 4D for use in other codes...

Selected References

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Questions?