

Assessment of Acoustic Liner Hole Clustering in a Normal-Incidence Impedance Tube

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Characterization of nonuniform, tightly packed, facesheet hole layouts (defined here as “hole clustering”) is shown in a normal-incidence, no-flow environment, both experimentally and analytically. Nineteen additively manufactured acoustic liner samples are tested in a normal-incidence impedance tube at various frequencies, sound pressure levels (SPLs), and source types (tonal and broadband). These samples vary in geometry, including the use of both uniformly distributed and hole-clustered facesheets. Resonant frequency differences between these two facesheet types are documented and shown for all cases. An impedance prediction model is developed, relying on radiating piston theory to capture impedance changes associated with hole clustering in a no-flow environment. This includes samples with both constant- and variable-depth chambers backing the facesheet. It is shown experimentally that a nonuniformly distributed, hole-clustered facesheet can provide improved lower-frequency absorption compared to a standard uniformly distributed facesheet. This behavior can be accurately predicted at low to mid SPLs (≤ 120 dB) for frequencies up to 3000 Hz, at least for the acoustic liner geometries and aeroacoustic environment considered in this study.

Nomenclature

a_T, b_T	= impedance tube or chamber width and height	u_{1H}, u_{2H}	= acoustic particle velocity at bottom and top of hole, normalized by c
c, ρ	= sound speed and density of air	u_0, u_1, u_2	= acoustic particle velocity at the three nodes of the impedance model, normalized by c
C_D, C_P, C_V	= perforate discharge coefficient and specific heat at constant pressure and volume	u_i, u_j	= surface acoustic particle velocities of the i th and j th holes, normalized by c
d_C, h	= chamber diameter and depth	v_{mn}, w	= intermediate values used in radiation impedance calculation
d, ϕ, t_S	= facesheet hole diameter, open area ratio, and thickness	x_i, x_j, y_i, y_j	= horizontal (x) and vertical (y) coordinates of the i th and j th holes within the tube
H_m, H_n, H'_m, H'_n	= intermediate values used in radiation impedance calculation	Γ, ζ_C	= propagation constant and characteristic impedance
i	= imaginary number, $\sqrt{-1}$	δ_I, δ_E	= interior and exterior total mass-end correction terms
$J_q()$	= Bessel function of the first kind with order q	δ_{ij}	= total mass-end correction term for i th hole due to j th hole
k, k_{mn}, ω, t	= free-space and axial wavenumber, angular frequency, and time	ζ_R, θ, χ	= single hole radiation impedance, liner surface resistance and reactance, normalized by ρc
N_C	= number of channels in the liner sample	ζ, β	= liner surface impedance and admittance of the acoustic liner sample, normalized by ρc
$N_{H,R}, N_{H,C}$	= number of holes in the radiation impedance environment and a given channel	ζ_i, β_i	= surface impedance and admittance of the i th perforated hole, normalized by ρc
n	= intermediate value used in impedance calculation	ζ_k, β_k	= surface impedance and admittance of the k th channel, normalized by ρc
p, u	= acoustic pressure and particle velocity normal to the acoustic liner surface	ζ_{ij}	= mutual-radiation impedance of the i th hole due to the j th hole, normalized by ρc
p_0, p_1, p_2	= acoustic pressure at the three nodes of the impedance model, normalized by ρc^2	θ_{NL}, θ_S	= single hole nonlinear resistance and viscous loss resistance at surface, normalized by ρc
p_{1H}, p_{2H}	= acoustic pressure at bottom and top of hole, normalized by ρc^2	μ, κ	= air dynamic viscosity and thermal conductivity
p_B	= iterated acoustic pressure at the backplate of the liner, normalized by ρc^2	Ω	= ratio of acoustically active surface area to total surface area of liner
s, σ^2, γ	= shear wavenumber, Prandtl number, and ratio of specific heat of air		
$T_{11}, T_{12}, T_{21}, T_{22}$	= impedance model transmission coefficients		

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I. Introduction

FUTURE aircraft are trending toward larger-diameter engines and nacelles of a shorter length for increased fuel efficiency. This negatively impacts the effectiveness of acoustic liners in mitigating radiated fan noise, not only because of reduced treatment area but also because dominant fan noise frequencies are lower. Typically when lower-frequency absorption is needed, a larger acoustic liner volume is required. However, this adds to the weight of the aircraft, and typically there is inadequate space available.

Therefore, it is critical to assess potential solutions for improving acoustic liner lower-frequency absorption that do not require additional space or weight.

A single-degree-of-freedom (SDOF) acoustic liner is shown in Fig. 1, where it is composed of a perforated facesheet with uniformly distributed holes over honeycomb chambers with a rigid backing. The acoustic characteristics of a liner can be defined by its surface impedance (ζ), which is the ratio of acoustic pressure (p) to acoustic particle velocity (u) normal to the liner surface. This is shown mathematically as

$$\zeta = \frac{p}{\rho c u} = \theta + i\chi \quad (1)$$

where the terms are normalized by the characteristic impedance of air (ρc), and the impedance is defined in the $e^{i\omega t}$ convention, where ω is the angular frequency and t is time. The surface impedance contains both resistive (θ) and reactive (χ) components, with contributions from both the facesheet and chambers.

Additionally, there are radiation impedance contributions from the facesheet holes, impacted by the surrounding aeroacoustic environment. This includes both self-radiation and mutual-radiation impedance. Self-radiation impedance is due to the radiated acoustic field that a perforated hole creates acting on itself, while mutual-radiation impedance is caused by the radiated acoustic field of other holes acting on the hole of interest. For small hole diameters and the frequencies of interest in this study (≤ 3000 Hz), the self-radiation resistance can be neglected. This is a common assumption [1] due to the radiation resistance of an isolated hole scaling roughly with d^2 , while the radiation reactance scales with d , where d is the perforated hole diameter [2]. Additionally, as will be shown later in this article, numerous previous studies experimentally have demonstrated that the mutual-radiation resistance is a minor contribution and can also be neglected for this study. Hence, the radiation impedance is assumed to be purely reactive and, for a single hole, modeled as

$$\zeta_R \approx ik(\delta_I + \delta_E) \quad (2)$$

where k is the free-space wavenumber and $\delta_I + \delta_E$ is the total mass-end correction term. Note that the total mass-end correction is a summation of the interior (δ_I) and exterior (δ_E) mass-end correction terms, since the hole radiates on both sides of the sheet and possibly into two different environment types, e.g., a honeycomb chamber or impedance tube. This total mass-end correction term contains the contributions of both self-radiation and mutual-radiation effects. For instances where the facesheet hole distribution is nonuniform, holes can have different total mass-end correction terms, and these terms can be modeled with or without frequency dependence, depending on the frequencies of interest and environment to which it is exposed.

The acoustic field radiated from the perforated holes can be modeled similarly to that radiated by massless pistons. Rayleigh [3] developed an expression for the mass-end correction of an isolated piston radiating into semi-infinite environment. This expression, modified for a perforated hole radiating into semi-infinite environment on either side of the sheet, is

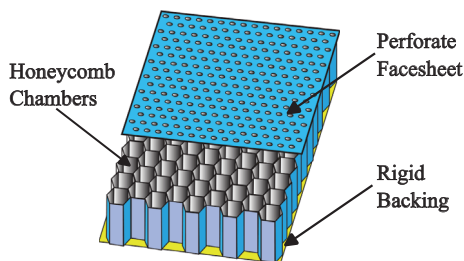


Fig. 1 Single-degree-of-freedom (SDOF) liner with a uniformly distributed perforated facesheet.

$$\delta_I + \delta_E = \frac{8}{3\pi} d \approx 0.85d \quad (3)$$

Fok [4] and Nesterov [5], theoretically and experimentally, respectively, developed expressions for the mass-end correction terms of a centered hole radiating into an infinitely long tube of finite cross-section. Both expressions are fairly similar, with the Nesterov [5] expression being

$$\delta_I + \delta_E = 0.85 d \left(1 - 1.47 \sqrt{\phi} + 0.47 \sqrt{\phi^3} \right) \quad (4)$$

where ϕ is the open area ratio (OAR) of the facesheet. Note that the end correction decreases as the OAR increases. Ingard [6] later computed similar end correction terms for centered holes within infinitely long tubes of different finite cross-sectional shapes. Similar to Fok [4] and Nesterov [5], he developed an approximated expression that works for multiple tube and perforated shapes up to an OAR value of 0.16, shown as

$$\delta_I + \delta_E = 0.85 d \left(1 - 1.25 \sqrt{\phi} \right) \quad (5)$$

Note that due to source-imaging effects from radiation near walls [2], the expressions from Fok [4], Nesterov [5], and Ingard [6] can be used to predict mass-end correction terms of uniformly distributed perforated sheets in a no-flow environment. This applies to both the interior (honeycomb chamber) and exterior (impedance tube) mass-end correction terms. Ingard [6] took this further and computed mass-end correction terms of perforated holes with circular cross-section that are both noncentered in the tube and exposed to the radiated noise of a second hole, i.e., mutual-radiation impedance between holes. Ingard [6] notes an important detail: the maximum reactance (and hence lowest resonant frequency) of a perforated sheet is when all open area is concentrated at a single hole, and two holes will approach this value as they move closer together. This demonstrates the potential opportunity of nonuniformly distributed perforated sheets for improved low-frequency performance. Ingard's mutual-radiation impedance work was later expanded upon by Ih [7] to include multiple hole and tube shapes, with holes at arbitrary positions within the cross-section of the tube.

The aforementioned studies all focused on radiation impedance of either one or two holes. Pritchard [8] demonstrated radiation impedance for multiple radiators on a plane using the relation

$$\zeta_i = \sum_{j=1}^{N_{H,R}} \zeta_{ij} \frac{u_j}{u_i} = ik \sum_{j=1}^{N_{H,R}} \delta_{ij} \frac{u_j}{u_i} \quad (6)$$

where $N_{H,R}$ is the number of holes in the radiation impedance environment. The total radiation impedance of the i th hole (ζ_i) can be computed through a summation of mass-end correction terms (δ_{ij}) associated with the mutual-radiation impedance the i th hole has with every other hole (ζ_{ij}), including itself ($i = j$, self-radiation). These terms are weighted by the ratio of surface particle velocities between the i th (u_i) and j th (u_j) holes. If the holes are radiating with an equivalent source strength and in phase with one another, this ratio is simply one.

A potential design consideration to improve the lower-frequency absorption of an acoustic liner is to group the existing set of facesheet holes closely together into nonuniform, tightly packed regions, defined in this study as "hole clustering." Figure 2 shows examples of this design change for a single chamber. Grouping holes close together and nonuniformly increases the mutual-radiation impedance of the facesheet, increasing the liner reactance and lowering the resonant frequency. While there are other ways geometrically to lower the resonant frequency of a liner, most have the byproduct of either changing the resistance or increasing the liner volume. At least for normal incidence in no flow, hole clustering has minimal impact on resistance and theoretically no impact on liner weight or volume. In short, hole clustering should in theory approach the reactance of a single large hole with the same open

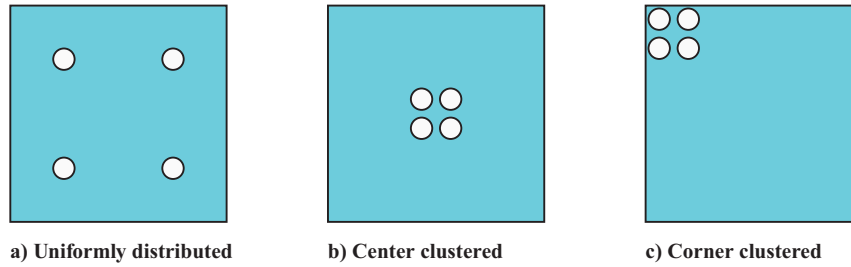


Fig. 2 Proposed hole-clustering design changes. Adapted with permission from Kreitzman and Jones [13], a work of the U.S. Government.

area (as mentioned earlier), but do so without much impact to the resistance. Large holes on a facesheet also tend to negatively impact the drag (and possibly self-noise) of an engine nacelle [9,10]. In terms of applicability to an aircraft engine nacelle, hole clustering may be ideal for aft-fan acoustic liners, as it is possible that the nonuniformity of the facesheet in an inlet could cause similar mode-scattering effects as liner splices [11,12].

Figure 2 shows that the hole-clustered pattern can be center or corner clustered. The corner-clustered design change takes advantage of increased radiation loading due to the chamber walls, and because the holes are near the corners, they also interact more strongly with holes from different chambers on the exterior side of the facesheet. This further increases radiation impedance and lowers the resonant frequency compared to center clustering. Figure 3 shows that the walls act as imaged sources, effectively increasing the number of holes in close proximity. Hence, placing hole-clustered regions near the chamber corners should yield the lowest resonant frequency, due to more holes effectively clustered together for the same OAR.

Previous work has demonstrated the effectiveness of hole clustering for a single large chamber in a normal-incidence impedance tube. Hersh and Walker [14] experimentally assessed “clustered orifices” at low sound pressure levels (SPLs) in a normal-incidence tube. This was done via testing a variety of facesheets, keeping the OAR constant (less than 0.02) while varying the hole diameter, number of holes, and hole spacing. Results were compared via an interaction parameter, defined as the center-to-center spacing between holes divided by the hole diameter, and the smallest interaction parameter tested was 1.5. It was shown that lowering the interaction parameter for a given OAR lowered the resonant frequency with only minor effects to the resistance. It could also be seen that more holes clustered together for a given OAR lowered the resonant frequency further.

Tayong [15] also assessed various samples with different hole spacings for a single chamber, again keeping the OAR constant (less than 0.01 in this study) while looking at higher SPL values (up to 140 dB). This allowed nonlinear effects to be assessed, particularly because of the low OAR. This yielded similar findings to Hersh and Walker [14]. A model was also developed based on the equivalent fluid approach for a perforated plate provided by Atalla and Sgard [16]. A geometrical tortuosity was defined based on work done for granular materials [17] to describe the modification to the propagation

paths due to the small regions between the holes. A constant must be deduced for each sample based on the experimental methodology described in Jaouen and Becot [18]. The predicted results compared well to the experimental measurements using this model.

Wang et al. [19] performed a detailed computational fluid dynamics study, looking at a variety of perforated facesheet configurations, including adjustments to the hole spacing (interaction parameters down to 1.2) and aspect ratio of the hole-clustered regions. A low-order model was developed from this high-fidelity data and subsequently validated with an experimental data set that had a wide range of OARs at a low SPL. Javareshkian [20] also analyzed the effect of hole spacing both analytically and experimentally for low-OAR perforated sheets (~ 0.01) and interaction parameters down to 1.5. Models were developed to help predict the hole-spacing behavior based on underlying theory and finite-element studies. Additionally, Lympny [21] analyzed acoustic damping due to propellant injectors in a rocket engine and used a mode-matching method to accurately predict impedance changes due to injector position differences. Similar findings as the hole-clustering studies were documented.

Kreitzman and Jones [13] performed a detailed experimental study in a normal-incidence impedance tube, comparing constant-depth liners with uniform and hole-clustered facesheet distribution patterns. This was done for frequencies up to 3000 Hz and for tone excitations of 120 and 140 dB. Unlike previous studies, hole-clustered regions were placed over an array of chambers rather than a single chamber, with OAR values similar to typical commercial aircraft nacelle acoustic liners. Significant resonant frequency shifts were achieved for chambers of more conventional sizes (up to 0.5" diameter) as well as larger sizes (up to 2" diameter) when hole clustering was implemented, with minimal impact on resistance. It was also shown that corner-clustered liners yielded a significant benefit compared to center-clustered liners.

The intent of the current article is to expand on the findings documented in Kreitzman and Jones [13]. Nineteen additively manufactured SDOF acoustic liners, eight of which are variable depth, are tested in a normal-incidence impedance tube at a variety of source excitations, including 100–140 dB tones as well as 120 and 140 dB broadband. These samples are meant to cover an OAR and chamber diameter range to allow for more holes per chamber for common OARs. Every uniformly distributed liner geometry includes counterparts that are either center clustered or corner clustered. The experimental data for these samples are complemented by an impedance model meant to predict the hole-clustering effect in a no-flow, normal-incidence environment, both for constant- and variable-depth liners. The rest of the article is organized in the following manner: Section II shows the theory behind the impedance prediction model. After this, Sec. III provides details on the test rig used for the experiment, as well as geometric information on each sample that is tested. Section IV shows trends in the experimental data related to the different samples and source excitations. Comparison of the experimental data to the prediction model is then shown in Sec. V. Finally, Sec. VI provides a closing summary and some future work planned on the topic.

II. Impedance Prediction Model

The impedance prediction model used for this study is based on theory from the NASA Zwikker and Kosten Transmission Line

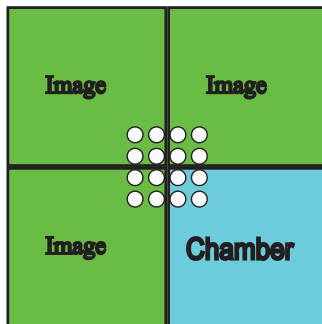


Fig. 3 Effect of chamber walls on the corner-clustered design. Adapted with permission from Kreitzman and Jones [13], a work of the U.S. Government.

(ZKTL) impedance prediction code [22–24], where acoustic pressures and velocities are computed by stepping through the acoustic liner from the backplate to the liner surface. Note that the model shown below is specifically for computing surface impedance due to tone excitation (single frequency) in the absence of grazing flow. Figure 4 shows the transmission line layers for a single acoustic liner channel, where the thickness of the perforated sheet has been exaggerated for clarity. For this particular study, an acoustic liner channel refers to a single chamber and the associated perforated facesheet on top.

The prediction model also implements an iterative method at the backplate of the liner to adjust the acoustic pressure until the root-mean-square (rms) pressure at the facesheet surface matches the prescribed SPL [25]. Therefore, this method attempts to account for acoustic particle velocity effects associated with the facesheet. The backplate is assumed rigid, with the acoustic pressure and velocity starting values given by

$$\begin{pmatrix} p_0 \\ u_0 \end{pmatrix} = \begin{pmatrix} p_B \\ 0 \end{pmatrix} \quad (7)$$

where these values are normalized by ρc^2 and c , respectively. The value p_B is the current iterated guess for the backplate acoustic pressure. This value updates after every iteration utilizing a simple bisection method. As shown in Fig. 4, the channel is composed of an air chamber and perforated sheet. For a given air chamber, the acoustic pressure and velocity at the top of the chamber (p_1, u_1) are calculated via transmission coefficients in conjunction with the acoustic pressure and velocity at the bottom of the chamber, shown as

$$\begin{pmatrix} p_1 \\ u_1 \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} p_0 \\ u_0 \end{pmatrix} \quad (8)$$

The values for the transmission coefficients are computed using [24]

$$\begin{aligned} T_{11} = T_{22} &= \cosh(k\Gamma h), \quad T_{12} = \zeta_c \sinh(k\Gamma h), \quad \text{and} \\ T_{21} &= \zeta_c^{-1} \sinh(k\Gamma h) \end{aligned} \quad (9)$$

where h is the chamber depth, Γ is the propagation constant, and ζ_c is the characteristic impedance. To compute the propagation constant and characteristic impedance, one must compute the shear wavenumber (s), Prandtl number (σ^2), and ratio of specific heats (γ) using

$$s = \frac{d_C}{2} \sqrt{\frac{\rho\omega}{\mu}}, \quad \sigma^2 = \mu \frac{C_P}{\kappa}, \quad \text{and} \quad \gamma = \frac{C_P}{C_V} \quad (10)$$

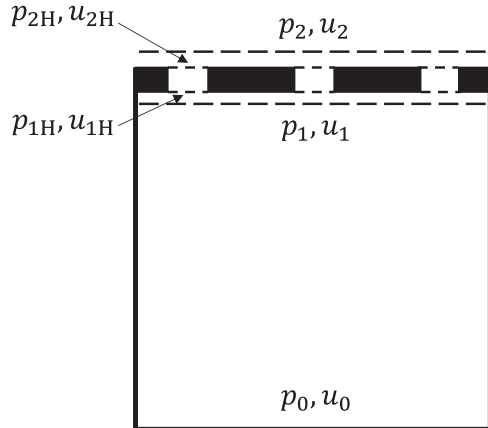


Fig. 4 Transmission line layers in the acoustic liner impedance prediction model. Adapted with permission from Kreitzman and Jones [25], a work of the U.S. Government.

where d_C is the chamber diameter, μ is the dynamic viscosity of air, κ is the thermal conductivity of air, and C_P and C_V are the specific heats of air at constant pressure and volume, respectively. The propagation constant is then calculated using [24]

$$\Gamma = \sqrt{\frac{J_0(i^{3/2}s)}{J_2(i^{3/2}s)}} \sqrt{\frac{\gamma}{n}} \quad (11)$$

where n is computed using

$$n = \left[1 + \frac{\gamma - 1}{\gamma} \frac{J_2(i^{3/2}\sigma s)}{J_0(i^{3/2}\sigma s)} \right]^{-1} \quad (12)$$

Once the propagation constant is known, the characteristic impedance is calculated via [24]

$$\zeta_c = \frac{-i J_0(i^{3/2}s)}{\Gamma J_2(i^{3/2}s)} \quad (13)$$

In addition to the air chamber, a similar process is used to compute the transfer impedance of the top layer facesheet [26,27]. To do this, it is assumed that at the transition from an air chamber to the i th perforated hole, continuity of acoustic pressure is maintained while the acoustic particle velocity increases due to conservation of mass, shown analytically as

$$p_{1H} = p_1 \quad \text{and} \quad u_{1H} = \frac{u_1}{\phi} \quad (14)$$

where ϕ is the OAR of the facesheet directly over the channel, and (p_{1H}, u_{1H}) are the acoustic pressure and normal velocity just inside the hole. Once (p_{1H}, u_{1H}) are defined, Eqs. (8–13) are used again to calculate the acoustic pressure and velocity at the other side of the hole, noting that instead of the chamber diameter d_C and depth h , the perforated sheet hole diameter d and thickness t_s are used. This yields the acoustic pressure and velocity at the other side of the hole, or (p_{2H}, u_{2H}).

Additional terms are included to more accurately account for the perforated hole transfer impedance [25]. The nonlinear resistance of a single perforated hole due to the turbulent mixing loss of the acoustic waves exiting the holes is accounted for using [27–29]

$$\theta_{NL} = \frac{0.98}{4\sqrt{2}C_D} (|u_{1H}| + |u_{2H}|) \quad (15)$$

where C_D is the discharge coefficient of the hole. Based on previous studies [25], a C_D value of 0.85 is assumed. Note that the particle velocity has been scaled down by $\sqrt{2}$ to utilize the rms value instead, and the particle velocities at the bottom and top of the hole are averaged together in the formulation. The resistance due to viscous losses at the surface of the facesheet (both interior and exterior) as the acoustic waves bend into the perforated hole is also accounted for using [6,30]

$$\theta_s = \frac{\sqrt{2}}{2} \sqrt{\frac{k\mu}{\rho c}} \quad (16)$$

Finally, in order to account for the radiation impedance of the perforated hole, one can assume that radiation resistance is negligible (at least for the frequencies and liner geometries considered in this study), and therefore the radiation impedance of a perforated hole is the same as Eq. (2) and shown again here as

$$\zeta_R = ik(\delta_I + \delta_E) \quad (17)$$

For a perfect uniformly distributed perforated sheet subjected to a tone at a frequency much lower than the cuton frequency of the first higher-order mode in the impedance tube, the summation

of interior and exterior mass-end correction terms is approximated as [6]

$$\delta_I + \delta_E = 0.85d(1 - 1.25\sqrt{\phi}) \quad (18)$$

which is the same as Eq. (5) and accounts for the self-radiation and mutual-radiation impedance of a given hole. Every perforated hole is assumed to have the same total mass-end correction, and the term itself is frequency-independent. Note that when the acoustic liner has partitions between channels that are not negligible in thickness, this assumption breaks down. However, for the purposes of this study, the partitions are thin enough that this assumption is still used in the prediction process for uniformly distributed liners. When the perforated sheet is nonuniformly distributed, such as hole-clustered liners, the computation of the mass-end correction terms utilizes the research originally conducted in Ingard [6] and expanded upon by Ih [7]. Here, perforations are modeled as massless pistons radiating into an infinitely long tube. Ih [7] took the acoustic potential solution of this massless piston fluctuating and radiating into this assumed inviscid environment. From this, mass-end correction terms are computed utilizing the acoustic pressure solution of this radiated noise signature acting both on the piston itself as well as other radiating pistons. This mass-end correction term on one side of the i th perforate due to the j th perforate is [7]

$$\delta_{ij} = i \frac{\pi}{a_T b_T} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{4}{v_{mn} k_{mn}} \frac{J_1^2\left(w \frac{d}{2}\right)}{w^2} H_m H_n H'_m H'_n \quad (19)$$

where the coordinate system for this computation is shown in Fig. 5. It is assumed that all holes on the perforated sheet are of the same diameter d . In this calculation, a_T and b_T represent the horizontal (x) and vertical (y) dimensions, respectively, of the impedance tube (or chamber) cross-section. Additionally, $v_{m0} = v_{0n} = 0.5$, $v_{mn} = 0.25$, (m, n) represents the horizontal and vertical mode orders of the impedance tube (or chamber), and

$$\begin{aligned} H_m &= \cos\left(\frac{m\pi x_i}{a_T}\right), & H_n &= \cos\left(\frac{n\pi y_i}{b_T}\right), \\ H'_m &= \cos\left(\frac{m\pi x_j}{a_T}\right) & \text{and} & \quad H'_n = \cos\left(\frac{n\pi y_j}{b_T}\right) \end{aligned} \quad (20)$$

where (x_i, y_i) and (x_j, y_j) are the coordinates for the center of the i th and j th holes, respectively, within the radiation environment of interest (impedance tube or chamber).

The primes on the summations in Eq. (19) indicate that the plane wave, $(m, n) = (0, 0)$, should be ignored since this is a purely resistive component. The axial wavenumber (k_{mn}) is defined as

$$k_{mn} = \sqrt{k^2 - \left(\frac{m\pi}{a_T}\right)^2 - \left(\frac{n\pi}{b_T}\right)^2} \quad (21)$$

and the w term is

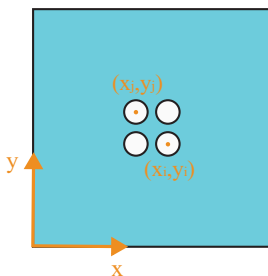


Fig. 5 Coordinate system for the radiation impedance calculation.

$$w_{mn} = \sqrt{\left(\frac{m\pi}{a_T}\right)^2 + \left(\frac{n\pi}{b_T}\right)^2} \quad (22)$$

This mutual-radiation, mass-end correction computation for the i th hole [see Eq. (19)] is performed with every hole in the given radiation environment, including itself. As mentioned previously using Eq. (6), these mass-end correction terms are then summed to achieve the total value for the i th hole. This computation is performed for both the interior (δ_I) and exterior (δ_E) radiation environments and summed at the end. In the case of this research, the interior environment is inside the chamber, while the exterior environment is that portion exposed to the impedance tube. Also note that this computation for the total mass-end correction term can be used for uniformly distributed perforated facesheets as well, although for the OARs considered in this study, one will get an answer very close to Eq. (18).

Note that if the frequencies of interest are sufficiently below the cuton frequency of the first higher-order mode in the duct, one can improve computational efficiency of this method by assuming $k \approx 0$ for Eq. (21), much like Eq. (18) does. This makes the mass-end correction term frequency-independent and avoids having to compute mass-end correction terms for every frequency of interest. For this particular study, this simplifying assumption is made, noting that the first higher-order mode in the impedance tube of interest cuts on around 3400 Hz, while frequencies up to 3000 Hz are analyzed. However, even at 3000 Hz, the total mass-end correction term is only predicted to change by about 1.5% when the free-space wavenumber is included in Eq. (21), at least for a typical hole-clustered sample, as shown in Kreitzman and Jones [13].

Equation (19) contributes to the computation of both the self-radiation and mutual-radiation impedance of a given perforated hole, where the self-radiation impedance case occurs when $i = j$. The summation in Eq. (19) is selected to go up to a mode order of $(m, n) = (100, 100)$. The inclusion of any modes higher than (100, 100) does not significantly impact predicted impedance spectra. For instance, adjusting the highest mode order in the summation from (100, 100) to (250, 250) only changes the total mass-end correction terms by $\sim 0.5\%$. It is likely that less modes could have been used in this computation; a full convergence study was not done here. The number of modes needed for a given computation may change depending on the geometry of the radiation environment, so a brief assessment of convergence should always be performed.

Each perforated hole may have a different radiation impedance, so this process must be performed for every hole on the facesheet, for both the interior and exterior radiation impedance environments. For the i th hole, the final surface impedance is

$$\zeta_i = \frac{p_{2H}}{u_{2H}} + \theta_{NL} + \theta_S + \zeta_R \quad (23)$$

The surface impedance of each hole in a given channel must be combined together with the nonporous part of the sheet in order to obtain the effective impedance. To do this, a summation of admittances is performed, shown as

$$\beta_k = \frac{\phi}{N_{H,C}} \sum_{i=1}^{N_{H,C}} \beta_i \quad (24)$$

where $\beta_i (= \zeta_i^{-1})$ is the admittance of the i th hole, β_k is the admittance of the k th channel, and $N_{H,C}$ is the number of perforated holes for a given channel. The effective surface impedance of the k th channel is then simply the inverse of the effective admittance, or $\zeta_k = \beta_k^{-1} = p_2/u_2$. One can continue the transmission line process by computing p_2 , where u_2 is simply $u_{2H}\phi$. For acoustic liners that vary in impedance across the face of the sample (e.g., variable-depth liners or any liner with a finite partition thickness), one can compute an effective impedance much like what is done for the perforate. To do this, Eq. (24) is modified to

$$\beta = \frac{\Omega}{N_C} \sum_{k=1}^{N_C} \beta_k \quad (25)$$

where N_C is the number of channels in the sample, and Ω is the ratio of the acoustically active surface area to the total surface area of the sample. The Ω value is included to account for the partition thickness between channels. The final surface impedance of the acoustic liner is then $\zeta = \beta^{-1}$.

III. Description of the Test Rig and Samples

A. Normal Incidence Tube (NIT) Test Rig

The NASA Langley Normal Incidence Tube (NIT) is a 2" × 2" waveguide of approximately 36" in length and shown in Fig. 6. It uses six BMS Speaker 4599ND 2" compression acoustic drivers to generate a sound field that impinges on the acoustic liner surface. There are three microphones flush-mounted in the walls of the rig. One is a reference microphone 0.25" away from the sample surface that is used to provide feedback in setting the appropriate total SPL (incident plus reflected) for the test. Two other microphones, located 2.50" and 3.75" from the liner surface, are mounted in a rotating plug so that their positions can be interchanged [31]. This allows

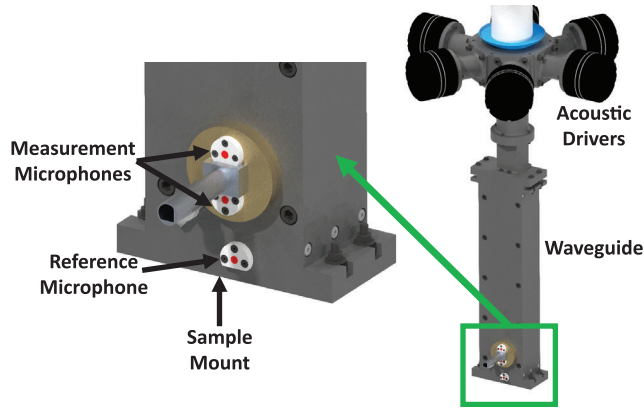


Fig. 6 Normal Incidence Tube (NIT) test rig. Reprinted with permission from Kreitzman and Jones [34], a work of the U.S. Government.

errors associated with the amplitudes and phases due to mismatched microphone calibrations to be eliminated during the two-microphone method process [32,33].

The NIT allows for four different source types: stepped-sine, swept-sine, broadband, and multitone at frequencies of 400–3000 Hz [31,35]. The highest frequency tested in the NIT is kept sufficiently low so that only plane waves propagate within the waveguide. For this particular test, the swept-sine source is used at SPL levels of 100, 120, and 140 dB. Additionally, the broadband source is used at a partial overall sound pressure level (POASPL) of 120 and 140 dB. Sufficient averages are taken in the broadband acquisition to smooth out results. The multipoint method [33,36] is used to compute the impedance using a weighted residual technique to minimize the error between measured data and an assumed one-dimensional standing wave pattern. Note that when only two microphones are used, this method becomes deterministic, and the error between measured and predicted data is zero. With regard to uncertainty, 95% confidence intervals on the NIT normalized impedance spectra should be less than 0.02 and 0.04 for SPL values of 120 and 140 dB, respectively, for the samples considered in this study [37]. Additionally, ambient air parameters within the impedance tube stay relatively consistent over time; the characteristic impedance of air changes only about 2.5% over the course of a year.

B. Sample Descriptions

Table 1 shows the geometric parameters of 19 acoustic liner samples that are tested, noting that the target hole spacing of each sample is the center-to-center distance between the facesheet holes. Also, the chamber size is referring to the side length of the square cross-section. Depictions of the sample facesheets are shown in Fig. 7. Each sample is additively manufactured via stereolithography (SLA) as one part. This improves sample rigidity and helps avoid facesheet rattle during testing.

For the constant-depth samples, every acoustic liner is printed without a backplate to allow resin to drain out of the chambers. During testing, these samples are simply press-fitted onto a flat metal surface to act as a backplate. For the variable-depth samples, only the longest chambers are printed without a backplate. The smaller chamber depths are printed with 0.1" thick backplates. The holes associated with the facesheet are large enough that resin can still drain through them. Any excess dried resin

Table 1 Test sample geometric parameters

Sample number	Target chamber size	Target OAR	Target hole spacing	Holes per chamber	Hole distribution ^a	Chamber depths	Chamber grouping ^b
1	0.375"	0.0536	0.169"	6	UND	2.0"	— —
2	0.375"	0.0536	0.050"	6	CEC	2.0"	— —
3	0.375"	0.0536	— —	1	UND	2.0"	— —
4	0.375"	0.1072	0.118"	12	UND	2.0"	— —
5	0.375"	0.1072	0.050"	12	CEC	2.0"	— —
6	0.483"	0.0485	0.168"	9	UND	2.0"	— —
7	0.483"	0.0485	0.050"	9	CEC	2.0"	— —
8	0.483"	0.0485	0.050"	9	COC	2.0"	— —
9	0.483"	0.0862	0.126"	16	UND	2.0"	— —
10	0.483"	0.0862	0.050"	16	CEC	2.0"	— —
11	0.483"	0.0862	0.050"	16	COC	2.0"	— —
12	0.483"	0.0485	0.168"	9	UND	2.0", 1.0"	UG
13	0.483"	0.0485	0.050"	9	COC	2.0", 1.0"	UG
14	0.483"	0.0485	0.168"	9	UND	2.0", 1.5", 1.0", 0.5"	UG
15	0.483"	0.0485	0.050"	9	COC	2.0", 1.5", 1.0", 0.5"	UG
16	0.483"	0.0485	0.168"	9	UND	2.0", 1.0"	SG
17	0.483"	0.0485	0.050"	9	COC	2.0", 1.0"	SG
18	0.483"	0.0485	0.168"	9	UND	2.0", 1.5", 1.0", 0.5"	SG
19	0.483"	0.0485	0.050"	9	COC	2.0", 1.5", 1.0", 0.5"	SG

^aUND, uniformly distributed; CEC, center clustered; COC, corner clustered.

^bUG, unlike chambers grouped; SG, similar chambers grouped.

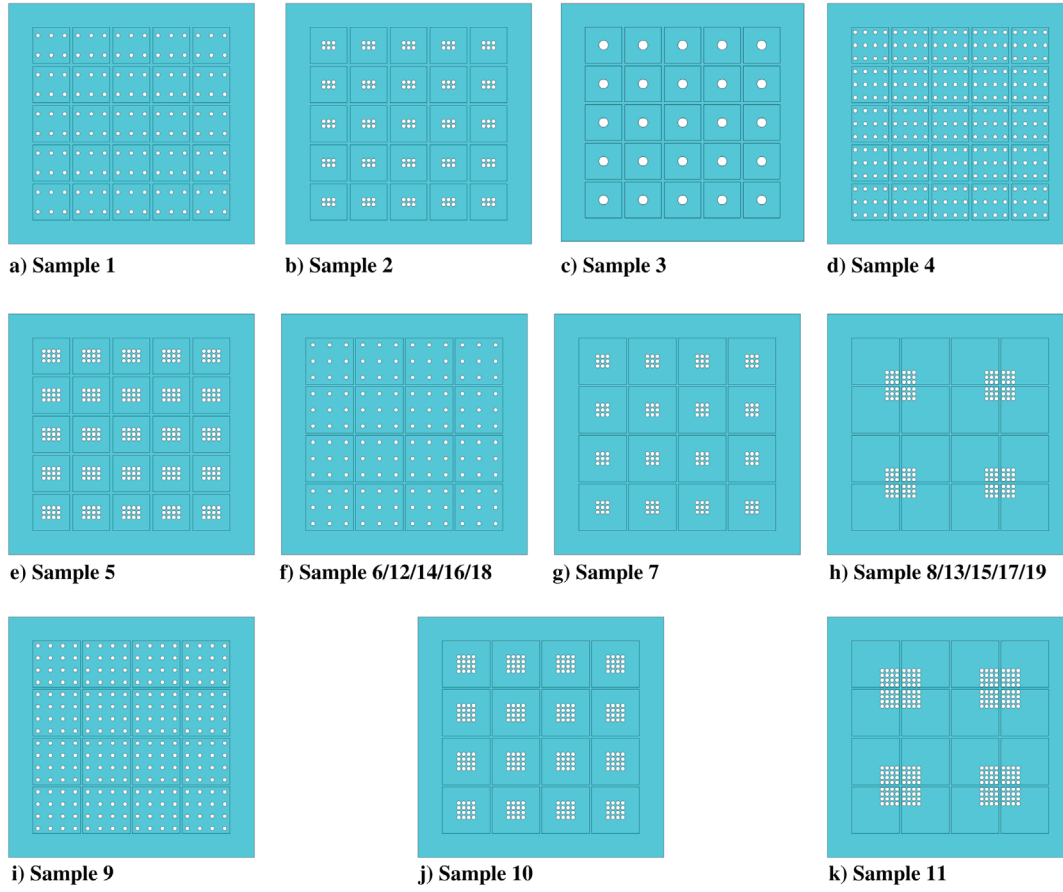


Fig. 7 Test sample facesheets showing differences between uniformly distributed and hole-clustered layouts. Adapted with permission from Kreitzman and Jones [13], a work of the U.S. Government.

within the hole is easily removed with a pin gauge. After cleanup of the parts, backs of the chambers are sealed with two layers of aluminum foil tape to avoid communication between chambers. Finally, these samples are also then press-fitted onto a flat metal surface during testing.

Every sample has a facesheet thickness of 0.03" and a hole diameter of 0.04". Two different chamber cross-sectional sizes are assessed to evaluate liners that have similar OARs but more holes per chamber. There are three different hole-distribution patterns analyzed, denoted as uniformly distributed (UND), center clustered (CEC), and corner clustered (COC) in Table 1. All hole-clustered liner samples have center-to-center hole spacings of 0.05" that, similar to previous studies, result in an interaction parameter of 1.25. This was found in Kreitzman and Jones [13] to provide good results and be practical for additive manufacturing. Additionally, all constant-depth liner samples (samples 1–11) have a chamber depth of 2.0". The impact of hole clustering when using different constant depths is discussed later in the article.

Samples 12–19 are the variable-depth samples, with Table 1 showing the unique chamber depths. The selection of these depths was arbitrary, i.e., specific frequencies were not targeted. Of these samples, there are two different kinds of chamber groupings. "Similar chambers grouped" (SG) refers to chambers of a similar depth being placed next to one another. "Unlike chambers grouped" (UG) refers to chambers of different depths placed side by side. Figure 8 shows the distinction for the variable-depth samples. All samples are assessed for quality via measurements with calipers, depth gauges, and pin gauges to ensure that liner geometry is close to targeted values. These measured geometric parameters are used in the prediction modeling process.

IV. Experimental Results

A. Constant-Depth Samples

Due to the vast amount of data taken, only portions of the data are shown to illustrate the various trends. Figure 9 shows the surface

2"	1"	2"	1"
1"	2"	1"	2"
2"	1"	2"	1"
1"	2"	1"	2"

a) Two-depth UG

2"	2"	1"	1"
2"	2"	1"	1"
1"	1"	2"	2"
1"	1"	2"	2"

b) Two-depth SG

2"	1.5"	2"	1.5"
0.5"	1"	0.5"	1"
2"	1.5"	2"	1.5"
0.5"	1"	0.5"	1"

c) Four-depth UG

2"	2"	1.5"	1.5"
2"	2"	1.5"	1.5"
0.5"	0.5"	1"	1"
0.5"	0.5"	1"	1"

d) Four-depth SG

Fig. 8 Chamber-depth layouts for the variable-depth samples.

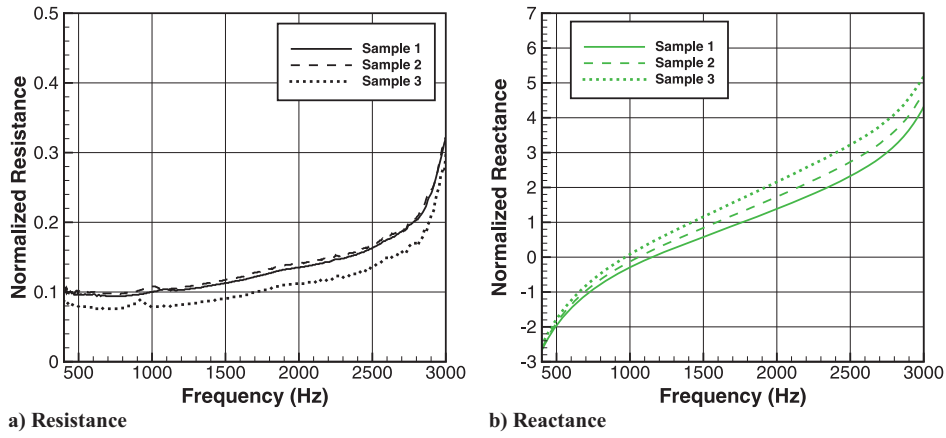


Fig. 9 Normalized impedance vs frequency (Hz) for samples 1–3 excited by a 100 dB tone. Sample 1 is uniformly distributed; sample 2 is center clustered; sample 3 is a single large hole per channel.

impedance of samples 1–3 for 100 dB tone excitation. Samples 1 and 2 show the impact of center clustering the holes, where there is a resonance shift to lower frequencies due to the increased radiation impedance of the center-clustered sample. The resonant frequency is defined as where reactance spectra cross zero with a positive slope. The resistance spectra of the two samples, however, are very similar, indicating that hole clustering largely impacts reactance with only minor effects to resistance in a no-flow environment. Sample 3 is plotted with these results to demonstrate the theoretical maximum resonance shift that can occur (one single large hole per chamber). Sample 2 approaches the reactance spectrum of sample 3 but does not achieve as much of a radiation impedance increase. However, due to a much larger hole size, sample 3 does not have the same resistance as the other two samples. This, coupled with the fact that larger holes are detrimental to aircraft nacelle drag and present increased risk to liner self-noise, demonstrates the positive impact that hole clustering could have relative to large facesheet holes. Future efforts need to assess the hole-clustering impact in flow to gain a better understanding of the true benefits for engine nacelle use, as it is well known that the reactance is altered in a grazing-flow environment [38].

Figure 10 shows the impact of both center and corner clustering, using samples 6–8 at 100 dB tone excitation. Corner clustering further lowers the resonant frequency compared to simply center clustering the holes. Added radiation loading due to the chamber walls, as well as holes being in closer proximity to holes of other chambers, are the primary contributors to this added benefit. Note that resistance is still only minimally impacted, although more strongly than samples 1 and 2 shown earlier, likely due to minor radiation resistance increases. Section V will expand on these results to provide further clarity on the findings, using the impedance

prediction model to assess the impact of hole spacing and number of holes clustered together.

Figure 11 shows a reactance comparison between tonal and broadband excitation, using samples 9–11. Again as before, the effect of hole clustering is evident, as the resonant frequency drops going from uniformly distributed to center clustered to corner clustered. The reactance differences between source excitation cases are minimal, as the tonal and broadband results mostly overlap with one another. There are some slight differences due to how the acoustic particle velocity effect presents itself for each source, meaning that 140 dB results are a bit more impacted. In general, though, tonal and broadband reactance spectra for a given sample are quite similar.

It is also of interest to assess the resistive differences at higher SPL excitations (120 and 140 dB) and different source types (tone and broadband). Figure 12 shows these differences using samples 9–11. Focusing first on tone excitations, the resistance spectra for different samples stay relatively consistent, other than the peak resistance due to particle velocity jetting, which shifts due to the resonant frequency shifting between samples. There does appear to be a slight increase in peak resistance magnitude as one goes from uniformly distributed to center clustered to corner clustered. It is possible that vortices shedding off the hole edges may be interacting with vortices from other holes as they get closer to one another. The broadband resistance spectra are more flat due to acoustic energy being spread across the frequency range of interest [35]. Due to this energy spread, the resistance spectra basically overlap between samples, once again indicating that hole clustering largely affects reactance with minimal impact on resistance in a no-flow environment.

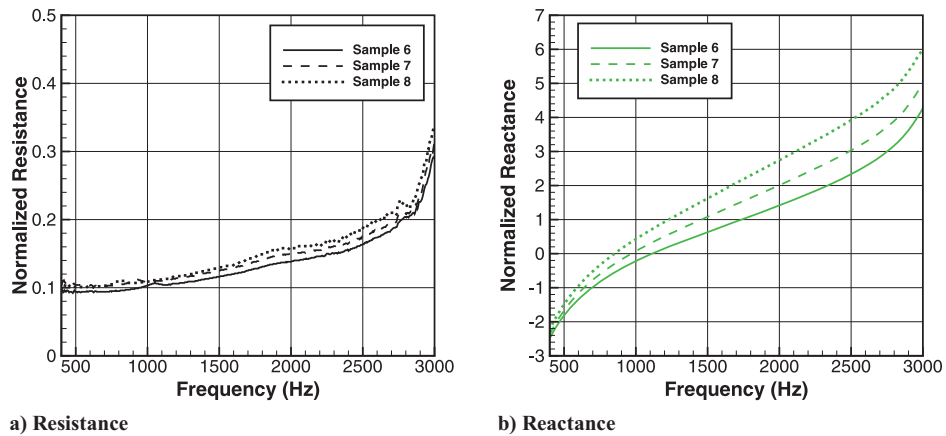


Fig. 10 Normalized impedance vs frequency (Hz) for samples 6–8 excited by a 100 dB tone. Sample 6 is uniformly distributed; sample 7 is center clustered; sample 8 is corner clustered.

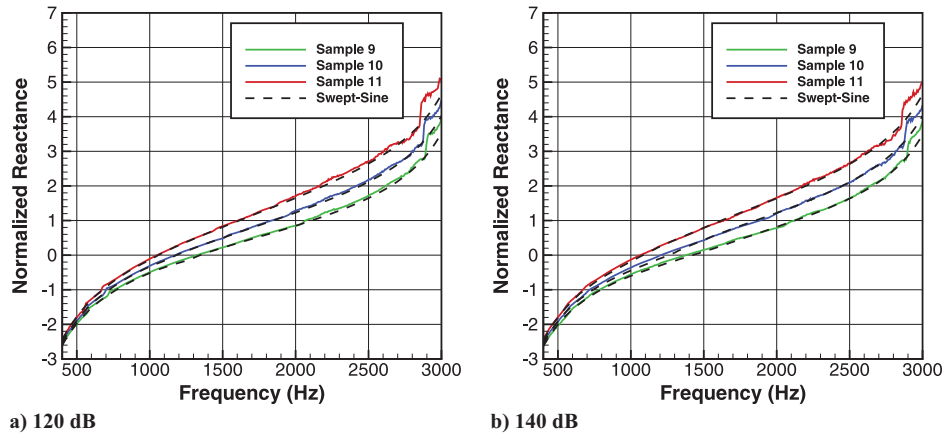


Fig. 11 Normalized reactance vs frequency (Hz) for samples 9–11 excited by tones and broadband at 120 and 140 dB. Sample 9 is uniformly distributed; sample 10 is center clustered; sample 11 is corner clustered.

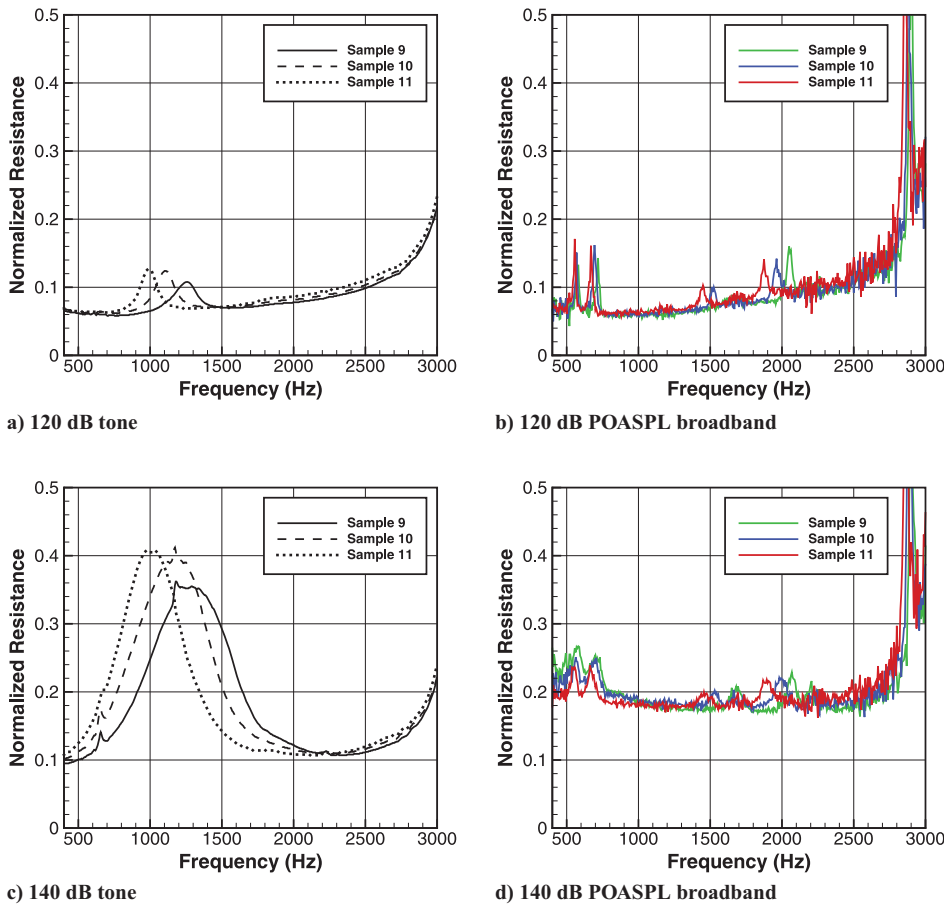


Fig. 12 Normalized resistance vs frequency (Hz) for samples 9–11 excited by tones and broadband at 120 and 140 dB. Sample 9 is uniformly distributed; sample 10 is center clustered; sample 11 is corner clustered.

It is clear that modifying an acoustic liner facesheet so that holes are nonuniformly clustered instead of uniformly distributed lowers the resonant frequency of the liner in a no-flow, normal-incidence environment, especially if the liner is corner clustered. In order to be more thorough, resonant frequency characteristics for all constant-depth samples tested are shown, specifically resonance differences between uniformly distributed and hole-clustered liners. Table 2 shows this for every source type and level. The “Baseline” column represents the uniformly distributed liner with its associated sample number, while the “Clustering change” column indicates the sample numbers that have the same geometry but with the hole-clustered distribution instead. The resonant frequency shifts are defined as positive when the clustering change

lowers the resonant frequency compared to its uniformly distributed counterpart.

These values are specific to the test samples in this study. Changes will occur for different hole diameters, OARs, and most importantly, chamber depths. Recall that the chamber depth selected for this study is 2.0”, which is typically quite large for modern aircraft engine nacelles. Since radiation impedance is a function of the free-space wavenumber [see Eq. (2)], it is more dominant at higher frequencies. Acoustic liners with smaller chamber depths will therefore see more of a resonance shift when clustering the holes on the facesheet. This was proven in a normal-incidence environment by Kreitzman and Jones [13], where both 2.0” and 1.0” chamber depth samples were tested with identical facesheets,

Table 2 Resonant frequency shifts for hole-clustered samples relative to their respective uniformly distributed baselines at different source types

Baseline	Clustering change	Tonal (SPL)			Broadband (POASPL)	
		100 dB	120 dB	140 dB	120 dB	140 dB
1	2	93.9 Hz	100.8 Hz	72.6 Hz	95.1 Hz	90.1 Hz
4	5	111.6 Hz	113.0 Hz	109.4 Hz	115.0 Hz	118.8 Hz
6	7	146.3 Hz	159.6 Hz	147.1 Hz	144.4 Hz	171.0 Hz
6	8	264.2 Hz	283.0 Hz	319.4 Hz	264.1 Hz	325.4 Hz
7	8	117.9 Hz	123.4 Hz	172.3 Hz	119.7 Hz	154.4 Hz
9	10	156.7 Hz	162.8 Hz	162.7 Hz	148.3 Hz	176.0 Hz
9	11	278.2 Hz	286.4 Hz	340.5 Hz	273.3 Hz	315.4 Hz
10	11	121.5 Hz	123.6 Hz	177.8 Hz	125.0 Hz	139.4 Hz

both uniformly distributed and center-clustered facesheets. Despite using a 2.0" chamber depth in this study, large resonance shifts are still achieved, sometimes greater than 300 Hz when corner clustering is involved.

A simple way to approximate the resonant frequency shifts of smaller chamber depths is to correct the 2.0" chamber depth data. By assuming that the liner reactance of the chambers alone is $-\cot(kh)/\Omega$, one can modify the data by removing this term and adding a term with a new depth value. Although being modeled as an infinitely long tube, a change in the chamber depth could potentially impact the perforated radiation impedance due to reflections from the rigid end of the chamber. Ih [7,39] presented a modification to the end correction computation to account for this. A brief analysis of this modification demonstrated that the chamber depths of interest in this article (down to 0.5") likely would not contribute to any significant differences associated with the end correction computation. Table 3 shows the projected resonant frequency shifts assuming a chamber depth of 1.5" and 1.0", using samples 6–8 at 100 dB tone excitation. The 2.0" chamber depth experimental data are also shown for comparison. Shifts in the resonant frequency increase as chamber depth gets smaller, as expected. Going from a uniformly distributed to corner-clustered liner backed by a 1.0" chamber depth yields a resonant frequency shift of almost 500 Hz compared to a ~250 Hz shift for a 2.0" chamber depth.

Returning to the 2.0" chamber depth experimental data in Table 2 and focusing first on 100 dB tone excitation to minimize facesheet nonlinear effects, one can make a number of observations. For one, for similar OARs, the samples with more holes per chamber experience greater mutual-radiation impedance impacts and hence resonance shifts. This is evidenced by sample 2 shifting the resonant frequency by 93.9 Hz compared to sample 1, whereas sample 7 shifts it by 146.3 Hz compared to sample 6. Additionally, samples with higher OARs experience slightly higher resonance shifts when hole clustering is implemented. This applies to center and corner clustering. This, like the chamber depth effect, is partially due to the fact that liners with higher OAR values resonate at higher frequencies, where radiation impedance is more significant and therefore makes more impact when altered.

Once the facesheet radiation impedance starts to be impacted by particle velocity jetting (120 and 140 dB), similar trends exist in Table 2, with some nuances. For a given SPL, the trends between geometries are similar to other SPL cases. Additionally, since 120 dB tones only produce minor nonlinear effects, resonance shifts for the 120 dB tone excitations are similar to those of the 100 dB tone excitation. Most differences in the resonance shifts between the 120 and 140 dB tone cases can be attributed to the frequency ranges

where particle velocity plays a strong role in lowering the reactance around resonance. Adding a high amount of radiation impedance to the facesheet steepens the reactance spectrum (see Fig. 10, for example), which means that the frequency range around resonance where particle velocity effects are significant narrows. The 140 dB reactance spectrum should gradually approach that of the 120 dB as more radiation effects are present. Therefore, resonance shifts between hole-clustering and uniformly distributed samples for 140 dB may be higher at times than 120 dB.

For the 120 and 140 dB broadband cases, the geometric trends shown previously hold. The 120 dB broadband excitation resonant shifts are very similar to the 100 dB tone excitation shifts. Again, since particle velocity effects are spread out in frequency for the broadband excitation, impact on the actual resonant frequency is lowered, and hence it does exhibit similar resonance shifts as the 100 dB tone excitation.

B. Variable-Depth Samples

It is important to also assess the effects of hole clustering for acoustic liners meant to attenuate a broad range of frequencies, such as a variable-depth liner. Figure 13 shows reactance spectra for samples 12–15 at 100 dB tone excitation. There is a clear shift to lower frequencies when corner clustering is employed on the facesheet, and the broadband behavior of the liner (frequencies where reactance is close to zero) is relatively maintained, although it is a reduced frequency range compared to the uniformly distributed liner. As mentioned earlier, due to the steepening of the reactance spectra when radiation impedance is increased, the broadband behavior is lessened when hole clustering is implemented. When the liners are excited by a 140 dB tone, antiresonances in the reactance spectra are less prominent, as shown in Fig. 13. Additionally, resonance shifts between uniformly distributed and corner-clustered liners are close to 500 Hz for the 140 dB source excitation cases.

Placement of the chambers within the impedance tube sample is also important, as it alters the radiation impedance of the liner. This is especially true for hole-clustered samples. Figure 14 shows the impact of chamber placement on reactance spectra for uniformly distributed and hole-clustered samples. Note that while the effects are present for all liner samples, impact is greater on the hole-clustered liners. The SG liners achieve lower-frequency resonance, and this is due to enhanced mutual-radiation effects that they experience from clustering chambers of a similar depth. This increases the radiation impedance of hole clustered liners due to the in-phase radiation that the closely coupled holes experience.

V. Prediction Results

The model shown in Sec. II is used to predict the no-flow impedance of constant- and variable-depth liners, with uniformly distributed, center-clustered, and corner-clustered facesheet configurations. This model does not take into account the reduction in radiation impedance when particle velocity effects are significant. Hence, reactance predictions are only compared to 100 dB tone excitation spectra, although these predictions should mostly hold

Table 3 Projected resonant frequency shifts for samples 6–8 at different chamber depths for 100 dB tone excitation

Baseline	Clustering change	2"	1.5"	1.0"
6	7	146.3 Hz	200.2 Hz	280.6 Hz
6	8	264.2 Hz	352.9 Hz	491.7 Hz
7	8	117.9 Hz	152.7 Hz	211.1 Hz

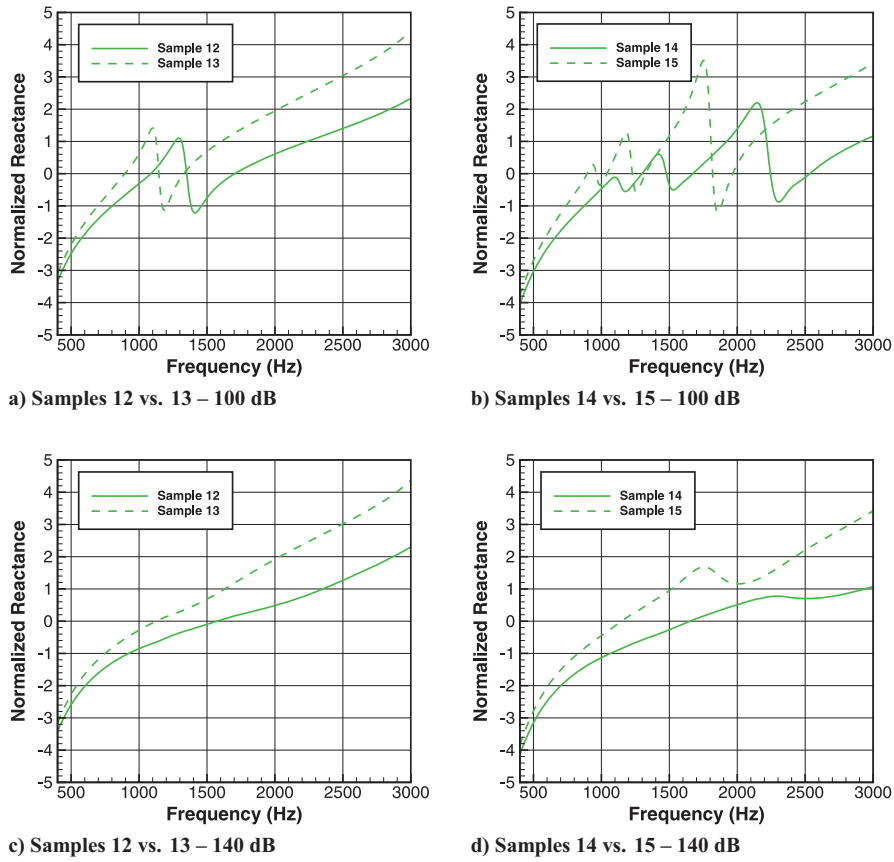


Fig. 13 Normalized reactance vs frequency (Hz) for samples 12–15 excited by both a 100 and 140 dB tone. Samples 12 and 14 are uniformly distributed, while samples 13 and 15 are corner clustered.

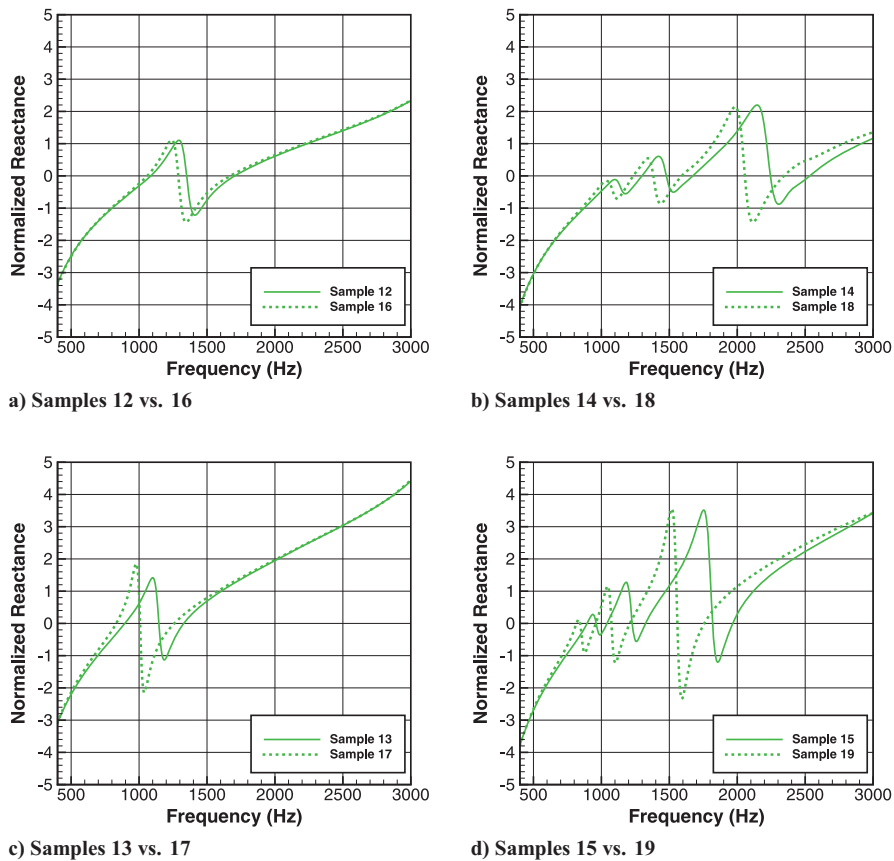


Fig. 14 Normalized reactance vs frequency (Hz) for samples 12–19 excited by a 100 dB tone. Samples 12–15 are UG variable-depth liners, while samples 16–19 are SG variable-depth liners.

for 120 dB tone excitation as well. The model does, however, account for resistive changes due to different SPL excitations, and this is better approximated by predicting and using the SPL at the liner surface rather than the reference microphone measurement [35].

Results for reactance predictions at 100 dB for samples 1–9 are shown in Fig. 15. The prediction model effectively tracks the reactance increases as one goes from uniformly distributed to center clustered to corner clustered, including different OARs, hole count per chamber, and chamber cross-sectional size. The difference between prediction and measurement is slightly increased at the higher frequencies. This is likely due to minor modeling errors associated with the mass reactance within the perforated holes, as errors are seen for uniformly distributed samples as well. As mentioned earlier, all samples have a target facesheet thickness of 0.03" and hole diameter of 0.04". Future work should continue to analyze this model for other facesheet geometries.

Figure 16 shows resistance predictions compared to experiment for samples 6 and 8 at tone excitations of 100, 120, and 140 dB. The predictions compare fairly well and accurately track the peak resistance shift due to corner clustering. As noted earlier, there does appear to be some slight increases in the nonlinear resistance when hole clustering is applied, and this is not accounted for in the model. The model does, however, accurately predict the narrowing of the resistance peak for the hole-clustered liner due to the steepening of the reactance spectra from added radiation impedance.

Since there is no variation in hole spacing for the center- and corner-clustered experimental samples, the impedance prediction model is used to assess resonance shifts for different spacings. Samples 6–8 are used as baseline cases, where the interaction parameter (hole center-to-center spacing to diameter ratio) is used, looking at various values going from 1.000 to 3.125. Note that the hole-clustered experimental samples have an interaction parameter

of 1.25, and an interaction parameter of one indicates that the edges of the holes are just touching one another. To better visualize the trends, exponential curve fits are added to the results. Like before, resonant frequency shifts are defined positive when resonance is lowered compared to the uniformly distributed (sample 6) resonant frequency. The results are shown in Fig. 17.

The impact of hole clustering becomes increasingly significant as hole spacing is reduced, such that an exponential curve fit, while not perfect, does a reasonable job at describing the behavior. The impact of clustering as it pertains to the number of holes in close proximity is also demonstrated, as the corner-clustered case yields increased resonance shifts for equivalent hole spacings, as was seen in the experimental data. The trends in Fig. 17 are also helpful in identifying where hole clustering may no longer be viable if the holes need to be further spaced out, for instance due to structural concerns. It is interesting to note that a 200 Hz resonance shift can still be achieved through corner clustering with an interaction parameter of 1.5, or 0.02" edge-to-edge distance (double the experimental distance).

Recall from earlier [see Eq. (6)] that the total radiation impedance of a given hole is the summation of its self-radiation and mutual-radiation impedance with every other hole. That mutual-radiation impedance is dependent on the ratio of surface particle velocities that the given hole has with every other hole. For constant-depth samples, all holes are assumed to radiate in phase with one another, so this ratio is one. For variable-depth samples, one can make the same basic assumption that was applied in Schiller and Jones [40]: perforated holes have no mutual-radiation impact on one another if they are associated with different chamber depths. This is not entirely true but is a good approximation for the purposes of this model. If one uses this assumption for variable-depth liners, the uniformly distributed samples can no longer use the simplified mass-end correction computation [Eq. (18)] and must use the more generalized solution [Eq. (19)]. Implementing these assumptions in

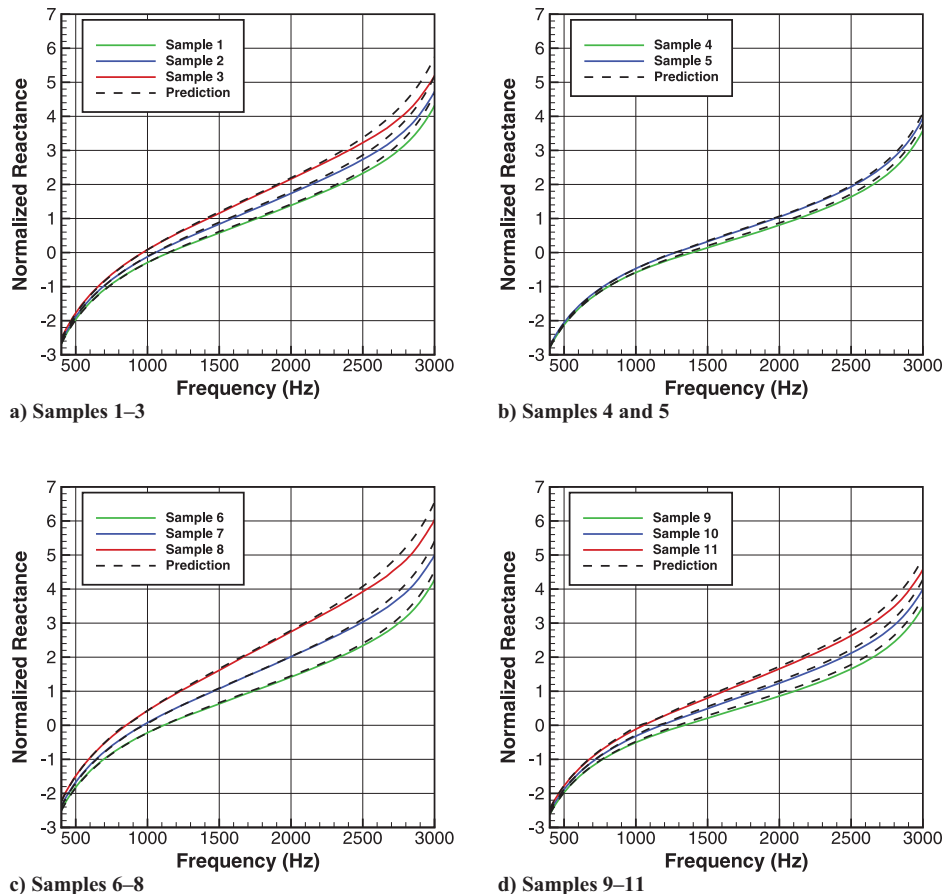


Fig. 15 Normalized reactance predictions vs frequency (Hz) for samples 1–11 (constant depth), excited by a 100 dB tone.

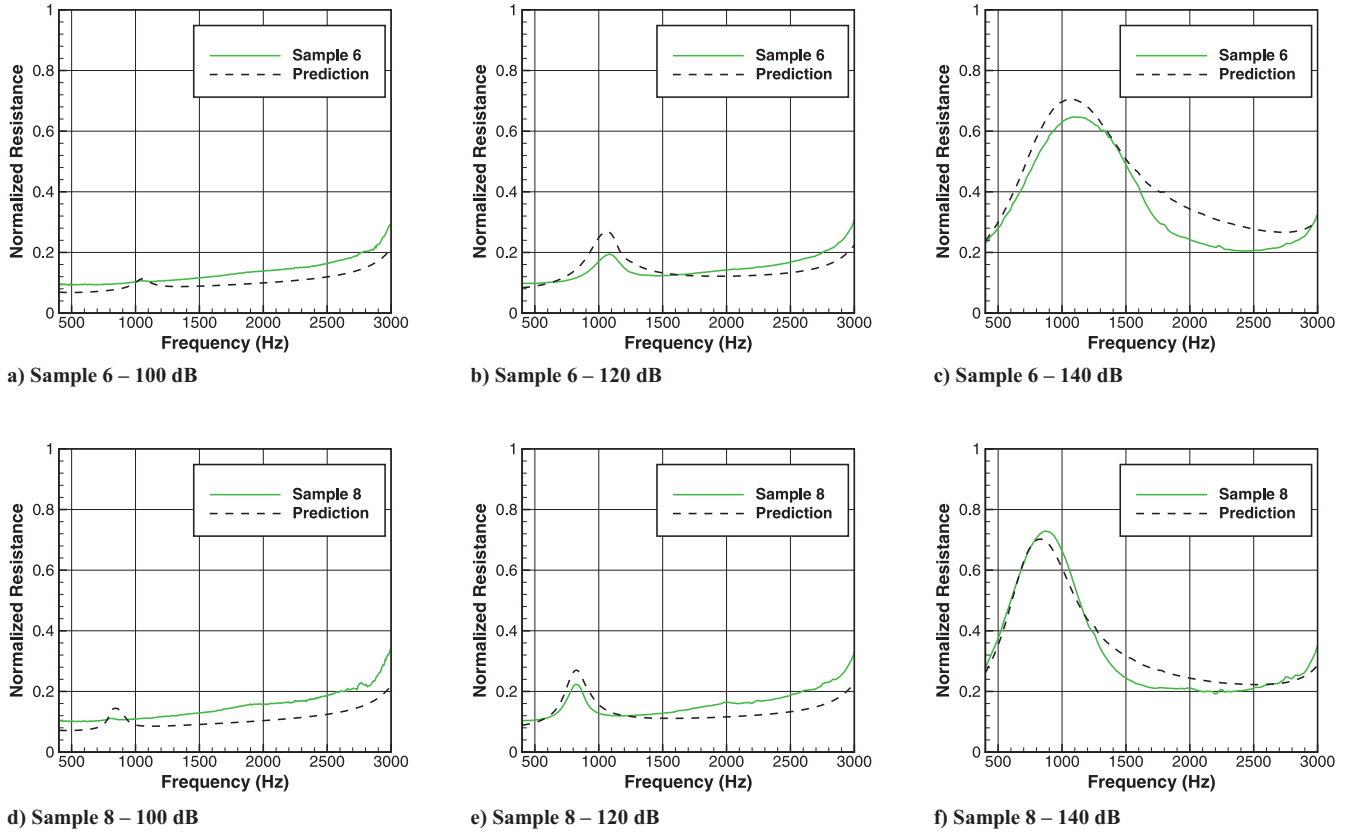


Fig. 16 Normalized resistance predictions vs frequency (Hz) for samples 6 and 8 (constant depth), excited by 100, 120, and 140 dB tones.

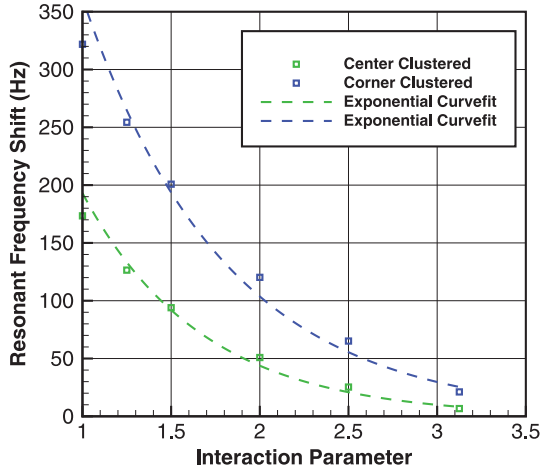


Fig. 17 Effect of hole spacing and number of holes clustered together for a given OAR, using samples 6–8 as the baseline configurations.

the model leads to predicted results for the eight variable-depth samples shown in Figs. 18 and 19. In general, good agreement is seen between predicted and experimental data, despite the simplifying assumptions. This is true for both UG and SG liners. Similarly, Fig. 20 shows resistance predictions compared to experimental data for samples 12–15 at 100 dB tone excitation. The shift in antiresonant peaks due to hole clustering is well predicted. One would expect further improvements to the predictions if the simplifying assumptions are no longer made. It is important to reiterate that this impedance model is for a no-flow environment, and modifications would have to be made for grazing-flow environments. Such efforts have been done previously [41] and usually amount to scaling the no-flow end correction values based on the Mach number in the duct.

VI. Summary and Next Steps

Adjusting the hole pattern distribution of a perforated sheet acoustic liner from uniform to highly nonuniform (i.e., clustered) lowers the resonant frequency of the acoustic liner in a no-flow environment. This effect is enhanced when hole-clustered patterns are placed near common chamber corners. This resonance shift is coupled with only minor changes to the resistance, and these resistance changes are even smaller when the liner is excited by a broadband source, where particle velocity effects are spread throughout the frequency range of interest. Nineteen additively manufactured acoustic liners (both constant and variable depth) with varying OAR, chamber size, and hole spacing were designed, printed, and tested in the NASA Langley NIT to verify these trends. These liners were excited by different acoustic sources, including broadband and tonal excitations up to 140 dB. Variable-depth liners also showed lower resonant frequencies when hole clustering was implemented, but the frequency range where the liner was highly absorptive was also slightly narrowed due to increased radiation impedance steepening the reactance spectra. Overall, hole clustering on the facesheet provides a simple approach to lower the peak absorption frequency of an acoustic liner while avoiding added weight and volume. However, these trends need to be reassessed in a grazing flow tube to better understand the hole-clustering impact in a more realistic aeroacoustic environment.

An impedance model was developed to accurately account for the acoustic changes due to hole clustering in a no-flow environment. The model relies on theory for pistons radiating in an infinitely long tube, leveraging previous work from Ingard [6] and Ih [7] to compute mass-end correction terms for the radiation impedance. The model accurately predicts the impedance of constant-depth liners for different OARs, chamber sizes, and clustering locations (center and corner) for a chamber depth of 2" at frequencies up to 3000 Hz. The model also, through an assumption that channels with different chamber depths do not interact, adequately predicts the impedance of variable-depth liners, both with uniformly distributed

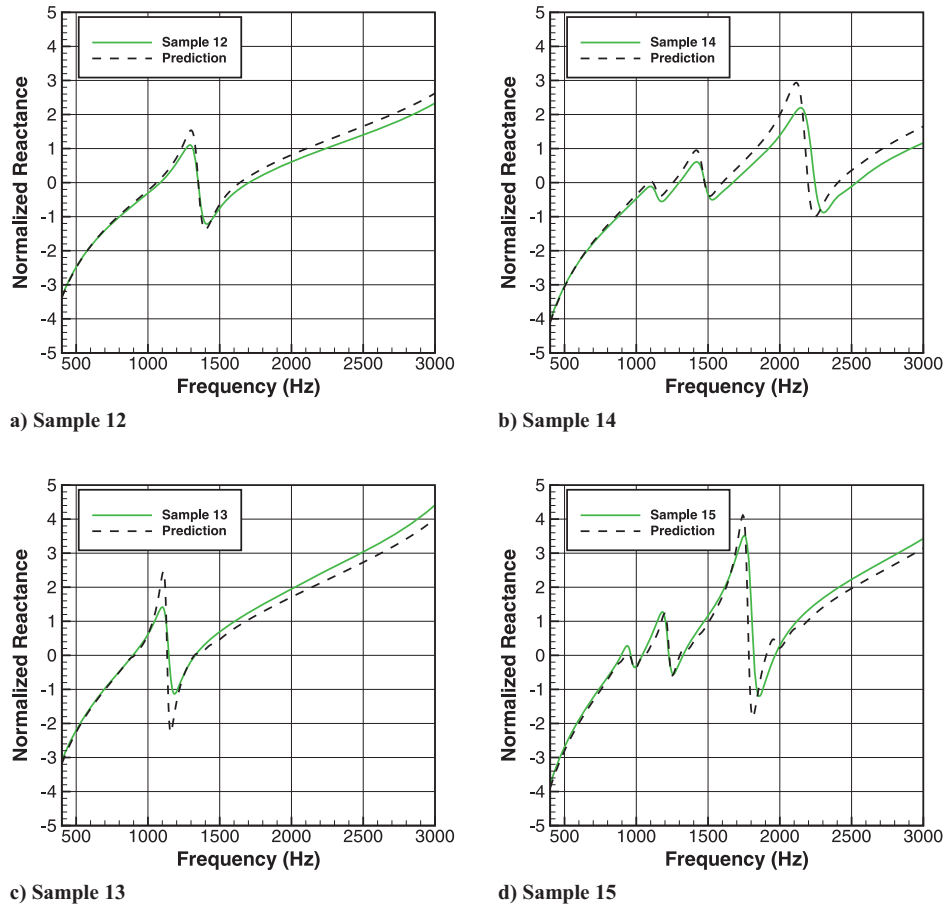


Fig. 18 Normalized reactance predictions vs frequency (Hz) for samples 12–15 (UG variable depth) excited by a 100 dB tone.

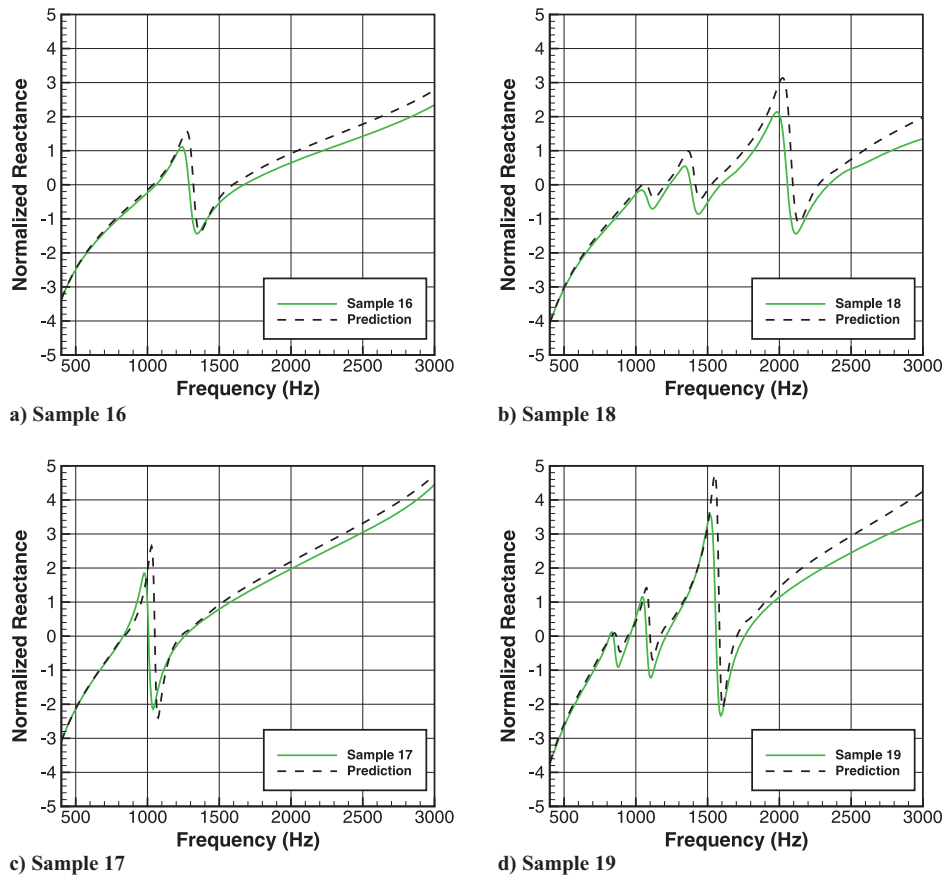


Fig. 19 Normalized reactance predictions vs frequency (Hz) for samples 16–19 (SG variable depth) excited by a 100 dB tone.

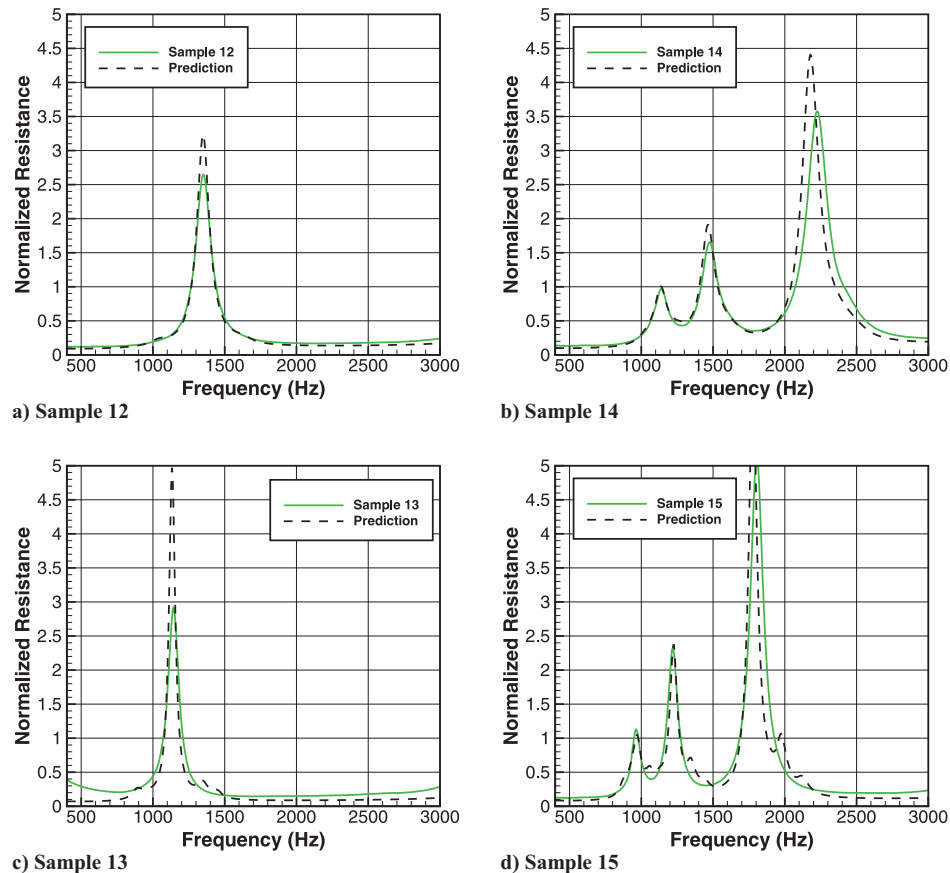


Fig. 20 Normalized resistance predictions vs frequency (Hz) for samples 12–15 (UG variable depth) excited by a 100 dB tone.

and hole-clustered facesheets. This includes capturing differences in the impedance spectra due to chamber location.

Efforts are underway to further validate the model with a wider variety of hole-clustered liners, i.e., different hole diameters, spacings, facesheet thicknesses, and chamber depths. This may also include smaller chamber depths to assess the impact of radiation impedance changes associated with reflections due to the rigid backing. It would also be valuable to assess the accuracy of the model at higher frequencies, and the NASA Langley High Intensity Modal Impedance Tube [42] could be used for such testing. Efforts should also take place to further improve the reactance modeling and incorporate nonlinear effects due to source excitation intensity, as currently only the modeled resistance is impacted. Additionally, improvements can be made to models of hole-clustered, variable-depth liners, including the explicit use of the surface particle velocity ratio between holes. The hole-clustering concept is also actively being researched in flow to better understand acoustic characteristics for potential aircraft engine nacelle usage. This will result in improved hole-clustering impedance models where flow effects are taken into account. As mentioned earlier, updated flow models will likely be slight modifications to the no-flow model presented in this article.

Finally, hole clustering could be employed beyond the typical facesheet application. For example, embedded perforated elements within the liner chambers could take advantage of hole clustering. This could provide lower-frequency absorption to a liner meant for a wider bandwidth of performance. An embedded perforate would also avoid flow effects, and the use of no-flow impedance models would be applicable. Future studies should continue to assess other applications beyond the typical facesheet over honeycomb chambers.

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