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Development of System Identification Testing Method for Stirling Convertors

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July 2025

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Summary

This report presents a new method of identifying accurate system representations of free-piston Stirling convertors (FPSCs) to support FPSC controller development. Previously, these system representations were identified by measuring as many physical parameters of the FPSC as possible, then calculating the ones that were impractical to measure. However, this previous method generated system representations that produce an error higher than 20% for one of the most important parameters. Furthermore, because this previous method requires the use of multiple software tools and expertise in several different fields to obtain accurate calculations of the physical parameters, it can be very time-consuming. In contrast, the new proposed method can be performed in a few hours to generate a more accurate system representation with an error of less than 1.5%, improving accuracy by a factor greater than 13. As a result, it is expected that the proposed method would significantly reduce the time needed to develop a reliable FPSC controller for upcoming flight projects.

Introduction

An accurate system representation of a free-piston Stirling convertor (FPSC) is essential for developing a reliable controller to regulate the piston motion and properly manage the power generated by a FPSC. To this end, researchers at NASA Glenn Research Center and Johns Hopkins University's Applied Physics Laboratory have been working to build such a representation for years by linearizing the nonlinear FPSC at the FPSC's target operating temperature (Ref. 1). Once the FPSC is expressed in a linearized state-space representation, linear system analysis methods such as the Bode plot or root locus can be used to analyze the control properties of the FPSC.

To represent the FPSC in the linearized state-space representation, several physical parameters of the FPSC must be measured or calculated. Although most of the parameters can be measured during the development process of the FPSC, some parameters, such as piston/displacer pressure factors and piston/displacer delta pressure factors, are impractical to measure. As a result, to date, those parameters have been calculated using tools in Sage software for engineering modeling and optimization (Gedeon Associates) and the MATLAB® (The MathWorks, Inc.) programming and computing environment (Ref. 2). Although this method computes those unmeasurable parameters using theoretical equations, recent efforts have shown that such developed state-space models show finite errors, including an error greater than 20% for one of the important parameters of the Advanced Stirling Convertor (ASC). Furthermore, when this method was applied to the Sunpower Robust Stirling Convertor (SRSC) to generate a state-space representation, the analyzed system stability of the representation could not represent the system behavior of the actual SRSC effectively. Finally, because this analytical method requires several experts in different fields of study to analyze the system to calculate the unmeasurable parameters, the initial computation and corrections typically require months or longer to complete.

In contrast, the new method presented in this report can improve the accuracy of the model and decrease the model development time significantly by conducting a simple system identification test on the FPSC. This report presents details on the new method and test setup, followed by the test results to demonstrate that the new method can generate much more accurate state-space models than the existing model in a much shorter time.

Method

State-space representation of a linearized model of an FPSC (Refs. 3 and 4) can be expressed as shown in Equations (1) and (2).

$$\begin{bmatrix} \dot{X}_d \\ \ddot{X}_d \\ \dot{X}_p \\ \ddot{X}_p \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K_d + A_r P_d}{M_d} & \frac{A_d \Delta P_d}{M_d} & -\frac{A_r P_p}{M_d} & \frac{A_d \Delta P_p}{M_d} \\ 0 & 0 & 0 & 1 \\ -\frac{A_p P_d}{M_p} & 0 & -\frac{K_p + A_p P_p}{M_p} & -\frac{C_p}{M_p} \end{bmatrix} \begin{bmatrix} X_d \\ \dot{X}_d \\ X_p \\ \dot{X}_p \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ K_i / M_p \end{bmatrix} I_{alt} \tag{1}$$

$$V_b = \begin{bmatrix} 0 & 0 & 0 & K_e \end{bmatrix} \begin{bmatrix} X_d & \dot{X}_d & X_p & \dot{X}_p \end{bmatrix}^T \tag{2}$$

where:

A_d = displacer area, m^2
A_p = piston area, m^2
A_r = displacer rod area, m^2
C_p = piston damping, N/(m/s)
I_{alt} = alternator current, A
K_d = displacer spring constant, N/m
K_e = back emf constant, V/(m/s)
K_i = magnetic force constant, N/A
K_p = piston spring constant, N/m
M_d = displacer mass, kg
M_p = piston mass, kg
P_d = displacer pressure factor, Pa/m
P_p = piston pressure factor, Pa/m
ΔP_d = delta displacer pressure factor, Pa/(m/s)
ΔP_p = delta piston pressure factor, Pa/(m/s)
V_b = back-EMF, V
X_d = displacer displacement, m
$\dot{X}_d$ = displacer velocity, m/s
X_p = piston displacement, m
$\dot{X}_p$ = piston velocity, m/s

To generate a state-space representation of an FPSC, all the parameters listed here must be known. Most of these parameters can be measured during the development process of the FPSC and provided by the manufacturer; however, measuring some parameters, such as C_p, K_p, P_d, P_p, ΔP_d, and ΔP_p, is impractical, so additional modeling efforts are required to calculate those parameters in general. Such efforts are described in Reference 5, where Sage FPSC models and MATLAB® code are used to calculate them using known mechanical and thermal properties of the FPSC. Although this method has been predicting the performance of the FPSC very accurately, the system-level prediction tends to be less accurate. For example, to assess the model accuracy, the generated state-space model is connected to a tuning capacitor and an AC bus voltage in SPICE simulation software, and the output parameters—$X_p, X_d, V_{alt}, I_{alt}$, and P_{alt}—are compared with test data. When assessed, the difference is approximately 10 to 20% in general. Occasionally, the system in the simulation is unstable when it should be stable according to the test data, requiring more thorough and accurate modeling efforts. This process is repeated multiple times until the system becomes stable, which could take from months to a year.

Researchers at NASA Glenn and the Applied Physics Lab developed a system identification testing method to address these problems. In this method, the piston amplitude is step-changed to identify those nonmeasurable parameters by changing the control voltage within a very short duration to see how quickly the piston amplitude follows the commanded control voltage. This will reveal important information on the system stability, such as poles and zeros, which is critical for controller development. Figure 1 shows the electrical schematic of the test setup. The FPSC is connected to a tuning capacitor and an AC bus in series. The following signals are monitored and collected at a high rate during testing: the piston and displacer positions, alternator voltage/current, tuning capacitor voltage, and AC bus voltage. After that, the FPSC is motored at the room temperature and the heater at the heat source is slowly ramped up to a target temperature to transition from motoring to power generation. Once it reaches the steady state at the target temperature, the piston amplitude is adjusted to 3 mm and the setup is now ready for the step response test illustrated in Figure 2.

The piston amplitude is step-changed from 3 to 4 mm within a fraction of a second, then step-changed back to 3 mm within 10 s to maintain the steady-state thermal conditions. During this step response test, all monitored signals are collected at a high rate using a DL850 ScopeCorder® (Yokogawa Corporation of America) data acquisition recorder. These collected signals are then plugged into the MATLAB® System Identification Toolbox™. More specifically, *greyest*, a built-in MATLAB® function for estimating ordinary differential equations (ODEs) of a linear grey box model, is used to estimate the unknown modeling parameters of the linearized FPSC model with known parameters based on Equations (1) and (2) (Ref. 6). This MATLAB® function then provides the best estimates of those unknown parameters by finding the unknown parameters that will result in the best curve-fitting of the step response of the linearized model into

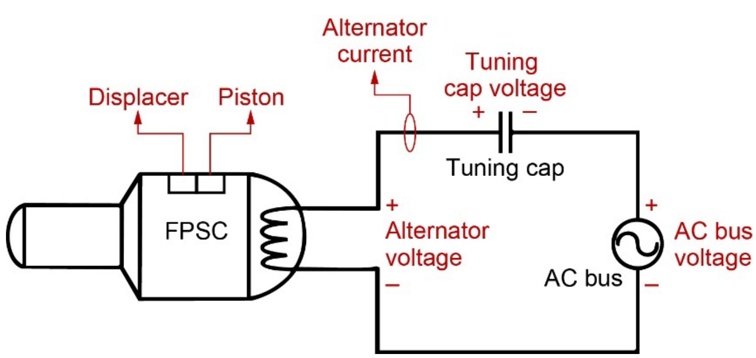
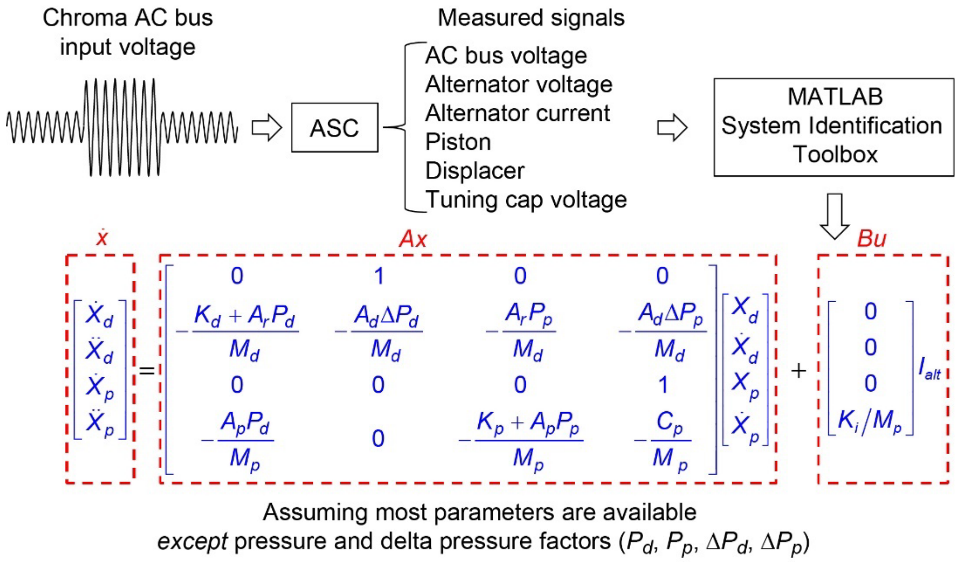


Figure 1.—System identification measurements.



$$\begin{bmatrix}\dot{X}_d\\ \ddot{X}_d\\ \dot{X}_p\\ \ddot{X}_p\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0\\ -\frac{K_d+A_rP_d}{M_d} & -\frac{A_d\Delta P_d}{M_d} & -\frac{A_rP_p}{M_d} & -\frac{A_d\Delta P_p}{M_d}\\ 0 & 0 & 0 & 1\\ -\frac{A_pP_d}{M_p} & 0 & -\frac{K_p+A_pP_p}{M_p} & -\frac{C_p}{M_p}\end{bmatrix}\begin{bmatrix}X_d\\ \dot{X}_d\\ X_p\\ \dot{X}_p\end{bmatrix}+\begin{bmatrix}0\\0\\0\\K_i/M_p\end{bmatrix}I_{alt}$$

Figure 2.—Step response test block diagram.

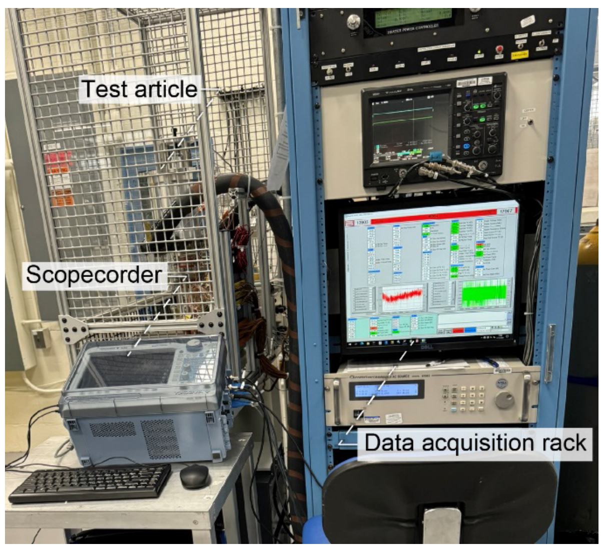


Figure 3.—System identification test setup.

the actual step-response data. Once those unknown parameters are found, they are plugged into the state-space representation of the linearized FPSC model to perform the verification process in the simulation. The overall test setup, consisting of a test article (FPSC), a data acquisition rack, and a DL850ScopeCorder® recorder, is shown in Figure 3. The test article used in this report is ASC-E2 #2, which Sunpower provided to NASA Glenn in 2010 to support the Advanced Stirling Radioisotope Generator (ASRG) flight project. This article has been extensively used for research and development activities for FPSC controllers. The instrumentation in the data acquisition rack can acquire high-rate data at 7 kHz and low-rate data at 0.5 Hz. Furthermore, the rack has the tuning capacitors and the AC bus to provide the FPSC power path, and the failsafe protection circuit (FPC) to protect the FPSC from piston overstroke (Figure 1). Finally, the DL850 ScopeCorder® recorder can collect and display 32-channel signals at up to 100 MS/s. In the test setup, the key parameters—piston and displacer positions, alternator voltage and current, tuning capacitor voltage, and AC bus voltage—were measured at 100 kS/s.

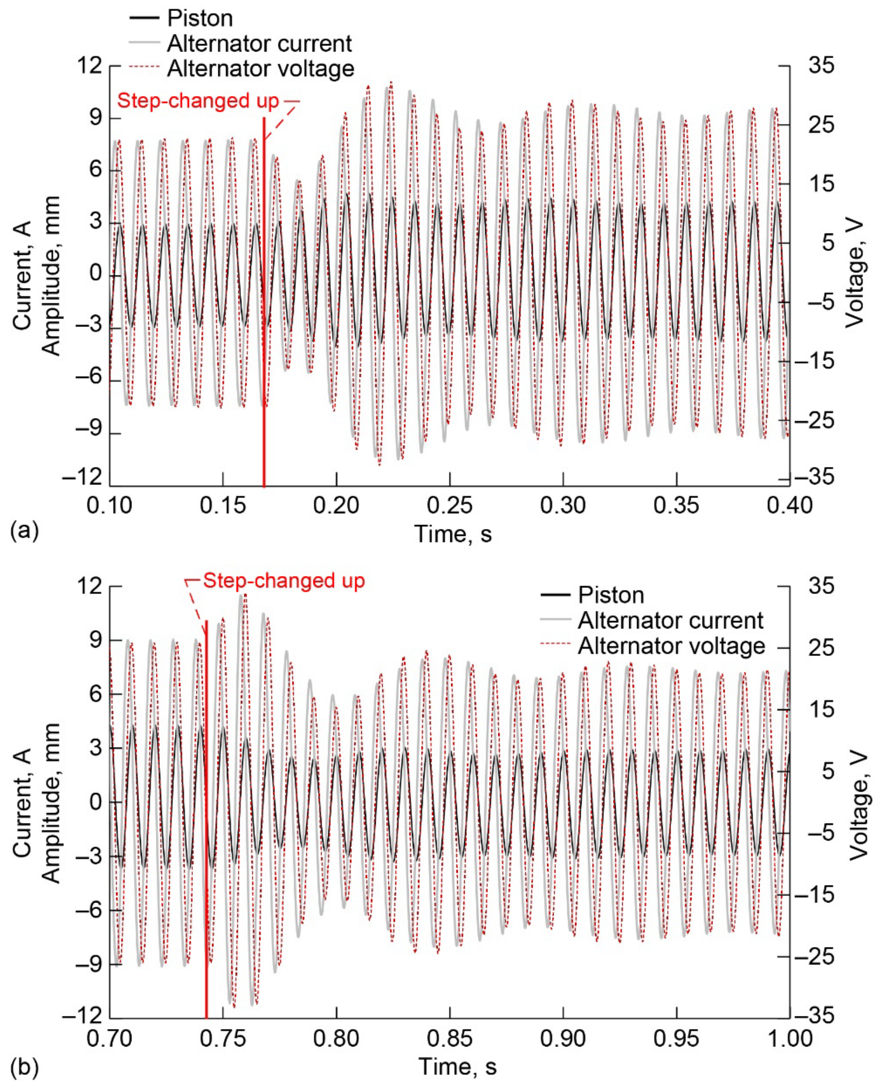


Figure 4.—Results of system identification testing. (a) During step-up from 3 to 4 mm. (b) During step-down from 4 to 3 mm.

Test Results

Results of the system identification testing are shown in Figure 4. The piston amplitude was step-changed from 3 to 4 mm (Step-Up), then back to 3 mm (Step-Down) within 10 s to prevent any significant changes in the thermal conditions of the FPSC. It is clearly shown that during the step changes, the piston and other signals take approximately 5 to 10 engine cycles to reach a steady state at the final piston amplitude. More specifically, when the Step-Up procedure was performed, the piston amplitude increased to 4.417 mm momentarily during the transient response. Similarly, the alternator voltage and current, which are normally 17 V and 6.7 A, increased to 22.9 V and 8.7 A momentarily during the transient response. These transient responses, as well as steady-state responses, are unique for each convertor; therefore, a state-space model of a specific convertor linearized at these specific thermal conditions can be estimated. For an accurate estimation, this set of step response data was processed using a MATLAB® function and plugged into the MATLAB® System Identification Toolbox™.

More specifically, given that most physical parameters of the convertor are known except for pressure and delta pressure factors for the displacer and the piston (P_d, P_p, ΔP_d, and ΔP_p), which are impractical to measure, the convertor model is considered a grey-box model; therefore, the test data and the known parameters were plugged into the MATLAB® *greyest* function to estimate P_d, P_p, ΔP_d, and ΔP_p. After this process was complete, the estimated variables were used to build the new model, which was used in LTspice® (Analog Devices, Inc.), a circuit SPICE simulation tool, to validate its accuracy. Figure 5 illustrates a simple circuit schematic in which ASC-E2 #2 was connected to a tuning capacitor (840 µF) and an AC bus at 102.2 Hz. The old model, which was developed using simulation tools only, was also used to compare the accuracy of some critical signals between the old and new models. Figure 6 shows the alternator voltage and alternator current waveforms from test data, simulation data using the new model, and simulation data using the old model. Clearly, the alternator voltage and current signals in the simulation using the new model match the test data more accurately than those signals in the simulation using the old model. Table I provides a comparative summary.

The simulation results using the new model show far fewer errors than the results using the old model. More specifically, the least accurate parameter in the simulation using the old model was the alternator current and its error was greater than 20%. On the other hand, the error in the alternator current signal using the new model was reduced to 0.42%, an improvement by a factor of more than 55, validating the accuracy of the new model using the proposed system identification test method.

TABLE I.—TEST AND SIMULATION RESULTS

Parameter	Test results	Old model	New model
X_p	2.99 mm	3.07 mm (+2.7%)	2.99 mm (0.0%)
V_{alt}	11.98 V_{rms}	10.56 V_{rms} (–11.9%)	12.15 V_{rms} (+1.4%)
I_{alt}	4.76 A_{rms}	3.64 A_{rms} (–23.5%)	4.74 A_{rms} (–0.42%)
P_{alt}	41.4 W	33.51 W (–19.1%)	41.9 W (+1.2%)

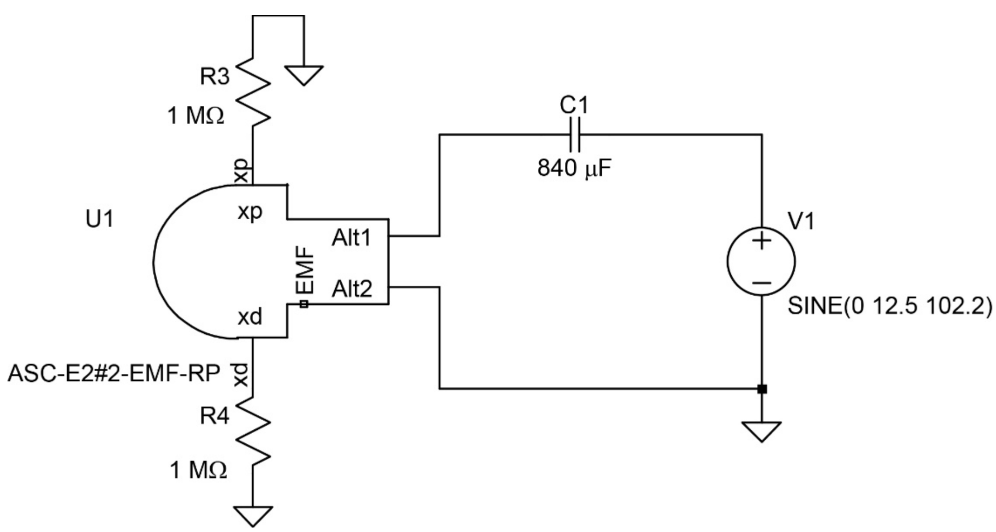


Figure 5.—Simple circuit schematic to validate new model in LTspice®.

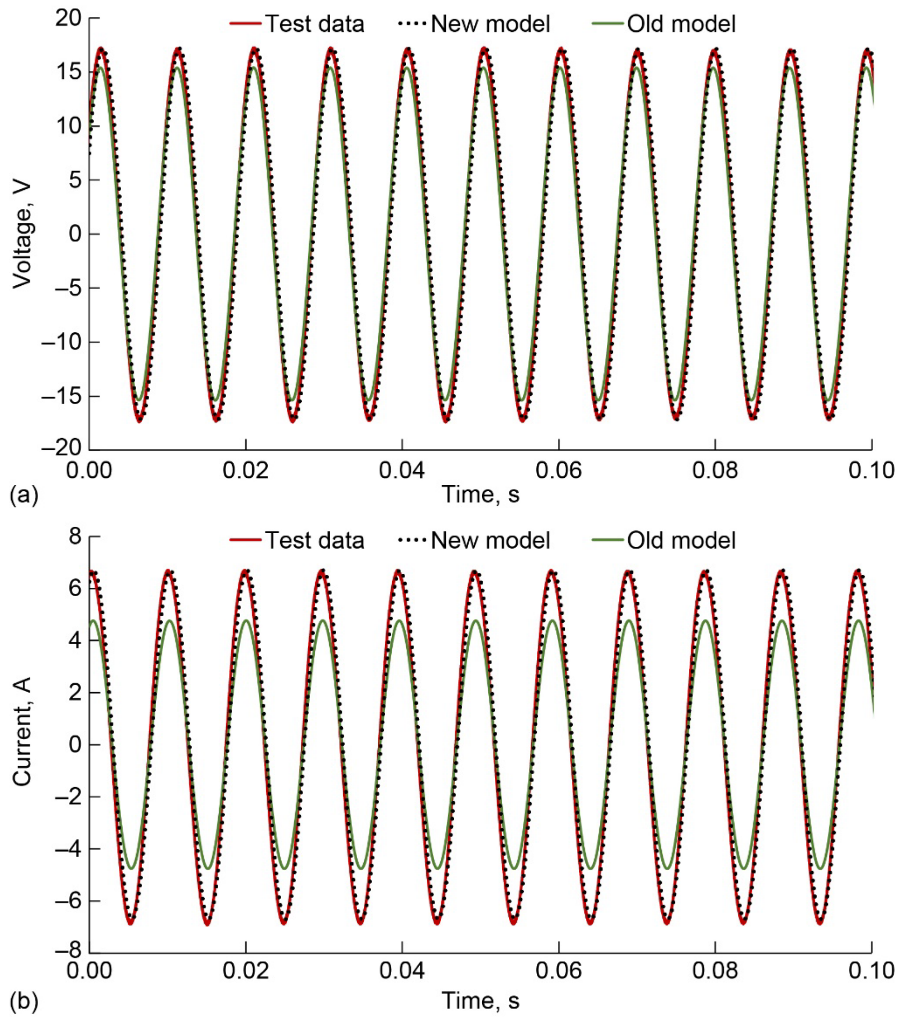


Figure 6.—Simulation results. (a) Old model. (b) New model.

Conclusion

In this report, a new system identification method for free-piston Stirling convertors (FPSCs) was proposed and a model developed based on the method was validated through a simulation and a test. Unlike the existing method, in which critical variables for a linearized FPSC model are computed by using simulation tools such as Sage and often takes a long time for analysis and validation process, the proposed method is capable of generating a linearized model quickly and accurately. In this method, the piston amplitude was step-changed from 3 to 4 mm (Step-Up) and then back down to 3 mm (Step-Down), and the piston and displacer signals, alternator voltage and current, tuning capacitor voltage, and the AC bus voltage were recorded during the step changes. After that, those signals, as well as known physical variables of the FPSC, were plugged into the MATLAB® System Identification Toolbox™ to estimate unknown variables of interest such as pressure factors and delta pressure factors. Finally, the estimated variables were used to build a linearized FPSC model and an LTspice® simulation was performed to validate the accuracy of the linearized FPSC model where the accuracy of the simulation was improved by a factor of 55 when the new model was used when compared to when the old model was used. For future work, the system identification method will be performed on other FPSCs such as the Sunpower Robust Stirling Convertor (SRSC) to support current and future projects.

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