

TIME-OF-FLIGHT FREE MODEL PREDICTIVE CONTROL APPLICATION FOR POLYHEDRAL ASTEROID LANDINGS

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In this work, an application of model predictive control (MPC) for polyhedral-gravity asteroid landings is presented. The MPC utilizes a pre-computed, fixed time-of-flight (ToF) reference trajectory generated through successive convex approximations (SCvx) to minimize a ToF free cost function. The implementation of the MPC is designed to be computationally cheap enough for application on board the asteroid landing craft. The approach is demonstrated by running simulations of landings on various asteroids, comparing the fuel expenditure between the reference and MPC trajectories, as well as the computation time versus the simulation time.

INTRODUCTION

Asteroids and other similarly sized celestial bodies present a unique set of challenges for scientists and engineers around the globe. Several missions have already undertaken the daunting task of performing landing procedures on small bodies. Namely, NASA's Near Earth Asteroid Rendezvous (NEAR) mission¹, Japan Aerospace Exploration Agency's (JAXA) Hayabusa², NASA's OSIRIS-Rex³, and the European Space Agency's (ESA) Rosetta mission and Philae lander⁴. While the aforementioned missions all had varying mission success criteria, the motivation behind studying asteroids is unanimous; small bodies like asteroids have many unique properties that have proven difficult to address.

The challenges of performing proximity operations and landing on small bodies include the uncertain gravity models. The irregular shape and mass distribution of the asteroids, when compared to well-understood planetary orbital and landing procedures, drastically alter the dynamics of the spacecraft in question. Even though we can characterize the asteroids before launch, we still can encounter significant uncertainties in the dynamics models that drastically increase in effect the closer a lander is to the body surface. Multiple previous small-body missions (e.g., Hayabusa) utilized "rehearsal" procedures before the touchdown as a means to better understand the risks and the potential sources of error, and for preliminary data collection. The existence of these rehearsals, however, still is not sufficient to ensure the mission's success. Hayabusa's first two touchdowns resulted from autonomous obstacle avoidance failures, causing the lander to impact the surface and bounce twice⁵. With a more adaptive capability to characterize the dynamic uncertainties of the small bodies and respond to them in real-time, we could potentially substantially increase the mission success rate of small-body landing missions. The landing craft must come equipped with the

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mechanisms to adapt the trajectory in response to gravitational or other uncertainties encountered during a small-body mission.

The proposed solution to the problem of necessary adaptive control within small body landings is an application of model predictive control (MPC). As a concept, MPC involves predicting the future behavior of a system based on potential control laws across a given time horizon. MPC repeatedly solves an optimization problem to minimize a cost function computed with state and control information along the shifting prediction horizon. The computation of minimum cost implicitly defines the control law of the system⁶. MPC formulations are popular due to their ability to handle constraints in the optimization problem and because MPC requires little maintenance for its produced results⁷. Due to MPC's flexibility in cost function creation, MPC can be a computationally cheap alternative to other control algorithms and also operate in a time-of-flight (ToF) free regime. A computationally quick control algorithm can enable rapid response to trajectory deviations due to uncertainties and a ToF free formulation ensures that any additional corrective burns are not penalized as suboptimal trajectories. Both of these factors enable MPC to be employed on board a spacecraft rather than rely on ground computations. MPC's online capability, flexibility in formulation, and adaptability of the system under uncertainties make it an attractive option for the asteroid landing problem.

BACKGROUND

A brief background on the derivation of the gravity model employed in this work is provided.

The Polyhedral Gravity Model

While many other gravity models exist for modeling the gravity field as an asteroid, namely mass concentrations (mascons) or spherical harmonics, the model chosen for this work is the polyhedral gravity model. There are several reasons for this selection. Mainly, the polyhedral gravity model has the potential to be the most accurate gravity model if enough facets are considered. If the asteroid is poorly-characterized prior to a landing mission, a low-resolution polyhedral model can still be used. Further, the polyhedral has no mathematical region of divergence like spherical harmonics and the polyhedral gravity model computes a term denoted the "solid angle" that informs as to whether the spacecraft is outside, on, or inside the asteroid's polyhedral volume with no additional computational effort⁸. The main downside to the polyhedral gravity model is its computational expense.

As outlined by Werner and Scheeres (1996), the equation for computing the gravitational attraction of a polyhedral shape model is as follows:

$$\nabla U = -G\rho \sum_{e \in edges} \mathbf{E}_e \mathbf{r}_e \cdot L_e + G\rho \sum_{f \in faces} \mathbf{F}_f \mathbf{r}_f \cdot \omega_f \quad (1)$$

Where G is the gravitational constant, ρ is the assumed constant asteroid density, $\mathbf{r}_e$ is the vector from the spacecraft location to a point on edge e . Similarly, $\mathbf{r}_f$ is the vector from the spacecraft location to a point on face f . The other quantities used in equation 1 are computed as follows according to Werner and Scheeres (1996):

$$\mathbf{E}_e = \hat{\mathbf{n}}_A(\hat{\mathbf{n}}_{12})^T + \hat{\mathbf{n}}_B(\hat{\mathbf{n}}_{21})^T \quad (2)$$

Where faces A and B share a common edge of vertices 1 and 2, $\hat{\mathbf{n}}_A$ and $\hat{\mathbf{n}}_B$ are the normal vectors of faces A and B , and $\hat{\mathbf{n}}_{12}$ and $\hat{\mathbf{n}}_{21}$ are unit normals with respect to faces A or B and the shared edge in opposing directions. $\mathbf{E}_e$ is a 3×3 matrix. These quantities can be visualized in Figure 1.

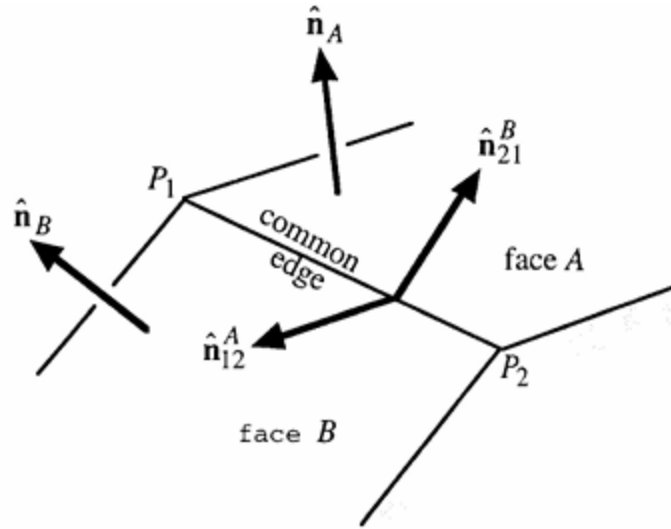


Figure 1. Polyhedral Gravity Edge Normal Vectors⁸.

$$L_e = \ln \frac{r_{e1} + r_{e2} + e_{12}}{r_{e1} + r_{e2} - e_{12}} \quad (3)$$

Where r_{e1} and r_{e2} are the distances between the spacecraft location and the two vertices making up the shared edge and e_{12} is the length of the shared edge.

$$\mathbf{F}_f = \hat{\mathbf{n}}_f \hat{\mathbf{n}}_f^T \quad (4)$$

$$\omega_f = 2 \tan^{-1} \frac{\mathbf{r}_1 \cdot (\mathbf{r}_2 \times \mathbf{r}_3)}{r_1 r_2 r_3 + r_1 (\mathbf{r}_2 \cdot \mathbf{r}_3) + r_2 (\mathbf{r}_3 \cdot \mathbf{r}_1) + r_3 (\mathbf{r}_1 \cdot \mathbf{r}_2)} \quad (5)$$

Where $\hat{\mathbf{n}}_f$ is the normal vector of the face f . $\mathbf{F}_f$ is a 3×3 matrix. $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_3$ are the vectors from the spacecraft location to vertices 1, 2, and 3 of the face f . r_1 , r_2 , and r_3 are their magnitudes. These quantities can be visualized in Figure 2. ω_f is the solid angle. If $\omega_f = 0$, the spacecraft is outside the asteroid volume. If $\omega_f = 2\pi$, the spacecraft is on the asteroid surface. If $\omega_f = 4\pi$, the spacecraft is inside the asteroid volume.

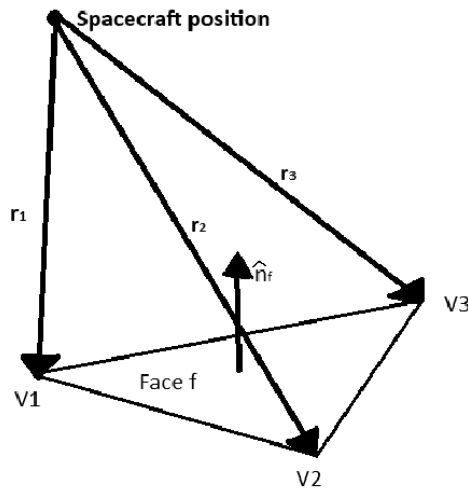


Figure 2. Polyhedral Gravity Face Vectors.

Because the solid angle is computed as part of the polyhedral gravitational attraction vector, determining whether the spacecraft is inside the asteroid or not is essentially free additional information. This information can be used to constrain the spacecraft's position relative to the asteroid in the MPC algorithm while being less constricting than a glide slope constraint.

METHODS

To ensure the MPC follows a fuel-efficient trajectory to the landing site, a pre-computed reference trajectory is generated on the ground using the same mission parameters intended for the MPC. This reference trajectory is computed using an application of successive convex approximations (SCvx). The MPC's cost function can be designed to utilize the reference trajectory and to be ToF free to enable potentially more fuel-efficient trajectories to be found.

The Reference Trajectory

SCvx works by breaking a highly nonlinear trajectory into N linear subproblems where the terminal conditions of one problem are the initial conditions for the next subproblem. All these subproblems are solved synonymously until all N fall within some set tolerance⁹.

Work by Pinson and Lu (2018) formulates an SCvx optimization problem for asteroid landings with a combination of spherical harmonics and Bessel functions for the nonlinear asteroid gravity. This formulation is as follows:

$$\min -q(t_f) \quad (6)$$

$$s. t. \quad \dot{\mathbf{r}} = \mathbf{v} \quad (7)$$

$$\dot{\mathbf{v}} = \mathbf{a}_t - 2\boldsymbol{\omega} \times \mathbf{v} - \dot{\boldsymbol{\omega}} \times \mathbf{r} - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + \nabla \mathbf{U}(\mathbf{r}) \quad (8)$$

$$\dot{q} = -\frac{1}{v_{ex}} a_{tm} \quad (9)$$

$$\|\mathbf{a}_t\| \leq a_{tm} \quad (10)$$

$$T_{min} e^{-q_0} (1 - (q - q_0) + 0.5(q - q_0)^2) \leq a_{tm} \quad (11)$$

$$a_{tm} \leq T_{max} e^{-q_0} (1 - (q - q_0)) \quad (12)$$

$$\|\mathbf{r} - \mathbf{r}_f\| \cos \theta - (\mathbf{r} - \mathbf{r}_f)^T \hat{\mathbf{n}} \leq 0 \quad (13)$$

$$q \geq q_{dry} \quad (14)$$

$$\mathbf{r}(0) = \mathbf{r}_0, \mathbf{v}(0) = \mathbf{v}_0, q(0) = \ln(m_{wet}) \quad (15)$$

$$\mathbf{r}(t_f) = \mathbf{r}_f, \mathbf{v}(t_f) = \mathbf{v}_f, t_f \text{ given} \quad (16)$$

$$q_0 = \ln(m_{wet} - \frac{T_{min}}{v_{ex}} \Delta t) \quad (17)$$

Where q is the natural log of the mass and equation 6 maximizes the fuel at the end of the trajectory. Equations 7 and 8 are the state dynamics with respect to an asteroid rotating at an angular velocity $\boldsymbol{\omega}$. $\nabla \mathbf{U}(\mathbf{r})$ is the nonconvex gravity from spherical harmonics and Bessel functions. Equation 9 outlines the mass flow rate with respect to exhaust velocity and the slack variable a_{tm} . Exhaust velocity is computed with the specific impulse and gravity: $v_{ex} = i_{sp} g_0$. Equation 10 defines the inequality condition for the slack variable constraining the total acceleration magnitude. Equations 11 and 12 are Taylor series expansions of the rocket equation to constrain the thrust to always fall between some set T_{min} and T_{max} . Equation 17 defines the q_0 term at which the fuel expenditure

expands around. Note that Δt is the time since the start of the simulation. Equation 13 is a glide slope constraint, disallowing the spacecraft from moving below the plane define by the surface normal of the landing site. Equation 14 ensures that the mass can never fall below the dry mass. Equations 15 and 16 outline the initial and final conditions of the simulation.

The presented formulation is still nonconvex and nonlinear due to the $\nabla U(\mathbf{r})$ term in the dynamics. The solution to convexifying and linearizing the problem is the implementation of SCvx. The dynamics are converted into a simple set of linear equations of the full state.

$$\dot{\mathbf{x}}^{(k)} = \mathbf{A}(\mathbf{r}^{(k-1)})\mathbf{x}^k + \mathbf{B}\mathbf{u}^{(k)} + \mathbf{c}(\mathbf{r}^{(k-1)}) \quad (18)$$

The full state vector is constructed as $\mathbf{x} = [\mathbf{r}^T, \mathbf{v}^T, q]^T$ and the full control vector is constructed as $\mathbf{u} = [\mathbf{a}_t^T, a_{tm}]^T$. The superscript k denotes the SCvx iteration. The matrix $\mathbf{A}$ and vector $\mathbf{c}$ depend on the spacecraft's position at the current timestep from the previous SCvx iteration. $\mathbf{B}$ is a constant matrix. The use of the previous iteration's state information in the assembly of $\mathbf{A}$ and $\mathbf{c}$ linearizes the dynamics. Equation 18 replaces equations 7 and 8 from the previous formulation. In a single SCvx iteration, convex optimization problems are solved for all N subproblems that assemble into the full trajectory until a full trajectory solution is found. Using the information from this iteration, another iteration is run solving the same problem until the difference between the previous and current trajectories is below some set tolerance¹⁰.

While Pinson and Lu (2018) utilize the spherical harmonics and Bessel function approach to asteroid gravity, this work utilizes the polyhedral gravity model. As such, $\mathbf{A}$ and $\mathbf{c}$ take on new values and the glide slope constraint in equation 13 is replaced by the solid angle constraint presented in equation 20 that ensures the spacecraft never enters the asteroid's volume.

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \omega^2 & 0 & 0 & 0 & 2\omega & 0 & 0 \\ 0 & \omega^2 & 0 & -2\omega & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^{(k-1)} \quad \mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1/v_{ex} \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{\partial U}{\partial r_x} \\ \frac{\partial U}{\partial r_y} \\ \frac{\partial U}{\partial r_z} \\ 0 \end{bmatrix}^{(k-1)} \quad (19)$$

$$\omega_f^{(k)} \leq 2\pi \quad (20)$$

Where ω is the asteroid angular velocity now assumed to be entirely in the z-axis direction and the three nonzero components to the $\mathbf{c}$ vector are the x, y, and z components of the polyhedral gravitational acceleration $\nabla U(\mathbf{r})$ from the previous SCvx iteration. Note that on the first iteration of SCvx, the spacecraft position is assumed to be stationary at the initial position. Every iteration beyond this uses the spacecraft position from the previous SCvx iteration.

The MPC Cost Function

Model Predictive Control works by solving a repeated optimization problem across some set prediction horizon. The goal is to minimize a cost function that accumulates value across the duration of the prediction horizon. Once the minimum cost is found from the MPC optimizer, the optimal control solution is applied to the system and then the prediction horizon shifts forward one time step and the process repeats until the spacecraft hits some set conditions to break the MPC loop.

Designing an appropriate cost function is more complex than penalizing fuel expenditure. To ensure the spacecraft makes sufficient progress towards the terminal state, the cost must also account for position and velocity of the spacecraft so that the spacecraft does not just choose to hover in space to burn no fuel. The MPC cost function is outlined in equation 21.

$$\min_{\mathbf{u}} J = \frac{1}{H} \sum_{i=1}^H \left(\frac{d_{min,i}(\mathbf{r}_i, \mathbf{r}_{r,i}, \mathbf{u}_i)}{l_{max}(\mathbf{r}_r)} + \frac{\|\mathbf{v}_i - \mathbf{v}_{r,i}\|}{v_{r,0,tangential}} + \frac{e^{q_{r,f}} - e^{q_i(\mathbf{u}_i)}}{m_{wet} - e^{q_{r,f}}} \right) \quad (21)$$

Where J is the cost function and H is the number of time steps in the prediction horizon. $d_{min,i}(\mathbf{r}_i, \mathbf{r}_{r,i}, \mathbf{u}_i)$ is the minimum distance at time step i to the nearest segment of the reference trajectory. This works by parameterizing the position between the two vertices making up the nearest segment of the reference trajectory to determine where along that segment the minimum distance is. By using minimum distance to the segment over minimum distance to the nearest reference vertex, the formulation can be more easily made to be ToF free. Why a ToF free cost function is important will be explained in the next subsection. Due to the parameterization, $d_{min,i}$ can be a function of the spacecraft's position on the nearest reference segment at time step i , $\mathbf{r}_{r,i}$, in addition to the spacecraft position at time step i in the MPC simulation, $\mathbf{r}_i$, and the optimal control vector at time step i , $\mathbf{u}_i$. $\|\mathbf{v}_i - \mathbf{v}_{r,i}\|$ is the magnitude of the difference between the spacecraft velocity in the MPC simulation and the velocity of the spacecraft at the nearest reference segment at time step i . $e^{q_{r,f}}$ is the final mass remaining on the reference trajectory. $e^{q_i(\mathbf{u}_i)}$ is the mass remaining up to time step i in the MPC simulation.

In order to ensure no one term dominates the other terms, each of the three terms are normalized by a specific quantity. The first term, the minimum distance term, is normalized by the max segment length along the reference trajectory $l_{max}(\mathbf{r}_r)$. Having a segment length as the normalization is important as this quantity will scale with the time step size. The second term, the velocity difference term, is normalized by the initial reference trajectory ground track speed. Because the optimization is in the rotation frame, this is simply the initial tangential velocity of the reference trajectory. As long as the spacecraft does not start on a synchronous orbit, this normalization is valid. The final term, the fuel expenditure term, is normalized by the fuel burned along the entire reference trajectory computed by subtracting the mass remaining at the end of the reference trajectory from the wet mass of the spacecraft. Normalizing by fuel burned along the reference trajectory will enable the user to quickly assess how well the fuel consumption of MPC compared to the reference trajectory.

This cost function design has many advantages. The minimum distance term will equal zero if perfectly aligned with the reference and positive otherwise. The velocity difference term will equal zero if perfectly aligned with the reference and positive otherwise. The fuel expenditure term will start negative and approach zero as fuel burns. The term will equal zero if fuel expenditure perfectly matches the reference and will go positive if the MPC fuel expenditure is higher than the reference. This enables the user to quickly check how well the MPC is performing. If the cost ends negative, the spacecraft is guaranteed to have used less fuel than the reference. If the cost ends equal to zero, the MPC perfectly matched the reference. If the cost ends positive, the spacecraft may have used less fuel and landed far from the landing site or landed at a velocity different from the desired landing site velocity. The other possibility is that the spacecraft simply burned more fuel than the reference trajectory.

The Importance of a Free-ToF Cost Function

As discussed in the breakdown of the MPC's cost function, the MPC can be run ToF free since the cost function assesses the minimum distance to a reference trajectory segment rather than vertex. Because of the complex shape and gravity models of asteroids, dynamics

mismodeling is a certainty. Such mismodeling could cause the spacecraft to deviate from the reference trajectory despite using the same control profile as the reference. The spacecraft does not necessarily need to expend additional fuel to correct these deviations, however. If the cost function assessed distance to the nearest reference trajectory vertex, a time step that has the spacecraft remain on a reference trajectory segment but only move half the distance of the segment would incur a significant penalty in the cost function and would expend unnecessary fuel to “catch up” and arrive at the landing site in the fixed time. The free ToF formulation outlined in the previous section would have no penalties for falling behind since it remained on a segment. In other words, the spacecraft can follow another trajectory similar to the reference trajectory to account for dynamics mismodeling in order to still arrive near the landing site with little to no additional fuel expenditure.

The MPC Algorithm

The MPC algorithm is simply a repeated optimization problem minimizing the cost function outlined in the previous section. Due to the computational expense of polyhedral gravity, however, the MPC prediction horizon can not use the polyhedral gravity model to propagate the state through the horizon if it is desired to compute the optimal control on board the spacecraft. To ensure the MPC optimization problem can be solved in real time, a lower resolution gravity model is used. A simple point mass gravity model is used in the MPC prediction horizon as outlined in equation 22.

$$\nabla U_{pred}(\mathbf{r}) = \frac{\mu}{r^3} \mathbf{r} \quad (22)$$

Where μ is the gravitational parameter of the asteroid. Spherical harmonics and Bessel functions could also be used as the gravity model¹⁰. Pseudocode for the MPC algorithm with “spacecraft” abbreviated “s/c” is as follows:

t = 0

SET list of potential segments to all reference trajectory segments

WHILE s/c far from landing site OR s/c too fast relative to landing site AND s/c outside asteroid:

 SET initial control guess along prediction to control along reference for time step t to t+H

 COST = 0

 FOR each time step in the prediction horizon

 PROPAGATE state using low resolution gravity model (point mass)

 COMPUTE minimum distance to nearest reference segment at new state

 COST += minimum distance / max segment length

 COST += velocity difference between current state and nearest reference segment / initial reference tangential velocity

 IF nearest reference segment close enough to s/c

 Remove nearest reference segment for the rest of the prediction horizon

 ENDIF

 COST += (fuel burned at end of reference – MPC fuel burned thus far) / fuel burned at the end of the reference trajectory

 COST = Cost/prediction horizon length

ENDFOR

COMPUTE control vector that minimizes the COST using the initial control guess

```

    PROPAGATE dynamics one time step using the optimal control and polyhedral gravity
    IF nearest reference segment close enough to s/c
        REMOVE nearest reference segment for the rest of the MPC simulation
    ENDIF
    t += 1
ENDWHILE

```

The key to ensuring the spacecraft make sufficient progress along the MPC is through the removal of considered segments in the minimum distance computation. If the spacecraft is within some distance of a segment after state propagation, that segment is removed from the list of segments considered in the minimum distance computation. The MPC loop will continue until the spacecraft lands close enough to the landing site at a close enough relative speed or until the spacecraft crashes into the asteroid surface.

PRELIMINARY RESULTS

Using the simulation parameters presented in Table 1, the reference trajectory and the MPC with respect to the reference trajectory are computed. 6489 Golevka is chosen as the asteroid of interest due to having a readily available polyhedral model¹¹. All figures containing a 3-D plotting of the trajectory are in an inertial frame with respect to Golevka.

Initial conditions selected to be one kilometer away from the landing site and at a speed much higher than the circular orbit velocity. Recall that the reference trajectory is a fixed ToF problem, so $t_f = N\Delta t$ where Δt is the duration of a time step. The SCvx iteration tolerance is the maximum allowable difference between the vertices of two consecutive iterations. If every vertex in a generated SCvx solution differs from the vertices in the previous iteration by less than this tolerance, the SCvx loop stops. If the spacecraft's minimum distance to a segment within the MPC is below the MPC segment removal distance, that segment is removed from the list of checked segments.

Figure 3 shows the polyhedral model of Golevka as well as the computed reference trajectory to the selected landing site $\mathbf{x}_f$. Figure 4 plots the time history of the spacecraft's state information and other variables. The thrust profile takes on a well-studied bang-bang control shape, maximum-minimum-maximum. Figure 5 shows both the reference trajectory and the MPC's trajectory. Figure 6 plots the time history of the spacecraft's x, y, and z position in the MPC as well as the magnitude of the position difference between the reference trajectory and MPC. Figures 7 and 8 plots similar data, but for velocity and control respectively. Figure 9 plots the mass and thrust history, the magnitude of their differences through time, and the altitude and distance to the landing site. Figure 10 plots the time history of the MPC cost. Table 2 contains various results from both the reference and MPC simulations. The tangential and radial velocity magnitude differences at the landing site are positive if the MPC is faster than the reference and negative if the velocity is slower than the reference.

While the reference trajectory followed a well-known thrust profile, the MPC has multiple distinctions from the reference trajectory despite no present uncertainties. The most prominent difference between the reference and MPC trajectories is the spike in error around 230 seconds into the simulation. The position error rises to 25 cm, the velocity error to 3 cm/s, and the control profile by axis sees erratic spiking. This error is due to the different gravity models used between the prediction horizon and true state propagation in the MPC. Recall that, in the prediction horizon, a low-resolution gravity model is used. The optimal control for that prediction horizon under that

low-resolution gravity is the output used when propagating the true state with respect to the different, higher-resolution polyhedral gravity model. While the two gravity models give similar accelerations, the optimal control outputs are different from one another. This causes state errors to slowly increase over the simulation time. Once these errors grow large enough, the cost to correct the error with fuel expenditure becomes cheaper than simply allowing the error to continue growing, so a brief spike of around five Newtons can be seen before returning to the reference trajectory thrust profile. This small deviation in thrust also leads the MPC to take an extra time step to land and expend more fuel than the reference trajectory, hence the positive final cost.

Work is still being completed to implement and simulation realistic asteroid uncertainties. The choices of uncertainties, the breakdown of these uncertainties mathematically, the MPC simulations they are implemented in, and a discussion of the MPC's performance with present uncertainties will be shown in the full conference paper.

Table 1. Simulation Parameters.

$\mathbf{x}_0$ (m, m/s)	[934.3131, -667.3665, 400.4199, 0.793, 1.443, -0.312]
$\mathbf{x}_f$ (m, m/s)	[205.461, -68.676, 0.000, 0.0199, 0.0595, 0.0000]
Dry & wet mass (kg)	1000, 1400
Min & max thrust (N)	20, 80
μ of Golevka (m^3/s^2)	14.01582
ω of Golevka (hr/rev)	6.026
N, # of subproblems	80
Δt (s)	5
Specific impulse (s)	225
SCvx iteration tolerance (m)	0.5
H, prediction horizon steps	5
MPC segment removal distance (m)	1.0

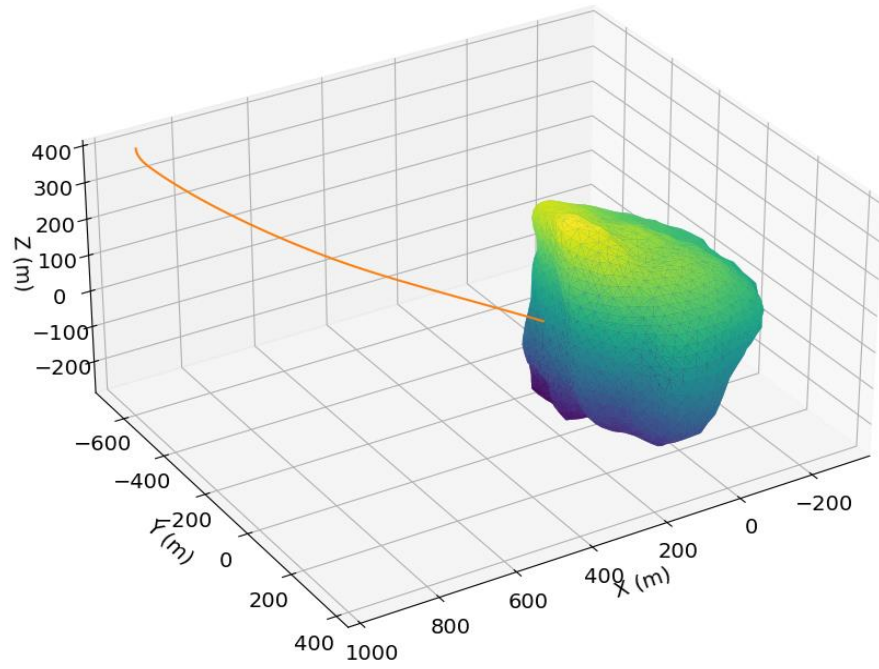


Figure 3. Reference Trajectory to 6489 Golevka.

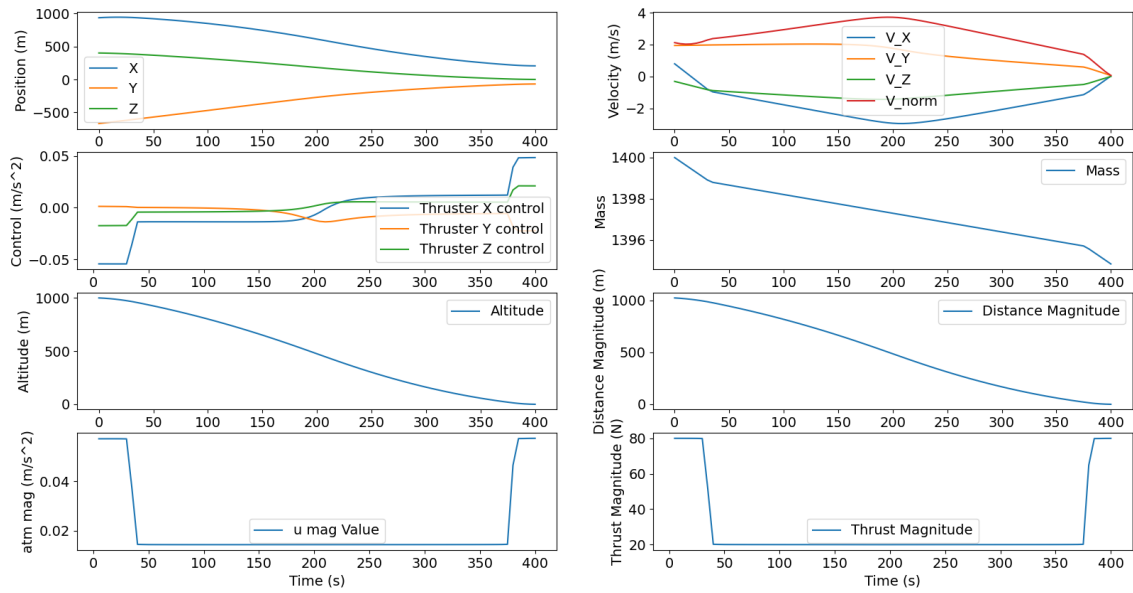


Figure 4. State History for the Reference Trajectory.

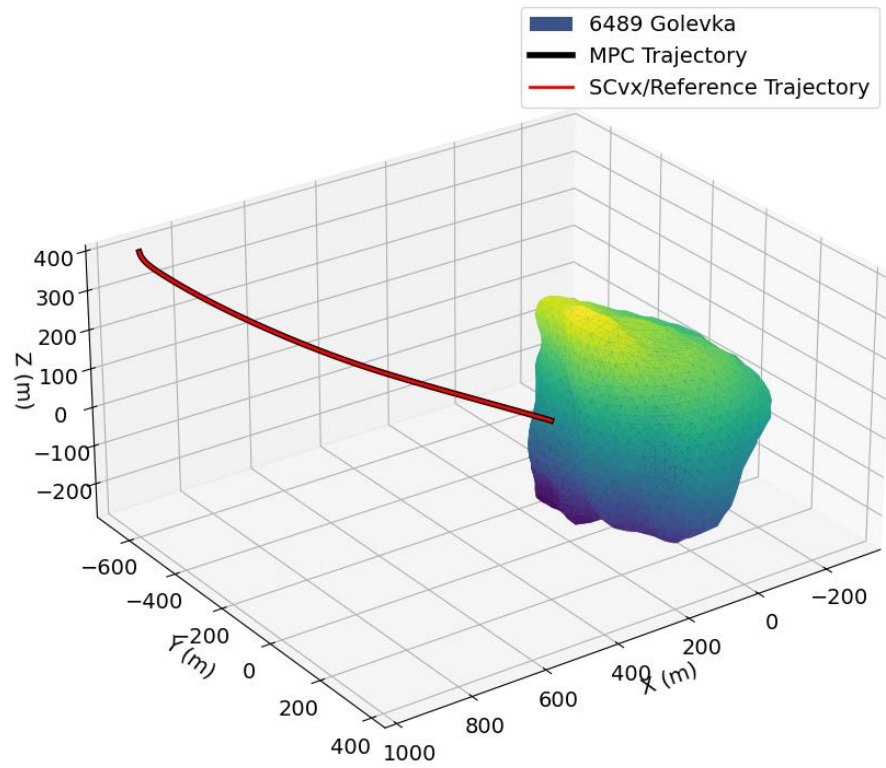


Figure 5. MPC and Reference Trajectories.

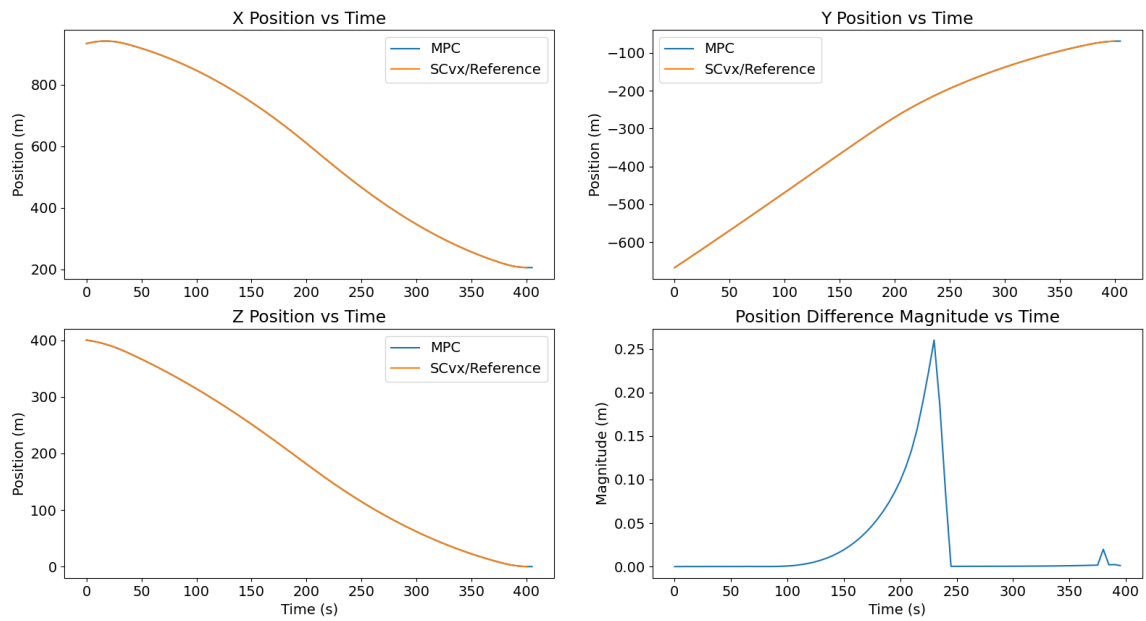


Figure 6. MPC Position History by Axis and Magnitude of Error.

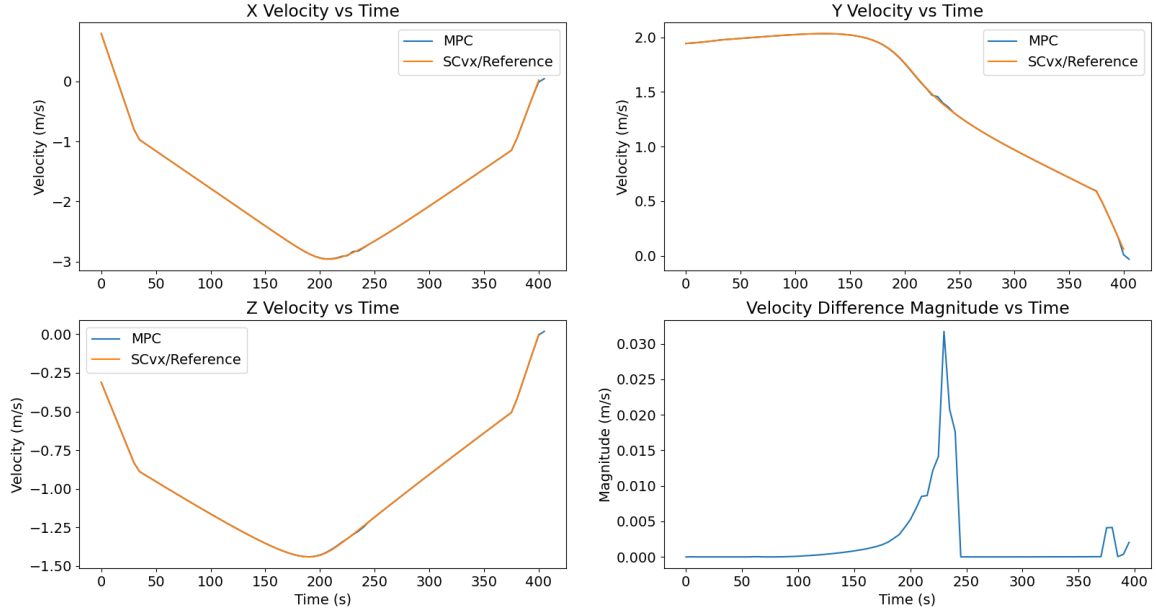


Figure 7. MPC Velocity History by Axis and Magnitude of Error.

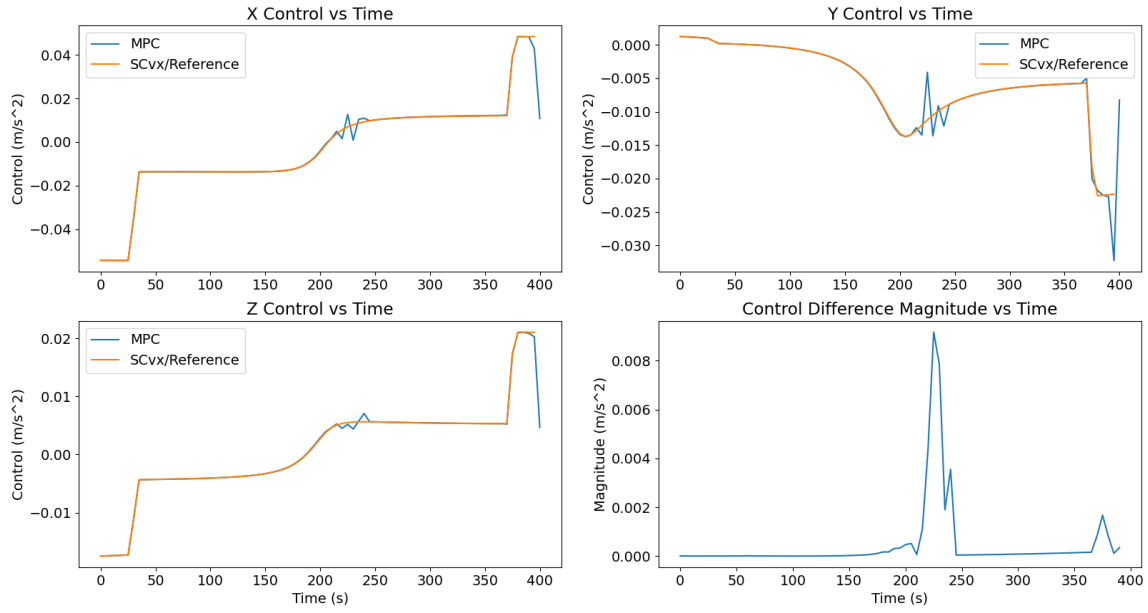


Figure 8. MPC Control History by Axis and Magnitude of Error.

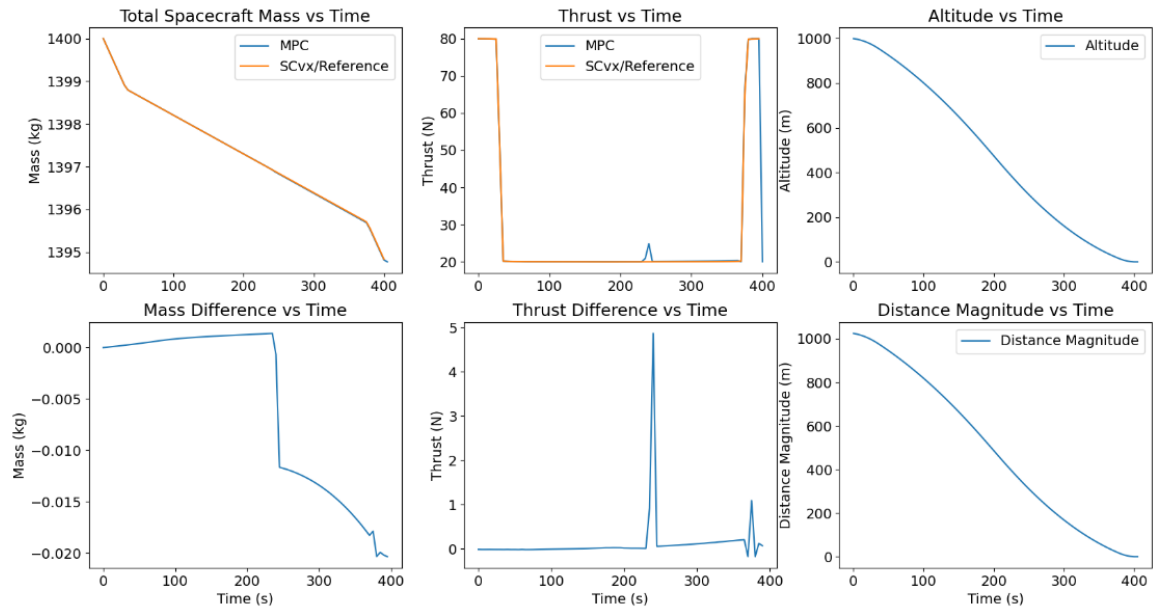


Figure 9. MPC Mass and Thrust History, Mass and Thrust Difference History, and Altitude and Distance to the Landing Site.

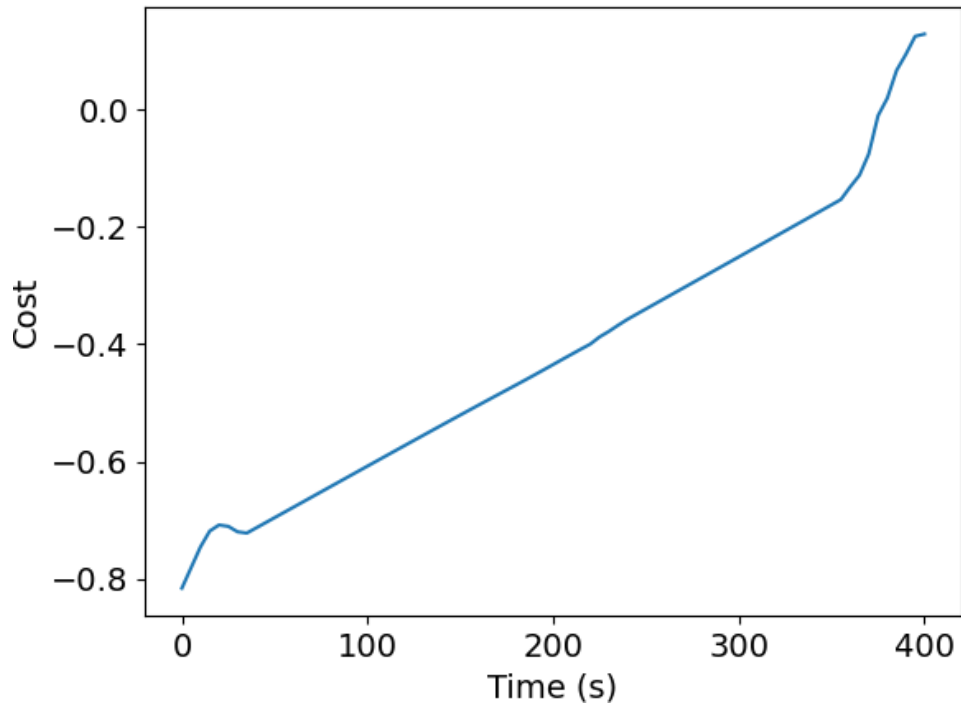


Figure 10. MPC Cost History.

Table 2. Simulation Results.

Fuel burned by Reference (kg)	5.1610
Fuel burned by MPC (kg)	5.2268
Reference ToF (s)	400
MPC ToF (s)	405
MPC runtime (s)	57
MPC terminal distance to landing site (m)	0.07455
MPC terminal tangential velocity with respect to landing site (m/s)	-0.03796
MPC terminal radial velocity with respect to landing site (m/s)	0.05098

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