Highlights

The Atmospheric Radiative Transfer Simulator ARTS, Version 2.6 — Deep Python Integration

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- The radiative transfer simulator ARTS is now deeply integrated with the Python programming language and available as a standard conda Python package
- It allows the simulation of remote sensing observations and energy fluxes throughout the terrestrial and solar spectral range.

The Atmospheric Radiative Transfer Simulator ARTS, Version 2.6 — Deep Python Integration

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Abstract

The atmospheric radiative transfer simulator ARTS is a software for computing atmospheric absorption, scattering, the transfer of radiation through an atmosphere, and sensor characteristics. It is written in C++ and can simulate remote sensing observations and radiative energy fluxes. The article describes version 2.6 of the software. There are numerous changes compared to the last ARTS publication, the most striking being that the program is now controlled by Python scripts, which is convenient and allows for great flexibility. The article discusses the ARTS history, the theory behind the computations of absorption and radiative transfer, available solvers for atmospheres with scattering, the computation of energy fluxes and heating rates, and the built-in system for inverting remote observations to atmospheric state variables by optimal estimation. ARTS is publicly available, open source, and free of charge.

Keywords: atmospheric radiative transfer simulator; ARTS; absorption; scattering; line-by-line; energy fluxes *PACS:* 42.68.Ay, 42.68.Ca, 42.68.Ge, 42.68.Mj

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1 1. Introduction

2 1.1. Basics

The Atmospheric Radiative Transfer Simulator ARTS is a software, written primarily in C++, for calculating atmospheric absorption and scattering, and for simulating the transfer of radiation through planetary atmospheres. Philosophically, what sets it apart from other such programs is its very wide scope and its flexibility, unmatched by any other software that the authors are aware of.

ARTS can do radiative transfer simulations in 1D, 2D, and 3D spherical atmospheres, spherical geometry being a key feature for simulating limb observations. It is used for simulating remote sensing observations and retrieval, but also to compute radiative fluxes and heating rates as a reference for faster radiation schemes in atmospheric circulation models for climate research and weather forecasting.

The radiative transfer is fully polarised, allowing simulation of up to four Stokes components, which is important for simulating sensors with polarisation capabilities, and even sometimes for correctly simulating simple polarisation sensors observing at frequencies where the radiation is polarised, for example by Zeeman splitting.

ARTS also does analytical or semi-analytical Jacobians (derivatives of the simulated observation with respect to changes in atmospheric state or model parameters), and these can be used in a built-in optimal estimation method (OEM) implementation for atmospheric state retrieval.

A significant part of ARTS deals with the calculation of absorption spectra from line-by-line spectroscopic data catalogues. In line with the overall philosophy, also this part is very flexible, allowing for example calculations with broadening gases other than air, to the extent that broadening parameters are available. This is important for simulating radiation on other planets.

Additionally, other types of absorption can be added, including predefined continua, measured absorption cross-sections, and collision-induced absorption spectra.

For simulations with scattering, a number of different scattering solvers are available, including a native scheme based on lambda-iteration (DOIT), a native Monte-Carlo scheme, and also well-known schemes such as DIS-ORT. There also is a sophisticated system for specifying the single scattering properties that are needed for these simulations.

38 1.2. History

Historically, ARTS started in 2000 as a collaboration between Patrick
Eriksson (Chalmers) and Stefan Buehler (then University of Bremen). The
program was open source from the start and many others have made important contributions over the years.

Important early milestones were the addition of water vapor continuum
absorption by Thomas Kuhn [1] and the iterative scattering solver by Claudia
Emde and Sreerekha T. R. [2, 3]. Eriksson et al. [4] describes the first OEM
implementation (in Matlab) and Buehler et al. [5] gives the first ARTS
overview.

Corv Davis [6] added a Monte Carlo scattering solver and Christian 48 Melsheimer [7] did a first validation and intercomparison with other radiative 40 transfer models. Buehler et al. [8] describes the first application of ARTS 50 for radiative flux and cooling rate calculations, and Eriksson et al. [9] de-51 veloped the matrix sensor representation that is still in use today. Buehler 52 et al. [10] developed a method to represent integral radiation quantities by 53 a few representative frequencies, Buehler et al. [11] describes the handling 54 of absorption in lookup tables, and Eriksson et al. [12] is the second ARTS 55 overview paper. 56

In 2014, Richard Larsson added code to handle Zeeman splitting [13, 14].
This code was revised over the years and Zeeman splitting coefficients were
updated for oxygen and other species [15, 16].

For calculations with ice particle scattering, dedicated databases of single scattering data for different particle shapes and sizes were developed for ARTS for both randomly oriented [17] and specifically oriented [18] particles. Very recently, polynomial fits to HITRAN absorption cross-section data [19] were developed to allow simulations with a large number of halocarbon species [20].

In general, ARTS release versions have an even last digit, development 66 versions an odd last digit. The most recent ARTS overview paper was [21], 67 describing Version 2.2, and this article is about Version 2.6. The most user-68 visible change between these versions is the Python integration, but there 69 also have been numerous other improvements, such as a new core to calcu-70 late atmospheric absorption, updates to the clear-sky radiative transfer core, 71 new scattering solvers, and the capability to do optimal estimation retrievals 72 inside ARTS. Tables 1 and 2 concisely list the most notable changes from 2.2 73 to 2.4 (Table 1) and from 2.4 to 2.6 (Table 2). 74

Key changes between $2.2 \ {\rm and} \ 2.4$

Other changes worth mentioning

- New improved format for line-by-line data
- Non-LTE (pure-rotational non-overlapping, and non-chemical cases)
- Dedicated methods for heating rate calculations (see Section 5)
- Basic simulations of radars (both single and multiple scattering, see Section 4.2.2)
- Radio link calculations not supported in this version
- Interfaces to DISORT and RT4 scattering solvers (see Section 4.2)
- Jacobian for new quantities: spectroscopic variables and particle properties (hybrid solver, see Section 4.2)
- Optimal estimation inversions inside ARTS (see Section 6)
- TELSEM and TESSEM surface models
- PyARTS: Python bindings for ARTS

• Radiative transfer code (except Monte Carlo) completely revised, including: higher consistency between modules, higher calculation efficiency, Jacobian of atmospheric variables now fully analytical

- Absorption code revised: support for new line shapes, performance improvements (see Section 3)
- New and extended system for defining particle size distributions (see Section 4.1)
- DOIT scattering solver improvements: optimized pressure grid, convergence acceleration, optional precalculated first-guess field
- New sensor setup for passband-type meteorological millimeter instruments (sensor_responseMetMM)
- New single scattering database for randomly oriented [17] and specifically oriented [18] ice particles

Table 1: Changes between ARTS 2.2 (described in last overview paper [21]) and ARTS 2.4 (last release).

Key changes between 2.4 and 2.6	Other changes worth mentioning		
• Extension to shortwave radiation (see Section 2.2), including new clear-sky solver with support for shortwave	• Improved flux calculations with DISORT (faster, easier and more accurate)		
radiation (iyClearsky), and addition of molecular scattering (clear-sky Rayleigh scattering)	• Recent Rosenkranz absorption models added (PWR2021)		
	• MT_CKD 4.0 Water continuum added		
• Support of measured cross-section data (e.g. HITRAN absorption cross-sections, see [20] for details)	• Automatic download of ARTS spectral line catalog (see example in Section 1.4)		
• Improved DISORT interface	• New workspace method surfaceFlatRvRhEvEh, to calculate		
• Much improved Python integration, including examples (see Section 1.4)	'surface_rmatrix 'and 'surface_emission' when emissivity and reflectance are provided externally.		

Table 2: Changes between ARTS 2.4 (last release) and ARTS 2.6 (this version).

⁷⁵ 1.3. ARTS use — literature analysis

It is interesting to see for what applications ARTS is used in practice.
For this, we did a search on Web of Science for all articles that cite one of
the three primary ARTS papers [21, 12, 5], between 2018 and June 2023.
This yielded 134 publications. Out of these, 29 just mention the program.
Often they are about other radiative transfer software (for example [22, 23])
or spectroscopy (for example [24, 25]).

The remaining 105 publications actually used ARTS for their work. The 82 largest group of these (52 publications) deals with remote sensing of the 83 clear atmosphere (without scattering in the radiative transfer setup). Out 84 of these, many used ground-based microwave radiometers for profiles of at-85 mospheric trace gases and temperature. Examples include tropospheric wa-86 ter [26], stratospheric and mesospheric water [27, 28, 29, 30], stratospheric 87 and mesospheric ozone [31, 32, 33, 34, 35], mesospheric carbon monoxide 88 [36, 37, 38], temperature [39, 40], and wind [41, 38]. 89

Another significant group, still within the clear-sky category, are publications with existing or planned satellite sensors, with examples including operational meteorological microwave and infrared sounders [42, 43, 44, 45, 46, 47, 48, 49, 20] and microwave to infrared limb sounders [50, 51, 52, 53, 54, 55, 56, 16]. In addition to these, yet another interesting clear-sky application is the retrieval of the rotational temperature from stratospheric and lower thermospheric O₂ airglow emissions [57, 58].

The other big group of publications that used ARTS (38 publications out 97 of 105) deals with all-sky observations of the atmosphere and/or the retrieval 98 of hydrometeor properties, so they require radiative transfer simulations with 99 scattering. Applications here include passive microwave to sub-mm wave 100 sensors, radars, and their synergy [59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 101 70, 71, 72, 73, 74]. ARTS is here not only used on its own, but also as a 102 reference to train or validate faster more approximative models, in particular 103 for data assimilation in numerical weather prediction [e.g., 75, 47]. Many 104 studies are in preparation of the upcoming Ice Cloud Imager (ICI) mission 105 on Metop Second Generation [76, 77, 78, 79]. Also, several focus specifically 106 on polarised observations [17, 80, 18, 81]. Other studies focus on terahertz 107 sensors [82, 83, 84, 85, 86] or on infrared sensors [87]. 108

A growing application area is the simulation of atmospheric infrared energy fluxes and associated heating rates in the context of climate modeling [20, 88, 89, 90, 91]. Although ARTS so far was not capable of simulating radiative transfer in the solar part of the spectrum, a few studies used its capability to compute atmospheric absorption at these frequencies [92, 93, 94].
The overwhelming majority of use is for Earth, but there are also reported
applications for Venus [95] and Mars [96]. Also, while the overwhelming
majority of studies is in the context of atmospheric science, there is reported
use for retrieving surface temperature [97, 98].

In summary, ARTS has been used for a wide range of applications. The vast majority are remote sensing applications, with atmospheric energetics picking up in recent years. Applications with and without scattering are almost equally frequent, with a slight majority for clear-sky. A big majority of applications is passive, but radar also plays a certain role [62, 69, 60].

In terms of frequency region, microwave to sub-mm applications were most common, terahertz and infrared applications were less frequent, and solar applications rare (as expected). This distribution aligns well with the historical development of ARTS, which started as a microwave program and over the years expanded to higher and higher frequencies.

128 1.4. Python integration

In order to allow the widest possible range of different absorption and 129 radiative transfer simulations, ARTS uses the concept of workspace variables 130 (which hold quantities of interest, such as a vector of simulation frequencies) 131 and workspace methods (which operate on the workspace variables to for 132 example calculate absorption). In the past, calculations were specified in 133 ARTS' own – quite primitive – scripting language. This is still possible, 134 but for most users it will be much more convenient to use pyarts, the new 135 Python interface to these workspace variables and methods, and thus specify 136 the calculation in the form of a Python script. 137

It is worth pointing out here, that pyarts and ARTS really are the same 138 program. Pyarts just is the name of the Python module. The C++ header 139 library pybind11 [99] is used to automatically generate Python interfaces to 140 the C++ workspace methods. So, pyarts is not ARTS translated to Python, 141 but a python interface to the C++ ARTS methods, and the ARTS workspace. 142 To work with ARTS in Python, the user has to import module pyarts 143 and create an object of type Workspace. Workspace variables and methods 144 are simply member variables and member functions of that object. 145

Just enter the name of a workspace variable in an interactive Python session to see its content, or use Python's print() function. In some cases, you may have to append '.value' after the variable name to access its contents.



Figure 1: Absorption cross section of water vapor computed with the Python code in Table 3. The heavy line is a geometric running mean (averaging logarithmic values consistently with the plot's logarithmic scale).

In the ipython shell, type a question mark at the end of a workspace method
or variable name to see its built-in documentation. You can also find it online
on the ARTS homepage.

An overall design goal for pyarts was that it should feel 'pythonic' — 152 natural for any user familiar with other python packages. Pyarts allows all 153 computations that the traditional interface allowed, but with much greater 154 convenience and flexibility. We therefore recommend to use this new inter-155 face, even though the old one is still maintained for now (it may be phased 156 out at some point in the future). To ease the transition, we provide also a 157 simple method that parses traditional ARTS controlfiles and translates them 158 into python. 159

To convey a flavor of how it is to use pyarts, Table 3 gives a complete code example, which computes the absorption cross section spectrum of water vapor and displays it in a simple plot, shown in Figure 1.

In Code Section 1, it is made sure that the ARTS spectral line catalog is available (if not, it is downloaded automatically), and then the ARTS workspace object is created.

Code Section 2 deals with specifying the absorption species to consider, in this case the line-by-line spectrum of water vapor according to the ARTS

```
1 import matplotlib.pyplot as plt
2 import numpy as np
3 import pyarts
5 # 1) Prepare ARTS workspace
6 pyarts.cat.download.retrieve() # Download ARTS catalogs and set search path
7 ws = pyarts.workspace.Workspace() # Initialize ARTS
9 # 2) Set up absorption species and read catalog data
10 ws.abs_speciesSet(species=["H20, H20-SelfContCKDMT400, H20-
      ForeignContCKDMT400"])
11 ws.ReadXML(ws.predefined_model_data, "model/mt_ckd_4.0/H20.xml")
12 ws.abs_lines_per_speciesReadSpeciesSplitCatalog(basename="lines/")
13 ws.abs_lines_per_speciesCutoff(option="ByLine", value=750e9)
14
15 # 3) Set up line-by-line calculation
16 ws.lbl_checkedCalc() # Check that the line-by-line data is consistent
17 ws.propmat_clearsky_agendaAuto() # Set up propagation matrix calculation
18
19 # 4) Initialize required workspace variables
20 ws.stokes_dim = 1 # Unpolarized
21 ws.jacobian_quantities = [] # No derivatives
22 ws.select_abs_species = [] # All species
23 ws.rtp_mag = [] # No magnetic field
24 ws.rtp_los = [] # No particular LOS
25 ws.rtp_nlte = pyarts.arts.EnergyLevelMap() # No NLTE
26
27 # 5) Set up the frequency grid and the atmospheric conditions
28 f_grid_kayser = np.linspace(10, 2400, 30000) # Frequency grid in Kayser
29 ws.f_grid = pyarts.arts.convert.kaycm2freq(f_grid_kayser) # Convert to Hz
30 ws.rtp_pressure = 1000e2 # 1000 hPa
31 ws.rtp_temperature = 295  # At room temperature
32 ws.rtp_vmr = [0.02] # H20 VMR
33
34 # 6) Calculate the absorption cross section
35 ws.AgendaExecute(a=ws.propmat_clearsky_agenda) # Call the agenda
36 xsec = ws.propmat_clearsky.value.data.flatten() / (
      ws.rtp_vmr.value[0] * ws.rtp_pressure.value
37
      / (pyarts.arts.constants.k * ws.rtp_temperature.value)
38
39 ) * 10000 \# Convert absorption coefficients to cross sections in cm^2
40
41 # 7) Plot the absorption of this example
42 fig, ax = plt.subplots(figsize=(8, 4))
43 ax.semilogy(f_grid_kayser, xsec, lw=0.2, alpha=0.5, color="#932667")
44 def rolling_mean(x, w=1000):
      return np.convolve(x, np.ones(w) / w, "valid")
45
46 ax.semilogy(rolling_mean(f_grid_kayser), 10 ** rolling_mean(np.log10(xsec)),
       lw=2, color="#932667")
47 ax.set_xlabel("Wavenumber / cm$^{-1}$")
48 ax.set_ylabel("Absorption cross section / cm$^2$ molecule$^{-1}$")
49 ax.set_xlim(f_grid_kayser.min(), f_grid_kayser.max())
50 ax.spines[["right", "top"]].set_visible(False)
51 fig.savefig("h2o-xsec.pdf")
52 plt.show()
```

Table 3: Python example to compute and display the absorption cross section of water vapor.

catalog (based on HITRAN), and the MT_CKD 4.0 self- and foreign con-168 tinuum of water vapor [100]. It also sets the correct cutoff to the lineshape 169 function for the line spectrum, which is very important in order to get cor-170 rect total absorption when the continuum models are used. (This procedure 171 is explained in numerous articles, for example in the historical overview by 172 Shine [101].) Note that ARTS uses SI base units throughout, so the canonical 173 cutoff value of $25 \,\mathrm{cm}^{-1}$ becomes 750e9 Hz. All function calls in this code sec-174 tion are calls of ARTS workspace methods (which have detailed descriptions 175 in the ARTS online documentation). 176

Code Section 3 sets up how to actually compute the absorption. The workspace method propmat_clearsky_agendaAuto, which is new in this ARTS release, is very convenient for this. It creates an agenda with just the right workspace methods for the task, based on the selection of absorption species in the workspace variable abs_species, which was set in the previous code section.

Code Section 4 deals with some workspace variables that are needed for advanced features, such as the magnetic field, which is used for calculations with Zeeman splitting. Here we just set them all to empty, because we do not want to use them in this calculation.

Code Section 5 defines the input quantities for our calculation, that is, 187 pressure, temperature, the volume mixing ratio of water vapor, and also the 188 frequency grid. The latter is set first in wavenumber units (Kayser) then 189 converted to the ARTS internal SI base unit of Hz. This is a nice example 190 illustrating the convenience of using python as the control language: The 191 conversion is done by a lower level helper function, not a workspace method, 192 that is also exposed to Python by pybind11. Note that one could also simply 193 program the conversion, or any other transformations that one might want 194 to apply, in Python directly. 195

Code Section 6, finally, does the actual calculation. The agenda that was built automatically in Code Section 3 is executed, and it stores its results in the workspace variable for the propagation matrix **K**, called propmat_clearsky (more on **K** in the next section).

In this case, because the workspace variable stokes_dim was set to 1 in Code Section 4, **K** contains only a single element (see Equation 4), corresponding to the extinction coefficient (here identical to the absorption coefficient, since there is no scattering).

We then extract that value, convert it from an absorption coefficient to an absorption cross section, and store it in the variable xsec. For the unit conversion, we divide by the product of volume mixing ratio and total number
density (which we compute from pressure and temperature using the ideal gas
equation). Normal Python functionality comes handy for this transformation
also. A factor 10⁴ comes from the conversion from m² (SI base unit) to the
more conventional cm² that we use for plotting.

In Code Section 7, all that is left to do is to create the line plot that is displayed in Figure 1, using Python's powerful matplotlib module.

Hopefully, this quite explicit example helps to convey a feeling for what can be done with this new radiative transfer tool, by freely mixing the specialized ARTS functions with general python code. For more examples see the ARTS online documentation, in particular 'getting started' there. The location is given in Section 'Obtaining ARTS' at the end of this article.

218 2. Radiative transfer

219 2.1. Theory

The core equation for ARTS' internal radiative transfer solver is the vector Schwarzschild equation

$$\frac{\mathrm{d}\vec{\mathrm{I}}}{\mathrm{d}s} = -\mathbf{K}\left(\vec{\mathrm{I}} - \vec{\mathrm{J}}\right) \tag{1}$$

where \vec{I} is the four component Stokes vector, \vec{J} is the source term, and \vec{K} is the propagation matrix, which describes how the radiation is modified along an infinitesimal path distance ds.

The equation formally looks exactly as the original scalar Schwarzschild equation [102], but has a wider scope, not only because **K** is a matrix and \vec{I} and \vec{J} are vectors, but also because **K** and \vec{J} include the effect of scattering, whereas Schwarzschild thought only about absorption and thermal emission. The advantage of casting the radiative transfer differential equation in this form is that its integral form is known.

One key implementation difference of ARTS version 2.6 compared to version 2.2 is that it now uses a dedicated data type for the propagation matrix **K**, which greatly speeds up the calculation of polarized radiative transfer by simplifying both matrix inversions and matrix exponential calculations.

We assume in ARTS and in this subsection that **K** and \vec{J} are approximately constant over a small enough distance $r = |s_{i+1} - s_i|$, where *i* is some discrete positional index. In fact, we discretize ARTS in a level-by-level manner and both **K** and \vec{J} are simply the average of the surrounding levels when a

layer is constructed, explicitly, $\mathbf{K} = (\mathbf{K}_i + \mathbf{K}_{i+1})/2$ and $\vec{J} = (\vec{J}_i + \vec{J}_{i+1})/2$. Given this, the transmission through the atmosphere between two positions can be written as

$$\vec{\mathbf{I}}_{i+1} = \exp\left(-\mathbf{K}r\right)\vec{\mathbf{I}}_i \tag{2}$$

when the source term is negligibly small, or

$$\vec{\mathbf{I}}_{i+1} = \exp\left(-\mathbf{K}r\right)\left(\vec{\mathbf{I}}_{i} - \vec{\mathbf{J}}\right) + \vec{\mathbf{J}}$$
(3)

²³³ when the source term must be taken into account.

The propagation matrix has in other places been called the extinction matrix or the attenuation matrix. We prefer the term propagation matrix, because this matrix also deals with the effect of Faraday rotation, which just moves energy between different polarization states (higher Stokes components), but does not decrease the total intensity (first Stokes component), so that **K** is not always associated with extinction. The physical unit of **K** is m^{-1} .

There are only 7 independent variables in the 4×4 propagation matrix

$$\mathbf{K} = \begin{bmatrix} A & B & C & D \\ B & A & U & V \\ C & -U & A & W \\ D & -V & -W & A \end{bmatrix},$$
(4)

so only these are kept. The Zeeman effect uses all seven components, Faraday
rotation only uses the U-component, and all the other line-by-line, continua,
and collision-induced-absorption models of ARTS only use the A-component,
corresponding to the scalar extinction coefficient for unpolarized radiation,
which would be the sum of absorption coefficient and scattering coefficient.

The matrix exponential $\exp(-\mathbf{K}r)$ is computed after a rewrite as

$$\exp\left(-Ar\right)\exp\left(\mathbf{K}'\right),\tag{5}$$

where \mathbf{K}' is as Equation 4 but with A = 0 and scaled already by r (this is the same solution as found in [103]). We can rewrite the remaining matrix exponential using the Cayley-Hamilton theorem as

$$\exp\left(\mathbf{K}'\right) = c_0 \mathbf{I} + c_1 \mathbf{K}' + c_2 {\mathbf{K}'}^2 + c_3 {\mathbf{K}'}^3, \tag{6}$$

where c_0 - c_3 are four coefficients that can be found using eigenvalue decomposition. The eigenvalues (λ) are found from solving the characteristic polynomial,

$$\begin{array}{rcl}
0 &=& \lambda^4 + b\lambda^2 + c \\
b &=& U^2 + V^2 + W^2 - B^2 - C^2 - D^2 \\
c &=& -(DU - CV + BW)^2 \\
s &=& \sqrt{b^2 - 4c} \\
x^2 &=& \sqrt{\frac{s-b}{2}} \\
y^2 &=& \sqrt{\frac{s-b}{2}},
\end{array}$$
(7)

where x^2 and y^2 are the positive and negative parts of the solutions for λ^2 , respectively, as their practical real values. The coefficients of Equation 6 are found from the set of equations

$$\begin{array}{rclrcl}
e^{x} &=& c_{0} &+& c_{1}x &+& c_{2}x^{2} &+& c_{3}x^{3} \\
e^{-x} &=& c_{0} &-& c_{1}x &+& c_{2}x^{2} &-& c_{3}x^{3} \\
e^{iy} &=& c_{0} &+& ic_{1}y &-& c_{2}y^{2} &-& ic_{3}y^{3} \\
e^{-iy} &=& c_{0} &-& ic_{1}y &-& c_{2}y^{2} &+& ic_{3}y^{3} \\
\end{array} \tag{8}$$

which yields

$$c_0 = \frac{x^2 \cos y + y^2 \cosh x}{x^2 + y^2} \tag{9}$$

$$c_1 = \frac{x^2 \frac{\sin y}{y} + y^2 \frac{\sinh x}{x}}{x^2 + y^2} \tag{10}$$

$$c_2 = \frac{\cosh x - \cos y}{x^2 + y^2} \tag{11}$$

$$c_3 = \frac{\frac{\sinh x}{x} - \frac{\sin y}{y}}{x^2 + y^2}.$$
 (12)

Lastly, it is important to keep some limits in mind as $x \to 0$ or $y \to 0$ as the hyperbolic and the trigonometric functions are not computationally stable while they are divided by some numbers close to 0. For convenience, these

limits are

$$c_{0}: \lim_{x \to 0} c_{0} = \lim_{y \to 0} c_{0} = 1$$

$$c_{1}: \lim_{x \to 0} c_{1} = \lim_{y \to 0} c_{1} = 1$$

$$c_{2}: \lim_{x \land y \to 0} c_{2} = \frac{1}{2}$$

$$c_{3}: \lim_{x \land y \to 0} c_{3} = \frac{1}{6}$$

$$\lim_{x \to 0} c_{3} = \frac{1}{y^{2}} - \frac{\sin y}{y^{3}}$$

$$\lim_{y \to 0} c_{3} = \frac{\sinh x}{x^{3}} - \frac{1}{x^{2}},$$
(13)

where $x \wedge y \to 0$ means that both variables approach zero. We have opted for a value of 0.001 as a limit of actual 0 of x and y.

As a comment on the numerics of this matrix exponential solution, we 248 are not sure how stable the method really is, but it is very fast. The work by 249 Moler and Van Loan [104] gives the scaling and squaring method with the 250 Padé approximation as one of the most effective algorithms at hand. The 251 algorithm above is more than an order of magnitude faster in our experience. 252 The differences between the methods in computed results are small if the 253 maximum value in \mathbf{K}' is small (less than 0.1) but grow out of control if this 254 is large (above 10). We cannot judge which method is then better, but we do 255 note that the Padé approximation algorithm tends to return infinities when 256 the algorithm above still returns large but finite values. In any case, since an 257 atmospheric layer with too much absorption is a poorly designed atmospheric 258 layer, for the sake of radiative transfer using the faster algorithm above is 259 simply better. 260

The source function is first computed as

$$\vec{\mathbf{J}} = \mathbf{K}^{-1} \left(\vec{\alpha} B_P + \vec{\mathbf{J}}_n + \vec{\mathbf{J}}_s + \vec{\mathbf{J}}_{\odot} \right), \tag{14}$$

where $\vec{\alpha} = \{A, B, C, D\}^{\mathsf{T}}$ is the absorption vector, B_P is Planck's function, \vec{J}_n is the emission correction due to non-local thermodynamic equilibrium, \vec{J}_s is the additional particulate scattering source term, and \vec{J}_{\odot} is the additional solar scattering term.

When the atmosphere is in local thermodynamic equilibrium, and there is no Sun, and scattering is not considered, this simply translates to $\vec{J} = \{B_P, 0, 0, 0\}^{\mathsf{T}}$. This can be seen from noting that $\mathbf{K}^{-1}\mathbf{K}$ is the unit matrix and $\vec{\alpha}$ is just the first column of \mathbf{K} . The \vec{J}_n is computed as the additional emission caused by non-local thermodynamic equilibrium added up for every absorption species. Note also that the split above is done mostly for pedagogical reasons. Inside ARTS \vec{J}_n and \vec{J}_s are treated as the same variable, but \vec{J}_{\odot} is still treated separately.

The background source term, \vec{I}_0 , depends on the background of the prop-273 agation path. Three such backgrounds are allowed: (1) surface, (2) space, 274 and (3) a cloud box. The surface in particular might itself recursively spawn 275 a new set of radiative transfer calculations to compute the incoming radia-276 tion. For propagation paths with space as background, the sun is added as 277 background source if the propagation path in reverse direction hits the solar 278 disc. This allows to resolve the shape of the sun or to track the sun through 279 the atmosphere. 280

281 2.2. Simulations with a solar source term

In this subsection we will give a brief overview about simulations with a solar source. There will be also an upcoming article dedicated to this. The simulation of solar radiation in ARTS is based on the assumption that the distance between the Sun and any position within the atmosphere or on the surface is much larger than the radius of the Sun. This means that the angular variability of radiation leaving the surface of the Sun can be neglected and the incoming spectral radiance is assumed to be parallel (collimated beam approximation, [105]). For a position at the top of the atmosphere (TOA), the incoming spectral radiance (in units of W m⁻² sr⁻¹ Hz⁻¹) in direction Ω at TOA can be described as

$$I_{s,TOA}\left(\Omega\right) = F_{s,TOA}\delta\left(\Omega - \Omega'\right) \tag{15}$$

with Ω' the direction from the center of the Sun to TOA and $F_{s,TOA}$ the spectral irradiance (in units of W m⁻² Hz⁻¹) at TOA and δ the Dirac function. The spectral irradiance $F_{s,TOA}$ at TOA is

$$F_{s,TOA} = \int_{0}^{2\pi} \int_{0}^{\alpha_{s,TOA}} I_s \cos\theta \sin\theta d\theta d\phi$$

= $\pi I_s \sin^2 \alpha_{s,TOA}$
= $F_s \sin^2 \alpha_{s,TOA}$, (16)

with $\alpha_{s,TOA}$ the solar angular radius at TOA, F_s the solar spectral irradiance at the position of the Sun in space and I_s the hemispherically isotropic radiance leaving the surface of the Sun. The factor $\sin^2 \alpha_{s,TOA}$ takes into account that the irradiance is changing with the inverse square of the distance as it is

$$\sin^2 \alpha_{s,TOA} = \frac{r_s^2}{d_{s,TOA}^2 + r_s^2}$$
(17)

with r_s the radius of the sun and $d_{s,TOA}$ the distance between TOA and the center of the sun. Inserting (16) in (15) results in

$$I_{s,TOA}\left(\Omega\right) = F_s \sin^2 \alpha_{s,TOA} \delta\left(\Omega - \Omega'\right) \tag{18}$$

This approach makes it possible to use the same solar spectrum for dif-282 ferent distances of the sun, for example to simulate radiation for different 283 planets or different orbital configurations. The only thing, that needs to be 284 changed is the actual distance between the sun and the planet. The incoming 285 spectral radiance is automatically adjusted. Note that F_s is formally defined 286 at the center of the sun, although it could of course never be observed there. 287 At the surface of the sun I_s is already reduced by a factor of 1/2, according 288 to Equation 17. 289

ARTS assumes an elliptical geometry. This has some implications com-290 pared to a plane parallel geometry. For example, the distance between the 291 sun and TOA is not unique but depend on the geographical position. In 292 Figure 2 the distance R_A at TOA location A is greater than the distance R_B 293 at TOA location B. This also results in different angular radii, because the 294 angular radius α_s depends on the distance. The resulting differences due to 295 the different geographical positions are small, in the order of 0.1 W m^{-2} , but 296 they can be important for reference simulations, especially when comparing 297 to simulations with other geometries. Furthermore, the direction Ω' from 298 the sun to TOA depends on the geographical position because the radiative 290 transfer is relative to a local coordinate system. For example, the local so-300 lar incidence angles Θ_A and Θ_B in Figure 2 differ, which results in different 301 directions Ω' . 302

In addition to the irradiance spectrum, a solar source is defined by the radius of the Sun, its distance to the planet and the geographical position where the Sun is at zenith on the planet. A solar spectrum can be defined as a black body with an effective emission temperature or as an arbitrary user-defined spectrum.

There are two solvers in ARTS capable of simulations with a solar source term: First, there is ARTS' internal clear-sky solver iyClearsky. It is a 3D fully polarized radiative transfer solver for spherical geometry. It includes



Figure 2: Sketch of the sun planet geometry. R_A and R_B are the distance from the TOA locations A and B to the sun. $r_{TOA,A}$ and $r_{TOA,B}$ are the distance between the center of the planet and the TOA locations A and B. z_A and z_B define the local vertical directions and d_{sp} is the distance between the position of the Sun and the center of the planet.

first order molecular scattering from solar sources but no scattering of particulates or scattering from thermal sources.

Second, there is CDISORT [106], which is DISORT 2.1 [105] ported to 313 C and for simplicity called DISORT from here on. It is a 1D non-polarized 314 radiative transfer solver for plane parallel atmospheres and can handle mul-315 tiple scattering. In contrast to iyClearsky, DISORT can have only one solar 316 source. As DISORT is a plane parallel solver and ARTS assumes a spherical 317 geometry, it needs to be run for a specific geographic position because the 318 local solar zenith angle, which DISORT internally needs, is calculated from 319 the specific geographic position and the geographic zenith position of the 320 Sun. 321

For both solvers molecular scattering (Rayleigh scattering) is provided by a parametrization from M. Callan, University of Colorado [105] based on the results of Bates [107] for the scattering cross sections and the Rayleigh phase matrix including depolarization from Hansen et al. [108].

Line shape	Parameters
Doppler	G _D
Lorentz	G_0, D_0
Voigt	G_D, G_0, D_0
Speed-dependent Voigt	G_D, G_0, D_0, G_2, D_2
Hartmann-Tran	$G_D, G_0, D_0, G_2, D_2, f_c, \eta$

Table 4: Summary of available line shape operators in ARTS and their input parameters. G_D is the Doppler width, G_0 is the speed-independent pressure width, D_0 is the speed-independent pressure frequency shift, G_2 is the speed-dependent pressure width, D_2 is the speed-dependent pressure frequency shift, f_c is the frequency of velocity changing collisions, and η a correlation factor. Except for G_D , the other parameters depend on temperature as detailed in Table 5. See Tran et al. [109] for details, we follow them in our definitions.

326 3. Absorption

327 3.1. Computing absorption from spectroscopic data

How ARTS computes absorption coefficients by summing up spectral lines based on spectroscopic data has been completely re-implemented. The set of operators and multiplications for each spectral line is

$$\alpha = (1 + G_{lm} - iY_{lm}) S(T, p, \cdots) N(\nu, \cdots) F(\nu, \cdots), \qquad (19)$$

where α is the complex absorption coefficient, whose real part is the attenuation and imaginary part is the dispersion. The G_{lm} and Y_{lm} parameters are the second and first order line mixing coefficients, the *S* operator computes the line strength, the *N* operator renormalizes the line shape, and the *F* operator computes the line shape. Furthermore, *i* is the imaginary unit, *T* is temperature, *p* is pressure, and ν is frequency.

There are many variants and combinations of these operators available. For the line shape F, Table 4 gives a summary of available operators and what user input they depend on, and Table 5 lists what temperature dependence models for these parameters are supported.

As one example, the adaptation of default HITRAN line-by-line data for ARTS [24] uses

$$\alpha = \frac{x_s p}{kT} S_i \frac{Q(T_0)}{Q(T)} \exp\left(E_i \frac{T - T_0}{kTT_0}\right) \frac{\nu \left(\exp\left[h\nu/kT\right] - 1\right)}{\nu_i \left(\exp\left[h\nu_i/kT_0\right] - 1\right)} F_v(\cdots), \quad (20)$$

Model	Formulation
T0	X_0
T1	$X_0 \left(\frac{T_0}{T}\right)^{X_1}$
T2	$X_0 \left(\frac{T_0}{T}\right)^{X_1} \left[1 + X_2 \log\left(\frac{T}{T_0}\right)\right]$
Τ3	$X_0 + X_1(T - T_0)$
T4	$\left[X_0 + X_1 \left(\frac{T_0}{T} - 1\right)\right] \left(\frac{T_0}{T}\right)^{X_2}$
T5	$X_0 \left(\frac{T_0}{T}\right)^{0.25+1.5X_1}$
AER	$X(T = 200) = X_0, X(T = 250) = X_1, X(T = 296) = X_2,$
	$X(T = 340) = X_3$, linear interpolation
DPL	$X_0 \left(\frac{T_0}{T}\right)^{X_1} + X_2 \left(\frac{T_0}{T}\right)^{X_3}$
POLY	$X_0 + X_1T + X_2T^2 + X_3T^3$

Table 5: Temperature dependence of line shape parameters in Table 4. The names are as enumerated inside ARTS. The coefficients that are provided by the user are X_0 , X_1 , X_2 , and X_3 . T is the current temperature and T_0 is the reference temperature.

where the terms before the $F_v(\dots)$ line profile operator represent the S and 338 N operations of Equation 19, x_s is the volume mixing ratio of the molecule 339 in question, k is Boltzmann's constant, S_i is the reference line strength of the 340 absorption line as provided by HITRAN, Q is the total internal partition sum 341 operator for some temperature [110], T_0 is the reference temperature of the 342 line (for HITRAN always 296 K), E_i is the HITRAN lower state energy level 343 of the absorption line, ν is the frequency at which absorption is sampled, 344 ν_i is the HITRAN reference line center of the absorption line, and h is the 345 Planck constant. HITRAN does not provide any line mixing parameters by 346 default, so the $1 + G_{lm} - iY_{lm}$ term of Equation 19 disappears completely. 347

The $F_v(\dots)$ line profile operator in this example case is

$$F_v(\cdots) = \frac{1}{\sqrt{\pi}G_D} w \left(\frac{\nu - \nu_i - D_0 + G_0}{G_D}\right)$$
(21)

$$G_D = \frac{\nu_i}{c} \sqrt{\frac{2000RT}{m_s}} \tag{22}$$

$$G_0 = x_s \gamma_{i,s} p\left(\frac{T_0}{T}\right)^{n_{i,a}} + (1 - x_s) \gamma_{i,a} p\left(\frac{T_0}{T}\right)^{n_{i,a}}$$
(23)

$$D_0 = \delta \nu_{i,a} p \tag{24}$$

where w is the Faddeeva function (the convolution of the Doppler line profile 348 and the Lorentz line profile [111]), c is the speed of light, m_s is the molar 349 mass of the molecule in question, R is the universal gas constant, $\gamma_{i,s}$ is 350 the HITRAN self broadening coefficient, $\gamma_{i,a}$ is the HITRAN air broadening 351 coefficient, $n_{i,a}$ is the HITRAN air broadening temperature exponent, and 352 $\delta \nu_{i,a}$ is the HITRAN air pressure shift. In Equation 22, a factor 1000 comes 353 in because we give the molar mass m_s in grams per mole, and R and T in SI 354 units (as all other quantities). 355

³⁵⁶ Comparing the expressions of Equations 21 to 24 to Tables 4 and 5 shows ³⁵⁷ that with the standard HITRAN data we are using a Voigt line shape with ³⁵⁸ the T1 temperature model for the parameters going into the pressure width ³⁵⁹ G_0 and the T0 temperature model for the parameter going into the pressure ³⁶⁰ shift D_0 .

By default, HITRAN does not provide the more complete Hartmann-Tran profile parameters [109], but ARTS can in principle compute the Hartmann-Tran profile if the parameters are supplied. For this the users must provide the parameters listed in the last row of Table 4 in a form fitting on of the temperature models of Table 5 and set the appropriate line profile operator. One additional complication not covered by any of the expressions above occurs when the atmosphere is considered in non-local thermodynamic equilibrium as this will contribute to the \vec{J}_n term of Equation 14. Yamada et al. [112] describes the basics of the implementation in ARTS to compute non-local thermodynamic equilibrium.

371 3.2. Other absorption

Besides spectral line by spectral line absorption calculations, ARTS also 372 includes state of the art absorption continua, such as CKD_MT up to Version 373 4 for water vapor (from Version 4 on CKD_MT water vapor continuum coef-374 ficients are stored as external data and distributed via the HITRAN website. 375 so any later version will also work). Furthermore, HITRAN collision-induced 376 absorption ([113], last HITRAN data access July 2022) and absorption cross 377 section data [19] are also available, for example for halocarbons as described 378 in [20]. Continua and other absorption functions will continue to be updated 379 as new data become available. 380

381 4. Scattering

This section deals with simulations where scattering by particles within the atmosphere is considered. Particles here refers to hydrometeors (liquid or frozen cloud and precipitation particles) or aerosols. ARTS 2.6 handles for the first time also molecular scattering (for UV/visible radiation) but this is to be described in another article. This section starts by outlining the input data required, and ends with an overview of available scattering solvers.

388 4.1. Particle properties

The combination of shapes and sizes of aerosols and hydrometeors is basically unlimited and, as a consequence, the core manner in ARTS to describe particle properties is generic by design. It is up to the user to decide what set of scattering elements to include in the calculations. The definition of a scattering element has two parts, the single scattering properties of the element and the associated number density.

395 4.1.1. Single scattering data

ARTS has its own format for single scattering data. These data represent 396 the extinction, absorption and scattering function, gridded as a function of 397 frequency and temperature, of the scattering element. For a limited set of 398 shapes, including spheroids and cylinders, the single scattering data can be 399 calculated with a relatively low calculation burden by the T-matrix method. 400 ARTS contains an interface to such code, the one by [114]. For more complex 401 shapes, the scattering data have to be calculated externally. For hydrome-402 teors and calculations at frequencies between 1 and 900 GHz, the ARTS 403 infrastructure contains an extensive database of precomputed values. This 404 ARTS single scattering database consists of two parts. The broadest selection 405 of habits (i.e., shape model) is offered for the standard assumption of totally 406 random orientation (TRO), for details see [17]. The ARTS format can also 407 represent particles that just have azimuthally random orientation (ARO). 408 The corresponding part of the database contains data for two habits [18]. 409 Data from other databases covering cloud ice particles have been converted 410 to the ARTS format and been applied. We are not aware of any application 411 of ARTS involving scattering by aerosols, but this should in principle also be 412 possible. 413

414 4.1.2. Particle number densities and size distributions

The user can opt to directly import particle number densities for the 415 chosen scattering elements, if such are at hand from an external source or 416 are calculated on the Python side. However, more common is the case that 417 the user has bulk properties, such as condensate mass concentrations, and 418 from this wants to generate particle number concentrations that follow a 419 particle size distribution (PSD). To simplify this process, a number of PSD 420 parameterisations are included in ARTS. For clarity, it should be noted that 421 a PSD is a continuous size distribution (for example in units of $\#/(m^3 \cdot m)$), 422 while derived particle number densities are values integrated/binned in size 423 (for example in units of $\#/m^3$). 424

First of all, there is a set of methods for treating the PSD as a modified gamma distribution (MGD), implemented following the nomenclature and equations of [115]:

$$n(x) = N_0 x^{\mu} e^{-\Lambda x^{\gamma}}, \qquad (25)$$

where the four MGD parameters are N_0 , μ , Λ , and γ , and x is the measure on size. To allow analytical expressions for all needed operations, the

relationship between size and mass must follow a power-law:

$$m = ax^b. (26)$$

As long as Equation 26 can be fulfilled, x can represent any selection of size. For example, to let x represent mass, both a and b have to be set to 1. See [115] for further choices.

The MGD methods allow that N_0 , μ , Λ and γ all vary throughout the 428 atmosphere, but can also be set to be constant. That is, the user can operate 429 with up to four moments. There are methods for expressing one moment as 430 bulk mass $[kg/m^3]$, possibly combined with a second bulk property. This 431 second moment can be number density, mean particle mass, mean size or 432 median size. There is also a MGD method handling the common assumption 433 of a power-law relationship between N_0 and Λ . The generic PSD methods 434 include also two methods for setting up mono-dispersive distributions. 435

There are several PSD methods specific for hydrometeors. For rain the PSDs of [116] and [117] are included. The classical rain PSD of [118] is covered by the general MGD methods. For ice hydrometeors, the parameterisations of [119], [120], [121] and [122] are included. Also included are two PSD schemes matching multimoment microphysics schemes common in atmospheric models [123, 124].

All these methods can provide the derivative of the PSD with respect to free parameters (that is, the moments used). This is a prerequisite to perform OEM inversions of observations involving scattering. However, just two of the scattering solvers can actually use this feature, see below.

446 4.2. Scattering solvers

Algorithms performing radiative transfer in the presence of scattering are 447 referred to as scattering solvers. Common to these algorithms is that they 448 solve the problem that, due to scattering, radiation traveling in a given direc-449 tion depends on the radiation in all other directions at the same point. Ex-450 pressed in the mathematics of section 2.1, the computation of the \vec{J}_s term in 451 Equation 14 would require an integral over the radiation Stokes vector I from 452 all incoming directions, weighted by the scattering phase matrix, making the 453 simple solution of integrating along a single line of sight, as in Equation 3, 454 impossible. 455

⁴⁵⁶ Different scattering solvers employ different strategies to overcome this. ⁴⁵⁷ In doing so, they use their own internal representation of the radiation field,

Name	Vector	3D	PP	Flux	Comment
DISORT	No	No	Yes	Yes	Only Lambertian surface
DOIT	Yes	Yes	No	Yes	3D not recommended
Hybrid	No	Yes	No	No	Can be used with OEM
MC	Yes	Yes	No	No	Choice for detailed 3D
RT4	Yes	No	Yes	Yes	No multi-threading

Table 6: ARTS radiance scattering solvers. The columns are, starting from left: short name of the solver, if vector (polarized) radiative transfer can be made, if 3D geometry covered, if plane parallel, if suitable for flux calculations, and notable limitation or feature. Solvers having Yes below Vector can also handle particle orientation (ARO). Comments refer to the implementation in ARTS, and should not be taken as generally true in the case of third party solvers.

and their own intrinsic approximations and simplifications, such as scalar 458 (unpolarized) radiation only, one-dimensional atmospheres, and often plane-459 parallel geometry. It goes beyond the scope of this text to discuss the different 460 solver strategies, good overviews are given for example in [125, 126, 127, 128]. 461 Instead, the goal here is to describe the solvers that are available in ARTS. 462 along with their strengths and limitations, where limitations include those 463 that are intrinsic to the solver and those related to our implementation or 464 the coupling to ARTS. 465

ARTS comes with two types of scattering solvers, a set for calculating scattering of thermal emission, resulting in radiances, and two methods for simulating radar reflectivity measurements. These are described in separate subsections below.

470 *4.2.1.* Radiances

The discrete ordinate iterative (DOIT) [2] and the Monte Carlo (MC) 471 [6] solvers were developed directly for ARTS and were introduced by [12]. 472 More recently, interfaces to two external scattering solvers have been added, 473 namely DISORT and RT4. The MC module is restricted to simulate remote 474 sensing observations, while DISORT, DOIT and RT4 provide the full radi-475 ance field and can thus also form the basis for flux calculations (Sec. 5). 476 Table 6 gives a summary of the scattering solvers covered by this section and 477 their individual limitations. 478

The DISORT approach [129] is arguably the most established scattering solver in our field and exists in several implementations. ARTS is coupled to the code of [106]. In contrast to other scattering solvers in ARTS, DISORT is limited to unpolarized (scalar) calculations and thus also to totally random
particle orientation (TRO).

The RT4 solver [130] fits better with the general functionality of ARTS. It handles polarized radiation and scattering data for both TRO and ARO and it can be coupled to ARTS own description of specularly reflecting surfaces. A limitation of RT4 compared to DISORT is that scaling with respect to the forward scattering peak is missing. RT4 also can not yet be used with multi-threading inside ARTS, mainly as RT4 comes as FORTRAN code.

Both DISORT and RT4 assume a plane-parallel atmosphere (in other 490 words an infinite planet radius), while remaining parts of ARTS operate with 491 a spheroidal reference geoid. Another deviation to ARTS is that RT4 and 492 DISORT take layer-means as input, while ARTS operates with point values 493 (assuming a linear variation, in each spatial dimension, between grid points). 494 To overcome this difference, an averaging is performed between the altitude 495 levels of ARTS. That is, if the ARTS pressure grid has n points, DISORT 496 and RT4 will be used with n-1 layers. This difference in discretization can be 497 significant if comparing radiances between scattering solvers inside ARTS. 498

To extend the usage of the scattering solvers of 1D character, an inde-499 pendent beam approximation (IBA) wrapper method has been implemented. 500 This method allows to apply the 1D solvers in simulations of remote sensing 501 data done using 2D and 3D atmospheres. The IBA strategy is to calculate 502 the propagation path through the 2D or 3D atmosphere, and interpolate the 503 atmospheric and surface fields to the points of the path. After the inter-504 polation, the obtained values are used to create a, likely slanted, 1D view 505 of the atmosphere. The final radiances for the simulation are obtained by 506 interpolating, in angles, the radiation field given by the scattering solver 507 based on the obtained 1D view. By applying IBA repeatedly it is possible 508 to incorporate inhomogenities inside the footprint of microwave sensors with 509 relatively small errors compared to full 3D MC calculations [72]. For obser-510 vations resulting in that the propagation path does not cover all altitudes 511 (such as airborne upward observations), the atmosphere is sampled vertically 512 to cover the missing altitude range, starting from the observation point. 513

⁵¹⁴ None of the scattering solvers mentioned above provides the Jacobian, ⁵¹⁵ which is a prerequisite for OEM-type retrievals (see Section 6). As a first step ⁵¹⁶ to overcome this limitation, a hybrid method has been implemented. In short, ⁵¹⁷ either DISORT, DOIT or RT4 is first run to obtain the full radiance field. ⁵¹⁸ The hybrid method performs an integration following Equation 3, where the ⁵¹⁹ precalculated radiance field is used to calculate the scattering source term (\vec{J}_s) in Equation 14). The general approach for deriving the Jacobian in ARTS has been extended to cover this case, but with the limitation that the incoming radiation field is assumed constant. That is, the provided Jacobian ignores the fact that a change at one point in the atmosphere can affect the scattering source term at another point. Thus the Jacobian is not fully exact, but tests showed that retrieval convergence can be achieved in most cases and the method has been applied successfully in remote sensing applications [68, 69].

The hybrid method is fully integrated into ARTS. The internal functions 527 of ARTS for performing pencil beam radiative transfer have been extended 528 to handle a precalculated scattering source term, as required by the hybrid 529 method. When decreasing the amount of scattering along the pencil beam, 530 the hybrid method converges to the standard clear-sky emission solver. As 531 an internal solver, it operates with a spherical planet. The hybrid approach 532 calculates the radiance and the Jacobian for a specific observation direction. 533 In this sense it is similar to the MC module, in contrast to the other, full, 534 scattering solvers that calculate the full radiation field. Since the hybrid 535 method needs a full radiation field as input, one should see it simply as a 536 way to complement a solution from any of the other ARTS scattering solvers 537 with a Jacobian. 538

We will end this subsection with a usage example: Fox et al. [131] used 539 ARTS to evaluate the representation of ice clouds in a Numerical Weather 540 Prediction (NWP) model. The Monte Carlo scattering solver was used 541 to simulate passive sub-millimeter wave brightness temperatures from 3D 542 NWP model input fields using ice crystal scattering properties from the 543 ARTS database [17], and these were compared to airborne observations. 544 The flexibility permitted by the wide range of ice crystal habits provided 545 in the ARTS scattering database, and the different PSD parameterisations 546 described above, make it possible to achieve good consistency between the 547 representation of cloud microphysics in the NWP model and radiative trans-548 fer simulations. 549

Figure 3 shows an example of the simulated and observed brightness 550 temperatures between 157 and 874 GHz, and the NWP model hydrometeor 551 fields used as input to the simulations. This example used the scattering 552 properties of the large column aggregate particle type to represent the cloud 553 ice. The study demonstrated that ARTS is capable of simulating realistic 554 brightness temperatures across the full range of frequencies used, when ap-555 propriate ice crystal scattering models are selected, although in this example 556 there are some discrepancies, particularly at 664 and 874 GHz in the vicinity 557

of the convective core. These are probably caused by errors in the NWP hydrometeor fields, although it is also possible that different microphysical assumptions are required in different regions of the cloud.

561 4.2.2. Radar measurements

ARTS contains two methods for simulating atmospheric monostatic radar observations. Their complexity depends strongly on whether multiple scattering is significant or not. As a consequence, there are two distinct radar methods.

For **single scattering** only simulations, the calculation is very straightforward, and the core task is to determine the bulk backscattering and weigh it with the two-way transmission:

$$\vec{\mathbf{I}}_b = \mathbf{T} \mathbf{Z}_b \mathbf{T} \vec{\mathbf{I}}_t, \tag{27}$$

where \vec{I}_b is the Stokes vector for the returned radar pulse, \vec{I}_t is the unit Stokes vector describing the polarisation state of the transmitted pulse, Z_b is the bulk scattering matrix for the point of concern in the back-scattering direction, and **T** is the Mueller transmission matrix for the distance between the radar transmitter/receiver and the backscattering point.

The transmission matrix for the away and return directions can theoret-571 ically differ (for vector calculations, not for scalar ones), but this should be 572 of no practical concern and is ignored here. Furthermore, the method allows 573 to scale the extinction going into **T**. This is a scalar value, r_e , with one as 574 default. By setting $r_e = 0$, the unattenuated return pulse is obtained. At 575 least for some situations, a full calculation with multiple scattering ends up 576 roughly halfways between $r_e = 0$ and $r_e = 1$ [132], and setting $r_e \approx 0.5$ could 577 be considered for approximating the neglection of multiple scattering. 578

This single-scattering method provides the full Jacobian. That is, the Jacobian includes derivatives with respect to both \mathbf{Z}_b and \mathbf{T} . OEM retrievals using this method are found in papers by Pfreundschuh et al. [68, 69].

For multiple scattering simulations, the calculation becomes more complex. For simulating atmospheric profiling pulse radar, the calculation of scattering events has to be considered within the context of pulse propagation time. Additionally, to avoid overestimating multiple scattering effects and properly account for the relationship between beam size and multiple scattering, simulations require a finite antenna pattern [133, 134].

Following studies by Marzano et al. and Battaglia et al. [135, 134], ARTS uses a Monte Carlo approach to account for multiple scattering effects from



Figure 3: Top panel: Simulated (solid lines) and observed (dashed lines) brightness temperatures between 157 and 874 GHz from a passive airborne radiometer viewing a cloudy scene. Bottom: Cross-section of the cloud fields from the NWP model used as input to the simulations. The colours represent the ice, rain and graupel water contents, and the gray contours represent the cloud liquid water content.

hydrometeors in the atmosphere. All scattering events have the potential
to contribute to the simulated backscatter profile, so backward ray tracing
would not produce any computational advantage unlike those realized for
passive sensing [for example 6]. Therefore, ray tracing initiates from the
transmitter using traditional forward Monte Carlo sampling.

Each discrete contribution to the radiative transfer solution is initiated by drawing two random numbers from a bivariate Normal distribution with means aligned with the sensor boresight and standard deviations related to the antenna full-width, half-maximums (FWHM) of the E- and H-planes of the antenna pattern, where $\sigma = FWHM/2.3548$. This procedure determines the direction of the line-of-sight for the radiative transfer contribution under the assumption of a Gaussian antenna pattern.

Next to calculate propagation path length, a random number r_p drawn 602 from a uniform distribution represents the scalar path transmission corre-603 sponding to the extinction of the first Stokes element I. Starting with a 604 scalar transmission coefficient $t_I = 1$, the path transmission matrix is ac-605 cumulated while $t_I > r_p$. For cases in which the bulk extinction matrix 606 is block-diagonal due to contributions from azimuthally-random particles, 607 transmission includes the effects on cross-polarization based on the state of 608 the Stokes vector at the beginning of the propagation path. When $t_I \leq r_p$, 609 propagation ends and the accumulated transmission matrix, and correspond-610 ing path length, are stored. 611

After path length and transmission calculation, the bulk scattering prop-612 erties are calculated at the propagation path end. A random number r_a is 613 drawn from a uniform distribution and compared with the single scattering 614 albedo $\alpha_{\rm ss}$ (the ratio of scattering cross-section C_{sca} over extinction cross-615 section C_{ext}). If $\alpha_{ss} < r_a$, propagation is terminated at an extinction event; 616 otherwise, the monostatic backscatter contribution is recorded for the radar 617 range bin corresponding to the calculated path length. This contribution 618 includes the normalized transmission in the transmit direction, to account 619 for polarization mixing, return transmission, and weighting for the receiver 620 Gaussian antenna pattern. 621

After accounting for the scattering event, the process restarts, this time with uniform random variables r_{ze} and r_{az} providing the zenith and azimuth scattering directions, and a new starting (scattered) Stokes vector \vec{I}_s is calculated from the product of the phase matrix Z and the incident Stokes vector \vec{I}_i :

$$\vec{\mathbf{I}}_s = \mathbf{Z}(\theta_i, \theta_s, \Delta \phi) \vec{\mathbf{I}}_i \tag{28}$$

where θ is the zenith angle and $\Delta \phi$ is the difference of the scattered and incident azimuth angles. Subscripts *i* and *s* correspond to incident and scattered, respectively.

At subsequent scattering events, bistatic scattering for the incident and 625 scattered angles contributes to the accumulated backscattering at the range 626 bin corresponding to the total distance traveled before the scattering event. 627 Path tracing ends, at an absorption event, when I falls below a predetermined 628 threshold, or when the maximum specified scattering order is reached. A 629 scattering order of 1 is equivalent to single scattering. For multiple scattering 630 simulations, the suggested scattering order is 20 based on use across a range 631 of cloud types. 632

Figure 4 shows an example simulation, a hypothetical space-based nadir-633 pointing W-band radar, comparing single scattering (top panel) and multiple 634 scattering (middle panel) simulations, with differences shown at the bottom. 635 The top simulation uses the ARTS single scattering solver convolved with a 636 Gaussian antenna pattern such that the field of view at the surface is 1 km. 637 As expected, the largest multiple scattering enhancement corresponds to the 638 convective cells and the surrounding regions. Multiple scattering results in 639 excess reflectivity where the W-band signal is otherwise being extinguished, 640 but this large enhancement is decorrelated from the vertical structure of the 641 cloud along the radar line of sight. There are two features to note. The 642 multiple scattering simulations are noisy at cloud edges and miss some thin 643 clouds captured in the single scattering simulation. This is due to the large 644 number of ray traces, on the order of 5e5, needed to resolve the simulations. 645 While runtime is linear with the number of ray traces, the ray tracing routine 646 is not trivial in terms of runtime. Thus, this simulator should be used only 647 when multiple scattering is suspected. 648

⁶⁴⁹ 5. Radiative energy flux and heating rate calculations

650 5.1. Fluxes

⁶⁵¹ ARTS is suitable for accurate reference calculations of atmospheric ener-⁶⁵² getics: radiative energy fluxes and associated heating rates. As an example, ⁶⁵³ Roemer et al. [136] used it to investigate the longwave radiative feedback ⁶⁵⁴ from a spectrally resolved perspective. To this end, spectral irradiance (\mathcal{L}_{ν}) ⁶⁵⁵ at the top of the atmosphere was calculated for a set of idealised atmo-⁶⁵⁶ spheres, using the existing interface between ARTS and the single-column ⁶⁵⁷ radiative-convective equilibrium model konrad [90, 137]. Figure 5 shows some



Figure 4: Top panel: W-band single scattering simulation of convective cells and associated anvil. Middle panel: Multiple scattering simulation of same cloud field as top. Bottom panel: Difference of middle and top showing the enhancement due to multiple scattering.



Figure 5: Simulated spectrally resolved outgoing longwave radiation \mathcal{L}_{ν} as a function of wavenumber ν for idealised atmospheres with different surface temperatures $T_{\rm s}$. Shown are the spectra at a spectral resolution of $0.1 \,\mathrm{cm}^{-1}$ (thin lines, perhaps invisible in printed version) and the 20 cm⁻¹ moving averages (thick lines).

of those \mathcal{L}_{ν} spectra between 10 cm^{-1} and $2,500 \text{ cm}^{-1}$ for surface temperatures of 268 K, 288 K, and 308 K, representing the spatial variations of Earth's surface temperature. Those simulations were then used to better understand the spectral longwave feedback derived from satellite observations.

⁶⁶² Another very recent usage example of this capability is the study by ⁶⁶³ He [138] that used ARTS for reference calculations of instantaneous $4xCO_2$ ⁶⁶⁴ forcing at different surface temperatures.

Spectral irradiance is calculated by integrating the normal component of 665 spectral radiance over one hemisphere [139, Equation 2.53], where spectral 666 radiance is the first element of the Stokes vector introduced in Equation 1. 667 Integrating spectral irradiance over frequency then gives the total irradiance, 668 that is, the total radiative energy flux in units of W/m^2 . We define fluxes 660 as a directed quantity in the context of 1D atmospheres, positive fluxes are 670 directed upwards, negative fluxes are directed downwards, and the net flux 671 is defined as the sum of the upward and downward flux. 672

ARTS has several ways to calculate radiation fluxes. For clear-sky fluxes there is an internal method, which uses the internal clear-sky radiative transfer solver to calculate spectral radiances. It assumes a plane parallel atmo⁶⁷⁶ spheres, for consistency with the all-sky solvers described below.

For all-sky fluxes, one can in principle use any of the discrete ordinate 677 solvers available in ARTS, that is DISORT, RT4 and DOIT to calculate 678 spectral radiances and then integrate them as mentioned above using in-679 ternal integration methods. Due to the higher complexity when handling 680 scattering, all-sky flux simulations are in general several times slower than 681 clear-sky flux simulations. To mitigate this, we recommend to use DISORT 682 as it is much faster than RT4 and DOIT and advanced features of the other 683 solvers like polarization are of less interest for flux calculations. Furthermore, 684 ARTS supports a dedicated DISORT mode for simulating fluxes, in which 685 the integration over the hemisphere is done internally and very efficiently. 686

The flux simulation can be done with an arbitrary number of zenith angles over which the angular integration is done, and an arbitrary number of frequencies over which the spectral integration is done. As a rule of thumb, to get an accuracy in the order of 1 Wm^{-2} for the fluxes, the number of zenith angles should be at least 6 and the number of frequencies should be in the order of several thousand. For reference calculations the number of frequencies should be even in the order of several ten thousand.

The left-hand columns of Figures 6 and 7 illustrate ARTS' capability of simulating long wave and short wave net fluxes for different atmospheric conditions. Figure 6 displays the results for a tropical atmosphere over the eastern Pacific with a thin liquid water cloud on top of the boundary layer and Figure 7 for a summer atmosphere over the North Atlantic with various cloud layers and types. Reference flux calculations for all-sky and clear-sky were done using ARTS-DISORT with 30,000 frequencies and 10 streams.

An interesting option for efficient flux calculations is that ARTS ships 701 with a set of representative frequencies and associated quadrature weights 702 derived by Paulina Czarnecki [140]. Stated very briefly, the idea is that a 703 weighted mean over the spectral flux at these few frequencies gives an ac-704 curate estimate of the total flux. They are identified from high spectral 705 resolution reference calculations for a diverse set of atmospheres by a com-706 bination of simulated annealing and linear regression, a method that was 707 originally developed for efficiently simulating satellite observations [10]. In 708 this case there are 64 representative frequencies each for longwave and short-709 wave. The calculations with this method are $30,000/64 \approx 470$ times faster 710 than the reference calculations, results are marked with dots in Figures 6 and 711 7, illustrating the close agreement. 712



Figure 6: Simulation results for a modeled tropical atmosphere over the eastern Pacific with a thin but dense liquid water cloud on top of the boundary layer (peak liquid water content 0.4 g/m^3 at 1.3 km altitude, liquid water path 500 g/m^2). Solid lines show the reference setup and dots show the fast setup. The blue shaded area indicates the position of the liquid water cloud. Top left: All sky and clear sky long wave net flux. Bottom left: All sky and clear sky short wave net flux. A positive net flux denotes a net upward flux and a negative net flux a net downward flux. Top right: All sky and clear sky long wave heating rates. Bottom right: All sky and clear sky short wave heating rates.



Figure 7: Simulation results for a modeled summer atmosphere over the North Atlantic. It contains a thin low level liquid water cloud, a mid level mixed phase cloud, and a high level ice cloud (total liquid water path 1450 g/m^2 , frozen water path 93 g/m^2). Solid lines show the reference setup and dots show the fast setup. The blue shaded areas indicate the position of the liquid water clouds and the gray shaded areas the position of the frozen clouds. Top left: All sky and clear sky long wave net flux. Bottom left: All sky and clear sky short wave net flux. A positive net flux denotes a net upward flux and a negative net flux a net downward flux. Top right: All sky and clear sky long wave heating rates. Bottom right: All sky and clear sky short wave heating rates.

713 5.2. Heating rates

Based on the fluxes, ARTS also calculates radiative heating rates. Assuming hydrostatic equilibrium, the heating rate H is defined as

$$H = \frac{g}{c_p} \frac{dF_{net}}{dp},\tag{29}$$

where g is the gravitational acceleration, c_p is the mass specific heat capacity 714 at constant pressure and F_{net} is the net flux. The derivative is internally 715 approximated with central differences and for the edges with a polynomial 716 interpolation, which both are second order accurate. With this approxima-717 tion, the heating rates are calculated on the same grid as the atmospheric 718 state. The gravitational acceleration q depends on altitude and latitude and 719 is calculated internally (formally the function takes also longitude as input, 720 but this is currently not used). The specific heat capacity c_p theoretically 721 depends on temperature (and for a non-ideal gas it could even depend on 722 pressure) and has to be provided by the user. For practical purposes, the 723 temperature and pressure dependence results from the temperature and pres-724 sure dependence of the atmospheric composition. But since c_p varies only 725 weakly for typical atmospheric conditions on Earth, it can be approximately 726 set to a constant. In the following example, c_p is set constant to the mass 727 specific heat capacity of dry air $c_{p,air} = 1005.7 \,\mathrm{J\,kg^{-1}K^{-1}}$ and g is varying 728 with altitude. 729

The right columns of Figures 6 and 7 show the all-sky and clear-sky long 730 wave (top) and short wave (bottom) heating rates for the selected example 731 cases. In Figure 6 the liquid water cloud causes strong cooling in the long 732 wave at the cloud top and weaker but also significant heating at the cloud 733 bottom; in the short wave it causes some heating at the cloud top and sup-734 presses the clear sky heating below. Dots again mark the fast approximation 735 using the representative frequencies and weights. The very good agreement 736 shows that the fast scheme is a very attractive option when spectral infor-737 mation is not needed. In Figure 7 the clouds cause strong cooling in the long 738 wave at the cloud tops and weaker but also significant heating at the cloud 739 bottoms except for the lowest cloud, where no heating occurs. In the short 740 wave the clouds cause significant heating at the cloud tops in the middle and 741 upper troposphere and suppress the clear sky heating below. At the low-742 est cloud the magnitude of sw-netflux is too small to create any significant 743 amount of heating. 744

Note that the reason why the fast scheme works so well for heating rates
is that heating rates were included as a training target, in addition to fluxes,
in the derivation of the frequencies and weights [140]. This is necessary,
because small fluctuations in the flux can have a large impact on its altitude
gradient, the heating rate. This is particularly true at higher altitudes where
the heat capacity is small due to low pressure.

Note also, though, that the training for the representative frequencies 751 and weights was completely based on clear-sky simulations. It is therefore 752 not completely self-understood that they would work equally well for all-sky 753 simulations, as the figure seems to indicate, although we had hypothesized 754 that this would be the case. The argument in favor of this is that clouds tend 755 to make the radiation field more homogeneous across different frequencies, 756 which makes the exact positions of the quadrature frequencies less critical. 757 We have not yet done a proper quantitative evaluation of the accuracy of the 758 fast scheme for all-sky simulations, but these first results seem promising. 759

Finally, it is worth mentioning that the representative frequencies and weights so far were only trained for present-day variations of water vapor, ozone, temperature and different CO_2 levels. There is work in progress on expanding that to variability in all greenhouse gases but for now the scheme will probably not work well for large perturbations in parameters not covered in the above list, for example for a significantly different methane concentration.

⁷⁶⁷ 6. Optimal estimation retrievals

ARTS has supported retrievals since its first version by providing the Jacobian, but there has not been any built-in retrieval method. The standard alternative has been Qpack [4], providing a Matlab implementation of the optimal estimation method (OEM, [141]). OEM has now been integrated into ARTS, and, compared to [4], more efficient calculations and options can be offered.

774 6.1. Overview

The OEM is based on a Bayesian formulation of the inverse problem of finding an atmospheric state \vec{x} consistent with a vector \vec{y} of remote sensing observations using a forward model $F: \vec{x} \to \vec{y}_f$ that allows simulating observations corresponding to a given atmospheric state \vec{x} . The OEM is based on the assumptions that (1) the error affecting the observations is bias-free Gaussian noise with covariance matrix \mathbf{S}_e and that (2) a priori knowledge of the atmospheric state \vec{x} can be described using a Gaussian distribution with a priori state \vec{x}_a and a priori covariance matrix \mathbf{S}_a . By application of Bayes' theorem, the posterior distribution, which fully describes the solution of the inverse problem, is found to be [141, Equation 2.24]:

$$p(\vec{x}|\vec{y}) = \exp\left(-\frac{1}{2}(F(\vec{x}) - \vec{y})^T \mathbf{S}_e^{-1}(F(\vec{x}) - \vec{y}) - \frac{1}{2}(x - \vec{x}_a)^T \mathbf{S}_a^{-1}(\vec{x} - \vec{x}_a) + \text{const.}\right)$$
(30)

If the underlying assumptions of the OEM hold true, the posterior distribution is Gaussian, too, and is fully specified by its mean and covariance matrix. The mean of the posterior distribution, which, due to the Gaussian nature of the posterior distribution, coincides with the maximum a posteriori estimator of \vec{x} , is typically found by minimizing the negative log-likelihood of (30), which is given by

$$-\log(p(\vec{x}|\vec{y})) = \frac{1}{2} (F(\vec{x}) - \vec{y})^T \mathbf{S}_e^{-1} (F(\vec{x}) - \vec{y}) + \frac{1}{2} (x - \vec{x}_a)^T \mathbf{S}_a^{-1} (\vec{x} - \vec{x}_a)$$
(31)

Finding the posterior mean state of the inverse problem thus boils down 785 to minimizing Equation 31. If the forward model F is linear, a global mini-786 mum of Equation 31 can be found in a single step using the Gauss-Newton 787 method. However, for most applications in atmospheric remote sensing the 788 forward model F is non-linear. In this case, Equation (31) has to be mini-789 mized iteratively. In addition to the Gauss-Newton (GN) method, the ARTS 790 OEM method also provides an implementation of the Levenberg-Marquardt 791 method (LM), which tends to be more stable for strongly non-linear forward 792 models, such as those involving scattering. 793

Apart from the evaluation of the forward operator F, the computationally most complex operation in the application of the GN and LM optimizers is the solving of a linear system of equations, having a size following the number of elements in \vec{x} . Since solving such a linear system of equations explicitly may become prohibitively expensive in terms of computation time and memory, all optimization methods in ARTS can be used with a conjugate gradient (CG) solver. The CG method solves the linear system iteratively and becomes computationally more efficient as the number of variables in \vec{x} grows large.

803 6.2. ARTS integration

The ARTS OEM method aims to retain most of ARTS' flexibility in terms of performing forward simulations. To provide a maximum of flexibility in terms of foward model calculations, the OEM module interfaces with the rest of ARTS by an agenda. This agenda typically performs the following steps:

1. Unpack the vector \vec{x} into the corresponding atmospheric fields and convert the elements to the forward model units,

2. perform the forward simulation,

3. apply required variable transformations to the Jacobian

⁸¹² Due to the flexibility of ARTS agendas this design allows most of ARTS' ⁸¹³ functionality to be used in a retrieval.

A principal benefit of the OEM integration into ARTS is that ARTS pro-814 vides built-in functions that map elements of \vec{x} to atmospheric fields and 815 back. Moreover, retrieval grids may deviate from the forward model grids. 816 Although the retrieval grid is not allowed to be broader than the correspond-817 ing forward model grid, it can cover a smaller range. Retrieved values are 818 mapped to the forward model grids by using linear interpolation inside the 819 ranges of the retrieval grids, and using nearest neighbour outside. That is, 820 values at end points of retrieval grids are assumed to be valid all the way 821 to end points of the forward model grids. ARTS also provides functional-822 ity to retrieve transformed variables and apply clipping, which helps avoid 823 unphysical states that may lead to errors in the forward model calculation. 824

825 6.3. Handling of a priori and observation error covariance matrices

The calculations required to perform an OEM minimization step involve 826 only the inverses of the covariance matrices \mathbf{S}_a and \mathbf{S}_e . Since, depending on 827 the retrieval problem at hand, the covariance matrices can grow relatively 828 large, ARTS allows them to be provided either as \mathbf{S}_a and \mathbf{S}_e or directly as 829 their inverses \mathbf{S}_a^{-1} and \mathbf{S}_e^{-1} , respectively. In conjunction with the CG solver, 830 this can drastically reduce memory requirements of the OEM calculation 831 step and thus allow for the retrieval of a larger number of variables and 832 observations simultaneously. 833

ARTS covariance matrices are represented as block-diagonal matrices. 834 where each block can be provided either as normal covariance matrix or its 835 inverse. Moreover, covariance matrix blocks can be dense or sparse matrices. 836 This design supports the basic use case in which the user simply provides the 837 covariance matrices but also allows for optimization of the memory footprint 838 of the inversion by providing pre-computed and potentially sparse covariance 839 matrices. The full covariance matrix can be represented as a single block, to 840 allow full generalisation in the specification of correlations between variables. 841

842 7. Summary

This article gave an overview of the capabilities and limitations of ARTS version 2.6.

ARTS can compute atmospheric absorption by gases efficiently and ac-845 curately across the entire spectrum, from the radiowave to the UV/visible 846 spectral range, including advanced features such as line mixing. Besides 847 spectral line by spectral line absorption calculations, it also includes state 848 of the art absorption continua, HITRAN collision-induced absorption, and a 849 simple polynomial model fitted to HITRAN absorption cross-sections, which 850 can be used for example for halocarbon species for which spectroscopic data 851 are typically not available [20]. 852

In the longwave spectral range, ARTS is established and well tested for both clear-sky and all-sky radiative transfer simulations. In the clear-sky case (without scattering) it uses its own native radiative transfer solver, which includes analytical Jacobians. It is fully polarized and includes advanced features such as an accurate treatment of Zeeman splitting.

For all-sky radiative transfer simulation, ARTS offers broad support to 858 cover microwave scattering due to hydrometeors. Inside this domain, the 859 main consideration for the future is to improve the calculation efficiency. It is 860 today costly to make simulations with a high number of scattering elements, 861 and one way forward is to allow specifying bulk scattering properties directly. 862 A full re-implementation of the handling of scattering data is ongoing to 863 open up for this development, as well as obtaining a more uniform code base 864 around the different scattering solvers. 865

Based on the radiative transfer simulations, ARTS can also perform energy flux and heating rate calculations for both clear-sky and all-sky conditions. Furthermore, it has built-in methods for optimal-estimation-type retrievals of atmospheric state properties from remote observations.

Very recently, ARTS was extended to include a solar source term, so 870 that it can simulate also clear-sky and all-sky shortwave radiative transfer 871 simulations and based on that shortwave energy fluxes and heating rates. 872 This new functionality is the subject of a separate article that is currently 873 in preparation. Calculations with the shortwave part have so far considered 874 molecular (Rayleigh) scattering and scattering by hydrometeors. Aerosol 875 scattering (and absorption) could in principle be handled as well, but so far 876 there are no ready-made databases of aerosol optical properties in the ARTS 877 format. 878

Last but not least, it should be mentioned that ARTS is also used as a teaching tool in the international Master program Atmospheric Science at University of Hamburg (https://www.mi.uni-hamburg.de/studium/20-atmo-science. html) for courses on radiation and climate and on remote sensing.

Obtaining ARTS and contributing

The ARTS homepage, providing extensive documentation, is https:// radiativetransfer.org. On GitHub, ARTS can be found at https:// github.com/atmtools/arts/releases. For archiving purposes, there is also a package of the exact version described in this article on Zenodo [142]. The easiest way for Python users to obtain ARTS is through conda ('conda install -c rttools pyarts'). Pyarts is the python interface to ARTS, providing full ARTS functionality.

ARTS has been extensively tested and used on macOS, Linux, and other Unix systems. It has currently not been used or tested on Windows and there is also no conda package for Windows. This is planned to change for the next major version.

We welcome contributions. The for us most convenient way to contribute is through GitHub. Contributions can for example be bug fixes or extensions to the ARTS core, but also documentation, usage examples or helper tools. Besides GitHub, a good way to communicate with other ARTS users and

developers are the ARTS mailing lists (https://www.radiativetransfer. org/contact).

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