

Use of Generalized Predictive Control for Flutter Suppression of the Tiltrotor Aeroelastic System Testbed (TRAST)

Thomas G. Ivanco
Flight Dynamics Branch
National Aeronautics and Space
Administration
Hampton, VA, USA

Robert P. Thornburgh
U.S. Army DEVCOM
Army Research Laboratory
Hampton, VA, USA

Andrew R. Kreshock
U.S. Army DEVCOM
Army Research Laboratory
Hampton, VA, USA

Martin K. Sekula
Aeroelasticity Branch
National Aeronautics and Space
Administration
Hampton, VA, USA

ABSTRACT

Generalized Predictive Control (GPC) is an advanced form of an adaptive control algorithm that uses experimentally acquired data to determine the input-output relationship of complex systems through a process called system identification. GPC has historically been employed for stability augmentation and vibration reduction of dynamically-scaled tiltrotor aircraft wind-tunnel models since the complex nature of these dynamic systems does not lend itself well to traditional control approaches. The present research expands upon previous analytical and experimental work with wind-tunnel experiments that utilize improved GPC techniques. These techniques improved controller robustness such that a working controller was stable across a multitude of model configurations and wind-tunnel conditions and successfully suppressed vibration and vehicle flutter. Advanced GPC (AGPC) enables self-adaptation of a traditional GPC control law. AGPC was also investigated during the present research but was not needed as anticipated because of the robustness resulting from improvements made to traditional GPC.

NOTATION

$1/rev$	=	one-per-revolution, normalized frequency
AGPC	=	advanced generalized predictive control
ARX	=	autoregressive moving average model with exogenous input
GPC	=	generalized predictive control
h_c	=	control horizon
h_p	=	prediction horizon
k	=	time step index parameter
Hz	=	Hertz, cycles per second
m	=	number of response measurements (outputs)
n	=	number of sine waves
OMP	=	Observer Markov Parameters
p	=	order of the ARX equation, number of past inputs and responses in model
Q	=	weighting matrix for control (penalizes control)
r	=	number of control inputs
R	=	weighting matrix for error (penalizes error)
s	=	number of time steps used in system identification
TDT	=	Transonic Dynamics Tunnel, NASA Langley Research Center
TRAST	=	TiltRotor Aeroelastic Stability Testbed
u	=	control input
y	=	response output
δ_3	=	pitch-flap coupling, angle of hinge-line rotation (“delta 3 angle”)

ε	=	error between target and measured response (equal to the response for a regulator)
ξ	=	system modeling error between predicted and actual system response (error in OMP)
φ	=	external noise and disturbances

INTRODUCTION

Tiltrotor aircraft are defining the state-of-the-art in vertical lift technology as they have the potential to greatly expand rotary-wing operational boundaries. However, they are often limited in forward flight speed due to complex coupled rotor and wing dynamic instabilities known as propotor pylon instability, often referred to as whirl flutter. The U.S. Army and NASA have recently developed a new wind-tunnel model, the TiltRotor Aeroelastic Stability Testbed (TRAST), shown in Figure 1, to investigate these aeroelastic propotor instabilities in the NASA Langley Research Center Transonic Dynamics Tunnel (TDT). TRAST was introduced and documented in Reference 1.

Presented at the Vertical Flight Society’s 81st Annual Forum & Technology Display, Virginia Beach, VA, USA, May 20-22, 2025. This is a work of the U.S. Government and is not subject to copyright protection in the U.S.

To counter the dynamic instabilities and to suppress modal vibration of tiltrotor aircraft models such as TRAST, active controls are utilized exploiting the inherent hydraulic actuators used for primary flight controls. Due to the complex rotor dynamics and nonlinear interactions, tiltrotor dynamic behavior is difficult to model. Therefore, adaptive controls that utilize experimentally acquired input-output relationships are pursued. Generalized Predictive Control (GPC) is an advanced adaptive controller, and its application to tiltrotor aircraft and other dynamic systems has been introduced and documented in Refs. 2-6. Advanced Generalized Predictive Control (AGPC) was first introduced and analytically investigated in Ref. 7 and was applied experimentally on TRAST in a hover test cell, documented in Refs. 8 and 9. AGPC is similar to GPC but enables the controller to self-adjust as flight or vehicle conditions change. This current paper is an extension of the work in Refs. 7-9 and presents wind-tunnel results qualifying the effectiveness of GPC and AGPC to suppress flutter on the TRAST model and the ability to safely continue operation beyond the flutter boundary. Additionally, the robustness of GPC was improved by using orthogonal multisine excitation (Ref. 10) during the system identification (system ID) process, using performance metrics introduced in Refs. 8 and 9, and using a notch filter to reject the one-per-rev (1/rev) noise generated by the rotor.

This paper will begin with a brief overview of the experimental setup and a review of the pertinent parts of GPC and AGPC. It will then describe the multisine excitation technique and the notch filter implementation. It will also review the performance metrics used to quantify controller performance. It will then demonstrate the effectiveness and robustness of the GPC controller. Finally, it will discuss and illustrate the performance of the AGPC controller.

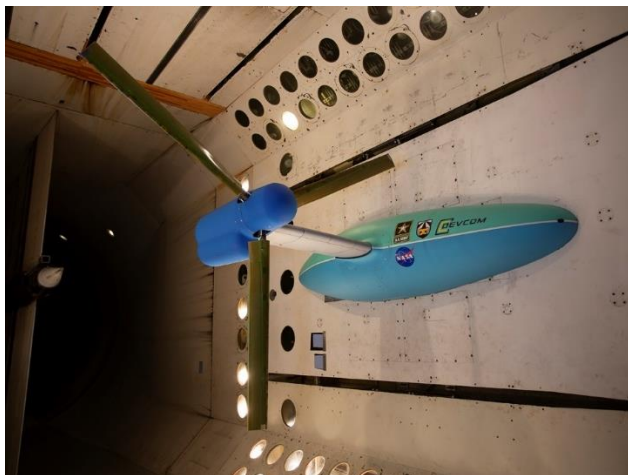


Figure 1. TRAST installed in the NASA Langley Transonic Dynamics Tunnel

EXPERIMENTAL SETUP

TRAST Model

The TRAST model was described in detail in Ref. 1 and described briefly in Refs. 7-9. During the latest TRAST entry in the TDT, various model configurations were tested including changing blade stiffness, pitch-flap coupling (δ_3 angle), and engine nacelle pitch and yaw spring stiffness. These parameters have significant impact on the flutter boundary of a tiltrotor vehicle. Blade flexibility affects a regressive lag mode of the rotor blade, that acts to dissipate rotor motion and has a damping effect. Stiffer blade configurations result in lower stability boundaries. Likewise, the δ_3 angle couples rotor flapping with nacelle pitch and yaw motions. Zero δ_3 angle causes the rotor disk to behave gyroscopically and maintain a consistent plane as the nacelle vibrates in the case of a gimballed rotor system such as TRAST. This decouples rotor flapping and is more stable from a flutter perspective. However, flight maneuvers will cause increased flapping which will likely limit the vehicle maneuverability. Therefore, typical designs find a compromise between flutter stability and maneuverability. Additionally, nacelle pitch and yaw stiffness determine the pitch and yaw frequencies of the engine nacelle that drives the dynamic motion of the rotor shaft. Changes in these stiffnesses will alter the aeroelastic modes and how they interact.

Several model design features enabled adjustment of model dynamic behavior. Blade stiffness was altered by testing two different blade sets. The baseline set consisted of a predominantly fiberglass construction and a stiffer set with an identical planform and weight distribution was constructed of carbon fiber. The δ_3 angle was adjusted by a novel design feature described in Ref. 1 that offsets the angle of the blade pitch-control links thereby changing the pitch-flap coupling. Nacelle pitch and yaw stiffnesses were controlled by two mechanical spring designs as illustrated in Figure 2. The first was a replaceable pitch spring used to join the pylon to the wing. Unlike a flight vehicle in which the pylon conversion angle is controlled by a displacement actuator, the TRAST pylon used a simple linear spring to simulate the stiffness of the flight actuator. A range of pitch springs with stiffnesses between approximately 3000 and 8000 lbs/in were manufactured and were changed during testing for parametric studies. In addition to the pitch spring, the second spring was a segmented diaphragm spring that connected the static side of the engine nacelle conversion bearing to the wing. By replacing the spring segments, the roll and yaw stiffness of the pylon were independently changed. The baseline diaphragm spring had a rotational stiffness of 20,000 in-lb/rad in both the roll and yaw directions. Replacing individual sections of the spring with half-stiffness segments yielded a 10,000 in-lb/rad yaw direction stiffness while maintaining the baseline 20,000 in-lb/rad roll direction stiffness. The various model configurations that were tested in the present study by

varying these parameters are shown in Table 1, and were tested over a range of airspeeds and rotor speeds.

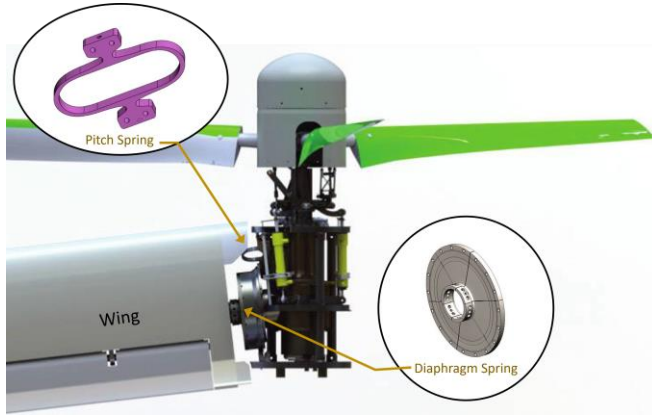


Figure 2. Illustration of TRAST model with spring attachments.

An electric motor and flexible driveshaft exist in the design to provide powered-rotor hover and forward-flight testing. For the data and results presented in the present work, the driveshaft coupling was disconnected to allow the rotor system to freewheel and operate in “wind milling” mode. Rotor speed, denoted in revolutions per minute (rpm), was controlled with collective pitch setting by the model pilot. As wind-tunnel speed was changed, rotor pitch was adjusted in real-time to maintain desired rpm. Three rotor speeds were tested in the present study: 909, 727, and 818 rpm corresponding to hover, cruise, and an intermediate rpm respectively.

Table 1. TRAST Configurations.

Name	Pitch Spring (lbs/in)	Yaw Spring (in-lbs/rad)	δ_3 (deg)	Blades
Alpha	4k	20k	-30	Fiberglass
Bravo	4k	20k	-20	Fiberglass
Charlie	4k	20k	-25	Fiberglass
Delta	4k	20k	-10	Fiberglass
Echo	4k	20k	0	Fiberglass
Foxtrot	4k	20k	0	Carbon
Golf	4k	20k	-30	Carbon
Hotel	4k	20k	-15	Carbon
India	4k	20k	-15	Fiberglass
Juliet	3k	20k	-20	Fiberglass
Kilo	5k	20k	-20	Fiberglass
Lima	5k	10k	-20	Fiberglass

Transonic Dynamics Tunnel

The NASA Langley Transonic Dynamics Tunnel (TDT) is a dedicated aeroelastic test facility. TDT is a continuous-flow, transonic tunnel with a maximum flow speed of Mach 1.2. It has a slotted 16 feet by 16 feet test section with cropped corners and can utilize either air or R-134a heavy gas as a

test medium. The test capabilities of the TDT are described in Ref. 11, and TDT is world unique in its ability to simulate rotorcraft performance and dynamic characteristics at speeds near the stability boundary of tiltrotor aircraft (Ref. 11). Applicable to tiltrotor research, TDT can simultaneously match Mach number, reduced frequency, and Froude number through the use of the R-134a test medium (Ref. 11). However, for the present paper, the TRAST model was characterized in air only. One important feature of the TDT is a set of bypass valves that can be impulsively opened to connect the back leg of the wind tunnel to the test section plenum. When opened, this enables a rapid speed reduction of the test section flow. This is of particular value in flutter testing as it prevents model failure when reaching a dynamic instability. A sketch of the TDT with key features of the wind tunnel annotated is shown in Figure 3.

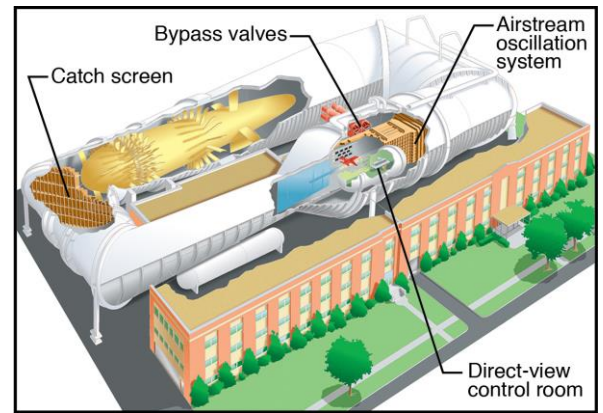


Figure 3. Sketch of the Transonic Dynamics Tunnel with cutaways illustrating key features.

ADVANCES IN THE IMPLEMENTATION OF GPC

GPC and AGPC Overview

The information presented in this subsection was presented in previous publications (Refs. 7-9) and a summary is also included here for reader convenience.

The concept of GPC was originally introduced by Clarke, Mohtadi, and Tuffs (Ref. 2) as a potential adaptive control algorithm. Reference 2 proposed the concepts of a prediction horizon (h_p), a control horizon (h_c) after which control commands become zero, and a least-squares observer parameter estimator (p), referred to as the order of the model. However, no details are presented in Ref. 2 regarding the parameter computation or system ID. Juang and Phan (Ref. 3) expanded this effort and explored various system ID processes of Autoregressive moving average model with exogenous inputs (ARX) models with multiple input multiple output (MIMO) systems and enabled unknown disturbance rejection. Building upon the ARX foundations of Ref. 3 and the GPC introduction of Ref. 2, Kvaternik, Juang and Bennet (Ref. 4) developed an objective function similar to optimal control theory and turned GPC into a practical and powerful MIMO controller. Reference 4 is a

foundational, albeit informal, paper describing the state-of-the-art of GPC. Reference 5 demonstrated the use of GPC experimentally with a tiltrotor aircraft and Ref. 6 further explored the use of GPC as a tracker for rigid-body control of a flight vehicle.

The derivation of the equations used in the modern GPC controller was presented in Ref. 7 and is further expanded upon in Ref. 8. For clarity, the governing equation is presented here as Equation 1. In this equation, y is the response output, u is the control input, k is the discrete time step index, the prediction horizon (h_p) is the limit of future state estimates, and the control horizon (h_c) is the limit of the finite control horizon after which the controls are assumed to be zero (Ref. 4). Past input and output values are denoted as u_p and y_p respectively and contain p values of u and y . The matrices T , A , and B are described in Refs. 7-9 and represent the Observer Markov Parameters (OMP). An experimental system ID process, as described in Refs. 7-9, is used to determine the OMP matrices. In this paper, “system ID” refers to a successfully computed set of OMP matrices with associated weight values rather than the process of identifying those matrices.

$$y_{h_p}(k) = Tu_{h_c}(k) + Ay_p(k-p) + Bu_p(k-p) \quad (1)$$

where, the subscripts h_p , h_c , and p denote the size of the respective vectors, and the parenthetical values represent the starting index.

Kvaternik et al. (Ref. 4) present the objective function shown here as Equation 2. Controls u are then determined from minimalization of this function as described in Refs. 4, and 7 and 8.

$$J = \varepsilon^T R \varepsilon + u_{h_c}^T Q u_{h_c} \quad (2)$$

where, ε is the error between the target response (y_T) and the predicted response $\varepsilon = y_T - y_{h_p}$, R is the weighting matrix for error, and Q is the weighting matrix for control input.

In the case of a regulator where the target response is zero, the error term ε simply becomes the response y .

Traditional GPC is a very effective controller that has been used in many dynamic systems (Refs. 4-9) and often outperforms other control options with regard to successful dynamic suppression and control robustness (Ref. 8). However, the performance of traditional GPC can erode as conditions change from those used to derive the system ID. To correct the controller for changing flight or vehicle conditions, the concept of advanced generalized predictive control (AGPC) was introduced and demonstrated in Refs. 7-9.

Reference 7 introduces AGPC and the definition of a system error term (ξ) that can be numerically determined from

measured data using Equation 3 after the system described by Equation 1 progresses at least h_p timesteps beyond k . The significance of this development is that a working controller can be improved or updated by changes to the OMP matrices done by passive observation of the input-output relationships during controller operation.

$$\begin{aligned} \xi &= y_{h_p \text{ actual}} - y_{h_p \text{ predicted}} + \{\varphi\} \\ &= y_{h_p \text{ actual}} - \left[Tu_{h_c \text{ actual}}(k) + Ay_p(k-p) + \right. \\ &\quad \left. Bu_p(k-p) \right] + \{\varphi\} \end{aligned} \quad (3)$$

where φ represents noise and external disturbances.

The subscript *actual* in Equation 3 indicates the actual measurements of response and control inputs after an additional h_p timesteps occur beyond k . If one were to assume that the noise and disturbances represented by φ follow a random process, then repeated computations should cancel these disturbances. If we continue to an additional s timesteps, then one would have $s-h_p$ computations of ξ to minimize the effect of φ . If φ is minimized and all the values of y and u shown in Equation 3 are now measured, then the only uncertainty that should remain is the error in the observer matrices T , A , and B . After these s timesteps, Equation 3 can be expanded into the matrix form shown in Equation 4 where the delta (Δ) operator indicates the error in the observer T , A , and B matrices where the control and state matrices are of width $s-h_p$. The Δ matrix values can then be determined by Equation 5 and the updated values of T , A , and B by Equation 6.

$$[\xi] = [\Delta T \mid \Delta A \mid \Delta B] \begin{Bmatrix} [u_{h_c \text{ actual}}] \\ [y_p(k-p)] \\ [u_p(k-p)] \end{Bmatrix} \quad (4)$$

$$[\Delta T \mid \Delta A \mid \Delta B] = [\xi] \text{pinv} \left(\begin{Bmatrix} [u_{h_c \text{ actual}}] \\ [y_p(k-p)] \\ [u_p(k-p)] \end{Bmatrix} \right) \quad (5)$$

$$\begin{aligned} T_{\text{new}} &= T_{\text{old}} + \Delta T \\ A_{\text{new}} &= A_{\text{old}} + \Delta A \\ B_{\text{new}} &= B_{\text{old}} + \Delta B \end{aligned} \quad (6)$$

Previous Investigations

Analytical results for the TRAST model showed that GPC performance would degrade as velocity continued significantly beyond that where the system ID was acquired (Ref. 7). This finding was similar to some previous implementations of GPC that showed sensitivity to large changes in velocity from that used to acquire a system ID

(Refs. 4 and 5) and was the impetus to the development of the AGPC algorithm (Refs. 7-9). GPC was successful at suppressing flutter and reducing vibration in all modes well beyond the flutter boundary in prior tests (Refs. 4, 5). However, the system ID was typically acquired close to the flutter boundary and IDs acquired far from it were observed to degrade in effectiveness as the boundary was reached, consistent with recent analytical results presented in Ref. 7.

Current Investigation

Similar behavior in system ID degradation was expected in the present investigation, but the demonstrated robustness of the traditional GPC controller was surprising in that extreme changes in velocity and even model configuration did not degrade the performance of the controller. This surprising increase in robustness is theorized to be the result of four factors. First, analytical models of complex tiltrotor systems are difficult to develop and even the open-loop behavior was not well-matched regarding stability boundaries. Therefore, the analytical predictions regarding controller robustness are not necessarily representative of the wind-tunnel model behavior. Second, in the present implementation, GPC employed orthogonal multisine excitation which is expected to improve the system ID process beyond that used in prior experimental investigations that utilized distinguishable random noise. Third, the present implementation of GPC utilized newly developed performance metrics to reliably quantify a good system ID and controller performance. Finally, fourth, the present implementation of GPC utilized a roving notch filter to reject the noise caused by 1/rev in-plane rotor vibration. Additional details follow.

Current Investigation - Multisine Excitation

Recent work in active controls for aircraft and aeroelastic stability has investigated the use of orthogonal multisine inputs in the system ID process (Refs. 12 and 13). Multisine inputs, as the name suggests, are a sum of multiple harmonic sine waves of different frequencies. The frequencies range from the lowest value of interest to the highest value of interest for the dynamic system in a series of discrete steps. Each command signal contains frequencies of differing discrete values so that they are mathematically orthogonal. Also, the frequency step between subsequent harmonic waves must be small enough to induce excitation of the modes of interest from each command signal.

Multisine inputs were generated based upon Equation 7.

$$u_i = \sum_{j=1}^n A \sin(2\pi k_{i,j}t + \phi_{i,j}) \quad (7)$$

where i denotes the number of the control channel (i.e. actuator #2), n is the number of sine summations, k is the frequency of the i^{th} channel at frequency step j , t is time (signal was constructed from 0 to 40 seconds), and ϕ is the phase of channel i at frequency step j .

Each phase was randomly chosen and was not optimized to minimize peak amplitudes as described in Refs. 12 and 13. Since the range of frequencies are large for an aeroelastic test, phase optimization is theorized to be less critical and random phasing typically provides adequate dispersion of peaks such that the time history does not contain large spikes where the phasing of multiple sine peaks coalesces as demonstrated in Refs. 8 and 9. For the current work, the starting frequency for each of the command signals was 0.1, 0.2, and 0.3 Hz respectively with a step size of 0.3 Hz for each signal and a maximum frequency of 40 Hz. With the given step size and frequency range, this yields an n value of 134. When specifying the maximum frequency, it is important to consider the Nyquist frequency (50 Hz in this example for a scan rate of 100 Hz) of the system otherwise the multisine signal could become aliased if frequencies become too high. Similarly, if the maximum frequency is too low, then high frequency behavior of the actuators or system will not be characterized, and the controller may become dynamically unstable at high frequencies.

In the present investigation, the use of orthogonal multisines improved the performance metrics (described below) of the controller but did not conclusively demonstrate better capability to suppress vibration or flutter. Controllers using distinguishable white noise, distinguishable implying different seeds in the random number generator such that each actuator had a unique signal, produced similar modal suppression capability.

Current Investigation - Performance Metrics

Performance metrics used to characterize and quantify controller performance were introduced in Refs. 8 and 9. These metrics include: the objective function magnitude, the rank of the objective function, the pairwise correlation coefficient between control channels and response channels, and the coefficient of determination. Details and examples are provided in Ref. 9, and a brief description is provided here for reader convenience.

The magnitude of the objective function (Equation 2) is a quantitative value describing how well a controller is functioning since the goal is to minimize the objective function. However, the quantitative value is dependent upon the test condition and a number without context is of little value. Therefore, knowledge of the system and test condition is required before this value has meaning.

Similarly, the rank of the objective function quantifies the number of linearly independent rows where each row corresponds to a time history of control inputs or response outputs. To characterize the input-output system behavior in a system ID, the controls and responses need to be unique and distinguishable. As described in Ref. 9, the autoregressive structure of the objective function matrix limits the maximum rank value because each row is not linearly independent from the neighboring rows. Therefore, it is introduced here that the maximum possible rank of the

objective function is limited to the product of the control horizon (h_c) and the sum of the number of control inputs (r) and response outputs (m) as expressed in Equation 8.

$$\text{Max Rank} = h_c(r + m) \quad (8)$$

As controls or response values become decreasingly less orthogonal, the rank of the objective function will decrease below its maximum possible value. During control implementation, non-orthogonality may be normal or desirable, such as for the suppression of an in-plane chord mode where the actuators act in unison. However, for determination of a system ID, orthogonality is required and an objective function rank near the maximum is therefore required. If the rank is too low, then orthogonal dither should be added to increase the rank to near the maximum level. In the present study, the control horizon consisted of 25 steps and there were three controls and responses respectively. This yields a maximum possible objective function rank of 150. System IDs were rejected when the rank fell below 145.

Similarly, the pairwise correlation coefficient between any two controls inputs or any two measurement responses should be low. The equation for correlation coefficient is shown in Equation 9 for comparison between any two control inputs (u). A similar equation can be cast for the pairwise correlation coefficient of response where the u -terms are replaced with y -terms. It is required that the correlation coefficient be below 0.9 between any two responses or any two controls (Refs. 10, 14, and 15). In the current investigation, the pairwise correlation coefficient for the control inputs are much lower when commanded with orthogonal commands and should be below 0.1.

$$ru_{ij} = \frac{(u_i - \bar{u}_i)^T (u_j - \bar{u}_j)}{\sqrt{(u_i - \bar{u}_i)^T (u_i - \bar{u}_i)} \sqrt{(u_j - \bar{u}_j)^T (u_j - \bar{u}_j)}} \quad (9)$$

where the subscripts i and j denote the control number, the bar ($\bar{}$) denotes the mean, and $i \neq j$.

Finally, the coefficient of determination was computed as shown in Equation 10 and discussed in Ref. 7. In this equation, the subscripts of *actual* and *predicted* refer to the measured system response versus the response predicted for each future time step in accordance with Equation 1, using coefficients from the applicable system ID.

$$R^2 = 1 - \frac{\sum (y_{hp\text{actual}} - y_{hp\text{predicted}})^2}{\sum (y_{hp\text{actual}} - \bar{y}_{hp\text{actual}})^2} \quad (10)$$

where the bar ($\bar{}$) denotes the mean value. The numerator of the fraction is often abbreviated as the residual sum square or *RSS* and the denominator as the total sum square or *TSS*. In the present work, the mean is normalized to zero and the

denominator can be simplified as the sum square of the actual response.

Current Investigation - Roving Notch Filter

During GPC implementation, it was noticed that significant controller noise existed at the $1/\text{rev}$ rotor revolution frequency. Since the swashplate controls have minimal effect on in-plane rotor vibration resulting from rotor imbalance, controller excitation at these frequencies is usually futile. Therefore, a “roving” notch filter was employed during later evolutions of GPC in the present work to eliminate the noise associated with rotor imbalance. The descriptor of roving refers to the center frequency of the notch filter being based upon the real-time measured rotational speed of the rotor system. A second-order notching Infinite Impulse Response (IIR) filter with a 3-dB attenuation bandwidth of 1.0 Hz was implemented. Figure 4 illustrates the notch filter performance for an example of 900 rpm, or 15 Hz.

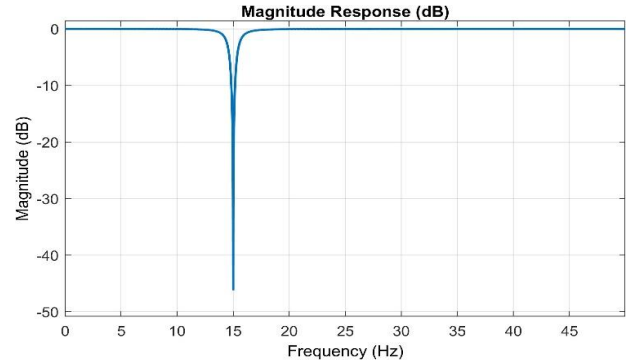


Figure 4. Roving IRR notch filter performance, example for 900 rpm (15 Hz).

EXPERIMENTAL RESULTS

Test Procedure

In the present study, wind-on data were acquired in the TDT for various model configurations and rotor speeds to document the open-loop stability boundary and to evaluate the ability of GPC and AGPC to improve damping and suppress instabilities.

Typical test iterations were a velocity sweep at a fixed rotor speed for a given model configuration. At each velocity step, model modes were intentionally excited with the control system and the free response was analyzed to determine modal damping. The in-plane wing bending mode was excited with the use of dynamic collective inputs to excite the wing in the fore-aft direction. Conversely, out-of-plane (vertical) wing bending, nacelle pitch, and nacelle yaw modes were excited with cyclic stirrs. The direction of the stir, either forward in the direction of rotor rotation or backwards opposite of the rotor rotation, was varied to

achieve the desired modal response. Reference 16 details the test campaign and the parametric investigation objectives.

Excitation amplitude was slowly increased to achieve the desired response magnitude. Once reached, the excitations were terminated in one of three ways. First, using a manual kill switch when the test engineer qualitatively thought the response magnitude reached an acceptable level. Second, with software when the measured wing bending-moment response of the target mode reached a predetermined amplitude. Finally, third, through software when a maximum time limit was reached. As the test progressed and various safety systems were validated, the test team typically used the second method in order to regulate the response amplitude during excitation. It was also discovered in testing that the stability boundary was sensitive to the amplitude. Amplitude dependency on stability boundary will be discussed in another publication and is beyond the scope of the present work.

An example excitation of wing out-of-plane bending using a positive cyclic stir is shown in Figure 5 for configuration Alpha at 727 rpm and a velocity of 65 knots. In Figure 5a are the actuator positions in inches relative to the data record mean, and in Figure 5b are the moment responses for out-of-plane bending, in-plane chord, and wing torsion.

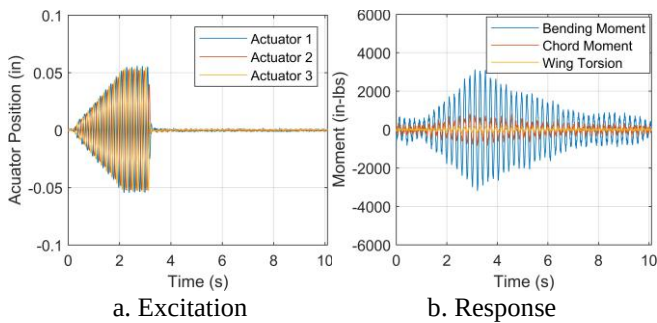


Figure 5. Example out-of-plane bending excitation, config Alpha, at 727 rpm, 65 kts.

At the beginning of the time history, the actuator positions are oscillated at the resonant frequency to excite the out-of-plane wing bending mode. Excitation amplitude was slowly ramped up over a period of two seconds. At approximately three seconds into the data record, the excitation stops once the response magnitude of bending moment reached approximately 3000 in-lbs. Following the excitation, the amplitude of the wing bending moment response dissipates gradually indicating this condition is below the stability boundary.

During early phases of testing, the effectiveness of GPC was uncertain, and the test team could not rely upon it to stabilize the model. The test technique used at the time to suppress flutter was to increase model collective pitch – to lower rotor speed – while simultaneously using the bypass valves of the TDT to rapidly reduce wind tunnel velocity. These methods removed the model from the flutter condition, but they did

not add modal damping and the stabilization of a flutter instability took time.

The time history of actuator position and bending moment response following a flutter instability is shown in Figure 6. For clarity, the time history of only one swashplate actuator (actuator 1) and only the wing bending moment (out-of-plane) response are superimposed as shown in Figure 6. Wing out-of-plane bending moment was chosen because this mode is the primary motion involved in the flutter instability for most configurations tested. Similarly, actuator 1 plays a large role in the pitch motion of the engine nacelle and this actuator is dominant in every excitation motion and nearly every modal suppression control. As seen in Figure 6, the instability was reduced, and the model returned to a safe condition. However, the model response continued to grow for a short period of time before reducing, and it took several minutes to reset the tunnel and model conditions following the bypass valve activation.

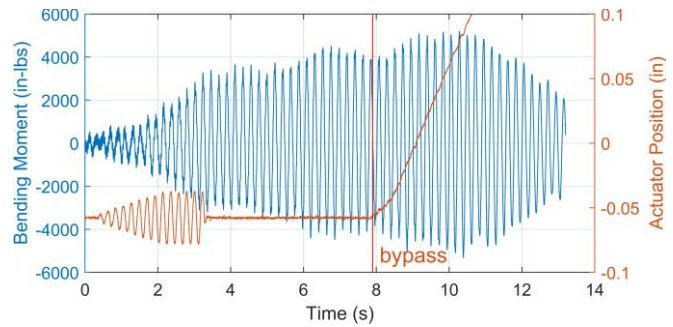


Figure 6. Flutter encounter, tunnel bypass and rpm used to arrest instability, config Alpha, at 727 rpm, 108 kts.

Traditional GPC Applied to TRAST – Modal Vibration

The employment of traditional GPC typically requires the acquisition of a system ID using the same model configuration and similar tunnel conditions to where the controller is intended to be employed. During the present study, system IDs were acquired for multiple configurations and conditions and used 10 seconds of data with an open-loop controller. The acquisition of a traditional system ID while closed loop is possible but may cause strange behavior and was avoided in the current work.

Model configuration Alpha was used to test the effectiveness of GPC to suppress modal vibrations. The time history of actuator 1 position and out-of-plane bending moment for a free response following a cyclic stir is shown in Figure 7 for the same data record as that presented in Figure 5. Easily observable in Figure 7 is a design feature that ramps the excitation amplitude to zero over a period of 0.25 seconds to smoothly turn off the excitation. This technique allows the vehicle to have a free response post-excitation but does not impulsively excite the vehicle as the commands are zeroed.

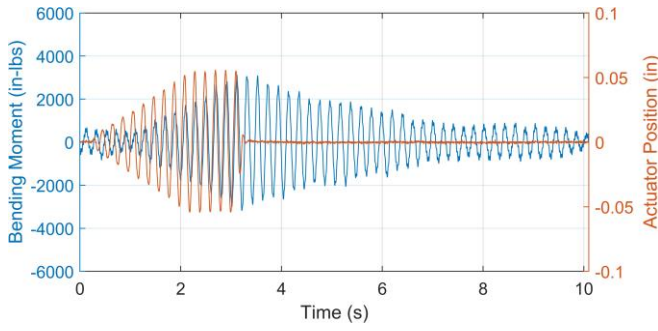


Figure 7. Free response following out-of-plane bending excitation, config Alpha, at 727 rpm, 65 kts.

To evaluate the effectiveness of GPC, modal excitation was achieved in the same manner by exciting modes in resonance. However, following excitation, GPC was enabled. The time history of actuator position and bending moment is shown in Figure 8 for this same condition of 727 rpm and 65 knots with configuration Alpha. The system ID used for GPC control in this data record was acquired with the same model configuration and rotor speed, but at a lower velocity of 35 knots. Similarly to the excitation, GPC control authority is slowly ramped up to 100% of authority over a period of one second.

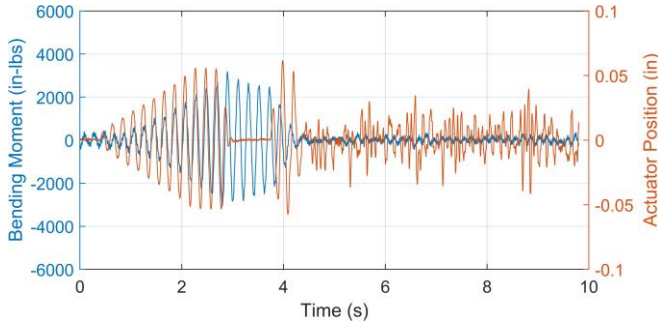


Figure 8. Modal suppression using GPC following bending excitation, config Alpha, at 727 rpm, 65 kts.

The bending moment time history response for these two data records, the free response and with GPC enabled, were superimposed as shown in Figure 9. One can observe that the GPC controller is very effective at suppressing the bending moment vibration response.

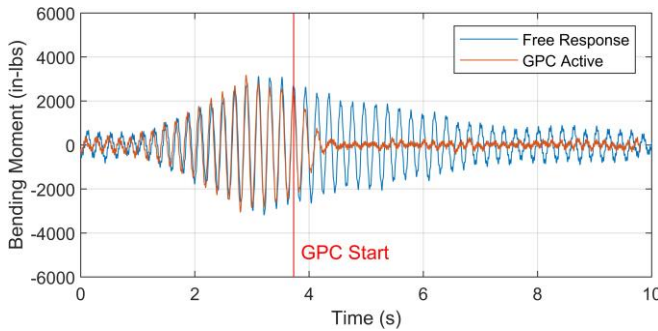


Figure 9. Overlay of open-loop free response and GPC suppression, config Alpha, at 727 rpm, 65 kts.

Subcritical testing below the flutter boundary, such as that shown here, enabled validation of the GPC controller and evaluation of its impact to modal damping prior to using it for flutter suppression. Additionally, suppression of modal vibration is of significant value in addition to dynamic stability and will improve ride quality and reduce material fatigue. Modal suppression is especially important for vehicles such as TRAST that exhibit low modal damping over a wide range of the flight envelope and incur significant vibration at many conditions.

The present GPC controller was also used to investigate its ability to suppress other modes of vibration in addition to the out-of-plane bending mode. Some analytical results predicted that the in-plane chord mode would become unstable at high velocity for configurations with increased out-of-plane bending stability. However, a chord instability was not encountered in experimental testing. Configuration India is one of the most stable, and it did not encounter a flutter instability before reaching model load limits in the wind tunnel. At the higher velocities, the damping of the in-plane chord mode was reduced, and sustained oscillations would occur following excitation. If model load limits would allow, it may have encountered a chord-mode instability at higher velocity.

Figure 10 contains a time history of position for actuator 1 and in-plane chord moment response of the vehicle for configuration India at 909 rpm and a velocity of 154 knots. For this data record, the excitation was a collective oscillation using the same excitation on all three actuators at the frequency of the in-plane chord mode. Collective excitation results in more control authority than a cyclic stir. Therefore, the magnitude of the actuator motion is less than that required for a cyclic stir. The excitation was automatically halted once a value of 4,000 in-lbs in moment response was reached. Inspection of Figure 10 reveals that the mode is stable, but lowly damped.

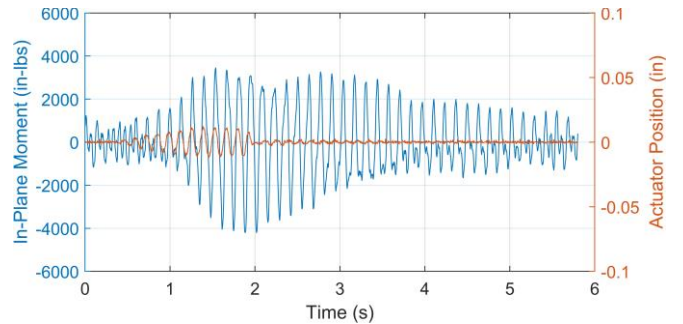


Figure 10. Free response following in-plane chord excitation, config India, at 909 rpm, 154 kts.

For comparison, Figure 11 contains a time history of the same actuator and in-plane chord moment with the same model configuration and at the same tunnel condition, but with GPC used to suppress the chord vibration following excitation. Inspection of Figure 11 and comparison to Figure 10 reveals that the GPC controller can effectively damp the

in-plane chord vibration. It is noteworthy that this same system ID was also used to damp the out-of-plane bending mode, and it was effective at both.

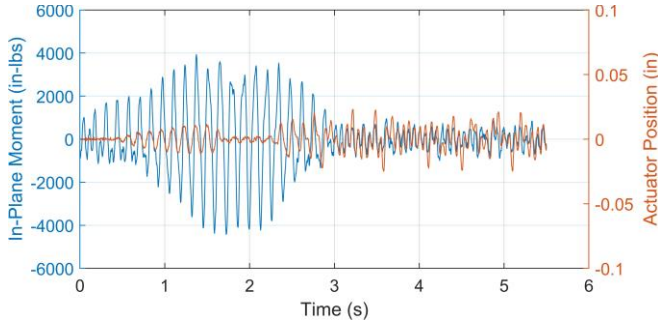


Figure 11. Modal suppression using GPC following chord excitation, config India, at 909 rpm, 154 kts.

As previously discussed, it was observed that $1/rev$ rotor noise was evident in the response and commands in some GPC iterations, such as that visible in the latter part of Figure 11. Therefore, the previously described roving notch filter was implemented to mitigate this noise that is otherwise uncontrollable. It is important to note that the notch filter was only implemented internally to the control law and did not affect the acquired data presented in the figures. To illustrate the filter capability, the free response time history is shown in Figure 12 and the GPC suppression following the excitation is shown in Figure 13 for configuration Bravo at 909 rpm and 100 knots. For both figures, the time history includes actuator 1 position and out-of-plane bending moment response.

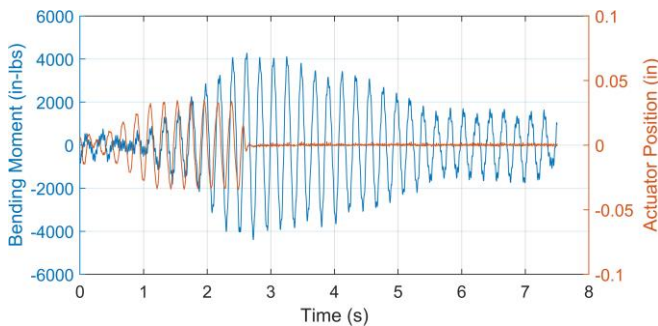


Figure 12. Free response following out-of-plane bending excitation, config Bravo, at 909 rpm, 100 kts.

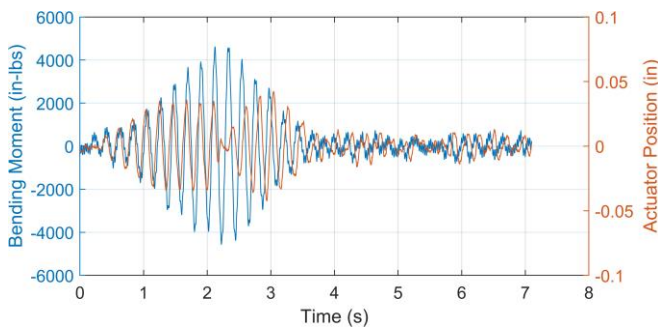


Figure 13. Modal suppression using GPC following bending excitation, config Bravo, 909 rpm, 100 kts.

As seen by comparison of Figure 12 and Figure 13, GPC is extremely effective at suppressing the modal vibration, and it does so with minimal control workload. Additionally, comparison of Figure 13 with Figure 8 or Figure 11 illustrates the significant improvement in elimination of $1/rev$ noise and the unnecessary control commands that the noise induces. The system ID used for the controller demonstrated in Figure 13 was acquired at the same tunnel conditions and with the same model configuration. Another implementation demonstrated in Figure 13 was the option to automatically start GPC following the excitation. Recall that the excitation has a ramp-down time of 0.25 seconds, and the GPC control authority is ramped up over a period of 1.0 seconds.

Traditional GPC Applied to TRAST – Flutter Suppression

Following successful modal vibration reduction with GPC, and increased confidence in the reliability of the controller, GPC was also used to suppress flutter. Figure 14 shows the time history of actuator position and wing bending moment response for a cyclic stick stir at a velocity of 145 knots that is in slight excess of the stability boundary. The model configuration used for this data set is configuration Bravo at 727 rpm and is the same as that used to derive the system ID. However, the system ID was acquired at 100 knots and a rotor rpm of 909. At approximately 2.5 seconds into the data record, the stick stir excitation stops. Following the excitation, the amplitude of the wing bending moment continues to increase indicative of the flutter instability. At approximately 5.5 seconds, the GPC controller is engaged and suppresses the vibration within approximately one second while tunnel conditions and rotor speed are held constant.

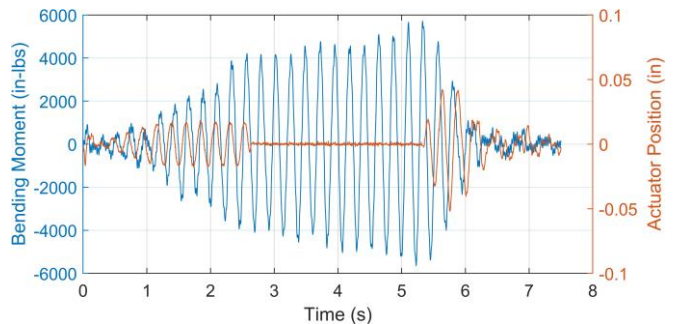


Figure 14. Flutter suppression using GPC with same config as that used to derive the system ID, config Bravo, at 727 rpm, 145 kts.

One can observe that the GPC controller is able to suppress the instability far faster than the traditional method of decreasing rotor speed and reducing tunnel velocity such as that demonstrated previously in Figure 6. Therefore, the use of GPC enabled the test team to test to higher vibration levels and explore the effect of amplitude on stability in addition to saving test time. Using GPC in lieu of traditional methods enabled the completion of all primary, secondary, and tertiary test objectives so that the test team completed

approximately three times as many configurations and test conditions than anticipated for this tunnel entry.

Traditional GPC Applied to TRAST – Controller Robustness

Controller robustness was further investigated during this testing by using the controller with the same system ID across a range of velocities and rotor speeds. A stable GPC controller with a single system ID was found to be effective at any velocity and rotor speed tested. It was then investigated if the same controller can be used across multiple model configurations.

Figure 15 shows the time history of actuator position and wing bending moment for a cyclic stick stir at a velocity of 138 knots that is also in slight excess of the stability boundary. The GPC controller is using the same system ID as that used for Figure 14, configuration Bravo. However, the model was changed to configuration Kilo which possess a stiffer nacelle pitch spring. This configuration is less stable, and the flutter boundary occurs sooner. In Figure 15, one can observe that the stick stir is terminated sooner and at a lower value of wing bending moment than for the event shown in Figure 14. Despite these differences, the GPC controller still suppresses the instability in approximately one second while tunnel conditions and rotor speed are again held constant.

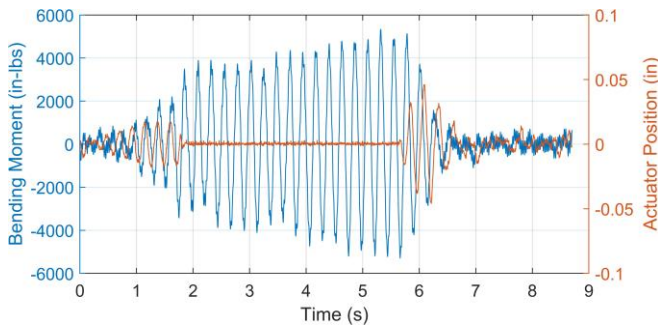


Figure 15. Flutter suppression using GPC from config Bravo system ID, with model config Kilo, at 727 rpm, 138 kts.

The effectiveness of the GPC controller to suppress vibrations and flutter across various velocities, rotor speed, and model configurations is unique to this research. Recall that changing the pitch spring changes the frequency of the nacelle modes and the associated flutter boundary, yet the same GPC controller parameters are still stable across this configuration change.

To further investigate controller robustness, a comparison of controller performance between configurations India and Bravo was conducted. The primary change between these configurations is the difference in δ_3 angle, or pitch-flap coupling, for which the flutter boundary is very sensitive. Recall, the open-loop free response following a cyclic stir of configuration Bravo at 909 rpm and 100 knots was shown in

Figure 12. The closed-loop GPC suppression of this modal excitation of configuration Bravo using the system ID acquired at this same model configuration and tunnel condition was shown in Figure 13 and is presented here again as Figure 16. Comparatively, the closed-loop GPC suppression of this modal excitation of configuration Bravo using a system ID acquired with configuration India at 110 knots and 909 rpm is shown in Figure 17. Surprisingly, the performance of the controller is nearly identical even with a significant model change.

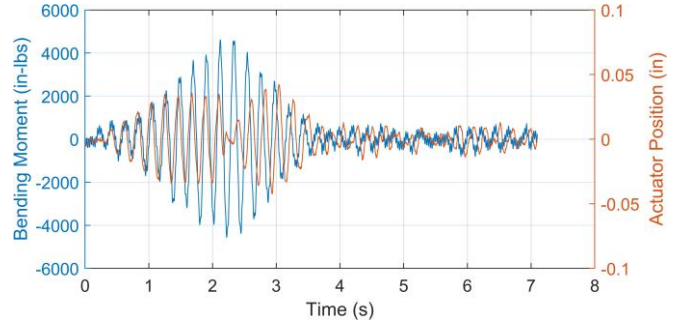


Figure 16. Modal suppression using GPC with same config as that used to derive the system ID, config Bravo, at 909 rpm, 100 kts.

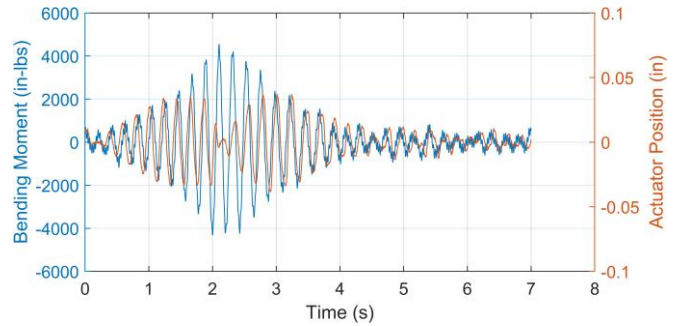


Figure 17. Modal suppression using GPC from config India system ID, with model config Bravo, at 909 rpm, 100 kts.

GPC performance was also investigated across configurations where the fiberglass rotor blades were replaced with carbon fiber blades thereby affecting the regressive lag mode of the rotor system. Recall from Table 1 that configuration Hotel utilized carbon fiber blades and configuration India utilized fiberglass blades. The open-loop free response following a cyclic stir of configuration India at 727 rpm and 120 knots is shown in Figure 18. The GPC suppression of this modal excitation of configuration India using the system ID acquired at this same model configuration and tunnel condition is shown in Figure 19. Similarly, the GPC suppression of this modal excitation of configuration India using a system ID acquired with configuration Hotel at 110 knots and 727 rpm is shown in Figure 20. The ability of the GPC controller to suppress modal vibration is nearly the same even with the different system ID acquired with carbon fiber blades. These data were acquired prior to the implementation of the notch filter and actuator noise is evident in Figure 20.

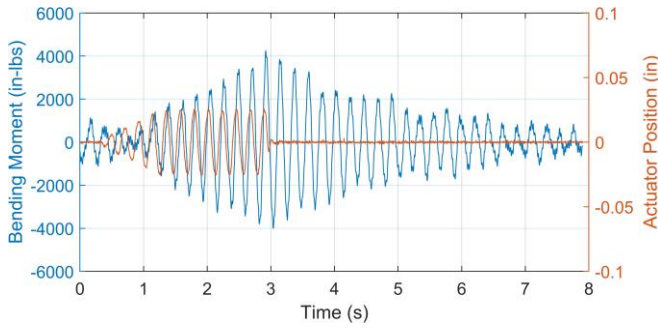


Figure 18. Free response following out-of-plane bending excitation, config India, 727 rpm, 120 kts.

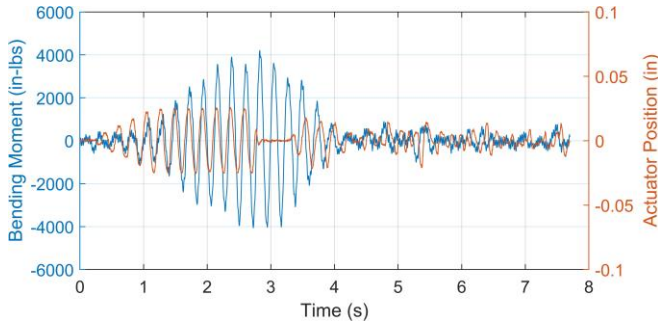


Figure 19. Modal suppression using GPC with same config as that used to derive the system ID, config India, at 727 rpm, 120 kts.

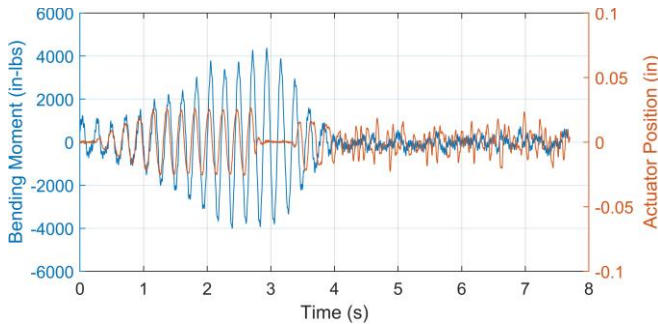


Figure 20. Modal suppression using GPC from config Hotel system ID, with model config India, at 727 rpm, 120 kts.

One may notice the differences in the beginning cycles of the time histories shown in Figure 18 through Figure 20. In one instance there is minimal vibration of the system and in the others, there is a notable vibration prior to the start of the cyclic stick stir. Additionally, the stir is initially out of phase with the vibration in Figure 18 requiring the excitation to last longer in order to build the modal response. In latter iterations of testing, GPC was also used to suppress vibration prior to the start of a modal excitation in order to improve consistency between data records.

The robustness of the GPC controller was further investigated across additional model configurations, rotor speeds, and tunnel velocities with a fixed system ID. A compilation of modal damping for the out-of-plane bending response is shown in Figure 21. The system ID used for all data records with GPC active was acquired with

configuration Bravo at 727 rpm and 100 knots. Modal damping was determined using an automated analysis of both the post-excitation free response and the GPC-augmented response, where a novel non-stationary signal processing method identified the appropriate time segments for each response type and determined their damping ratio (Ref. 16). Damping values with GPC engaged are denoted with a red circle and the free responses are denoted with a blue square. When the damping crosses zero and becomes negative, this is indicative of a flutter instability. The first encounter of flutter for each configuration is denoted with a vertical red line in Figure 21. As one can see, the changes in model configuration and rotor speed affect the flutter boundary significantly and they range from as low as 61 knots to as high as 147 knots, resulting in a factor of six difference in dynamic pressure. Yet, the same GPC controller with the same system ID (with the same gains, and weighting matrices) was able to stabilize the model in all cases. There was not a configuration, rotor speed, or velocity encountered during testing where GPC with this system ID did not work. While this level of robustness surprised the test team, the fundamental flutter mechanism (out-of-plane bending coupled with a nacelle pitch motion) and rotor controls of a tiltrotor aircraft are the same. Therefore, model configuration and tunnel test condition changes manifest as system phase and magnitude response changes. As demonstrated in Ref. 8, the magnitude and phase margins of GPC are much larger than traditional controllers because of the ability to utilize a time-history of past response and control inputs.

In many of the configurations presented in Figure 21, data were acquired far beyond the initial flutter encounter. As one can observe, some of these data showed a stable model condition, even for open-loop free response well beyond the initial flutter encounter. Recall as mentioned earlier, the stability boundary was found to be sensitive to the magnitude of excitation resulting from the non-linearities associated with proprotor aeroelastic behavior. Larger amounts of excitation were often less stable. Additionally, the model did not always behave in the same fashion with repeat data records. In some instances, the oscillations would damp out and in other instances they would not, as illustrated by the scatter in data shown in Figure 21. Regardless of this scatter, the performance of the controller to add damping and arrest an instability in all configurations at all velocities is clear.

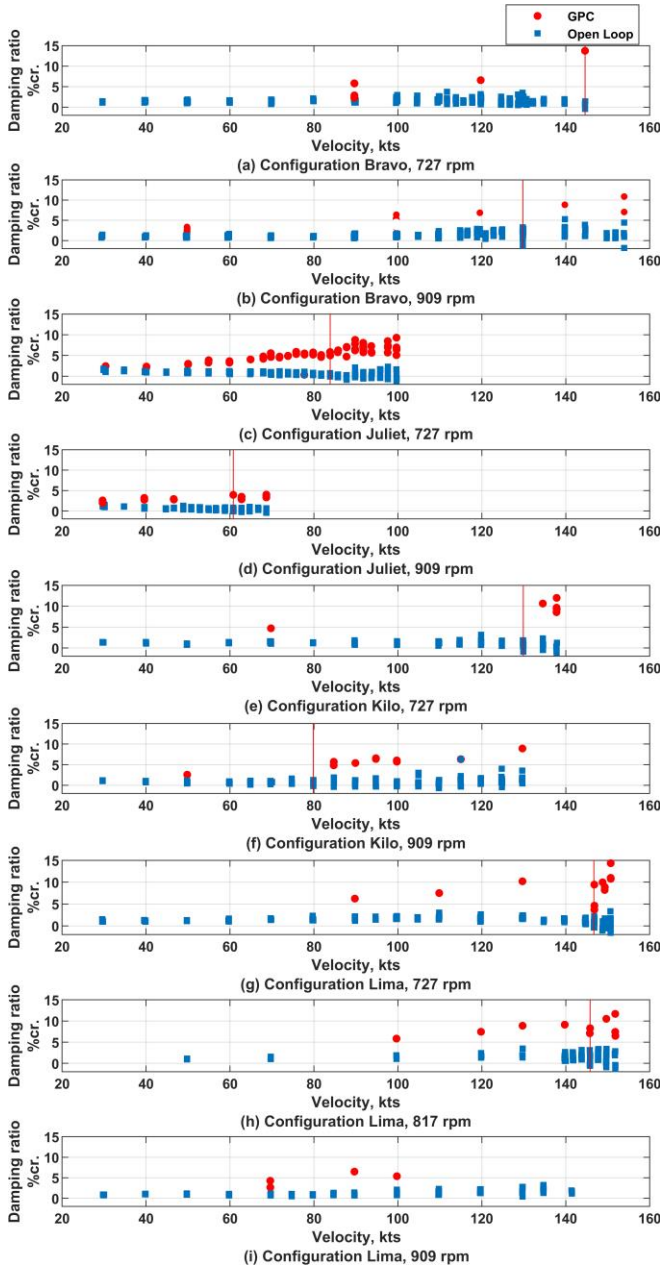


Figure 21. Damping of out-of-plane bending mode for multiple configurations, GPC using system ID from config Bravo at 727 rpm, 100 kts.

Advanced GPC Applied to TRAST

Similar to the investigations presented in Refs. 7-9, GPC effectiveness and the ability of AGPC to improve controller performance was evaluated. However, the performance of the traditional GPC control law was better than expected across a wide range of conditions. This made it difficult to evaluate AGPC since improving a controller that is already working well is difficult. Regardless, insights to AGPC performance can be made.

The open-loop steady response of the model for configuration India at 909 rpm and 110 knots is shown in Figure 22. The vibration and dynamic response of the model in this figure is the baseline vibration of the model at this stable condition. For comparison, the response of the model during an excitation to acquire a traditional system ID is shown in Figure 23 for this same configuration at this same tunnel condition. In order to characterize the input-output relationship, the model had to be excited above the baseline noise floor, and the increase in response is readily apparent in Figure 23 by comparison to Figure 22.

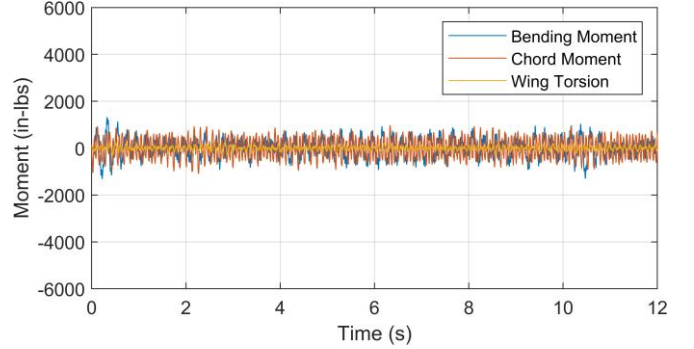


Figure 22. Open-loop steady-state response, config India, at 909 rpm, 110 kts.

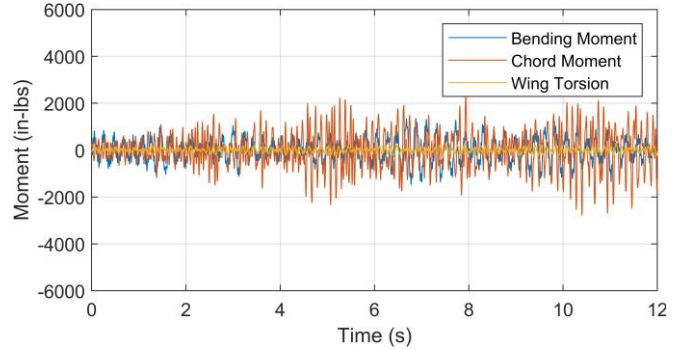


Figure 23. Traditional system ID, config India, at 909 rpm, 110 kts.

By comparison, an AGPC adaptation requires only a passive observer and does not require the excitation of the model beyond the open-loop noise floor. The time history of model response during an adaptive AGPC system ID update is shown in Figure 24. During this data record, the GPC algorithm is active using the system ID acquired during the data record shown in Figure 23. Additionally, dither was added to the control input in order to increase orthogonality to acceptable levels measured by the rank of the objective function and the magnitudes of the pairwise correlation coefficients. Even with dither added, the response of the system during an AGPC system ID update is below the baseline noise level of the system as seen by comparison of Figure 24 with Figure 22. The significance of this demonstration is that AGPC can continue to operate to suppress vibration and receive a system ID update with only negligible excitation making its use acceptable as a continuous vibration suppression system. Additionally, a

controller can be active and beyond a stability boundary while simultaneously updating the system ID thereby improving performance and theoretically enabling further expansion beyond a stability boundary where the controller would otherwise breakdown in effectiveness. This further expansion in boundary would not be possible with traditional system ID methods as one could not acquire a traditional system ID while open loop at an unstable condition.

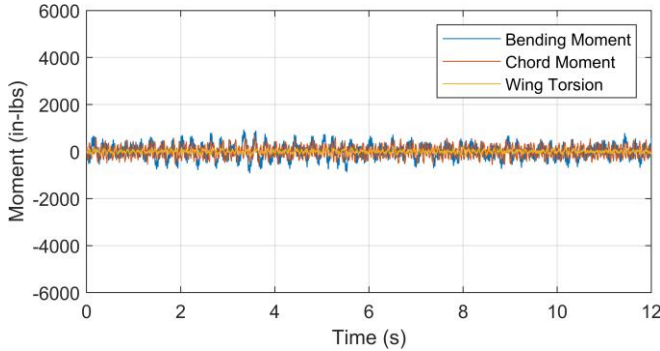


Figure 24. AGPC system ID, with dither applied, config India, at 909 rpm, 110 kts.

To demonstrate the performance of the traditional GPC and AGPC controllers for this configuration, multiple time histories of out-of-plane bending moment are superimposed from multiple data records as shown in Figure 25. The responses include open-loop and closed-loop out-of-plane bending moment response with traditional GPC and AGPC as indicated. These data were acquired at the same model condition as that used for the system ID and were windowed such that they are synchronized with the cessation of the stick-stir excitation. At this condition, the model is very stable, and the open-loop response dissipates quickly. For this condition, GPC offers minimal benefit but does show a measurable increase in damping. The AGPC controller produced better metrics, but the time history performance at this condition is comparable to the traditional GPC controller. Recall, it is the intent of AGPC to improve or “fix” a poorly working controller as the model or conditions change, and one should not expect a significant improvement when traditional GPC is performing well. Traditional GPC performed far better than anticipated and a condition where the traditional GPC controller performance started to deteriorate to highlight the benefit of AGPC was not encountered during this present investigation like it was in previous experimental testing (Ref. 9).

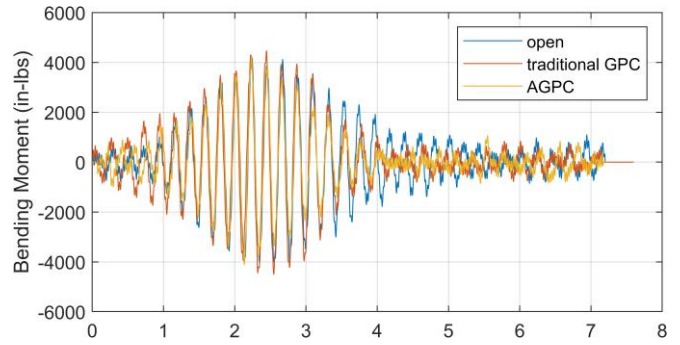


Figure 25. Overlay of open-loop free response, response with traditional GPC, and response with AGPC following excitation, config India, at 909 rpm, 110 kts.

To illustrate the ability of the AGPC controller to also stabilize an instability, the time history of bending moment response and actuator position is shown in Figure 26 for a flutter instability. This data record was acquired with configuration Golf at 909 rpm and 92 knots. As one can observe, the controller quickly suppresses the instability with minimal workload. It is not as aggressive as the controller used in Figure 14 or Figure 15, but the data record in Figure 26 is prior to the utilization of the notch filter and traditional GPC was also not as aggressive prior to notch filter use.

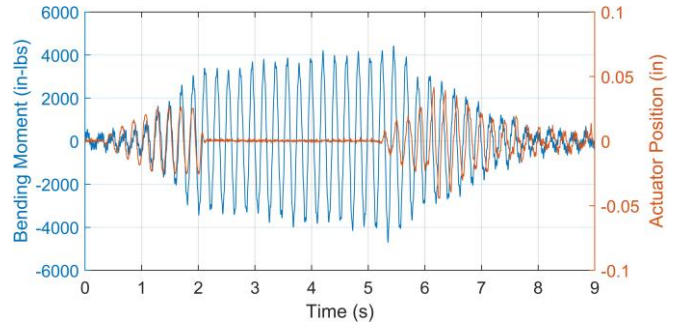


Figure 26. Flutter suppression with AGPC, config Golf, at 909 rpm, 92 kts.

CONCLUSIONS

GPC and AGPC were able to consistently and reliably stabilize the TRAST model. This capability enabled the model to be tested beyond the flutter boundary and reduced vibration at subcritical velocities by increasing modal damping. The use of orthogonal multisine excitation during the system ID provided slightly better performance of the controller than when random noise was used during the system ID. Also, the use of the roving notch filter made a significant increase to the robustness of the controller by filtering $1/rev$ vibration caused by the rotor imbalance that the controller could not influence. With these adjustments, traditional GPC demonstrated a robustness that far exceeded expectations. A single system ID was able to reliably stabilize the model over a wide range of wind-tunnel

conditions and model configuration changes. GPC, using a single system ID and weight settings, was effective despite changes in rotor speed, blade stiffness, pitch-flap coupling angle, and even with spring rate and natural frequency changes. There were no test conditions reached where the controller using this system ID was unstable. Finally, because of the reliability and stability of GPC, the test team was able to quickly suppress instabilities and could continue testing without changing model or wind-tunnel condition. Typically, when an instability is reached the test team reduces tunnel speed and rotor speed to stabilize the system. It then takes time to return to condition to acquire another data record. Using GPC in lieu of traditional methods enabled the completion of all primary, secondary, and tertiary test objectives so that the test team completed approximately three times as many configurations and test conditions than anticipated for this tunnel entry. It also enabled the acquisition of riskier data to investigate the effectiveness of excitation amplitude. Finally, AGPC was evaluated in the current study. Due to the improvements in robustness of traditional GPC, AGPC was not used to improve an otherwise degraded controller. However, it was demonstrated to be effective at vibration reduction and stabilization of flutter. It was also shown that the adaptive system ID can occur with the controller active and system vibration below the baseline noise level.

Author contact:

Thomas G. Ivanco, thomas.g.ivanco@nasa.gov
 Robert P. Thornburgh, robert.p.thornburgh.civ@army.mil
 Andrew R. Kreshock, andrew.r.kreshock.civ@army.mil
 Martin K. Sekula, martin.k.sekula@nasa.gov

ACKNOWLEDGMENTS

The authors would like to acknowledge the staff of the Transonic Dynamics Tunnel and the NASA Revolutionary Vertical Lift Technology (RVLT) Project who assisted in the setup and conduct of the experimental test. Additionally, the authors would like to acknowledge the expertise and guidance of Dr. Brett Newman from Old Dominion University, Norfolk, VA. Without the combined efforts of all involved, this work would not have been successful.

REFERENCES

- 1 Kreshock, A. R.; Thornburgh, R. P.; Wilbur, M. L. "Overview of the TiltRotor Aeroelastic Stability Testbed," In *AIAA SCITECH Forum*; American Institute of Aeronautics and Astronautics: San Diego, CA & Virtual, 2022. <https://doi.org/10.2514/6.2022-0566>.
- 2 Clarke, D. W.; Mohtadi, C.; Tuffs, P. S. Generalized Predictive Control—Part I. The Basic Algorithm. *Automatica* 1987, 23 (2), 137–148. [https://doi.org/10.1016/0005-1098\(87\)90087-2](https://doi.org/10.1016/0005-1098(87)90087-2).
- 3 Juang, J.-N.; Phan, M. "Deadbeat Predictive Controllers; NASA TM 112862," NASA TM 112862, NASA Langley Research Center, Hampton, VA, 1987. <https://ntrs.nasa.gov/citations/19970027551>.
- 4 Kvaternik, R. G.; Juang, J.-N.; Bennett, R. L. "Exploratory Studies in Generalized Predictive Control for Active Aeroelastic Control of Tiltrotor Aircraft," NASA TM 2000–210552; NASA Langley Research Center, Hampton, VA, 2000. <https://ntrs.nasa.gov/citations/20000115608>.
- 5 Kvaternik, R. G.; Piatak, D. J.; Nixon, M. W.; Langston, C. W.; Singleton, J. D.; Bennett, R. L.; Brown, R. K. An Experimental Evaluation of Generalized Predictive Control for Tiltrotor Aeroelastic Stability Augmentation in Airplane Mode of Flight. *Journal of the American Helicopter Society* 2002, 47 (3), 198–208. <https://doi.org/10.4050/JAHS.47.198>.
- 6 Kvaternik, R. G. "Generalized Predictive Control of Dynamic Systems with Rigid-Body Modes," NASA TM 2013–217976, NASA Langley Research Center, Hampton, VA, 2013.
- 7 Ivanco, T. G.; Kang, H.; Kreshock, A. R.; Thornburgh, R. P.; Newman, B. A. "Generalized Predictive Control for Active Stability Augmentation and Vibration Reduction on an Aeroelastic Tiltrotor Model," In *AIAA SCITECH Forum*; American Institute of Aeronautics and Astronautics: San Diego, CA & Virtual, 2022. <https://doi.org/10.2514/6.2022-0570>.
- 8 Ivanco, T. G.; Newman, Brett A.; Alberts, Thomas E.; Bawab, Sebastian Y.; Baysal, Oktay; Silva, Walter A. Advanced Generalized Predictive Control and Its Application to Tiltrotor Aircraft for Stability Augmentation and Vibration Reduction, Dissertation, Old Dominion University Libraries, Norfolk, VA, 2022. <https://doi.org/10.25777/QBQQ-EZ94>.
- 9 Ivanco, T. G.; Kreshock, A. R.; Thornburgh, R. P.; Wilbur, M. L.; Newman, B. A. "Experimental Testing of Advanced Generalized Predictive Control for Stability Augmentation and Vibration Reduction of Tiltrotor Aircraft," In *The Vertical Flight Society 79th Annual Forum*; The Vertical Flight Society: West Palm Beach, FL, 2023; pp 1–21. <https://doi.org/10.4050/F-0079-2023-18049>.
- 10 Klein, V.; Morelli, E. A. *Aircraft System Identification: Theory and Practice*; American Institute of Aeronautics and Astronautics, 2006.
- 11 Ivanco, T. G. "Unique Testing Capabilities of the NASA Langley Transonic Dynamics Tunnel, an Exercise in Aeroelastic Scaling," In *AIAA Ground Testing Conference*; American Institute of Aeronautics and Astronautics: San Diego, CA, 2013. <https://doi.org/10.2514/6.2013-2625>.
- 12 Heeg, J.; Morelli, E. A. "Evaluation of Simultaneous Multisine Excitation of the Joined Wing Aeroelastic Wind Tunnel Model," In *52nd AIAA/ASME/ASCE/AHS/ASC Structures, Structural*

Dynamics and Materials Conference; American Institute of Aeronautics and Astronautics: Denver, Colorado, 2011. <https://doi.org/10.2514/6.2011-1959>.

- 13 Morelli, E. A. Optimal Input Design for Aircraft Stability and Control Flight Testing. *Journal of Optimization Theory and Applications* 2021, 191 (2–3), 415–439. <https://doi.org/10.1007/s10957-021-01912-0>.
- 14 Simmons, B. M. "Efficient Variable-Pitch Propeller Aerodynamic Model Development for Vectored-Thrust eVTOL Aircraft," In *AIAA AVIATION Forum*; American Institute of Aeronautics and Astronautics: Chicago, IL & Virtual, 2022. <https://doi.org/10.2514/6.2022-3817>.
- 15 Simmons, B. M. "Evaluation of Response Surface Experiment Designs for Distributed Propulsion Aircraft Aero-Propulsive Modeling," In *AIAA SCITECH Forum*; American Institute of Aeronautics and Astronautics: National Harbor, MD & Online, 2023. <https://doi.org/10.2514/6.2023-2251>.
- 16 Kreshock, A. R.; Thornburgh, R. P.; Wilbur, M. L.; Kang, H.; Piatak, D. J.; Sekula, M. K. "Initial Whirl-Flutter Characterization of the TiltRotor Aeroelastic Stability Testbed," In *The Vertical Flight Society 79th Annual Forum*; The Vertical Flight Society: West Palm Beach, FL, 2023; pp 1–18. <https://doi.org/10.4050/F-0079-2023-18037>.