

Adaptive Atmospheric Entry Guidance via Neural Networks

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A common control strategy for atmospheric entry involves guiding the vehicle by modulating its bank angle. Bank maneuvers can be determined using predictor-corrector algorithms, where the current state is propagated to a final energy, and the distance between the target location and the propagated position is minimized by finding the optimal bank angle. Uncertainty in atmospheric density can degrade the propagation step, reducing targeting accuracy. This work addresses atmospheric uncertainty by introducing a new adaptive density model based on a recently proposed navigation solution. The navigation solution, originally designed in an open-loop fashion, estimates atmospheric density using an online-adapting neural network to correct for discrepancies between the onboard nominal model and true atmospheric density. The guidance algorithm uses this neural network, coupled with the nominal onboard model, to propagate the current state to the final target location during the prediction step of the Fully Numerical Predictor-Corrector Entry Guidance (FNPEG) framework. By incorporating this adaptive model, the accuracy of the prediction step is improved, allowing more precise guidance of the vehicle toward its target.

I. Introduction

The primary source of uncertainty during atmospheric entry is the limited knowledge of the true density [1]. Discrepancies between onboard atmospheric density models and the true density can lead to inaccurate state propagation, increasing the difficulty of targeting a final state through navigation and guidance algorithms. Since future robotic and human exploration missions require improvements in landing accuracy due to the need for increased landed mass [2],

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the development of precise entry navigation and guidance is crucial for their success. Addressing the uncertainty in atmospheric density can, therefore, bring improvements to future missions.

Numerical predictor-corrector (NPC) methods are commonly used to solve onboard guidance problems. In these algorithms, the current state is numerically integrated to a final state (prediction step), and a control input is determined by minimizing the error between the final state and a target state (correction step). Different NPC approaches have been applied to atmospheric entry applications, such as PredGuid [3–5] and the Fully Numerical Predictor-Corrector Entry Guidance (FNPEG) [6–8]. FNPEG is a bank control guidance algorithm that decouples longitudinal and lateral channels to target a desired location by adjusting the bank angle of entry vehicles. Since vertical lift is a function of the bank angle, adjusting it allows the vehicle to modify its range by flying through denser or less dense atmospheric conditions [9].

Although FNPEG has been shown to be suitable for entry applications [6–8, 10], this type of framework has certain limitations. An important limitation is that the prediction step assumes that the parameters in the dynamic models are known. This assumption can lead to inaccuracies in the prediction step when flying through uncertain environments, resulting in higher targeting errors. Therefore, the use of NPCs for atmospheric entry requires careful consideration of potential modeling errors. This issue is typically addressed by incorporating first-order fading memory filters to estimate atmospheric quantities such as lift or drag accelerations [10].

A different way to address uncertain environments is by using adaptive estimation [11, 12]. For example, if the navigation scheme incorporates adaptive estimation techniques, the prediction step in NPCs can take advantage of the adapted models instead of relying solely on nominal profiles. In previous work, Kalman filters [13] and recursive neural networks [14, 15] have been used to estimate atmospheric density, demonstrating that incorporating corrected nominal models into the prediction step of NPCs can increase overall targeting accuracy in atmospheric entry applications.

An adaptive filtering technique for atmospheric entry was recently developed, by using a neural network to estimate density onboard [16]. In this approach, a neural network is trained offline on a low-fidelity atmospheric density model, and its weights and biases are adapted online to account for discrepancies between the low-fidelity model and the true atmosphere. The adaptation step is set up as a maximum likelihood problem, minimizing the difference between predicted and true measurements with respect to network parameters. Although this approach was shown to successfully quantify uncertainty in both state estimates and atmospheric density, its derivation was performed in an open-loop framework, disregarding control inputs. This work builds on this estimation technique by developing guidance commands using the adapted atmospheric estimates and testing the filter’s consistency with control inputs.

The purpose of this work is two-fold*. First, we build on the success of the navigation solution presented in previous work [16] by incorporating the adaptive model into the prediction step of FNPEG. This enables FNPEG to improve

*This paper is an extension to the work presented at the 2024 AAS/AIAA Astrodynamics Specialist Conference [17].

targeting accuracy by using the adaptive model instead of relying on a nominal profile. Second, since the navigation solution was developed in an open-loop framework, this work validates the consistency of the estimation technique when applied to scenarios with control inputs. By closing the loop and incorporating bank angle control into the simulation, we demonstrate that the filter maintains consistency. To show this, we use two different atmospheric entry scenarios. The first, a robotic Mars entry mission, highlights the advantages of using the adapted model over nominal models determined offline. The second scenario, resembling an Earth re-entry mission, shows how the proposed adapted model can be combined with state-of-the-art adaptation techniques used in real entry scenarios. In both cases, the targeting and estimation errors are analyzed to evaluate the performance of the guidance algorithm and the consistency of the filter.

The remainder of this paper is organized as follows. Section II provides an introduction to atmospheric entry. The different atmospheric density models used in this work are discussed in Section III. The navigation solution is presented and summarized in Section IV. The new guidance solution, incorporating the adaptive model into the prediction step of FNPEG, is presented in Section V. The proposed technique is evaluated in a Mars entry scenario in Section VI, followed by an Earth re-entry problem in Section VII. Finally, Section VIII provides conclusions and future work.

II. Atmospheric Entry

The entry phase constitutes the longest segment of the entry, descent, and landing (EDL) sequence, thus playing a crucial role in determining landing precision [18]. This section summarizes the equations of motion that govern hypersonic flight and presents some of the sensors available for use during the entry phase, as used in [16].

A. Dynamics

During the entry phase, the entry vehicle can be modeled using a 3-DOF model [19–21]. This model describes the position of the vehicle using the planet-centric radius (r), latitude (ϕ), and longitude (θ) with respect to the planet-centered, planet-fixed frame [7, 18, 22].

$$\dot{r} = v \sin(\gamma), \quad (1)$$

$$\dot{\phi} = \frac{\cos(\gamma) \cos(\psi)}{r} v, \quad (2)$$

$$\dot{\theta} = \frac{\cos(\gamma) \sin(\psi)}{r \cos(\phi)} v. \quad (3)$$

The velocity components of the vehicle are described by the magnitude of its velocity vector (v), flight path angle (γ), and heading azimuth (ψ), relative to the geographic frame [22]. The flight path angle is negative-down, and the heading azimuth is defined by 0° pointing north and 90° pointing east. In the case of short-duration flights, planetary rotation effects may be considered negligible [18]. Therefore, by assuming a first-order gravity model and omitting the

planetary rotation terms, the velocity equations simplify to [22]

$$\dot{v} = -D - \frac{\mu}{r^2} \sin(\gamma), \quad (4)$$

$$\dot{\gamma} = \frac{1}{v} \left[L \cos(\sigma) - \frac{\mu}{r^2} \cos(\gamma) + \frac{v^2}{r} \cos(\gamma) \right], \quad (5)$$

$$\dot{\psi} = \frac{1}{v} \left[L \frac{\sin(\sigma)}{\cos(\gamma)} + \frac{v^2}{r} \cos(\gamma) \sin(\psi) \tan(\phi) \right], \quad (6)$$

Where σ represents the positive-right bank angle, and drag (D) and lift (L) accelerations are defined as [22]

$$D = \frac{1}{2} \rho(\cdot) v_\infty^2 \frac{C_d S}{m}, \quad (7)$$

$$L = \frac{1}{2} \rho(\cdot) v_\infty^2 \frac{C_l S}{m}. \quad (8)$$

Here, $v_\infty = v - W$, with W denoting wind velocity, C_d is the drag coefficient, C_l is the lift coefficient, S represents the area of the vehicle in direct contact with the atmosphere, and m is the mass of the vehicle. The term $\rho(\cdot)$ represents atmospheric density, indicating its dependence on various factors such as altitude. Considering the high speeds involved in the entry phase, wind velocities are significantly lower than the velocity of the vehicle ($W/v \ll 1$), making $v_\infty \approx v$ a valid assumption for most of the flight [14]. Figure 1 shows the position components with respect to the planet-centered, planet-fixed frame, and the velocity components with respect to the geographic frame.

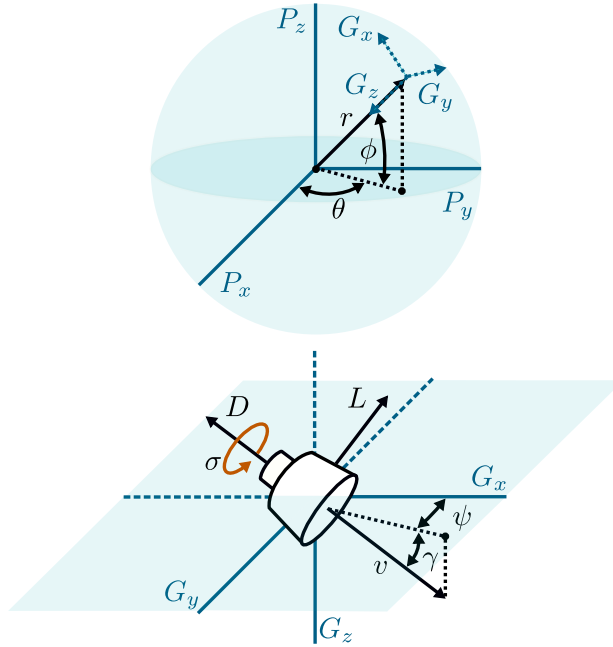


Fig. 1 Position with respect to the planet-centered planet-fixed frame (top), and velocity with respect to the geographic frame (bottom). Figure adapted from Giraldo-Grueso et al. [16]

B. Onboard Sensors

For this work, sensors onboard the spacecraft are used, including an inertial measurement unit (IMU), a cluster of pressure sensors, and a suite of thermocouples. In addition to these, global positioning system (GPS) measurements are used for the Earth re-entry scenario. These sensor selections are based on the configurations used in previous EDL missions [4, 18, 23–25].

The IMU measures non-gravitational accelerations relative to the body frame,

$$\tilde{\mathbf{a}}^b = T_v^b \mathbf{a}^v + \boldsymbol{\eta}_a, \quad (9)$$

where T_v^b is the velocity frame to body frame transformation matrix, $\mathbf{a}^v$ are velocity frame accelerations [18], and $\boldsymbol{\eta}_a$ is measurement noise, with

$$\mathbf{a}^v = \begin{bmatrix} -D & L \sin(\sigma) & L \cos(\sigma) \end{bmatrix}^T. \quad (10)$$

The pressure sensors are assumed to measure the dynamic pressure, defined as [18]

$$\tilde{q} = \frac{1}{2} \rho(\cdot) v^2 + \eta_q, \quad (11)$$

where $\eta_q \in \mathbb{R}$ is the measurement noise.

The thermocouples are assumed to reconstruct convective heating rate from temperature measurements (note that recent EDL missions have used heat flux sensors to directly measure heating rate [26, 27]), defined in this work by the Sutton-Graves relation [28, 29],

$$\tilde{Q}_s = k \left(\frac{\rho(\cdot)}{R_n} \right)^{\frac{1}{2}} v^3 + \eta_{\dot{Q}_s}, \quad (12)$$

where $k = 1.9027 \times 10^{-4} \text{ kg}^{1/2} \text{ m}^{-1}$ for Mars [29], $k = 1.7030 \times 10^{-4} \text{ kg}^{1/2} \text{ m}^{-1}$ for Earth [28] (assuming an atmospheric composition of 0.2320O₂-0.7680N₂), $R_n = 1.125 \text{ m}$ denotes the vehicle nose radius [30], and $\eta_{\dot{Q}_s} \in \mathbb{R}$ represents measurement noise. Although the precision and availability of the pressure and heating rate measurements may vary with flight regime, this work assumes available measurements throughout the entire entry phase.

In the case of Earth re-entry, GPS measurements may be available for altitudes $h \geq 80 \text{ km}$. Here, the GPS is assumed to measure the position of the vehicle in the Earth-Centered, Earth-Fixed (ECEF) frame such that [31]

$$\tilde{\mathbf{r}}_{\text{GPS}} = \begin{bmatrix} r \cos(\phi) \cos(\theta) & r \cos(\phi) \sin(\theta) & r \sin(\phi) \end{bmatrix}^T + \boldsymbol{\eta}_r, \quad (13)$$

where $\boldsymbol{\eta}_r \in \mathbb{R}^3$ represents the associated measurement noise.

III. Atmospheric Density Models

For entry missions, different approaches can be used to model atmospheric density, ranging from highly accurate models to simpler, less precise equations. In practice, the true atmospheric density is unknown, which means that the onboard computer does not have direct access to this exact information. This section presents the atmospheric density models used in this work. A simple exponential model is used as the nominal profile to represent the approximate information available pre-flight, while the NASA Global Reference Atmospheric Models (GRAMs) are used to simulate the true atmospheric density.

It is important to note that this setup is a simplification compared to real missions, where the nominal profile is typically more accurate than an exponential model, and the true atmosphere can behave differently than the GRAMs profiles. However, maintaining a consistent gap between the onboard model and the true atmospheric conditions reflects the uncertainty that can exist in real missions.

A. Exponential model

Modeling atmospheric density with an exponential model assumes isothermal equilibrium [32]. Given an atmosphere in isothermal equilibrium, the atmospheric density can be described as

$$\hat{\rho}_{\text{exp}} = \rho_0 \exp\left(-\frac{(r - r_0)}{h_s}\right), \quad (14)$$

where ρ_0 is the density at the reference radial position, r_0 is the reference radial position, and h_s is the atmospheric scale height. The model defined by (14) is used in this work as the pre-flight nominal atmospheric density.

B. Global Reference Atmospheric Models

GRAMs represent a significant improvement in atmospheric modeling compared to other approaches. These atmospheric models were initially developed to address the need for a reference atmosphere that offers geographical variation, coverage from surface to orbital altitudes, and seasonal and monthly variations of thermodynamic variables and wind components [33]. In addition, GRAMs have the ability to simulate spatial and temporal disturbances, which can be used to test the robustness of navigation and guidance solutions. Different GRAMs exist for various celestial bodies, such as Mars and Earth.

Mars-GRAM has been developed using data from the University of Michigan Mars Thermospheric General Circulation Model (MTGCM) and the NASA Ames Mars General Circulation Model (MGCM). This model has been validated against Radio Science and Thermal Emission Spectrometer (TES) data [34]. Here, Mars-GRAM is used to generate the true trajectories in the robotic Mars entry scenario.

Earth-GRAM has been constructed using a collection of empirical models that accurately represent atmospheric

density across different altitude ranges. For the lower atmosphere, Earth-GRAM uses data from the National Centers for Environmental Prediction (NCEP). For higher altitudes, it uses data from the Middle Atmosphere Program (MAP) [33]. For this work, Earth-GRAM is used to generate the true trajectories in the Earth re-entry scenario.

IV. Navigation

This section provides a summary of the adaptive filtering strategy in [16]. This summary is included in order for this work to be self-contained, but readers are encouraged to refer to the original paper for a more detailed explanation. This adaptive filtering technique consists of two stages: offline training and online adaptation.

A. Offline Training

A neural network is first trained to approximate density with respect to planet-centric radius, such that

$$\hat{\rho} = \mathcal{NN}(r, \xi), \quad (15)$$

where ξ are the parameters of the network. To train the network, training data is generated by using a least-squares exponential fit derived from Mars-GRAM or Earth-GRAM profiles, depending on the entry scenario. Using this exponential density model, a set number of entry trajectories are simulated and the planet-centric radius and atmospheric density are recorded. The network is then trained to match the density profile provided by the exponential model.

Note that this training setup is designed to be independent of a specific neural network architecture and pre-flight nominal model. If the network were to be trained with a higher fidelity nominal model (for example, including latitude and longitude dependence), obtaining training data from propagated trajectories ensures that the dataset is representative of the nominal model within the approximate entry trajectory. However, since the nominal model in this work is only a function of planet-centric radius, generating training data using propagated trajectories is equivalent to querying the exponential model at different altitudes along the approximate entry trajectory.

B. Online Adaptation

Since the exponential fit does not fully capture the true atmospheric density, which in this work is represented by the GRAMs profiles, the neural network is adapted online to better approximate the true density. Given the dynamics in Sec. II.A, the filter state is defined as

$$\mathbf{x} = [r \quad \phi \quad \theta \quad v \quad \gamma \quad \psi \quad B \quad L/D]^T, \quad (16)$$

where L/D is the lift-to-drag ratio and B is the inverse of the ballistic coefficient, both modeled as a random walk [18].

The measurement vector is defined as

$$\mathbf{y} = \left[\tilde{\mathbf{a}}^b \quad \tilde{q} \quad \tilde{Q}_s \mid \tilde{\mathbf{r}}_{\text{GPS}} \right]^T, \quad (17)$$

with the GPS measurement only available for the Earth re-entry problem when the altitude is greater than 80 km. Let the measurement model be denoted as

$$\mathbf{y}_k = h(\mathbf{x}_k, \rho_k) + \boldsymbol{\eta}_k, \quad (18)$$

where h refers to the measurement model of the available sensors, and $\boldsymbol{\eta}_k$ is white Gaussian measurement noise with covariance matrix R .

When a measurement ($\mathbf{y}_k$) is obtained, the log likelihood of the measurement is considered as a loss function for the parameters of the neural network, such that

$$\mathcal{L}(\xi_{k-1}) = \left[\mathbf{y}_k - h(\hat{\mathbf{x}}_k^-, \hat{\rho}_{k-1}) \right]^T R^{-1} \left[\mathbf{y}_k - h(\hat{\mathbf{x}}_k^-, \hat{\rho}_{k-1}) \right], \quad (19)$$

$$\hat{\rho}_{k-1} = \mathcal{NN}(\hat{\mathbf{x}}_k^-(1), \xi_{k-1}). \quad (20)$$

where $\hat{\mathbf{x}}_k^-$ is the prior state estimate obtained via an unscented Kalman filter [35] propagation augmented with a consider parameter [36, 37] to account for the uncertainty of the atmospheric density estimate. Therefore, the optimal network parameters are found by

$$\xi_k = \arg \min_{\xi_{k-1}} \mathcal{L}(\xi_{k-1}). \quad (21)$$

To minimize the loss function, the Adam optimizer [38] and back-propagation are used. Once the adapted parameters of the neural network have been obtained, a Schmidt-Kalman update is performed to obtain a posterior state estimate [36, 37, 39].

V. Guidance

Achieving high targeting accuracy during entry is fundamental for the descent and landing phases. Bank control has been used as a guidance strategy for maneuvering the vehicle during entry. Although various entry-guidance algorithms have been proposed, the Fully Numerical Predictor-Corrector Entry Guidance (FNPEG), introduced by Lu [7], has been widely discussed in the literature. This section summarizes FNPEG and introduces the new guidance solution incorporating the adaptive density model into the prediction step of FNPEG.

FNPEG is an entry guidance algorithm that determines the optimal bank angle of an entry vehicle given a final target state [7]. Since vertical lift is determined by the bank angle, adjusting the bank angle allows the vehicle to

modify its range by flying through denser or less dense atmospheric conditions [9]. FNPEG uses an NPC framework to calculate the optimal bank angle, aiming to minimize the range-to-go along the great-circle connecting the final location of the vehicle and the target location, at a set final energy. In FNPEG, the longitudinal and lateral channels are decoupled, which means that the magnitude of the bank angle can be calculated based on longitudinal equations. Once the magnitude of the bank angle is determined, the lateral channel determines the correct sign to bound the lateral error within a user-specified threshold [7].

A. Longitudinal Channel

Under the assumption that the lateral error is small, the magnitude of the bank angle is determined by minimizing the final range-to-go from a specified target latitude and longitude ϕ^*, θ^* . The final range-to-go is obtained by integrating the current state up to the final normalized energy e^* (prediction step), predetermined by a reference trajectory,

$$e^* = \frac{1}{r_n^*} - \frac{v_n^*}{2}, \quad (22)$$

where r_n^* and v_n^* are the normalized target planet-centric radius and velocity, respectively. In this work, to calculate the final range-to-go, the reduced and normalized longitudinal equations are integrated with energy as the independent variable such that [6]

$$\frac{dr_n}{de} = \frac{\sin(\gamma)}{D_n}, \quad (23)$$

$$\frac{d\gamma}{de} = \frac{1}{D_n v_n^2} \left[L_n \cos(\sigma) + \left(v_n^2 - \frac{1}{r_n} \right) \left(\frac{\cos(\gamma)}{r_n} \right) \right], \quad (24)$$

$$\frac{ds}{de} = -\frac{\cos(\gamma)}{r_n D_n}, \quad (25)$$

where $v_n = \sqrt{2(1/r_n - e)}$ and s is the range-to-go. The normalized variables are defined as

$$r_n = \frac{r}{r_p}, \quad (26)$$

$$v_n = \frac{v}{\sqrt{r_p g_p}}, \quad (27)$$

$$L_n = \frac{L}{g_p}, \quad (28)$$

$$D_n = \frac{D}{g_p}, \quad (29)$$

which follows from normalizing time by $\sqrt{r_p/g_p}$, that is, $\tau = t/\sqrt{r_p/g_p}$. For these equations, r_p refers to the mean radius of Mars or Earth, and g_p is the gravitational acceleration at r_p . With this setup, the bank angle is found by

minimizing the square of the integrated final range-to-go s_f^* (correction step),

$$\sigma^* = \arg \min_{\sigma} \frac{1}{2} s_f^* (e^*, \sigma)^2. \quad (30)$$

In addition, a linear parameterization of the bank angle profile with respect to energy is used in the prediction step to increase the robustness of the algorithm, following [6, 7]:

$$\sigma = \sigma^* + \frac{e - e_0}{e^* - e_0} (\sigma_f - \sigma^*), \quad (31)$$

where e_0 is the current energy, e is the energy along the trajectory, and σ_f is a user-defined parameter. Note that other parameterizations have also been studied, such as using exponential [40] or logistic [41] functions, or parameterizing the bank angle with respect to the range [10].

To find σ^* , the Newton-Raphson algorithm is used, where

$$\sigma_{i+1} = \sigma_i - \lambda_i \frac{s_f^* (e^*, \sigma_i)}{\partial s_f^* (e^*, \sigma_i) / \partial \sigma}, \quad (32)$$

with $\partial s_f^* (e^*, \sigma_i) / \partial \sigma$ calculated using numerical differentiation and λ_i is chosen to satisfy the Armijo condition [42]. The algorithm stops when $|s_f^* (e^*, \sigma_i)|$ reaches a user-specified threshold or when a set number of iterations have been completed.

B. Lateral Channel

The longitudinal channel determines the magnitude of the bank angle, as the longitudinal equations are only a function of the cosine of this angle. To determine the correct sign, the lateral logic described by Smith [43] is adopted. Once the magnitude of the bank angle is determined, the current state is integrated over time using the full 3-DOF equations, considering both signs of the bank angle, until the final target energy is reached. Once the final state is obtained for both solutions, the final crossrange error is calculated for each. Let χ_f^+ represent the final crossrange error for the solution integrated with the current bank angle sign, and let χ_f^- represent the final crossrange error for the solution integrated with the opposite bank angle sign. Then, if the ratio of the crossrange errors is greater than a threshold K , such that

$$\left| \frac{\chi_f^+}{\chi_f^-} \right| > K, \quad (33)$$

a bank reversal is commanded. A low value of K will result in more bank reversals, which can improve the robustness to prediction errors. However, too many bank reversals can substantially increase propellant consumption [43].

Once the magnitude and sign of the bank angle have been determined, the vehicle flies that bank angle until the next

guidance call, where the same process is repeated. This continues until the vehicle reaches the final energy.

C. Spherical Geometric Definitions

As FNPEG requires the calculation of crossrange error, range-to-go, and, for validation purposes, downrange error, it is important to address how these quantities are calculated in this work. Figure 2 shows the geometric definitions of these errors with respect to the planetary surface [8].

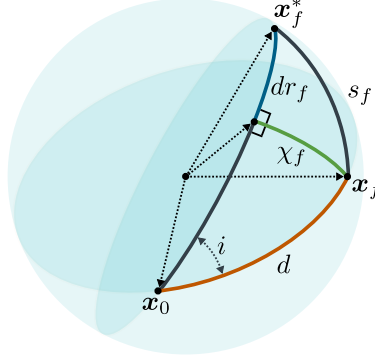


Fig. 2 Final crossrange error, downrange error, and range-to-go for the entry problem. Figure adapted from Lee et al. [8]

Using the spherical law of sines, it can be seen that the final crossrange error can be calculated as

$$\chi_f = \arcsin(\sin(i) \sin(d)), \quad (34)$$

where d is great-circle distance between the initial state and the final state, defined as

$$d = \arccos(\sin(\phi_0) \sin(\phi_f) + \cos(\phi_0) \cos(\phi_f) \cos(\theta_f - \theta_0)), \quad (35)$$

and i is the angle between d and the arc connecting the initial state and the target state,

$$i = \arccos\left(\left[\frac{\mathbf{x}_0 \times \mathbf{x}_f}{\|\mathbf{x}_0 \times \mathbf{x}_f\|}\right] \cdot \left[\frac{\mathbf{x}_0 \times \mathbf{x}_f^*}{\|\mathbf{x}_0 \times \mathbf{x}_f^*\|}\right]\right). \quad (36)$$

The final downrange error is calculated by using the spherical law of cosines such that

$$dr_f = s_0 - \arccos\left(\frac{\cos(d)}{\cos(\chi_f)}\right), \quad (37)$$

where s_0 is the initial range-to-go,

$$s_0 = \arccos\left(\sin(\phi_0) \sin(\phi_f^*) + \cos(\phi_0) \cos(\phi_f^*) \cos(\theta_f^* - \theta_0)\right). \quad (38)$$

And the final range-to-go is calculated as

$$s_f = \arccos \left(\sin(\phi_f) \sin(\phi_f^*) + \cos(\phi_f) \cos(\phi_f^*) \cos(\theta_f^* - \theta_f) \right). \quad (39)$$

Note that the calculated final range-to-go will only yield positive values. Therefore, to assess trajectory overshooting, the downrange error serves as a more informative metric. Furthermore, since all angles are expressed in radians, these quantities can be multiplied by the mean radius of the planet to quantify the distance dispersion. It is also important to distinguish between s_f and s_f^* . While s_f is determined using the geometric definitions presented, s_f^* is obtained by integrating longitudinal dynamics.

With these definitions, Fig. 3 illustrates the longitudinal and lateral channels in FNPEG, where the algorithm aims to minimize the final range-to-go while maintaining the final crossrange error within a specified tolerance.

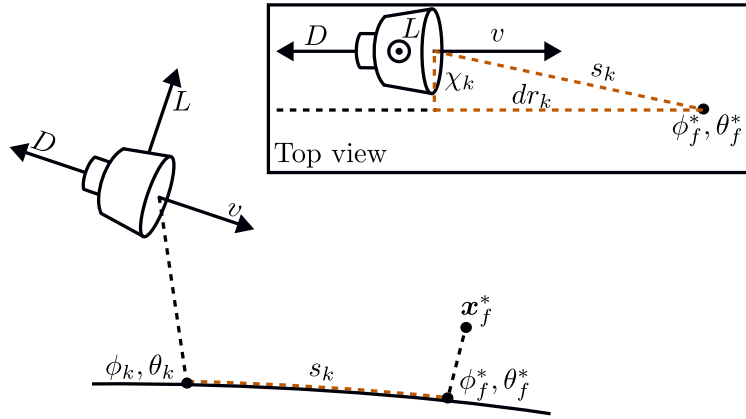


Fig. 3 Longitudinal and lateral channels in FNPEG.

D. Propagating the Trajectory

The guidance solution described involves propagating the current state estimate to a final state defined by the target energy. Therefore, the novelty of this work relies on the atmospheric density model used to propagate the guidance trajectories. The neural network used in the navigation filter is consistently being adapted to appropriately fit the atmospheric density within a local region of the filter's current state. In contrast, the exponential fit, used to train the initial neural network, is relatively precise at lower altitudes, as the exponential fit inherently aims to minimize errors in this region. Thus, for the prediction step (and lateral channel propagation), this work uses a linear combination of the neural network output and the exponential model, formulated as follows

$$\rho(r, e) = \alpha(e) \mathcal{NN}(r, \xi) + [1 - \alpha(e)] \rho_{\text{exp}}(r), \quad (40)$$

where α is an exponentially decaying factor with respect to the energy defined as

$$\frac{d\alpha}{de} = -\kappa \frac{\alpha}{v_n}, \quad (41)$$

with κ controlling the exponential decay rate. Equivalently,

$$\frac{d\alpha}{dt} = -\kappa \frac{\alpha D}{\sqrt{r_p} g_p}. \quad (42)$$

For every propagation $\alpha(e_k) = 1$, indicating that α always starts equal to one at the current energy. This implies that a high value of κ will result in a very rapid exponential decay of α , suggesting that the guidance algorithm will rely more on the exponential atmospheric model than on the neural network. In contrast, lower values of κ will make the guidance algorithm trust the neural network for a longer duration along the propagation of the trajectory. This linear combination has the added benefit of modularity, as the exponential model can easily be replaced with a different model, such as a corrected nominal used in real missions such as Orion skip re-entry [5].

VI. Mars Entry

In Mars EDL, the entry phase begins approximately 125 km above the Martian surface, when the vehicle first enters the atmosphere, and ends with the deployment of the parachute approximately 10 km above the surface [18]. This section presents a Monte Carlo test for a robotic Mars entry mission. The test involves simulating 1,000 different trajectories, where the state is estimated using the presented navigation scheme, and the guidance commands are calculated using FNPEG and the new adaptive density model. Each true trajectory is simulated using a different Mars-GRAM density profile. Since Mars-GRAM provides density data at discrete altitudes, cubic spline interpolation is used to determine density values at queried altitudes.

The purpose of this entry scenario is to validate the use of the network for propagating trajectories in the guidance algorithm. Therefore, for this scenario, the neural network is coupled with the exponential model, as presented in (40). It is important to note that the least-squares exponential model, used to train the neural network offline and to propagate the guidance trajectories, is fitted to different Mars-GRAM profiles that were not used in the Monte Carlo testing. However, in both cases, the Mars-GRAM profiles are generated by setting the density and wind random perturbations to represent a standard deviation of 2% to 45% of the unperturbed mean.

A. Monte Carlo Settings

This work uses the same Monte Carlo settings as detailed in the previous work [16], with the only difference being that the vehicle is given slightly more lift to improve controllability. The initial conditions, as described in Table 1,

correspond to the entry conditions for the Mars Science Laboratory (MSL).

Table 1 Entry conditions for the Mars entry scenario

Parameter	Value	Standard Deviation (3σ)
r_0 (m) [44]	3.5222×10^6	3.2066×10^1
ϕ_0 (deg) [44]	-0.3919×10^1	7.8100×10^{-4}
θ_0 (deg) [44]	1.2672×10^2	3.6700×10^{-4}
v_0 (m/s) [44]	6.0833×10^3	2.6059×10^{-2}
γ_0 (deg) [44]	-1.5489×10^1	4.0000×10^{-4}
ψ_0 (deg) [44]	9.3206×10^1	2.6800×10^{-4}
B_0 (m ² /kg) [18, 45]	7.1000×10^{-3}	4.8000×10^{-3}
L_0/D_0 (n.d.)	2.7000×10^{-1}	1.5178×10^{-1}

Each trajectory and measurement is corrupted by additive white Gaussian process noise. Tables 2 and 3 show the values used for the process and measurement noise, respectively. For this entry scenario, the available sensors are sampled at a frequency of 4 Hz. The consider parameter in the filter is modeled as an exponentially correlated random variable (ECRV) with $\tau = 5$, $P_{ss} = 1 \times 10^{-3}$, and an initial covariance of $P_{c,0} = 1 \times 10^{-10}$. Furthermore, for each simulated trajectory, the angle of attack is kept constant at $\alpha = -17^\circ$.

Table 2 Process noise statistics for the Mars entry scenario

Parameter	Standard Deviation (3σ)
r (m)	-
ϕ (deg)	-
θ (deg)	-
v (m/s)	3×10^{-1}
γ (deg)	2×10^{-3}
ψ (deg)	2×10^{-4}
B (m ² /kg)	1×10^{-5}
L/D (n.d.)	3×10^{-5}

Table 3 Measurement noise statistics for the Mars entry scenario

Parameter	Standard Deviation (3σ)
$\tilde{a}^b$ (μg) [21]	1×10^2
$\tilde{q}$ (% of reading) [23]	1
$\tilde{Q}_s$ (% of reading)	1

The final target state is obtained by propagating a nominal trajectory using the full 3-DOF equations with a randomly selected Mars-GRAM profile from the training set. The nominal trajectory is integrated using a bank angle of $\sigma = -50^\circ$ when $v(t) \geq 6000$ m/s, $\sigma = -40^\circ$ when $v(t) \leq 3000$ m/s, and a linear decay for 3000 m/s $\leq v(t) \leq 6000$ m/s (with the

sign reversed within this range). This trajectory is propagated until a final velocity of $v_f = 1100$ m/s is reached. This velocity marks the transition from range control to heading alignment [46]. Additionally, this target state is selected so that it is reachable by the guidance algorithm if no navigation errors are considered and the true atmospheric density is known. Table 4 shows the final target state and Fig. 4 shows the nominal altitude as a function of range-to-target, the nominal bank angle as a function of time, and the nominal flight trajectory that ends close to the Gale Crater. Note that this is a short-range mission, as it represents a direct entry.

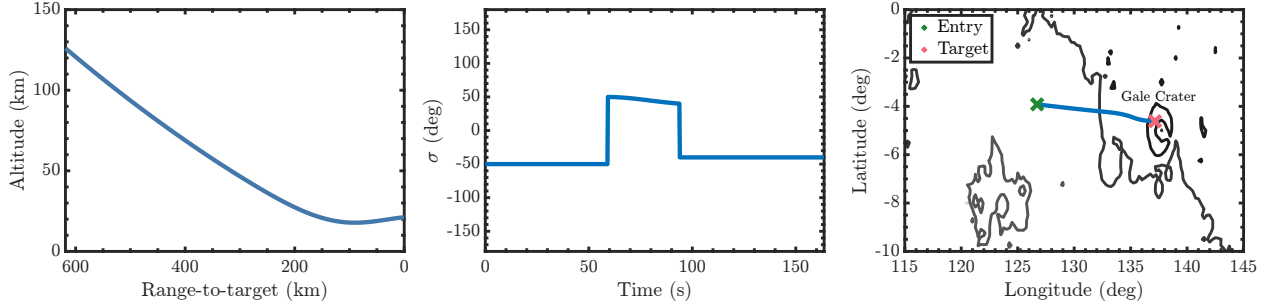


Fig. 4 Altitude as a function of range-to-target (left), bank angle as a function of time (center), and flight trajectory (right) for the Mars entry scenario.

Table 4 Final target state for the Mars entry scenario

Parameter	Value
r_f^* (m)	3.4167×10^6
ϕ_f^* (deg)	-0.4385×10^1
θ_f^* (deg)	1.3726×10^2
v_f^* (m/s)	1.1000×10^3

B. Guidance Algorithms

To assess the accuracy of the adaptive guidance algorithm, three different algorithms are tested and compared:

- The first algorithm, referred to as TG (True Guidance), has access to the true vehicle state and the true atmospheric density. This means that navigation errors are not considered. This algorithm serves as a baseline for the best achievable performance.
- The second algorithm, referred to as GEXP (Guidance with Exponential Model), uses the state estimates from the navigation filter but uses only the exponential density model in the prediction step of FNPEG and the crossrange calculations (i.e. $\alpha = 0 \forall e$ in (40), or equivalently $\kappa \rightarrow \infty$).
- The third algorithm, referred to as GADA (Guidance with Adaptive Model), uses the state estimates of the filter and the adaptive density model for prediction and crossrange calculations as in (40).

For each guidance algorithm, the maximum rate-of-change of the bank angle is set to $15^\circ/\text{s}$. In addition, FNPEG is

called at a frequency of 1 Hz once the sensed acceleration is greater than 0.2 times Earth’s gravity, or roughly 0.53 times Mars gravity ($\|\bar{a}^b\| > 1.96 \text{ m/s}^2$) [46, 47]. Before range control starts, the vehicle flies at a constant bank angle of $\sigma = -50^\circ$. For the longitudinal channel, the Newton-Raphson algorithm is used until one of the following conditions is met: $|s_f^*(e^*, \sigma_i)| \leq 0.1 \text{ km}$ or 10 iterations have been completed. For the lateral channel, $K = 3.5$, and bank reversals are performed following a minimum time solution. For the prediction step and lateral channel propagation, $\sigma_f = \sigma^*$, so that the bank angle follows a constant profile during propagation.

C. Monte Carlo Results

To compare the three solutions, the final crossrange error, the downrange error, and the range-to-go error are used [8]. Figure 5 shows the final crossrange and downrange errors for TG, GEXP, and GADA using two different values of κ . From the figure, it can be seen that GEXP has the highest downrange error, landing mostly outside a 2.5 km radius from the target. As the prediction step begins to rely more on the neural network, that is, as the value of κ decreases, the landing profile moves closer to the TG results. This behavior shows that the online adaptation of the neural network can capture the current atmospheric density values more accurately than the pre-flight nominal model. In this particular entry scenario, relying solely on the neural network for the prediction step ($\kappa = 0$) yields the closest performance to the TG results. This outcome is expected because the exponential model does not capture the true atmospheric density at high altitudes, showing that relying on inaccurate nominal models during prediction can be problematic. Table 5 shows the final mean downrange and crossrange errors along with their standard deviation.

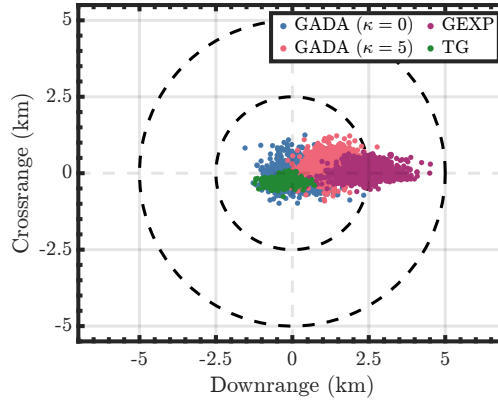


Fig. 5 Crossrange error vs. downrange error for different values of κ for the Mars entry scenario.

Figure 6 shows the final range-to-go cumulative densities for the three guidance solutions. This figure combines both crossrange and downrange errors to present a single performance metric. From this figure, it can be seen that as the prediction step (and lateral channel propagation) increasingly uses the neural network throughout the propagation of the trajectory, the final range-to-go begins to align with the TG solution. Additionally, the spread of the results is smaller when using the neural network compared to when using only the exponential model. Note that for this example, the

Table 5 Final downrange and crossrange errors for the Mars entry scenario

Algorithm	$\bar{dr}_f$ (km)	$\sigma(dr_f)$ (km)	$\bar{\chi}_f$ (km)	$\sigma(\chi_f)$ (km)
GADA ($\kappa = 0$)	0.037	0.506	-0.155	0.341
GADA ($\kappa = 5$)	1.193	0.492	0.197	0.340
GEXP ($\kappa \rightarrow \infty$)	2.401	0.624	0.016	0.212
TG	-0.290	0.267	-0.245	0.093

exponential model is not being adapted and does not accurately represent the atmospheric density conditions in which the vehicle is flying. Using an adaptive nominal model in the linear combination (40), as will be shown in the next section, can further improve the performance of the algorithm, as both the neural network and the adapted nominal model can represent the atmospheric density better than the pre-flight nominal.

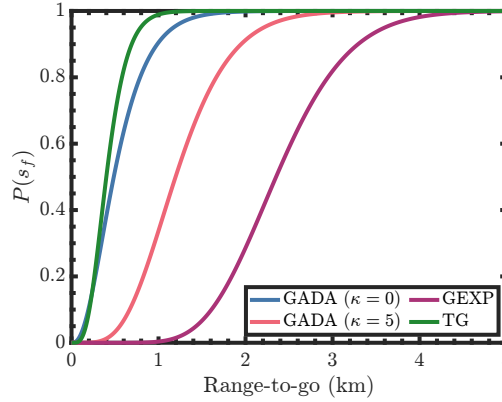
**Fig. 6 Final range-to-go for different values of κ for the Mars entry scenario.**

Figure 7 shows the average bank angle as a function of time for the three guidance solutions. For GEXP, the trajectory adopts slightly higher bank angles near the midpoint, indicating that the nominal model underestimates the true density at high altitudes. This results in faster energy depletion, which leads the vehicle to undershoot the target. For GADA, the bank profile is closer to TG, showing that the atmospheric estimate more closely resembles the true density. This allows the trajectory to align more closely with the TG, ending closer to the target. Note that some energy depletion could be mitigated by using $\sigma_f \neq \sigma^*$, but as shown in the next section, the linear bank angle profile cannot fully compensate for atmospheric mismatch.

As mentioned previously, the navigation technique was previously developed without considering any control input to the system [16]. This current work also serves as a sanity check to verify the filter's consistency when its state estimates are used for guidance. Figure 8 shows the estimation error as a function of time. While only 100 Monte Carlo runs are shown, the sample mean, filter covariance, and sample covariance are calculated using every run. As it can be seen, the estimates are very close to zero mean, and the filter's predicted covariance accurately matches the sample covariance of the Monte Carlo runs. These error plots closely resemble those presented in the previous work [16],

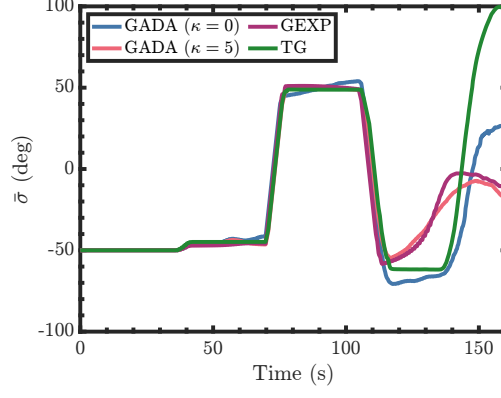


Fig. 7 Average bank angle as a function of time for the three guidance solutions for the Mars entry scenario.

although there is a higher estimation error when guidance starts, which arises from the fact that the bank angle is constantly changing. However, this shows that the estimation technique presented in previous work remains consistent in a closed-loop simulation.

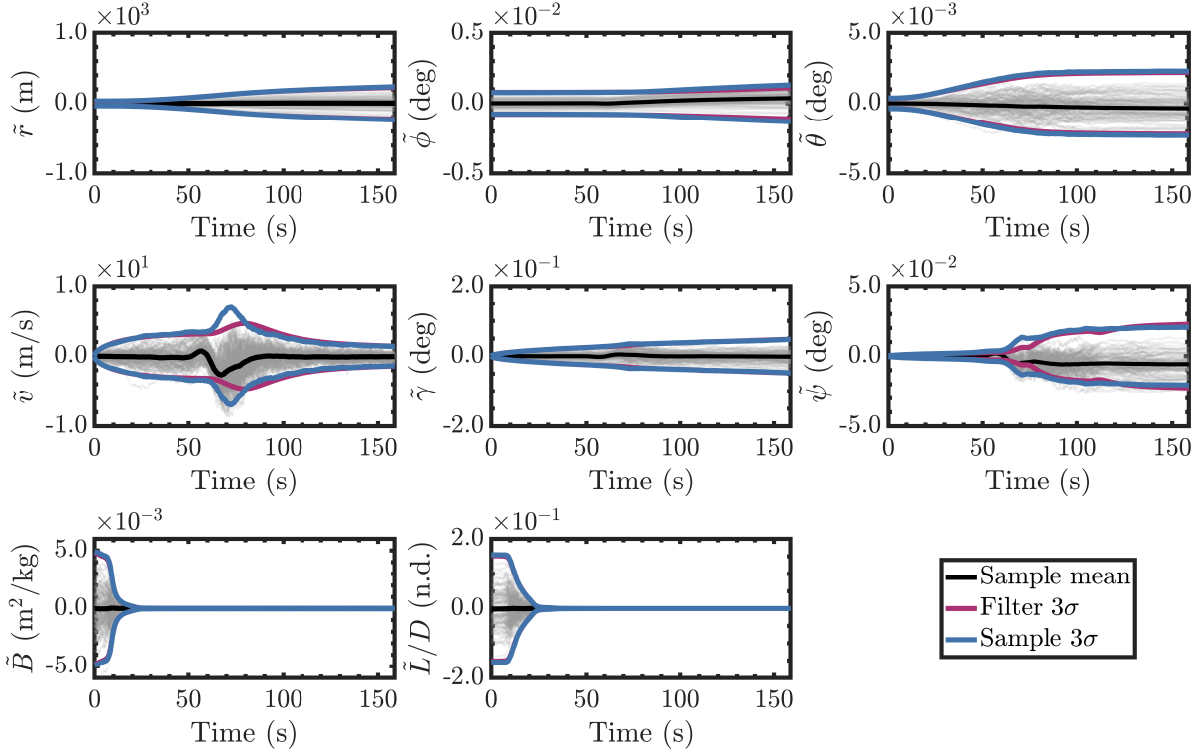


Fig. 8 Estimation error as a function of time for the Mars entry scenario. The gray lines show 100 Monte Carlo trajectories out of the 1000 runs.

VII. Earth Re-Entry

The linear combination in (40) provides the added advantage of modularity, enabling the exponential density model to be replaced with a different model for improved accuracy. In this section, a Monte Carlo test for an Earth skip re-entry scenario is presented. Additionally, the exponential model is improved with the atmospheric density adaptation used in the Orion skip re-entry mission [3, 5]. The purpose of this entry scenario is to compare the network with state-of-the-art adaptation techniques, and show how they can compliment each other. As in the Mars entry scenario, each true trajectory is simulated using a different Earth-GRAM density profile. The trajectory points are also interpolated using a cubic spline to approximate density values at the queried altitudes.

For this setup, the linear combination used to propagate the guidance trajectories is expressed as

$$\rho(r, e) = \alpha(e) \mathcal{NN}(r, \xi_k) + [1 - \alpha(e)] \hat{\mathcal{K}}_k \rho_{\text{exp}}(r), \quad (43)$$

where $\hat{\mathcal{K}}_k$ is a correction factor determined using a low-pass filter [3]

$$\rho_{\text{est},k} = 2 \frac{D_k}{\hat{B}_k \hat{v}_k^2}, \quad (44)$$

$$D_k = \frac{\|\tilde{\mathbf{a}}_k^v\|}{\sqrt{(\hat{L}/D_k)^2 + 1}}, \quad (45)$$

$$\hat{\mathcal{K}}_k = k_{\text{gain}} \frac{\rho_{\text{est},k}}{\rho_{\text{exp}}(\hat{r}_k)} + (1 - k_{\text{gain}}) \hat{\mathcal{K}}_{k-1}, \quad (46)$$

where k_{gain} is a user-defined gain. The estimates for $\hat{B}_k$, $\hat{v}_k$, and $\hat{L}/D_k$ are obtained from the navigation filter, meaning these quantities do not need to be estimated using low-pass filters as in previous approaches [10]. Note that this adaptation of the exponential model is linear, as it assumes a constant bias, whereas the adaptation of the neural network is nonlinear and has more degrees of freedom. Nevertheless, this re-entry scenario aims to show how the two adaptive models can be coupled to achieve the best possible results. As with the Mars entry example, the least-squares exponential model, used to train the neural network offline and to propagate the guidance trajectories, is fitted to different Earth-GRAM profiles that were not used in the Monte Carlo testing. The Earth-GRAM profiles are generated by setting random density and wind perturbations to represent a standard deviation from 1% to 30% of the unperturbed mean.

A. Monte Carlo Settings

The initial conditions used for this scenario, as shown in Table 6, correspond to the entry conditions for the Orion entry capsule.

The entry trajectories are simulated following the same procedure as for the Mars entry scenario, meaning that the dynamics are propagated with additive process noise and the measurements are obtained from the true state, corrupted

Table 6 Entry conditions for the Earth re-entry scenario

Parameter	Value	Standard Deviation (3σ)
r_0 (m) [4, 5]	6.5001×10^6	1.5240×10^2
ϕ_0 (deg) [4, 5]	-2.5829×10^1	2.0000×10^{-1}
θ_0 (deg) [4, 5]	-1.2008×10^2	2.0000×10^{-1}
v_0 (m/s) [4, 5]	1.0992×10^4	2.7432×10^1
γ_0 (deg) [4, 5]	-0.5664×10^1	1.0000×10^{-1}
ψ_0 (deg) [4, 5]	0.4654×10^1	0.5000×10^{-1}
B_0 (m ² /kg) [4, 48]	3.1000×10^{-3}	4.8000×10^{-3}
L_0/D_0 (n.d.) [4, 48]	3.0000×10^{-1}	1.5178×10^{-1}

with additive process noise as well. The values used for the process and measurement noise are reported in Tables 7 and 8, respectively. For this scenario, the sensors are sampled at a frequency of 2 Hz. The consider parameter is also modeled as an ECRV with $\tau = 5$, $P_{ss} = 1 \times 10^{-3}$, and an initial covariance of $P_{c,0} = 1 \times 10^{-10}$. The angle of attack also remains constant at $\alpha = -17^\circ$.

Table 7 Process noise statistics for the Earth re-entry scenario

Parameter	Standard Deviation (3σ)
r (m)	-
ϕ (deg)	-
θ (deg)	-
v (m/s)	3×10^{-2}
γ (deg)	2×10^{-4}
ψ (deg)	2×10^{-5}
B (m ² /kg)	1×10^{-5}
L/D (n.d.)	3×10^{-5}

Table 8 Measurement noise statistics for the Earth re-entry scenario

Parameter	Standard Deviation (3σ)
$\tilde{a}^b$ (μg) [49]	3×10^2
$\tilde{q}$ (% of reading) [23]	1
$\tilde{Q}_s$ (% of reading)	1
$\tilde{r}_{GPS}$ (m) [31]	18

In this case, the nominal trajectory is integrated using a bank angle of $\sigma = -75^\circ$ when $v(t) \geq 7500$ m/s, $\sigma = -30^\circ$ when $v(t) \leq 4500$ m/s, and a linear decay for $4500 \text{ m/s} \leq v(t) \leq 7500$ m/s (with the sign reversed within this range). For this re-entry scenario, the trajectory is propagated until a final velocity of $v_f = 304.8$ m/s is reached, which marks the transition from range control to heading alignment [3]. Table 9 shows the final target state and Fig. 9 shows the nominal altitude as a function of range-to-target, the nominal bank angle as a function of time, and the nominal flight

trajectory that ends close to the western coast of Baja California. Note that the skip re-entry profile makes this a longer-range mission compared to the previous Mars entry scenario presented. This introduces additional challenges for the guidance algorithm, as bank reversals become more difficult to time, and poorly optimized bank angle solutions can cause the vehicle to skip out of the atmosphere. However, since Earth’s atmosphere is denser than Mars’, this scenario has increased controllability because of the higher lift and drag accelerations. In addition, using GPS measurements can reduce navigation errors and improve overall target accuracy.

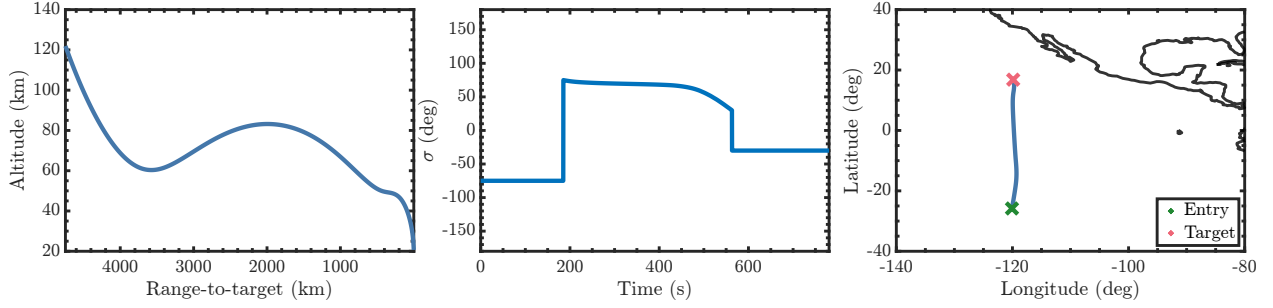


Fig. 9 Altitude as a function of range-to-target (left), bank angle as a function of time (center), and flown trajectory (right) for the Earth re-entry scenario.

Table 9 Final target state for the Earth re-entry scenario

Parameter	Value
r_f^* (m)	6.3989×10^6
ϕ_f^* (deg)	-2.1123×10^1
θ_f^* (deg)	-1.2008×10^2
v_f^* (m/s)	3.0480×10^2

B. Guidance Algorithms

For this re-entry scenario, three algorithms are also used to assess the accuracy of the adaptive atmospheric density model. However, instead of using the nominal exponential model, GEXP uses the adaptation presented at the beginning of the section and is therefore renamed Guidance with Orion Adaptation (GORI). Consequently, GADA now represents a linear combination of the neural network and the adapted exponential model. The three guidance algorithms are summarized below.

- The first algorithm, TG, represents the solution with access to the true state of the vehicle and the true atmospheric density.
- The second algorithm, GORI, uses the state estimates from the navigation filter but uses only the adapted exponential density model in the prediction step of FNPEG and the crossrange calculations (i.e. $\alpha = 0 \forall e$ in (43)). The adaptation is only performed when the sensed acceleration is greater than 0.05 times Earth’s gravity

($\|\tilde{\mathbf{a}}^b\| > 0.49 \text{ m/s}^2$) [3], with $k_{\text{gain}} = 0.9$.

- The third algorithm, GADA, uses the state estimates from the filter and the adaptive density model for the prediction and crossrange calculations as in (43).

In each guidance algorithm, the maximum rate-of-change of the bank angle is set to $15^\circ/\text{s}$, and FNPEG is called at a frequency of 1 Hz once $\|\tilde{\mathbf{a}}^b\| > 0.49 \text{ m/s}^2$ [3]. Before range control starts, the vehicle flies at a constant bank angle of $\sigma = -75^\circ$. For the longitudinal channel, the Newton-Raphson algorithm is used with the same convergence conditions as for the Mars entry scenario. For calculations of the lateral channel and cross range, $K = 3.5$, and bank reversals are also performed following a minimum time solution. For the prediction step and the propagation of the lateral channel, $\sigma_f = 55^\circ$.

C. Monte Carlo Results

Figure 10 shows the final crossrange and downrange error for the three guidance solutions, Fig. 11 shows the final range-to-go cumulative densities, and Table 5 shows the final mean downrange and crossrange errors with their standard deviation.

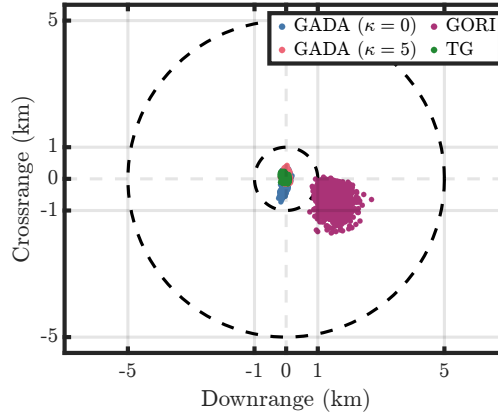


Fig. 10 Crossrange error vs. downrange error for different values of κ for the Earth re-entry scenario.

For this re-entry setup, a behavior similar to that in the previous section is observed. However, the overall errors are smaller due to the thicker atmosphere and improved controllability. In this scenario, due to the skip phase, not adapting the nominal model may cause the guidance algorithm to yield bank angles that result in the vehicle skipping out of the atmosphere. The results obtained with GORI show that adapting the nominal model helps bring the vehicle closer to the target, but it still undershoots. In the case of GADA with $\kappa = 0$, the landing profile moves closer to the TG solution. For this particular re-entry trajectory, the combination of both models, $\kappa = 5$, produces results closest to the TG solution. This behavior can be further explained by analyzing the bank profiles.

Figure 12 shows the average bank angle as a function of time for the three guidance solutions. Similarly to the results obtained for the Mars entry scenario, for GORI, the adapted model still underestimates the true density, causing

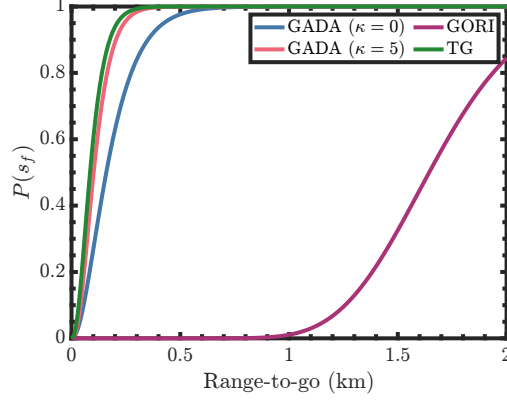


Fig. 11 Final range-to-go for different values of κ for the Earth re-entry scenario.

Table 10 Final downrange and crossrange errors for the Earth re-entry scenario

Algorithm	$\bar{dr}_f$ (km)	$\sigma(dr_f)$ (km)	$\bar{\chi}_f$ (km)	$\sigma(\chi_f)$ (km)
GADA ($\kappa = 0$)	-0.006	0.066	-0.056	0.207
GADA ($\kappa = 5$)	0.008	0.053	0.066	0.101
GORI ($\kappa \rightarrow \infty$)	1.497	0.312	-0.655	0.345
TG	-0.035	0.062	0.042	0.076

the vehicle to fly at higher bank angles, depleting energy faster and, consequently, undershooting the target. Note that in this case, the linear bank angle profile is used, but the mismatch in atmospheric density still leads to faster energy depletion. In the case of GADA with $\kappa = 0$, the atmospheric estimate slightly overestimates the true density, causing the vehicle to fly at shallower bank angles. This results in energy depletion occurring closer to the target, though the vehicle still lands closer to the true target than GORI. For GADA with $\kappa = 5$, the effect of both adaptations is compensated, and the bank angle profile follows the TG solution, landing closer to the target.

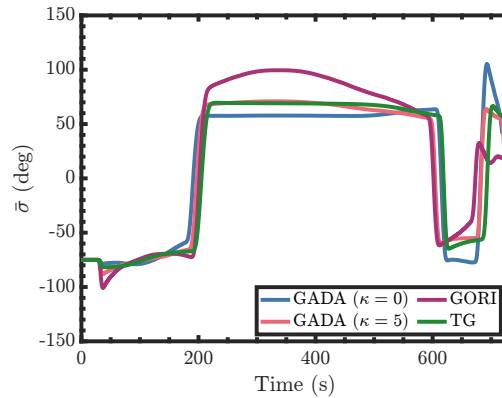


Fig. 12 Average bank angle as a function of time for the three guidance solutions for the Earth re-entry scenario.

It is important to note that the targeting error obtained with GORI could be reduced by improving the onboard nominal model to better capture the true atmospheric density. Common onboard models used for Earth re-entry include

the 1976 U.S. Standard Atmosphere model, a one-dimensional table look-up atmosphere of density with respect to altitude, and polynomial curve fits [3]. In such cases, the neural network could also be trained to match these models. However, this is not the primary goal of this research. The objective of this paper is to show that this new adaptive method can adapt to mismodeled dynamics more effectively than existing methods, improving the prediction step in numerical predictor algorithms. The results shown in this section suggest that common adaptation schemes may have a smaller domain of adaptation convergence and, therefore, require initialization closer to the truth than the neural network approach.

Lastly, Fig. 13 shows the estimation errors as a function of time. Similarly to the Mars entry scenario, the filter is consistent, with the estimation errors staying close to a zero mean and the filter's covariance aligning with the covariance of the Monte Carlo errors. In particular, the error decreases during the skip phase of the trajectory as GPS measurements are used, and the estimates recover quickly.

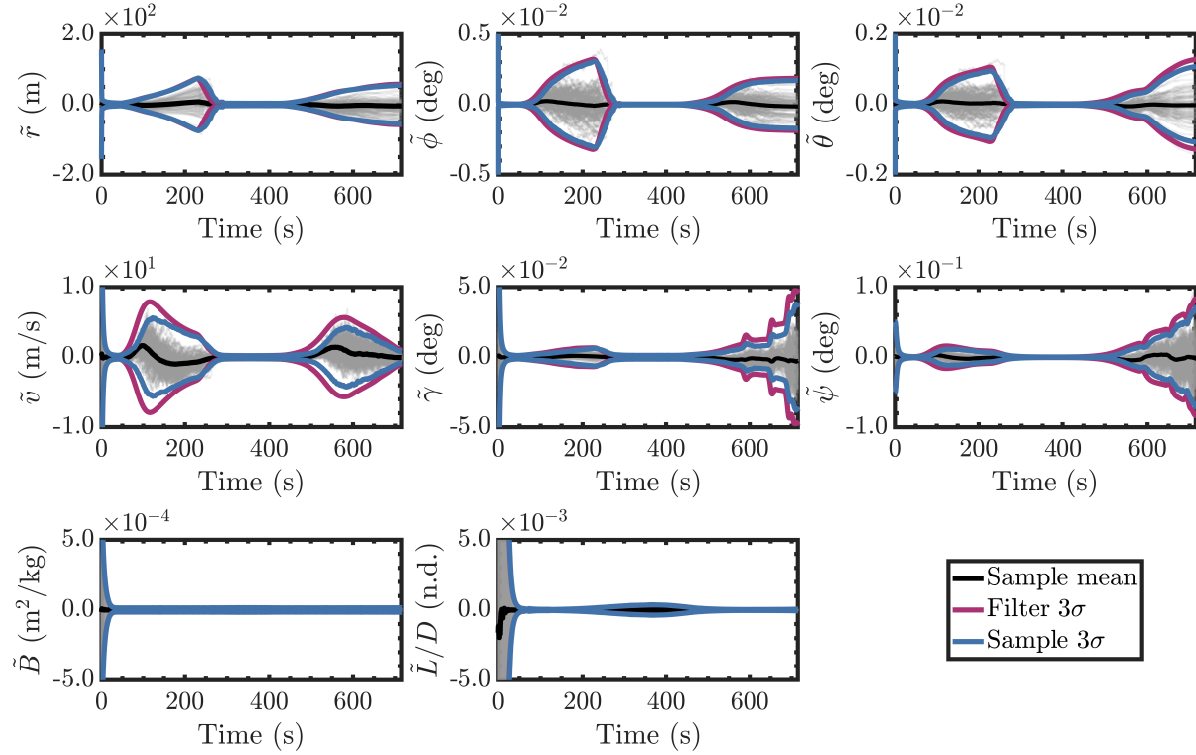


Fig. 13 Estimation error as a function of time for the Earth skip re-entry scenario. The gray lines show 100 Monte Carlo trajectories out of the 1000 runs.

VIII. Conclusions

In this work, a new atmospheric entry adaptive guidance algorithm is presented. This algorithm combines the adaptive neural network density model from a recently developed navigation solution [16] with an onboard nominal model, improving atmospheric density estimation for the prediction step and lateral channel propagation in FNPEG. By

including this adaptive model in FNPEG, the accuracy of the prediction step and the propagation of the lateral channel is improved, resulting in better targeting precision.

The proposed algorithm was tested through a Monte Carlo analysis in two different atmospheric entry scenarios. For the first scenario, a robotic Mars entry mission was considered. In this use case, the neural network was coupled with an exponential density model. The results showed that using the neural network for propagation can increase targeting accuracy as the network is adapted online to better represent the true atmospheric density. For the second scenario, an Earth re-entry mission was considered. For this application, the neural network was coupled with an online adaptive exponential model, similar to the techniques used in real re-entry missions. The results showed that using a combination of the two models can increase targeting accuracy, as both models are adapted simultaneously online.

In addition, the consistency of the navigation solution was evaluated in a closed-loop format. In both entry scenarios, the filter was shown to be consistent and robust to control inputs determined using the adaptive guidance algorithm.

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