Thurstone's law of comparative judgment applied to 5-point verbal annoyance responses from community noise studies

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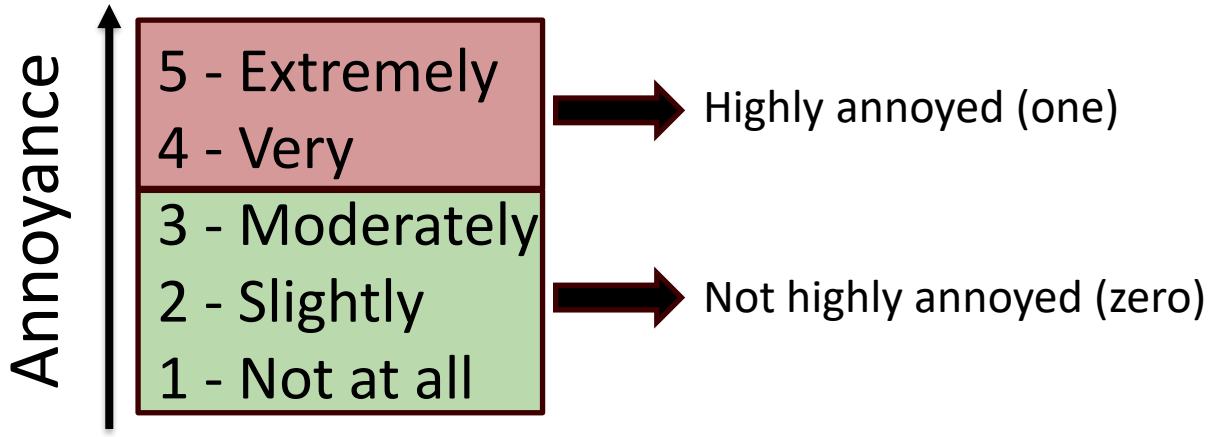
(Session 4pNS, Exposure Response and Community Tolerance Level)

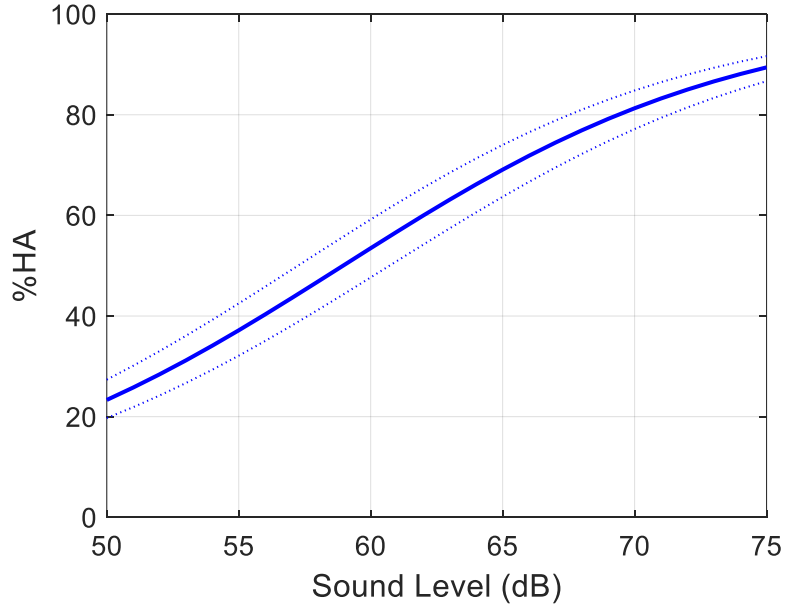
Overview

- Introduction
- Thurstone analysis and Law of Comparative Judgment
- Simulated community annoyance dataset
- Results

Perceptual response (dose-response) modeling

Common approach





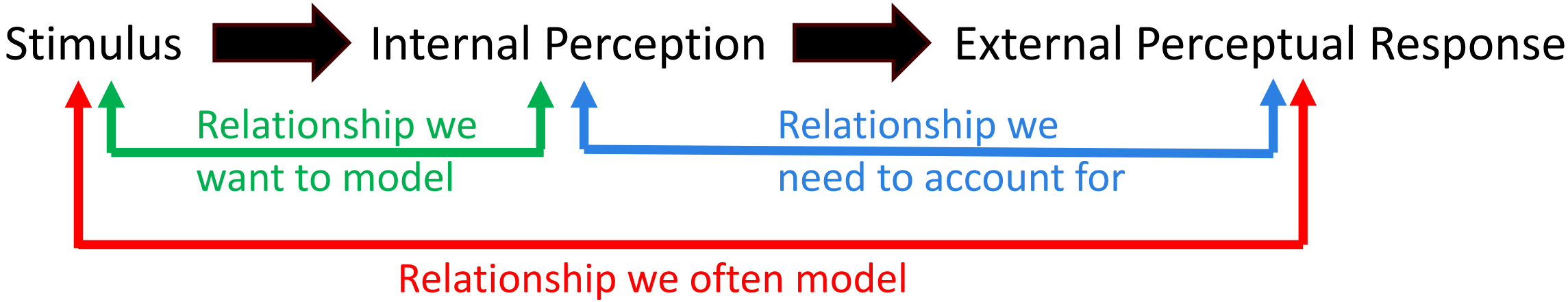
Potential limitations

- Dichotomization of 5-point responses
  - Highly annoyed or not
  - Discrete annoyance scale
- Equally spaced categories not guaranteed (ISO/TS 15666)

Motivation to use Thurstone analysis

- Continuous annoyance scale
- Just-noticeable-differences
- Evaluate spacing of response categories
- Internal annoyance perception

Focus on stimulus-to-internal perception relationship



- **Problem: people use response scales in different ways**[1]
  - Perception is an internal judgment
  - People differ in how they report internal judgments. They respond:
    - Differently for the same judgment
    - The same to different judgments

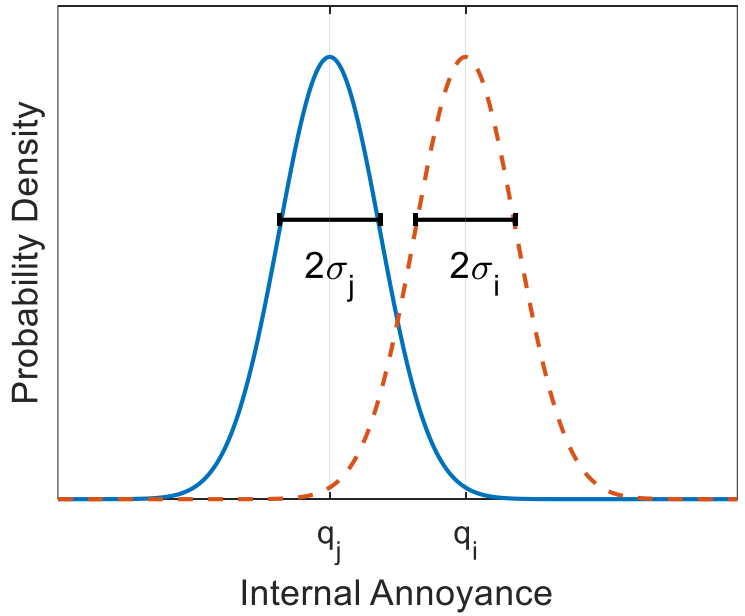
- **Possible solutions**
  - Multilevel analysis
  - Master scaling (Berglund, 1991)
  - Thurstone's Law of Comparative Judgment (1927)

[1] G.A. Gescheider, "Psychophysics", 1997

Basics of Thurstone's Law of Comparative Judgment[2]

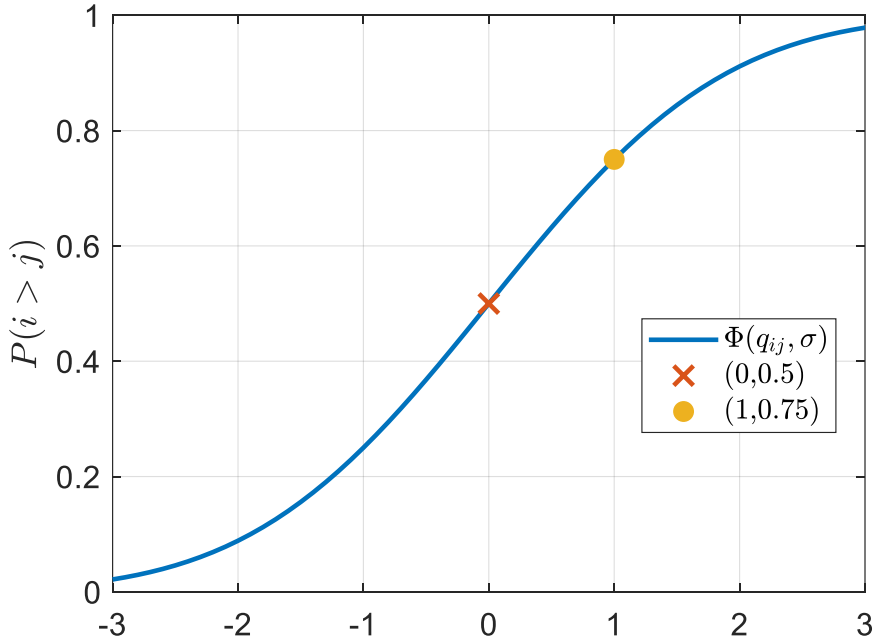
1. **Build preference matrix [C]**
   - **Entries c_{ij}: number of times sound i judged more annoying than sound j**
2. **Trial values for consensus annoyance opinion, q_i**
3. **Maximize probability**
4. **Output is annoyance value for each stimulus (Thurstone scale)**

Which sound was more annoying?



Difference limen

$q_{ij} = 1$ ➡ **75% consensus**



Difference in annoyance

$$q_{ij} = q_i - q_j$$

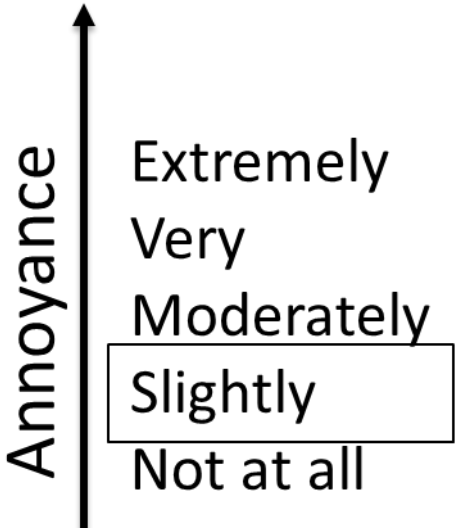
[2] Thurstone, "A law of comparative judgment," Psychological Review, 34, 273-286 (1927)

5-point responses and Law of Comparative Judgment[2]

- Community noise tests are not based on paired comparisons
- However, 5-point verbal responses can be interpreted in a paired comparison context[3]
- Apply Law of Comparative Judgment to get internal perception[4]

➢ **Law of Comparative Judgment applied to 5-point verbal annoyance scale**

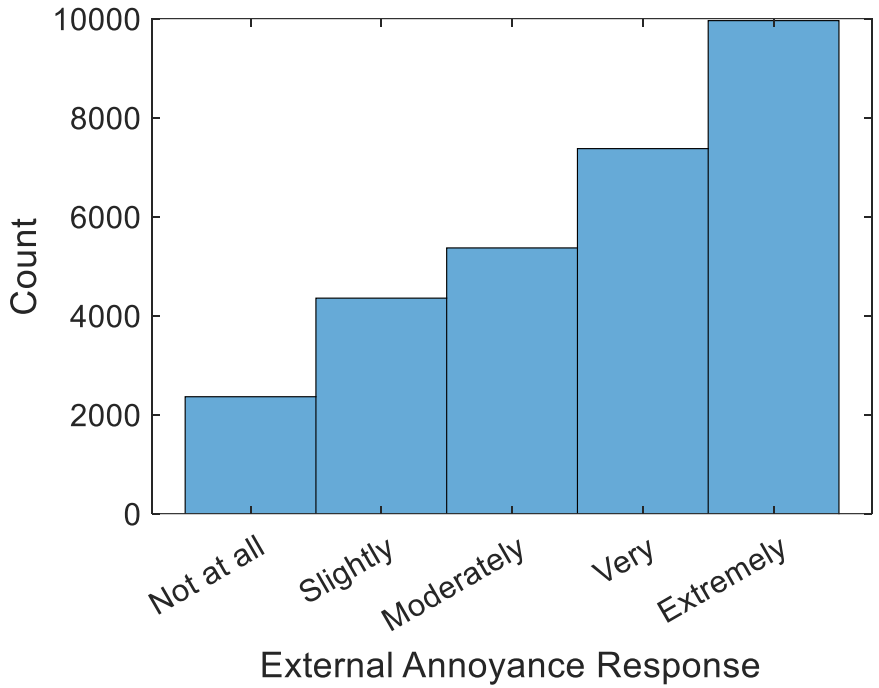
- Example response: "slightly"
- Judgment between what was heard and other verbal categories[3]
- What was heard:
  - More annoying than "not at all"
  - Less annoying than other categories

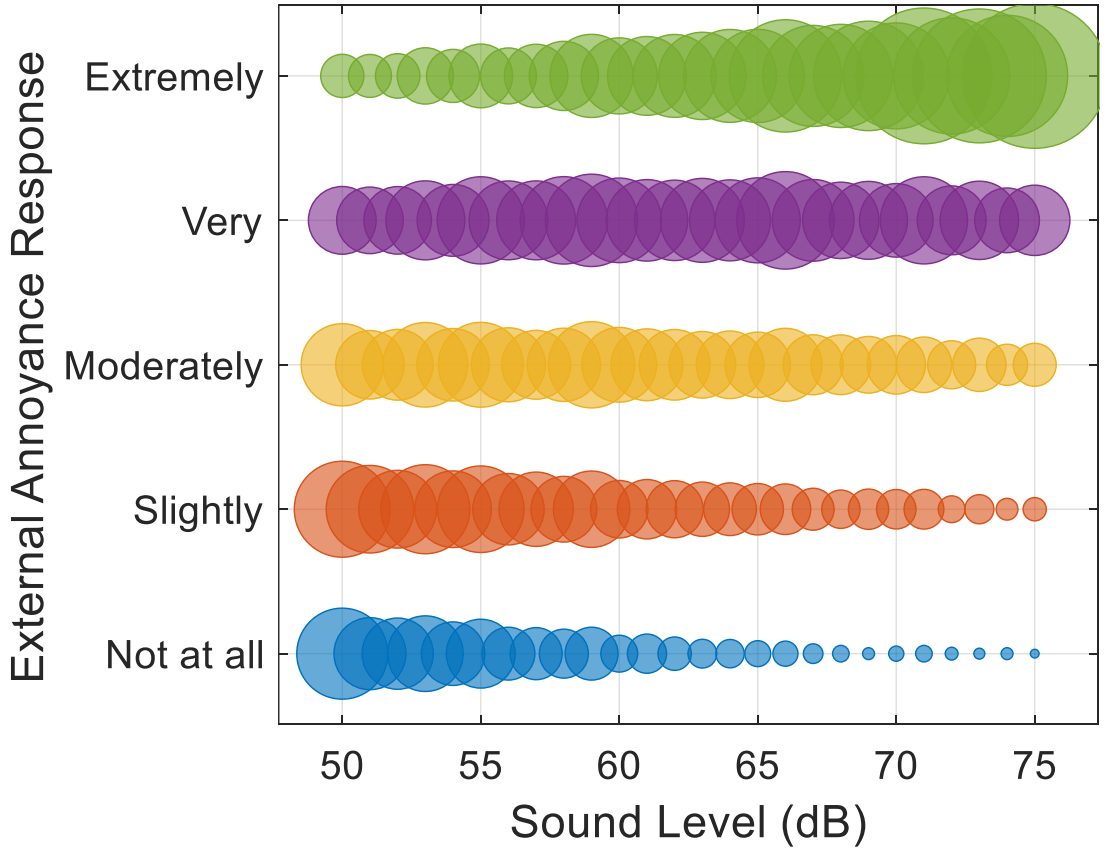


[2] Thurstone, "A law of comparative judgment," Psychological Review, 34, 273-286 (1927)
[3] Rimoldi et al., "The law of comparative judgment in the successive intervals and…," Educational Testing Service RB-54-5, 1954
[4] Perez-Ortiz and Mantiuk, "A practical guide and software for analysing paired comparison experiments," arXiv, 2017.

Application on a simulated dataset[5,6]

- **Dataset parameters***
  - 86 single events
  - 742 participants
  - 47% contribution rate





*Emphasis on simulated data. Demonstration purposes only. Interpretation for specific noise source not recommended.

[5] Doebler et al., "Dose error impacts on a collection of realistic dose-response curves…", InterNoise23

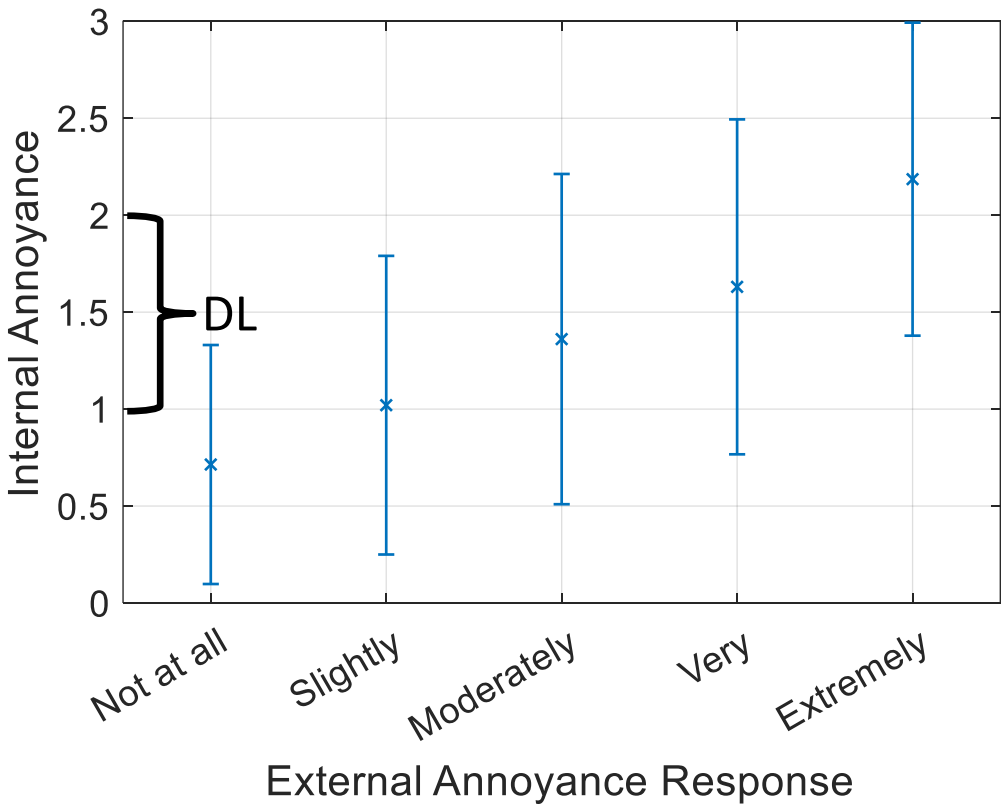
[6] Lee et al., "Bayesian statistical models for community annoyance survey data," J. Acoust. Soc. Am., 2020

Thurstone analysis gives internal annoyance perception

- **Each single event (SE) gets unique annoyance value**
  - Categorical based on SE ID
  - Sound level ignored
- **75% difference limen (DL) built-in[4]**
  - Δ(Internal Annoyance) = 1
- **Higher internal annoyance for higher 5-point verbal response**
- **Although relationship appears linear, spacing of categories is unknown**

- **Relationship we need to account for**

Internal Perception ➡ External Response



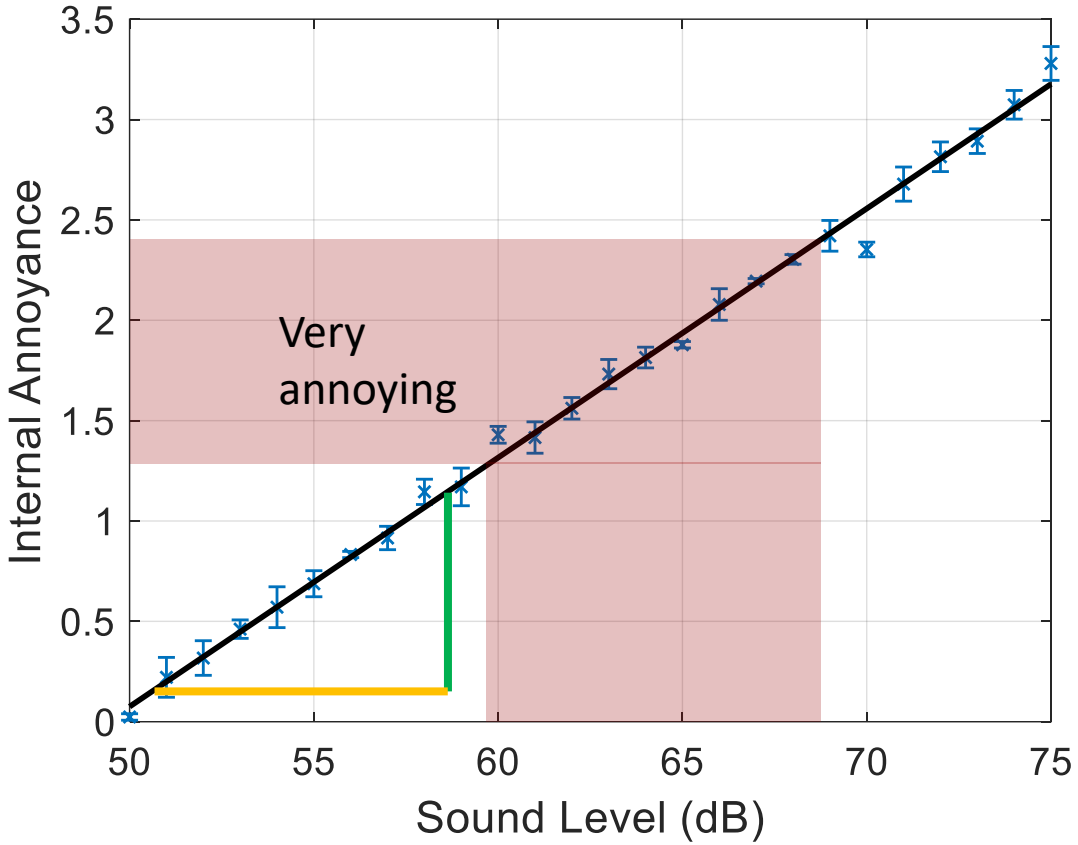
[4] Perez-Ortiz and Mantiuk, "A practical guide and software for analysing pairwise comparison experiments," arXiv Stat.AP, 2017

Model internal annoyance perception vs. sound level

- **Fechner's Law[1]**
  - Linear relationship between sensation and logarithm of stimulus
  - Applied to perception here
- **Internal annoyance highly correlated with sound level**
- **Just-noticeable-difference**
  - What is **ΔSPL** when **Δ(Annoyance) = 1**
  - 1 JND ≈ 8 dB *
- **Map verbal categories to sound level (e.g., "very")**

- **Relationship we want to model**

Stimulus → Internal Perception

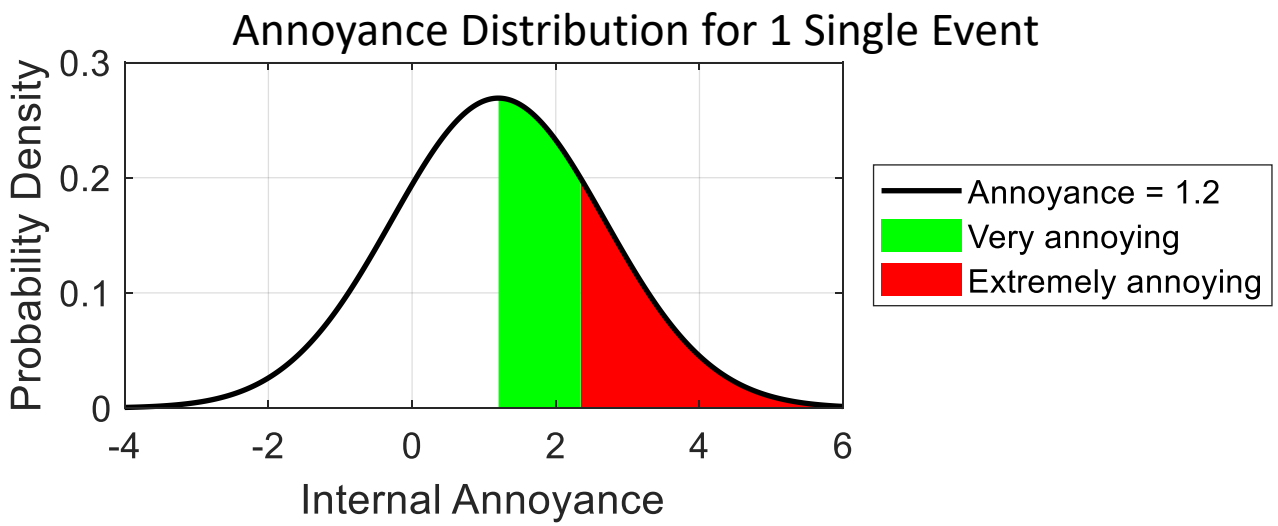


[1] G.A. Gescheider, "Psychophysics", 1997

Comparison of standard and Thurstone approaches

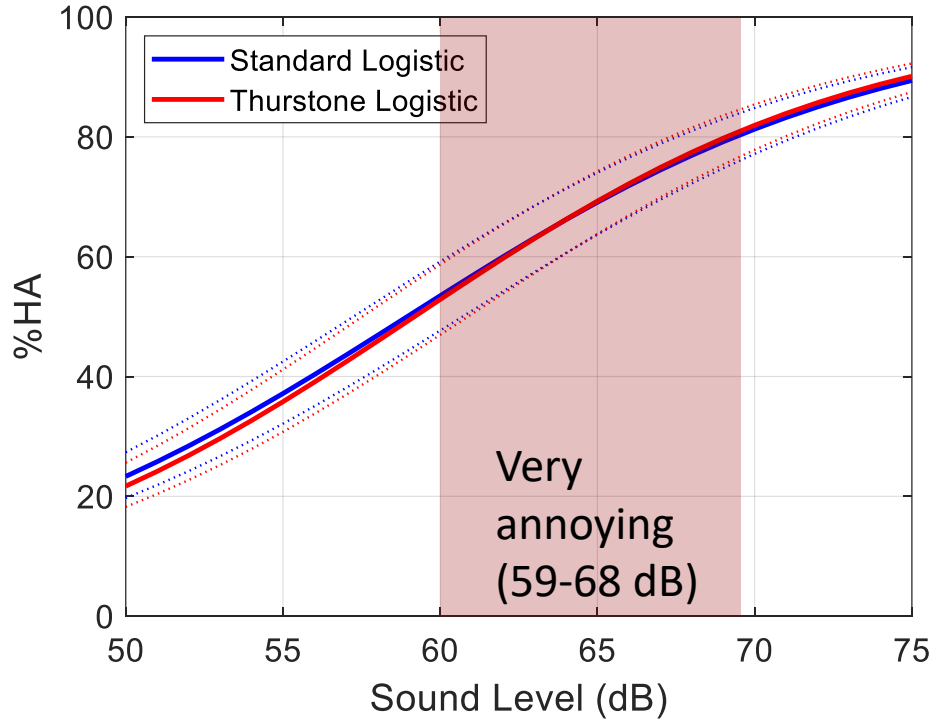
Thurstone Logistic Approach

- Probability density of annoyance for each single event
- Calculate percent highly annoyed
- Generate new binary response and do logistic regression



Relationship we often model

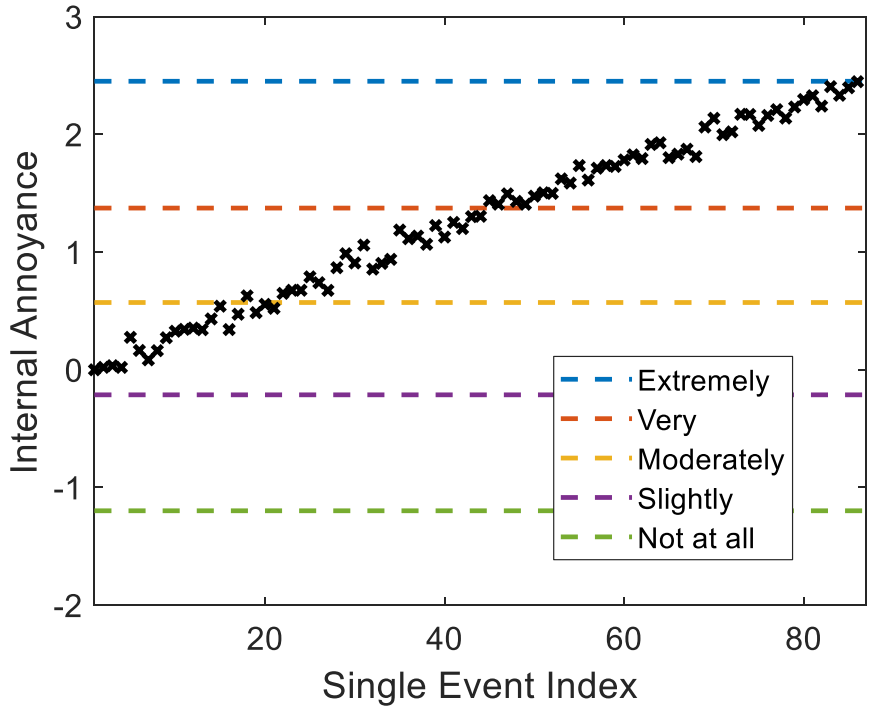
Stimulus ➡ External Response



- Both approaches agree
- Thurstone gives range of categories

Evaluating distance between verbal categories

- **Spacing may be sensitive to study demographics (ISO/TS 15666)**
  - Indicated by Lee et al. using ordinal regression (JASA, 2020)
  - Fidell and Teffeteller (JSV, 1981)



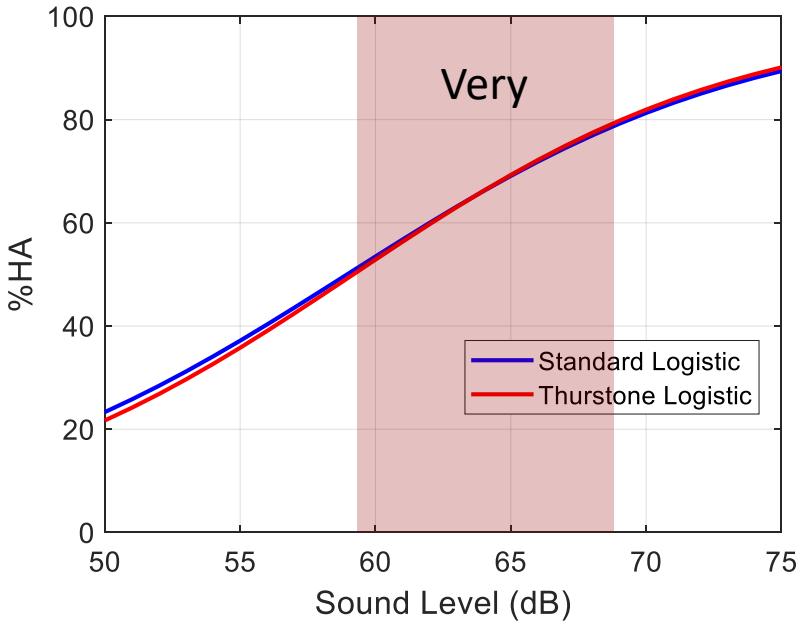
1 JND ≈ 8 dB

Interval	Difference in Annoyance	Change in Sound Level (dB)
'Extremely' – 'Very'	1.08	8.7
'Very' – 'Moderately'	0.80	6.5
'Moderately' – 'Slightly'	0.78	6.3
'Slightly' – 'Not at all'	0.99	7.9

- **Thurstone analysis**
  - Can quantify distance between 5-point verbal response categories
  - Use JND to determine change in sound level between response categories

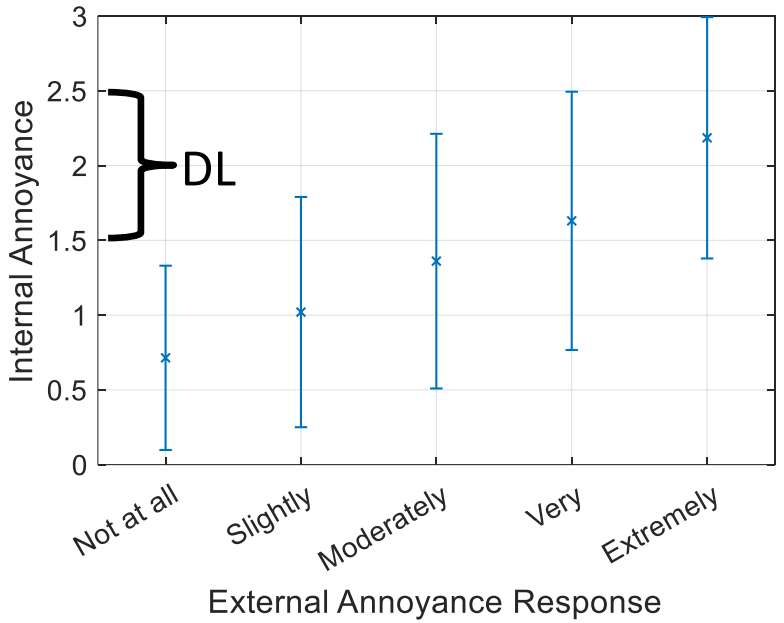
Summary

Relationship we often model



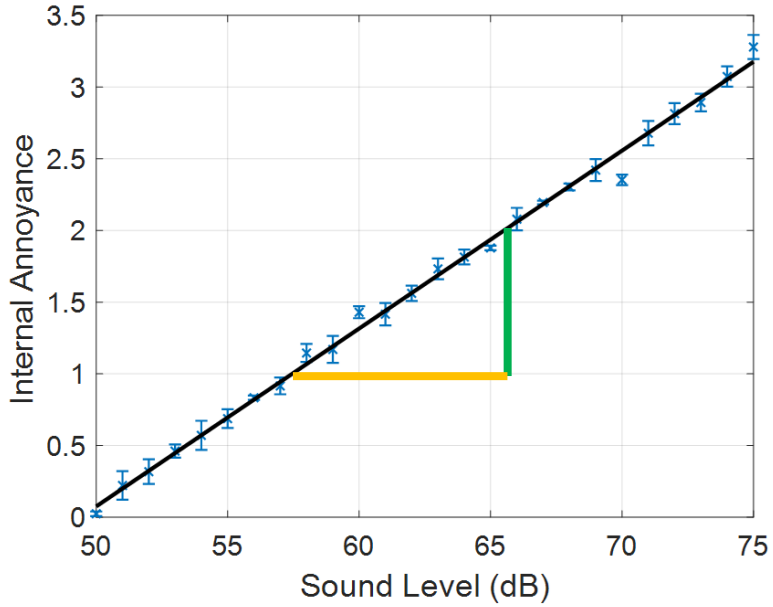
- Thurstone analysis agrees with standard approach
- Range of response categories found

Relationship we need to account for



- Continuous annoyance scale with built-in 75% difference limen (DL)

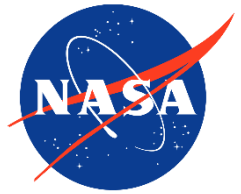
Relationship we want to model



- Determine JND for noise metrics

Additional insight: spacing of verbal categories can be quantified

Thank You



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