Heralded entanglement source analysis for cascaded parametric down-conversion

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Introduction

Several techniques are presented for high-fidelity Bell pair generation through heralded, multiplexed parametric down-conversion (PDC). Generalizes previous cascaded PDC [1] and zero-added-loss multiplexing (ZALM) schemes [2].

Closed form expressions for the **heralding rate and fidelity** of Bell pairs produced by **generalized cascaded PDC and ZALM sources** extend previous analyses [1-3].

Heralded Bell pairs based on 4-photon PDC emissions

In this work, we study generalized schemes to extract high-fidelity, **heralded** Bell pairs from a dual two-mode squeezed vacuum state (D-TMSV) state
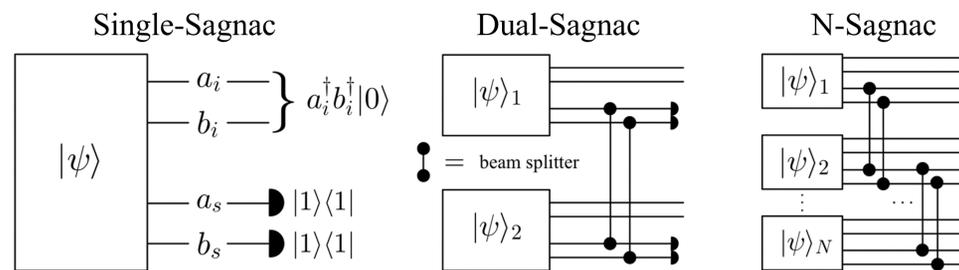
$$|\psi\rangle = (1 - \lambda) \sum_{n}^{\infty} \sum_{m}^{\infty} \lambda^{n/2}(-1)^m |n-m, m; m, n-m\rangle$$

arising from the simplified PDC interaction Hamiltonian

$$H_{int} = \kappa(a_s^\dagger b_i^\dagger - b_s^\dagger a_i^\dagger) + h.c.$$ entangling emissions in two dual-rail mode pairs.

Generalized cascaded parametric down-conversion

Heralding Bell pairs by cascading Sagnac-configured SPDC sources and performing a double-detection on 4-photon emissions:


Dual-Sagnac source: Studied by Dhara et. al. [1]. Two D-TMSV states are combined in an entanglement swap configuration. **A successful entanglement swap heralds the production of a Bell pair**. Due to probabilistic nature of SPDC, the heralded Bell pair has only 50% fidelity (errors occur when one of the D-TMSV states produces a double-pair). Can recover 100% fidelity Bell pair either with (1) vacuum-filtering at the receiver or (2) distillation of two heralded 50% fidelity pairs (right). **The raw heralded pair is unitarily equivalent (via linear optics) to an anti-correlated pair, or 2-photon N00N state**.

Single-Sagnac source: Simplification of the Dual-Sagnac scheme which requires only one D-TMSV source and no entanglement swap. A double-detection of opposite polarizations in the signal channel heralds the production of an anti-correlated pair in the idler channel. **This anti-correlated pair can be converted into the state produced by the Dual-Sagnac source by placing it through a 50:50 beam-splitter.**

"0-Sagnac" (TMSV + active mode-switch)
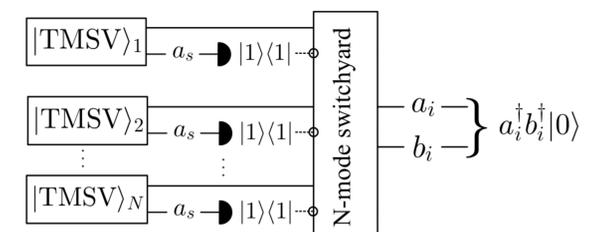
With active mode-switching, the efficiency of producing heralded anti-correlated pairs is enhanced quadratically.

Bell state projection/distillation
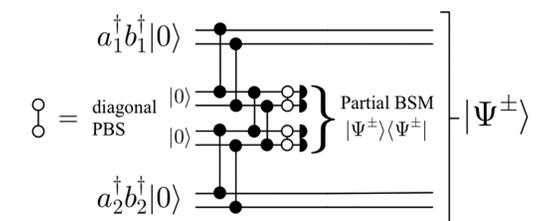
Two anti-correlated pairs can be combined in a linear optical interferometer to herald the production of a 100% fidelity Bell state (with success probability 1/8).

Heralding probability and fidelity of heralded pair state

By exploiting a symmetry of the Hamiltonian H_{int} to express the joint state $|\psi\rangle^{\otimes N}$ in the output modes of the N-port interferometer, the various traces of the density matrix projected by the detection POVM reduce to geometric sums over all high order emissions, yielding closed form expressions:

Parameters of study

μ = mean photon number per mode

η = detection efficiency

N = No. cascaded SPDC sources

α = PNR (0 ↔ no PNR to 1 ↔ PNR)
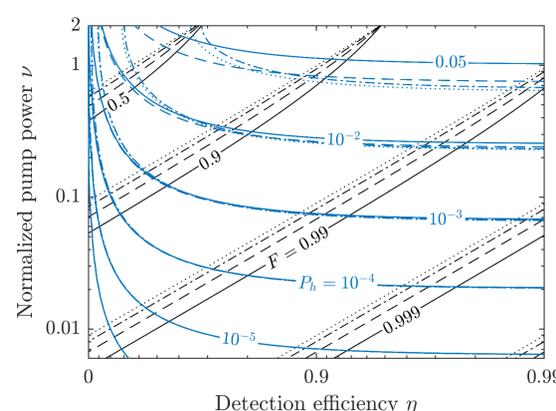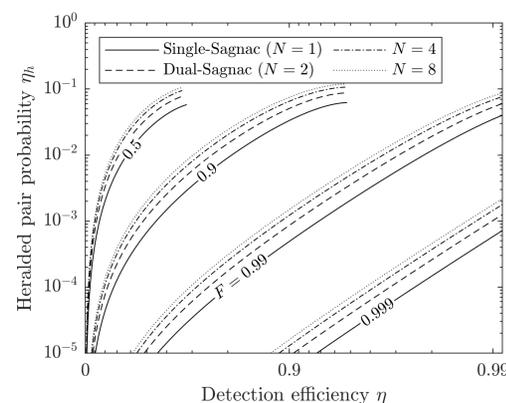
p_d = detector dark count probability

Detector model

PNR and dark counts modeled as array of k SPDs with dark count probability p_d, yielding derived detection parameters:

$$\alpha = (k-1)/k$$
$$\delta_0 = 1 - (1-p_d)^k$$
$$\delta_1 = kp_d(1-p_d)^{k-1}$$
$$\delta_2 = 1 - (1-p_d)^{k-1}$$

$$P_h^{(N)} = N^2 \frac{(\eta\mu)^2(1-\delta_0)^{2N-2}}{(1+\eta\mu)^{2N}(1+\alpha\eta\mu)^2}\left(1 - \delta_2 + \delta_1\frac{1+\alpha\eta\mu}{\eta\mu}\right)^2$$ (Heralded pair probability)

$$F^{(N)} = \frac{(1+\eta\mu)^{2N}(1+\alpha\eta\mu)^2}{(1+\mu)^{2N+2}}\frac{[\delta_1(1-\eta)\mu + (1-\delta_2)\eta\mu]^2}{[\delta_1(1+\alpha\eta\mu) + (1-\delta_2)\eta\mu]^2}$$ (Heralded pair fidelity)

Left panel shows the maximum heralded pair probability at a given fidelity, limited only by detection efficiency. Right panel shows the heralding probability and pair fidelity constrained by detection efficiency and pump power (PNR detection with no dark counts).

Summary

Assuming a large number M of N-Sagnac sources are multiplexed (as in the ZALM scheme [2]) the analysis yields a few key considerations:

- Limited only by detection efficiency, at a target fidelity F there is a small **gain in heralded pair probability with no. of sources N** (up to ~3 dB), at the cost of implementation complexity.
- Limited by detection efficiency and pump power, the heralding probability is independent of N, but there is a small **gain in fidelity by increasing N**, with diminishing returns up to $N \sim 4$.
- By implementing active mode-switching between M independent TMSV sources ("0-Sagnac" configuration), the heralding probability increases quadratically relative to the passive Single-Sagnac source, **reducing the multiplexing factor per pair:**

$$M \sim \left(P_h^{(N)}\right)^{-1} \quad \text{(passive N-Sagnac source)},$$
$$M \sim 2\left(P_h^{(1)}\right)^{-1/2} \quad \text{(actively-switched TMSV)}$$

References

[1] P. Dhara, S. Johnson, C. Gagatsos, P. Kwiat, S. Guha "Heralded-multiplexed high-efficiency cascaded source of dual-rail polarization-entangled photon pairs using spontaneous parametric down-conversion," *Phys. Rev. App.* **17**, (2021).
[2] K. C. Chen, P. Dhara, M. Heuck, Y. Lee, W. Dai, S. Guha, and D. Englund, "Zero-added-loss entangled-photon multiplexing for ground- and space-based quantum networks," *Phys. Rev. Appl.* **19**, 054029 (2023).
[3] J. H. Shapiro, M. G. Raymer, C. Embleton, F. Wong, and B. J. Smith, "Entanglement source and quantum memory analysis for zero-added-loss multiplexing," *Phys. Rev. Appl.* **22**, 044014 (2024).