Uncertainty Quantification with QUEST: Toward En-Route Noise Certification-by-Analysis

Marian Nemec

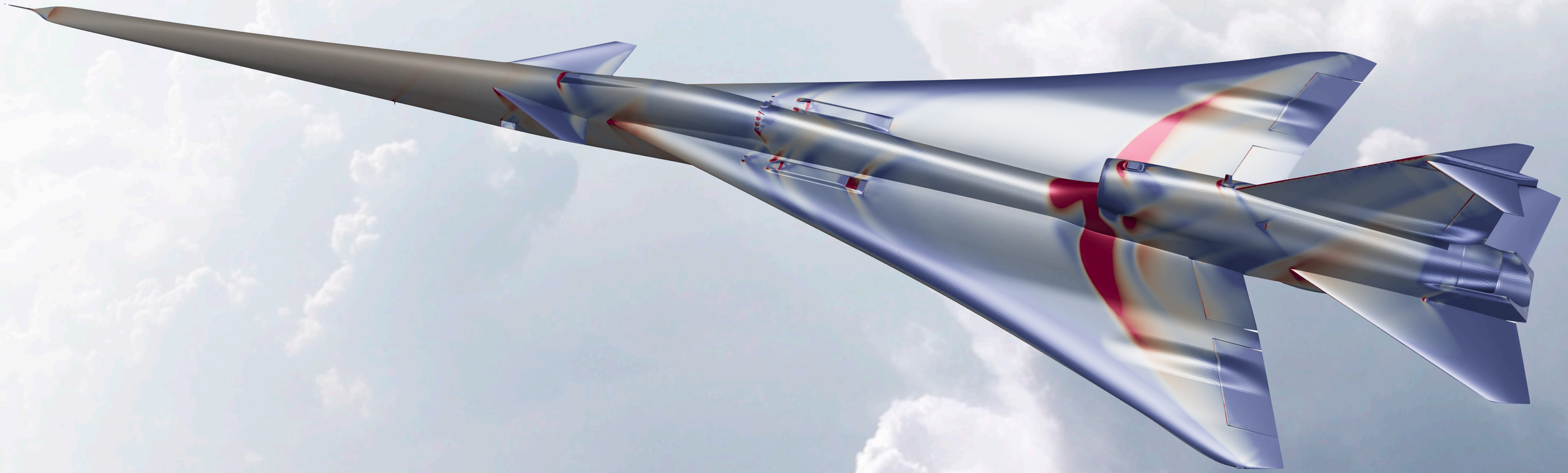
Joint work with
Michael Aftosmis and Sriram Rallabhandi

NASA Advanced Supercomputing Division, Ames Research Center

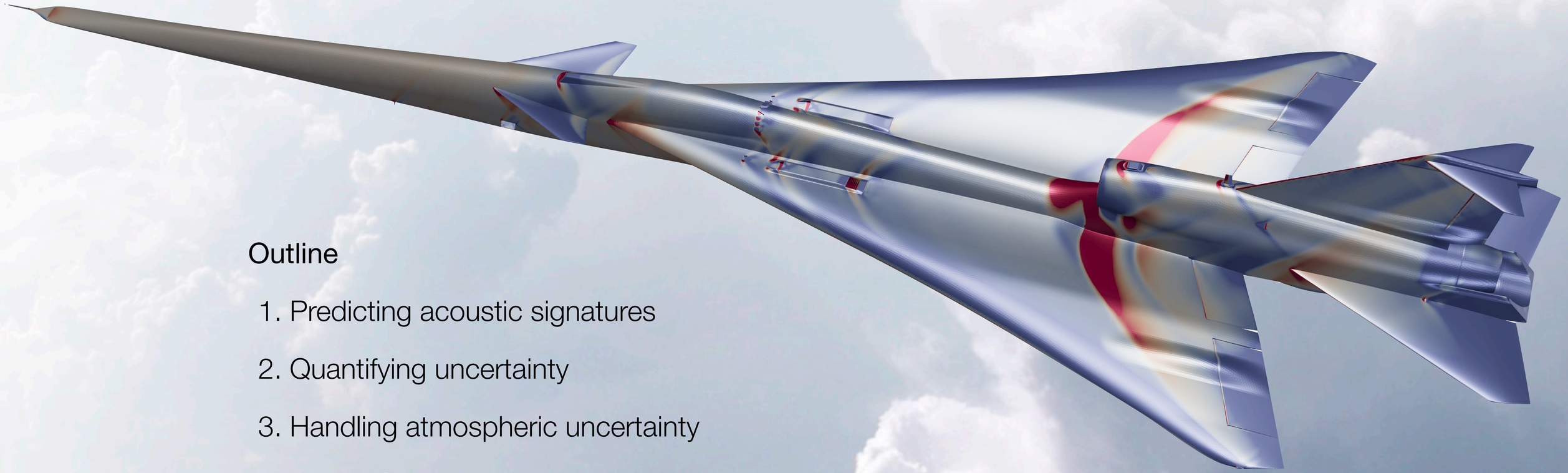


RCA High-Fidelity CFD Workshop
Lockheed-Martin Center for Innovation, Suffolk, VA
May 21, 2025

Goal is to quantify uncertainty in the level of noise reaching ground from supersonic aircraft at cruise conditions



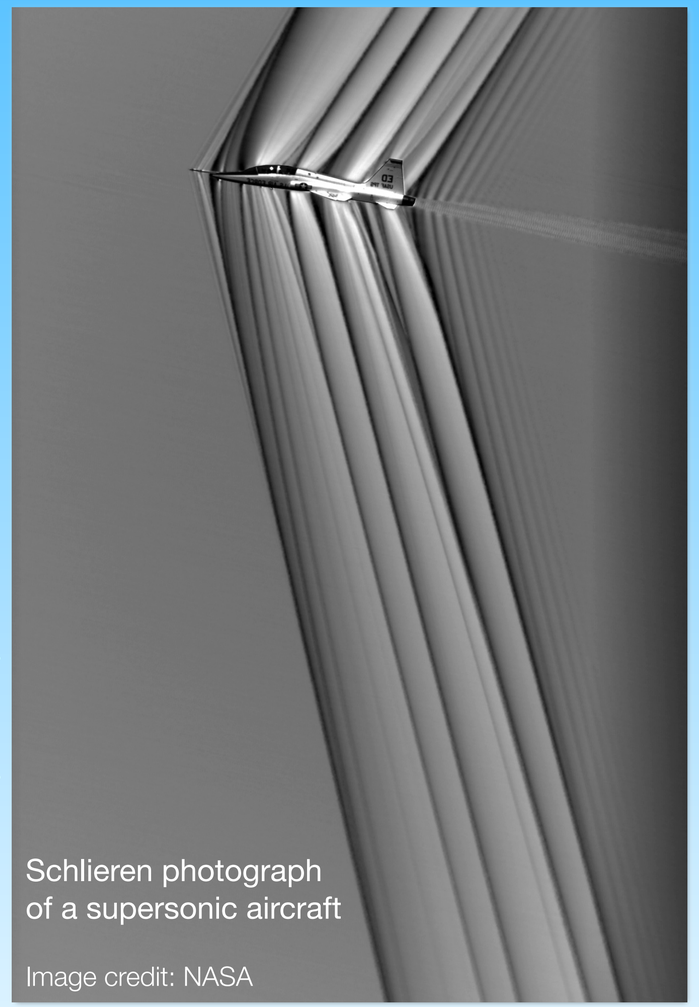
Goal is to quantify uncertainty in the level of noise reaching ground from supersonic aircraft at cruise conditions



Outline

1. Predicting acoustic signatures
2. Quantifying uncertainty
3. Handling atmospheric uncertainty
4. X-59 UQ

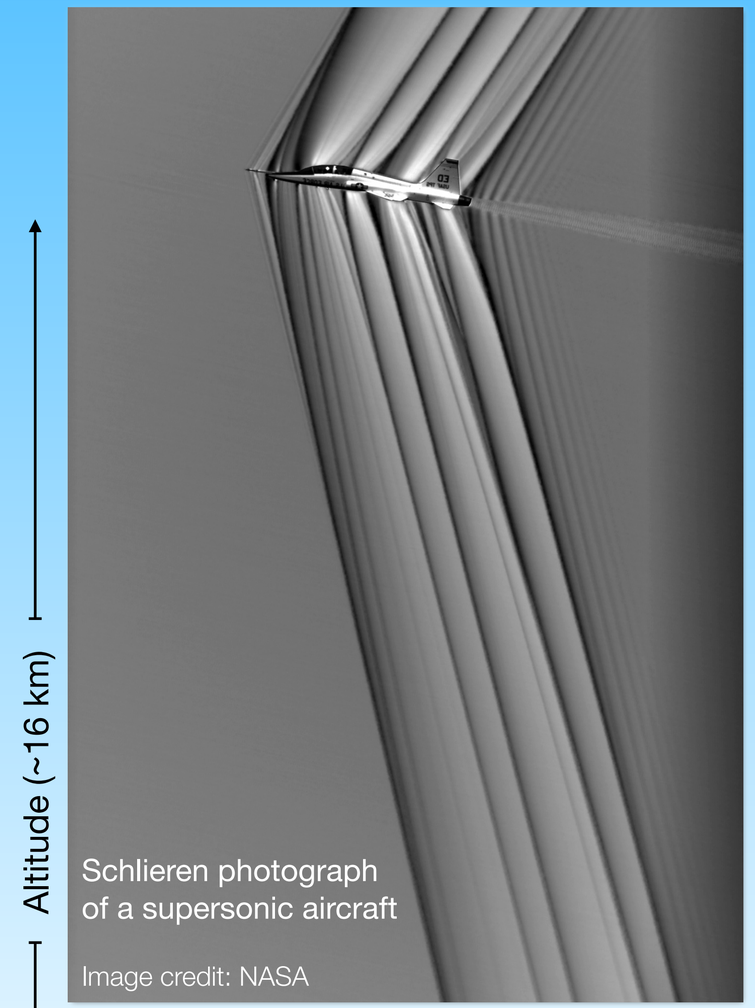
Sonic Booms

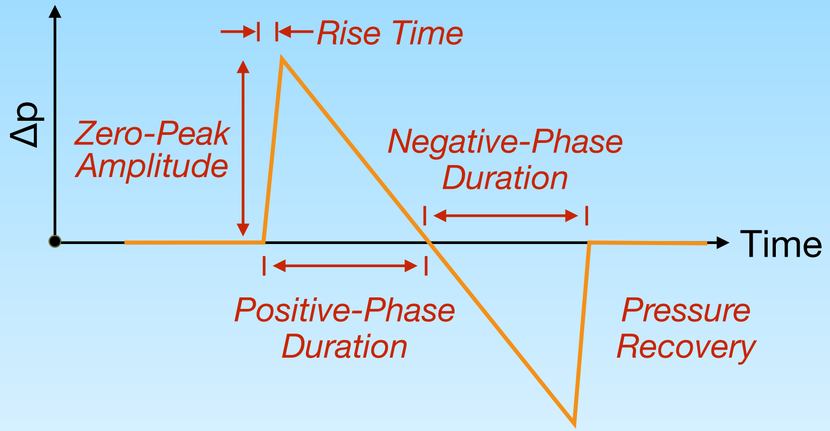

Schlieren photograph
of a supersonic aircraft
Image credit: NASA

Altitude (~16 km)
Ground Pressure
Signature

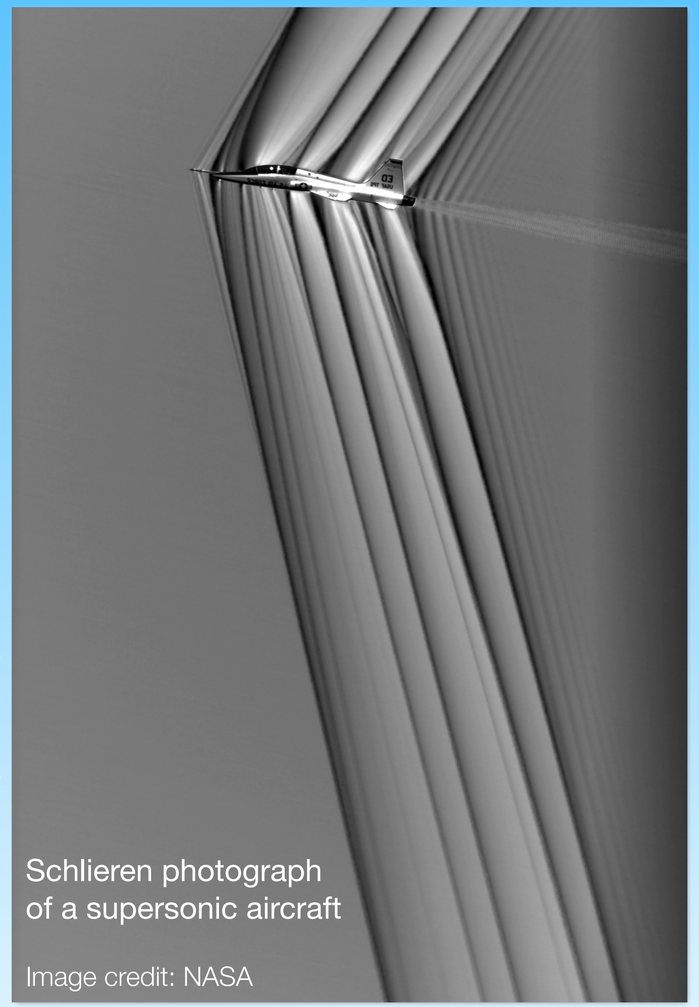
Sonic Booms

- Classical signatures are N-waves
- Sound characteristics are a function of the ground pressure signature

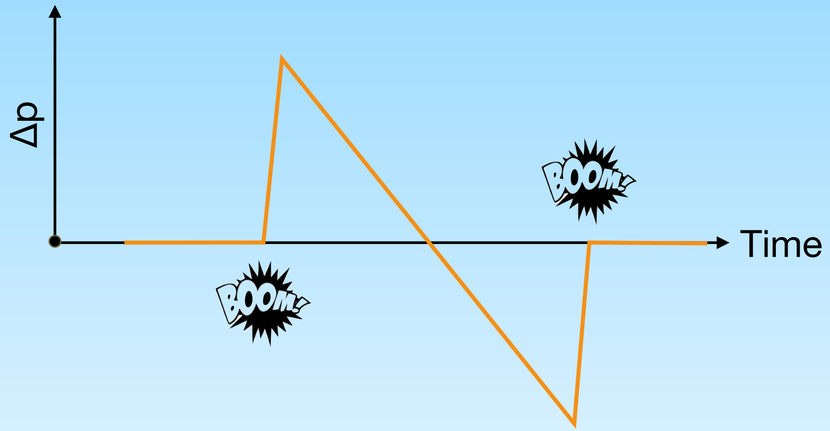




Sonic Booms



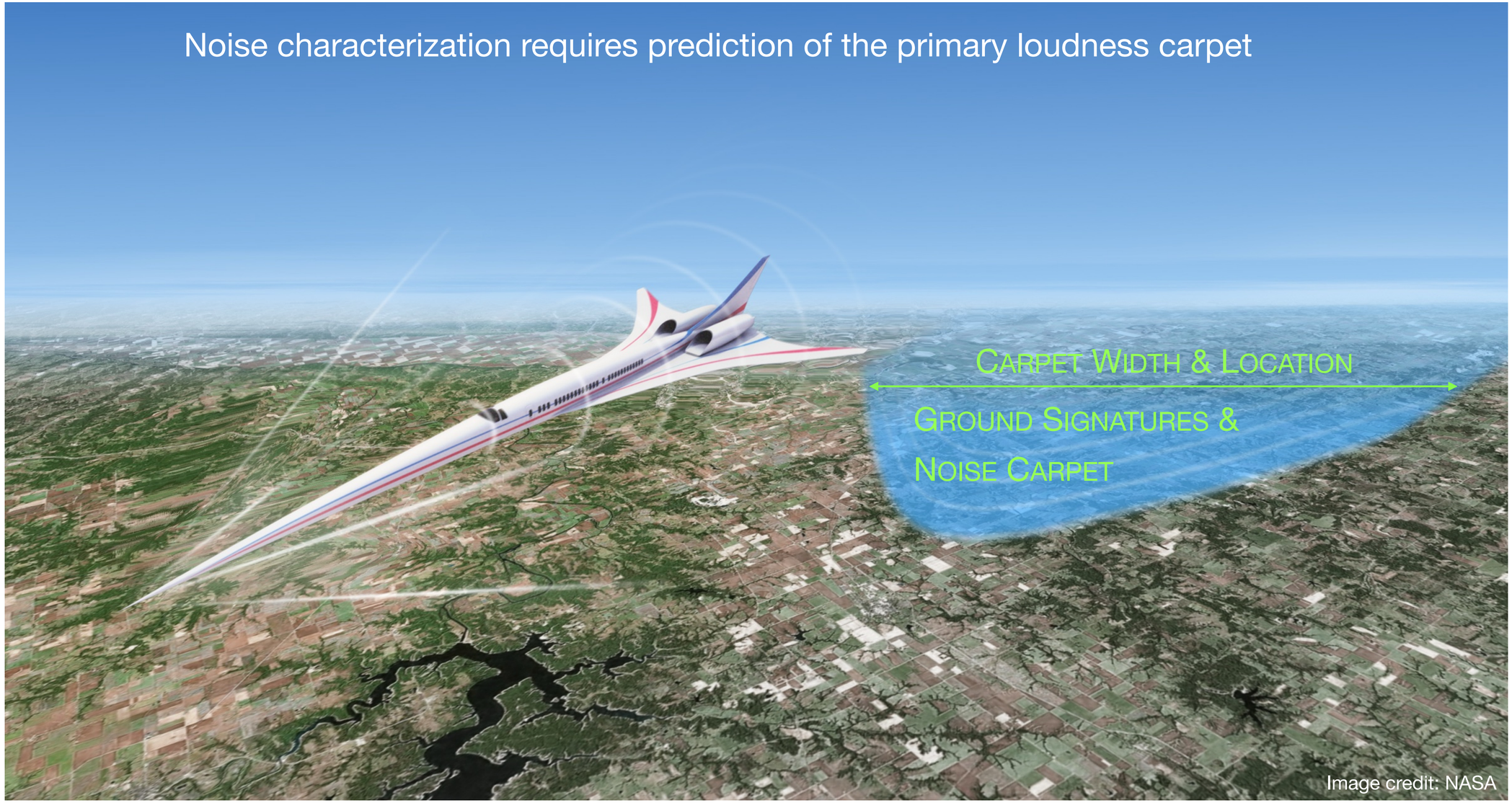
- Classical signatures are N-waves
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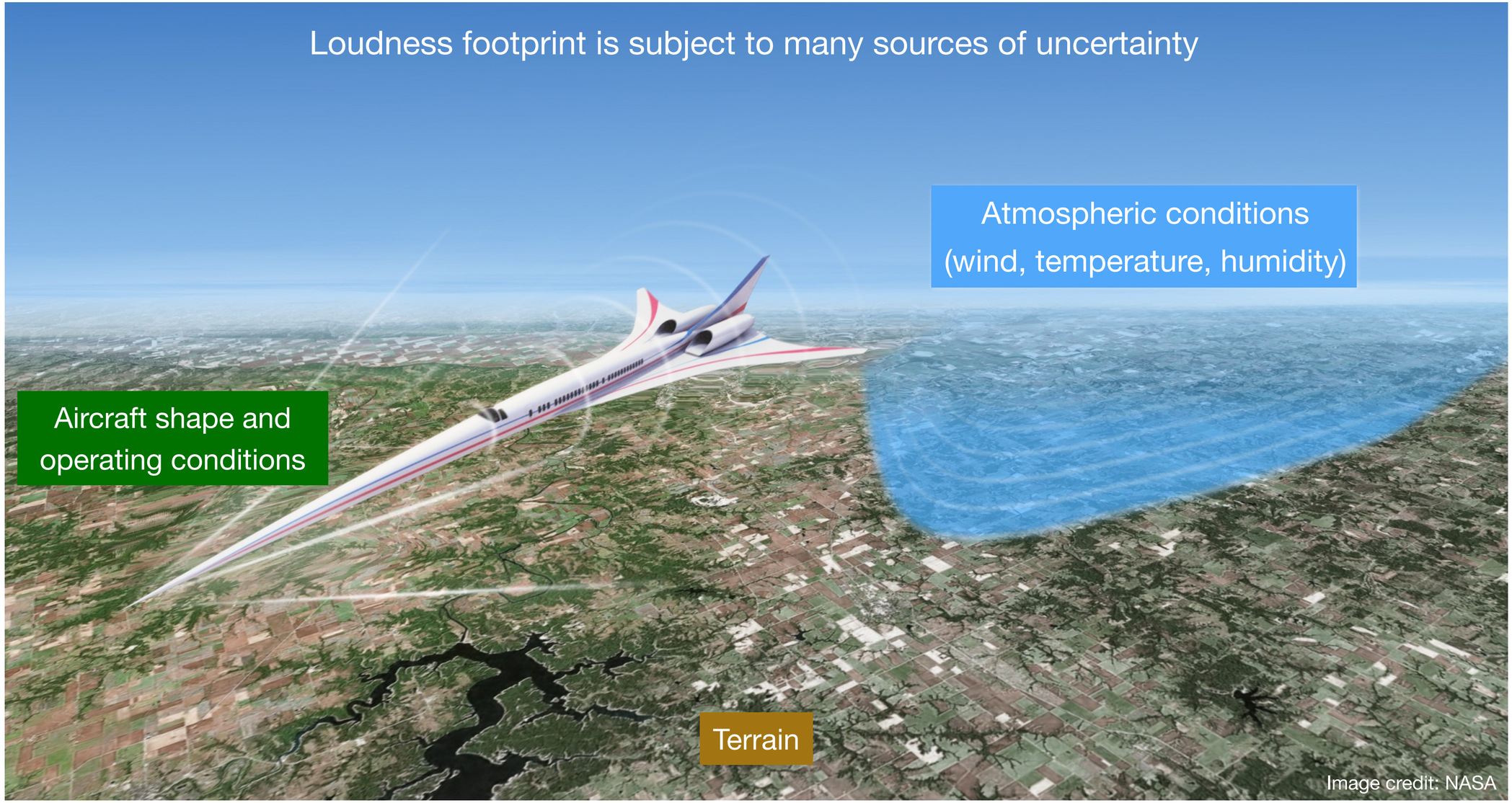
Loudness metrics (dB)

- Perceived Level PL
- Weighted Sound Exposure Level: A/B/C/D/ESEL
- Indoor Sonic Boom Annoyance Predictor
  - ISBAP = PL - 0.4201(CSEL-ASEL)

Predicting En-Route Noise



Predicting En-Route Noise



Objectives

1. Quantify uncertainty in low-boom outputs due to input sources of uncertainty

- Mean & standard deviation
- Probability density function
- Cumulative distribution function

- Noise metrics
  - Perceived Level (PL)
  - Sound Exposure Level (SEL)
  - Indoor Sonic Boom Annoyance Predictor (ISBAP)

- State of the atmosphere
  - Temperature and relative humidity
- Flight conditions
  - Mach number
  - Altitude



2. Assess and minimize sources of error
   - Directly apply UQ to high-fidelity simulations

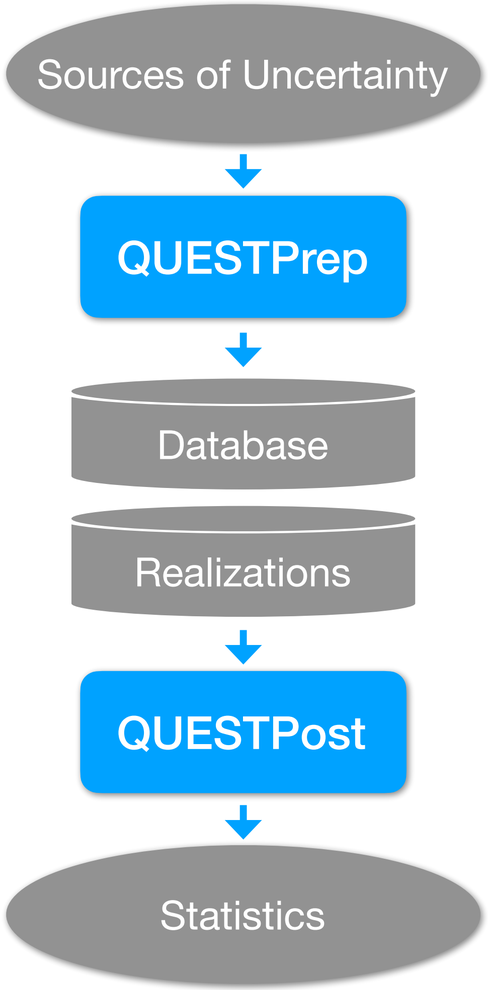
3. Minimize computational cost

Enable future en-route noise compliance through simulation-based analysis

Uncertainty Quantification with QUEST

Quantified Uncertainty with Error bias estimation Software Tools

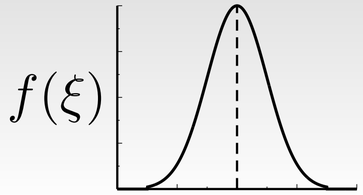
- Implements comprehensive set of UQ methods
  - Dense and sparse quadratures
  - Multi-level Monte-Carlo sampling
  - Kernel density estimation for PDF and CDF evaluation
  - Specialized numerics to handle stochastic discontinuities, e.g. those due to shocks
- Intrinsic post-processing of discretization errors
- Developed by Dr. Tim Barth in TTT-RCA
- Currently maintaining active QUEST server and webpage (NASA internal use only)



https://nas.nasa.gov/publications/software/quest/questresourcesV4.html

Evaluation of Statistics for Outputs of Interest

Input Probability Density Function



Expectation (Mean)

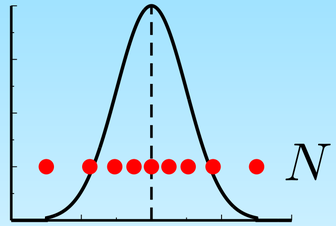
$$E[J(\xi)] = \int J(\xi) f(\xi) d\xi$$

Noise Output (PL)

Input Random Variable (Mach Number)

Numerical Approximation of Expectation

N - point Gauss-Patterson Quadrature



$$J_H(\xi_1), \ \ldots, \ J_H(\xi_N)$$

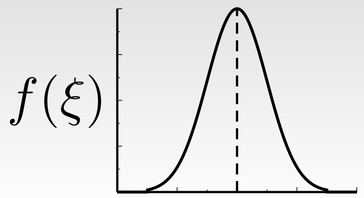
Approximate Outputs (Realizations)

$$\mu = E_N[J_H] = \sum_{i=1}^{N} (w \, J_H)_i$$

Quadrature Weights

Evaluation of Statistics for Outputs of Interest

Input Probability Density Function



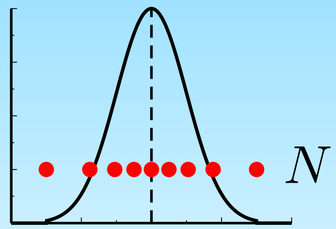
Expectation (Mean)

$$E[J(\xi)] = \int J(\xi) f(\xi) d\xi$$

Noise Output (PL)

Numerical Approximation of Expectation

N - point Gauss-Patterson Quadrature



$$J_H(\xi_1), \ \ldots, \ J_H(\xi_N)$$

Approximate Outputs (Realizations)

$$\mu = E_N[J_H] = \sum_{i=1}^{N} (w \, J_H)_i$$

Quadrature Weights

Numerical Error in Expectation



$$\varepsilon = E[J] - E_N[J_H] = E[\varepsilon_H] + \varepsilon_Q[J_H]$$

$\varepsilon_H = |J - J_H|$ Discretization Error

Error in Expectation Quadrature

Approach: Non-Intrusive Implementation

QUEST

Sources of Uncertainty

QUESTPrep

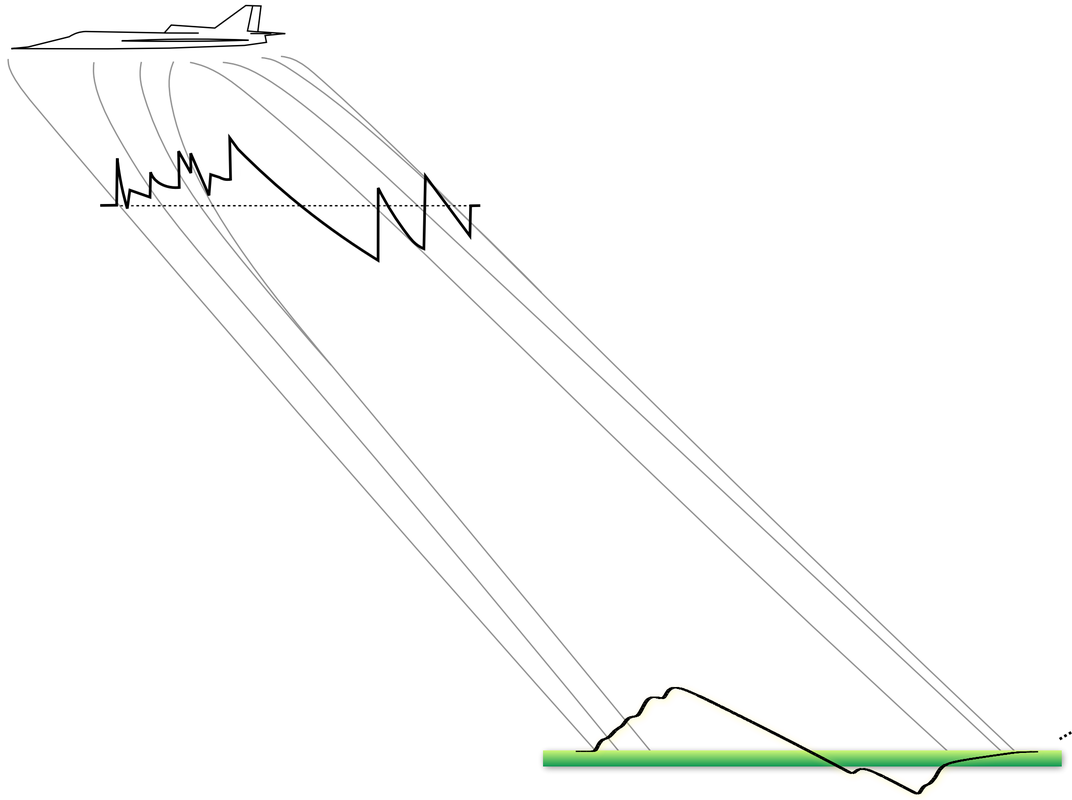
Database

Realizations

QUESTPost

Statistics

Acoustic Signature Simulation

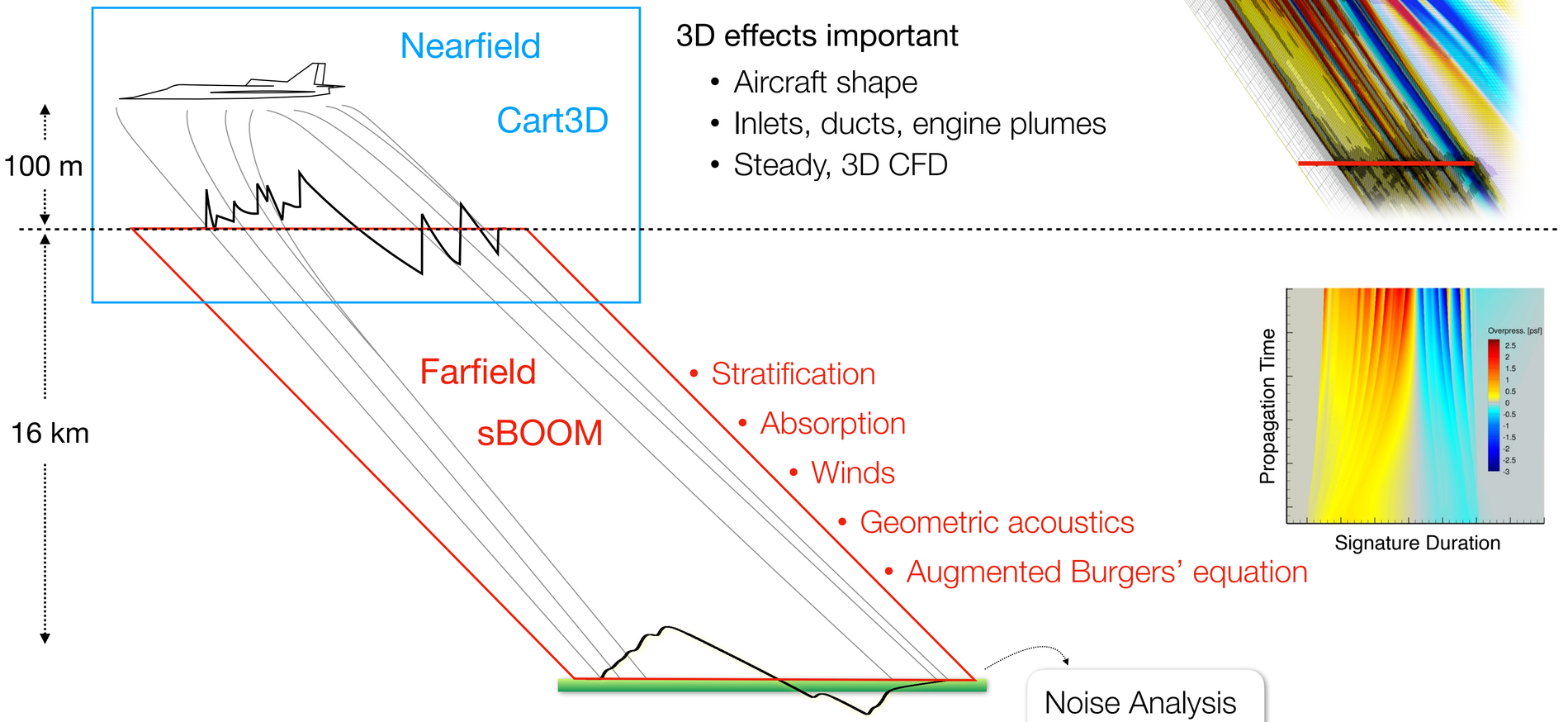


Noise Analysis

PL, ISBAP (dB)

Realizations, e.g. noise metric $J_H(\xi_1),\ J_H(\xi_2),\ \ldots,\ J_H(\xi_N)$

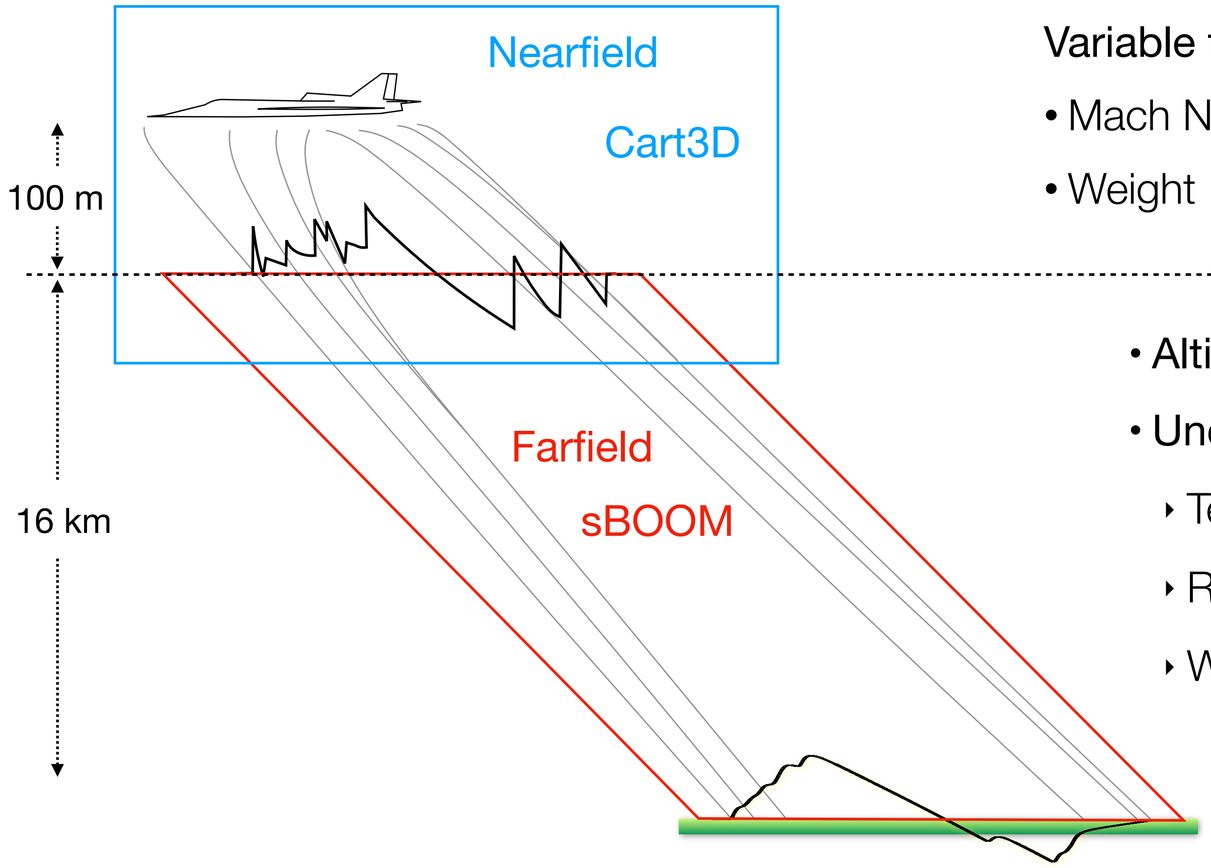
Approach: Nearfield-Farfield Decomposition



3D effects important

- Aircraft shape
- Inlets, ducts, engine plumes
- Steady, 3D CFD

Sources of Uncertainty



Variable flight conditions

- Mach Number
- Weight

- Altitude
- Uncertain state of atmosphere
  - Temperature
  - Relative Humidity
  - Wind

Atmospheric Observations

Current focus is on variability in atmospheric profiles at Edwards AFB

- Location for X-59 acoustic validation flights

Database of atmospheric profiles from European Centre for Medium-Range Weather Forecasts (ECMWF) Reanalysis v5 (ERA5)

- Selected observations from 2003 to 2022
- Selected August at 18:00 UTC (10 am PT)

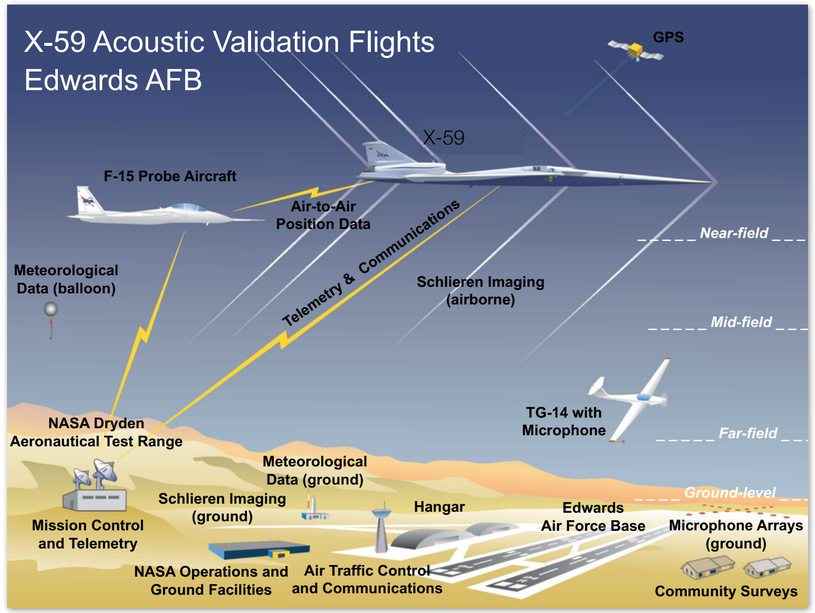
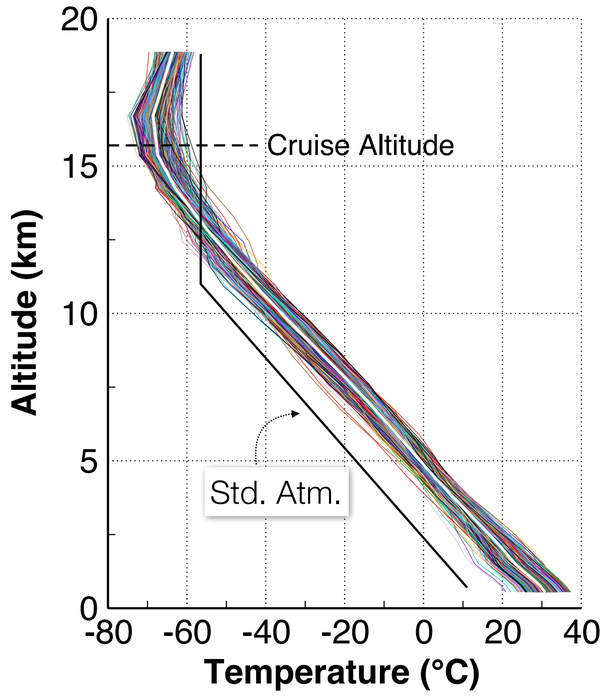
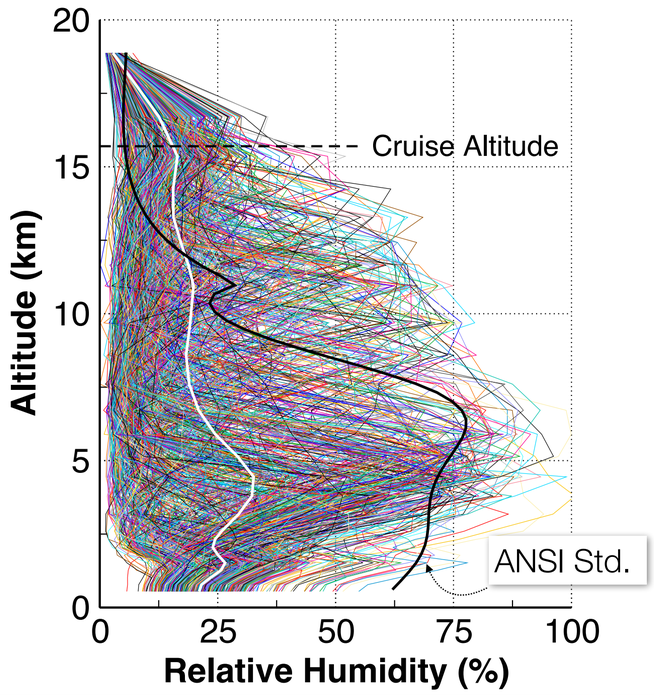


Image Credit: NASA

Edwards AFB
18:00 UTC (10 am PT)
ERA5 Dataset 2003 - 2022
620 profiles

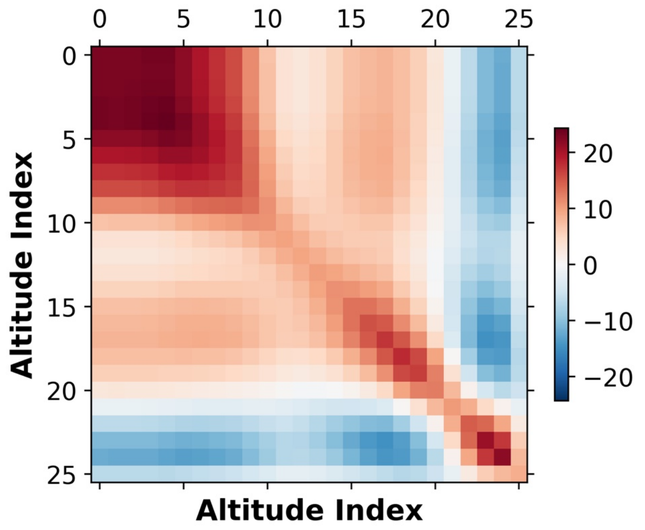




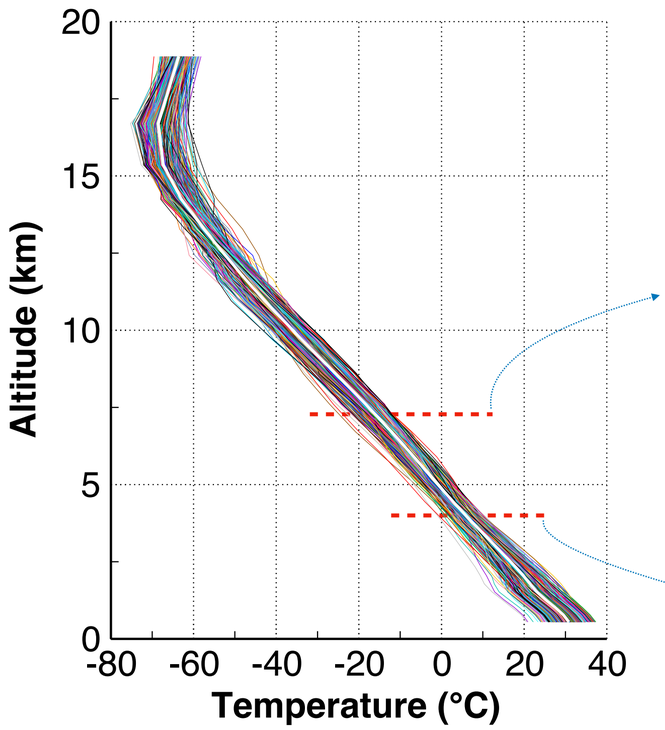
Properties of Atmospheric Profiles

Challenge is to accurately and efficiently model stochastic fluctuations in the atmosphere

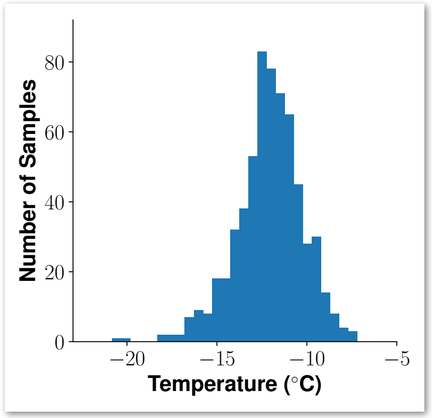
Atmospheric profiles are correlated fields

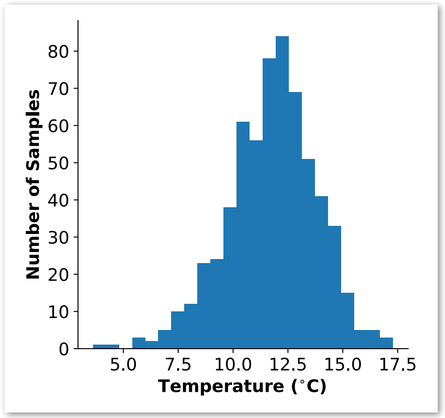


Temperature Covariance Matrix



Temperature profiles are normally distributed (Gaussian)



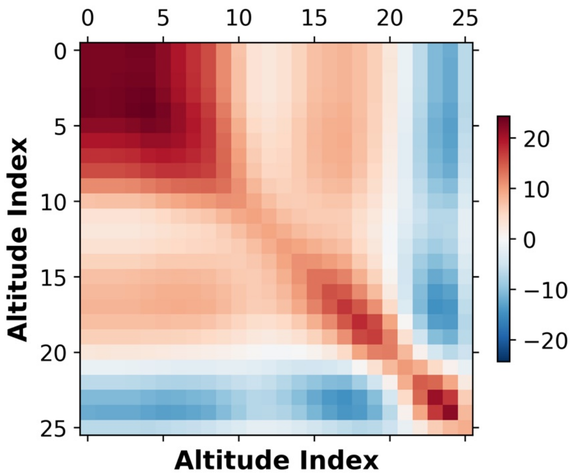


Find independent random variables that efficiently parameterize observed profiles

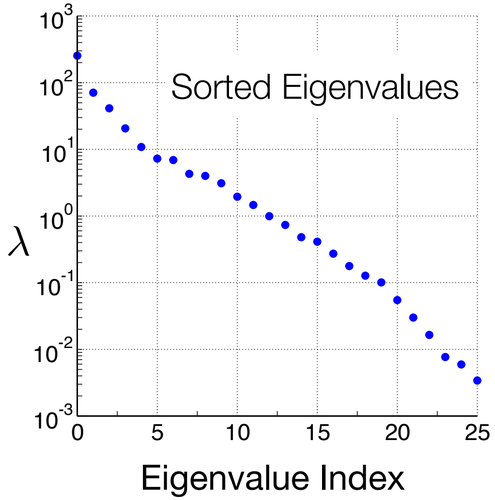
We seek a compact, low-dimensional representation

Karhunen-Loeve (KL) Expansion

- Optimal representation of Gaussian process correlated random fields
- Main idea is to use the covariance of the observed data to anchor the parameterization
  - Diagonalize discrete covariance matrix
  - Select largest eigenvalues to model dominant fluctuations of atmospheric properties



$$\mathbf{C} = \frac{1}{K-1}\mathbf{W}\mathbf{W}^{\mathrm{T}}$$



For an ensemble of discrete random-process atmospheric profiles

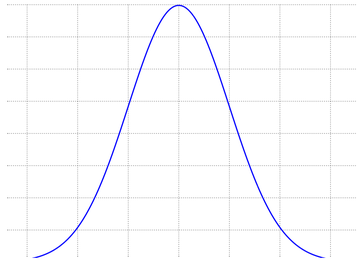
$$\mathbf{w} = \{\mathbf{w}_1, \ldots, \mathbf{w}_k, \ldots, \mathbf{w}_K\}$$

Parametric form of each profile

$$\widetilde{\mathbf{w}}_k = \boldsymbol{\mu}(\mathbf{w}) + \mathbf{w}'_k = \boldsymbol{\mu}(\mathbf{w}) + \sum_{i=1}^{M} \xi_i \sqrt{\lambda_i}\boldsymbol{\phi}_i$$

Covariance eigenpairs (λ, ϕ)

Mean Profile

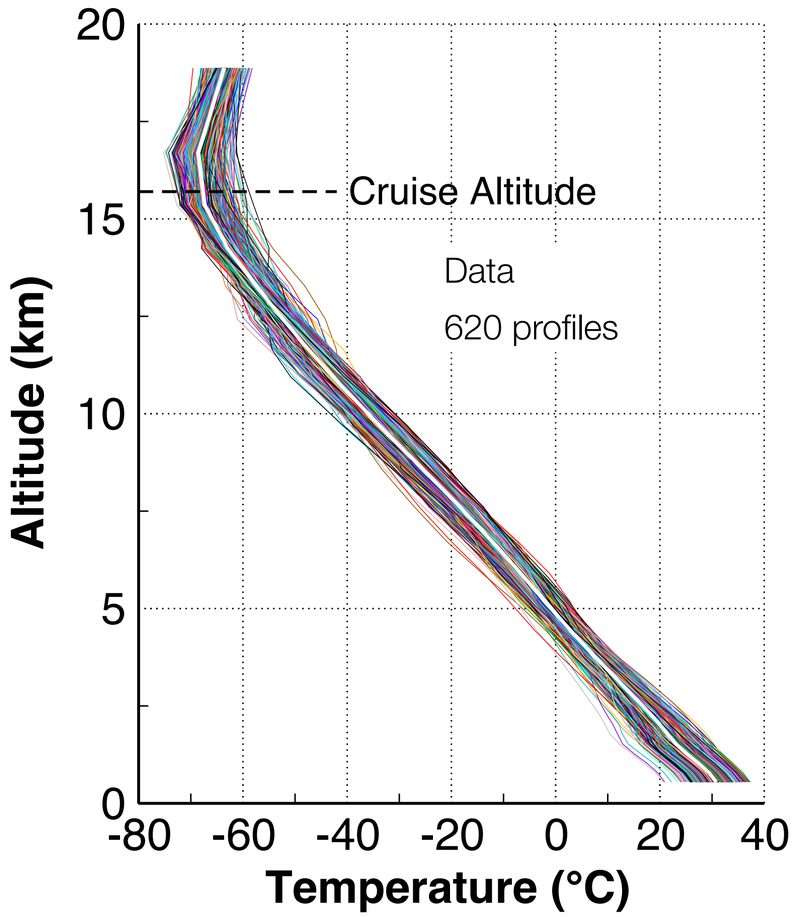


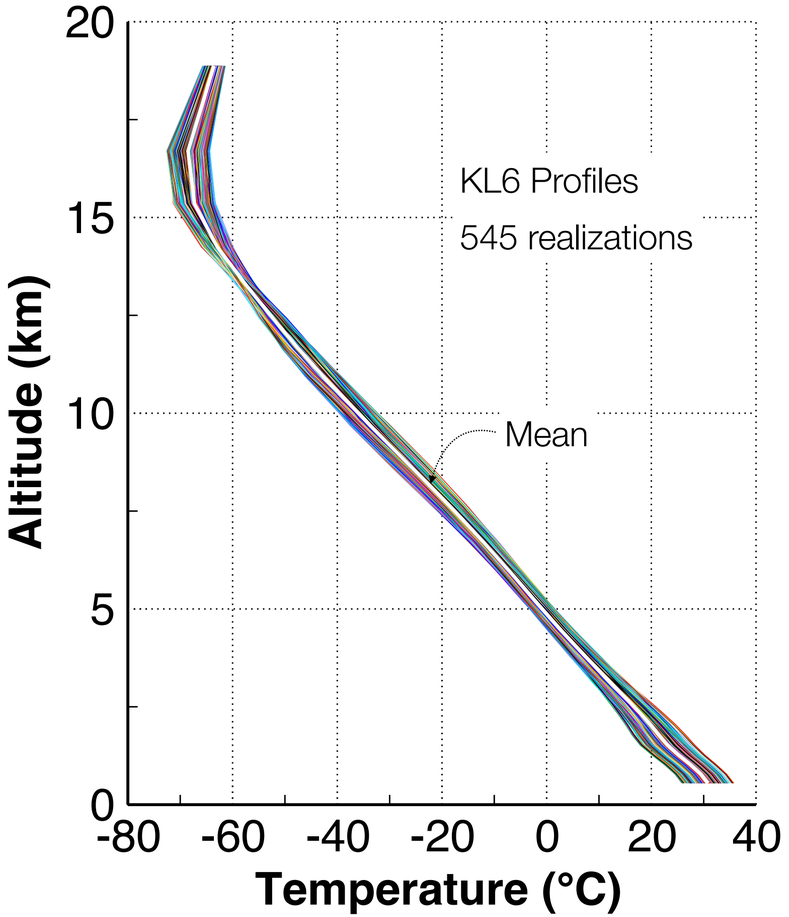
Independent Random Variables

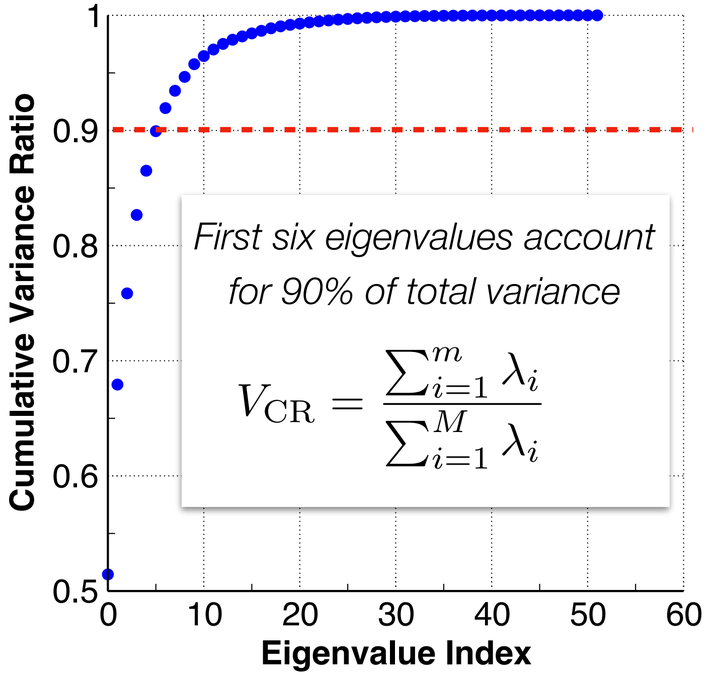
$$\xi_i = \mathcal{N}_{n\sigma}(\mu = 0, \sigma^2 = 1)$$

Observed and Modeled Temperature Profiles

Consider KL6 Expansion $\longrightarrow \widetilde{\mathbf{w}}_k = \boldsymbol{\mu} + \sum_{i=1}^{6} \xi_i \sqrt{\lambda_i} \boldsymbol{\phi}_i$



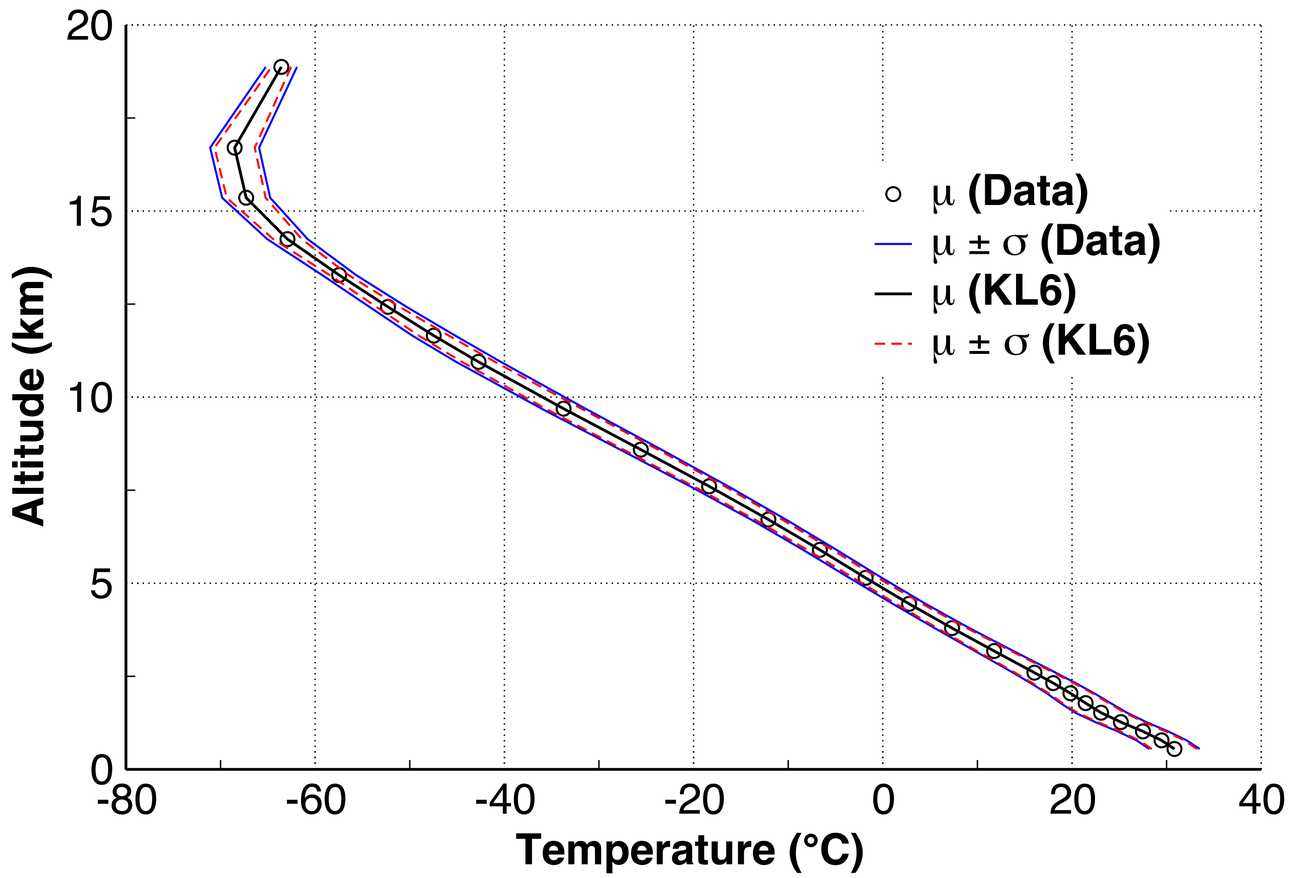


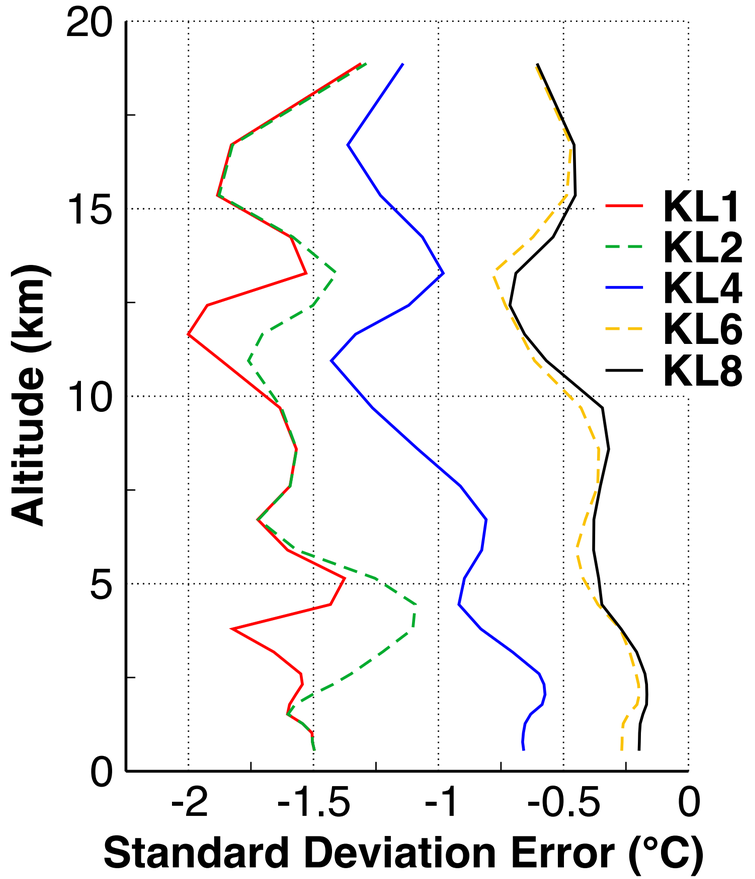


Good agreement between modeled and measured profiles

- Infrequent outliers not sampled well
- Excellent agreement near the mean

Errors in Temperature Statistics





Mean profiles agree exactly (by construction)

KL6 standard deviation errors are small

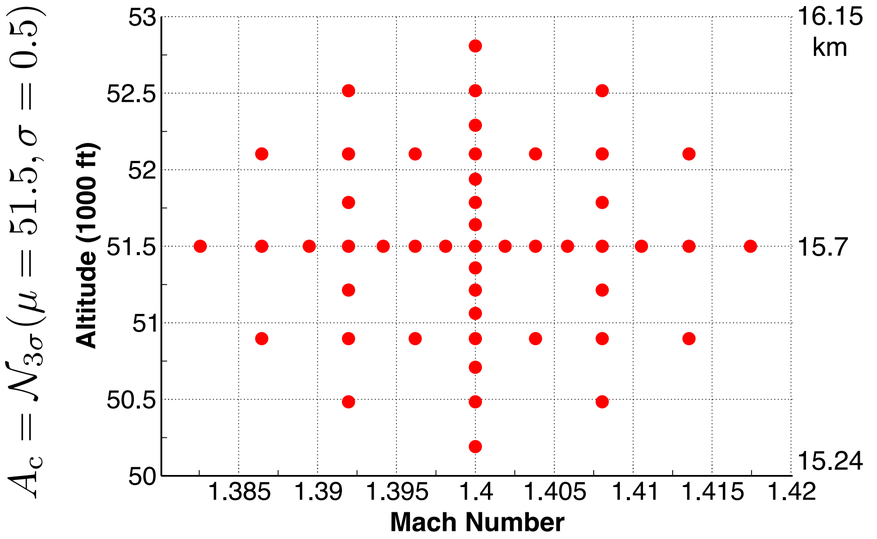
Error in standard deviation decreases as number of terms is increased

Uncertain Operating Conditions and Atmosphere

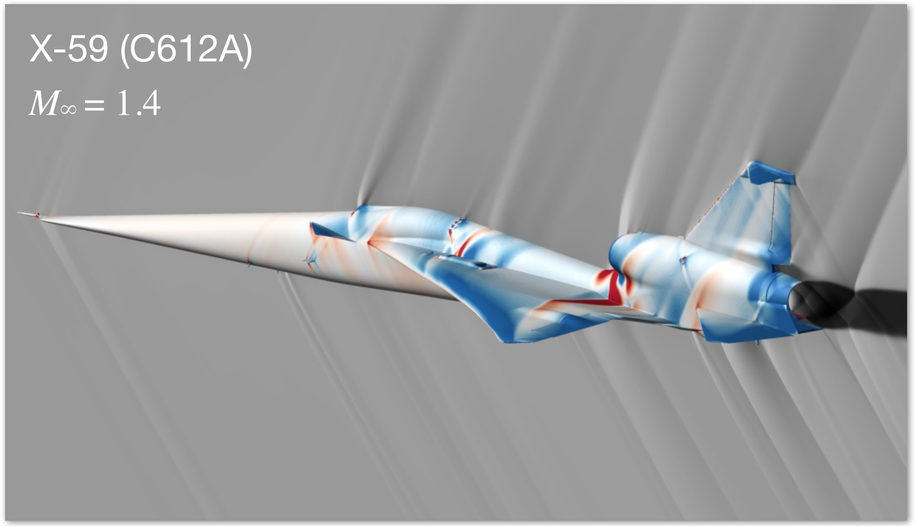
Characterize uncertainty in X-59 ground signature and loudness due to uncertainty in cruise Mach number and altitude, and the atmosphere

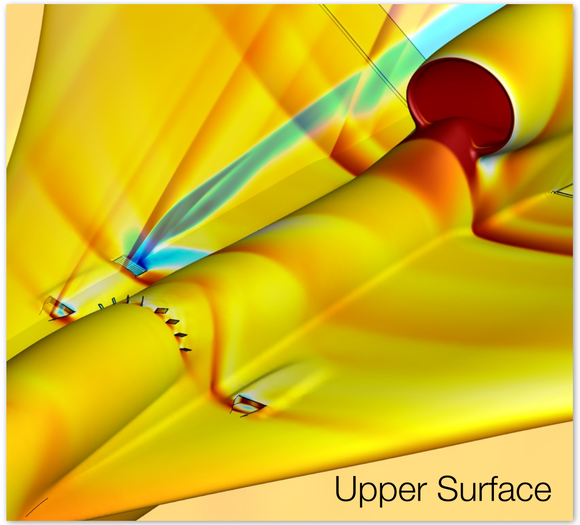
- 8 independent random variables
  - Assume normal distributions for Mach number (M_∞) and altitude (A_c)
  - KL-6 parameterization of T and RH profiles
- Assume flights at Edwards AFB in August, around 10 am PT

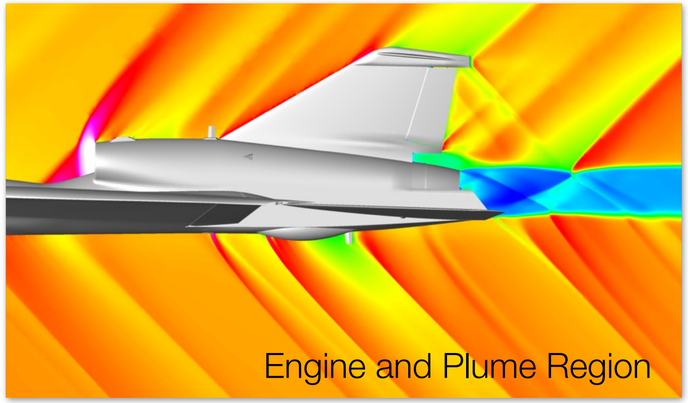
Level 4 Sparse Gauss-Patterson Quadrature



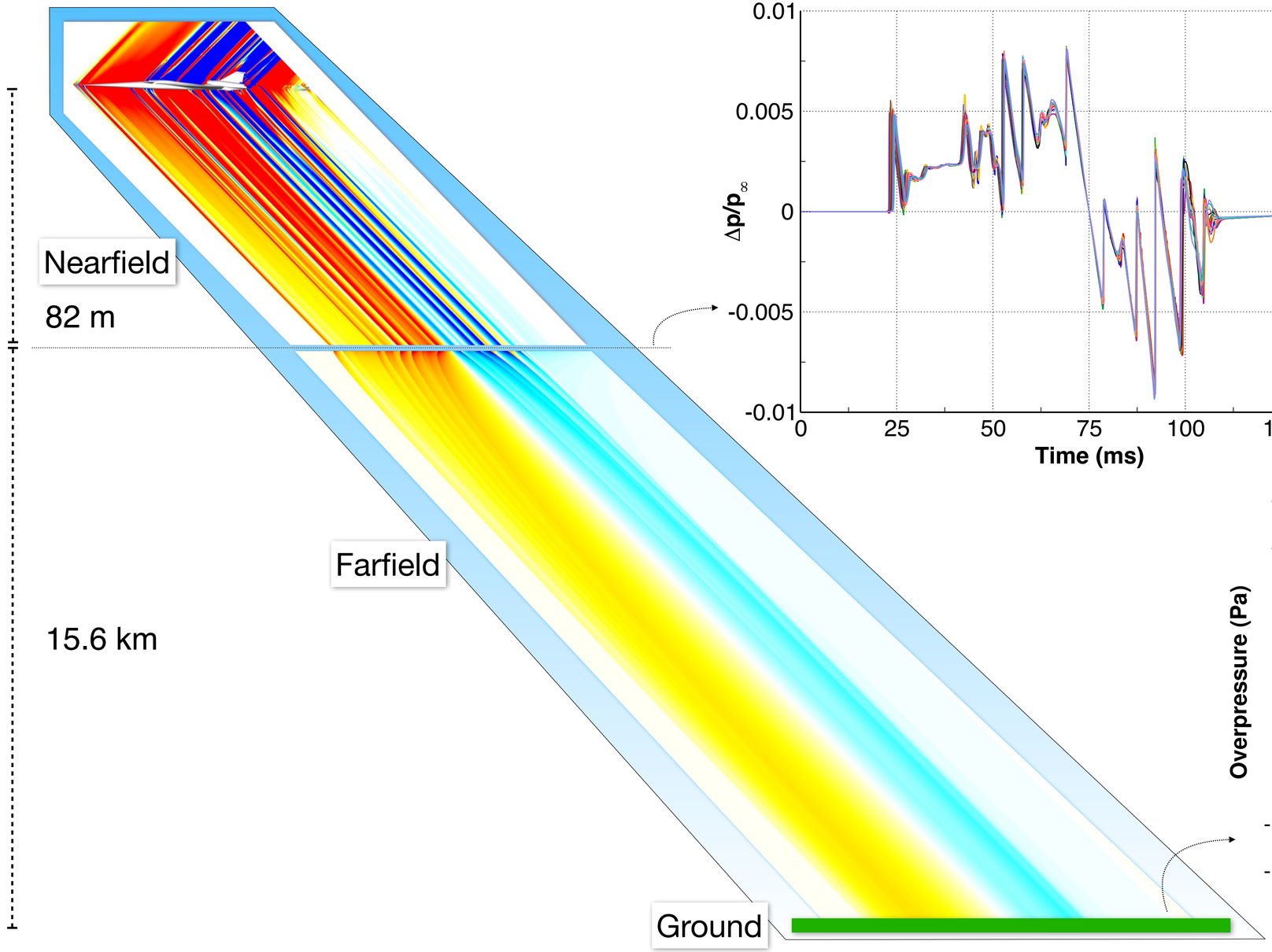
$$M_\infty = \mathcal{N}_{3\sigma}(\mu = 1.4, \sigma = 0.0066)$$







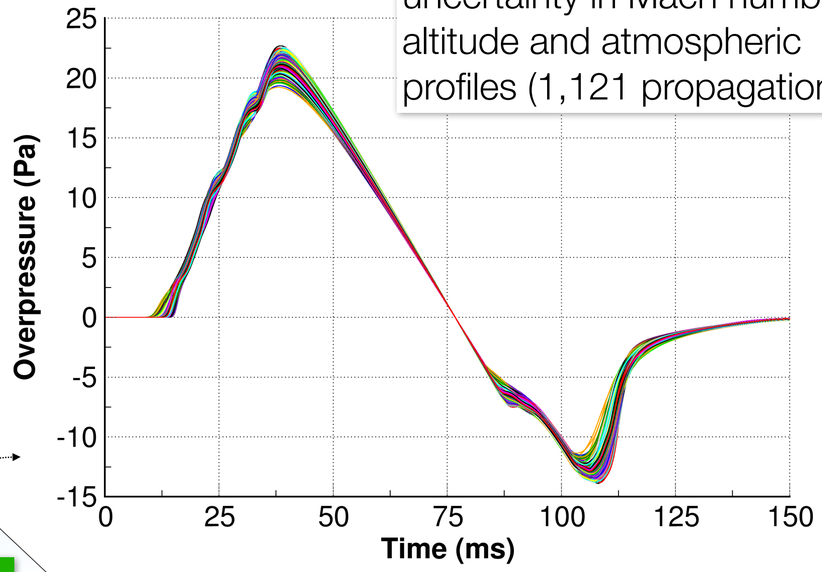
Uncertain Operating Conditions and Atmosphere



Nearfield signatures due to uncertainty in Mach number (15 solutions)

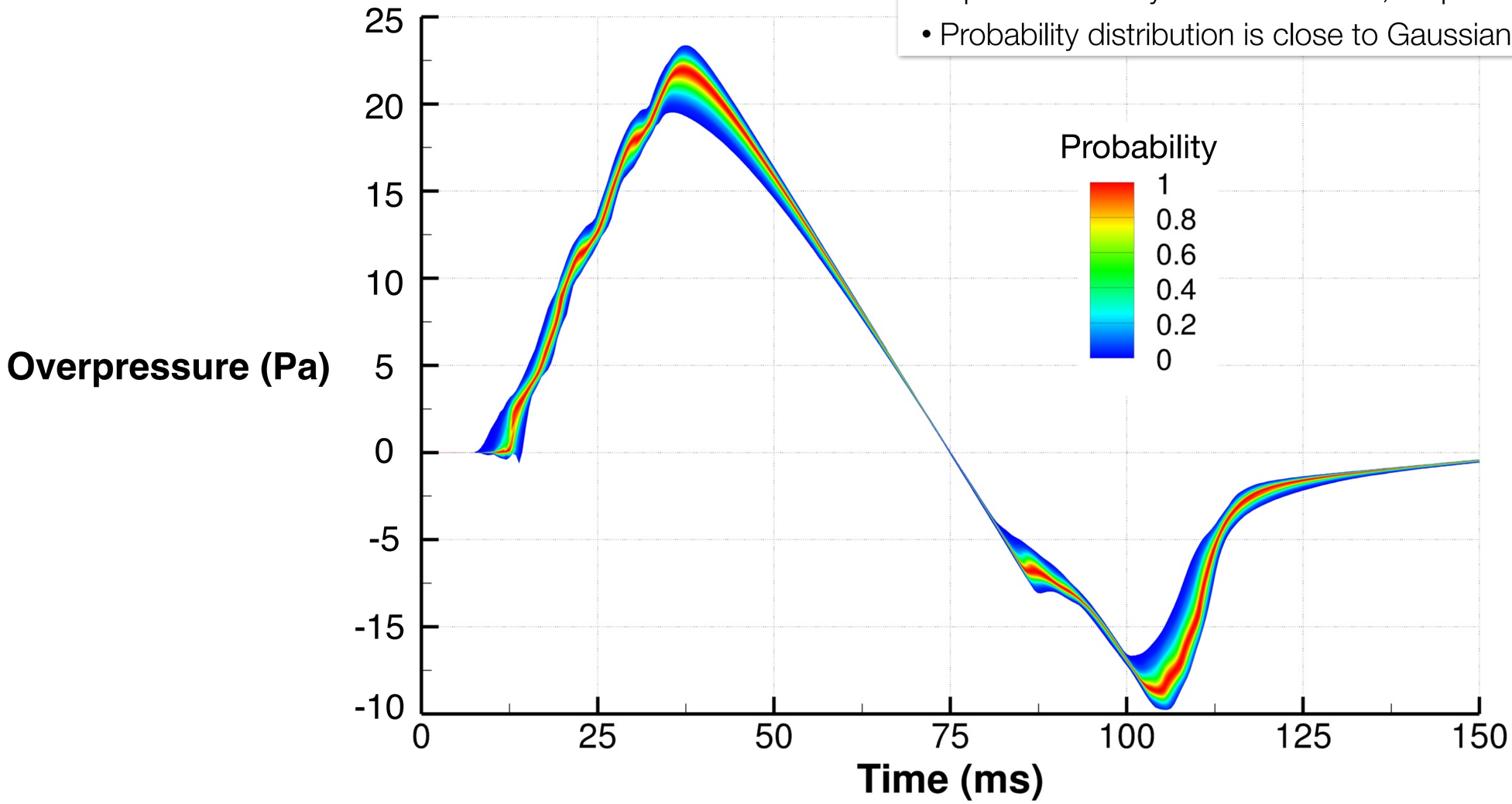
"Overnight" turnaround on small clusters frequently achieved in practice

Ground signatures due to uncertainty in Mach number, altitude and atmospheric profiles (1,121 propagations)

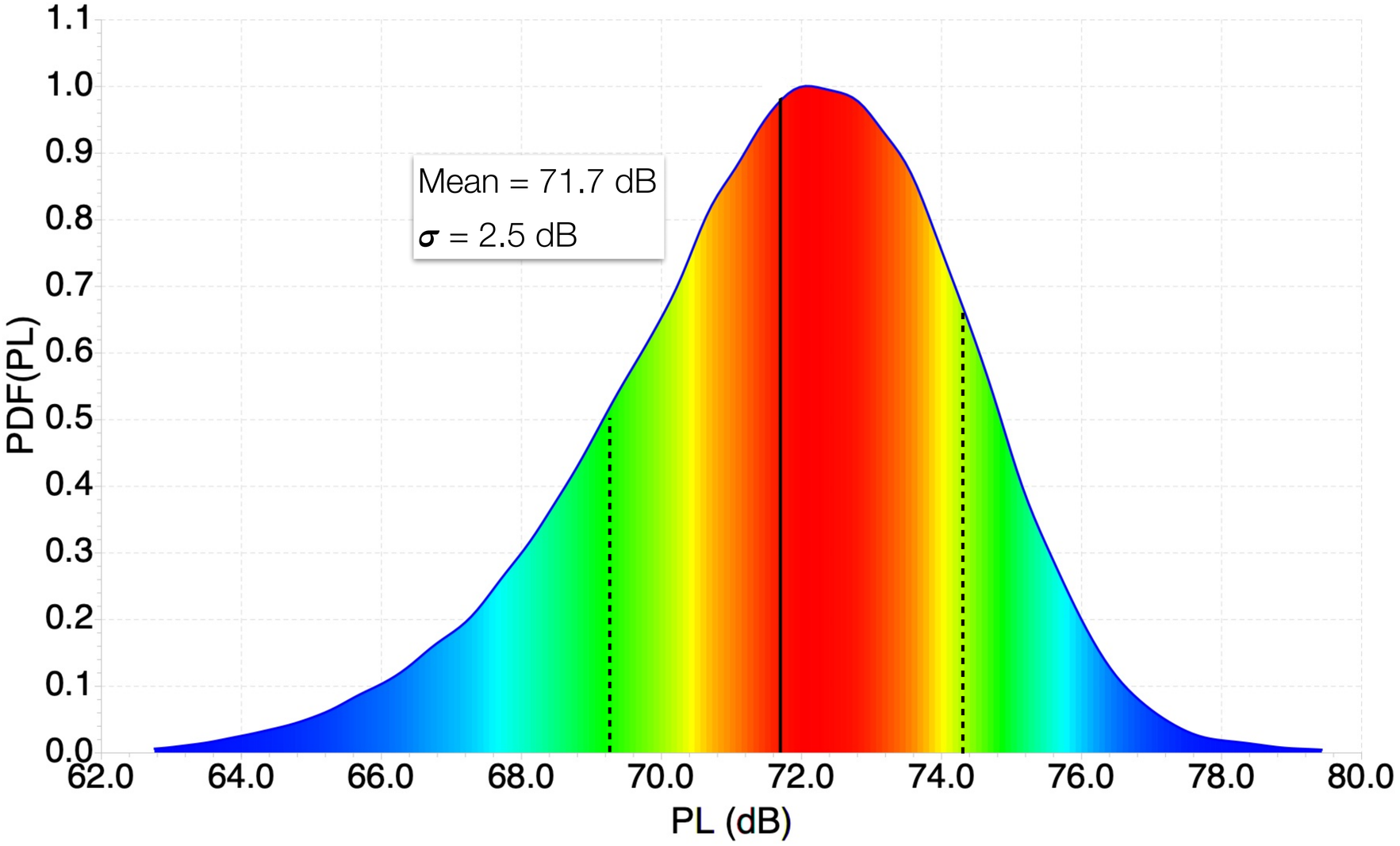


Ground Signature Probability Density

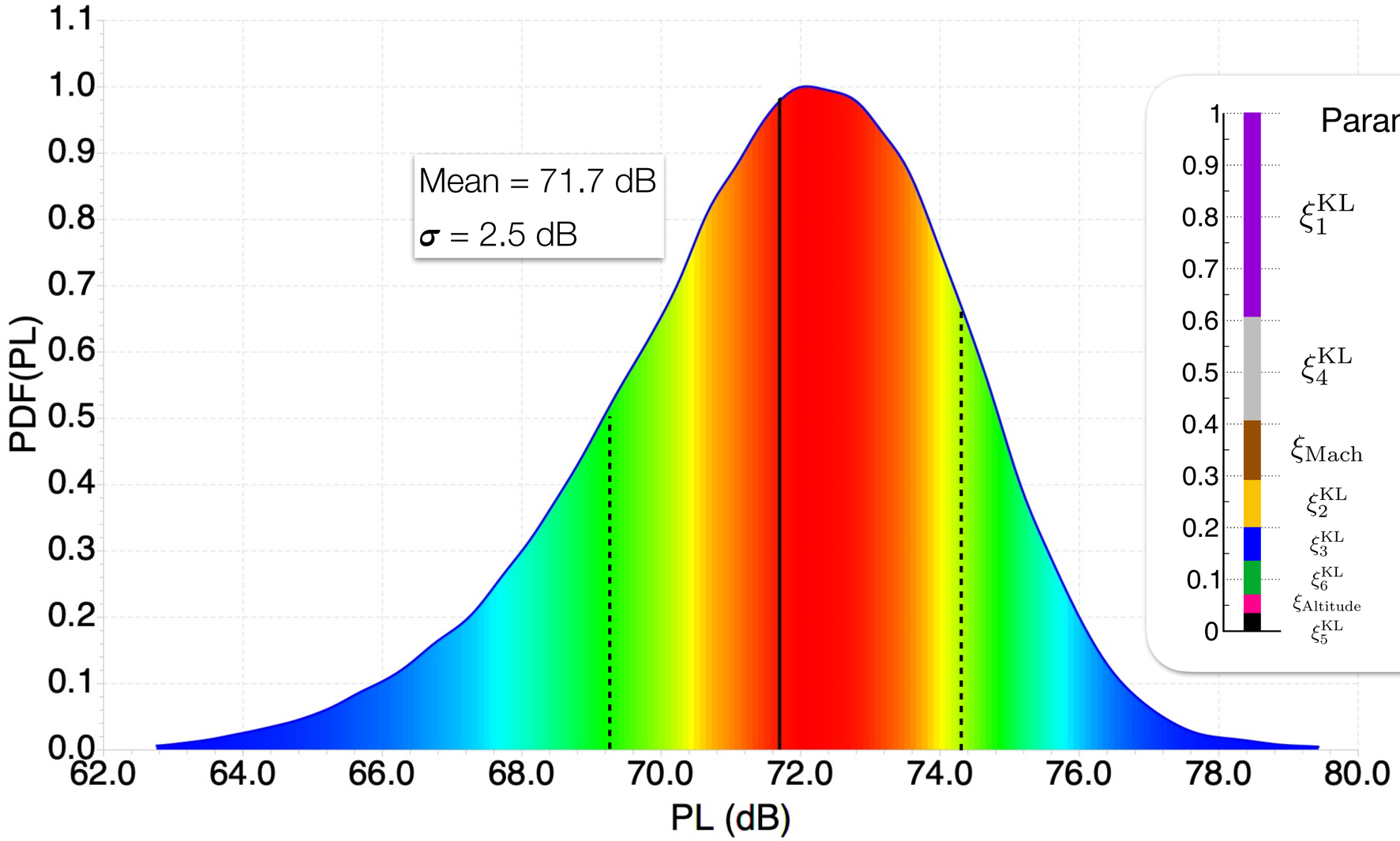
- Input uncertainty affects rise time, amplitude and signal duration
- Probability distribution is close to Gaussian



Perceived Loudness PDF

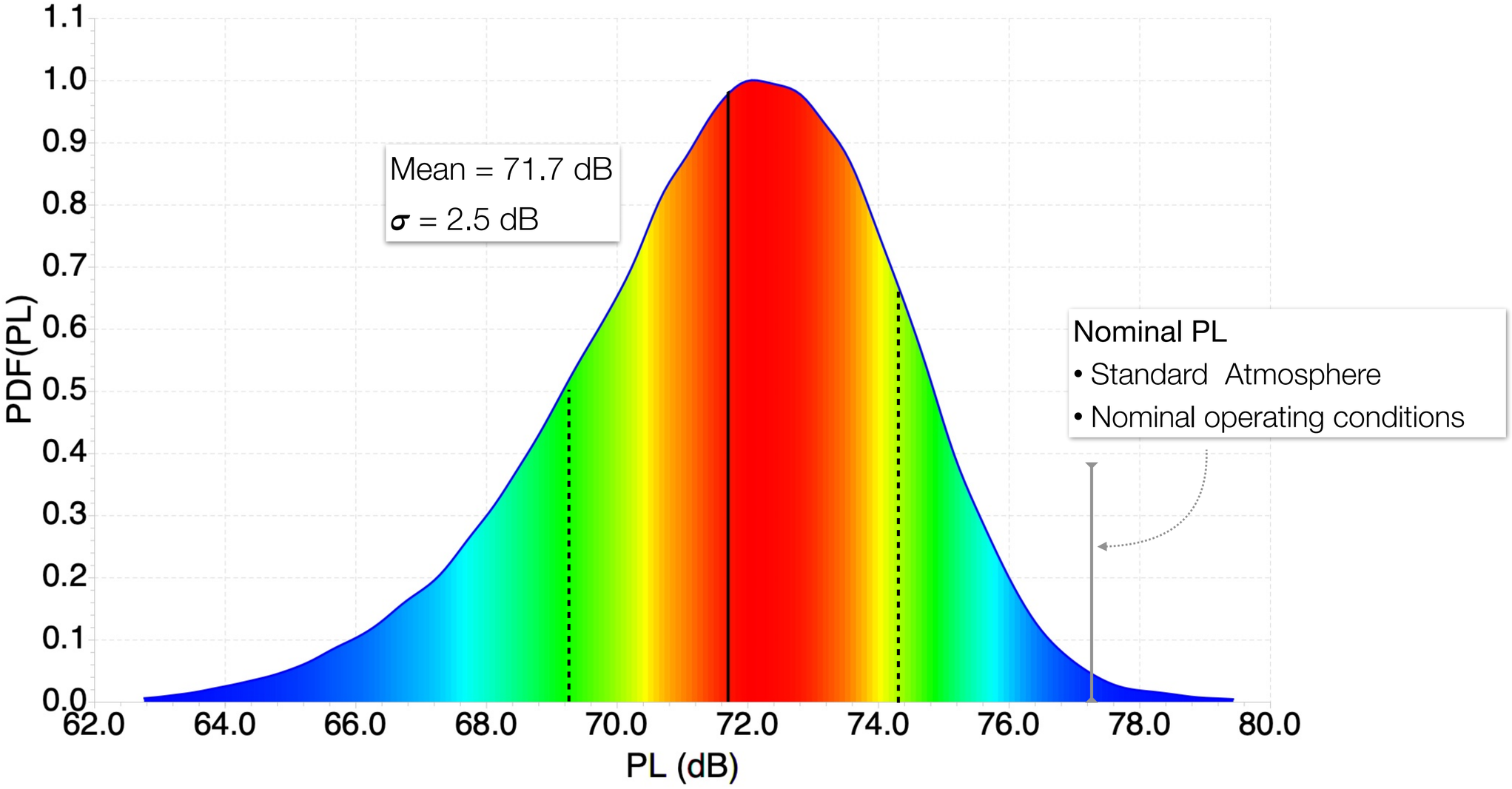


Ranking of Uncertain Parameters



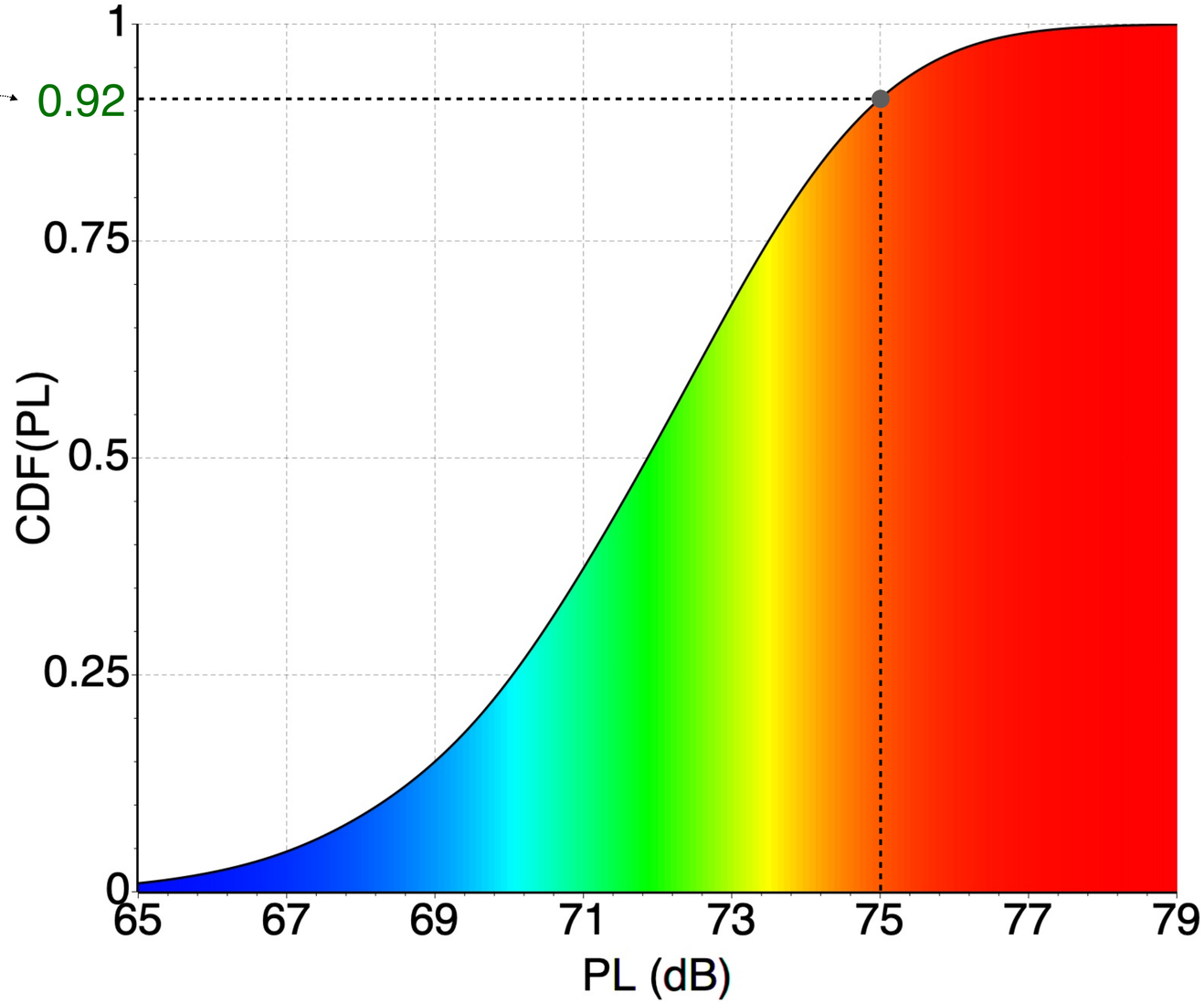
- Fractional contribution to standard deviation
- Atmospheric uncertainty most dominant (KL1,4)
- Influence of Mach number is moderate
- Altitude rank among lowest in current problem formulation

Nominal Perceived Loudness



Cumulative Distribution Function for PL

- 92% chance of success
  - Probability of exceeding 75 PL dB at Edwards AFB for flights in August at 18:00 UTC due to uncertainty in cruise Mach number, altitude and atmospheric conditions is 8%
- Estimate neglects variability in aircraft mass and trim, and atmospheric turbulence



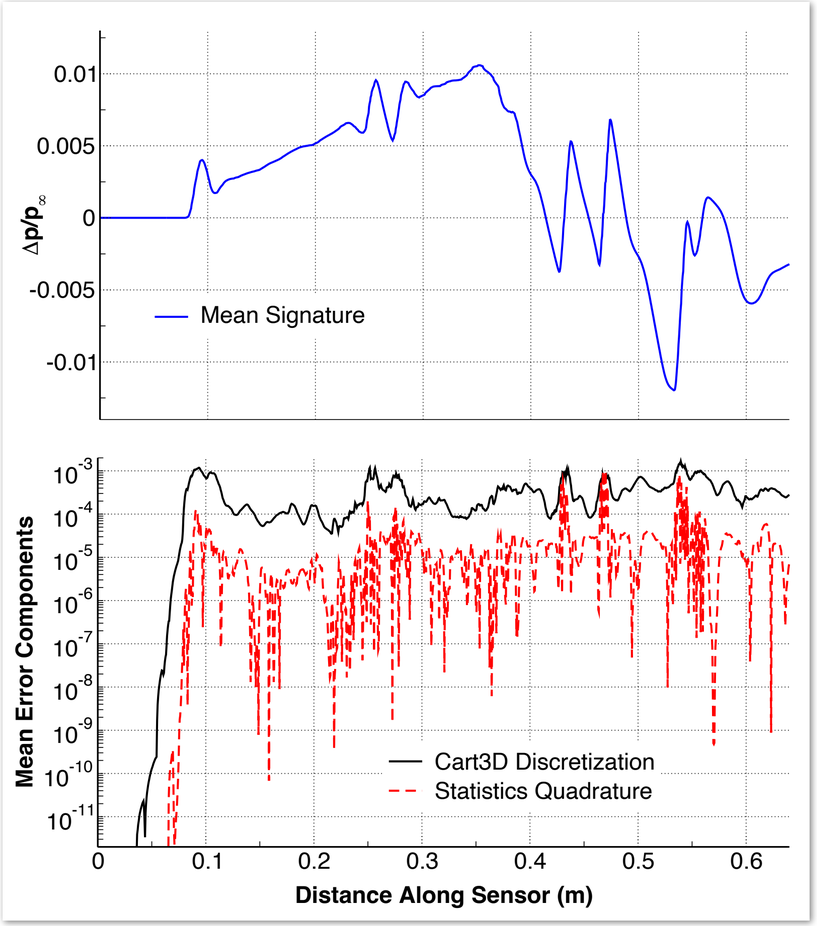
Summary

1. Developed an efficient parameterization of atmospheric profiles using Karhunen-Loeve expansion
   - Expansion with just 6 parameters is an excellent compromise between accuracy and efficiency
2. Atmosphere has a major influence on noise metrics
   - Standard deviation 4-8% in loudness metrics
   - Two terms of the KL expansion account for 60% of the uncertainty
3. A morning flight on any day in August has a 92% probability of not exceeding 75 PL dB

- **Demonstrated efficient uncertainty quantification for noise under conditions expected during test flights**
  - Affordable cost on real-world problems, despite direct use of high-fidelity simulations and complex aircraft geometry
- **Important step toward en-route noise certification by simulation-based analysis**

Next Steps

- Continue stewardship of current release
- Development of new capabilities
  - Ranking input uncertainties for higher dimensional outputs
  - Leveraging automated error analysis to control error bias
  - Benchmarking MLMC for large problems
  - Investigating model-form errors
- Applications beyond quiet supersonic flight



NPP Post-doc opportunity:

“Uncertainty Quantification for Aircraft Certification by Analysis”

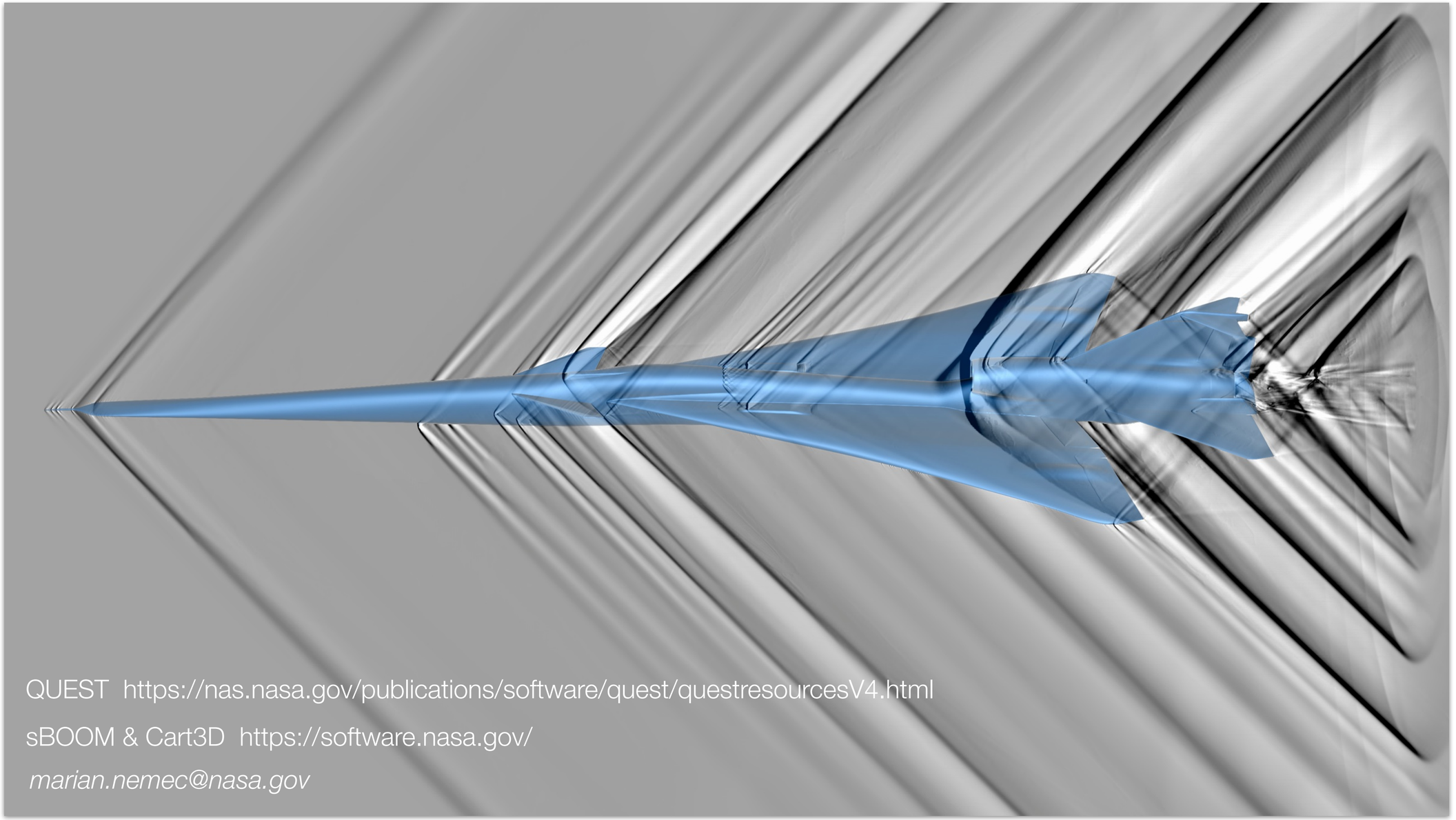
https://zintellect.com/Opportunity/Details/0133-NPP-NOV25-ARC-Aero

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- NASA's Transformational Tools and Technologies Project
- NASA's High-End Computing Program at Ames Research Center for computing resources



Questions



QUEST https://nas.nasa.gov/publications/software/quest/questresourcesV4.html

sBOOM & Cart3D https://software.nasa.gov/

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