

Formulation of an Uncertainty Analysis Methodology for Computational Fluid Dynamics of External Flows over Launch Vehicles

Avery White* and Kyle M. Hanquist[†]
*Department of Aerospace & Mechanical Engineering
University of Arizona
Tucson, AZ, 85721*

Paul A. Schallhorn[‡]
*NASA Launch Services Program
National Aeronautics and Space Administration
Kennedy Space Center, FL, 32899*

Michael Elmore[§]
*a.i. solutions, Inc.
Kennedy Space Center, FL, 32899*

Space launch vehicles are often engineered with large safety factors due to the scarcity and cost of ground tests. In addition, these vehicles often carry expensive and delicate payloads that must go through extreme launch conditions. Computational Fluid Dynamics (CFD) can be utilized to support the design of these vehicles but often have uncertainty and errors associated with them that are often not fully quantified. This paper provides a methodology and investigation for quantifying and estimating the uncertainty for CFD over space launch vehicles when no experimental data exists. A generic launch vehicle was used for this study. The primary output quantities of interest for the study are the aerodynamic coefficients. While there are numerous sources of uncertainty, this study investigates the impact of mesh quality, boundary conditions, and turbulence models. The largest uncertainty lies in the y-moment coefficient, where the y-axis is aligned with the angle of attack, and is due to perturbing the turbulence model for a zero-degree angle of attack configuration or perturbing the mesh quality for a six-degree angle of attack configuration. The smallest uncertainty is in the x-moment coefficient, with a zero percent uncertainty for all configurations.

Nomenclature

AoA	=	angle of attack
A_{ref}	=	reference area
c_{b1} , etc.	=	empirical constants in the turbulence model
C_D	=	drag coefficient
C_L	=	lift coefficient
C_p	=	pressure coefficient
C_{M_x}	=	x-moment coefficient
C_{M_y}	=	y-moment coefficient
C_{M_z}	=	z-moment coefficient
d	=	distance to the wall
E	=	total energy per unit mass

*Graduate Research Assistant, AIAA Student Member. ✉: stockdaleavery@arizona.edu

[†] Assistant Professor, AIAA Associate Fellow. ✉: hanquist@arizona.edu

[‡] VADR Technical Manager. ✉: paul.a.schallhorn@nasa.gov

[§] Fluids Analyst. ✉: michael.elmore@nasa.gov

$\bar{F}^c$	=	convective flux
F_D	=	drag force
F_L	=	lift force
$\bar{F}^v$	=	viscous flux
f_{v2} , etc.	=	empirical functions in the turbulence model
$g, r, \tilde{S}$	=	intermediate variables
k_e	=	turbulent kinetic energy
k	=	coverage factor
L_{ref}	=	reference length
M_x	=	x-moment
M_y	=	y-moment
M_z	=	z-moment
p	=	static pressure
$\mathcal{R}$	=	residual vector
r	=	CFD result
T	=	temperature
U	=	conservative variable
u_{input}	=	uncertainty from input parameters
u_{num}	=	numerical uncertainty
u_{val}	=	total uncertainty
v	=	freestream velocity
$\bar{v}$	=	velocity vector
X_i	=	input parameter
x_i	=	cartesian coordinates
X_L	=	lower value of result
X_U	=	upper value of result
Y_n	=	nested parameter of an input parameter
κ	=	thermal conductivity
μ	=	viscosity
μ_d	=	dynamic viscosity
ν	=	kinematic molecular viscosity
$\tilde{\nu}$	=	working variable of the turbulence model
ρ	=	fluid density
σ	=	turbulent Prandtl number
$\bar{\tau}$	=	viscous stress tensor
χ	=	intermediate variable
Ω	=	vorticity magnitude

I. Introduction

Space launch vehicles must face various harsh conditions, from launch and transition to upper-atmosphere flight. Ground tests of these space launch vehicles, while crucial to the engineering process, are typically expensive and time-consuming, so many preliminary tests are done using CFD [1]. As with any measurement, CFD comes with inherent uncertainty. It is important to differentiate between uncertainty and errors and we do so following AIAA CFD Guidelines [2]. Uncertainty is defined as:

“A potential deficiency in any phase or activity of the modeling process that is due to the lack of knowledge.”

Whereas error is defined as:

“A recognizable deficiency in any phase or activity of modeling and simulation that is not due to lack of knowledge.”

One example of uncertainty in CFD is turbulence modeling. Turbulence models in Reynolds-averaged Navier-Stokes (RANS) simulations model but do not mathematically resolve the turbulence scales. Large eddy simulation (LES)

resolves the larger turbulence scales while modeling the smaller scales at a larger computational cost, and direct numerical simulation (DNS) resolves all turbulence scales at an extreme computational cost [3]. Even within a RANS model, there are parameters and coefficients that would contribute to uncertainty. There exist methodologies for quantifying uncertainty in CFD of internal flows without experimental data [4] and methodologies for quantifying uncertainty in CFD when paired with experimental data for space launch vehicles [5] [6]. There is currently no standard methodology for evaluating uncertainty in CFD for space launch vehicles without the existence of experimental data [7].

Quantities of interest in CFD of launch vehicles include pressure and skin friction surface distributions, as well as aerodynamic coefficients [1]. Launch vehicles undergo a transition period shortly after liftoff that is particularly challenging to model with CFD. The user must provide the appropriate inputs to characterize their problem correctly. Some user inputs include geometry, mesh files, boundary conditions, flow conditions, governing equations, and numerical methods. This methodology evaluates each of these inputs and provides the corresponding uncertainty in the aerodynamic coefficients over a simple test case. To date, an uncertainty estimator for CFD over launch vehicles does not exist. The benefit to the community will be to eventually provide a "wrapper" that is agnostic of CFD solver and will quantify the uncertainty in CFD over launch vehicles without experimental data.

II. Methods

A. Computational Fluid Dynamics

The primary CFD software used for this work is SU2, a computational analysis and design package that has been developed to solve multiphysics analysis and optimization tasks using unstructured mesh topologies [8]. For this work, turbulent base flows are investigated. To simulate these flows, SU2 solves the compressible Navier-Stokes equations expressed in differential form as:

$$\mathcal{R}(U) = \frac{\partial U}{\partial t} + \nabla \cdot \bar{F}^c(U) - \nabla \cdot \bar{F}^v(U, \nabla U) = 0 \quad (1)$$

where the conservative variables, U , are the working variables and are given by:

$$U = \{\rho, \rho \bar{v}, \rho E\}^T \quad (2)$$

where ρ is the fluid density, $\bar{v}$ is the velocity vector, and E is the total energy per unit mass. The convective ($\bar{F}^c$) and viscous ($\bar{F}^v$) fluxes are given by:

$$\bar{F}^c = \begin{pmatrix} \rho \bar{v} \\ \rho \bar{v} \otimes \bar{v} + \bar{\bar{I}} p \\ \rho E \bar{v} + p \bar{v} \end{pmatrix} \quad (3)$$

and

$$\bar{F}^v = \begin{pmatrix} 0 \\ \bar{\bar{\tau}} \\ \bar{\bar{\tau}} \cdot \bar{v} + \kappa \nabla T \end{pmatrix} \quad (4)$$

where p is the static pressure, $\bar{\bar{\tau}}$ is the viscous stress tensor, T is the temperature, κ is the thermal conductivity, and μ is the viscosity, which is assumed to satisfy Sutherland's law as a function of temperature.

SU2 employs a median-dual finite-volume approach to solve the discretized governing equations. For this work, the convective fluxes are discretized using the Advection Upstream Splitting Method (AUSM) [9] flux vector splitting scheme. To achieve second-order accuracy in space, the upwind scheme of AUSM is used with MUSCL reconstruction [10]. Further details of the governing equations and numerical schemes of SU2 can be found in Ref. [8].

1. Aerodynamic Coefficients

The aerodynamic coefficients are the main output quantity of interest in this study. Uncertainty is calculated for the lift coefficient, the drag coefficient, the x-moment coefficient, the y-moment coefficient, and the z-moment coefficient, defined respectively as

$$C_L = \frac{F_L}{\frac{1}{2} \rho v^2 A_{ref}} \quad (5)$$

$$C_D = \frac{F_D}{\frac{1}{2}\rho v^2 A_{ref}} \quad (6)$$

$$C_{M_x} = \frac{M_x}{\frac{1}{2}\rho v^2 A_{ref} L_{ref}} \quad (7)$$

$$C_{M_y} = \frac{M_y}{\frac{1}{2}\rho v^2 A_{ref} L_{ref}} \quad (8)$$

$$C_{M_z} = \frac{M_z}{\frac{1}{2}\rho v^2 A_{ref} L_{ref}} \quad (9)$$

For this test case, the launch vehicle is aligned such that the x-moment is the rolling moment, the y-moment is the pitching moment, and the z-moment is the yawing moment and the angle of attack is in the y-z plane.

These coefficients are output by SU2 and are calculated as part of the compressible Navier-Stokes equations. SU2 calculates the forces and moments acting on the vehicle and then uses user-input reference values to calculate the coefficients from the respective forces and moments.

2. Turbulence Models

The Spalart-Allmaras (SA) and Shear Stress Transport (SST) turbulence models were used in this work [11] [12]. The SA model is a one-equation model that is given by the following equation:

$$\frac{\partial \tilde{\nu}}{\partial t} + u_j \frac{\partial \tilde{\nu}}{\partial x_j} = c_{b1} (1 - f_{t2}) \tilde{S} \tilde{\nu} - \left[c_{w1} f_w - \frac{c_{b1}}{k_e^2} f_{t2} \right] \left(\frac{\tilde{\nu}}{d} \right)^2 + \frac{1}{\sigma} \left[\frac{\partial}{\partial x_j} \left((\nu + \tilde{\nu}) \frac{\partial \tilde{\nu}}{\partial x_j} \right) + c_{b2} \frac{\partial \tilde{\nu}}{\partial x_i} \frac{\partial \tilde{\nu}}{\partial x_i} \right] \quad (10)$$

Definitions are given by the following equations:

$$\tilde{S} = \Omega + \frac{\tilde{\nu}}{k_e^2 d^2} f_{v2}$$

Where Ω is the magnitude of the vorticity, d is the distance from the field point to the nearest wall, and

$$f_{v1} = \frac{\chi^3}{\chi^3 + c_{v1}^3}$$

$$\chi = \frac{\tilde{\nu}}{\nu}$$

$$f_{v2} = 1 - \frac{\chi}{1 + \chi f_{v1}}$$

$$f_w = g \left[\frac{1 + c_{w3}^6}{g^6 + c_{w3}^6} \right]^{1/6}$$

$$g = r + c_{w2} (r^6 - r)$$

$$r = \min \left[\frac{\tilde{\nu}}{\tilde{S} k_e^2 d^2}, 10 \right]$$

$$f_{t2} = c_{t3} \exp(-c_{t4} \chi^2)$$

The constants are:

$$c_{b1} = 0.1355$$

$$\sigma = 2/3$$

$$c_{b2} = 0.622$$

$$k_e = 0.41$$

$$c_{w2} = 0.3$$

$$\begin{aligned}
c_{w3} &= 2 \\
c_{v1} &= 7.1 \\
c_{t3} &= 1.2 \\
c_{t4} &= 0.5 \\
c_{w1} &= \frac{c_{b1}}{\kappa^2} + \frac{1 + c_{b2}}{\sigma}
\end{aligned}$$

The SST model is a two-equation model that is given by the following equations:

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial(\rho u_j k)}{\partial x_j} = P - \beta^* \rho \omega k + \frac{\partial}{\partial x_j} \left[(\mu + \sigma_k \mu_t) \frac{\partial k}{\partial x_j} \right] \quad (11)$$

$$\frac{\partial(\rho \omega)}{\partial t} + \frac{\partial(\rho u_j \omega)}{\partial x_j} = \frac{\gamma}{\nu_t} P - \beta \rho \omega^2 + \frac{\partial}{\partial x_j} \left[(\mu + \sigma_\omega \mu_t) \frac{\partial \omega}{\partial x_j} \right] + 2(1 - F_1) \frac{\rho \sigma_{\omega 2}}{\omega} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j} \quad (12)$$

Where

$$\begin{aligned}
P &= \tau_{ij} \frac{\partial u_i}{\partial x_j} \\
\tau_{ij} &= \mu_t \left(2S_{ij} - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij} \right) - \frac{2}{3} \rho k \delta_{ij} \\
S_{ij} &= \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)
\end{aligned}$$

The turbulent eddy viscosity is computed from:

$$\mu_t = \frac{\rho a_1 k}{\max(a_1 \omega, \Omega F_2)}$$

Additional functions are given by:

$$\begin{aligned}
F_1 &= \tanh \arg_1^4 \\
\arg_1 &= \min \left[\max \left(\frac{\sqrt{k}}{\beta^* \omega d}, \frac{500\nu}{d^2 \omega} \right), \frac{4\rho \sigma_{\omega 2} k}{CD_{k\omega} d^2} \right] \\
CD_{k\omega} &= \max \left(2\rho \sigma_{\omega 2} \frac{1}{\omega} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j}, 10^{-20} \right) \\
F_2 &= \tanh \arg_2^2 \\
\arg_2^2 &= \max \left(2 \frac{\sqrt{k}}{\beta^* \omega d}, \frac{500\nu}{d^2 \omega} \right)
\end{aligned}$$

The constants are:

$$\begin{aligned}
\gamma_1 &= \frac{\beta_1}{\beta^*} - \frac{\sigma_{\omega 1} \kappa^2}{\sqrt{\beta^*}} \\
\gamma_2 &= \frac{\beta_2}{\beta^*} - \frac{\sigma_{\omega 2} \kappa^2}{\sqrt{\beta^*}} \\
\sigma_{k1} &= 0.85 \\
\sigma_{\omega 1} &= 0.5 \\
\beta_1 &= 0.075 \\
\sigma_{k2} &= 1.0 \\
\sigma_{\omega 2} &= 0.856
\end{aligned}$$

$$\beta_2 = 0.0828$$

$$\beta^* = 0.09$$

$$\kappa = 0.41$$

$$a_1 = 0.31$$

The impact of the uncertainty of these constants on the aerodynamic coefficients will be investigated in future work.

B. Uncertainty Analysis

The uncertainty analysis of this work utilizes the methodologies from Coleman and Steele [13] originally constructed for experimental approaches and Groves [14] for CFD of internal flows of payload fairing spacecraft environmental control systems (ECS). Quantifying the uncertainty in these CFD predictions is imperative as there are inherent uncertainties and errors associated with CFD with no standard approach to evaluating the CFD [7]. We will utilize the methodology Groves et al. [4], which was based on industry standards in experimental and numerical model validation and previously applied to ECS spacecraft configurations. One of the benefits of this methodology is that it can provide an uncertainty estimate when test data and validation are not available. An important aspect of this approach is the identification of input parameters and quantities of interest.

The total uncertainty, u_{val} , can be found in terms of the numerical uncertainty, u_{num} , and the uncertainty from the input parameters, u_{input} as follows:

$$u_{val} = k \sqrt{u_{num}^2 + u_{input}^2} \quad (13)$$

With some given degree of confidence provided by the coverage factor, k . The coverage factor is found from Student's t-distribution rather than assuming a normal distribution. The uncertainty from the input parameters can be found by a Taylor Series expansion about each input parameter. If the CFD result r is a function of J input variables X_J ,

$$r = r(X_1, X_2, \dots, X_J) \quad (14)$$

Equation 13 is the data reduction equation with the combined standard uncertainty given by

$$u_r = \sqrt{\left(\frac{\partial r}{\partial X_1}\right)^2 u_{X_1}^2 + \left(\frac{\partial r}{\partial X_2}\right)^2 u_{X_2}^2 + \dots + \left(\frac{\partial r}{\partial X_J}\right)^2 u_{X_J}^2} \quad (15)$$

Equation 15 can be written compactly as:

$$u_r = \sqrt{\sum_{i=1}^J \left(\frac{\partial r}{\partial X_i}\right)^2 u_{X_i}^2} \quad (16)$$

The uncertainty for each input parameter, u_{X_i} , was calculated using the method outlined by Groves Groves et al. [4]. For a CFD result, X_i , as before, with X_U being the upper value of the result and X_L being the lower value of the result.

$$u_{X_i} = \frac{1}{2}(X_U - X_L) \quad (17)$$

Equation 15 is useful if the input parameter does not also have a subset of its own parameters. In the case that the input parameter does depend on a series of other parameters (e.g. boundary conditions can be based on Mach number and atmospheric conditions), we can repeat the process for the input parameter,

$$X_i = X_i(Y_1, Y_2, \dots, Y_n) \quad (18)$$

The uncertainty of the input parameter will then be

$$u_{X_i} = \sqrt{\left(\frac{\partial r}{\partial Y_1}\right)^2 u_{Y_1}^2 + \left(\frac{\partial r}{\partial Y_2}\right)^2 u_{Y_2}^2 + \dots + \left(\frac{\partial r}{\partial Y_n}\right)^2 u_{Y_n}^2} \quad (19)$$

With the uncertainty of the subset of parameters calculated as in Equation 12 and the partial derivatives being calculated as follows:

$$\frac{\partial r}{\partial X_i} = \frac{1}{n} \left(\frac{\partial r}{\partial Y_1} + \frac{\partial r}{\partial Y_2} + \dots + \frac{\partial r}{\partial Y_n} \right) \quad (20)$$

This method is a method of moments probabilistic method of uncertainty quantification that uses a first-order first-moment and a second-order first-moment approximation on the mean value of the aerodynamic coefficient. This method typically gives results in a gaussian, or normal distribution about the mean value. Rather than assuming a normal distribution, the coverage factor will come from Student's t-distribution, which is used for experimental uncertainty calculations in which limited data is present. A table of coverage factors with 90% confidence from Student's t-distribution can be found in Table 1. The coverage factor will be based on the number of simulations completed for each configuration, with a corresponding degree of freedom.

Number of Cases	Degrees of Freedom	Confidence 90%
2	1	6.314
3	2	2.92
4	3	2.353
5	4	2.132
6	5	2.015
7	6	1.943
8	7	1.895
9	8	1.86
10	9	1.833
11	10	1.812
12	11	1.796
13	12	1.782
14	13	1.771
15	14	1.761
16	15	1.753
17	16	1.746
18	17	1.74
19	18	1.734
20	19	1.729
21	20	1.725
22	21	1.721
23	22	1.717
24	23	1.714
25	24	1.711
26	25	1.708
27	26	1.706
28	27	1.703
29	28	1.701
30	29	1.699
31	30	1.697
41	40	1.684
61	60	1.671
121	120	1.658
∞	∞	1.645

Table 1 Student t-distribution coverage factor values. [13]

III. Results

A. Test case description

A simple test case was constructed using a generic launch vehicle. The model was created using the Computer Aided Drafting (CAD) program SolidWorks. The shape of the nose cone was assumed to be a spherically blunted tangent ogive in the interest of simplicity. The length of the ogive was estimated to be 9.14 meters, the ogive radius was estimated to be 16.8 meters, and the radius of the spherically blunted cap was estimated to be 0.92 meters. The model is shown in Figure 1. The reference area shown in Equations 5 through 9 is the area of the base of the model, which was calculated to be approximately 22.90 meters, and the reference length used in the same equations is the length of the model, 66.30 meters. The engines and solid rocket boosters were excluded from this test case for simplicity but will be included in future work.

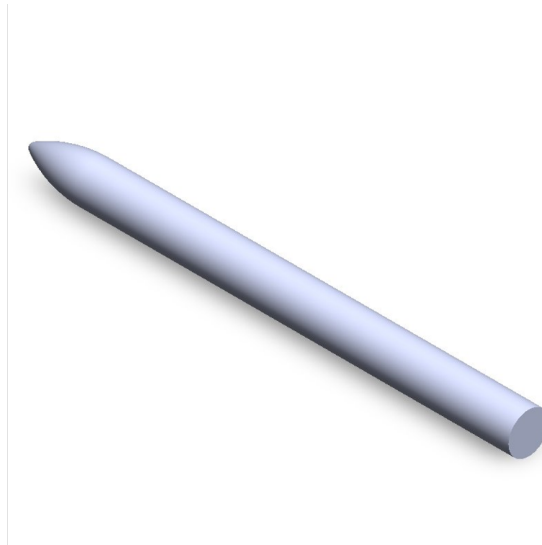


Fig. 1 Simplified model of a generic launch vehicle.

Mach 2 was chosen as the baseline for this model. Figure 2 was used to determine the approximate altitude of the launch at Mach 2. The freestream temperature and pressure were then found from the 1976 Standard Atmosphere Model [15].

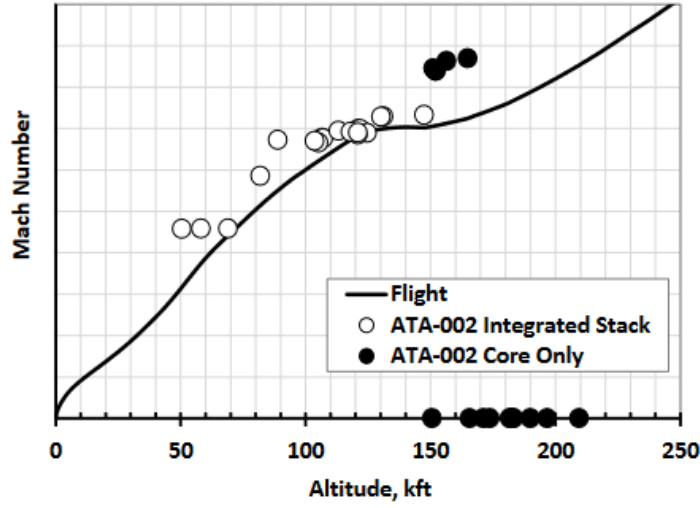


Fig. 2 Mach number vs. altitude for SLS [16].

The input parameters for this case included the mesh files, boundary conditions, and turbulence model. These inputs were loaded into SU2 to produce a solution. Equation 13 can then be expanded to give the uncertainty in the aerodynamic coefficients.

$$u_{coefficient} = k \left[\left(\left(\frac{\partial coefficient}{\partial mesh} \right)^2 u_{mesh}^2 \right) + \left(\left(\frac{\partial coefficient}{\partial boundary\ conditions} \right)^2 u_{boundary\ conditions}^2 \right) + \left(\left(\frac{\partial coefficient}{\partial turbulence\ model} \right)^2 u_{turbulence\ model}^2 \right) \right]^{\frac{1}{2}}$$

The mesh and boundary condition inputs are also parameter dependent, so Equation 19 is used to calculate u_{mesh} and $u_{boundary\ conditions}$ and Equation 20 is used to calculate $\frac{\partial coefficient}{\partial mesh}$ and $\frac{\partial coefficient}{\partial boundary\ conditions}$. The denominator of $\frac{\partial coefficient}{\partial turbulence\ model}$ is set to unity since the change in the turbulence model is not a numerical value. The parameters that were used to create the meshes will be explained in the following section. A baseline case was chosen to compare perturbations and apply the uncertainty analysis. The case corresponding to Mesh 1, Mach 2, and the SST turbulence model was chosen as the baseline case for this analysis. The coverage value of 1.833 was used because there are ten different CFD cases for each configuration, which are listed in Table 2.

Case Number	Parameter		Configuration	
			0 Degrees AoA	6 Degrees AoA
1	Mesh	Mesh 1	1	1
2		Mesh 2	2	2
3		Mesh 3	3	3
4		Mesh 4	4	4
5		Mesh 5	5	5
7	Boundary Condition	Mach - 5%	6	6
8		Mach +/- 0%	7	7
9		Mach + 5%	8	8
10	Turbulence Model	Spalart-Allmaras	9	9
11		Shear Stress Transport	10	10

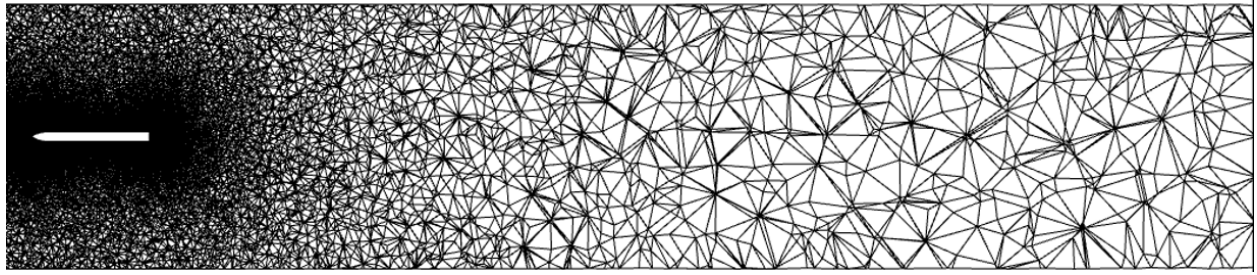
Table 2 Cases solved using CFD

B. Mesh Studies

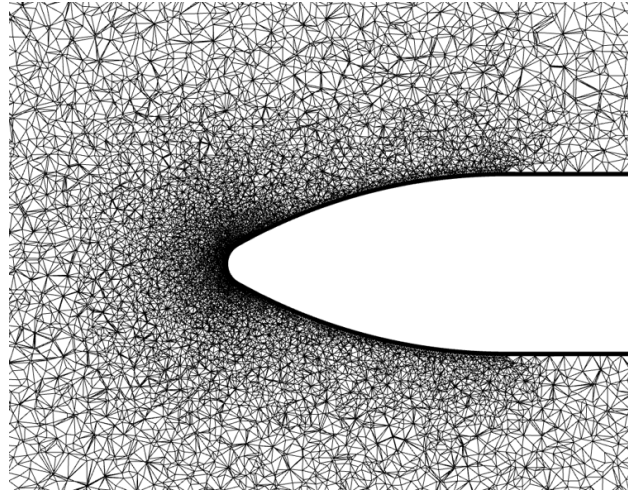
Five meshes were constructed, referred to as Mesh 1 through Mesh 5, respectively. Each of the meshes was made using Pointwise's T-Rex meshing feature, which builds layers of prisms near the surface before switching to tetrahedron elsewhere. Mesh 1 is the coarsest mesh, with 5,006,420 cells. Meshes 2-5 used different techniques to refine the meshes and the number of cells in each respective mesh is listed as follows:

- Mesh 2: 30,177,456 cells
- Mesh 3: 5,663,259 cells
- Mesh 4: 5,076,091 cells
- Mesh 5: 41,972,093 cells

The boundary layer is captured by a series of structured layers that are the same for all five meshes. The first cell spacing is 0.1 millimeter from the wall, and the growth rate is 1.1 for 50 layers. The first cell spacing was chosen to resolve the mesh to a y^+ of 1 on the surface. Details of the meshes are shown in Figures 3 through 7, respectively. The farfield of the mesh was designed to capture the shock wave and multiple body lengths of the wake. All the cases investigated so far are supersonic and an upwind numerical scheme is utilized, which allows for the inlet to be placed near the nose of the vehicle [17] [18]. These mesh studies are preliminary and intended to show the effect of different approaches and quality of meshes on the aerodynamic coefficients' uncertainty. More detailed mesh studies are planned for future work, which includes a better transition from the prism to tetrahedron layers, further surface refinement, mesh adaption, structured vs. unstructured meshes, and the impact of how large the mesh is (e.g., wake, farfield).

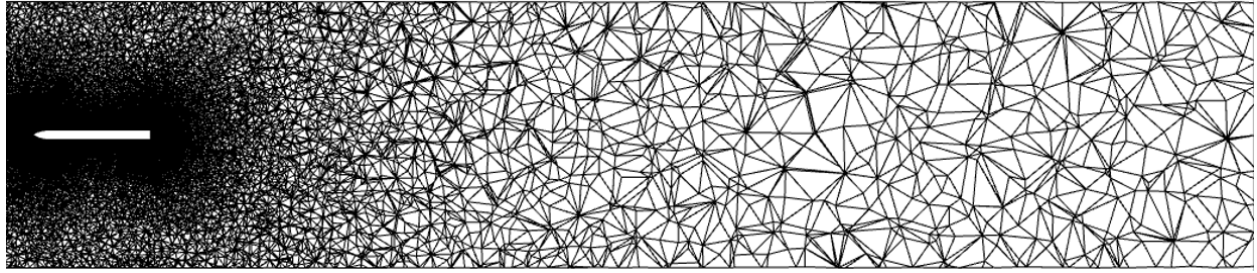


(a) Full mesh

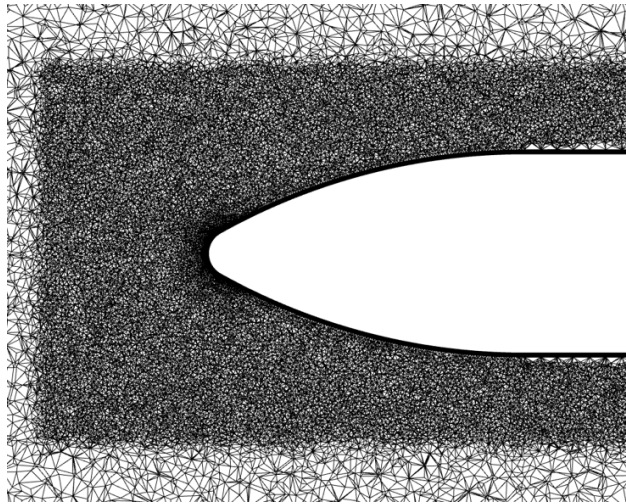


(b) Zoom on model

Fig. 3 Mesh 1 hybrid mesh of the generic model.

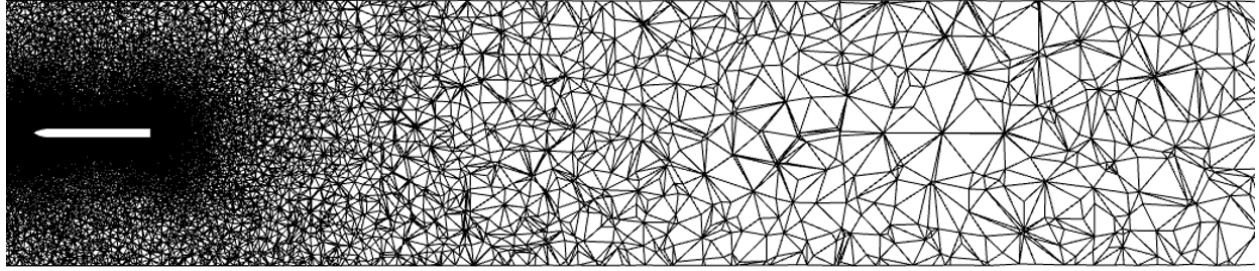


(a) Full mesh

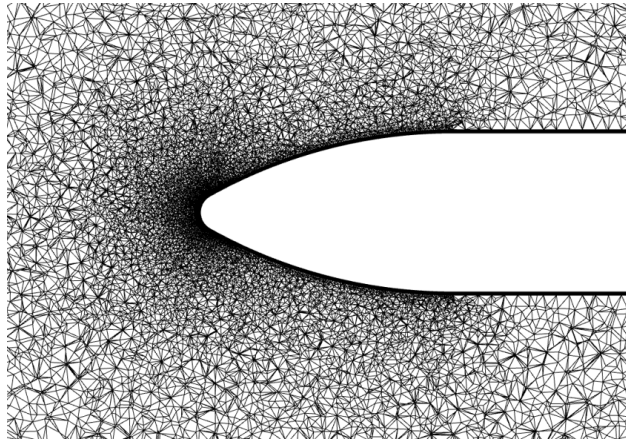


(b) Zoom on model

Fig. 4 Mesh 2 hybrid mesh of the generic model.

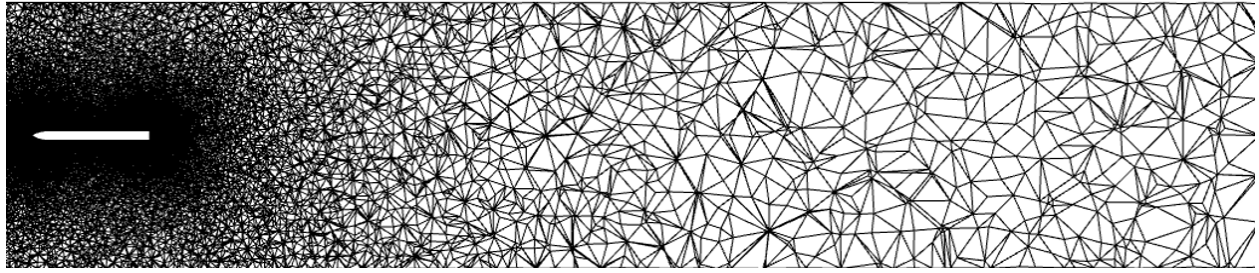


(a) Full mesh

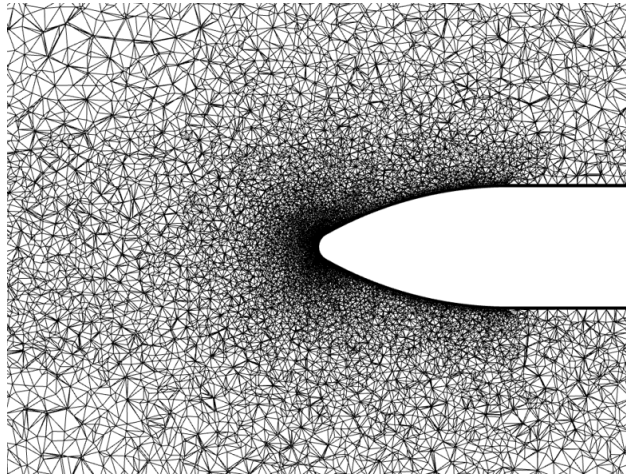


(b) Zoom on model

Fig. 5 Mesh 3 hybrid mesh of the generic model.

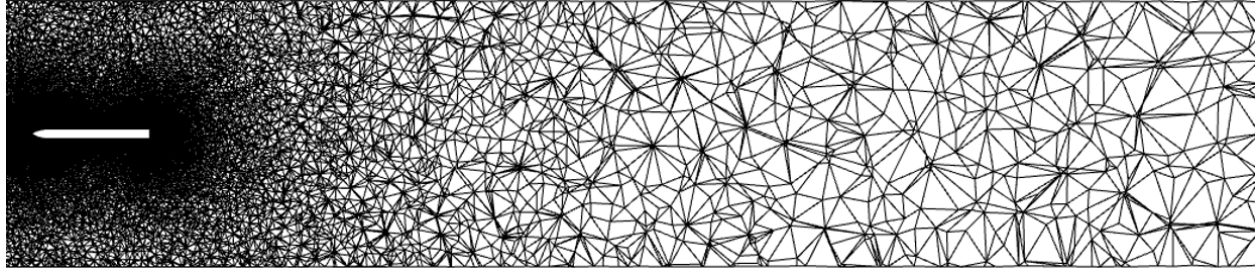


(a) Full mesh

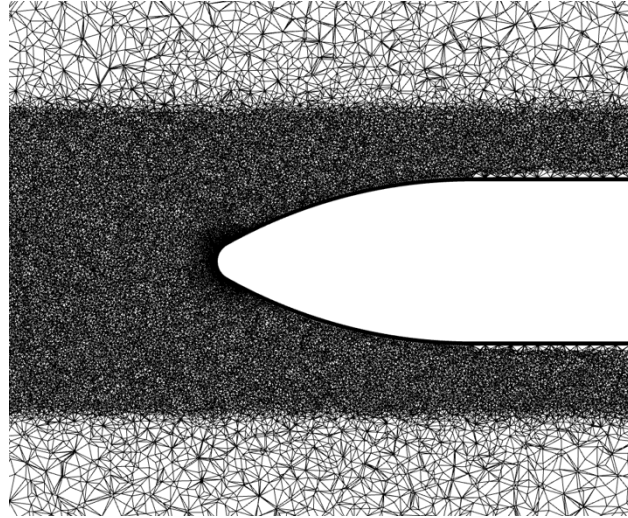


(b) Zoom on model

Fig. 6 Mesh 4 hybrid mesh of the generic model.



(a) Full mesh



(b) Zoom on model

Fig. 7 Mesh 5 hybrid mesh of the generic model.

C. CFD and Uncertainty Analysis

The aerodynamic coefficients of the model were chosen as a quantity of interest in this case. The results from the CFD modeling in SU2 are tabulated and shown in Tables 3 through Table 7. For the zero-degree angle of attack configuration, we expect to see about zero lift coefficient since the model is an axisymmetric shape. For the six-degree angle of attack configuration, however, we expect to see a nonzero lift coefficient as well as a higher drag coefficient since the frontal area decreases. The results shown in the following tables agree with these expectations.

	Mesh 1	Mach - 5%		Mach Baseline		Mach + 5%	
	Angle of Attack	0 degrees	6 degrees	0 degrees	6 degrees	0 degrees	6 degrees
Spalart-Allmaras	C_L			-0.001168	0.054496		
	C_D			0.040206	0.046212		
	C_{M_x}			0.000001	0.000004		
	C_{M_y}			0.030040	-1.526265		
	C_{M_z}			0.002632	0.001183		
Shear Stress Transport	C_L	0.000428	0.069069	0.000564	0.067977	0.001171	0.066865
	C_D	0.043195	0.049761	0.041294	0.047937	0.039244	0.046229
	C_{M_x}	0.000002	0.000005	0.000002	0.000005	0.000002	0.000005
	C_{M_y}	-0.024661	-2.069780	-0.025681	-2.016400	-0.029003	-1.964558
	C_{M_z}	0.007575	-0.004533	0.005838	-0.005253	-0.000787	-0.005864

Table 3 Tabulated aerodynamic coefficient data corresponding to Mesh 1.

	Mesh 2	Mach Baseline	
	Angle of Attack	0 degrees	6 degrees
Shear Stress Transport	C_L	0.000154	0.030719
	C_D	0.037881	0.041893
	C_{M_x}	0.000001	0.000002
	C_{M_y}	-0.004990	-0.495014
	C_{M_z}	-0.002349	-0.000902

Table 4 Tabulated aerodynamic coefficient data corresponding to Mesh 2.

	Mesh 3	Mach Baseline	
	Angle of Attack	0 degrees	6 degrees
Shear Stress Transport	C_L	0.001029	0.063555
	C_D	0.039892	0.047398
	C_{M_x}	0.000002	0.000005
	C_{M_y}	0.001806	-1.780981
	C_{M_z}	-0.004219	-0.002216

Table 5 Tabulated aerodynamic coefficient data corresponding to Mesh 3.

	Mesh 4	Mach Baseline	
	Angle of Attack	0 degrees	6 degrees
Shear Stress Transport	C_L	0.001522	0.067774
	C_D	0.040540	0.048020
	C_{M_x}	0.000001	0.000005
	C_{M_y}	-0.025546	-2.008672
	C_{M_z}	-0.004545	-0.002233

Table 6 Tabulated aerodynamic coefficient data corresponding to Mesh 4.

Shear Stress Transport	Mesh 5	Mach Baseline	
	Angle of Attack	0 degrees	6 degrees
	C_L	-0.000028	0.006393
	C_D	0.036864	0.039971
	C_{M_x}	0.000001	0.000003
	C_{M_y}	0.001062	0.681614
	C_{M_z}	-0.000123	0.002436

Table 7 Tabulated aerodynamic coefficient data corresponding to Mesh 5.

Contour plots of Mach contours and surface pressure were created using Tecplot. The baseline freestream Mach number was Mach 2; the contours corresponding to the SST turbulence model are shown in Figures 8 through 12. For the sake of brevity, contour plots for the six-degree angle of attack configuration are shown. Since the only difference between these plots is mesh quality, we expect the overall trends of the pressure coefficient and Mach number to be the same between all the contour plots. The main visual difference between the results is in the wake, which is a difficult area to model in CFD.

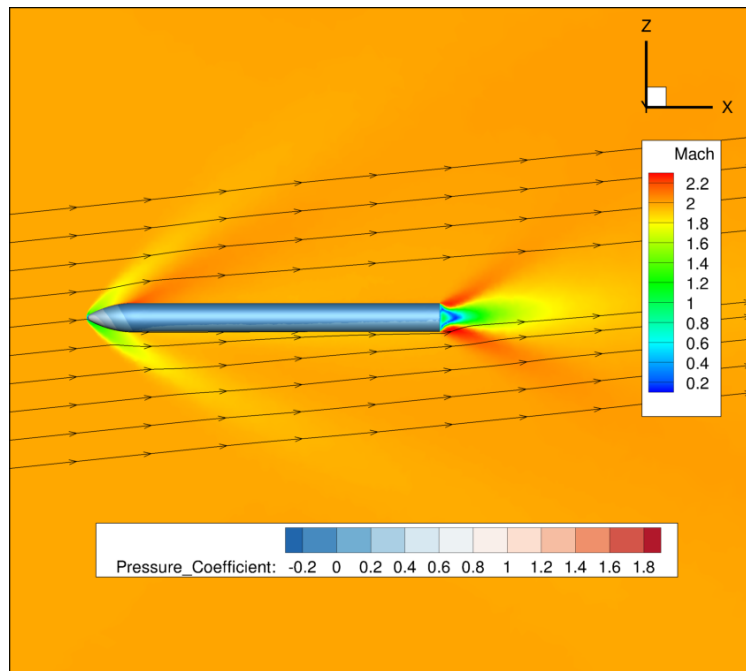


Fig. 8 Mach and pressure coefficient contours of Mesh 1 at the baseline freestream Mach number and 6 degree angle of attack configuration.

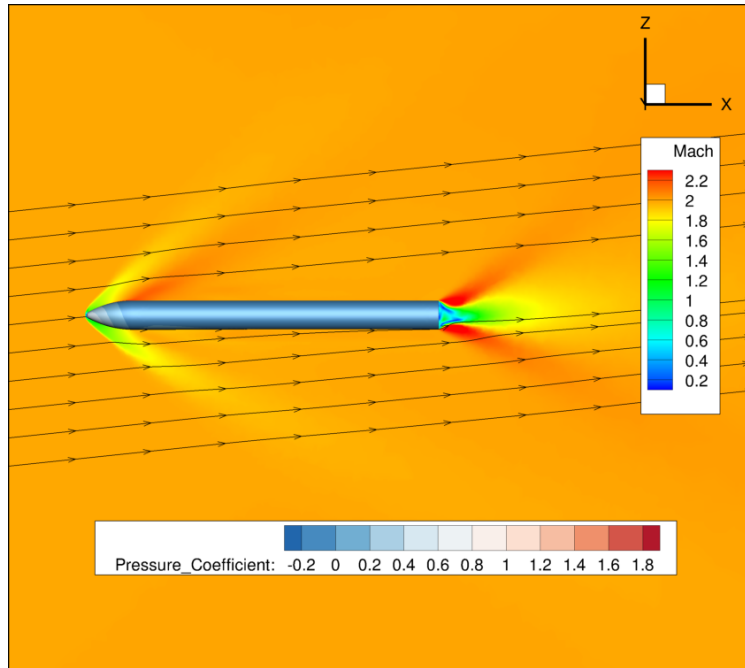


Fig. 9 Mach and pressure coefficient contours of Mesh 2 at the baseline freestream Mach number and 6 degree angle of attack configuration.

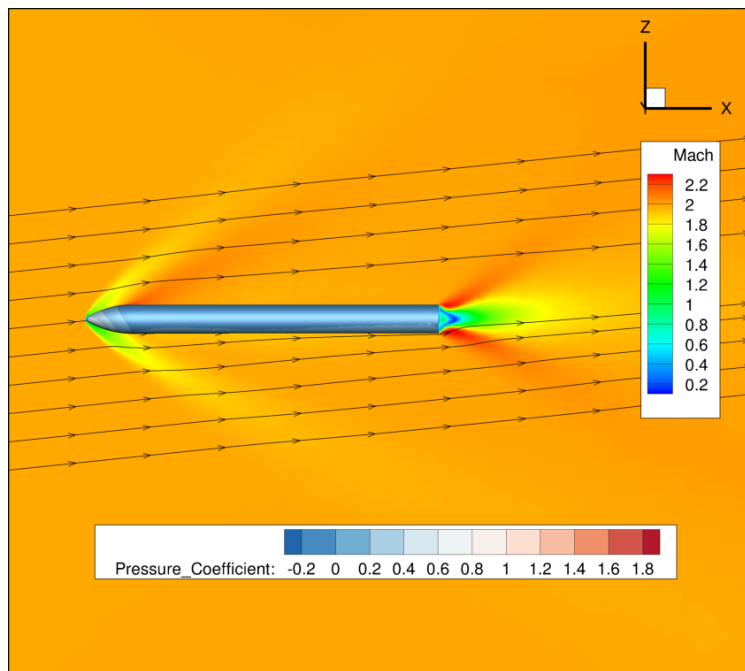


Fig. 10 Mach and pressure coefficient contours of Mesh 3 at the baseline freestream Mach number and 6 degree angle of attack configuration.

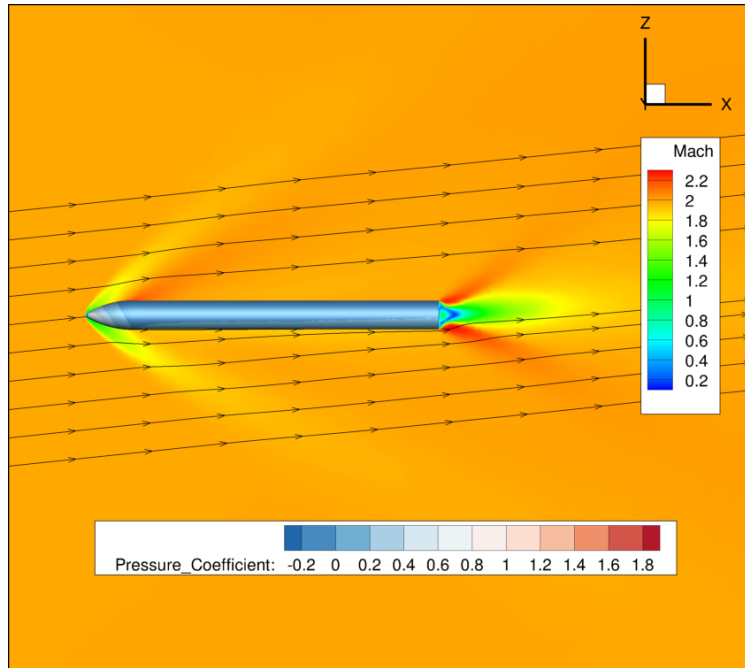


Fig. 11 Mach and pressure coefficient contours of Mesh 4 at the baseline freestream Mach number and 6 degree angle of attack configuration.

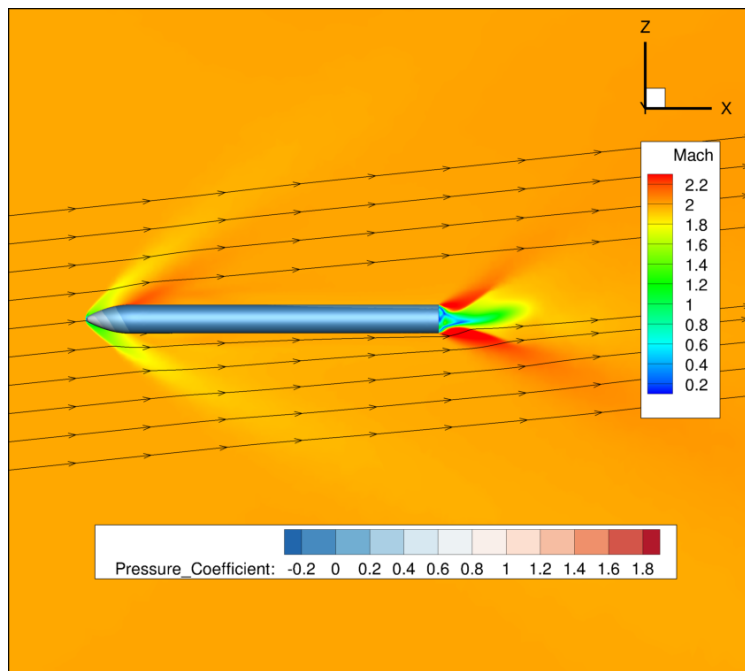


Fig. 12 Mach and pressure coefficient contours of Mesh 5 at the baseline freestream Mach number and 6 degree angle of attack configuration.

Difference contour plots were made by subtracting the results of two cases with the same mesh. Difference plots are shown comparing turbulence models and boundary conditions. "High" boundary condition comparison consists of subtracting the results for the Mach 2.1 case from the results for the Mach 2 case. "Low" boundary condition comparison

consists of subtracting the results for the Mach 1.9 case from the results for the Mach 2 case. The turbulence model and mesh correspond to the baseline case.

Figure 13 and Figure 14 show comparisons between turbulence models for Mesh 1 at the Mach 2 boundary condition. The largest difference in Mach number occurs at the base of the vehicle in the wake region and at the front of the vehicle near the stagnation point and the bow shock, so these parts of the vehicle are highlighted in these figures.

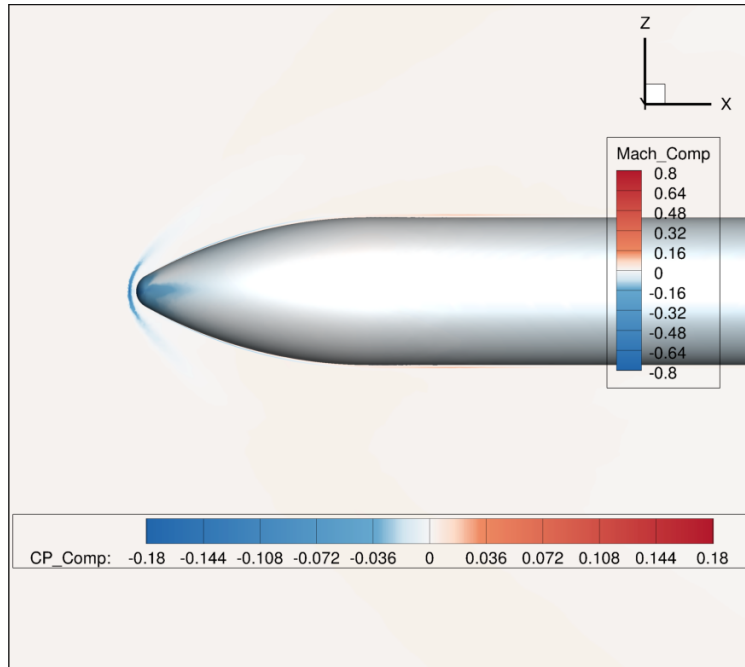


Fig. 13 Mach and pressure coefficient difference contours of the Mesh 1 at the baseline freestream Mach number turbulence model comparison.

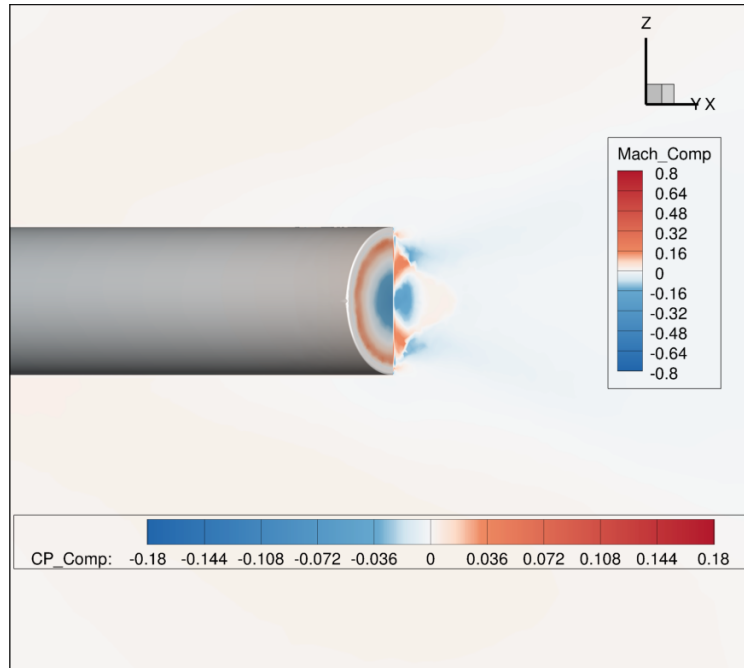


Fig. 14 Mach and pressure coefficient difference contours of the Mesh 1 at the baseline freestream Mach number turbulence model comparison.

Figure 15 and Figure 16 show comparison plots between boundary conditions for mesh 1 and the Spalart-Allmaras turbulence model. As before, there is a large difference in Mach number occurring in the wake region at the rear of the vehicle and at the front of the vehicle near the bow shock. Much of the same surface area of the vehicle is experiencing differences in the pressure coefficients for these cases, with the largest differences occurring at the front of the vehicle and along the very base of the vehicle. The overall magnitude of the pressure coefficient differences appears to align with the magnitude of the differences seen in Figure 13 and Figure 14 above.

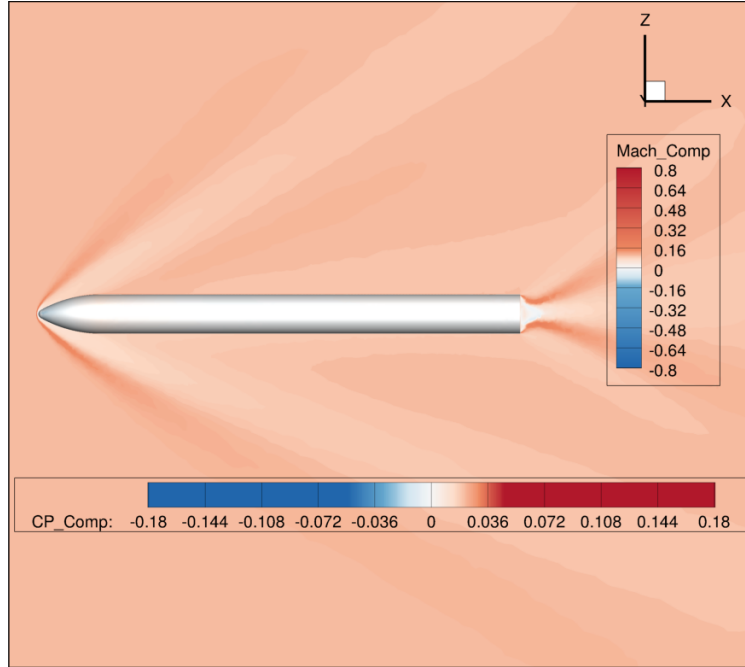


Fig. 15 Mach and pressure coefficient difference contours of Mesh 1 using the SA turbulence model high freestream comparison at 6 degrees angle of attack.

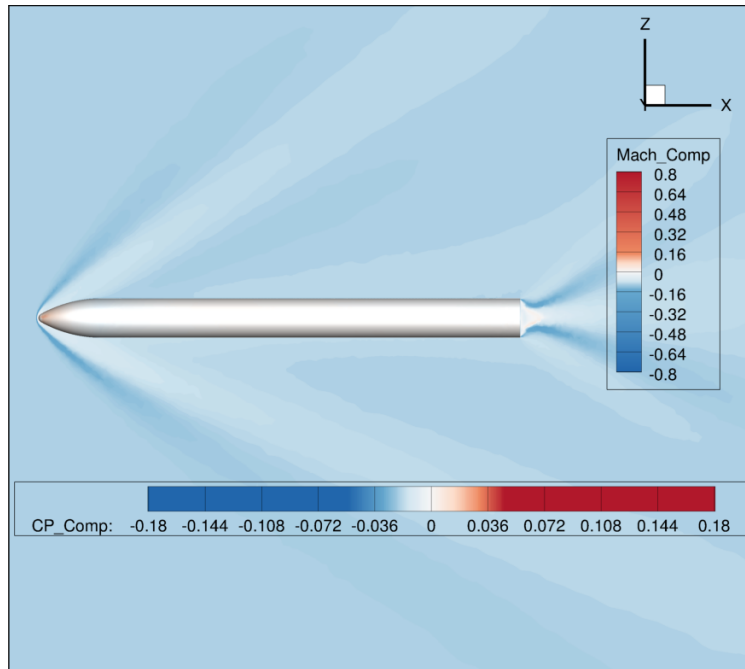


Fig. 16 Mach and pressure coefficient difference contours of Mesh 1 using the SA turbulence model low freestream comparison at 6 degrees angle of attack.

The uncertainty for each input parameter was calculated using Equation 12 for each output parameter. The results are shown in Table 8, Table 9, and Table 10. The largest uncertainty is in the y-moment coefficient from the mesh input parameter and the six-degree angle of attack configuration. The x-moment coefficient had the lowest uncertainty, with zero percent uncertainty contributions from each input parameter.

Mach Component Contribution		
	0 degrees	6 degrees
CL	0.03%	0.00%
CD	93.17%	52.12%
CM _x	0.00%	0.00%
CM _y	0.00%	0.02%
CM _z	98.07%	0.00%

Table 8 Uncertainty corresponding to the boundary condition input parameter.

The highest uncertainty from the Mach input parameter was the y-moment coefficient for the six-degree angle of attack configuration, followed by the z-moment coefficient for the zero-degree angle of attack configuration. Although there was above a three percent uncertainty in the y-moment coefficient in the six-degree angle of attack configuration due to the Mach number input parameter, there was a less than one percent uncertainty in the y-moment coefficient for the zero-degree angle of attack configuration.

TM Component Contribution		
	0 degrees	6 degrees
CL	99.97%	83.49%
CD	6.81%	47.24%
CM _x	100.00%	100.00%
CM _y	100.00%	0.19%
CM _z	1.91%	100.00%

Table 9 Uncertainty corresponding to the turbulence model input parameter.

The highest uncertainty from the turbulence model input parameter was in the y-moment coefficient for the zero-degree angle of attack configuration, followed by the y-moment coefficient for the six-degree angle of attack configuration. The turbulence model input parameter contributed the most to the uncertainty in the lift coefficient quantity of interest.

Mesh Component Contribution		
	0 degrees	6 degrees
CL	0.00%	16.51%
CD	0.02%	0.64%
CM _x	0.00%	0.00%
CM _y	0.00%	99.79%
CM _z	0.01%	0.00%

Table 10 Uncertainty corresponding to the mesh input parameter.

The highest uncertainty from the mesh input parameter was the y-moment coefficient for the six-degree angle of attack configuration, which was many orders of magnitude higher than any other uncertainty from any of the input parameters. This may be amended in future work when mesh refinement is more robust. Next, the partial derivatives of the aerodynamic coefficients were calculated with respect to the input parameters. These results are shown in Table 11, Table 12, and Table 13. The partial derivatives are overall very small values, indicating that small perturbations lead to small changes in the outputs.

Mach Partial Derivatives		
	0 degrees	6 degrees
C_L	7.43E-03	2.20E-02
C_D	3.95E-02	3.53E-02
C_{M_x}	0.00	0.00
C_{M_y}	4.34E-02	1.05E+00
C_{M_z}	8.36E-02	1.33E-02

Table 11 Partial derivatives corresponding to the Mach number input parameter.

TM Partial Derivatives		
	0 degrees	6 degrees
C_L	1.73E-03	1.35E-02
C_D	1.09E-03	1.73E-03
C_{M_x}	1.00E-06	1.00E-06
C_{M_y}	5.57E-02	4.90E-01
C_{M_z}	3.21E-03	6.44E-03

Table 12 Partial derivatives corresponding to the turbulence model input parameter.

Mesh Partial Derivatives		
	0 degrees	6 degrees
C_L	2.57E-04	2.33E-02
C_D	2.13E-03	3.78E-03
C_{M_x}	6.25E-07	1.88E-06
C_{M_y}	1.29E-02	9.52E-01
C_{M_z}	5.12E-03	2.72E-03

Table 13 Partial derivatives corresponding to the mesh input parameter.

The uncertainty values and partial derivatives corresponding to each input parameter were used to calculate the total uncertainty from Equation 13. The uncertainty corresponding to each input parameter is normalized to the raw CFD calculated value and is shown in Table 14.

Normalized Uncertainty		
	0 degrees	6 degrees
C_L	0.49%	0.27%
C_D	0.01%	0.01%
C_{M_x}	0.00%	0.00%
C_{M_y}	11.08%	250.58%
C_{M_z}	1.17%	0.72%

Table 14 Calculated uncertainty values for aerodynamic coefficients at 0 and 6 degrees angle of attack.

The highest uncertainty is in the y-moment coefficient for both the zero-degree and six-degree angle of attack configurations, with the remaining uncertainty values hovering around one percent of the baseline CFD result value. For

this study, increasing the fineness of the mesh had a large impact on the value of the y-moment coefficient, especially at nonzero angles of attack.

Bar charts of the total uncertainty and the uncertainty corresponding to each input parameter are presented in Figure 17, Figure 18, Figure 19, and Figure 20. The x-moment coefficient had zero percent uncertainty, so charts are not presented for that data. The uncertainty is not dominated by any one input parameter and the contributions from the input parameters varies by angle of attack as well as aerodynamic coefficient.

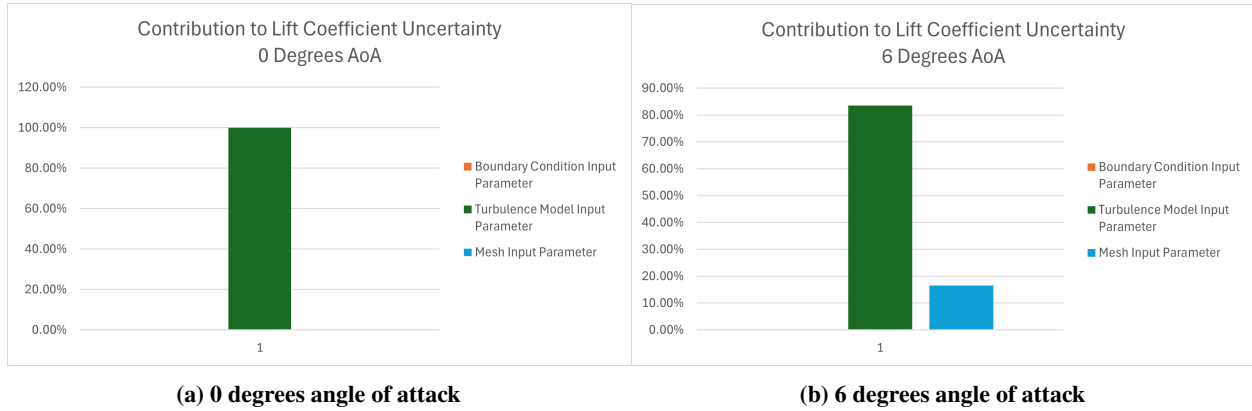


Fig. 17 Lift coefficient uncertainty.

The uncertainty in the lift coefficient was slightly higher for the zero-degree angle of attack configuration, with the highest contribution for both configurations coming from changing the turbulence model.

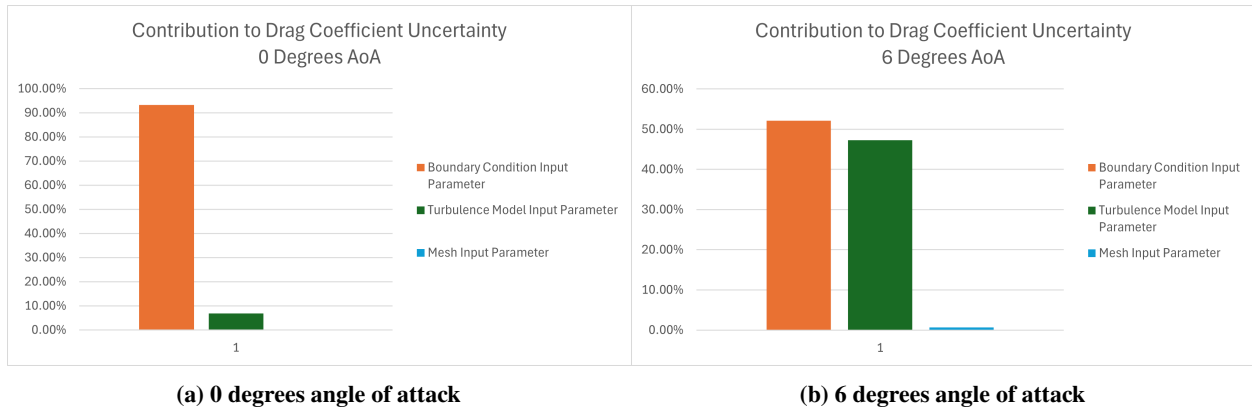


Fig. 18 Drag coefficient uncertainty.

The uncertainty in the drag coefficient was extremely low, with the highest contribution for both configurations coming from perturbing the freestream Mach number.

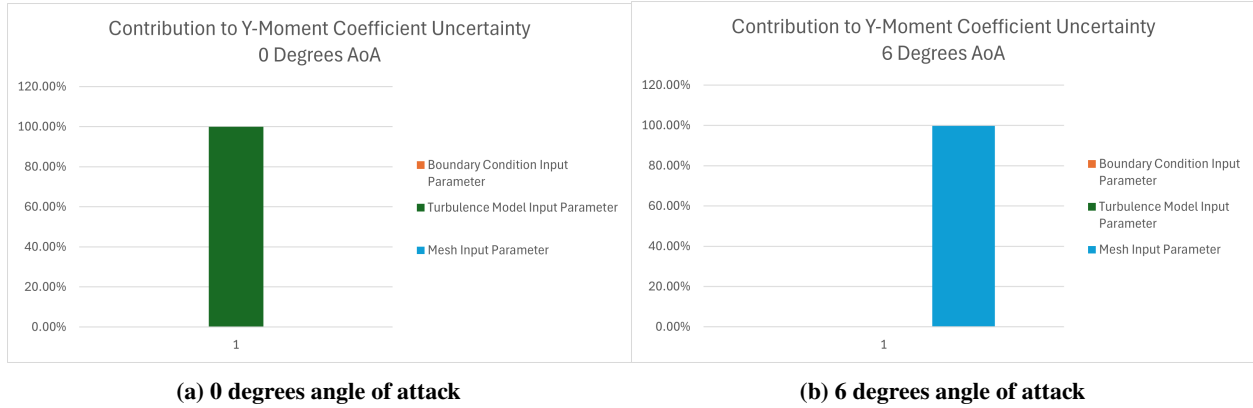


Fig. 19 y-moment coefficient uncertainty.

The y-moment coefficient had the highest uncertainty of all the output quantities. The largest contribution to this uncertainty was different for the two configurations, with the turbulence model contributing most to the zero-degree angle of attack configuration and the mesh quality contributing most to the six-degree angle of attack configuration.

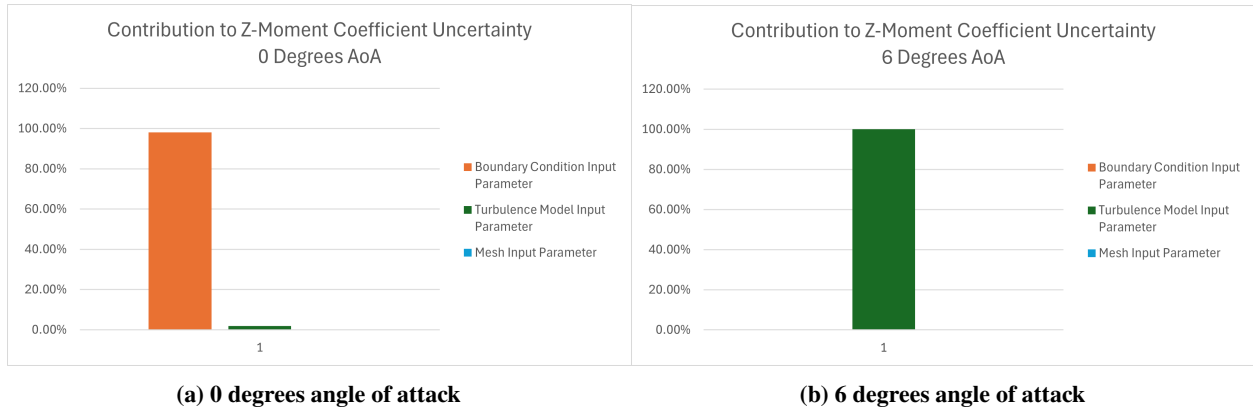


Fig. 20 z-moment coefficient uncertainty.

The z-moment coefficient had the second-highest uncertainty behind the y-moment coefficient for both configurations. The largest contribution to this uncertainty was also different for the two configurations, with the Mach number perturbation contributing most to the zero-degree angle of attack configuration and the turbulence model contributing most to the six-degree angle of attack configuration.

IV. Conclusion and Future Work

There is currently no standard for calculating the uncertainty of CFD over launch vehicles. The purpose of this work is to provide a tool for CFD users to use to properly characterize the uncertainty of CFD over launch vehicles using only numerical data. This tool will not eliminate the need for ground tests and validation, however, it may reduce the number of ground tests required for some launch vehicles and provide an uncertainty estimate when ground tests are unavailable or before any ground tests have been performed.

This work utilizes the methodology previously explored by Groves for CFD of internal flows and the methodology from Coleman and Steele for experimental approaches extrapolated to external flows over launch vehicles from liftoff through flight conditions. In the case that test data or validation is not available or has not yet been performed, this methodology combines Student's t-distribution with the Taylor Series Method for propagation of uncertainties to provide a reliable estimate of the uncertainty of CFD over launch vehicles. Although this toolbox will be used with SU2 initially, the intent is to make it work with other CFD solvers as well. Additional future work will include using other input

parameters (e.g., wall functions, surface roughness, CFD solver, trajectory, convective scheme, and additional boundary conditions) and continually refining the meshing process.

Acknowledgments

This work is supported by the NASA Launch Services Program via the Expendable Launch Vehicle Integrated Support (ELVIS) 3 contract, operated by a.i. solutions Inc. The authors would like to acknowledge the support provided by Scott Clark in managing this contract and valuable discussions with Michael Harris on the technical approach. This research was supported in part through computational resources and services provided by the University of Arizona's Research Data Center (RDC).

References

- [1] Ratnayake, N., Krist, S., Ghaffari, F., and Deere, K., "Computational Fluid Dynamics Methods used in the Development of the Space Launch System Liftoff and Transition Lineloads Databases," AIAA, Dallas, Texas, 2019.
- [2] Committee, C. F. D., "Guide: Guide for the Verification and Validation of Computational Fluid Dynamics Simulations (AIAA G-077-1998(2002))," American Institute of Aeronautics and Astronautics, Inc., Washington, DC, 1998. URL [10.2514/4.472855](https://doi.org/10.2514/4.472855).
- [3] Lucor, D., Xiu, D., Su, C., and Karniadakis, G., "Predictability and Uncertainty in CFD," *International Journal for Numerical Methods in Fluids*, Vol. 43, No. 5, 2003, pp. 463–596. <https://doi.org/https://doi.org/10.1002/fld.500>.
- [4] Groves, C. E., Ilie, M., and Schallhorn, P., "Computational Fluid Dynamics Uncertainty Analysis for Payload Fairing Spacecraft Environmental Control Systems," *52nd Aerospace Sciences Meeting*, AIAA Paper 2014-0440, National Harbor, Maryland, 2014. <https://doi.org/10.2514/6.2014-0440>.
- [5] Dalle, D., Rogers, S., Lee, H., and Meeroff, J., "Adjustments and Uncertainty Quantification for SLS Aerodynamic Sectional Loads," *AIAA Aviation Forum*, 2018. <https://doi.org/10.2514/6.2018-3640>.
- [6] Wignall, T., and Houlden, H., "Uncertainty Quantification for Launch Vehicle Aerodynamic Lineloads," *AIAA Scitech Forum*, 2020. <https://doi.org/10.2514/6.2020-1521>.
- [7] Roache, P. J., Ghia, K. N., and White, F. M., "Editorial Policy Statement on the Control of Numerical Accuracy," *Journal of Fluids Engineering*, Vol. 108, No. 1, 1986, p. 2. <https://doi.org/10.1115/1.3242537>.
- [8] Economou, T. D., Palacios, F., Copeland, S. R., Lukaczyk, T. W., and Alonso, J. J., "SU2: An open-source suite for multiphysics simulation and design," *AIAA Journal*, Vol. 54, No. 3, 2016, pp. 828–846. <https://doi.org/10.2514/1.J053813>, publisher: American Institute of Aeronautics and Astronautics Inc.
- [9] Liou, M. S., and Steffen, C. J., "A New Flux Splitting Scheme," *Journal of Computational Physics*, Vol. 107, No. 1, 1993, pp. 23–39. <https://doi.org/10.1006/JCPH.1993.1122>, publisher: Academic Press.
- [10] van Leer, B., "Towards the ultimate conservative difference scheme. V. A second-order sequel to Godunov's method," *Journal of Computational Physics*, Vol. 32, No. 1, 1979, pp. 101–136. [https://doi.org/10.1016/0021-9991\(79\)90145-1](https://doi.org/10.1016/0021-9991(79)90145-1).
- [11] Spalart, P. R., and Allmaras, S. R., "A One-Equation Turbulence Model for Aerodynamic Flows," *30th Aerospace Sciences Meeting and Exhibit*, AIAA, Reno, NV, 1992. <https://doi.org/https://doi.org/10.2514/6.1992-439>.
- [12] Menter, F. R., "Two-Equation Eddy-Viscosity Turbulence Models for Engineering Applications," *AIAA Journal*, Vol. 32, No. 8, 1994. <https://doi.org/https://doi.org/10.2514/3.12149>.
- [13] Coleman, H., and Steele, W. G., *Experimentation, Validation, and Uncertainty Analysis for Engineers*, John Wiley & Sons, 2009.
- [14] Groves, C. E., "Computational Fluid Dynamics Uncertainty Analysis for Payload Fairing Spacecraft Environmental Control Systems," Ph.D. thesis, University of Central Florida, Orlando, 2014.
- [15] "U.S. Standard Atmosphere, 1976," , 1976. URL https://www.ngdc.noaa.gov/stp/space-weather/online-publications/miscellaneous/us-standard-atmosphere-1976/us-standard-atmosphere_st76-1562_noaa.pdf.

- [16] Dufrene, A., MacLean, M., Holden, M., Mehta, M., and Seaford, M., “Space Launch System Base Heating Test: Experimental Operations and Results,” , Jan. 2016. URL https://web.archive.org/web/20190429090136id_/https://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/20160001817.pdf.
- [17] Herrera-Montojo, J., Fossati, M., and Minisci, E., “Shock-conforming mesh generation for aerodynamic analyses at supersonic regimes,” *Computers & Fluids*, Vol. 157, 2017, pp. 276–293. <https://doi.org/https://doi.org/10.1016>.
- [18] Candler, G., Barnhardt, M., Drayna, T., Nompelis, I., Peterson, D., and Subbareddy, P., “Unstructured Grid Approaches for Accurate Aeroheating Simulations,” *18th AIAA Computational Fluid Dynamics Conference*, AIAA Paper 2007-3959, 2007. <https://doi.org/6.2007-3959>.