

# NESC Trajectory Optimization Workshop - EDL Challenge Problem

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The NASA Engineering Safety Center (NESC) sponsored a trajectory optimization workshop in August 2024 at NASA Langley Research Center. As a result of this workshop, an effort began to formulate challenge problems in each of the domains of aeronautics, robotics, and entry, descent, and landing (EDL) to advance the state of the art of trajectory optimization. The plan is to disseminate the problems amongst academia and industry at an upcoming conference. This report details the challenge problem for EDL, specifically powered descent.

## I. Introduction

Powered-descent guidance (PDG) has received much attention in the entry, descent, and landing (EDL) guidance, navigation, and control (GN&C) community for the better part of the past 15 years with the various Mars missions [1] and more recently with Lunar exploration including NASA's Commercial Lunar Payload Services (CLPS) [2] and Human Landing System (HLS) programs [3]. The general trajectory optimization problem can be described as follows [4]:

$$\underset{x(t), u(t), t_f}{\text{minimize}} \quad J(x(t), u(t), t_f) \quad (1)$$

$$\text{s.t.} \quad \dot{x}(t) = f(x(t), u(t), t). \quad (2)$$

$$(x(t), u(t), t_f) \in C(t), \forall t \in [0, t_f], \quad (3)$$

$$x(0) \in X_0, x(t_f) \in \chi_f. \quad (4)$$

where the continuous states and control trajectories are the decision variables denoted by  $x(t)$  and  $u(t)$ . The final time  $t_f$  could either be free or fixed. In the context of the PDG problem, the objective defined in Eq. 1 generally attempts to minimize the propellant usage and/or landing error. The dynamics, represented by Eq. 2, can either be 3 degree-of-freedom (DoF) [5] or 6 DoF [6]. The state vector,  $x$ , consists of mass, position, velocity, attitude, and attitude rates. Standard state and control constraints defined by Eq. 3 include: angular rates, tilt from vertical, glideslope cone, thrust upper and lower bounds, thrust vector gimbal angle and gimbal rate limits, etc. Mission constraints can include state-triggered line-of-sight constraint to the landing site [7]. The resultant optimal trajectory needs to satisfy initial and terminal states which are represented by Eq. 4 where  $\chi_f$  is a set defined by requirements on vehicle states at touchdown, i.e. lateral position, velocity, attitude, attitude rate, etc.

For solving the PDG trajectory optimization problem offline, one standard approach is to use a commercial off-the-shelf nonlinear programming (NLP) solver such as IPOPT [8], Quickshot [9], or SNOPT [10]. Current and future planetary missions with ever-increasing complexity and requirements demand greater autonomy, necessitating a shift towards online guidance. Online PDG algorithms should possess the following key characteristics: speed, robustness, accuracy, and autonomy. Speed in that the guidance algorithm needs converge within a certain pre-allotted time which is especially critical for online applications. Accuracy in that the guidance algorithm needs to satisfy physics including nonlinear dynamics. Autonomy can be further described by reconfigurability, flexibility and robustness. The guidance algorithm needs to run online and be capable of taking into account mission updates such as a landing site redesignation command from the Crew.

The application of convex optimization has revolutionized the field of PDG which in essence transforms the landing guidance problem into a convex optimization problem that can be solved by a commercial conic solver [4,11–14] in real time. Prior to the breakthrough, planetary landings including Apollo, primarily relied on polynomial guidance [15–17]. While simple and robust, it is unable to address state, control, and mission constraints in the formulation of the optimal trajectory. Reference [18] demonstrated that a 3-DoF trajectory can be solved in under 2.5 seconds via lossless convexification (LCvx) with a custom interior-point-method (IPM) solver and on a radiation-hardened flight processor. Major aerospace companies have investigated leveraging the power of convex optimization for onboard trajectory optimization [19–23]. A plethora of other studies and flight tests can be found in Ref. [11] demonstrating that on-board optimization is capable of maximizing the performance envelope of the powered descent vehicle while still obeying physical limitations.

With the exception of Apollo, power descent vehicles on planetary missions have been all unmanned. For upcoming missions, such as Artemis, involving humans onboard, safety requirements are magnified. For safety and mission flexibility considerations, it is deemed advantageous to have an estimate of the reachability set or divert space (envelope) available to the Crew in real time to inform decision making. Formal definition of the constrained controllability and reachability sets (CRC) can be found in Ref.[24] and depicted in Fig 1. *“The constrained controllability set is the set of initial states which a predetermined landing site can be achieved. The constrained reachability set is the final set of position coordinates on the surface that can be reached from an initial condition.”* In other words, given certain initial states of the vehicle, characterize the region at which the vehicle can land while obeying the imposed state, controls, and mission constraints.

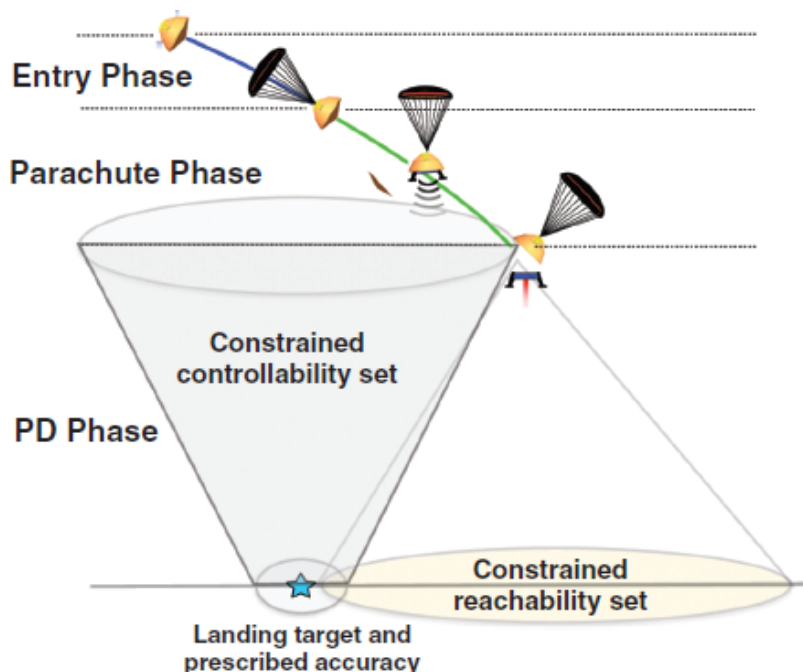


Figure 1. Illustration of Constrained Reachability and Controllability Sets (Source: Ref. [24])

Figure 2 is an illustration of a notional feasibility set described by an ellipse overlaid on top of a Lunar south pole terrain map. The region within the ellipse describes the set of feasible locations at which the

vehicle is capable of landing while satisfying the constraints and dynamics. Regions outside of the ellipse are considered infeasible as one or more of the constraints or the dynamics of the vehicle would have to be violated. Landing sites that are within and outside the feasibility set are distinguished by the color-coded circles. Having this information displayed to the Crew in real time would significantly improve mission flexibility as the Crew is no longer obliged to land at a single predetermined landing site and improves safety by not attempting to land anywhere outside of the feasible set. The feasibility set ebbs as the vehicle descends closer to the surface and propellant level diminishes.

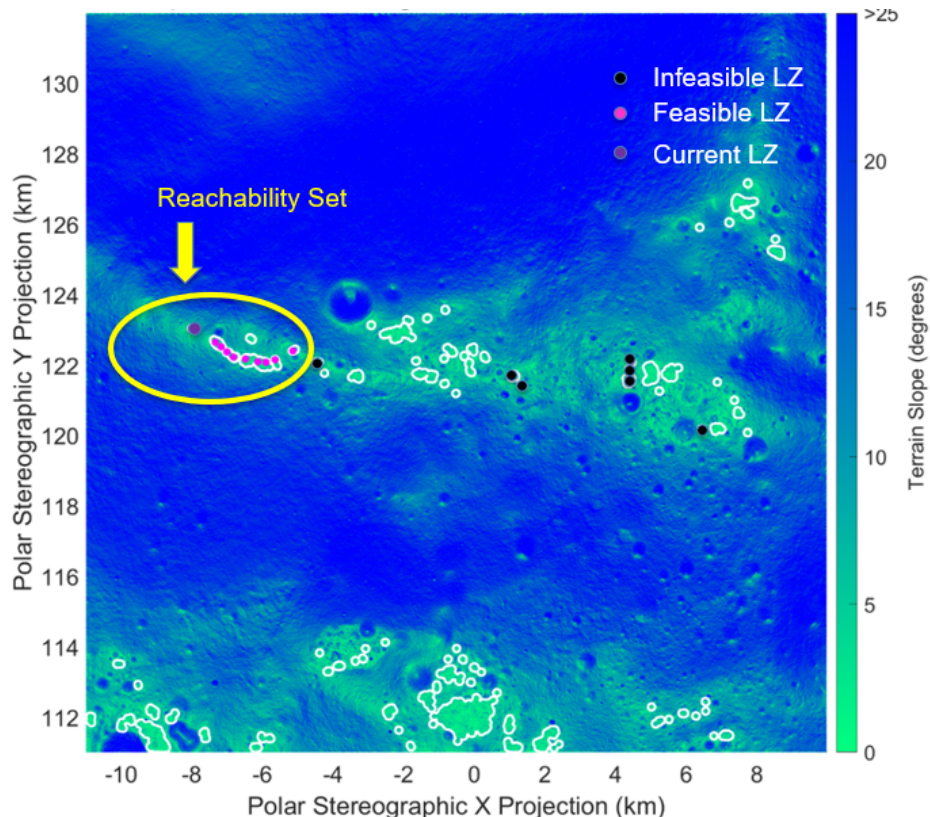


**Figure 2. Display of Reachability Set with Target Landing Sites**

Seminal papers [5, 17] show 3-DoF guidance based on LCVx vastly outperforming polynomial guidance of various flavors in terms of enabling access to a greater portion of the feasibility set. The method found in Refs. [24, 25] can be used to characterize the divert space by obtaining a conservative approximation of the CRC of final vehicle positions for a given time of flight. This method begins by losslessly convexifying the 3-DoF PDG problem and then solving a second-order cone program to obtain states on the boundary of the CRC. The convex hull formed by a set of boundary states is then used to approximate the CRC. The volume of the convex hull is expanded to better approximate the CRC by solving for additional boundary states. While this method can generate approximations of the feasible set that are arbitrarily close to the true set, it requires solving many optimization problems, one for each point added to the convex hull and the entire process must be repeated to obtain the divert space for different times of flight. The iterative nature of this approach requires extensive computational resources, thus it is not suitable for on-board computation given today's technology. Furthermore, this method does not account for vehicle attitude dynamics and constraints such as limits on the attitude, attitude rates, and engine gimbal angle that would reduce the size of the feasibility set.

Current state of art available in literature suggests that, for real-time applications, the runtime on a radiation-hardened flight computer for a single trajectory with 3-DoF dynamics via LCVx is on the order of 2.5 seconds [18]. For 6-DoF dynamics, Ref. [22] showed that with a dual quaternion formulation, runtime is on the order of 6 seconds for a trajectory with 10 nodes on a flight-like processor with custom IPM solver. Note: reported runtime increases with more nodes (12 seconds for 20 nodes) and state-triggered line-of-

sight constraints. Advances in custom first-order solvers such as the proportional integral projected gradient method (PIPG) are showing potential to significantly reduce runtimes. A  $\times 5\text{--}10$  improvement in runtime would allow for real-time application to produce a single trajectory, however, generating multiple optimal trajectories to form the boundaries of the feasibility set and characterize the maximum divert authority remains a challenge for a non-symmetric divert space shaped by complex system constraints..

## II. Problem Statement

In the original problem formulation laid out in Eqs. 1-4, the intent is to solve for a single trajectory that satisfies the dynamics, constraints, and initial/terminal conditions. The general trajectory optimization problem can be slightly modified when solving for the CRC:

$$\text{maximize } \eta_f \quad (5)$$

$$\text{s.t. } \dot{x}(t) = f(x, u, t). \quad (6)$$

$$(x(t), u(t), t_f) \in C(t), \forall t \in [0, t_f], \quad (7)$$

$$x(0) = X_0, \tilde{x}(t_f) \in \tilde{\chi}_f, \{r_2(t_f), r_3(t_f)\} \in \eta_f \quad (8)$$

In the modified formulation, the objective becomes finding the constrained feasibility set,  $\eta_f$ , defined by the *final* downrange and crossrange positions at touchdown,  $r_2$  and  $r_3$ , while satisfying the nonlinear dynamics (Eq. 6), and state/control/mission constraints (Eq. 7). This set may form a complex, non-symmetric region, reflecting the full range of feasible landing sites. The terminal condition constraints are modified such that a subset of  $x$  (excluding  $r_2$  and  $r_3$ ) denoted by  $\tilde{x}(t_f)$  is required to lie within  $\tilde{\chi}_f$ .  $\tilde{\chi}_f$  are the set of vehicle states that satisfy touchdown requirements, i.e. lateral velocity, attitude, attitude rate, etc (excluding  $r_2$  and  $r_3$ ). Perhaps, the most challenging aspect is being able to perform this procedure in real time onboard a processor that has far less computational power compared to a standard PC.

In Ref. [24], the upper bound on the time-of-flight used to generate the boundaries of the CRC via LCvx is determined by the propellant remaining. However, for vehicle configurations with thrust-to-weight (T/W) ratio greater than unity, the reachability set is not only determined by propellant remaining but also the current energy state of the vehicle. Figure 3 is a cartoon illustration of the reachability set at various times along the nominal trajectory for a lander configuration with  $T/W > 1$ . At a given time along the nominal trajectory, the trajectory optimization algorithm of choice can be called over a grid of touchdown locations to approximate the CRC. Key constraints can include vehicle tilt limit, thrust upper and lower bounds, and thrust vector gimbal limits. The region of feasibility at  $T = -50, -40$ , and  $-30$  seconds are portrayed by the blue, green, and orange ellipses which shrinks with time until touchdown. In reality, the outline of the CRC may not be ellipse shaped. To more accurately trace out the CRC, a finer mesh is warranted which undoubtedly increases runtime. While it is plausible to generate the feasibility set offline and store it as a lookup table (LUT) for displaying online, the challenge arises once the vehicle has deviated away from the nominal trajectory (e.g., the Crew initiates a divert to an alternative landing site). The feasibility set changes and the LUT which was generated from the nominal trajectory is no longer accurate. In other words, the CRC needs to be updated in real time given the current vehicle states, which may not lie on the nominal trajectory.

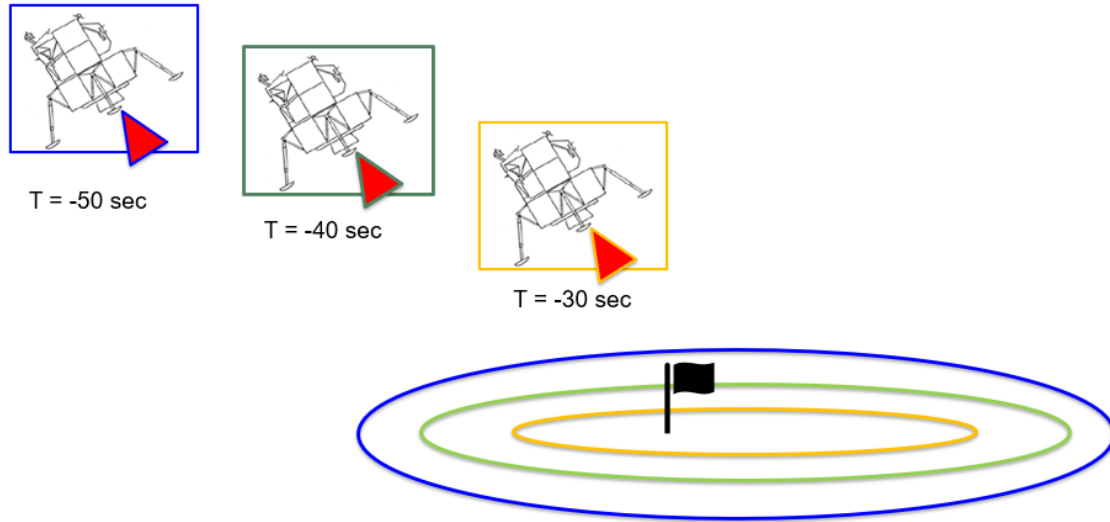


Figure 3. Cartoon Illustration of Reachability Sets along nominal trajectory for a lander with  $T/W > 1$

In summary, the challenge trajectory optimization problem pertaining to EDL as a result of the NESC sponsored workshop can be described in Eqs. 5 to 8. The goal is solve for the constrained reachability set in real time using 6 DoF vehicle dynamics model while subject to various state, control, and mission constraints.

## References

- <sup>1</sup><https://science.nasa.gov/planetary-science/programs/mars-exploration/> [accessed 18 September, 2024].
- <sup>2</sup><https://www.nasa.gov/commercial-lunar-payload-services/> [accessed 18 September, 2024].
- <sup>3</sup><https://www.nasa.gov/humans-in-space/human-landing-system/> [accessed 18 September, 2024].
- <sup>4</sup>Malyuta, D., Yu, Y., Elango, P., Acikmese, B., “Advances in Trajectory Optimization for Space Vehicle Control,” *Annual Rev. Control*, vol. 52, pp. 282-315, Dec. 2021
- <sup>5</sup>Acikmese, B., Ploen, S., “Convex Programming Approach to Powered Descent Guidance for Mars,” *Journal of Guidance, Control, and Dynamics*, vol. 30, No. 5, Sept-Oct 2007
- <sup>6</sup>Szmuk, Michael, Ackimese, B., “Successive Convexification for 6-DoF Mars Rocket Powered Landing with Free-Final-Time,” *Proceedings of the AIAA SciTech Forum 2018*, AIAA 2018-0617
- <sup>7</sup>Szmuk, Michael, Reynolds, T., Ackimese, B., “Successive Convexification for Real-Time Six-Degree-of-Freedom Powered Descent Guidance with State-Triggered Constraints,” *Journal of Guidance, Control, and Dynamics*, vol. 43, No. 8, August 2020
- <sup>8</sup>Wachter, A., Biegler, L., “On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming,” *Mathematical Programming*, vol. 106, March 2006
- <sup>9</sup><https://www.spaceworks.aero/software/quickshot/> [accessed 30 January, 2023].
- <sup>10</sup>Gill, G. E., Murry, W., and Saunders, M. A., “SNOPT: An SQP Algorithm for Large-Scale Constrained Optimization,” *SIAM Review*, vol. 47, No. 1, January 2005
- <sup>11</sup>Malyuta, D., Reynolds, T., Szmuk, M., Lew, T., Bonalli, R., Pavone, M., and Acikmese, B., “Convex Optimization for Trajectory Generation: A tutorial on Generating Dynamically Feasible Trajectories Reliably and Efficiently,” *IEEE Control Systems*, vol. 42, no. 5, October 2022
- <sup>12</sup>Toh, K., Todd, M., Tutuncu, R., “SDPT3 — a Matlab software package for semidefinite programming”, *Optimization Methods and Software*, Vol. 11, pp. 545–581, 1999.
- <sup>13</sup>Sturm, J., “Using SeDuMi 1.02, A Matlab toolbox for optimization over symmetric cones”, *Optimization Methods and Software*, Vol. 11, Issue 1-4, pp. 625-653, 1999, DOI: 10.1080/10556789908805766
- <sup>14</sup>MOSEK ApS, “MOSEK Optimization Toolbox for MATLAB”, 2022, <https://docs.mosek.com/latest/toolbox/index.html>
- <sup>15</sup>Cherry, G., “A General, Explicit, Optimizing Guidance Law for Rocket-Propelled Spaceflight,” *Proceedings of the AAS/ION Astrodynamics Guidance and Control Conference*, August 24-26, No. 64-638
- <sup>16</sup>Sostaric, R. R., and Rea, R. J., “Powered Descent Guidance Methods for the Moon and Mars,” *Proceedings of the AIAA Guidance, Navigation, and Control Conference and Exhibit 2005*, AIAA 2005-6287.
- <sup>17</sup>Ploen, R. S., Acikmese, B., and Wolf, A., “A Comparison of Powered Descent Guidance Laws for Mars Pinpoint Landing,” *Proceedings of AIAA/AAS Astrodynamics Specialist Conference and Exhibit*, Keystone, CO. August 21-24, 2006
- <sup>18</sup>Dueri, D., Acikmese, B., Scharf, D., and Harris, M., “Customized Real-Time Interior-Point Methods for Onboard Powered-Descent Guidance,” vol. 40, No. 2, February 2017
- <sup>19</sup>Mattingley, J., Boyd, S., “CVXGEN: a code generator for embedded convex optimization,” *Optim Eng*, vol. 13, 2012
- <sup>20</sup>Blackmore, L., “Autonomous Precision Landing of Space Rockets,” *Bridge*, vol. 4, no. 46, pp. 15-20, Dec. 2016

<sup>21</sup>Berning, W. A., Strohl, D. L., and Bieniawski, S., “Lossless Convex Guidance for Lunar Powered Descent,” *Proceedings of the AIAA SciTech Forum 2023*, AIAA 2023-2004.

<sup>22</sup>Strohl, D. L., Berning, W. A., Bieniawski, S., Doll, J., et. al. “Implementation of a Six Degree of Freedom Precision Lunar Landing Algorithm Using Dual Quaternion Representation,” *Proceedings of the AIAA SciTech Forum 2022*, AIAA 2022-1831.

<sup>23</sup>Schaffer, J., Owen, C., et. al. “Implementation and Testing of Convex Optimization-based Guidance for Hazard Detection and Avoidance on a Lunar Lander,” *Proceedings of the AIAA SciTech Forum 2024*, AIAA 2023-1584.

<sup>24</sup>Eren, U., Dueri, D., and Acikmese, B., “Constrained Reachability and Controllability Sets for Plan,” *Journal of Guidance, Control, and Dynamics*, vol. 38, No. 11, November 2015

<sup>25</sup>Srinivas, N., Vinod, A., Cairano, S., and Weiss, A., “Lunar Landing with Feasible Divert using Controllable Sets,” *Proceedings of the AIAA SciTech Forum 2024*, AIAA 2024-0324.