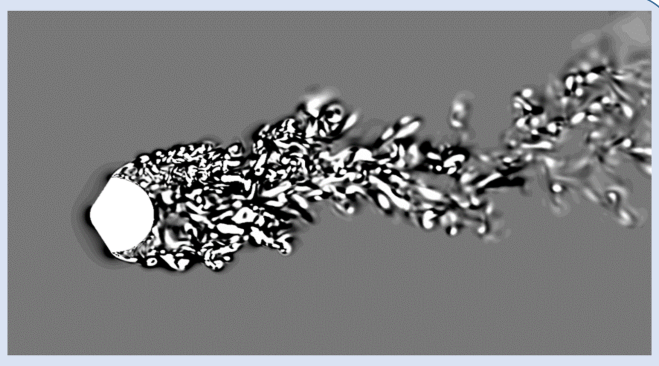
Unsteady Aerodynamic Modeling of Atmospheric Entry Vehicles in Subsonic Flow: A Frequency Response Approach

Hisham Shehata[1], Jehan Dastoor[2],
Bruce Owens[3], Eli Shellabarger[3] and Mark Schoenenberger[3]
[1] Analytical Mechanics Associates, [2] ASDL, Georgia Tech, [3] NASA Langley Research Center

IPPW, Stuttgart, Germany, 23-27 June 2025

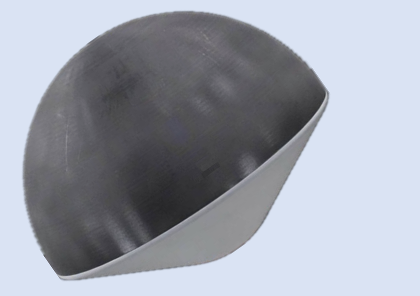
Motivation



- **Planetary missions** require an entry, descent, and landing (EDL) system to successfully deliver a payload to the surface.
- Dynamic instabilities tend to grow. Either the vehicle **(i) reaches a stable limit cycle**, or **(ii) the oscillations can grow** to where a stabilizing device may not be safe to deploy.

Characterizing unsteady wake dynamics under oscillatory input is an important part of aero-databasing, and the modeling and simulation of entry vehicle trajectories

Objectives

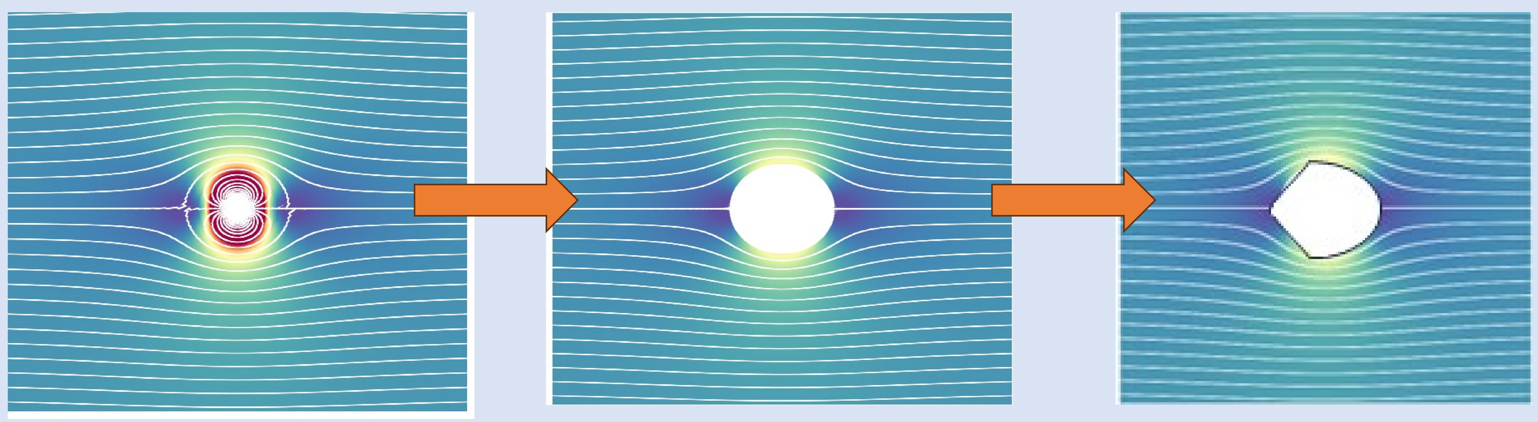


Utilize a data-driven frequency response approach to model unsteady aerodynamics (lift and moment) from experimental forced oscillation tests at NASA Langley's 12ft Low Speed Wind Tunnel

The chosen test article is a 60-degree spherical cone angle with a hemispherical backshell, inspired by past and current missions such as Mars Microprobe and Varda's Winnebago series

Classical Aerodynamic Theories

(1) Joukowski's transform: Conformal mapping from a simple geometric shape (circle) into a complex one (e.g., cambered airfoil, blunt bodies). The solution of velocity potential remains accurate.



(2) Kutta Condition : Bounded circulation is conserved between the wake and on the body. Flow leaves the trailing edge smoothly. Lift is proportional to flow circulation.

(3) Unsteady aerodynamic models: Theodorsen provides a frequency-dependent solution for the unsteady lift and pitch moment resulting from oscillations. Builds on the steady-lift concepts derived from Joukowski and Kutta condition and extends them to unsteady flow via potential flow theory.

Proposed semi-empirical model :

$$C_L = \underbrace{\frac{C_{L_\alpha} b}{2V_\infty^2}\left[U_\infty \dot{\alpha} - ba\ddot{\alpha}\right]}_{\text{Added Mass Effects}} + \overbrace{\underbrace{C_{L_\alpha}\left[\alpha + b\left(\frac{1}{2}-a\right)\frac{\dot{\alpha}}{U_\infty}\right]}_{\text{Quasi-Steady Term}} C_L(k)}^{\text{Unsteady Term}}$$

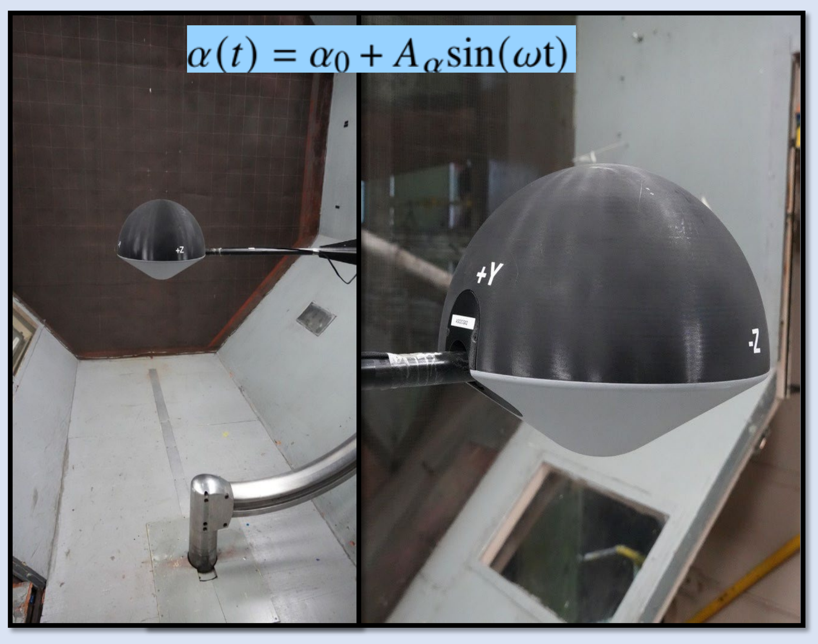
Transfer function accounts for wake effects

$$C_m = \underbrace{\frac{C_{L_\alpha} b}{2U_\infty^2}\left[-U_\infty b\left(\frac{1}{2}-a\right)\dot{\alpha} - b^2\left(\frac{1}{8}+a^2\right)\ddot{\alpha}\right]}_{\text{Added Mass Effects}} + \underbrace{C_{L_\alpha} b\left(a+\frac{1}{2}\right)\left[\alpha + b\left(\frac{1}{2}-a\right)\frac{\dot{\alpha}}{U_\infty}\right]}_{\text{Quasi-steady Term}} C_L(k)$$

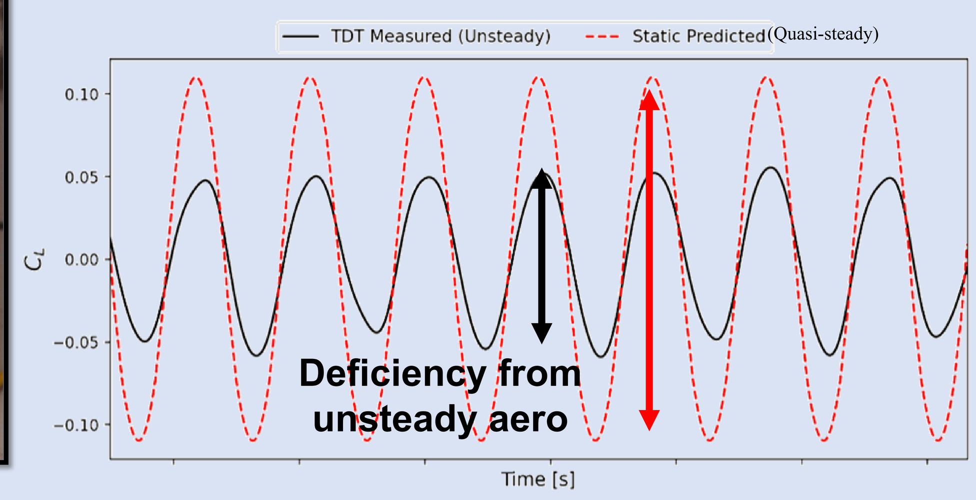
C_{L_α} = Lift-curve slope, U_∞ = Free-stream velocity | α = Angle of attack | b = Vehicle span | a = Normalized position of the axis of rotation from the center of the body | $C_L(k)$ = Transfer function

Technical Approach

NASA LaRC's 12ft Low Speed Wind Tunnel Forced Oscillation



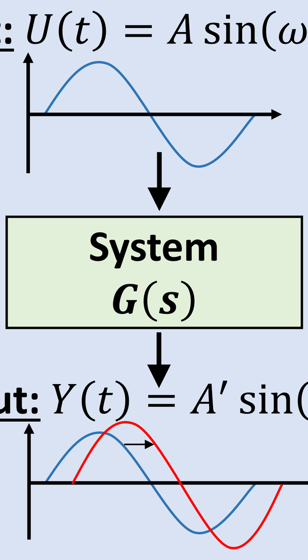
Mean Angle of Attack, α_0	Pitching amplitude, A_α	Reduced frequency, k
0°	10°	0.01 - 0.21



Frequency Response Analysis

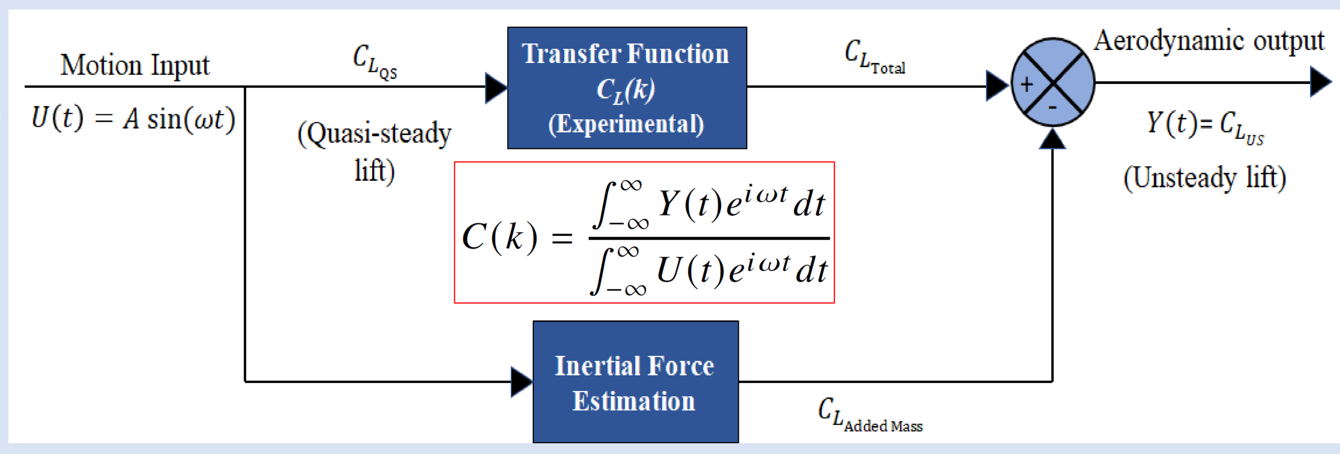
An approximated transfer function from forced oscillation measurements addresses unsteady aerodynamic analysis from a dynamical systems perspective

Input: $U(t) = A\sin(\omega t)$



Output: $Y(t) = A' \sin(\omega t + \phi)$

$$|G(s)| = \frac{A'}{A}$$
$$\angle G(s) = \phi$$



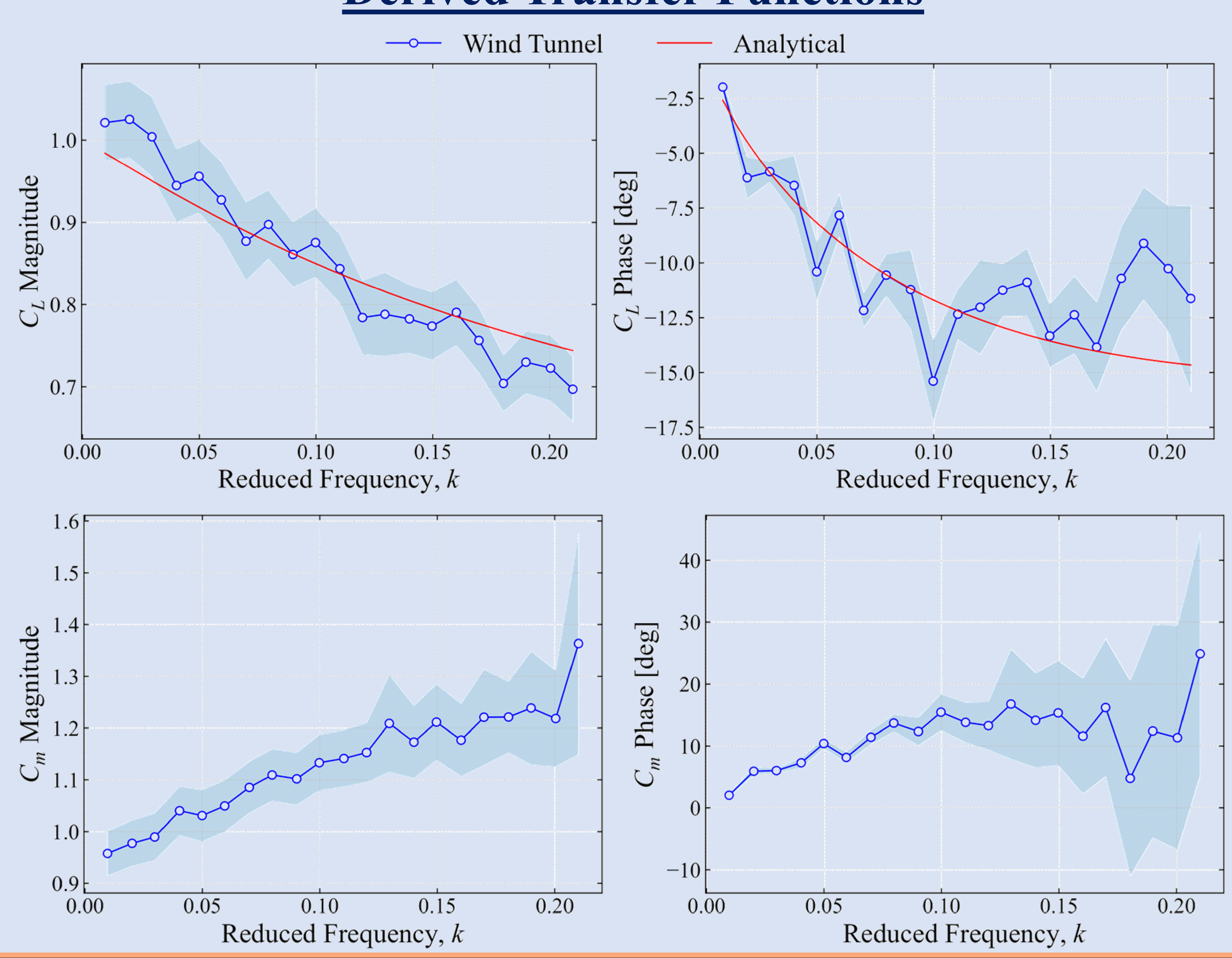
- The transfer function is a reduced frequency dependent complex number:

$$C(k) = F(k) + iG(k) \qquad C_L(k) = \frac{C_{L_{US}}(k)}{C_{L_{QS}}(k)}$$

- $C(k)$ contains information on the gain/deficiency and phase lag resulting from dynamic motion:

Results

Derived Transfer Functions

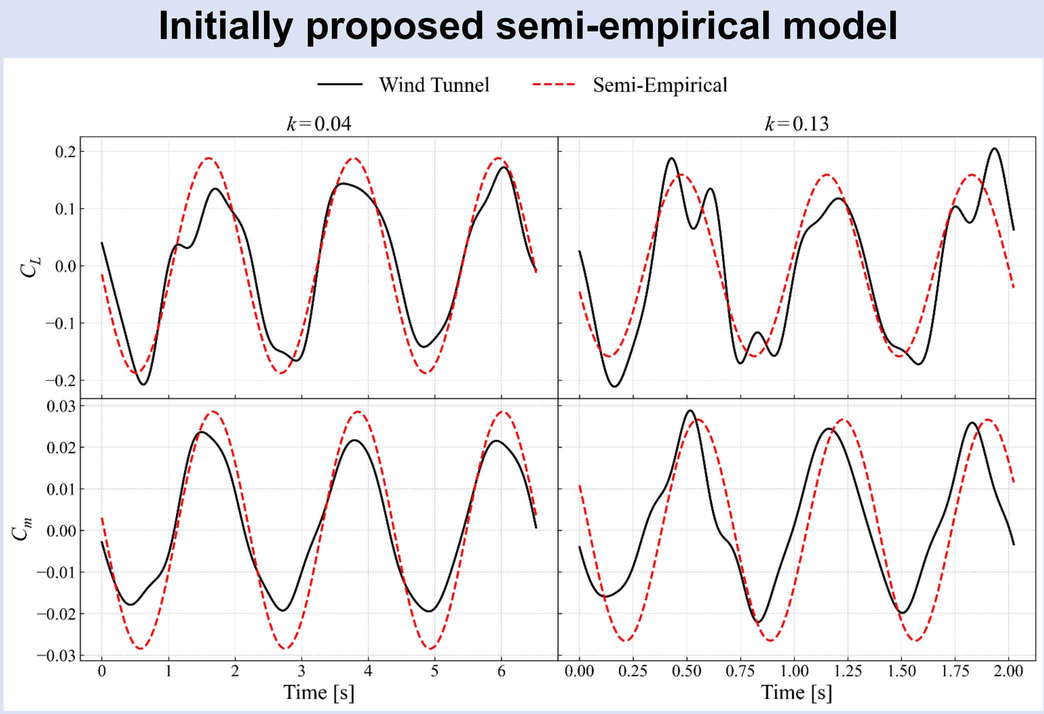


k	$C_L(k)$	$C_m(k)$	k	$C_L(k)$	$C_m(k)$
0.01	$1.021-0.035i$	$0.957+0.034i$	0.12	$0.767-0.163i$	$1.122+0.265i$
0.02	$1.019-0.109i$	$0.972+0.101i$	0.13	$0.773-0.154i$	$1.156+0.350i$
0.03	$0.999-0.102i$	$0.984+0.104i$	0.14	$0.768-0.148i$	$1.136+0.287i$
0.04	$0.939-0.106i$	$1.032+0.132i$	0.15	$0.753-0.178i$	$1.167+0.321i$
0.05	$0.940-0.173i$	$1.014+0.186i$	0.16	$0.772-0.169i$	$1.151+0.237i$
0.06	$0.919-0.126i$	$1.039+0.149i$	0.17	$0.734-0.181i$	$1.169+0.342i$
0.07	$0.857-0.185i$	$1.064+0.215i$	0.18	$0.692-0.131i$	$1.212+0.102i$
0.08	$0.882-0.164i$	$1.078+0.263i$	0.19	$0.721-0.116i$	$1.203+0.268i$
0.09	$0.844-0.167i$	$1.076+0.236i$	0.20	$0.711-0.129i$	$1.188+0.241i$
0.10	$0.844-0.232i$	$1.092+0.302i$	0.21	$0.682-0.141i$	$1.226+0.577i$
0.11	$0.824-0.180i$	$1.108+0.272i$			

Transfer functions describe the gain and the phase of the aerodynamic response due to oscillatory input motion as a function of frequency.

Lift and Pitch Moment Time-series Reconstruction

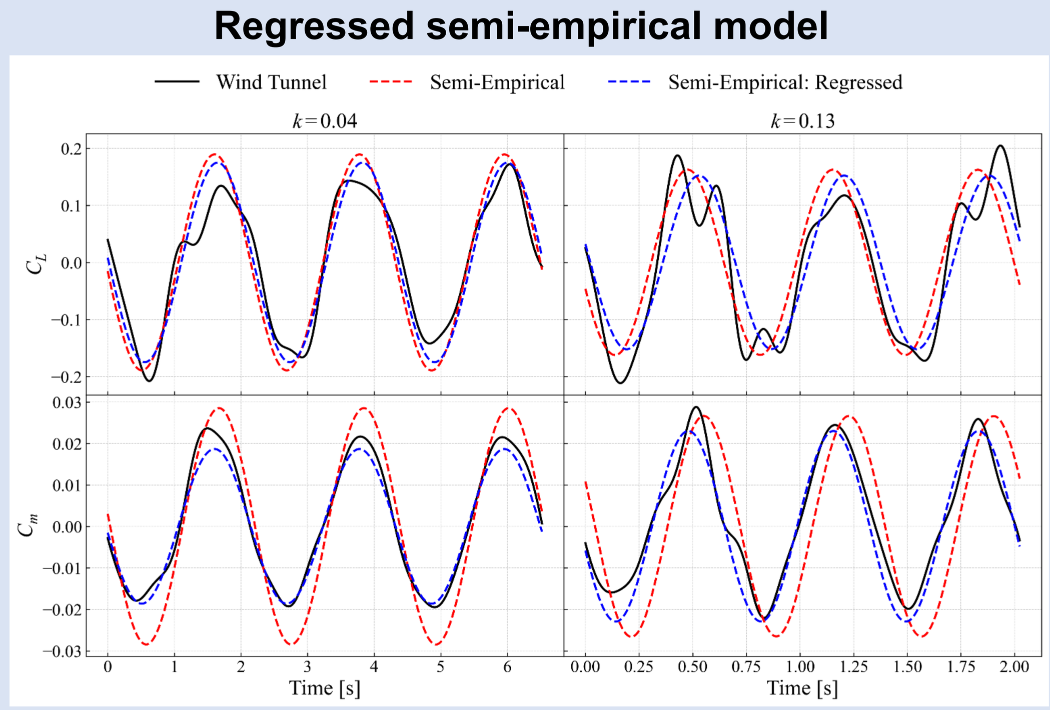
Initially proposed semi-empirical model



$$C_L = \frac{C_{L_\alpha} b}{2V_\infty^2}\left[U_\infty \dot{\alpha} - ba\ddot{\alpha}\right] + C_{L_\alpha}\left[\alpha + b\left(\frac{1}{2}-a\right)\frac{\dot{\alpha}}{U_\infty}\right] C_L(k)$$

$$C_m = \frac{C_{L_\alpha} b}{2U_\infty^2}\left[-U_\infty b\left(\frac{1}{2}-a\right)\dot{\alpha} - b^2\left(\frac{1}{8}+a^2\right)\ddot{\alpha}\right] + C_{L_\alpha} b\left(a+\frac{1}{2}\right)\left[\alpha + b\left(\frac{1}{2}-a\right)\frac{\dot{\alpha}}{U_\infty}\right] C_L(k)$$

Regressed semi-empirical model



$$C_L = C_1\frac{C_{L_\alpha} b}{2V_\infty^2}\left[U_\infty \dot{\alpha} - ba\ddot{\alpha}\right] + C_2 C_{L_\alpha}\left[\alpha + b\left(\frac{1}{2}-a\right)\frac{\dot{\alpha}}{U_\infty}\right] C_L(k)$$

$$C_m = C_3\frac{C_{L_\alpha} b}{2U_\infty^2}\left[-U_\infty b\left(\frac{1}{2}-a\right)\dot{\alpha} - b^2\left(\frac{1}{8}+a^2\right)\ddot{\alpha}\right] + C_4 C_{L_\alpha} b\left(a+\frac{1}{2}\right)\left[\alpha + b\left(\frac{1}{2}-a\right)\frac{\dot{\alpha}}{U_\infty}\right] C_m(k)$$

C_1 (C_L Added Mass)	C_2 (C_L Unsteady)	C_3 (C_m Added Mass)	C_4 (C_m Unsteady)
-2.77	0.924	-0.146	0.598

Conclusion

- Regression Analysis with a modified semi-empirical formulation from classical aerodynamic theories and frequency response. correctly predicts force and moments from wake dynamic effects.
- The approach can be used as a data reduction tool to extract ***Cmqs***

Acknowledgments

This work was supported by Entry Systems Modeling project. Support from Young Moon, Lauren Pegher, Zoe Hughes are appreciated.

Corresponding author: Sam Shehata; Hisham.m.shehata@nasa.gov

Submission ID: 109