AI/ML Applications to Transition and Turbulence Modeling

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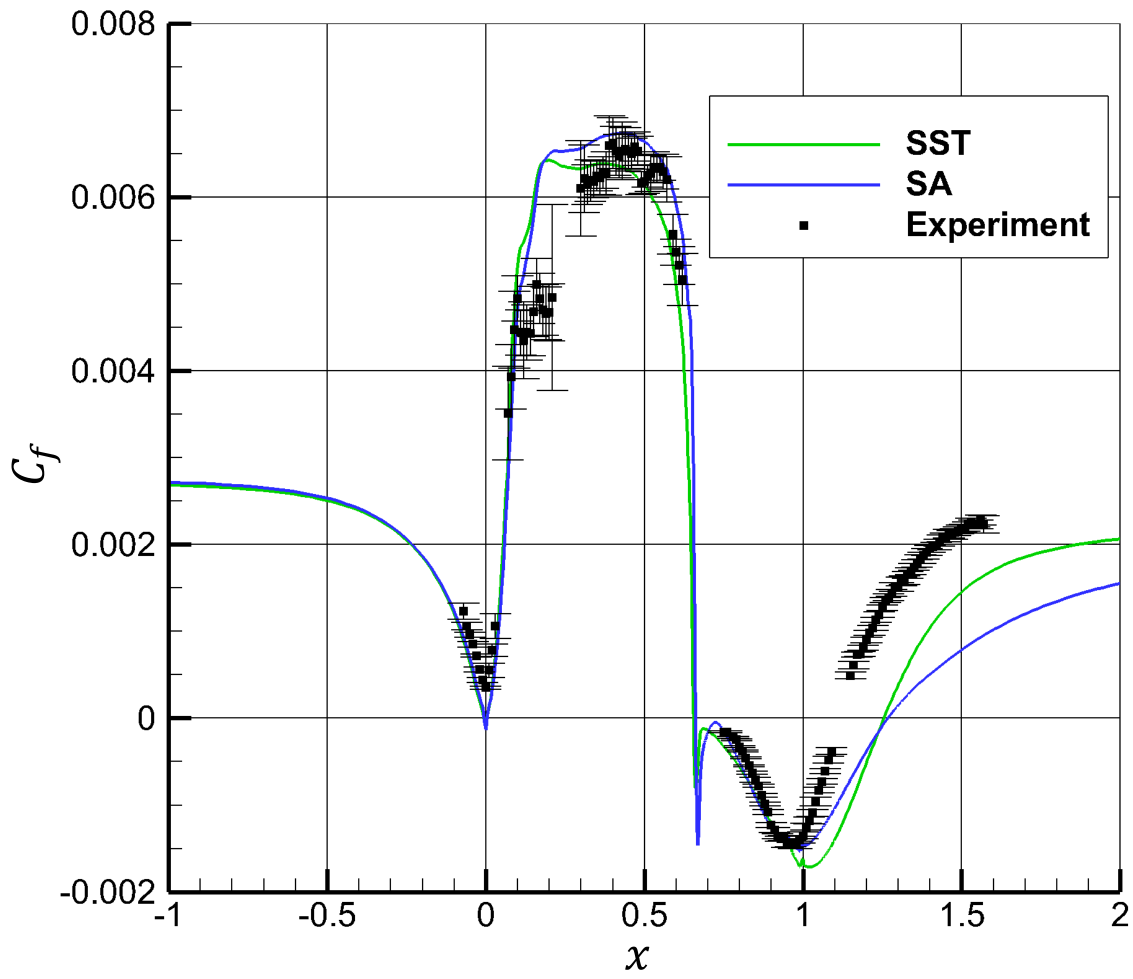
Data-driven Turbulence Modeling

Motivation

- Both SA and SST k-ω models overpredict the length of the separation bubble
- An underprediction of Reynolds stresses is responsible for the prediction of a delayed reattachment
- Both models accurately predict the skin friction behavior immediately after flow separation
- The SST k-ω model predicts a faster skin friction recovery compared to the SA model



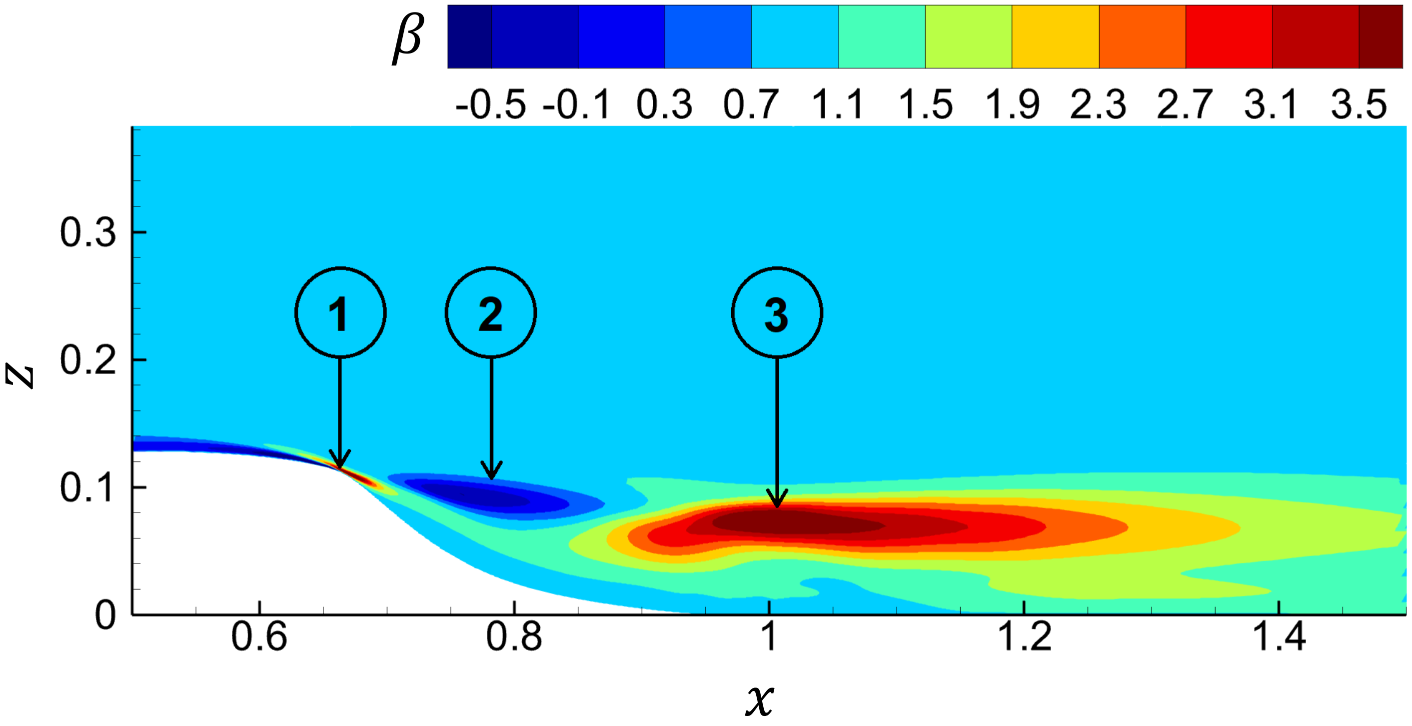
Inferring a model inadequacy field via Field Inversion using FUN3D

$$\frac{D\widetilde{\nu}}{Dt} = \beta(\boldsymbol{x}) c_{b1}\widetilde{S}\widetilde{\nu} + \frac{1}{\sigma}\left[\frac{\partial}{\partial x_j}\left((\nu+\widetilde{\nu})\frac{\partial\widetilde{\nu}}{\partial x_j}\right) + c_{b2}\frac{\partial\widetilde{\nu}}{\partial x_i}\frac{\partial\widetilde{\nu}}{\partial x_i}\right] - c_{w1} f_w \left(\frac{\widetilde{\nu}}{d_w}\right)^2$$

$$\boxed{\min_{\beta(\boldsymbol{x})} \left[\mathcal{C}(\beta(\boldsymbol{x})) + \lambda\mathcal{R}(\beta(\boldsymbol{x}))\right]}$$

$$\mathcal{C}(\beta(\boldsymbol{x})) = \left\| C_f^{\mathrm{pred}}(\beta(\boldsymbol{x})) - C_f^{\mathrm{data}} \right\|_2^2$$

$$\mathcal{R}(\beta(\boldsymbol{x})) = \|\beta(\boldsymbol{x}) - 1\|_2^2$$

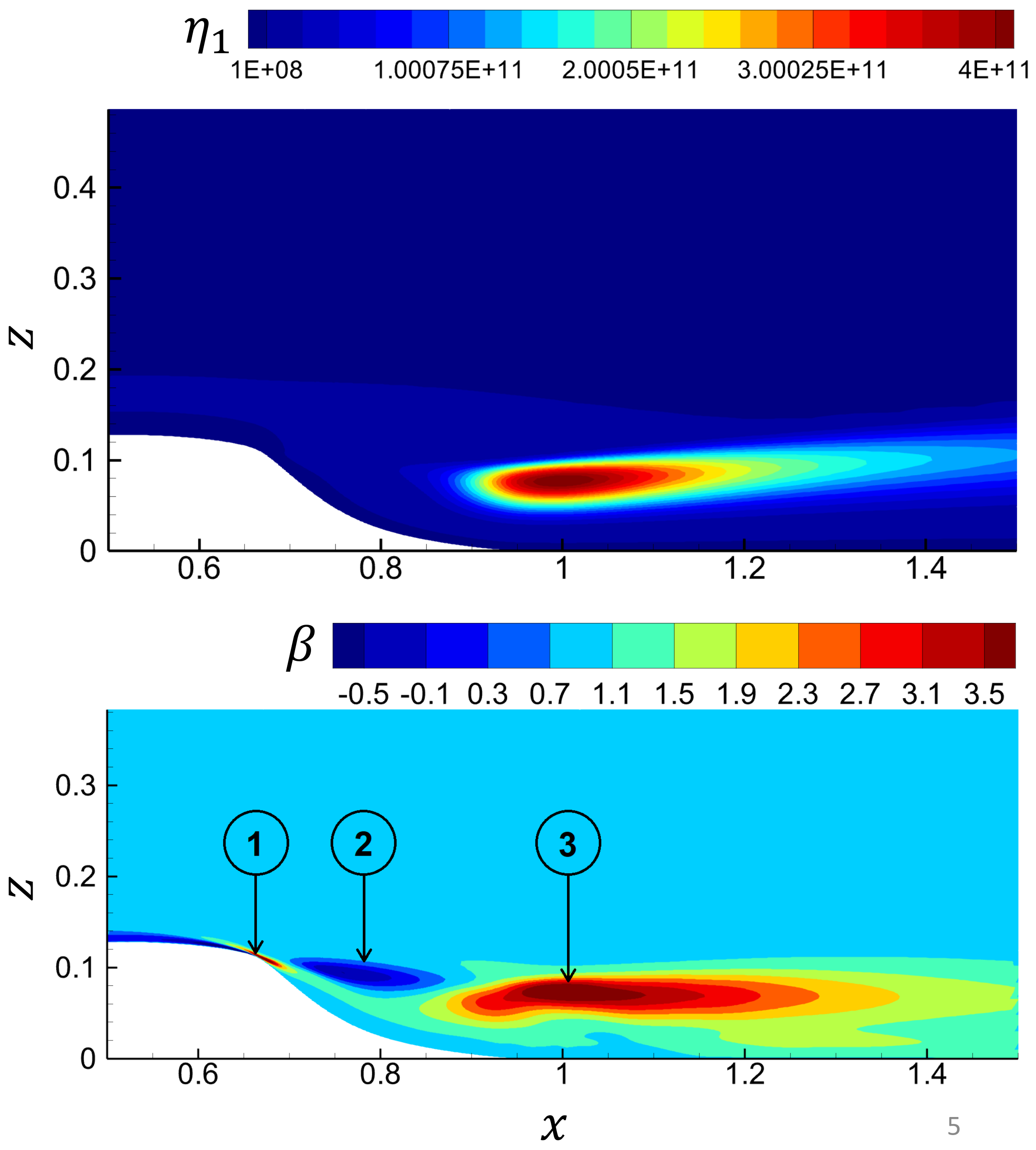


Feature Design for SA model

Vorticity Reynolds number: $Re_\Omega = \dfrac{d_w^2 \Omega}{\nu}$

Turbulence Reynolds number: $Re_t = \dfrac{\nu_t}{\nu}$

Feature candidate: $\eta_{r,1} = Re_\Omega Re_t^2$



Generalizability to different flow Reynolds numbers

Feature candidate: $\eta_{r,1} = Re_{\Omega} Re_t^2$

The parameters w_1 within an augmentation function $\beta(\eta_{r,1}; w_1)$ were calibrated based on the data obtained from field inversion that was performed on the hump case.

Hump case: **improves** ☺ Curved back-step: **no change** ☹ Periodic hill: **no change** ☹

A third quantity is needed within the feature to generalize to different flows.

Designing the analytic augmentation

Turbulence kinetic energy (TKE) Reynolds number: $Re_k = \dfrac{d_w\sqrt{k}}{\nu}$

Limiter to activate the augmentation only in regions of $Re_t > 10$: $\eta_r^\ell = \dfrac{1}{1+\exp(100-10Re_t)}$

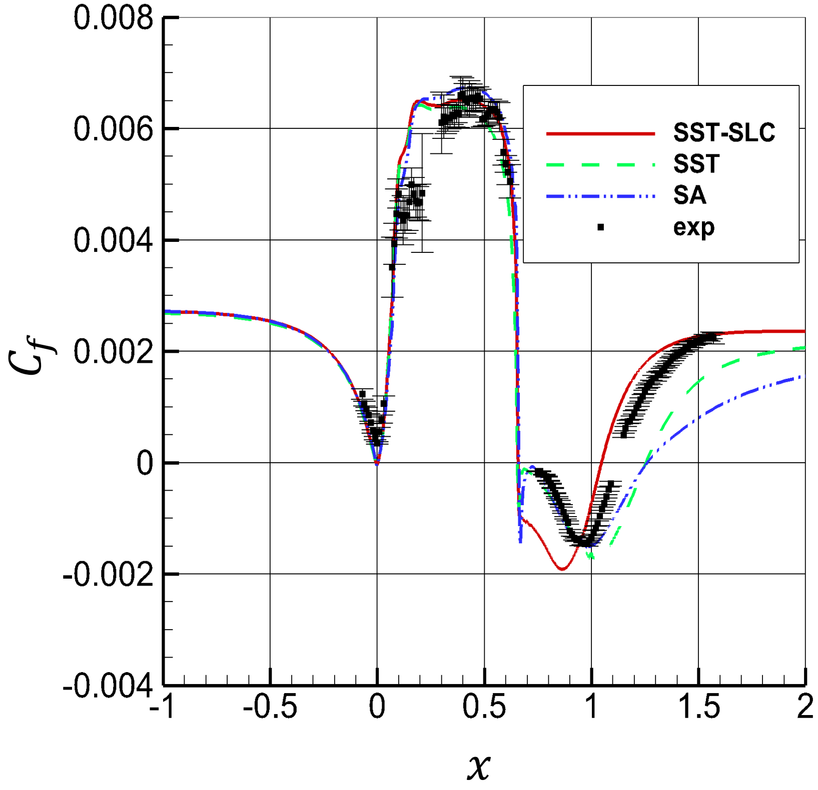
The feature was formulated via trial-and-error as follows: $\eta_r = \dfrac{Re_\Omega^{0.61} Re_t^{0.39}}{Re_k}\eta_r^\ell$

A simple closed-form expression for the augmentation (multiplied to the production term in the transport equation for k) is proposed as follows:

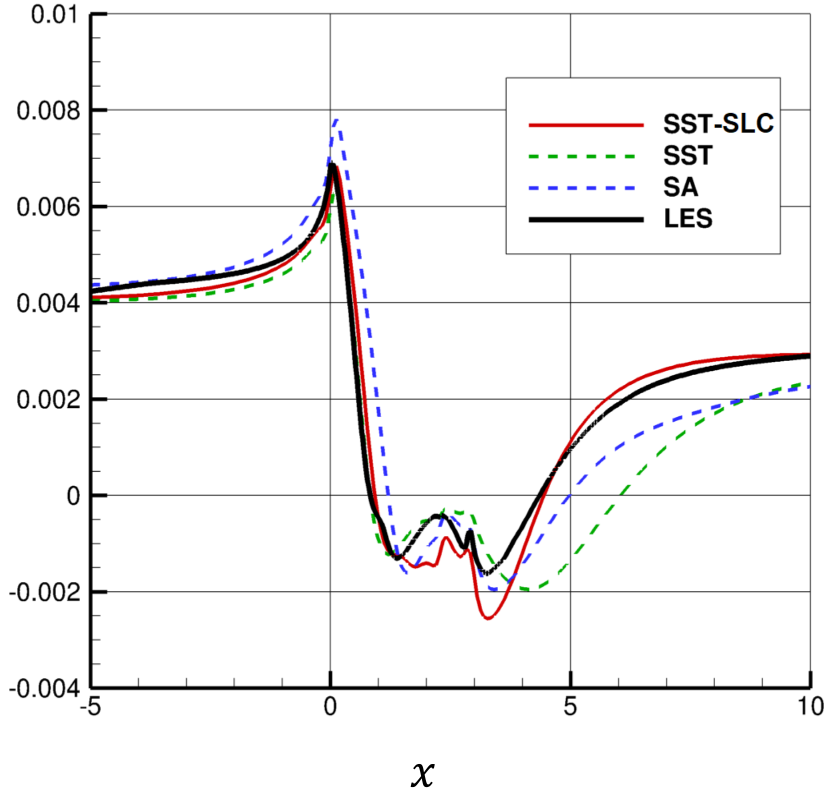
$$\beta = 1 + \frac{c_\beta^{\max} - 1}{1+\exp(100(c_{\eta_r} - \eta_r))}$$

where $c_{\eta_r} = 0.9$ and $c_\beta^{\max} = 5$ were found to be appropriate values for the constants.

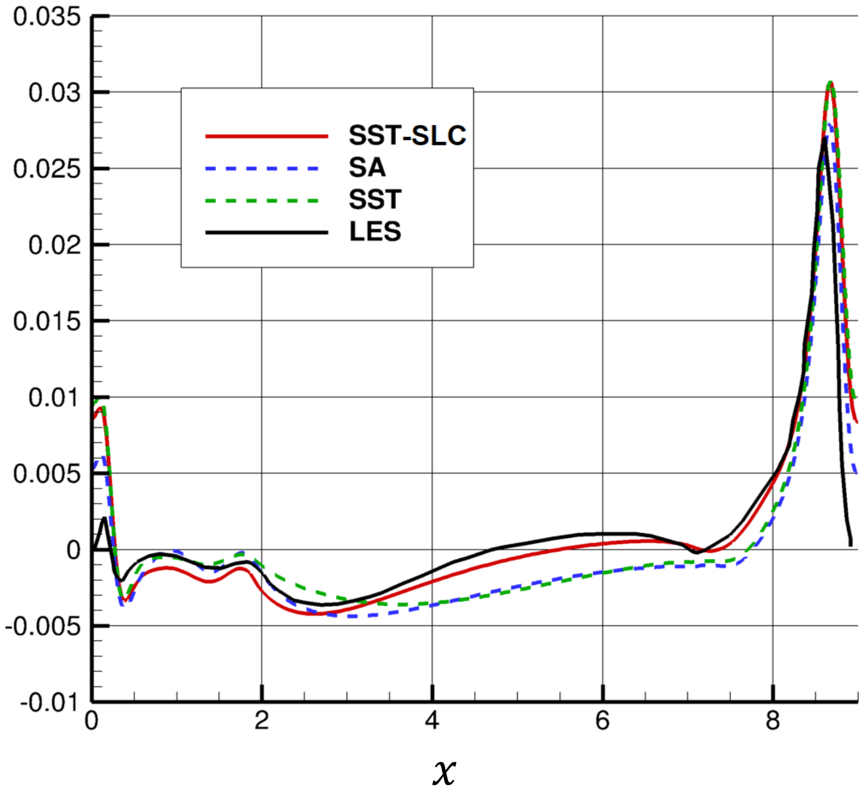
Results



2D NASA wall-mounted hump



2D Curved backstep



2D Periodic Hill

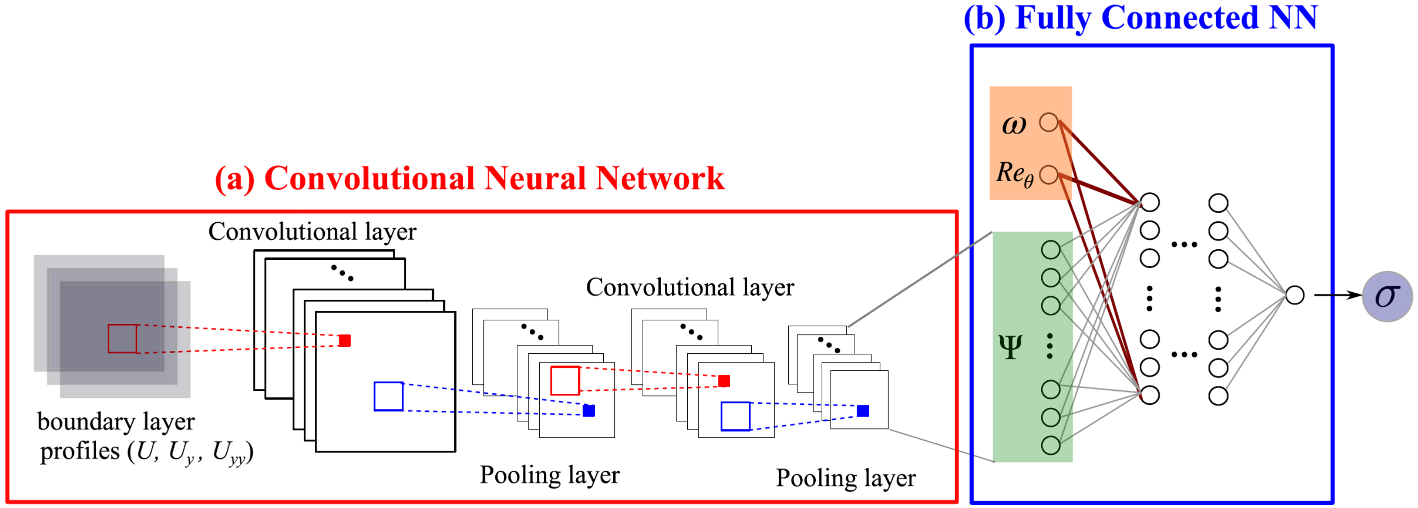
Related Publications

- Duraisamy, K., & Srivastava, V. (2025). Machine learning augmented modeling of turbulence. In *Data Driven Analysis and Modeling of Turbulent Flows* (pp. 311-354). Academic Press.
- Srivastava, V., Rumsey, C. L., Coleman, G. N., & Wang, L. (2024). On generalizably improving RANS predictions of flow separation and reattachment. In *AIAA SCITECH 2024 Forum* (p. 2520).
- Hildebrand, N., Srivastava, V., Zaki, T. A., & Choudhari, M. M. (2023, September). DeepONet-Assisted Optimization of Surface Topography for Transition Delay in A Mach 4.5 Boundary Layer. In *14th International ERCOFTAC Symposium on Engineering Turbulence Modelling and Measurements (ETMM14)* (No. 20230001917).
- Srivastava, V., Sulzer, V., Mohtat, P., Siegel, J. B., & Duraisamy, K. (2023). A non-intrusive approach for physics-constrained learning with application to fuel cell modeling. *Computational Mechanics*, *72*(2), 411-430.
- Srivastava, V., & Duraisamy, K. (2022). Towards a generalizable data-driven approach to predict separation-induced transition. In *12th International Symposium on Turbulence and Shear Flow Phenomena (TSFP12)*.
- Srivastava, V., & Duraisamy, K. (2021). Generalizable physics-constrained modeling using learning and inference assisted by feature-space engineering. *Physical Review Fluids*, *6*(12), 124602.

Stability-Based Surrogate Modeling of Transition

Modeling the Amplification of TS Instabilities in 2D BLs

- Existing data-driven models based on analytical curve fits (Drela & Giles, 1987) or rapid interpolation techniques
  - Not well-suited for large number of "stability modifiers"
  - Do not allow easy modifications for custom/new data
- CNN-encoder architecture provides a computationally efficient alternative to conventional fully connected networks
  - Can also enable physical interpretation of learned features of BL



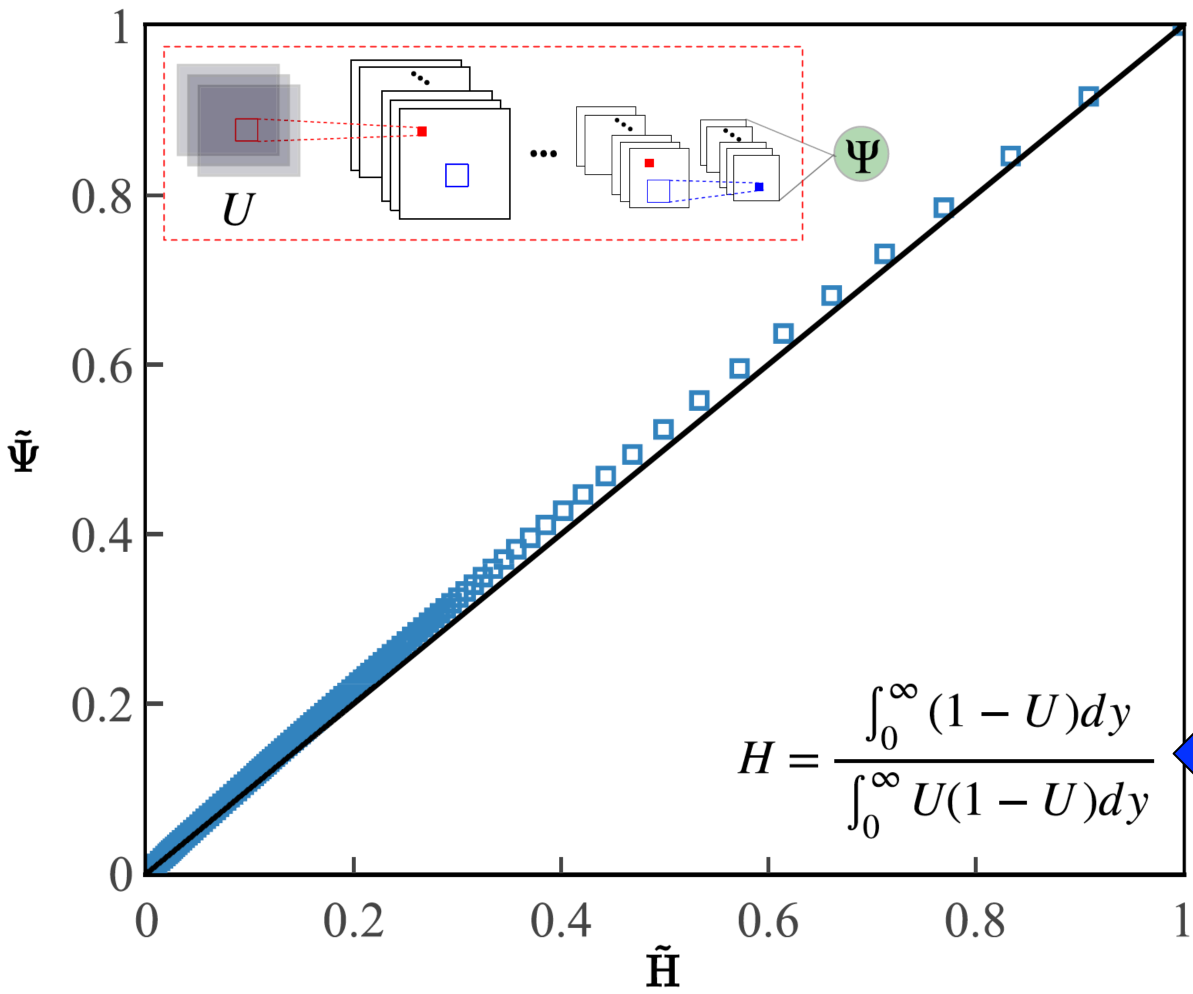
Encodes boundary layer profiles to latent physical parameters

Maps physical parameters to instability amplification rate

Zafar et al., Phys. Rev. Fluids, 2020

Physical Interpretation of CNN Based Latent Features

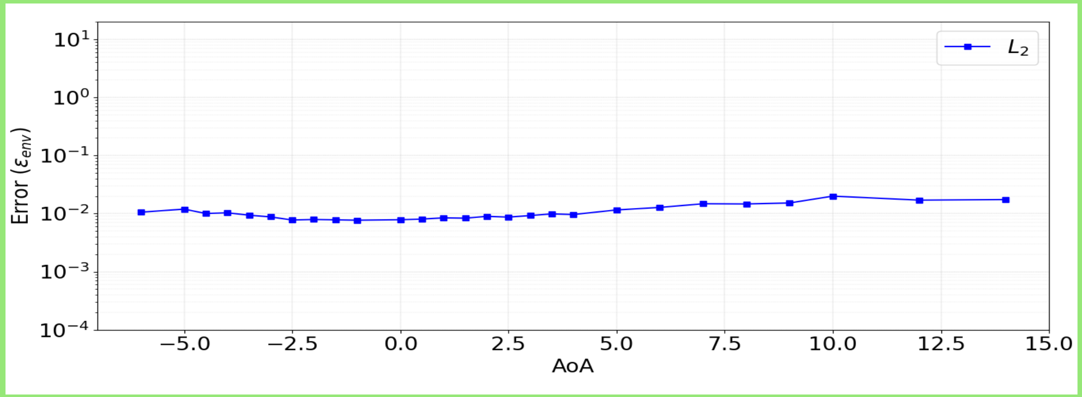
- Correlation between feature (Ψ) extracted by CNN from airfoil velocity profiles and profile shape factor (H) shows that **CNN is learning the velocity shape factor**



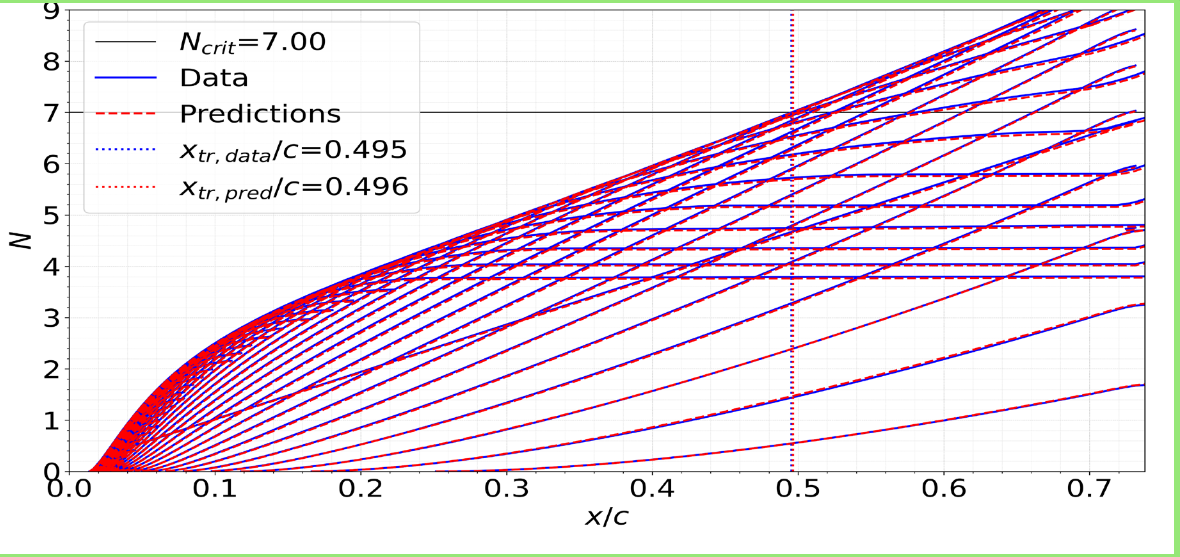
$$H = \frac{\int_0^\infty (1 - U)dy}{\int_0^\infty U(1 - U)dy}$$

Normalized shape factor

Surrogate Model for Crossflow Instability



(a) Average relative error in predicting envelope N-factor for several infinite-swept wing configurations with multiple combinations of Reynolds number, sweep angle, and angle of attack.

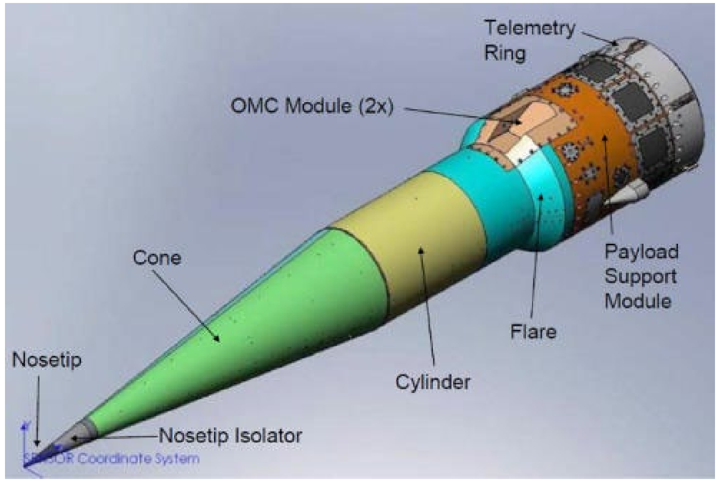


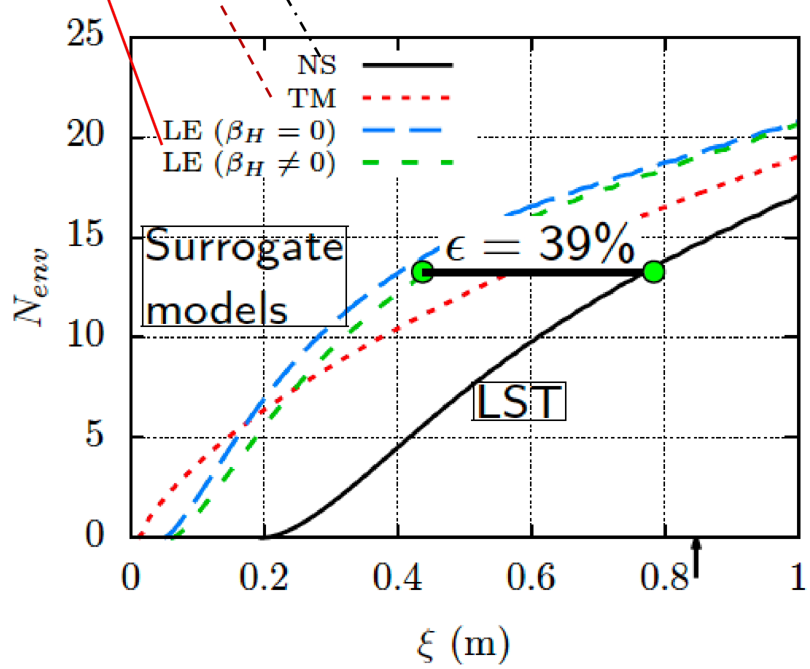
(b) Detailed model assessment for a canonical flow configuration: NLF(2)-0415 airfoil with 45-deg. Sweep, -4 deg. AoA abd Re_c = 3.2e6. Comparison of N-factor curves and resulting transition locations (N_{tr} = 7) based on neural network models (blue lines) with those based on direct stability computations (red curves, denoted as LST)

- Database of >105,000 different configurations for flows over infinite wings across 26 airfoil geometries and various angles of attack, Reynolds numbers and sweep angles

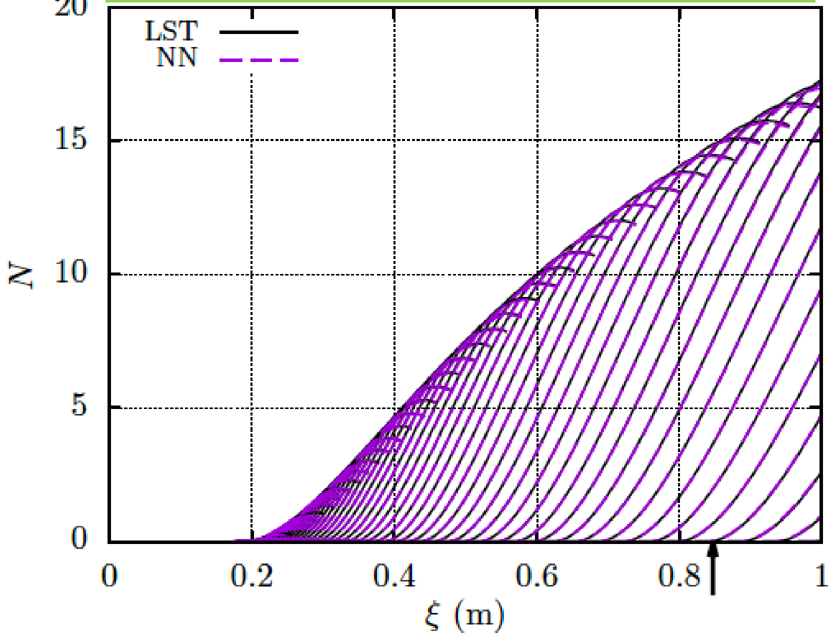
Surrogate Models Based on Deep Learning
HIFiRE-1 Flight Experiment: Ascent Phase (t = 21.5 sec)

N-factor envelopes:
NS: Navier-Stokes (NS)
TM: locally self-similar profiles based on Taylor-Maccoll post-shock conditions
LE: self-similar profiles based on local edge conditions (LE) with zero or nonzero Hartree pressure gradient parameter β_H





N-factor curves based on CNN



Related Publications

- Zafar, M., Xiao, H., Choudhari, M., and Paredes, P., “Recurrent Neural Network for End-to-End Modeling of Laminar-Turbulent Transition,” Data-Centric Engineering, Vol. 2, Oct. 2021, e17.
- Zafar, MI, Xiao, H, Choudhari, MM, Li, F, Chang, C-L, Paredes, P and Venkatachari, B, “Convolutional neural network for transition modeling based on linear stability theory,” Physical Review Fluids 5, 113903, 2020.
- Paredes, P, Venkatachari, B, Choudhari, MM, Li, F, Chang, C-L, Zafar, MI and Xiao, H, “Toward a practical method for hypersonic transition prediction based on stability correlations,” AIAA Journal 58(10), 4475–4484, 2020.
- Hildebrand, N., Choudhari, M., Srivastava, V., and Zaki, T., “DeepONet-Assisted Optimization of Surface Topography for Transition Delay in a Mach 4.5 Boundary Layer,” Proceedings of ETMM-14, Barcelona, Spain, Sep. 2023.