SOLAR SAIL ORBITS IN CISLUNAR SPACE FOR LUNAR SURFACE SURVEYING

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Repeatedly observing the lunar surface at close ranges possesses significant challenges. In this paper, we present results from an ongoing study that explores the feasibility of conducting comprehensive surveying of the entire lunar surface from an altitude of 10 km, within a 30-day response window. To address the high cost of traditional propulsion methods, we propose leveraging solar sail technology. Our preliminary findings indicate it is feasible to scan approximately one-eighth of the lunar surface within the specified altitude and time constrains by maintaining the spacecraft in the vicinity of the Earth-Moon Lagrange points and performing occasional low altitude excursions.

INTRODUCTION

Humanity is entering a new era of space exploration, driven by a surge in missions to the lunar surface with a specific interest in the lunar South Pole. The lunar South Pole shows great promise for scientific research and discovery due to evidence of its craters containing water. As the landing site for Artemis III, a more detailed mapping of the lunar terrain would provide better information on landing conditions thereby improving mission safety. Potential resources and geographic landmarks at the South Pole could also be better identified. Here we present results from an ongoing study evaluating the feasibility of surveying the entire lunar surface with the use of a solar sail. The proposed workflow targets a flyby altitude of ~10 km, with an on-demand revisit capability within 30 days. The use of a solar sail over traditional propulsion avoids the prohibitively high propellant masses that would result from a long duration operation as such. This study established a concept of operations (ConOps) with the following objectives:

1. Mobility for Cislunar Coverage: This objective focuses on understanding the extent of regions within the cislunar space that a solar sail-based spacecraft can effectively traverse. Sail-enabled station-keeping analyses are performed for multiple three-body orbit families.

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Specifically, this study focused on the halo, Lyapunov, and butterfly orbit families, assessing their viability in support of lunar surface observations.

2. Mobility for Lunar Surface Rapid Initial Visit Coverage: This objective it to quantify whether a solar sail can depart a multi-body orbit to perform a single low altitude pass of approximately 10-kilometers above the lunar surface. Invariant manifold structures are used for initial guess generation, then perturbations due to the Sun and the solar sail are applied. Additionally, accessible regions are mapped by finding what lunar latitudes and longitudes are reachable for a given initial staging orbit.
3. Mobility for Lunar South Pole Region Rapid Revisit Coverage: This objective expands upon the previous lunar coverage objective to determine if a solar sail orbiter can repeatedly perform low lunar altitude approaches. Analysis is performed to quantify what combinations of L1 and L2 orbits are needed to provide full lunar coverage.

By completing these objectives, this study seeks to characterize the strengths and limitations of using solar sail propulsion. This includes an assessment of solar sail capability. The approach for demonstrating the major components of cislunar mobility has three main phases. In phase 1 a station-keeping orbit is established around one of the Earth-Moon (EM) colinear Lagrange points (L1 or L2). Phase 2 completes a lunar transfer that passes as close as 10-kilometers above the lunar surface. Lastly, phase 3 maneuvers the spacecraft back to another station-keeping orbit about the opposite Earth-Moon Lagrange point. Therefore, a spacecraft that started at an L1 orbit would have its transfer end at L2. Determining if the lunar surface can be viewed in this manner is a key focus in this study. Once feasibility is demonstrated, the extent of observable lunar latitudes and longitudes is quantified. Station-keeping around EM L1 and L2 will demonstrate mobility for cislunar coverage, while lunar transfers will demonstrate the spacecraft's ability to provide rapid lunar surface coverage. The ability to repeatedly transfer to both Lagrange points and continuously complete lunar flybys will confirm the spacecraft's capability to revisit the lunar surface. Figure 1 below illustrates the complete process in six steps.

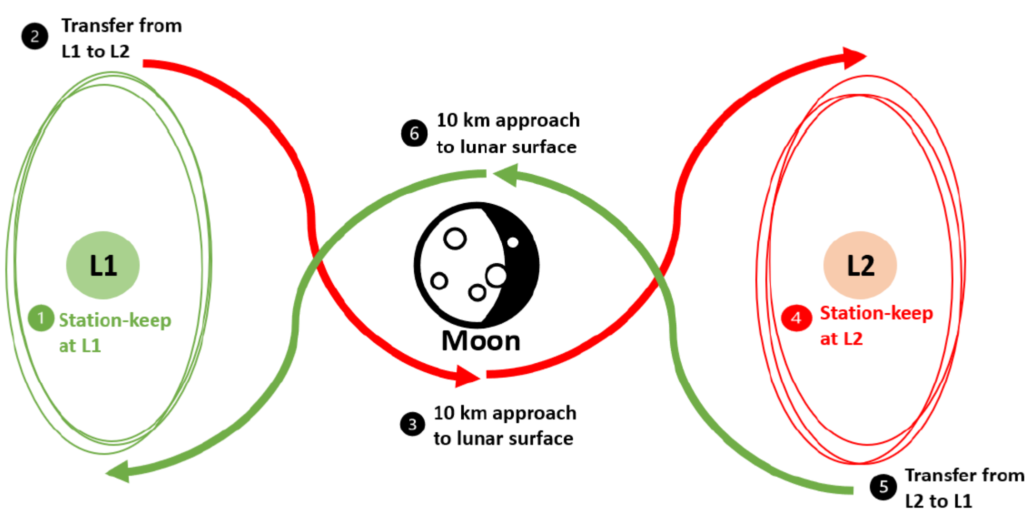

Figure 1. Concept of Operations.

SPACECRAFT AND MODELING ASSUMPTIONS

For this study, an ideal, flat plate solar sail is modeled with performance characteristics analogous to NASA's Near-Earth Asteroid Scout (NEA Scout). This is quantified as a sail lightness parameter of 0.0094 and characteristic acceleration of 0.056 mm/s^2.

The primary astrodynamics models used for trajectory design in this study are the Circular Restricted Three-Body Problem (CR3BP) and the Bi-Circular Restricted Four-Body Problem (BR4BP). The CR3BP treats the Earth and Moon as bodies in circular orbits about the system's barycenter, shown in Figure 2. Adding complexity, the BR4BP introduces the Sun as a fourth body into the system. The Sun revolves clockwise about the origin of the Earth-Moon rotating frame, emulating the motion of the EM system orbiting about the Sun. Note that the BR4BP utilizes the same basic vectors as the EM CR3BP. In Figure 3, the Sun's revolution relative to the frame is tracked by the variable θ_{Sun}. This is the angle formed between the Sun's position vector, $\vec{r}_{Sun}$ and the Earth-Moon line. In this diagram, the sailcraft is modeled as a massless particle (P3). The solar sail normal vector is denoted by $\hat{n}_{sail}$. The vector used for thrust pointing and control, is referenced as $\hat{u}_{sail}$ and is opposite the sail normal vector. The position of the Sun relative to the sailcraft position ($\vec{r}_{P3}$) is denoted as $\vec{r}_{Sun/P3}$. Finally, the angle between the sail control vector ($\hat{u}_{sail}$) and the Sun relative vector ($\vec{r}_{Sun/P3}$) is known as the sun incidence angle (SIA). These vectors are depicted below in Figure 3. Note that the Sun is not featured in this diagram due to its position being so far from the other bodies in the system. The angular rotation of the frame and angular rotation of the Sun are denoted by $\vec{\omega}$ and $\vec{\omega}_{Sun}$, respectively. For clarity, Figure 3 does not show L3, L4, and L5.

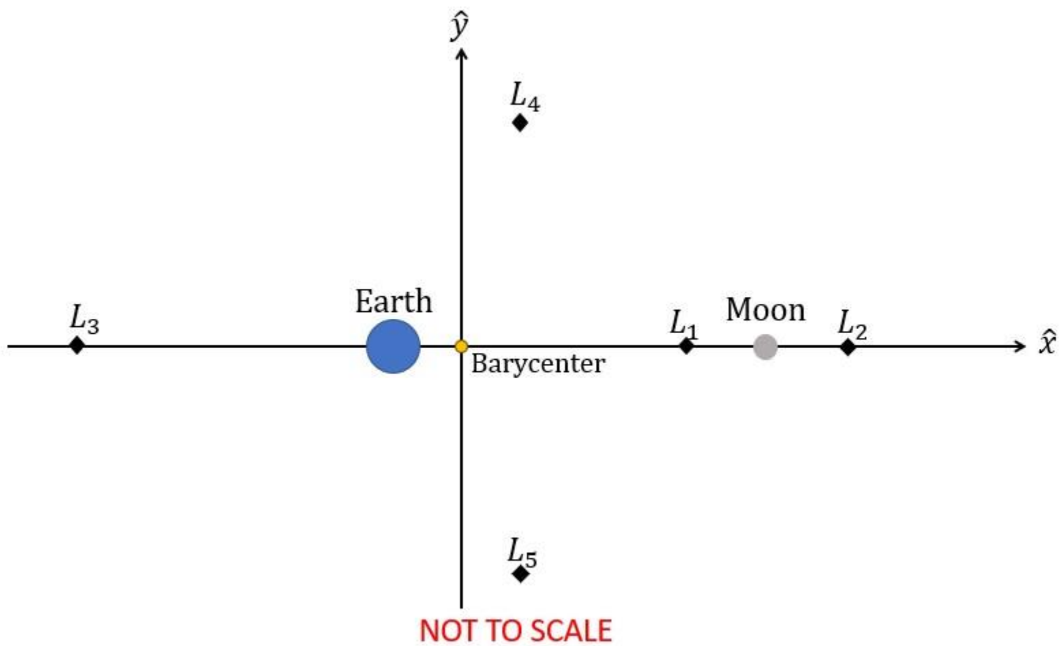

Figure 2. Earth-Moon CR3BP Frame.

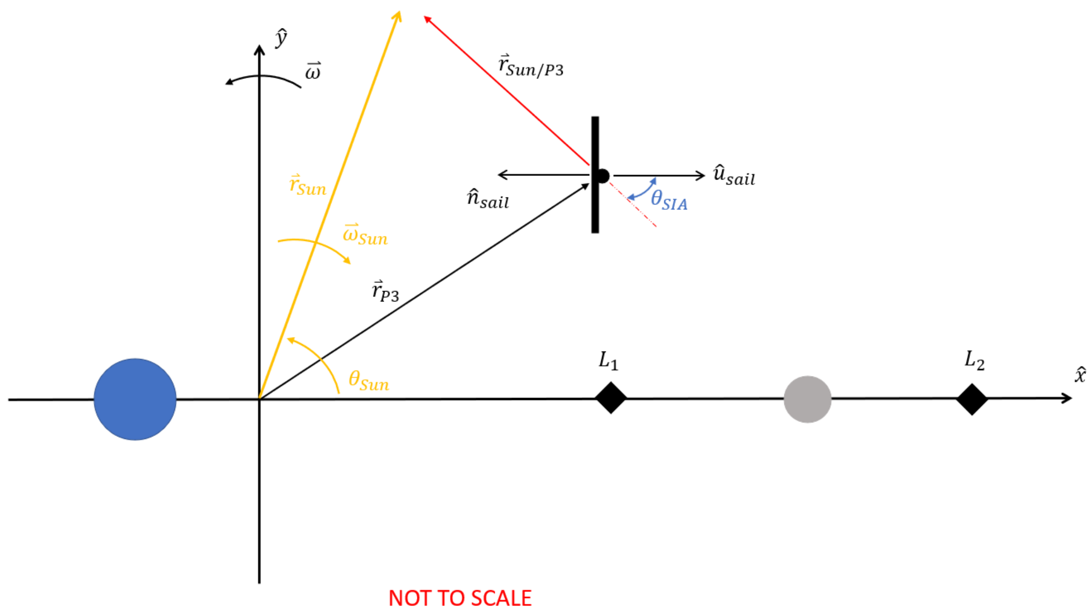

Figure 3. Earth-Moon-Sun Sail BR4BP Frame.

The primary mission design tool used for this study is the Astrodynamics Software and Science Enabling Toolkit (ASSET). ASSET is an open-source trajectory design and optimal control solver developed by the University of Alabama. The tool employs a collocation method with its own Parallel Spare Interior Point Optimizer (PSIOPT) to solve complex, optimal control problems via direct methods. Analysis performed in ASSET has been used for a range of programs, including NASA's Artemis missions and the upcoming Space Weather Investigation Frontier (SWIFT) proposal.

STATION-KEEPING ORBIT STUDY

Demonstrating the capability of a spacecraft to station-keep a multi-body orbit is a complex problem due to the chaotic gravitational regime of the Earth and Moon system. The CR3BP presents five equilibrium points, called Lagrange points. Localized dynamics allow for periodic solutions centered about these equilibrium points. These stationary orbits can be categorized into orbit families such as the halo, Lyapunov, axial, and butterfly families. Introducing a solar sail to these dynamics artificially shifts the equilibrium point location as a function of the spacecraft's position and time. These additional complications are all still within the CR3BP. By modeling within the BR4BP, the effects of the Sun's gravity are introduced into the system, adding further complexity. In general, the periodic orbits that exist in the CR3BP are no longer viable within the BR4BP. As such, station-keeping can only be attempted for quasi-periodic halo-like and Lyapunov-like orbits.

A spacecraft's ability to station-keep a three-body orbit is heavily dependent upon the stability of its orbit. If an orbit is stable, the effects of a perturbation will dampen out, returning the spacecraft to its original orbit. However, for an unstable orbit, a perturbation grows exponentially. This causes the spacecraft to diverge and over time, a perturbed vehicle in an unstable orbit will depart from its original path.

An orbit can be categorized as stable, unstable, or neutrally stable. Determining stability characteristics first require the use of methods from Dynamical Systems Theory. A crucial mathematical structure required for assessing stability is the orbits monodromy matrix. A monodromy matrix is created by integrating the state transition matrix of a periodic trajectory for one complete revolution. The resulting 6-D monodromy matrix will possess six eigenvalues. Of these, there will be one trivial unit pair as a result of periodic motion. The remaining four eigenvalues will consist of two complex conjugate pairs. An eigenvalue pair on the unit circle is associated with neutral stability. If the real component of an eigenvalue is greater than 1, it represents an unstable mode with exponential growth of perturbations. Conversely, an eigenvalue less than 1 is affiliated with stable motion. While this information is valuable for determining stability criterion, a more concise metric is desirable. To address this need, the eigenvalue with the largest real component can be identified as the "stability index" to compare different orbits. An orbit with a small stability index is considered more stable than an orbit with a large stability index.

Dynamics about the Lagrange points in the CR3BP do not yield any stable orbits. At best orbits can be neutrally stable. The halo orbits tend to have stability indices ranging from 1.00 to 1180.4, while Lyapunov orbits span a range of 49.6 to 1,337.7. This study originally investigated the halo and Lyapunov orbits of the Earth-Moon L1 and L2 points. However, the study later shifts to focusing particularly on the halo orbit family due to the inherent instability of Lyapunov orbits. Initial attempts at station-keeping a Lyapunov orbit with a solar sail were unable to successfully generate more than two complete revolutions.

Approach

To station-keep with a solar sail for multiple revolutions within the BR4BP, each revolution of a multi-body orbit needed to be solved individually. This was done by first taking an initial condition for a ballistic three-body orbit within the CR3BP and propagating in the BR4BP. Next, the mirror theorem was leveraged by constraining the first and last point of the orbit to lie on the x-z plane. Initial and final velocity was constrained to be exclusively in the y-direction. Modeling a BR4BP solar sail transfer utilized incremental steps in complexity. Having a separate optimal control problem for each revolution of the orbit helped simplify the optimal control problem and prevented it from becoming too large. This makes it easier for the optimizer to find consistent solutions. Once the trajectories for the desired number of revolutions had all been solved individually, one final optimal control problem was created. This new OCP enforces continuity between each revolution, essentially stitching the orbits together so that the spacecraft's states maintained continuity. No objective function was set, and an additional constraint was added to limit the solar sail's sun incidence angle to be less than or equal to 90 degrees. This constraint prevents a modeling error of having the solar sail thrust away from the Sun.

The initial approach for demonstrating station-keeping did not solve for each revolution individually. Instead, the optimal control problem attempted to solve for multiple revolutions at once. It used a signed distance field (SDF) constraint to keep the spacecraft within a certain distance from the nominal orbit family. A signed distance field is a volumetric representation of the desirable space for the spacecraft to stay within. That region is comprised of all the orbits within a chosen orbit family, as seen in Figure 4a. To implement this within ASSET, an inequality constraint was utilized to assess how far the spacecraft was from being within the SDF. If the spacecraft was on or inside of the surface of the SDF, a value less than or equal to zero was returned. The area outside the SDF corresponded to values greater than zero. Figure 4b depicts a horizontal slice of the 3-D SDF to illustrate this. If values of less than or equal to zero were returned to the inequality constraint, the constraint would be satisfied. This ensures the spacecraft stays in the specified region of space. With this approach only three to four revolutions could be solved for before the spacecraft would diverge. This complication led to the use of solving for revolutions individually.

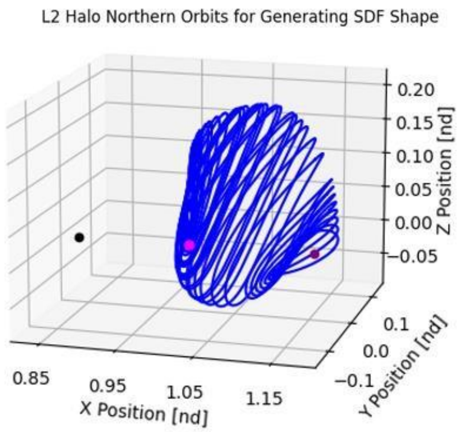

Figure 4a. Trajectory Make-Up of SDF.

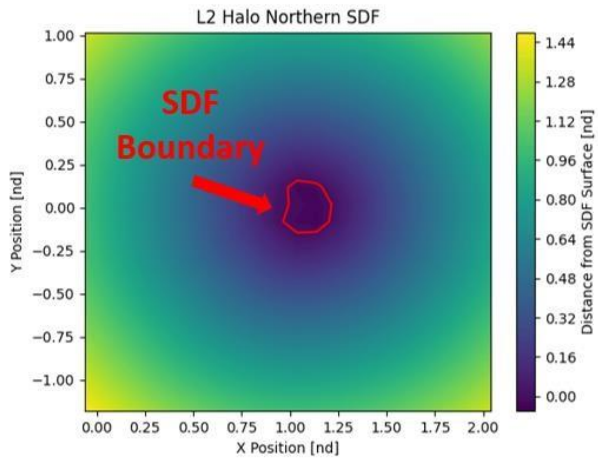

Figure 4b. SDF Slice Values.

Results

Initial conditions for the L1 halo station-keeping orbits were taken from the Jet Propulsion Lab Three-Body Orbit database. Station-keeping was attempted with orbits of varying stability indices to determine how unstable the initial orbit could be before sail-enabled orbit maintenance became untenable. This is done *before* analyzing lunar transfer feasibility. This bounds the manifold structures that will be used for initial transfers. The most unstable orbit that could be station-kept around L1 had a stability index of 103. The initial conditions of this orbit are listed in non-dimensional units below in Table 1. Figure 5 shows the station-keeping trajectory for 12 revolutions. The total time for these 12 revolutions to be completed is 145.84 days, with each revolution taking 12.15 days on average. The average perilune distance is 38,694 km and the average apolune distance is 78,141 km.

Table 1. Initial Condition for L1 Halo Northern Orbit.

x_0 [LU]	0.833
y_0 [LU]	-1.601e-27
z_0 [LU]	0.133
vx_0 [LU/TU]	2.544e-15
vy_0 [LU/TU]	0.246
vz_0 [LU/TU]	1.0292e-14

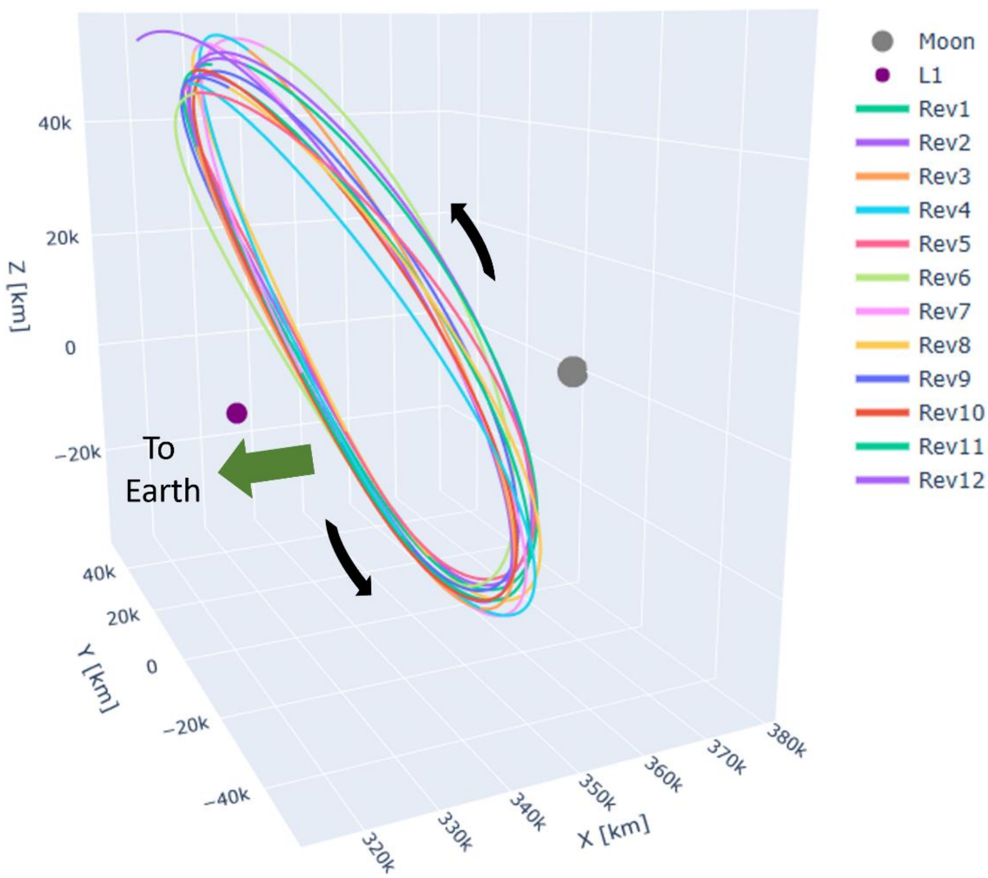

Figure 5. L1 Halo Northern Station-keeping Orbit.

During station-keeping the pointing of the solar sail follows a pattern that repeats every 31 days which corresponds to the period it takes the Sun to complete an entire revolution in the BR4BP frame. During this cycle, the spacecraft spends an average of 16.5 days, half the period of the Sun's revolution, with the sail turned edge on to the Sun, generating no acceleration. This half of the cycle corresponds to the duration of time where the Sun lies on the positive side of the y-axis, beginning when the Sun lies on the negative y-axis at an angle of 270º relative to the Earth-Moon line and ending once the Sun aligns with the positive y-axis at 90º. As the Sun begins to have an angle greater than 90º relative to the Earth-Moon line, the sun incidence angle decreases and reaches a minimum, almost becoming fully perpendicular to the incoming solar radiation pressure at an angle of around 270º, which corresponds to the Sun lying to the left of the Earth on the negative x-axis. After this point, the sun incidence angle increases until it returns to maintaining 90º and the pattern begins again as illustrated in Figure 6 below. The fact that the solar sail is only utilized when the Sun is at angles between 90º and 270º indicates the constant acceleration produced by the solar sail is not needed to maintain station-keeping and could create additional perturbations that force the solar sail out of its orbits. One important thing to note is that the solar sail model used for this study does not take into consideration any eclipsing caused by the Earth. When the Sun's relative angle to the Earth-Moon line is 270º and the solar sail has a near-zero sun incidence angle, the sail generates almost its max acceleration. How the sun incidence angle changes during each station-keeping revolution can be seen in Figure 7. Each station-keeping revolution is denoted by a different color.

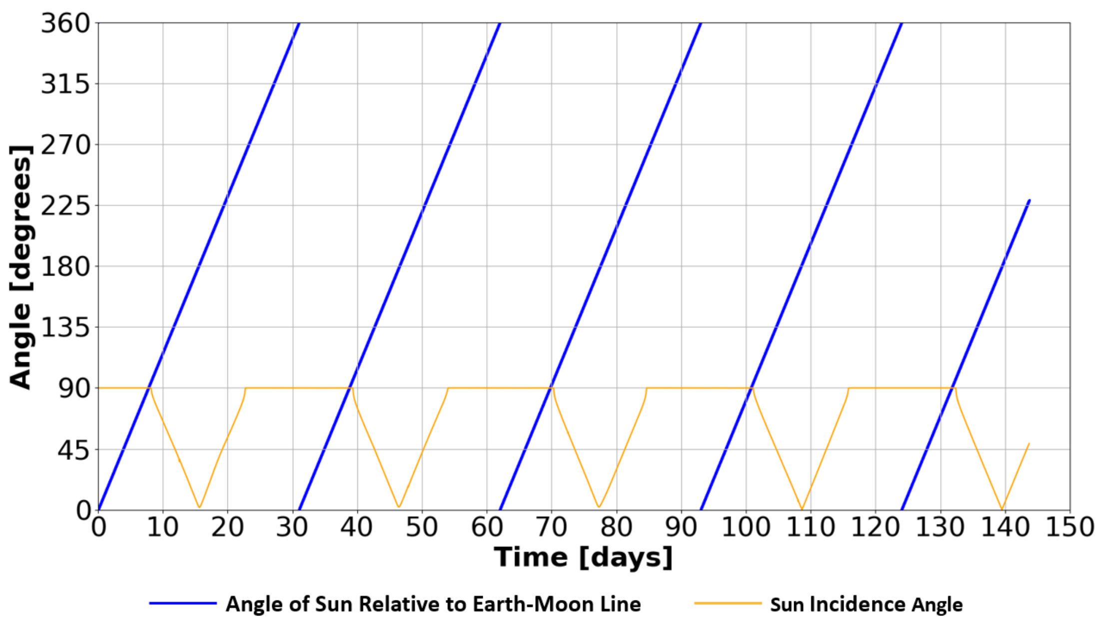

Figure 6: Relationship between Sun Incidence Angle and the Angle of the Sun Relative to the Earth-Moon Line.

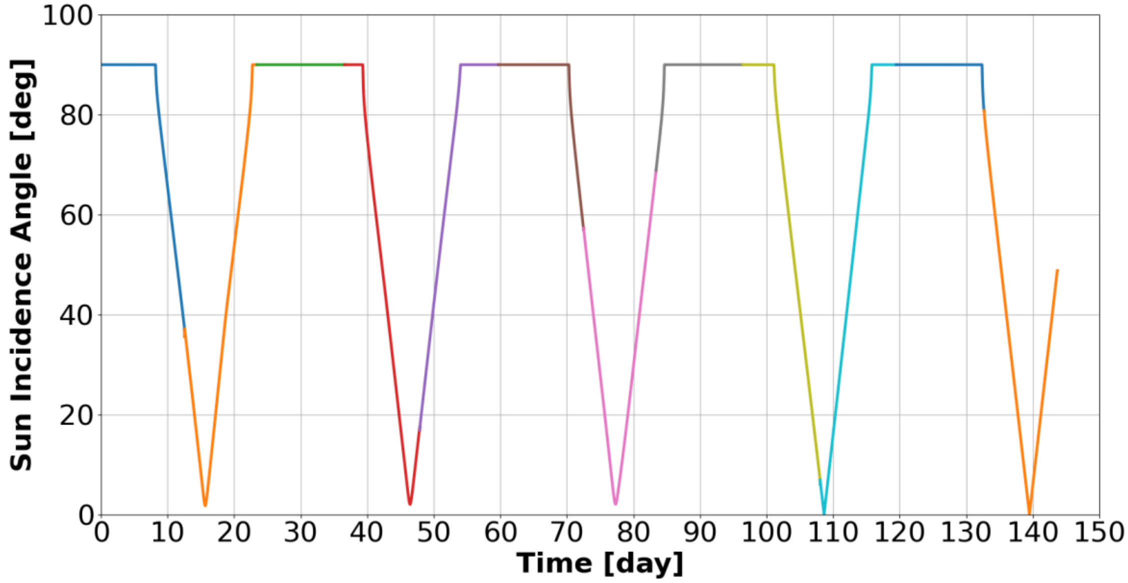

Figure 7: Spacecraft Sun Incidence Angle over Time.

LUNAR TRANSFER STUDY

To focus on the second objective of the study: mobility for lunar surface rapid initial visit coverage, transfers from the L1 station-keeping orbit to a point approximately 10-kilometers above the lunar surface were attempted. For these transfers, the unstable invariant manifold of the station-keeping orbit was leveraged to generate initial guesses.

Within the CR3BP, periodic orbits are unstable, with some being at most marginally stable. Perturbations applied to a spacecraft in one of these orbits will cause the spacecraft to deviate and eventually leave its nominal position in the orbit. The unstable invariant manifold is the collection of these trajectories as the spacecraft is perturbed in the direction of the orbit's unstable eigenvector along different points of the orbit. Using ASSET, the unstable manifold in the CR3BP was generated by perturbing both the position and velocity of the spacecraft by 300 kilometers at fifteen points along the orbit which can be seen below in Figure 8. Note that Point 0 and Point 14 are the same. Each trajectory within the manifold was propagated forward in time until the trajectory reached an x-z plane crossing with a relatively close approach of the Moon. The unstable manifold generated for the L1 station-keeping orbit can be seen in Figure 9.

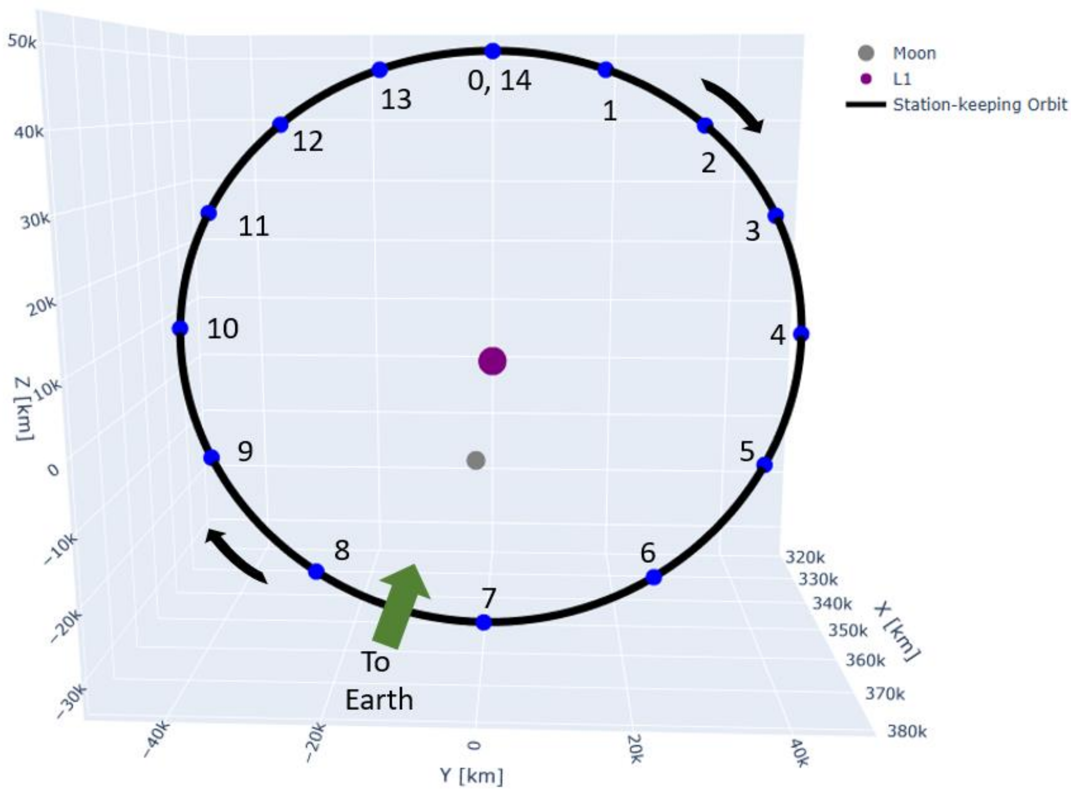

Figure 8: Manifold Departure Points.

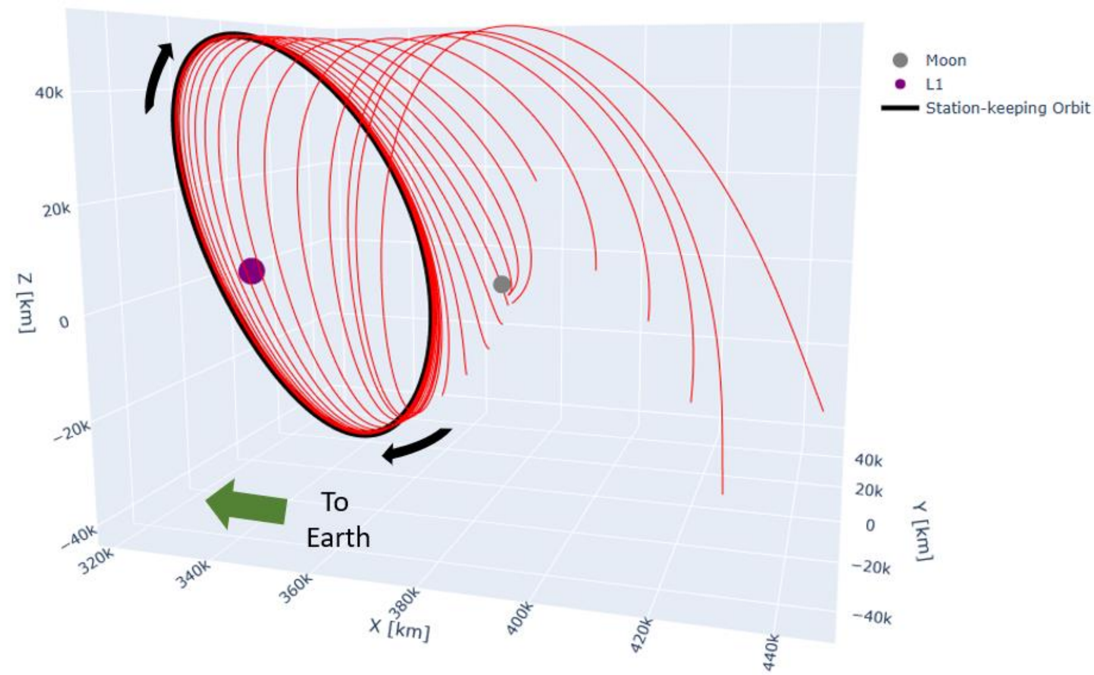

Figure 9: Unstable Manifold for Station-keeping Orbit.

Approach

Two optimal control problems were used to solve for the transfer. The first was set up by using the manifold trajectory segment that naturally has the closest approach to the Moon as the initial guess. Constraints were placed so that the spacecraft must start on the station-keeping orbit from the departure point the manifold trajectory corresponds with and end at a 10-kilometer lunar altitude. Because the initial guess used is a ballistic transfer within the CR3BP, the goal of the first optimal control problem was to simply generate the same path and end at the same distance using the sail in the BR4BP. The solved solution to the first optimal control problem was then used as the initial guess for the second problem that targeted hitting a final 10-kilometer altitude at the end point.

Results

By completing the described process, a transfer from the L1 station-keeping orbit to a point with a 10-kilometer altitude above the Moon was found. For the transfer depicted in Figure 10, the entire duration takes 11.07 days. Like the station-keeping orbit, there are periods of time during the transfer where the sail points edge-on to the Sun indicating that constant additional acceleration is not needed to counteract any perturbations or maneuver the spacecraft.

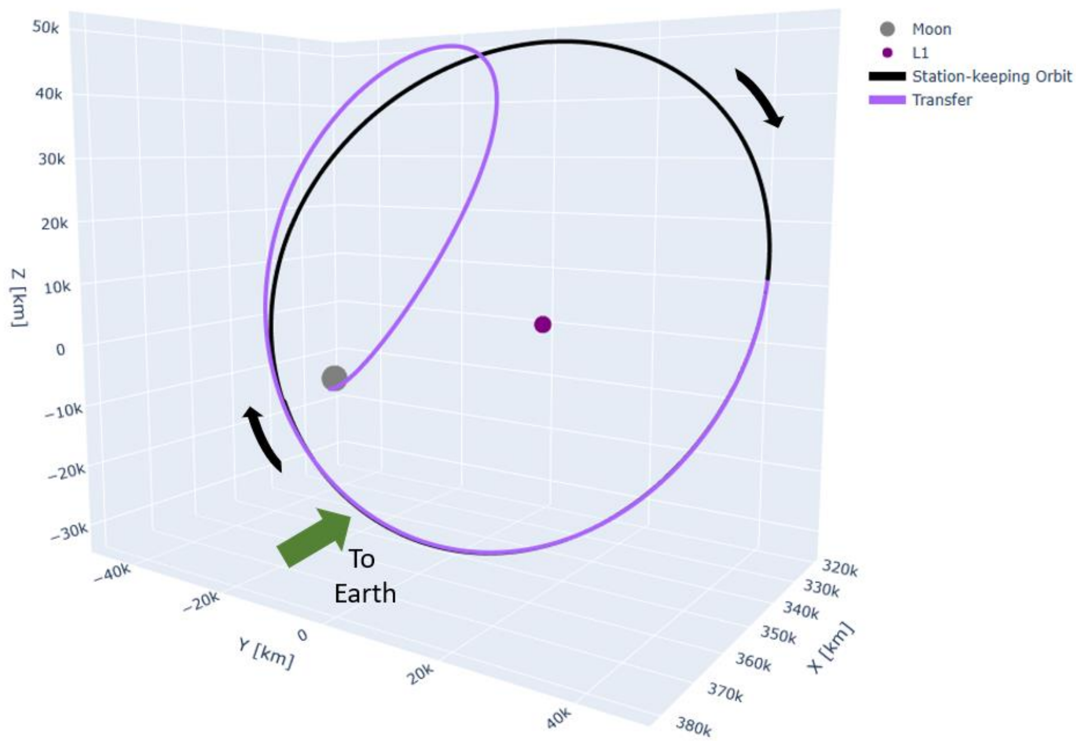

Figure 10: Solved Lunar Transfer.

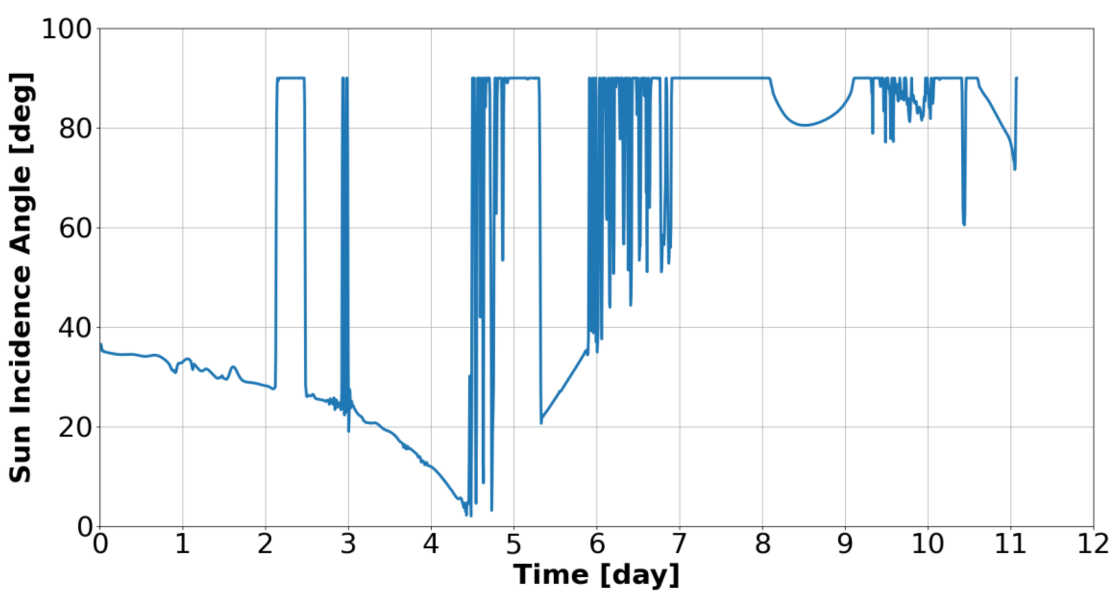

Figure 11. L1 Transfer Sun Incidence Angle over Time.

LATITUDE LONGITUDE STUDY

To determine what coverage these transfers from the station-keeping orbit could provide, the above process was repeated but incorporated an additional constraint at the end point to have the spacecraft hit a specific latitude and longitude combination. Latitude values ranged from -90º to +90º in 10º increments while latitudes ranged from -180º to 180º and were changed in 20º increments. Fifteen points along the halo orbit were still used for creating the manifold. A tolerance of 0.001º was applied to the latitude and longitude constraints.

Results

Of all the latitude and longitude combinations tested, the spacecraft was able to transfer to seven different combinations. Figure 12 below shows the eight solved transfers that resulted from leaving at the fifteen different points. The spacecraft was able to complete two transfers to a latitude longitude combination of -50º and -140º utilizing departure points 0 and 3. All the latitudes of the transfer end points reached were -50º, -40º, -30º, -20º, and 0º. For longitudes, the ones reached were -180º, -160º, -140º, and -100º. These latitude and longitudes provide coverage of the lower quadrant of the Moon on the side further away from L1 which is better seen in Figure 13. Of the fifteen departure points, four were viable for these lunar transfers. Leaving from point 0 resulted in access to the higher latitudes and transfer times of around fourteen days. Points 3 and 4 are in the upper right quadrant of the station-keeping halo orbit. Starting from these points resulted in transfers to the midrange latitudes and shorter transfers ranging from around eleven to twelve days. These transfers were shorter due to the spacecraft spending less time on the station-keeping before departing. Table 2 below details the latitude and longitude reached by each transfer, the point at which the transfer begins at, and the transfer duration.

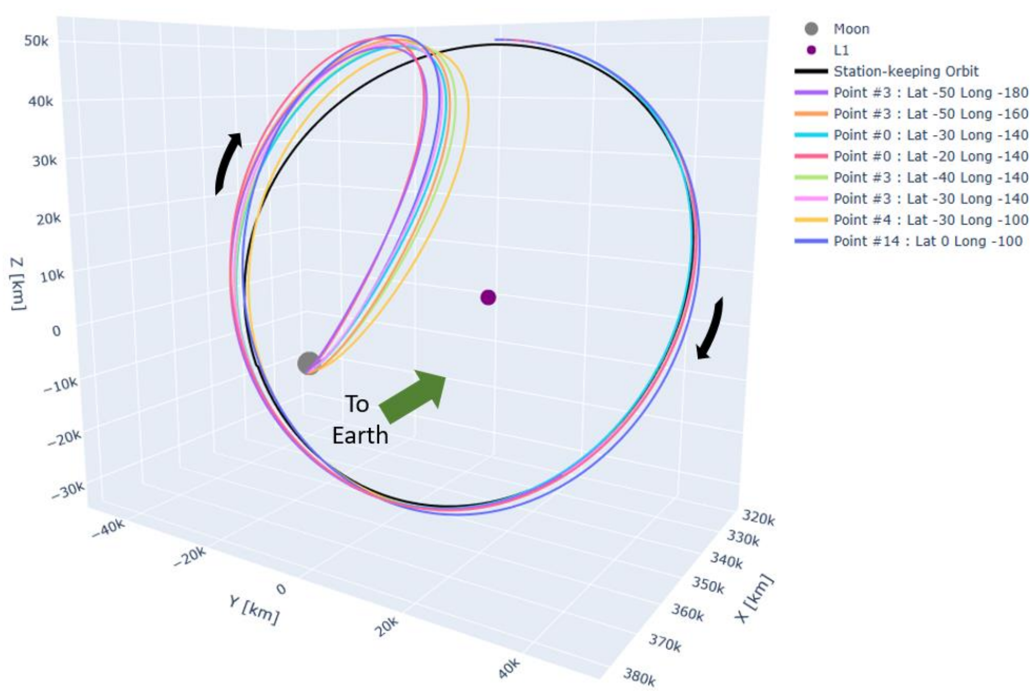

Figure 12: Transfers to Different Lunar Coordinates at L1.

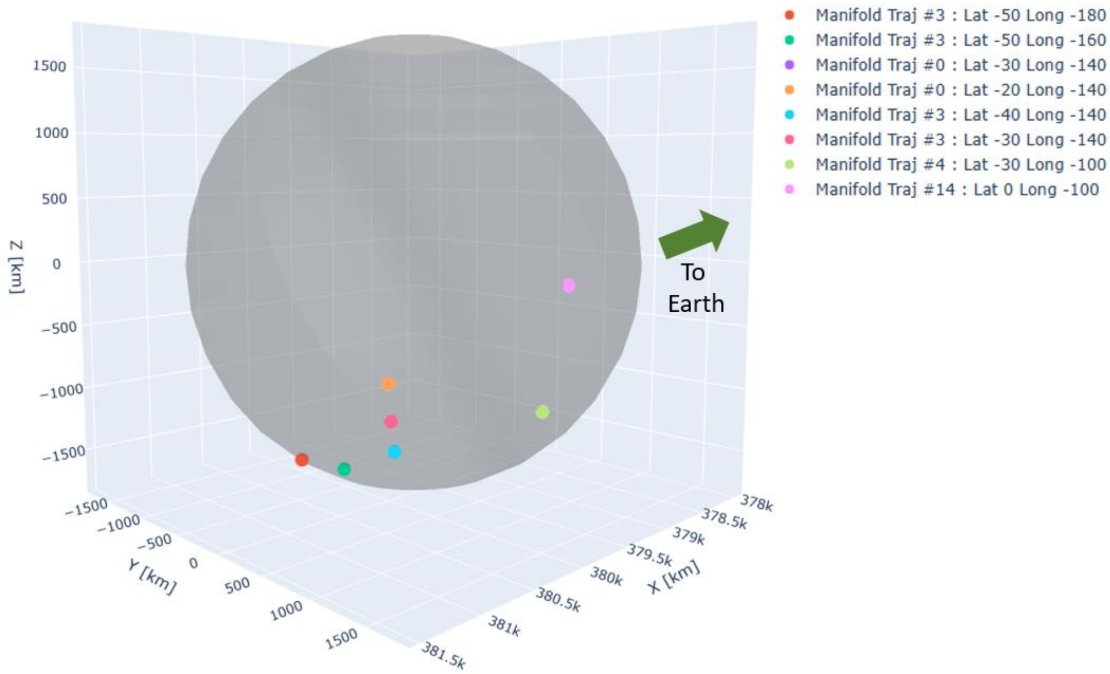

Figure 13: Transfer End Points in Relation to Moon.

Table 2: Summary of Solved Transfers.

Latitude [deg]	Longitude [deg]	Departure Point	Duration [days]
-50	-180	3	11.48
-50	-160	3	11.75
-30	-140	0	14.25
-20	-140	0	14.05
-40	-140	3	11.75
-30	-140	3	11.66
-30	-100	4	10.92
0	-100	14	14.24

HOMOCLINIC TRANSFERS

Having shown that low altitude transfers to the Moon are possible with a solar sail, work was completed to determine if the solar sail could transfer back to the original station-keeping orbit. For this analysis, a two-phase optimal control problem was used. Both the stable and unstable manifold of the station-keeping orbit were generated and propagated to the point where they cross the x-z plane. The unstable and stable manifold can be seen in Figure 14. This allowed for the Mirror Theorem to be applied which states that due to the time symmetry inherently within the CR3BP equations there is a second solution to any solution reflected over the x-axis.[5] These trajectory segments provide a free return to the optimal control problem as an initial guess. Figure 15 below showcases the specific manifold segments chosen for the optimal control problem chosen specifically for how close they approach the Moon and line up due to the Mirror Theorem. Initial attempts at the transfer were completed with no constraints on the sail's sun incidence angle before attempting with the less than or equal to 90º limit.

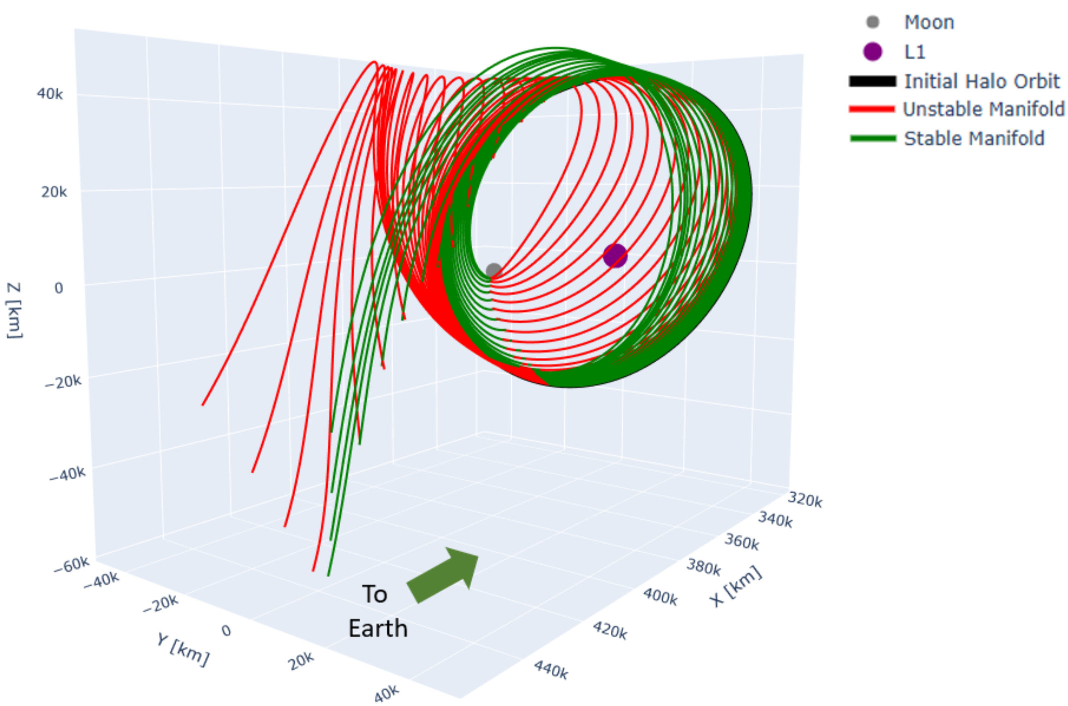

Figure 14: Unstable and Stable Manifolds for Homoclinic Transfer.

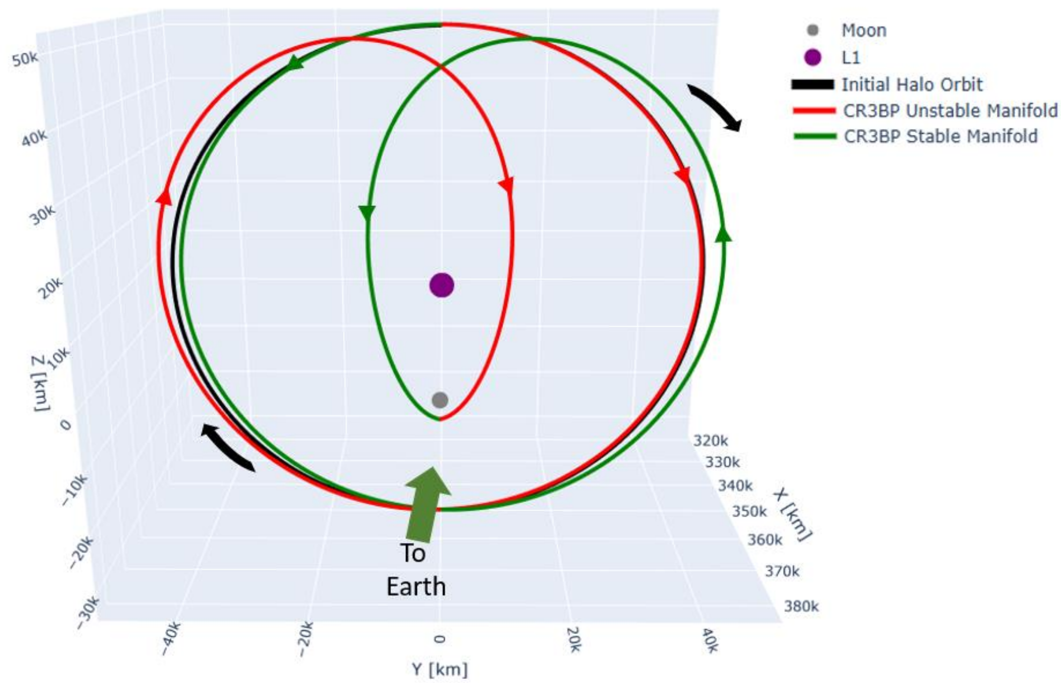

Figure 15: L1 Halo Northern Initial Homoclinic Transfer Guess Set Up.

Results

With the solar sail's sun incidence angle constrained to remain less than or equal to 90º the homoclinic transfer could not be solved however this transfer was managed with an unconstrained sail which can be seen below in Figure 16. Multiple periods of the homoclinic transfer have the spacecraft exceed the limit illustrated in Figure 17. Notably around fifteen days into the transfer when the spacecraft completes its 10-kilometer approach of the Moon, the sun incidence angle has a massive jump to above 90º. A sun incidence angle greater than 90º in the way the sail is modeled within ASSET is the equivalent of the sail thrusting in the direction of the Sun. This denotes that the spacecraft is attempting to negate the acceleration it has due to the influence of the Moon with the sail itself.

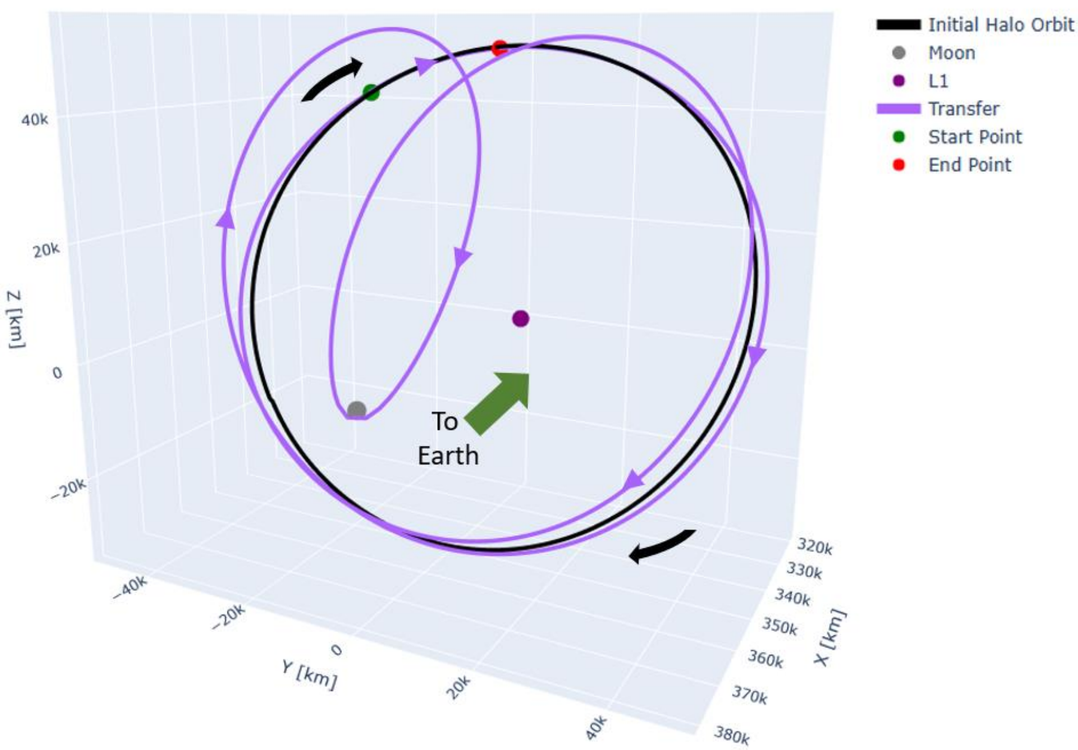

Figure 16: L1 Halo Northern Homoclinic Transfer with Unconstrained Sail.

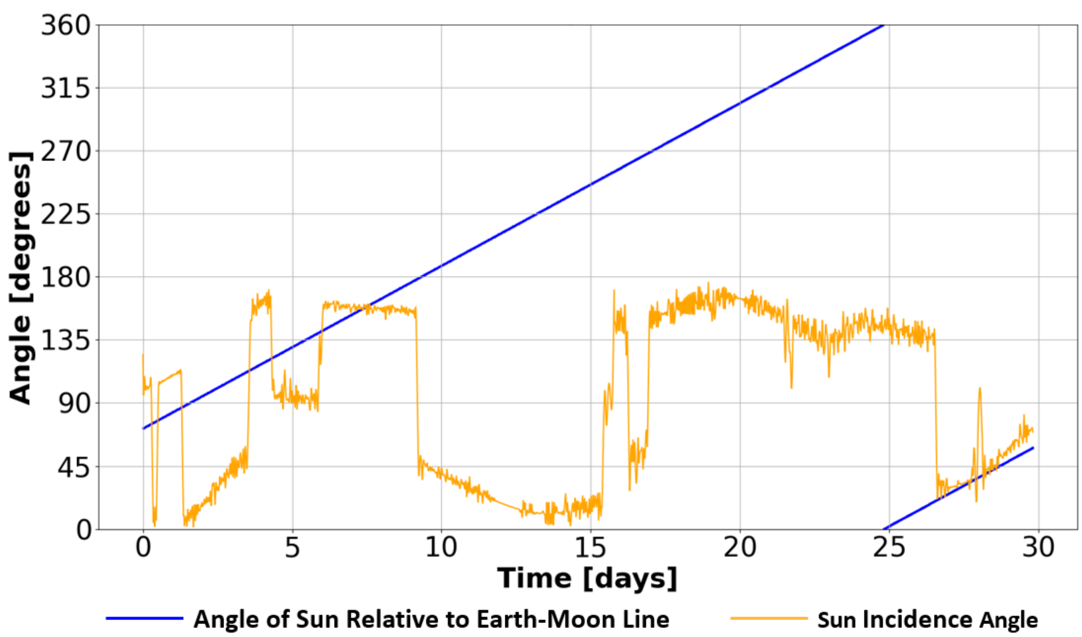

Figure 17: Sun Incidence Angle over Time for Homoclinic Transfer with Unconstrained Sail.

CONCLUSION

Through this study, the application and mobility of a solar sail in cislunar space has been greatly expanded upon. It has been shown that a solar sail can station-keep a quasi-periodic halo orbit centered on Earth-Moon L1. Despite the chaotic nature of the BR4BP, the solar sail is able to mitigate the perturbations from the Sun. The ability to station-keep extends beyond neutrally stable orbits and can be accomplished in unstable orbits with stability indices as great as 103. Similar work has been done to confirm that station-keeping around L2 can be accomplished. With regards to surveying the lunar surface, it is possible to transfer from these station-keeping halo orbits to complete a pass of the lunar surface with an altitude as low as 10 kilometers within 30 days. The coverage provided by these passes would allow for viewing approximately one eighth of the lunar surface. Transferring back to a station-keeping orbit utilizing homoclinic transfers poses difficulties specifically with counteracting the gravitational effects of the Moon. Utilizing heteroclinic transfers may provide an alternative method for completing these transfers but future work needs to be done to confirm this. With these current results, the utilization of a solar sail in cislunar space for surveying may be possible.

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