



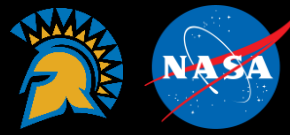
Modular Trajectory Analysis Tool for Skip-Out and Aerocapture Maneuvers

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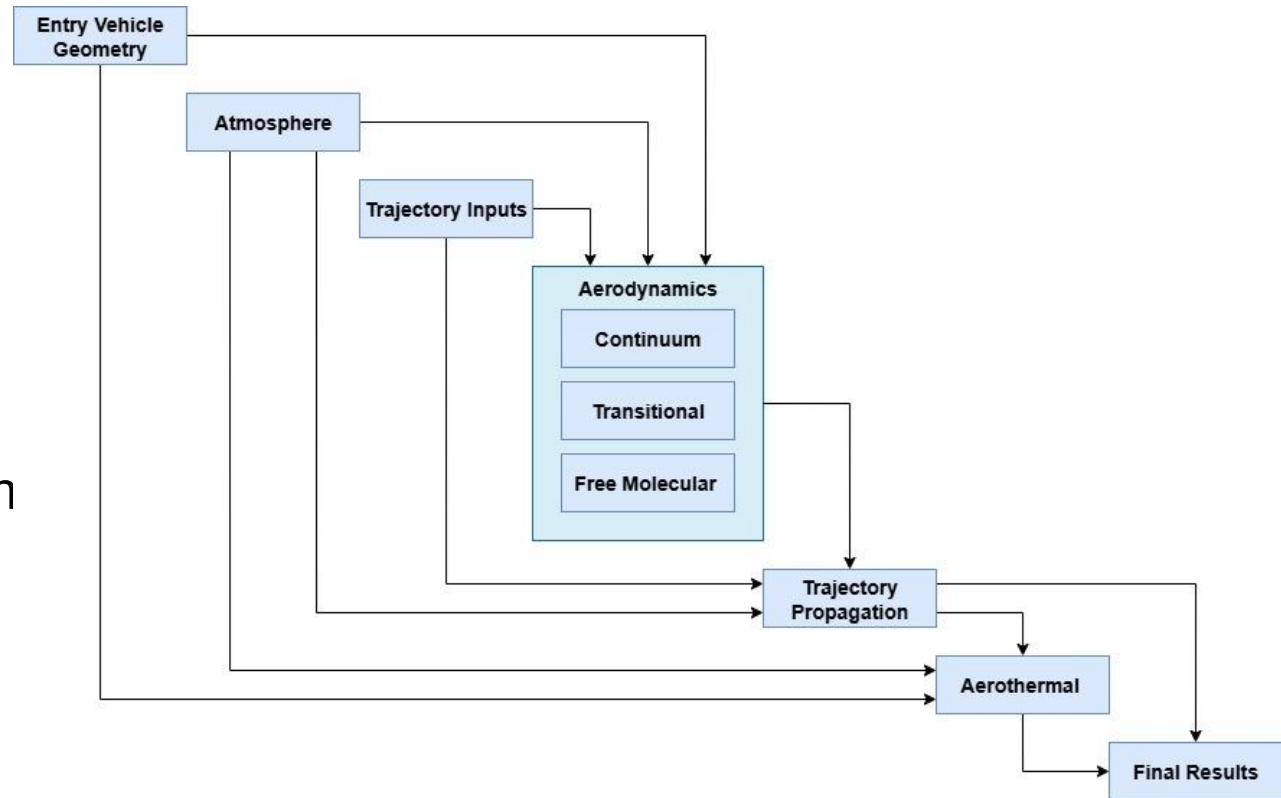




Project Overview

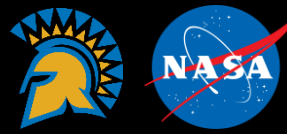


- Project was completed for an M.S. in Aerospace Engineering at SJSU.
- A 3DOF trajectory simulation was developed to study multi-pass aerocapture as an orbit insertion technique.
 - Initial goal was to compare with traditional, “single-pass” aerocapture.
 - Written entirely in MATLAB utilizing modern object-oriented programming techniques.
 - While various trajectory software tools already exist, programming a simulation from scratch re-enforces fundamental EDL relevant concepts. The most valuable takeaway from this project has been the learning experience.

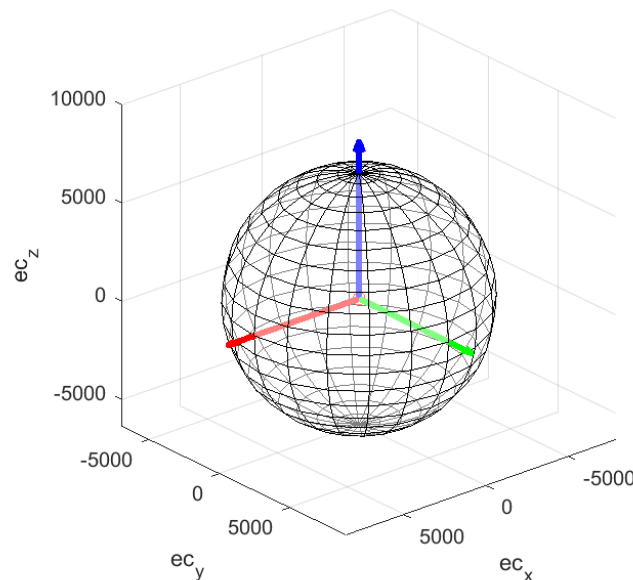




Orbital Mechanics



- Cartesian, equatorial reference frame (ECI) was chosen.
- 2-body, J2 gravity model, no other perturbing body forces on vehicle.
- Program utilizes a patched conic method to stitch together trajectory segments and supports three different physics types, coast, atmospheric flight, and propulsive maneuvers.
- 3DOF, vehicle modeled as a point mass, no rigid body or attitude dynamics.
- Solver utilizes a Runge Kutta 4/5th order variable step integrator to solve the equations of motion.
 - Widely used technique for trajectory propagation



$$F_g = \frac{Gm_1m_2}{r^2}$$

Newton's Universal Law of Gravitation

$$\mathbf{r} = \begin{bmatrix} X \mathbf{ec}_x \\ Y \mathbf{ec}_y \\ Z \mathbf{ec}_z \end{bmatrix}$$

Position Vector

$$\mu = GM$$

Gravitational Constant Definition

$$\ddot{\mathbf{r}} = \frac{\mu}{r^3} \mathbf{r}$$

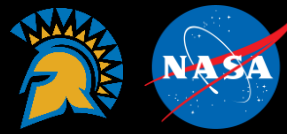
2-body Equation of Motion

$$\ddot{\mathbf{r}} = \frac{\mu}{r^3} \mathbf{r} + \frac{F\dot{\mathbf{r}}}{mr}$$

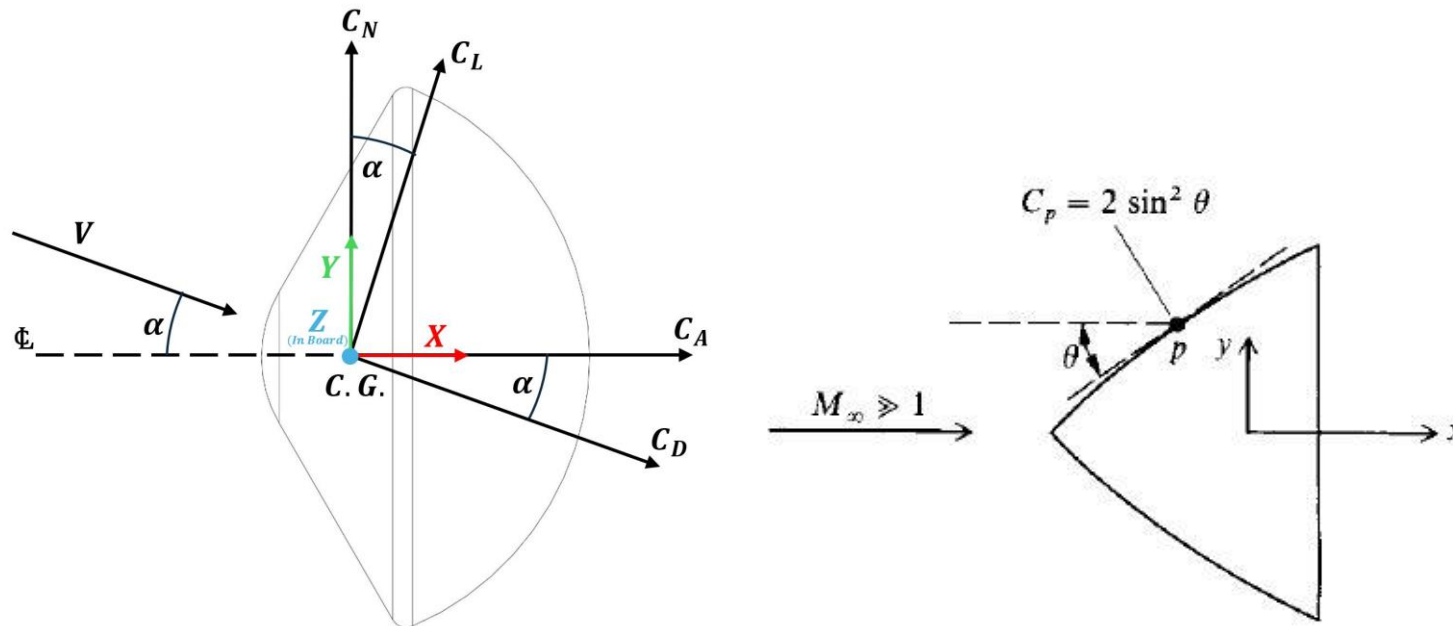
Equation of Motion for Propulsive Maneuvers



Aerodynamics Modeling



- The conceptual design nature of this analysis tool necessitated fast, reliable method of estimating vehicle aerodynamics.
- Newtonian mechanics are utilized for continuum flow and a Maxwellian particle collision model is used for free molecular flow.
 - Newtonian methods have been used since the 1950's for modeling hypersonic flows over blunt bodies and are utilized by modern tools such as CB-AERO and TRAJ.
- Pressure and shear coefficients (C_p , C_τ) depend on local surface inclination angle and various freestream properties.
- Analytical expressions can be derived for simple axisymmetric shapes. A sphere-cone consists of concatenated expressions for a sphere and frustum.
 - Expressions are derived by taking the surface integral over the vehicle body.
- Moment coefficients can also be computed given a reference length.



Continuum

$$\begin{bmatrix} C_A \\ C_N \\ C_S \end{bmatrix} = \frac{1}{A_{ref}} \iint_S C_p \begin{bmatrix} \mathbf{n}^T \hat{\mathbf{x}} \\ \mathbf{n}^T \hat{\mathbf{y}} \\ \mathbf{n}^T \hat{\mathbf{z}} \end{bmatrix} dA$$

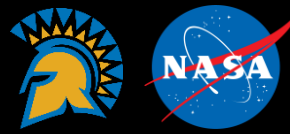
$$C_p = C_{p_{max}} \sin^2 \theta$$

Free Molecular

$$\begin{bmatrix} C_A \\ C_S \\ C_N \end{bmatrix} = \frac{1}{A_{ref}} \iint_S (C_p \hat{\mathbf{n}} + C_\tau \hat{\mathbf{t}}) dA$$



Transitional Aerodynamics



- Transitional or rarefied flow has characteristics of both free molecular and continuum flow. A non-dimensional parameter called the Knudsen number is used to define each flow regime.
- A general guideline is a $K_n < 0.01$ defines continuum flow and $K_n > 10$ defines free molecular flow.
 - These limits often require case by case refinement and comparison with empirical data.
- Transitional flow can be approximated through a bridging function which assumes any local or global aero-coefficient is a weighted average of the free molecular and continuum limits.
 - A sine squared bridging function was used for the trajectory sim in this project.
 - More advanced bridging functions exist but require a DSMC anchor point at the center of the transitional regime.

$$K_n = \frac{\lambda}{L}$$

Knudsen Number

$$\lambda = \frac{k_b T}{\sqrt{2} \pi d^2 p}$$

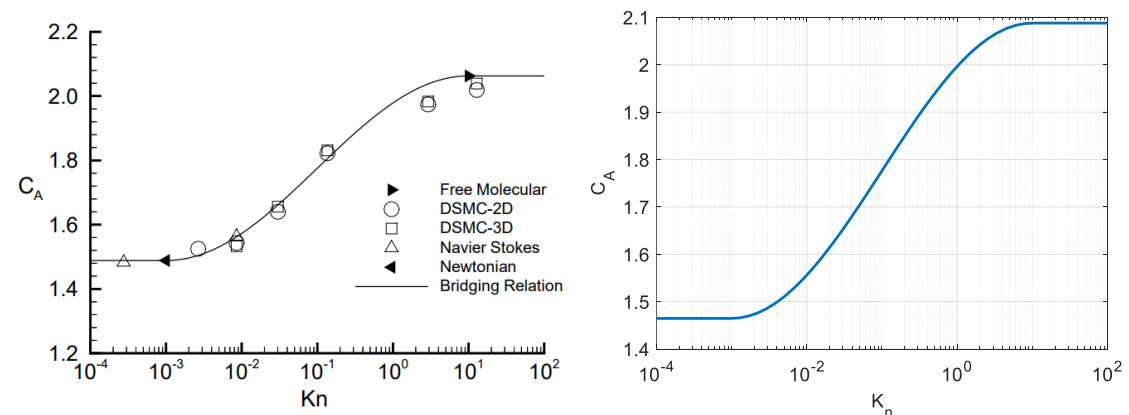
Mean Free Path

$$C_{tr} = P_b C_{fm} + (1 - P_b) C_{cont}$$

$$P_b = \sin^2 \psi$$

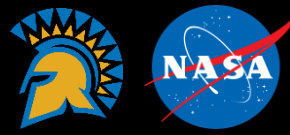
$$\psi = \pi (a_1 + a_2 \log_{10} K_{n\infty})$$

Sine Squared Bridging Function Formulation

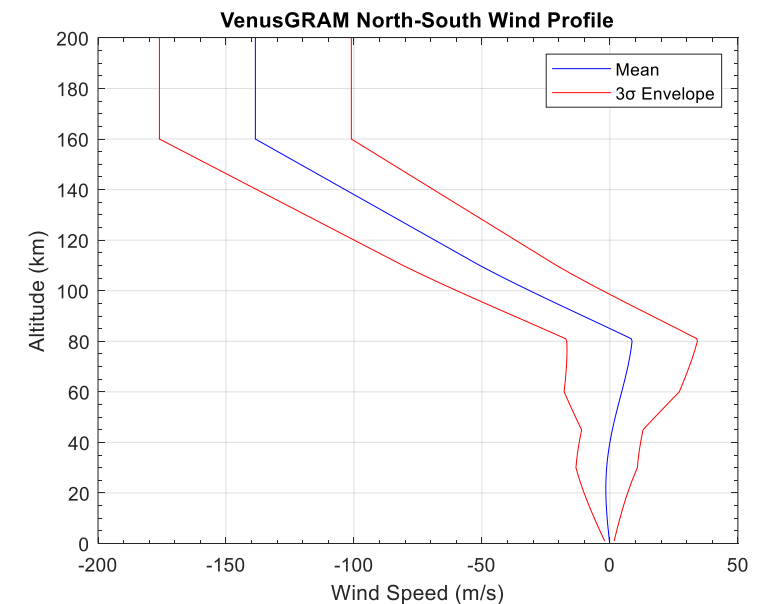
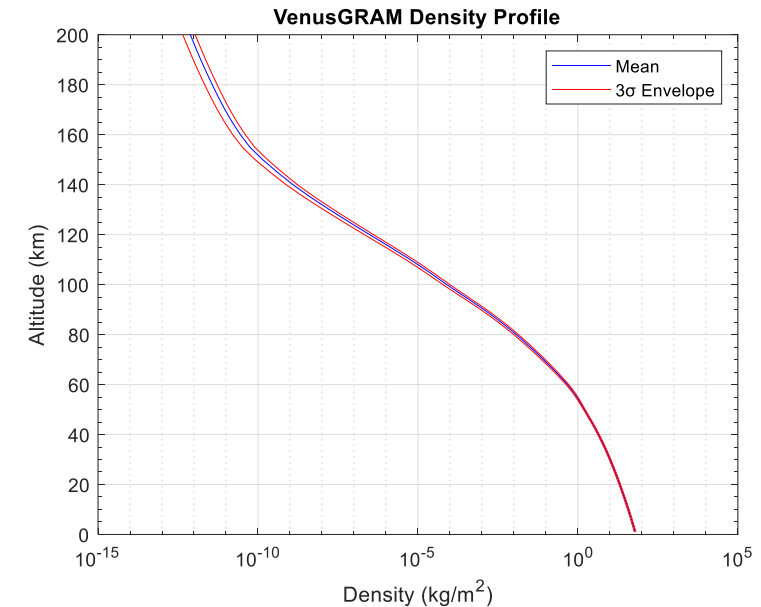




Atmospheric Modeling

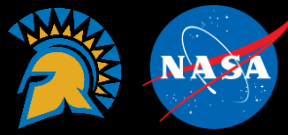


- The NASA developed Global Atmospheric Reference Model (GRAM) is an engineering design tool that can compute planetary atmospheric conditions utilizing ephemeris calculations and a host of empirical data.
- Supported Planets
 - Earth, Mars, Venus, Jupiter, Uranus, Neptune, Titan (SaturnGRAM in development)
- Key GRAM implementation details
 - GRAM model queried at each trajectory time step for maximum accuracy
 - Temporal, spatial variations and winds are modeled within GRAM
 - Atmospheric uncertainties and perturbations are supported through Monte Carlo runs
 - Constant vehicle orientation, 3DOF, no GNC modeling or uncertainties at this point





Aerothermal Modeling



- The Sutton Graves approximation was used as the primary stagnation heating rate metric for this project.
 - $q_s = k \left(\frac{\rho}{R_n} \right)^{\frac{1}{2}} V^3$ (convective only)
 - Simple, fast, and easy to implement, values for k readily available for various planetary bodies.
- Modern Brandis correlations have also been implemented for CO₂, N₂, and H₂/He atmospheres. Often require species mole fractions meaning decreased computational efficiency.
- To determine the thermodynamic state at the stagnation point, an iterative process is necessary utilizing the continuity equations.

*Shock relations
(solved iteratively)*

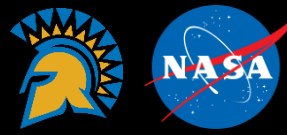
$$\left\{ \begin{array}{l} p_2 = p_1 + \rho_1 u_1^2 \left(1 - \frac{\rho_1}{\rho_2} \right) \\ h_2 = h_1 + \frac{u_1^2}{2} \left[1 - \left(\frac{\rho_1}{\rho_2} \right)^2 \right] \end{array} \right.$$

*Thermochemical
Equilibrium
Calculations*

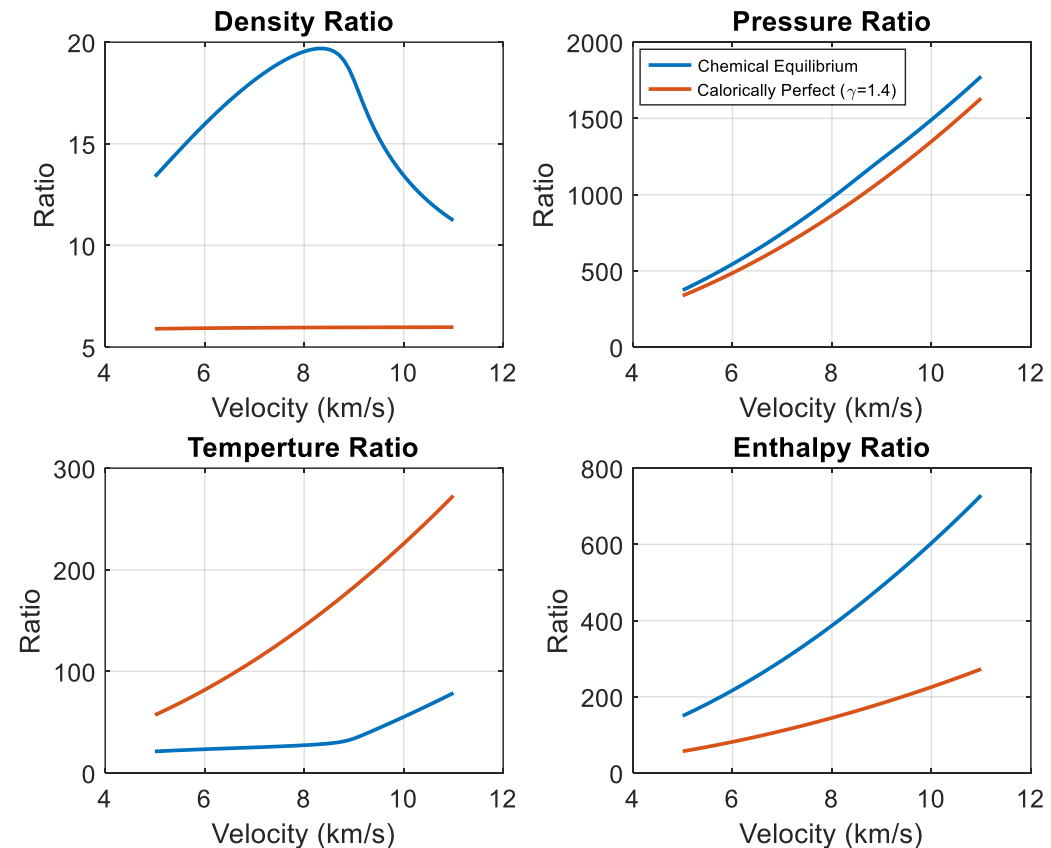
$$\left\{ \begin{array}{l} G_i = H_i - TS_i \\ \Delta G^{p=1} \equiv \sum_i v_i G_i^{p_i=1} \\ \prod_i p_i^{v_i} = e^{-\frac{\Delta G^{p=1}}{RT}} \\ K_p(T) = \prod_i p_i^{v_i} \\ p_t = \sum_i p_i \\ h = \sum_i c_i h_i = \sum_i \eta_i H_i \end{array} \right.$$



Aerothermal Modeling



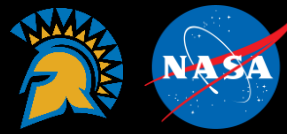
- The chemical equilibrium calculation is computationally tedious and is employed in the industry standard, NASA developed CEA software.
- The trajectory tool as part of this project utilizes symbolic math to reduce the set of non-linear partial pressure equations for fast convergence.
 - The NASA ARC TS developed TRAJ code will pre-generate a thermodynamic Mollier diagram for efficiency.
 - In many cases, the isentropic pressure ratio (Raleigh-Pitot formula) is a good enough approximation.
- The trajectory program supports Monte Carlo runs which can generate a normal distribution of aero heating environments based on trajectory and GRAM atmospheric dispersions.



- $P_1 = 1 \text{ Pa}$
- $T_1 = 216.4 \text{ K}$
- 0.79 mole N_2
- 0.21 mole O_2

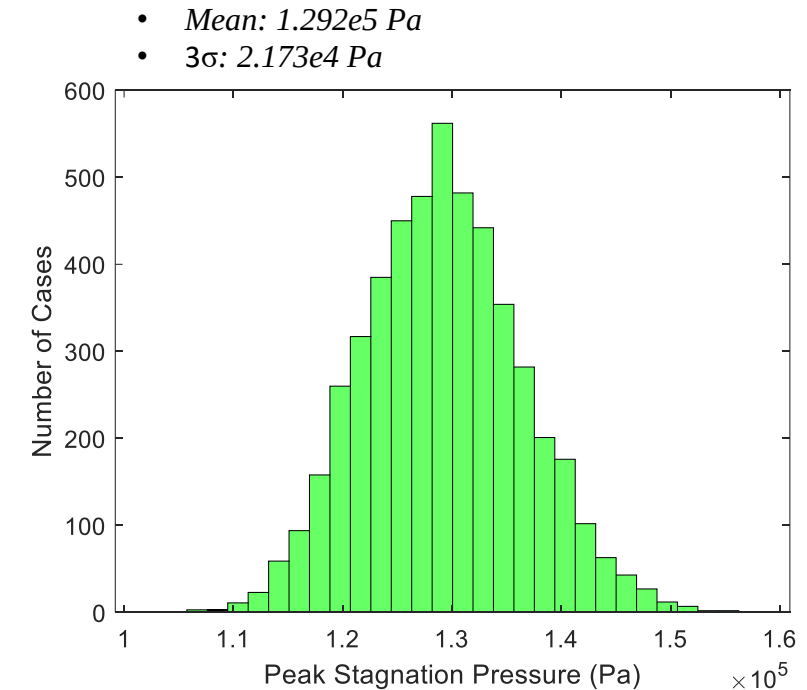
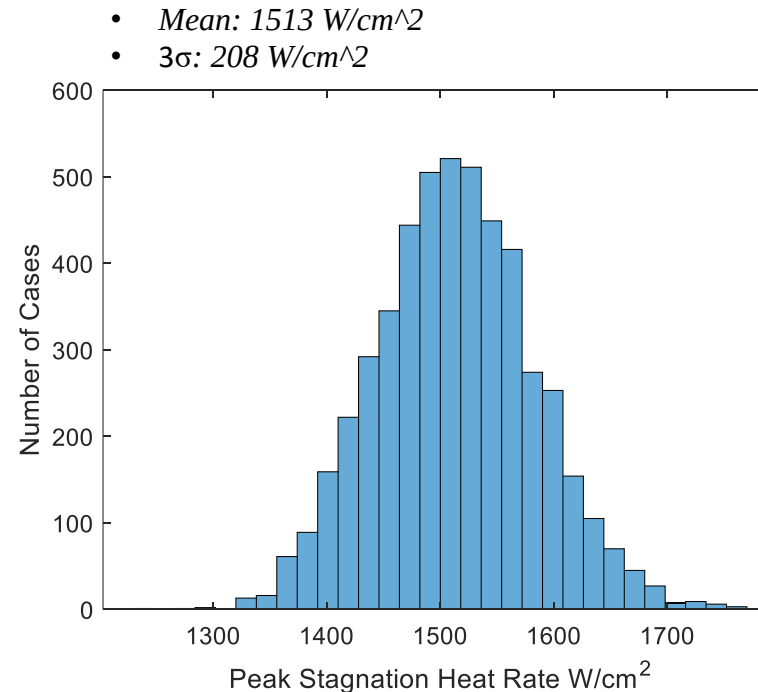


Uncertainty Modeling

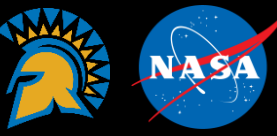


- A Monte Carlo Analysis was run for a small Venus entry vehicle to test the perturbation capabilities in GRAM.
- 5000 runs were completed in around 13 mins on a 12-core i9 machine, parallelization possible but not utilized.
- Potential use as a margin policy if 3σ edge case aerothermal environments are fed into a material response program.
 - Pressure, temperature and species mole fraction dispersions are not yet supported for VenusGRAM.

Monte Carlo Test Case		
Number of Runs	5000	
Julian Date	2460809 (14-May-2025)	
Mass	17 kg	
Cone Half Angle	45°	
Diameter	0.43 m	
Nose Radius	0.118 m	
Dispersions		
Parameter	Mean	Variance
Velocity	11 km/s	1σ=0.05 km/s
EFPA	-10°	1σ=0.1°
Longitude	130°	No Dispersion
Latitude	20°	No Dispersion
Azimuth	91°	No Dispersion
Winds	GRAM	GRAM 3σ
Density	GRAM	GRAM 3σ



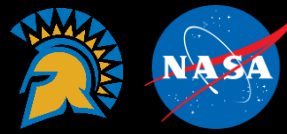
Move to Backup



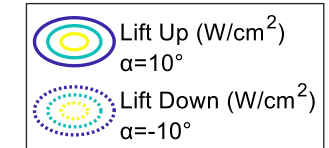
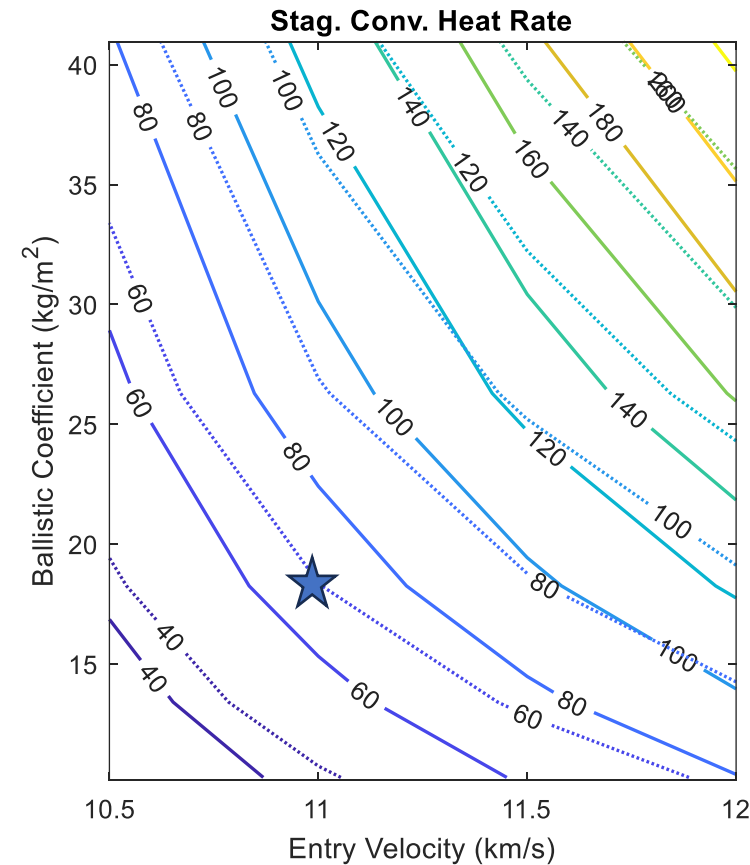
Trajectory Sample Cases



Multi-Pass Aerocapture at Venus



- A conceptual design space was generated for a low ballistic coefficient multi-pass aerocapture vehicle at Venus.
- A “sweet spot” for the initial apoapsis altitude can be found through optimization which for Venus is ~150000-250000 km.
 - Low B_C range chosen to allow for deployable TPS options.
- Numerous test trajectories were validated against traditional aerocapture studies in literature and with the NASA ARC developed TRAJ program.



- **Initial orbit apoapsis is 175000 km**
- Vehicle diameter varied from 2-4 m
- Nose radius is $\frac{1}{4}$ the vehicle diameter
- 70° Sphere Cone
- $\alpha=10^\circ$ / -10°
- 0° Lat, 170° Long, 150 km
- Heating can vary by ~5-7% depending on entry long/lat and JD

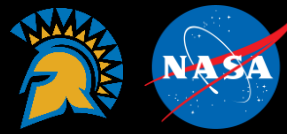


Primary Venus Validation Reference:

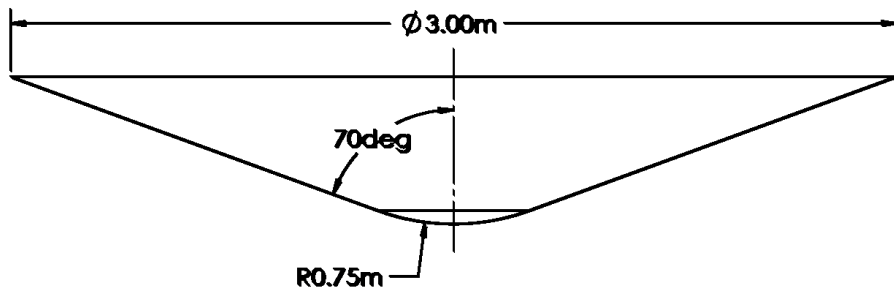
Lugo, R. A., Dutta, S., Matz, D., Johnson, B. J., Pensado, A. R., Roelke, E., Aguirre, J. T., and Powell, R., "Performance Analysis of SMALLSAT Aerocapture at Venus," AIAA SCITECH 2023 Forum, Jan. 23–27, 2023, National Harbor, MD. <https://doi.org/10.2514/6.2023-0878>



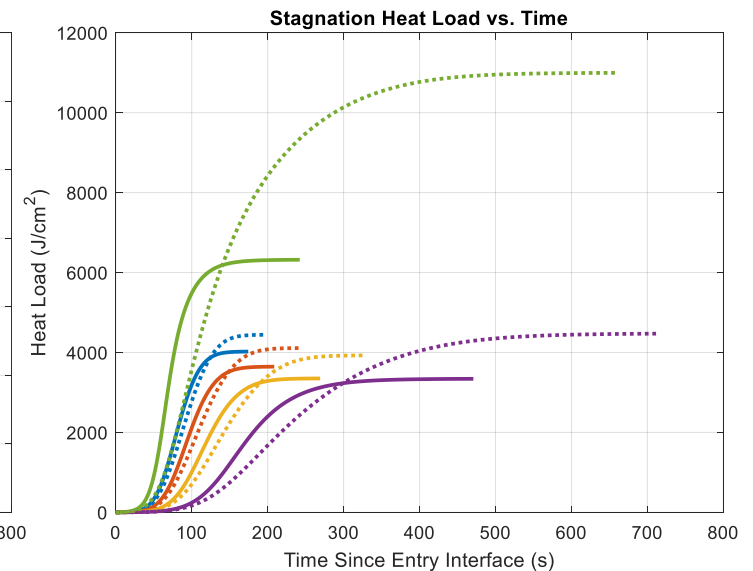
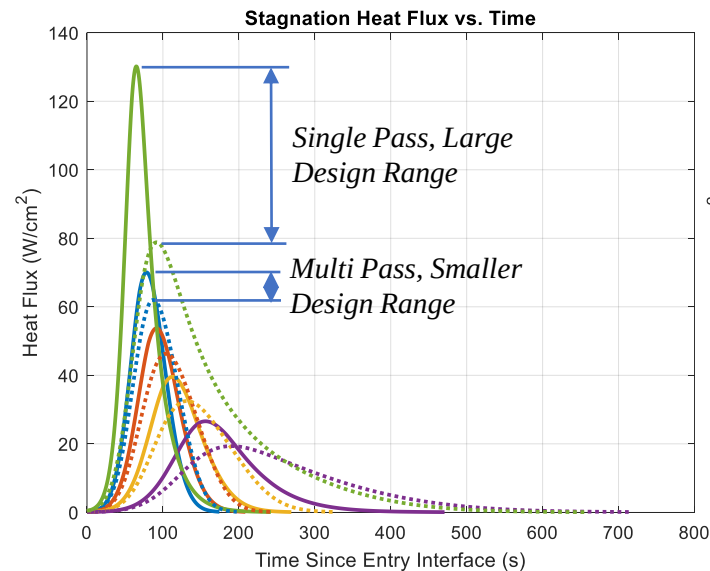
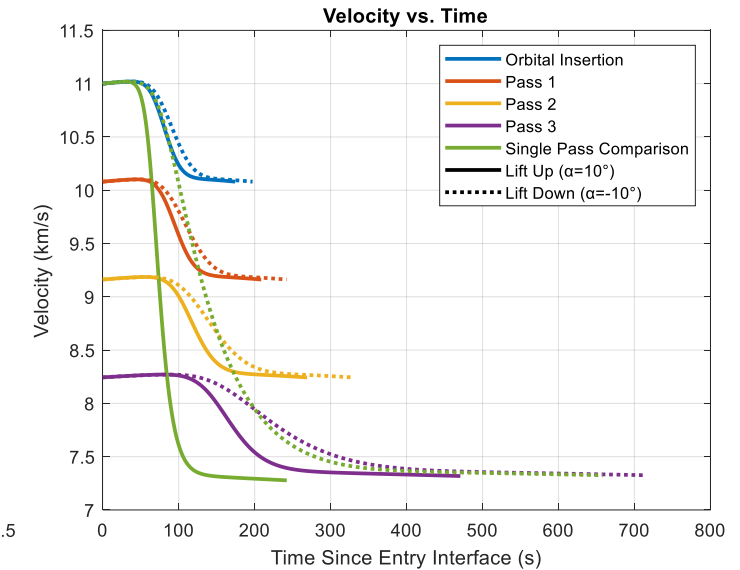
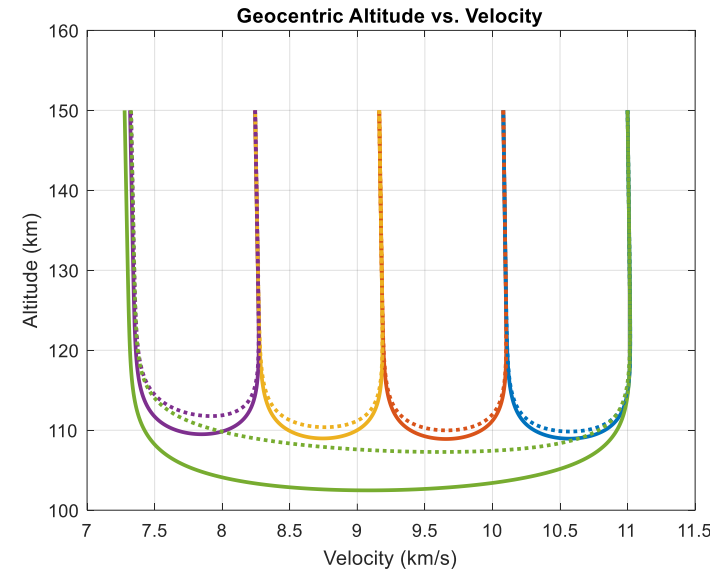
Multi-Pass Aerocapture at Venus



- 500 km final target science orbit, 4 total atmospheric entries.
- A mission planning routine determines the optimal number of atmospheric passes and TCM's to reach the final target orbit.
- The multi-pass offers a narrower range of aerothermal environments that are more benign than the single pass.
- A future study might include a Monte Carlo analysis to determine whether a multi-pass approach is less sensitive to atmospheric or trajectory uncertainties.

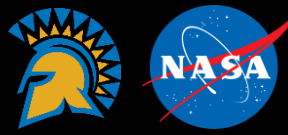


- $B_C = 17.55 \text{ kg/m}^2$ at $\alpha=0^\circ$
- Corner radius and backshell geometry not modeled for this run, but are supported by Newtonian panel method





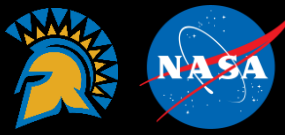
Summary and Next Steps



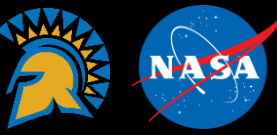
- A Novel EDL mission design tool has been developed that has the capability to study a wide range of entry problems to assess viability.
 - The program can optimize and simulate small propulsive maneuvers and con-ops for complex orbital insertion methods such as multi-pass aerocapture.
- A stretch goal for this software tool would be to model interplanetary transfers and directly couple aspects like the launch window, launch vehicle, and gravity assists to the aerothermal and entry environment at the destination.
- Some technical short-term improvements
 - N-body
 - SPICE integration
 - Translation to Python/C++
 - Expanded comparison with TRAJ, additional comparisons with other tools such as POST2, GENESIS,
 - 6DOF, mass properties, stability
 - Expanded radiative heating correlations



Acknowledgements



- Paul Wercinski, NASA mentor, NASA ARC, Entry Vehicles and Systems Branch
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- Periklis Papadopoulos, SJSU Advisor
 - For valuable advice and feedback for this project and throughout my M.S. degree.
- David Hash, Mairead Stackpoole, Ethiraj Venkatapathy, Todd White, NASA ARC, Entry Systems Division
 - For valuable feedback and technical insight during rehearsal presentation.



Questions?



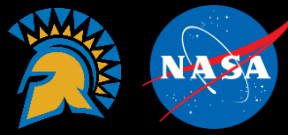
Backup



Backup



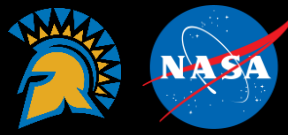
Why?



- A variety of trajectory simulation software exist such as POST, GENESIS, FAST, etc.
- Copernicus and several other tools exist for interplanetary mission design and maneuver optimization.
- Most importantly, programming a physics simulation from scratch re-enforces fundamental mathematics knowledge, modeling, and programming techniques that are all relevant to EDL and space systems in general.
 - Also, building a completely new codebase/architecture is rare, legacy programs are often used for 20+ years. While they can be continuously updated, there are often fundamental limitations and out of date implementations that remain.
 - Bloating out an architecture by adding modules, wrappers, and libraries while leaving original source untouched can lead to lower readability, original authors often retire or move on.
- At its core, 2-body gravitational mechanics isn't difficult to simulate in a modern development environment like MATLAB or Julia.
- Additional fidelity and physics can easily be added without starting over or throwing off the existing models.
- While traditional aerocapture has been simulated numerous times in literature, a simulation combining multiple atmospheric entries, propulsive maneuvers, and trajectory optimizations is not as common.



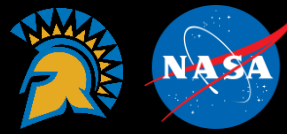
Multi-Pass Aerocapture at Venus



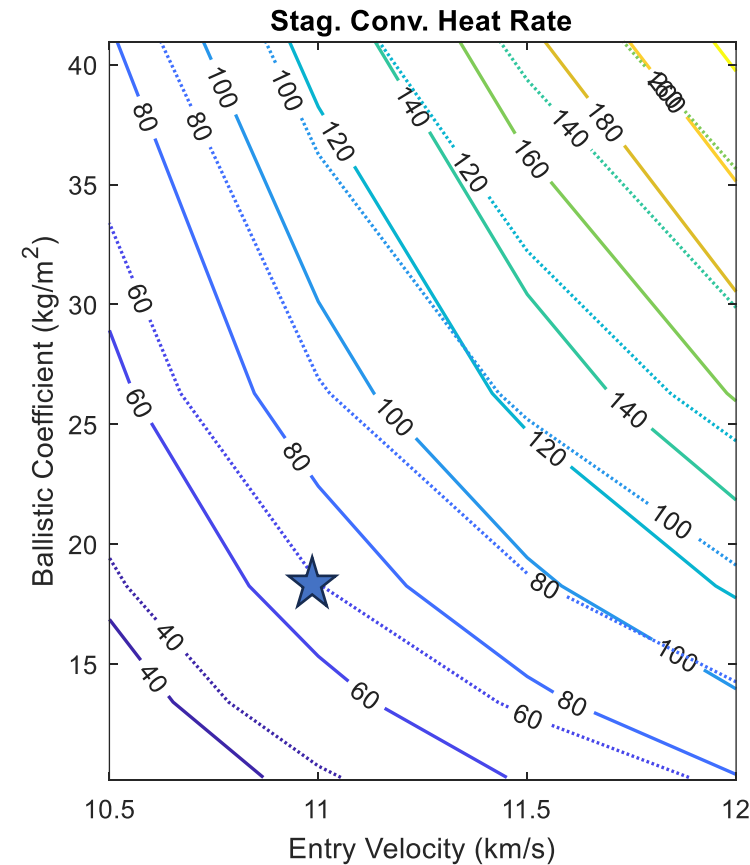
- Several planetary bodies were evaluated for the benefits of a multi-pass approach.
 - Uranus and Neptune, while popular for traditional aerocapture, aren't applicable for a multi-pass apoapsis reduction due to the presence of natural satellites for free down-pumping maneuvers and limited science at ultra low orbits.
 - The Uranus moon Titania orbits at $\sim 550,000$ km, which is near the ΔV limit of a bounded elliptical orbit.
- Venus has no natural satellite and has ample science objectives in low (~ 500 km) orbits. The trajectory tool was used to explore the feasibility of a small-sat, multi-pass aerocapture mission.
- A weighting function was used to evaluate the optimal aerocapture post-exit altitude based on stagnation heating rate and orbital period.



Multi-Pass Aerocapture at Venus



- One of the drawbacks of a multi-pass approach is increased mission duration.
 - Orbital period varies non-linearly with apoapsis altitude.
- Shaving off less ΔV will produce a milder aerothermal environment but also a higher apoapsis and longer orbital period.
 - A “sweet spot” can be found through optimization which for Venus is an apoapsis of ~150000-250000 km.
- Using this data, a conceptual design space can be generated for a low ballistic coefficient aerocapture vehicle at Venus targeting a 500 km science orbit.

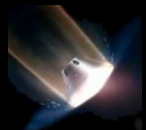


- **Initial orbit apoapsis is 175000 km**
- Vehicle diameter varied from 2-4 m
- Nose radius is $\frac{1}{4}$ the vehicle diameter
- 70° Sphere Cone
- $\alpha=10^\circ$ / -10°
- 0° Lat, 170° Long, 150 km
- Heating can vary by ~5-7% depending on entry long/lat and JD
- Low B_C range chosen to allow for deployable TPS options



Primary Venus Validation Reference:

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Venus Multi-Pass Additional Results



Maneuvers Summary		
Maneuver	Duration (s)	ΔV (m/s)
Lift Up		
Interplanetary TCM	10.49	0.525
1 st Periapsis Adjust	0.128	0.006
2 nd Periapsis Adjust	2.000	0.100
3 rd Periapsis Adjust	10.15	0.507
Final Periapsis Raise	190.0	116.3
Lift Down		
Interplanetary TCM	12.17	0.610
1 st Periapsis Adjust	0.194	0.010
2 nd Periapsis Adjust	2.141	0.110
3 rd Periapsis Adjust	8.137	0.407
Final Perigee Raise	177.1	108.2
Totals	Lift Up	Lift Down
Mission Duration (days)	5.461	5.468
Maneuvers ΔV (m/s)	117.4	109.4
Propellant Usage (kg)	7.832	7.304

Spacecraft Input Parameters	
Parameter	Value
Initial Mass	200 kg
High Thrust System	120 N
Low Thrust System	10 N
ISP (both systems)	300 s
Trim Angle of Attack (α)	10°/-10°
C_D at $\alpha=10^\circ/-10^\circ$	1.550
C_L at $\alpha=10^\circ/-10^\circ$	± 0.237
Diameter	3 m
Nose Radius	0.75 m
Sphere Cone Angle	70°
B_C at $\alpha=0^\circ$	17.55 kg/m ²
B_C at $\alpha=10^\circ/-10^\circ$	18.26 kg/m ²
Final Target Altitude	500 km

Interplanetary Orbital Elements	
Parameter	Value
Eccentricity (e)	1.3074
Semi Major Axis (a)	-2.0005e4 km
Inclination (i)	0°
Argument of Periapsis (ω)	0°
Long. of Ascending Node (Ω)	0°
True Anomaly (θ)	137°
Hyperbolic Excess Velocity	4.0298 km/s
Julian Date	2455504
Initial Target Apoapsis (km)	200000

Aero-Pass Summary			
Pass	q_{max} (W/cm ²)	J_s (J/cm ²)	ΔV (km/s)
Lift Up			
Insertion	70.05	4022	0.916
1 st Pass	53.74	3645	0.914
2 nd Pass	39.55	3350	0.912
3 rd Pass	26.58	3340	0.892
Lift Down			
Insertion	61.66	4416	0.916
1 st Pass	46.05	4085	0.915
2 nd Pass	32.30	3903	0.914
3 rd Pass	19.22	4445	0.912

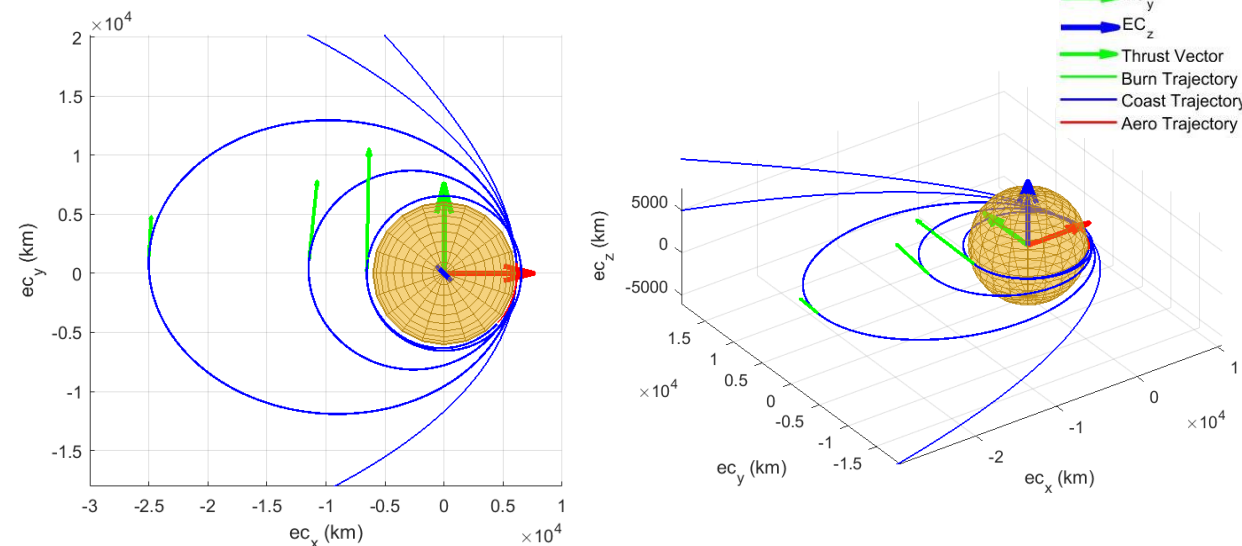
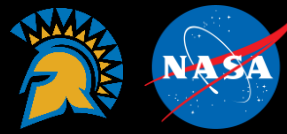


Figure 6.15 Venus Deployable TPS Additional Trajectory Views

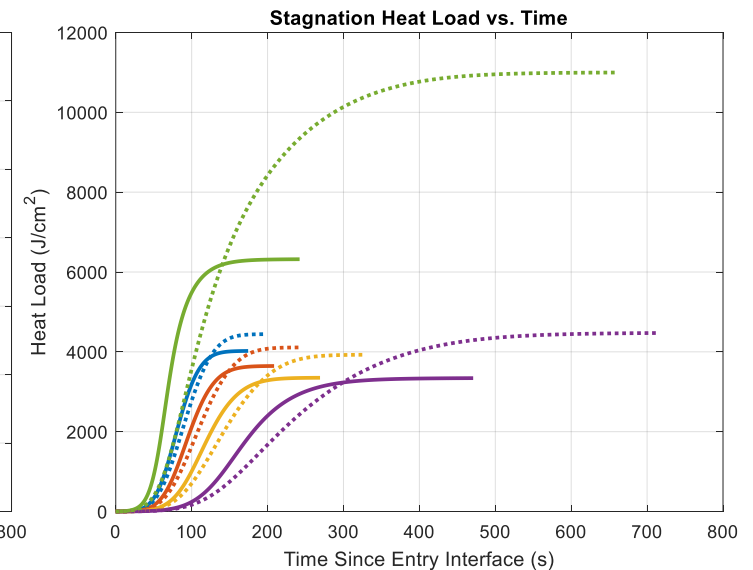
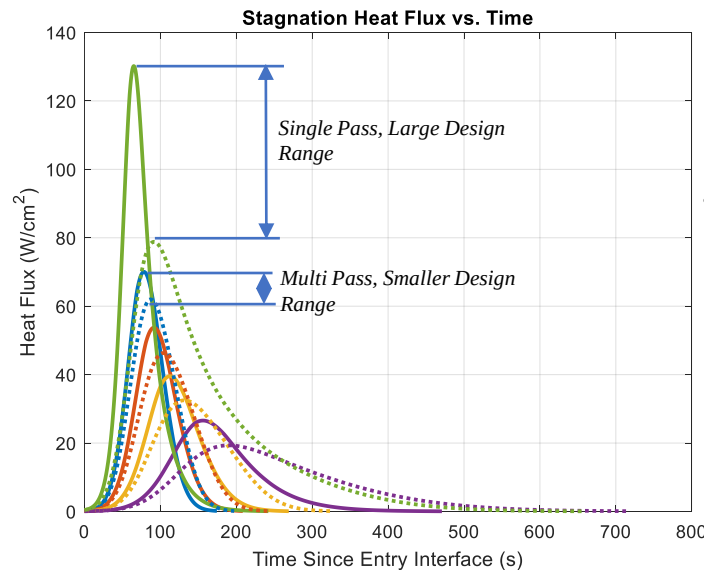
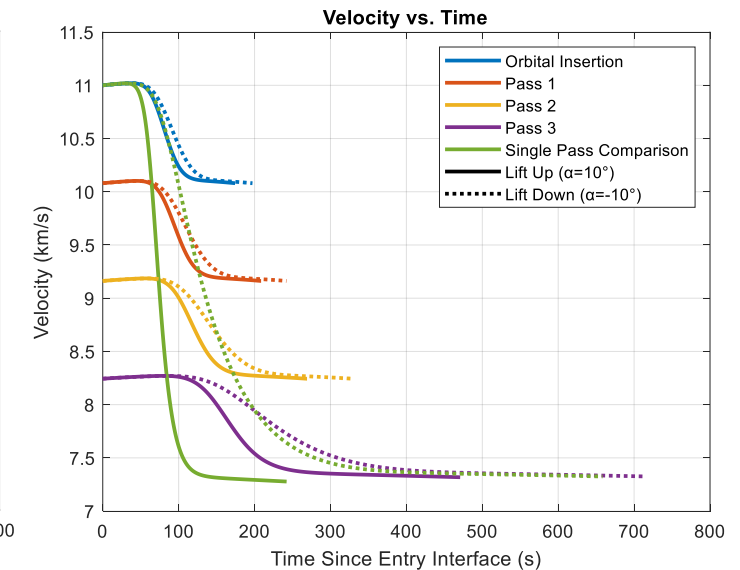
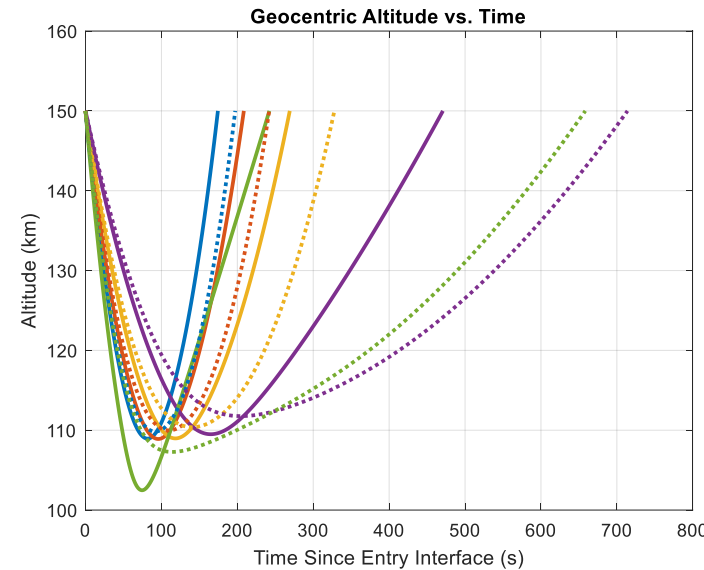


Multi-Pass Aerocapture at Venus



- An adaptive mission planning module was developed for the trajectory program that determines the optimal number of atmospheric passes to reach the final target orbit.
 - The program balances the ΔV of each pass with the bounding initial orbital insertion
 - Periapsis correction maneuvers are optimized and executed after each pass
 - A final periapsis raise maneuver is executed after the final target apoapsis is achieved

Spacecraft Input Parameters	
Parameter	Value
Initial Mass	200 kg
High Thrust System	120 N
Low Thrust System	10 N
ISP (both systems)	300 s
Trim Angle of Attack (α)	$10^\circ/-10^\circ$
C_D at $\alpha=10^\circ/-10^\circ$	1.550
C_L at $\alpha=10^\circ/-10^\circ$	± 0.237
Diameter	3 m
Nose Radius	0.75 m
Sphere Cone Angle	70°
B_C at $\alpha=0^\circ$	17.55 kg/m^2
B_C at $\alpha=10^\circ/-10^\circ$	18.26 kg/m^2
Final Target Altitude	500 km





Peak Heating and Orbital Period Tradeoff

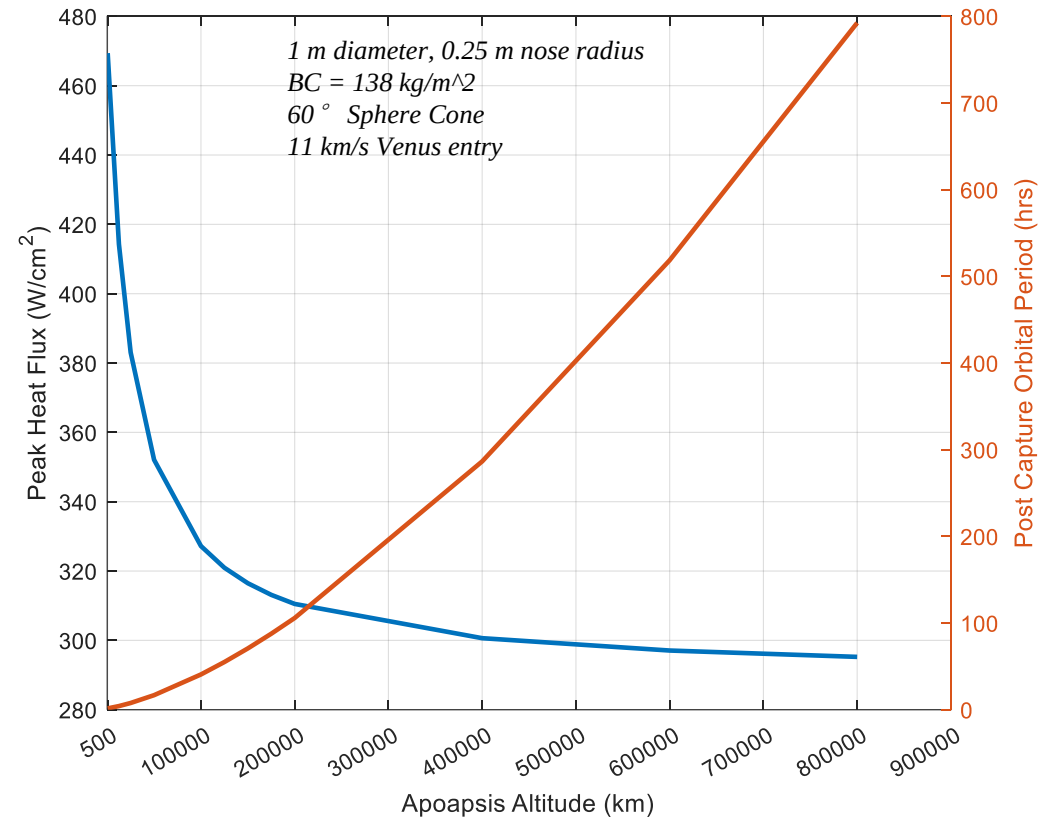
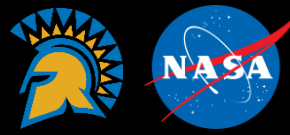
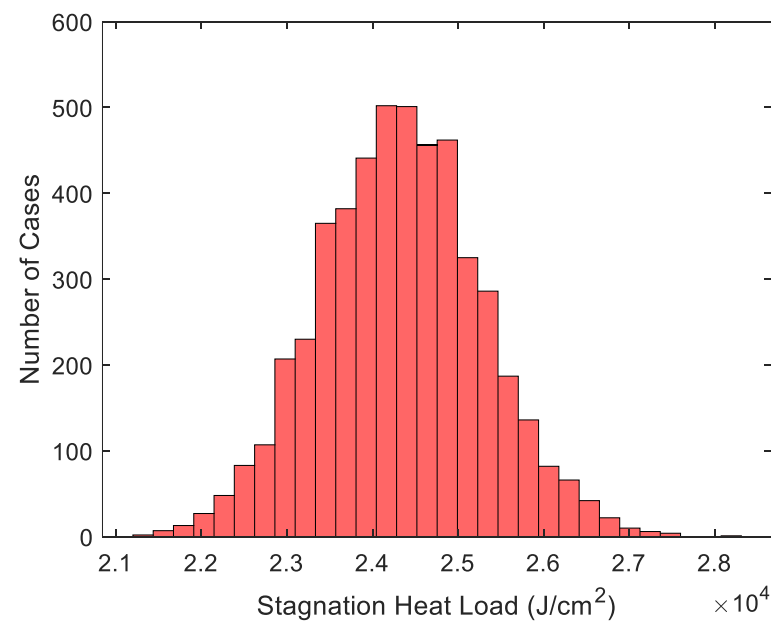
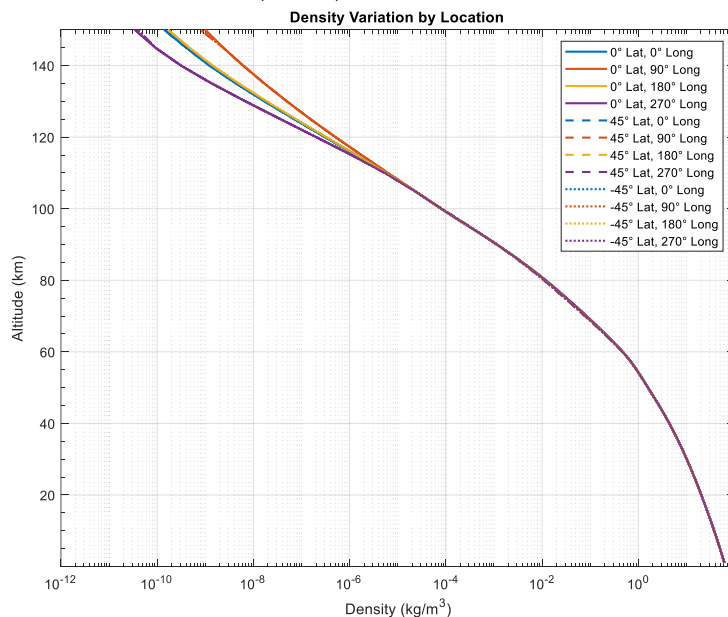
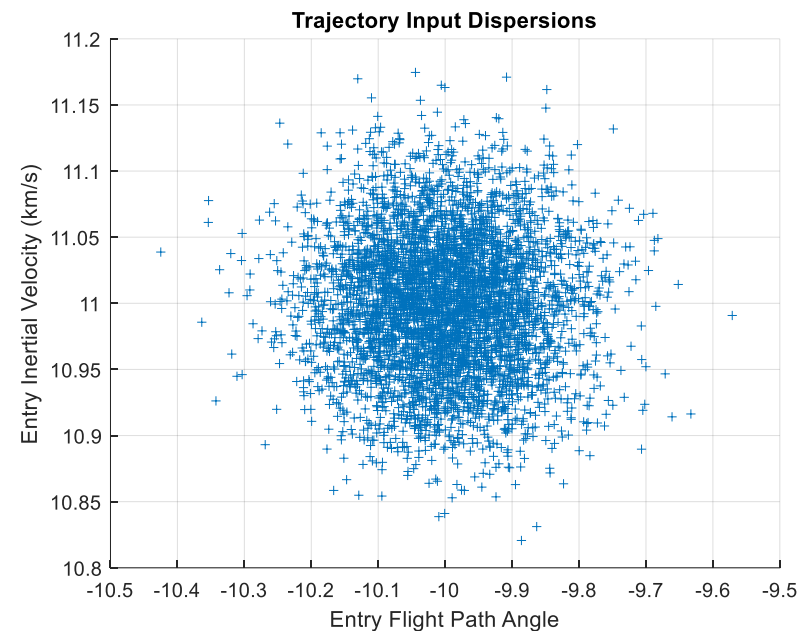
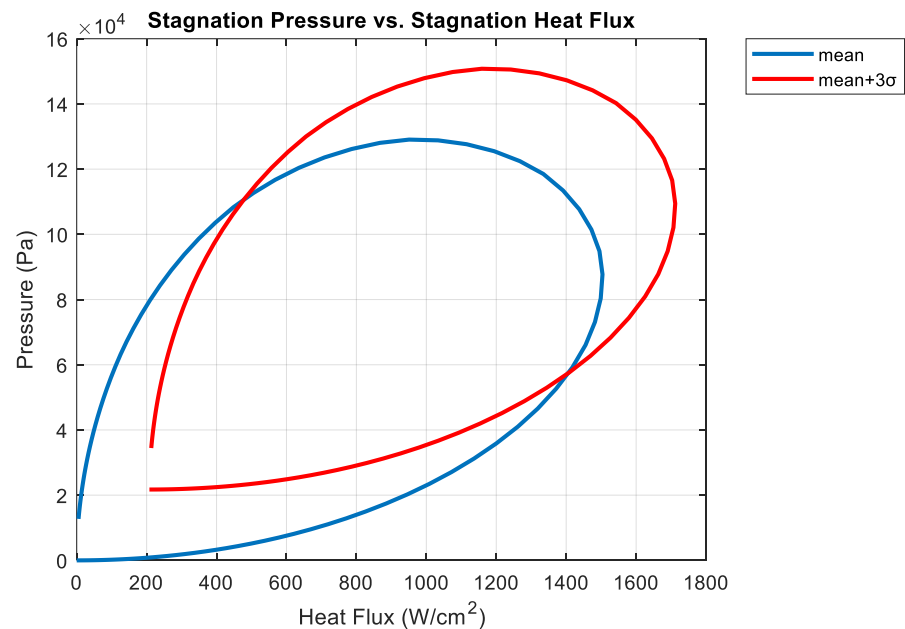


Figure 6.5 Peak Stag. Heat Flux to Orbital Period Tradeoff



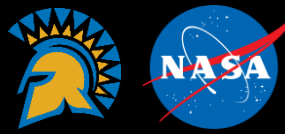
Monte Carlo Test Case Additional Details



- Mean: $2.4325 \times 10^4 \text{ J}/\text{cm}^2$
- 3 σ : $2.8471 \times 10^4 \text{ J}/\text{cm}^2$

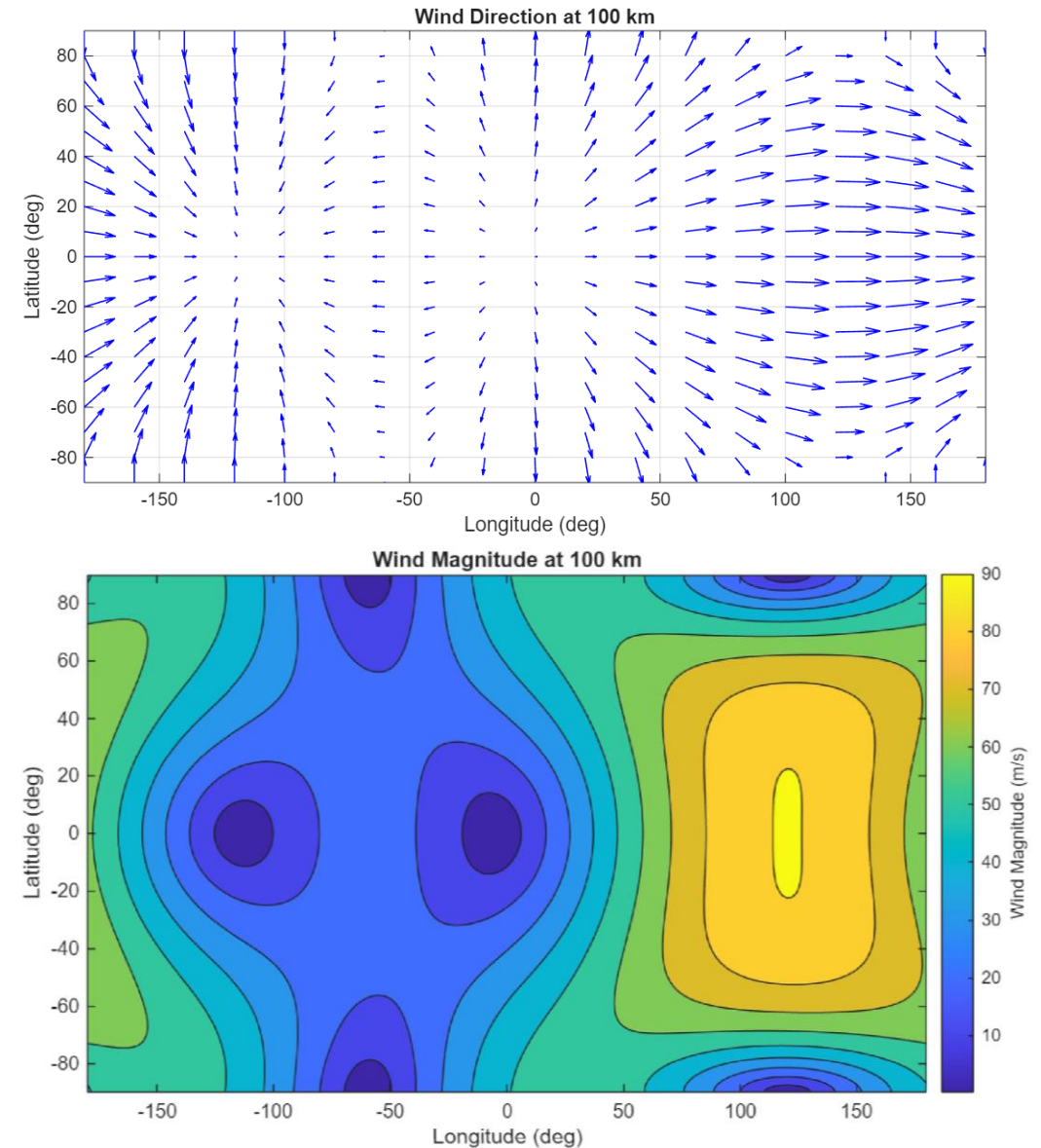


Atmospheric Modeling

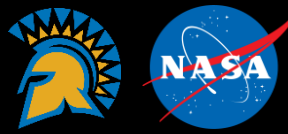


- Venusian winds modeled within GRAM were studied in detail.
 - The winds on Venus are significant at higher altitudes (<200 m/s) and are faster than the planet's rotation.
- GRAM outputs winds in three components, north to south (nsw), east to west (eww), and vertical (vw) in the east, north, zenith (ENZ) frame.
 - A great deal of time was spent in reference frame conversion handling and efficiency for this project (QXx is the ENZ to ECI conv. Matrix).

$$v_{rel} = V_{eci} - \underbrace{\begin{bmatrix} 0 \\ 0 \\ \omega \end{bmatrix} \times R_{eci}}_{\text{Coriolis Effect}} - \underbrace{Q_{Xx} \begin{bmatrix} -eww \\ -nsw \\ vw \end{bmatrix}}_{\text{Winds}}$$

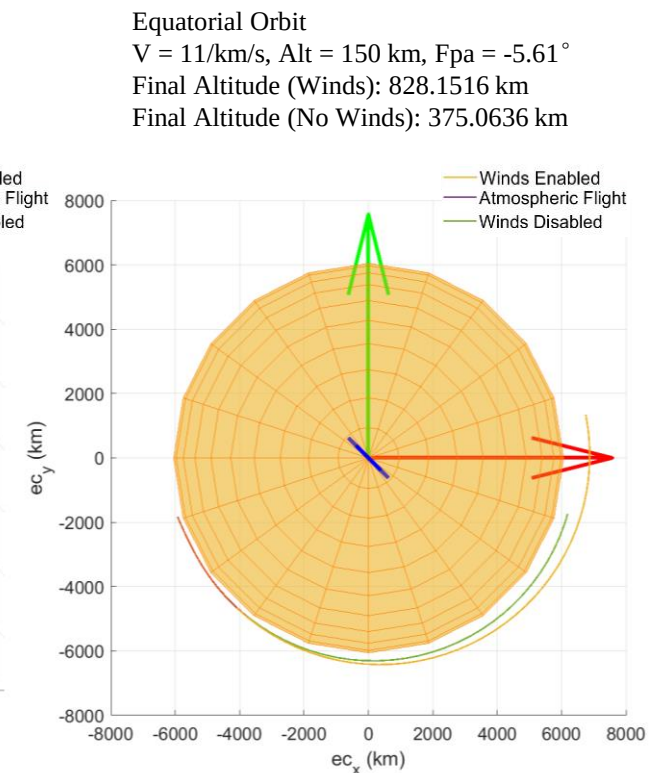
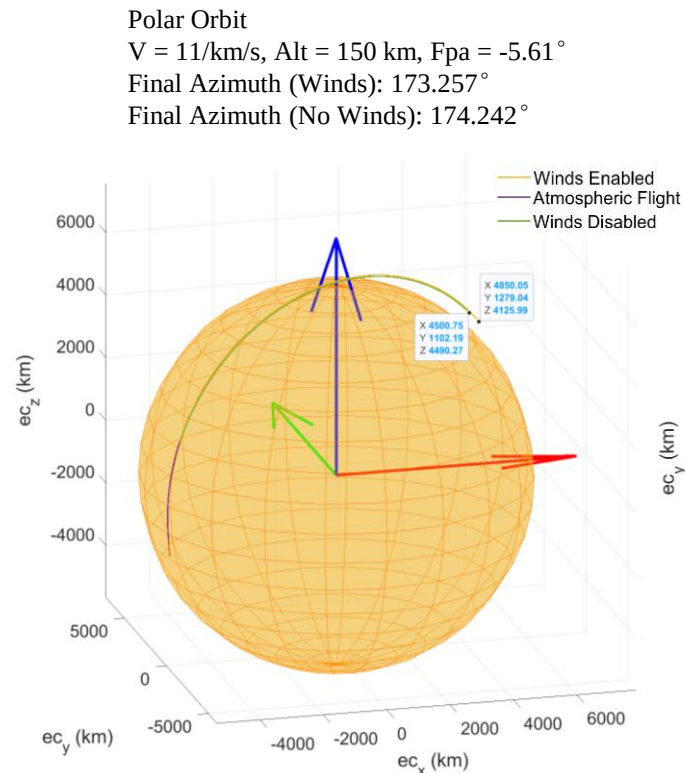
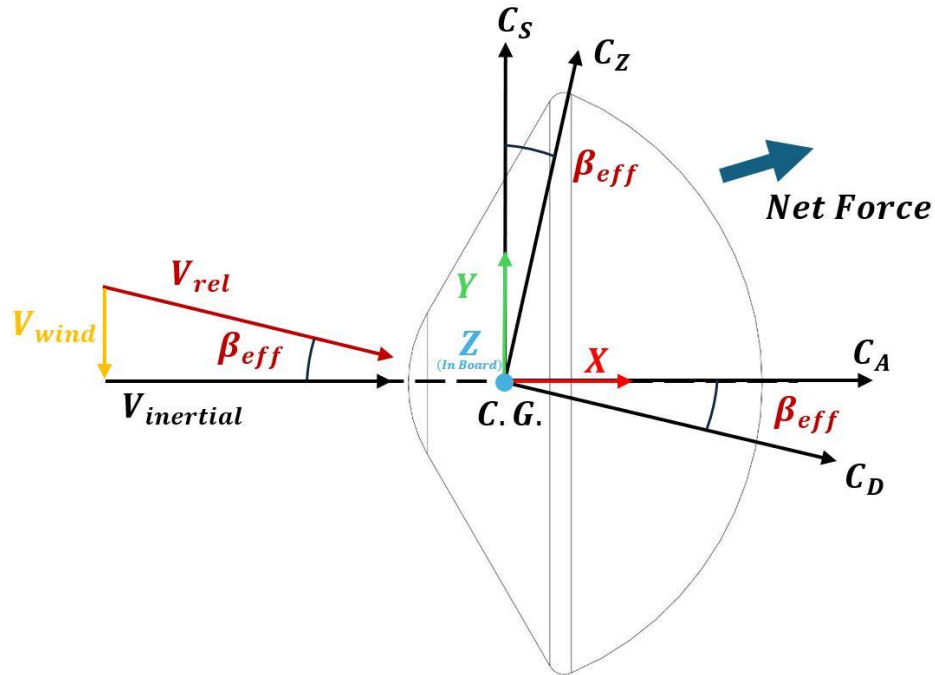


Venusian Wind Magnitude and Direction (JD 2459138)



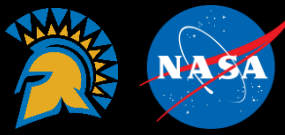
Atmospheric Modeling

- With a sufficiently blunt body ($\sim 60^\circ$ half angle) the C_L vs. α lift slope is reversed compared to a slender body or traditional airfoil. The same analogy applies for sideslip β .
- This effect results in the entry vehicle being pulled in the direction of the wind, almost like a sailboat sailing upwind.
 - Crosswind and tailwind trajectory dispersions were tested for a small 60° sphere cone entry vehicle at Venus.
 - Effect is significant for Aerocapture post-exit altitudes.

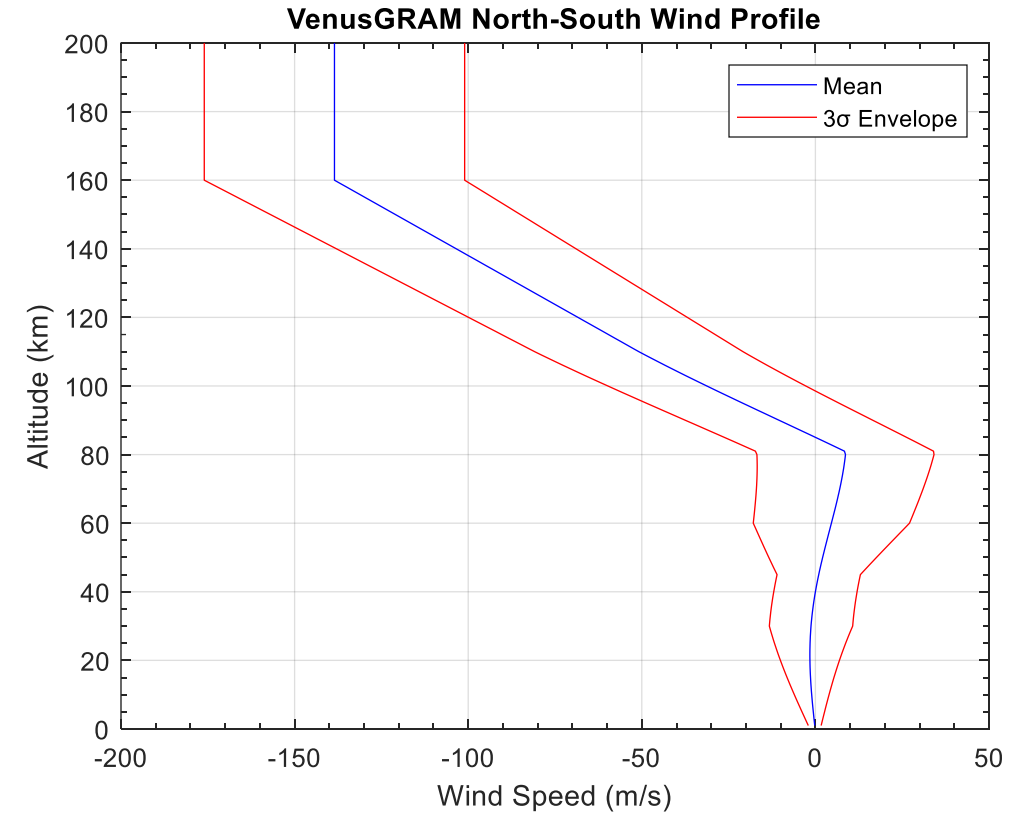
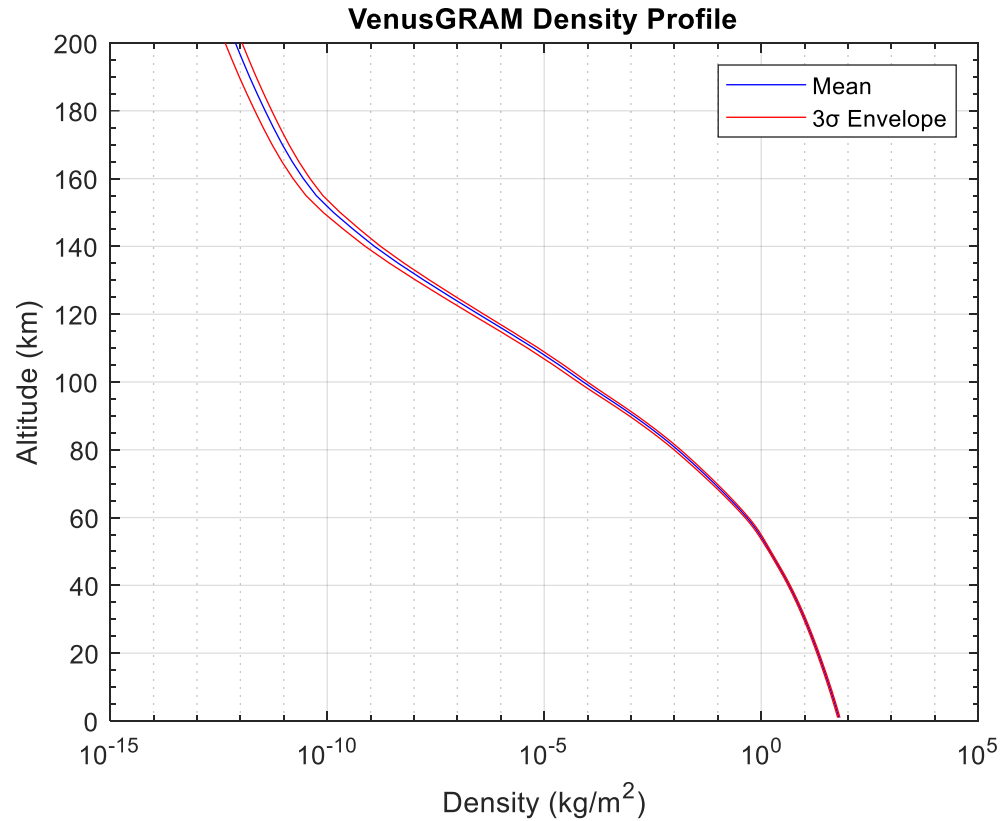




Atmospheric Modeling

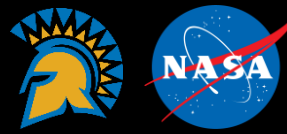


Long: 225, lat: 60, Date: 2020, 5, 12, 16, 0, 0.0

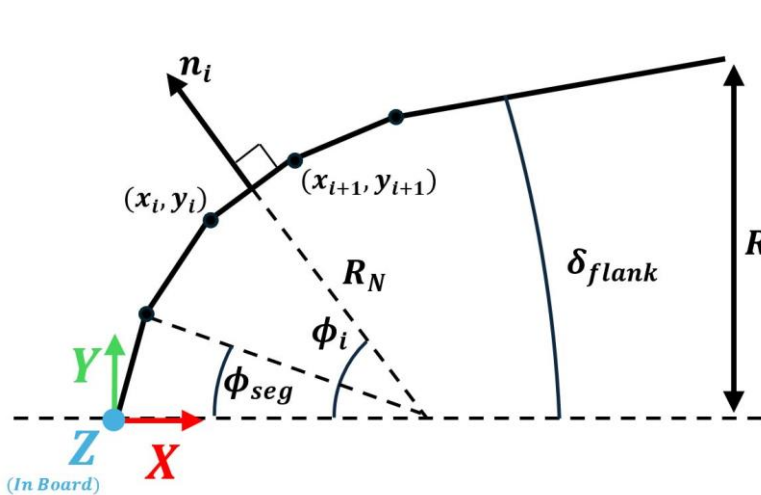




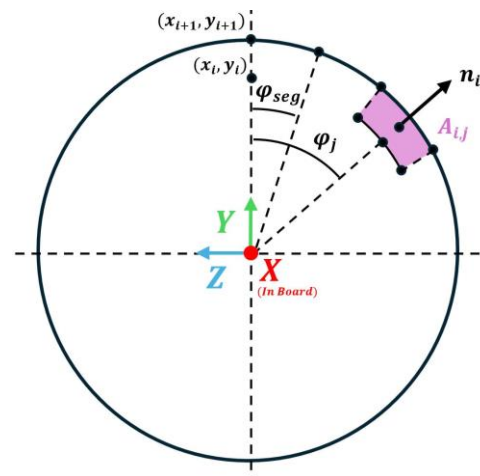
Gridding Methods



- Both the free molecular and continuum aerodynamics depend on the local inclination angle of the vehicle surface.
- Analytical expressions exist for simple shapes, however for this project a CB-AERO style grid generator was developed to allow for more complex geometries in the future.
- The coefficients form an expression for the perturbing acceleration vector due to aero forces. This is a simple addition to the equations of motion, additional perturbations due to spherical harmonics or solar radiation pressure can also be added.



Blunt Body Side View Discretization



Nose View Discretization

$$\begin{bmatrix} C_D \\ -C_L \\ C_Z \end{bmatrix} = \frac{C_{pmax}}{A_{ref}} \sum_{i=2}^{N_{seg}} \sum_{j=2}^{M_{seg}} (N_{i,j} \hat{x})^2 \begin{bmatrix} N_{i,j} \hat{x} \\ N_{i,j} \hat{y} \\ N_{i,j} \hat{z} \end{bmatrix} A_{i,j}$$

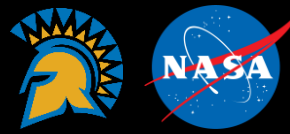
$$\sin \theta = \frac{V_{\infty}}{|V_{\infty}|} \cdot \mathbf{n} = N_{i,j} \hat{x}$$

$$\mathbf{P}_{vfr} = \frac{1}{2} \rho_{\infty} |\mathbf{v}_{rel}|^2 \begin{bmatrix} -C_D \\ C_Z \\ C_L \end{bmatrix} \quad \ddot{\mathbf{r}} = \frac{\mu}{r^3} \mathbf{r} + \mathbf{P}$$

Perturbing acceleration vector, coordinate transformation from relative velocity frame (VFR) to ECI frame not shown here for simplicity



Panel/Grid Method Expanded Relations



$$\mathbf{N}_{i,j} = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{n}_{i,j}$$

$$\begin{bmatrix} C_D \\ -C_L \\ C_Z \end{bmatrix} = \frac{C_{p_{max}}}{A_{ref}} \sum_{i=2}^{N_{seg}} \sum_{j=2}^{M_{seg}} (N_{i,j} \hat{\mathbf{x}})^2 \begin{bmatrix} N_{i,j} \hat{\mathbf{x}} \\ N_{i,j} \hat{\mathbf{y}} \\ N_{i,j} \hat{\mathbf{z}} \end{bmatrix} A_{i,j}$$

$$\frac{V_\infty}{|V_\infty|} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \sin\theta = \frac{V_\infty}{|V_\infty|} \cdot \mathbf{n} = N_{i,j} \hat{\mathbf{x}}$$

$$\begin{bmatrix} C_A \\ C_N \\ C_S \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}^T \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix}^T \begin{bmatrix} C_D \\ -C_L \\ C_Z \end{bmatrix}$$

$$\phi_{seg} = \frac{\frac{\pi}{2} - \delta_{flank}}{N_{seg}} \quad \phi_i = \frac{\phi_{seg}(2i-1)}{2}$$

$$x_{i+1} = R_N - R_N \cos(\phi_{seg} i)$$

$$y_{i+1} = R_N \sin(\phi_{seg} i)$$

$$\mathbf{n}_i = \begin{bmatrix} \cos\phi_i \\ \sin\phi_i \\ 0 \end{bmatrix}$$

Radial Segment Expressions

$$\varphi_{seg} = \frac{2\pi}{M_{seg}} \quad \varphi_j = \frac{\varphi_{seg}(2j-1)}{2}$$

$$\mathbf{n}_{i,j} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi_j & \sin\varphi_j \\ 0 & -\sin\varphi_j & \cos\varphi_j \end{bmatrix} \mathbf{n}_i$$

$$A_{tot} = \pi d(y_{i+1} + y_i)$$

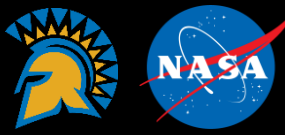
$$d_{seg} = \sqrt{(y_{i+1} - y_i)^2 + (x_{i+1} - x_i)^2}$$

$$A_{i,j} = \frac{A_{tot}}{N_{seg}}$$

Circumferential Segment Expressions



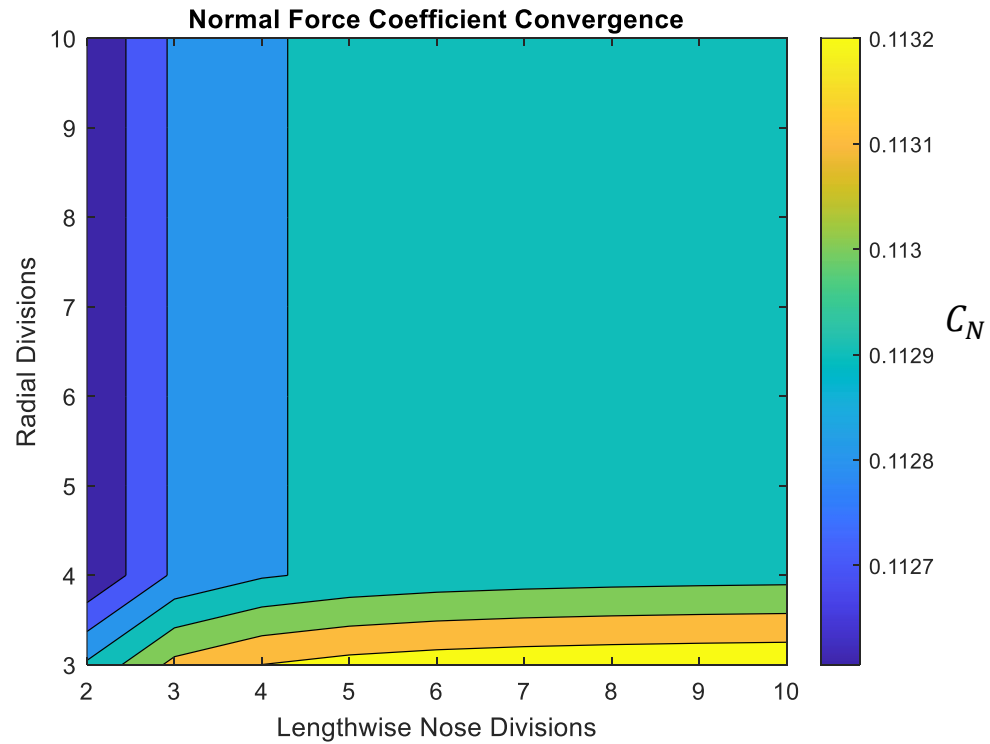
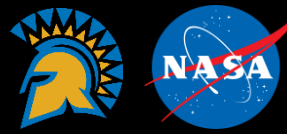
Free Molecular Flow Expressions



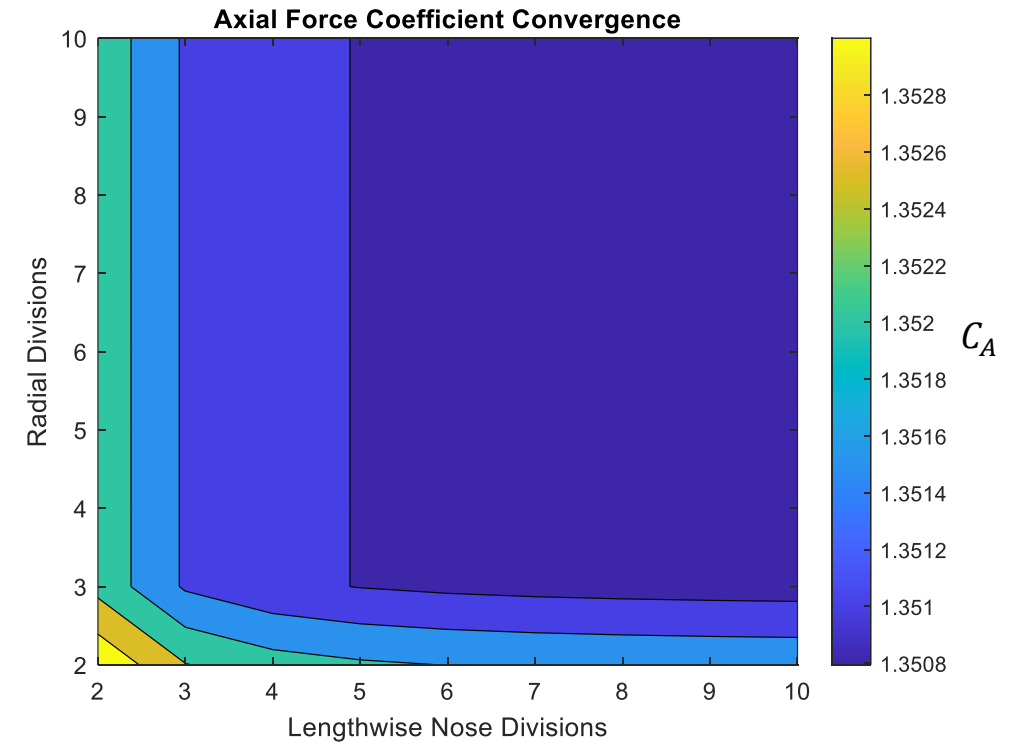
- $$C_p = \frac{1}{s^2} \left[\left(\frac{2 - \sigma_N}{\sqrt{\pi}} s \sin(\theta) + \frac{\sigma_N}{2} \sqrt{\frac{T_W}{T_\infty}} \right) e^{-(s \sin(\theta))^2} + \left\{ (2 - \sigma_N)(s \sin(\theta))^2 + \frac{1}{2} + \frac{\sigma_N}{2} \sqrt{\frac{\pi T_W}{T_\infty}} s \sin(\theta) \right\} (1 + \operatorname{erf}(s \sin(\theta))) \right]$$
- $$C_\tau = \frac{\sigma_N \cos(\theta)}{s \sqrt{\pi}} \left[e^{-(s \sin(\theta))^2} + \sqrt{\pi} s \sin(\theta) (1 + \operatorname{erf}(s \sin(\theta))) \right]$$
- $$S = \frac{V_\infty}{\sqrt{2RT_\infty}}$$



Panel/Gridding Method Convergence



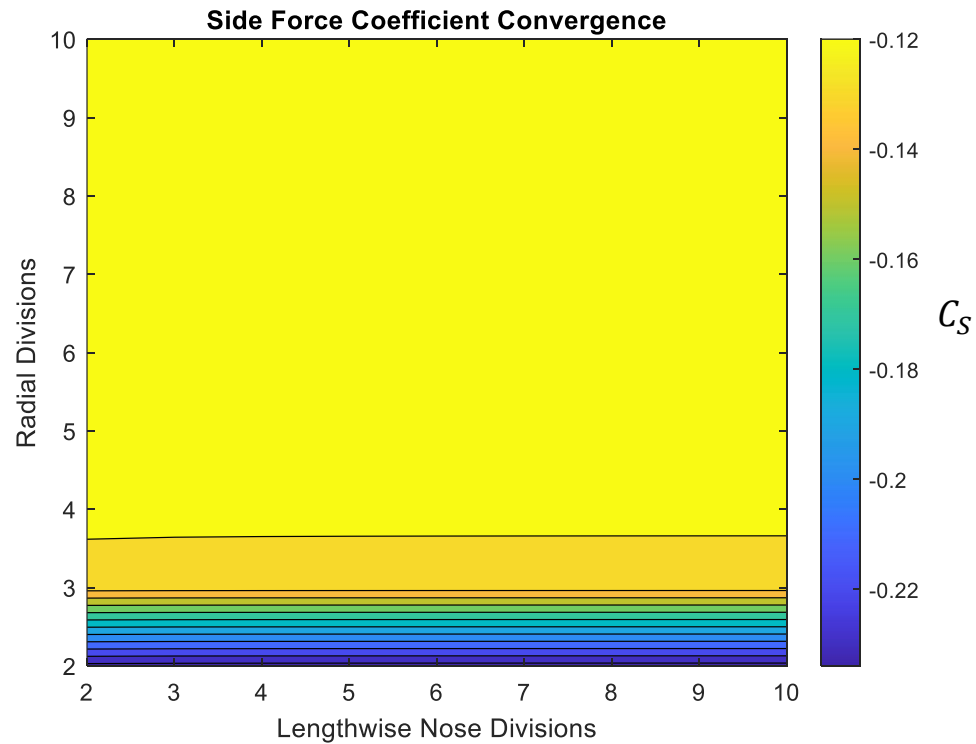
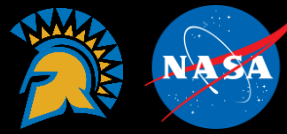
[Figure 9.5](#) Normal Force vs. Number of Divisions (60° sphere cone)



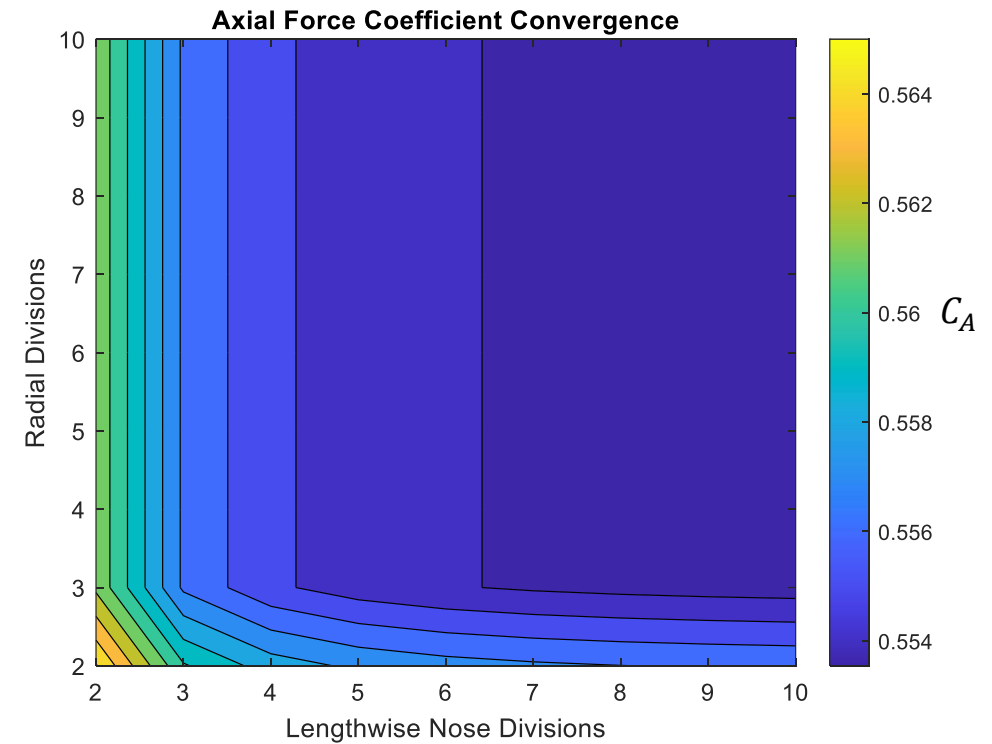
[Figure 9.6](#) Axial Force vs. Number of Divisions (60° sphere cone)



Panel/Gridding Method Convergence



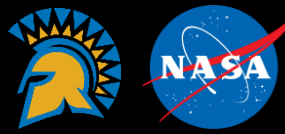
[Figure 9.7](#) Side Force Convergence (60° sphere cone)



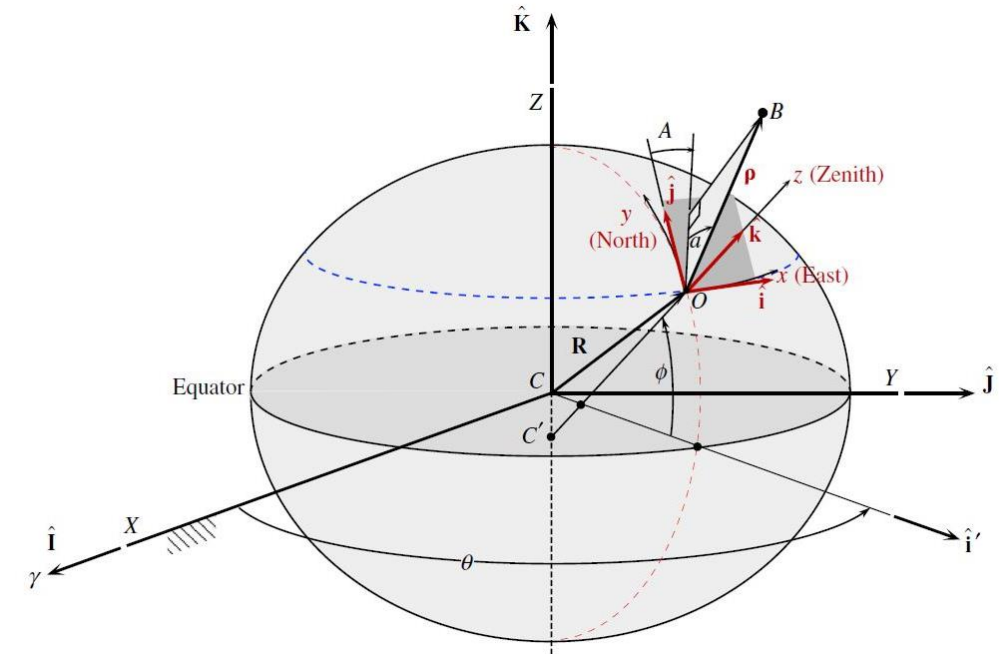
[Figure 9.8](#) Axial Force vs. Number of Divisions (25° sphere cone)



Reference Frames Overview



- $Q_{enz} = \begin{bmatrix} \cos(fpa)\sin(Az) & -\cos(Az) & -\sin(fpa)\sin(Az) \\ \cos(fpa)\cos(Az) & \sin(Az) & -\sin(fpa)\cos(Az) \\ \sin(fpa) & 0 & \cos(fpa) \end{bmatrix}$
- $Q_{vfi} = \begin{bmatrix} \cos(\beta)\cos(\alpha) & \sin(\alpha) & \cos(\beta)\sin(\alpha) \\ \sin(\beta)\cos(\alpha) & \cos(\beta) & -\sin(\beta)\sin(\alpha) \\ -\sin(\alpha) & 0 & \cos(\alpha) \end{bmatrix}$
- $\mathbf{v}_{enz} = Q_{enz}\mathbf{v}_{vfi}$
- $\mathbf{v}_{eci} = Q_{Xx}Q_{enz}Q_{vfi}\mathbf{v}_{vfr}$
- $\beta_{eff} = \sin^{-1}(\mathbf{v}_{vfr}\hat{\mathbf{y}})$
- $\alpha_{eff} = \sin^{-1}(\mathbf{v}_{vfr}\hat{\mathbf{z}})$



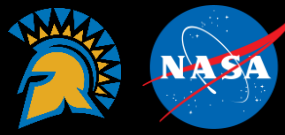
$$[Q]_{Xx} = \begin{bmatrix} -\sin \theta & \cos \theta & 0 \\ -\sin \phi \cos \theta & -\sin \phi \sin \theta & \cos \phi \\ \cos \phi \cos \theta & \cos \phi \sin \theta & \sin \phi \end{bmatrix}$$

Figure 4.16 ENZ and ECI Reference Frames [22]

- Planetocentric vs. Planetodetic is important and must be considered in modeling!
- Geocentric is planetocentric referring to “Earth”, often used incorrectly.



Reference Frames Overview



- $$\mathbf{v}_{rel} = \underbrace{\mathbf{V}_{eci} - \begin{bmatrix} 0 \\ 0 \\ \omega \end{bmatrix}}_{\text{Coriolis Effect}} \times \mathbf{R}_{eci} - \underbrace{Q_{Xx} \begin{bmatrix} -eww \\ -nsw \\ vw \end{bmatrix}}_{\text{Winds}}$$
- $$\mathbf{P}_{vfr} = \frac{1}{2} \rho_{\infty} |\mathbf{v}_{rel}|^2 \begin{bmatrix} -C_D \\ C_Z \\ C_L \end{bmatrix}$$
- $$\mathbf{P}_{eci} = Q_{Xx} Q_{enz} Q_{vfi} \mathbf{P}_{vfr}$$
- $$\ddot{\mathbf{r}} = \frac{\mu}{r^3} \mathbf{r} + \mathbf{P}_{eci}$$
- Assume atmosphere rotates with planet
- vfr : velocity frame relative, vfi : velocity frame inertial. This is also known as the LVLH (Local vertical local horizontal) frame.
- Recall ECI frame is fixed and does not rotate

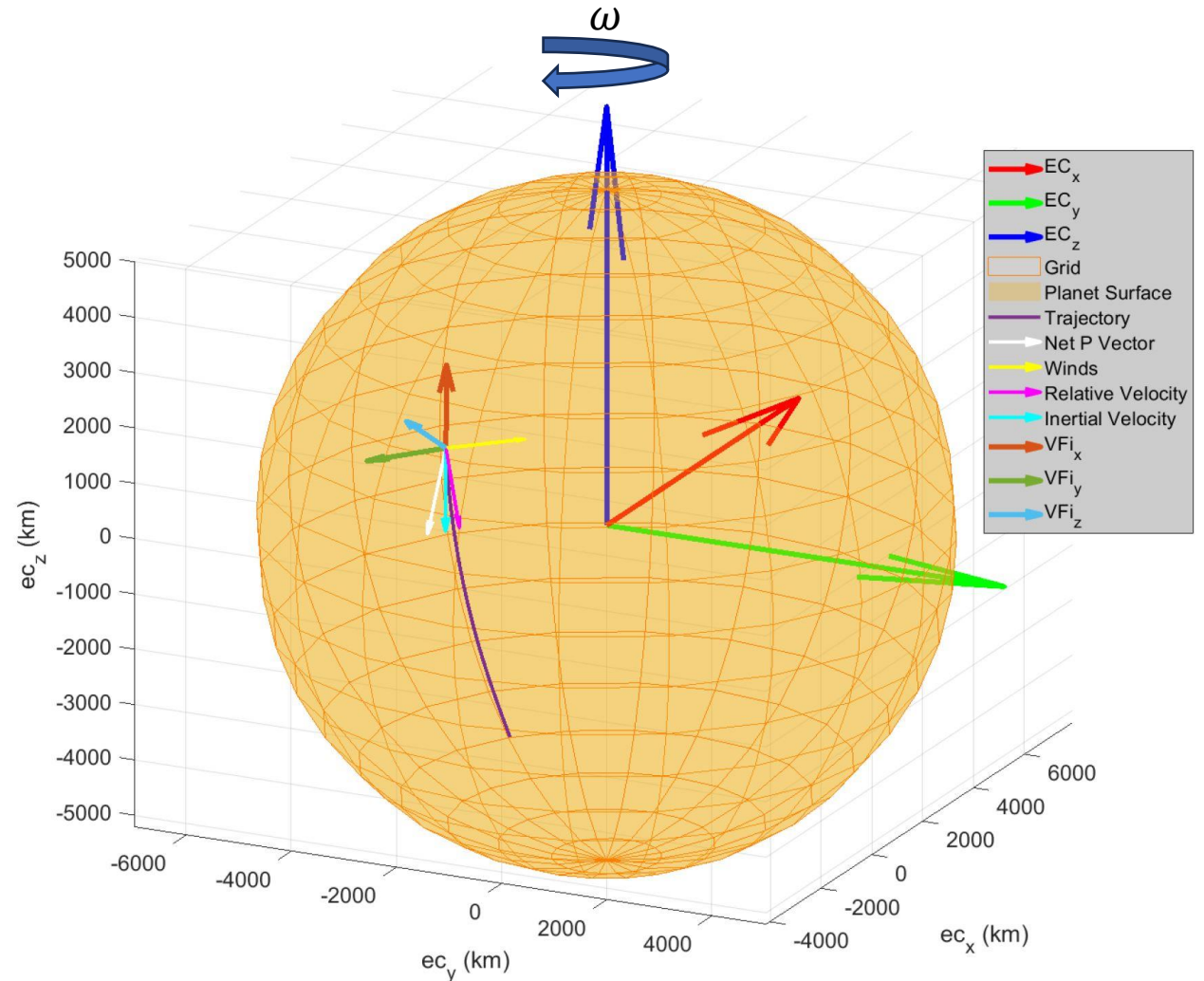
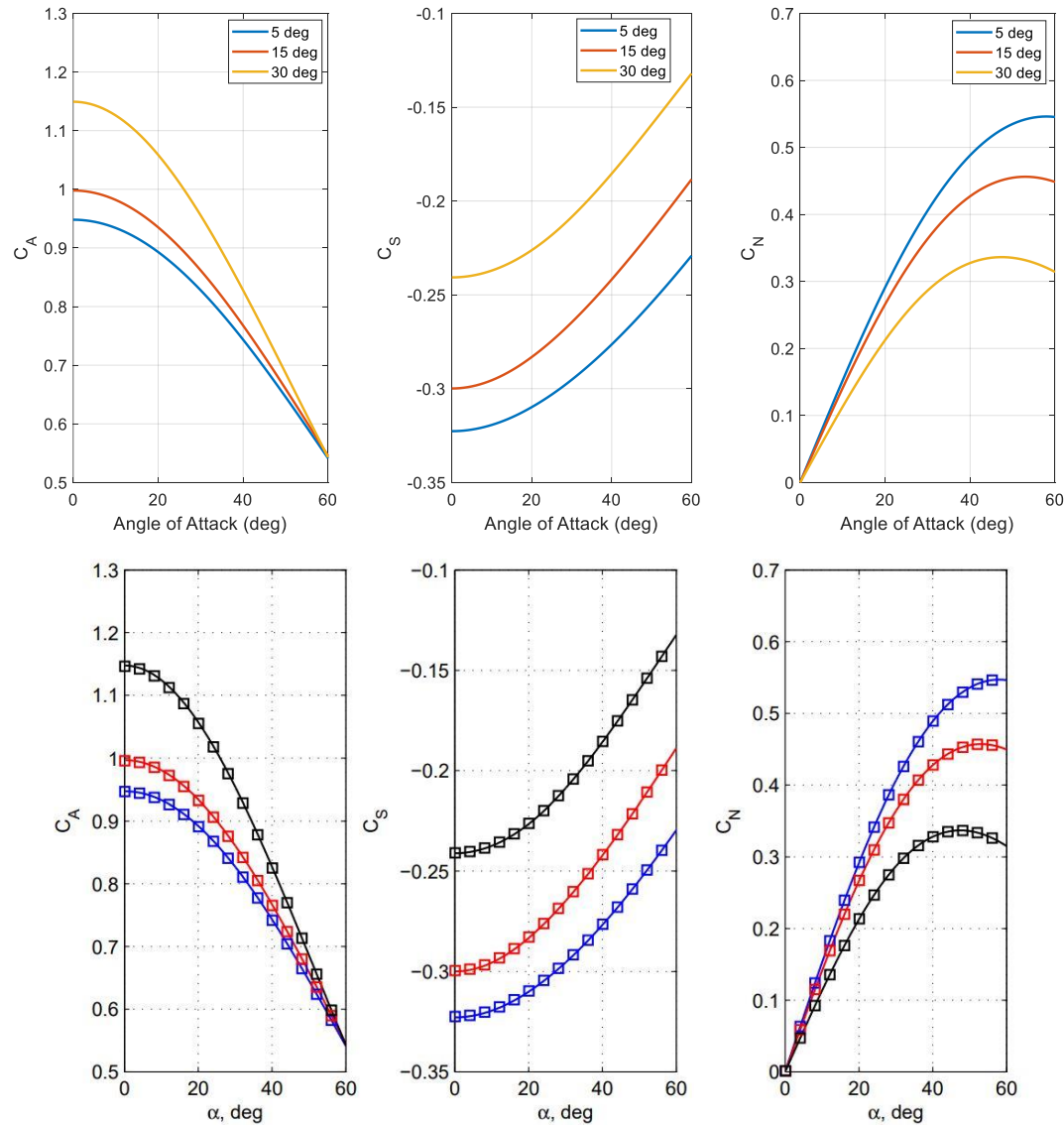
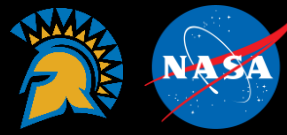


Figure 4.18 Inertial Velocity Frame and Forces on Entry Vehicle

Exaggerated, non-physical conditions to improve vector illustration



Validations of Newtonian Panel Method



Grant, M., and Braun, R., "Analytic hypersonic aerodynamics for conceptual design of entry vehicles," 48th AIAA Aerospace Sciences Meeting Including the New Horizons Forum and Aerospace Exposition, Jan. 4-7, 2010, Orlando, FL. <https://doi.org/10.2514/6.2010-1212>

Figure 10. Spherical Segment Force Coefficient Validation, $\beta = 20^\circ$.



Validations of Free Molecular Panel Method

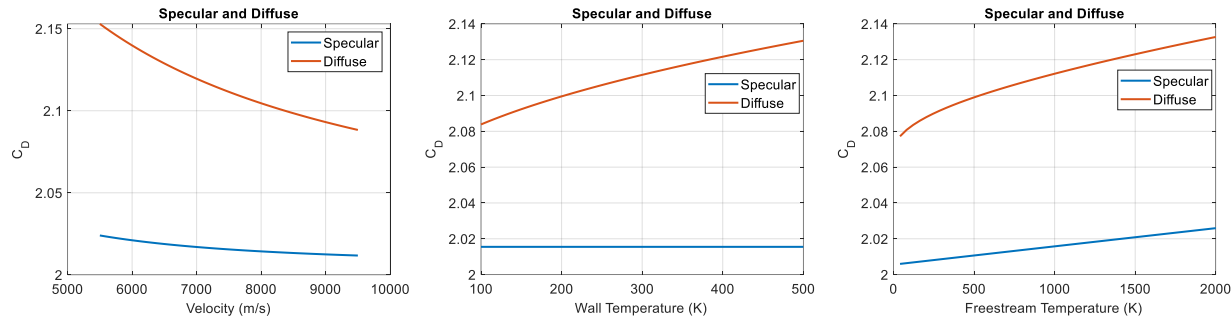
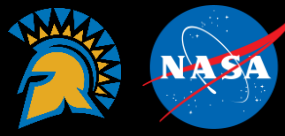


Figure 9.11 Sphere Drag Coefficient Validation Results

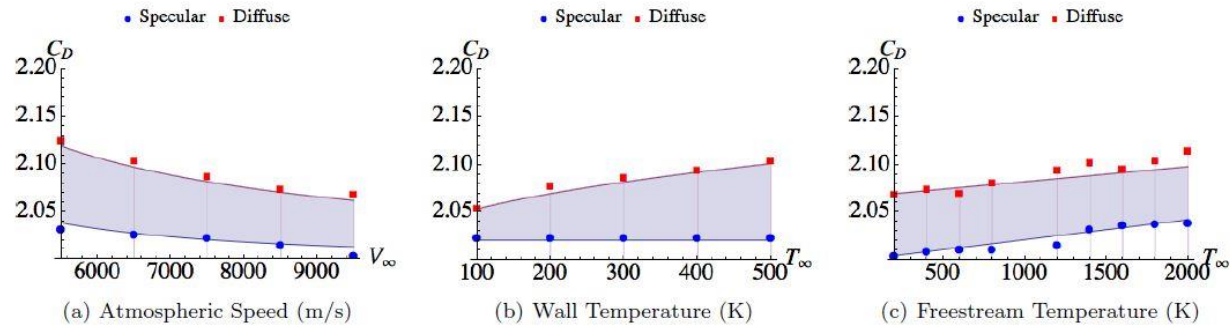


Figure 9.10 Sphere Drag Coefficient Results from [32]

- Kenneth A. Hart, Kyle R. Simonis, Bradley A. Steinfeldt, and Robert D. Braun. "Analytic Free-Molecular Aerodynamics for Rapid Propagation of Resident Space Objects," Journal of Spacecraft and Rockets 2018 55:1, 27-36 <https://doi.org/10.2514/1.A33606>
- Kenneth A. Hart, Soumyo Dutta, Kyle Simonis, Bradley A. Steinfeldt and Robert D. Braun. "Analytically-derived Aerodynamic Force and Moment Coefficients of Resident Space Objects in Free-Molecular Flow," AIAA 2014-0728. AIAA Atmospheric Flight Mechanics Conference, January 13-17, National Harbor, Maryland, USA 2014. <https://doi.org/10.2514/6.2014-0728>

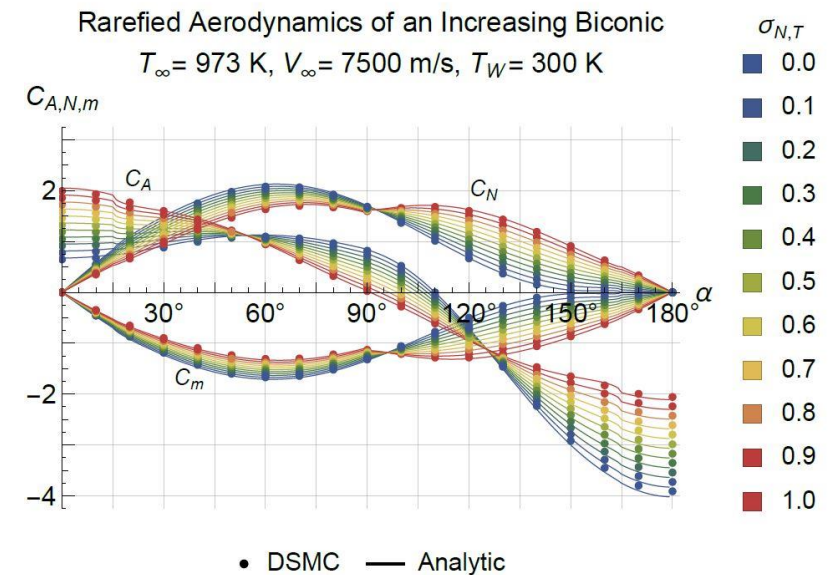
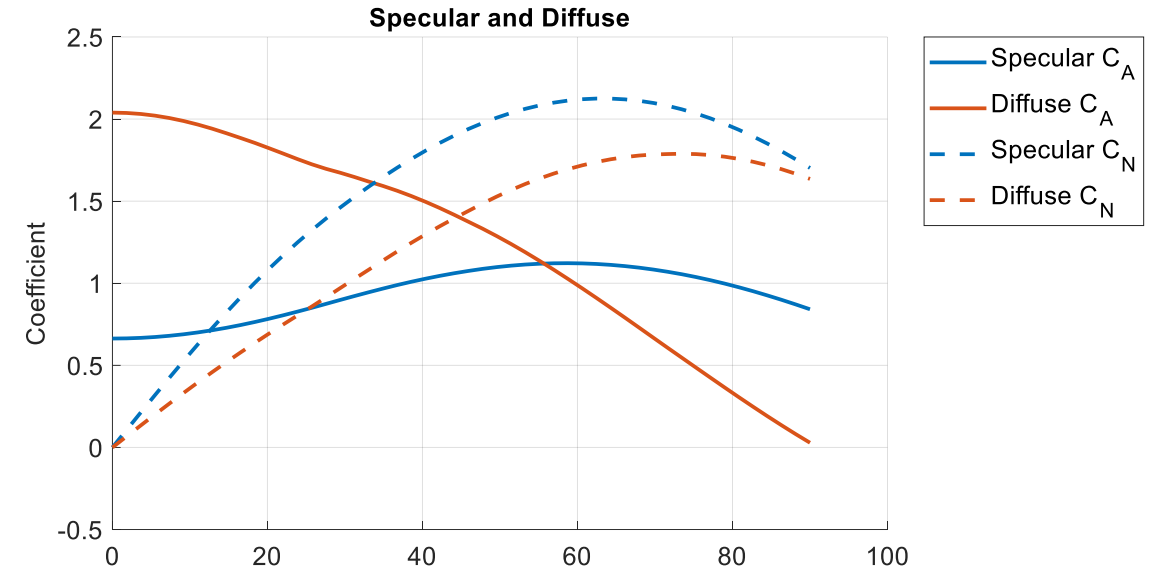


Figure 9.14 Increasing Bi-conic Results from [33]



Validations of Free Molecular Panel Method

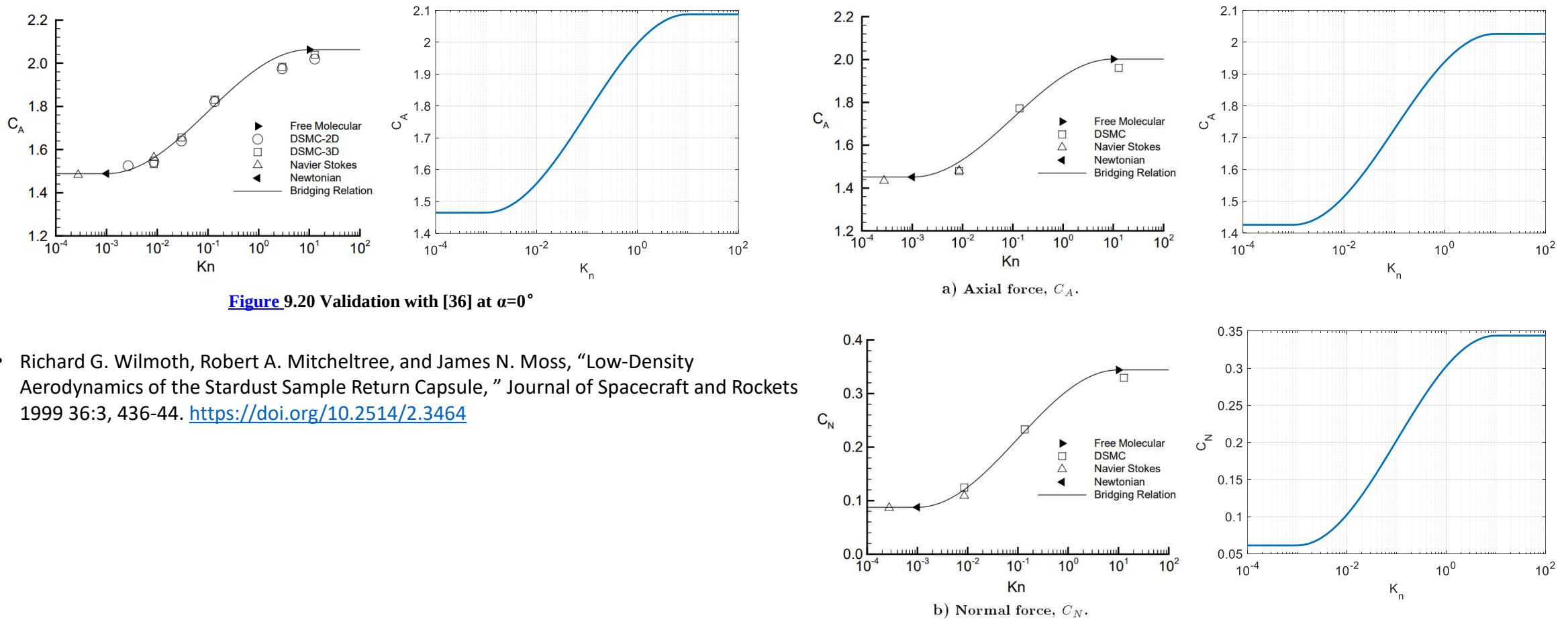
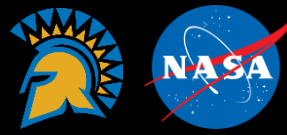


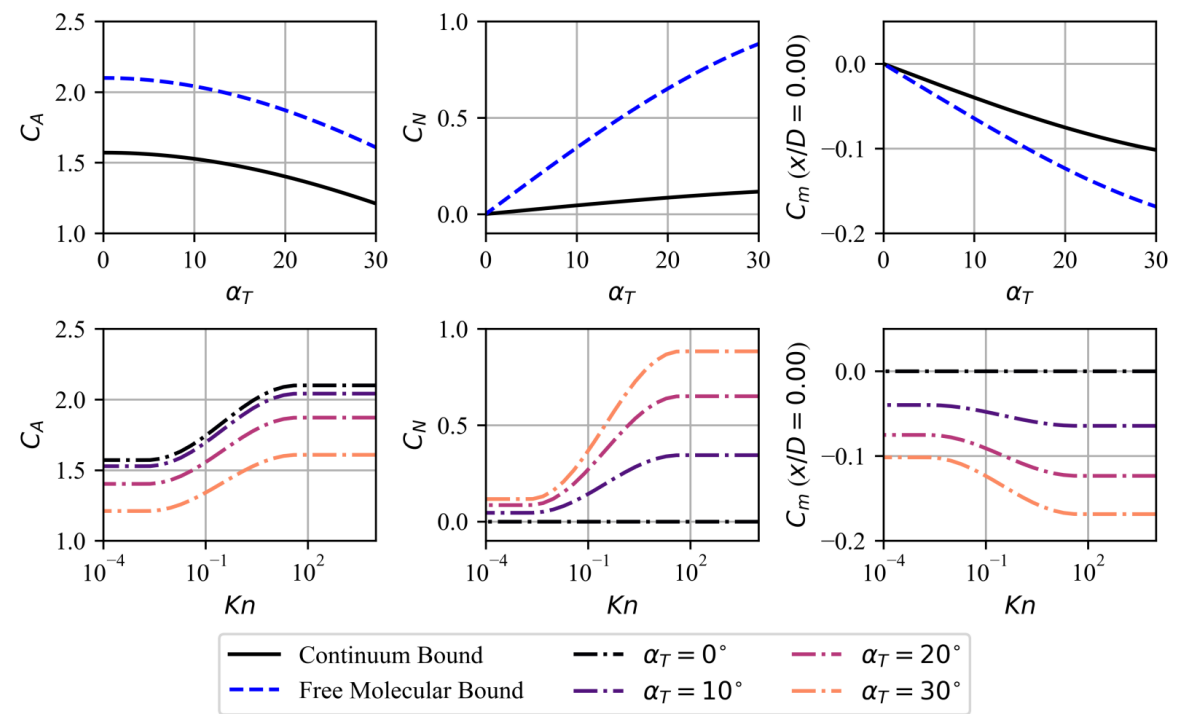
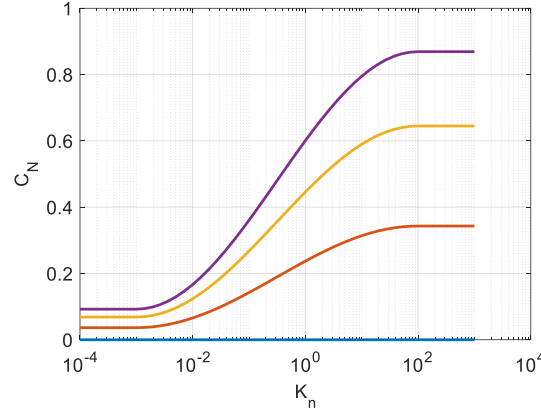
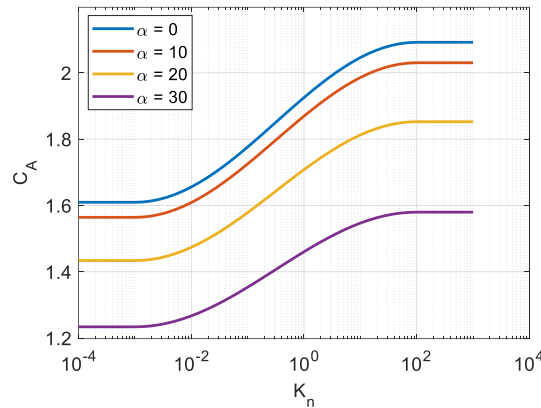
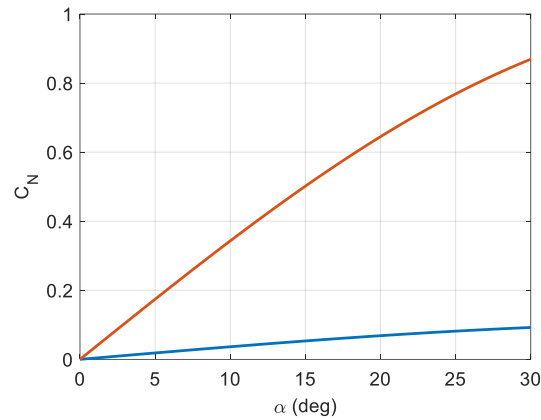
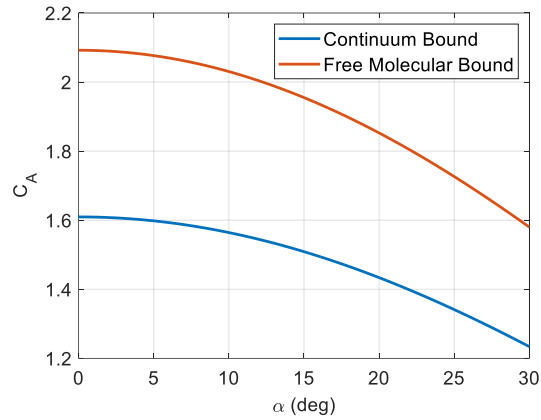
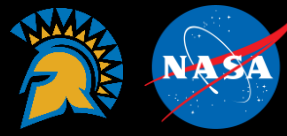
Figure 9.20 Validation with [36] at $\alpha=0^\circ$

- Richard G. Wilmoth, Robert A. Mitcheltree, and James N. Moss, "Low-Density Aerodynamics of the Stardust Sample Return Capsule," Journal of Spacecraft and Rockets 1999 36:3, 436-44. <https://doi.org/10.2514/2.3464>

Figure 9.19 Validation with [36] at $\alpha=10^\circ$



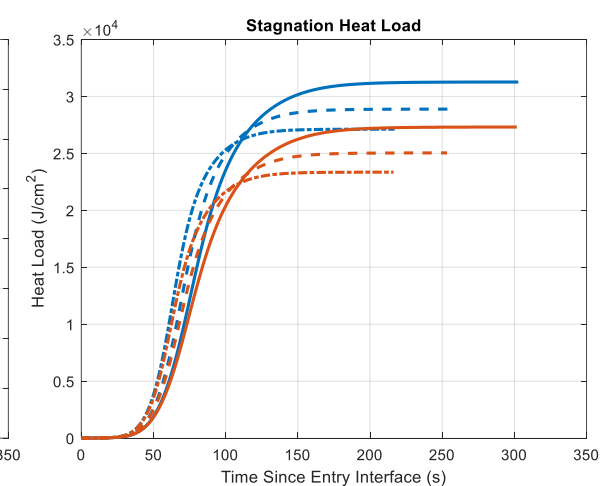
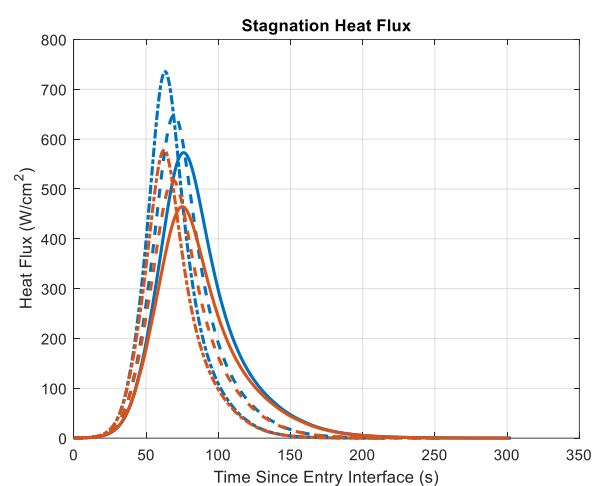
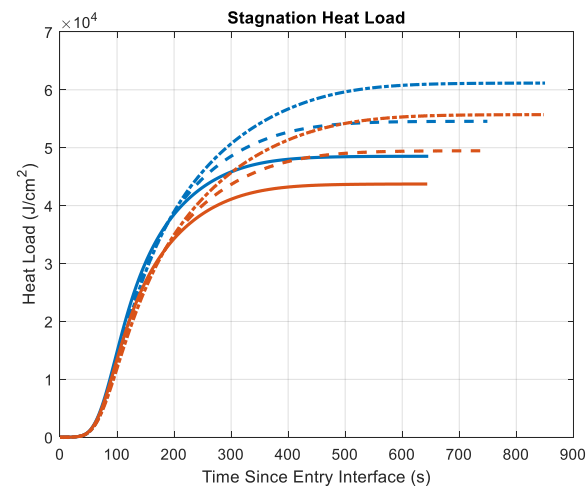
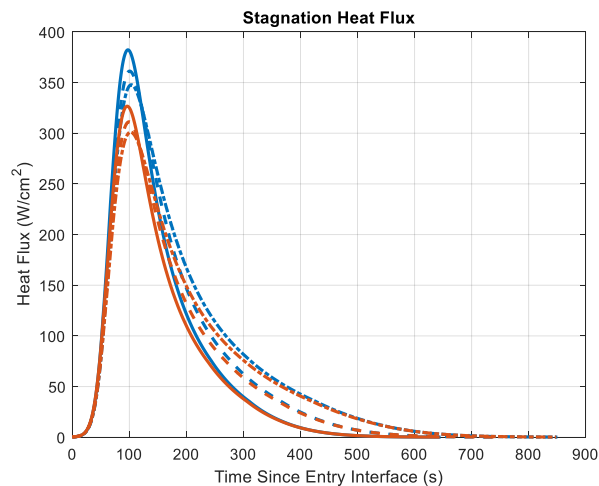
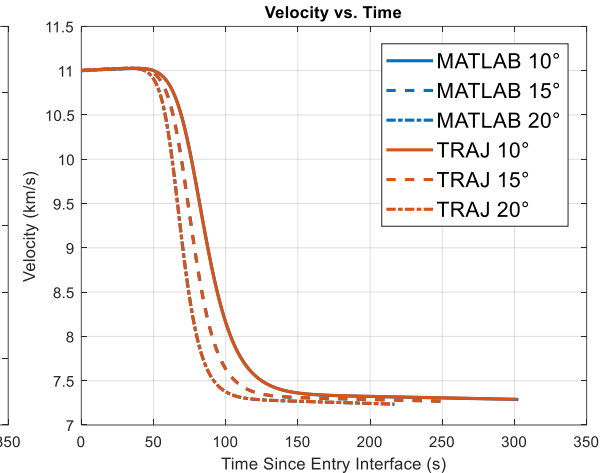
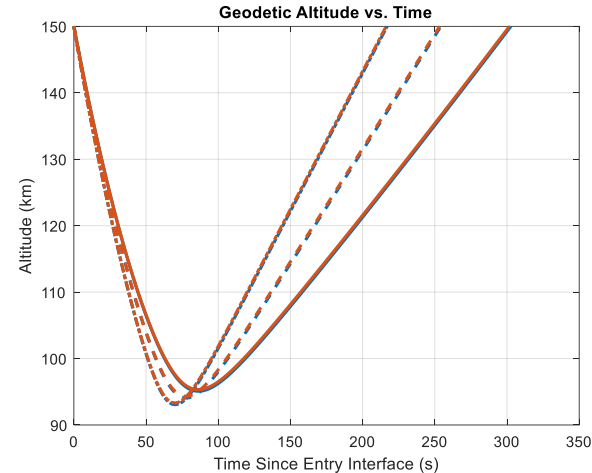
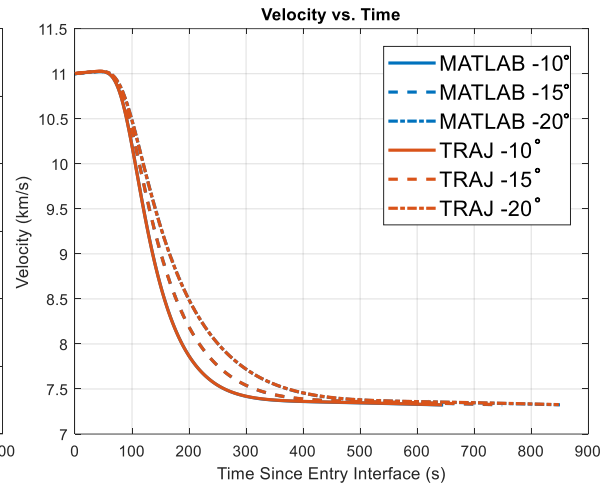
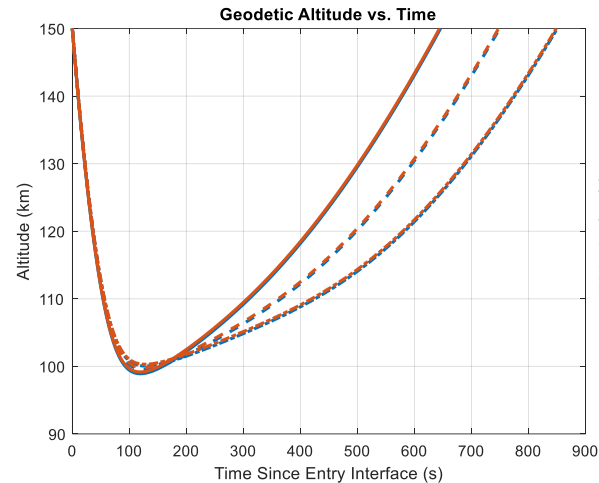
Validations of Free Molecular Panel Method



- Shellabarger, E., Scoggins, J. B., Hinkle, A. D., Dutta, S., Deshmukh, R., Patel, M., and Agam, S., "Aerodynamic Implications of Aerocapture Systems for Uranus Orbiters," AIAA SCITECH 2024 Forum, Jan. 8-12, 2024, Orlando, FL. <https://doi.org/10.2514/6.2024-0950>

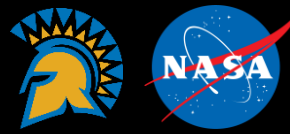


Trajectory Validations with TRAJ





Trajectory Validations with TRAJ



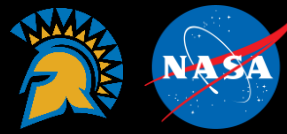
Aerocapture Trajectory at Venus	
Velocity	11 km/s
Azimuth/Heading	-90°
Longitude	305.61606°
Latitude	5.170641°
Julian Date	2456755
Mass	150 kg
Cone Half Angle	60°
Diameter	1 m
Nose Radius	0.25 m
Target Apoapsis	500 km

Results				
Lift Down	Resulting EFPA (deg)	Peak Convective Heat Flux (W/cm^2)	Peak Radiative Heat Flux (W/cm^2)	Total Heat Load (Conv. J/cm^2)
Lift Up				
MATLAB (-10°)	-5.424	382.25	-	48521.66
TRAJ (-10°)	-5.410	326.70	11.16	43108.42
MATLAB (-15°)	-5.368	361.27	-	54560.16
TRAJ (-15°)	-5.354	302.84	9.320	48836.38
MATLAB (-20°)	-5.329	347.80	-	61161.99
TRAJ (-20°)	-5.315	301.17	9.590	55053.46
MATLAB (10°)	-5.976	572.89	-	31264.45
TRAJ (10°)	-5.962	464.67	24.49	26578.55
MATLAB (15°)	-6.248	648.38	-	28883.92
TRAJ (15°)	-6.234	518.04	32.14	24242.16
MATLAB (20°)	-6.592	735.47	-	27123.57
TRAJ (20°)	-6.576	578.46	66.14	22462.20

- Shellabarger, E., Scoggins, J. B., Hinkle, A. D., Dutta, S., Deshmukh, R., Patel, M., and Agam, S., "Aerodynamic Implications of Aerocapture Systems for Uranus Orbiters," AIAA SCITECH 2024 Forum, Jan. 8-12, 2024, Orlando, FL. <https://doi.org/10.2514/6.2024-0950>

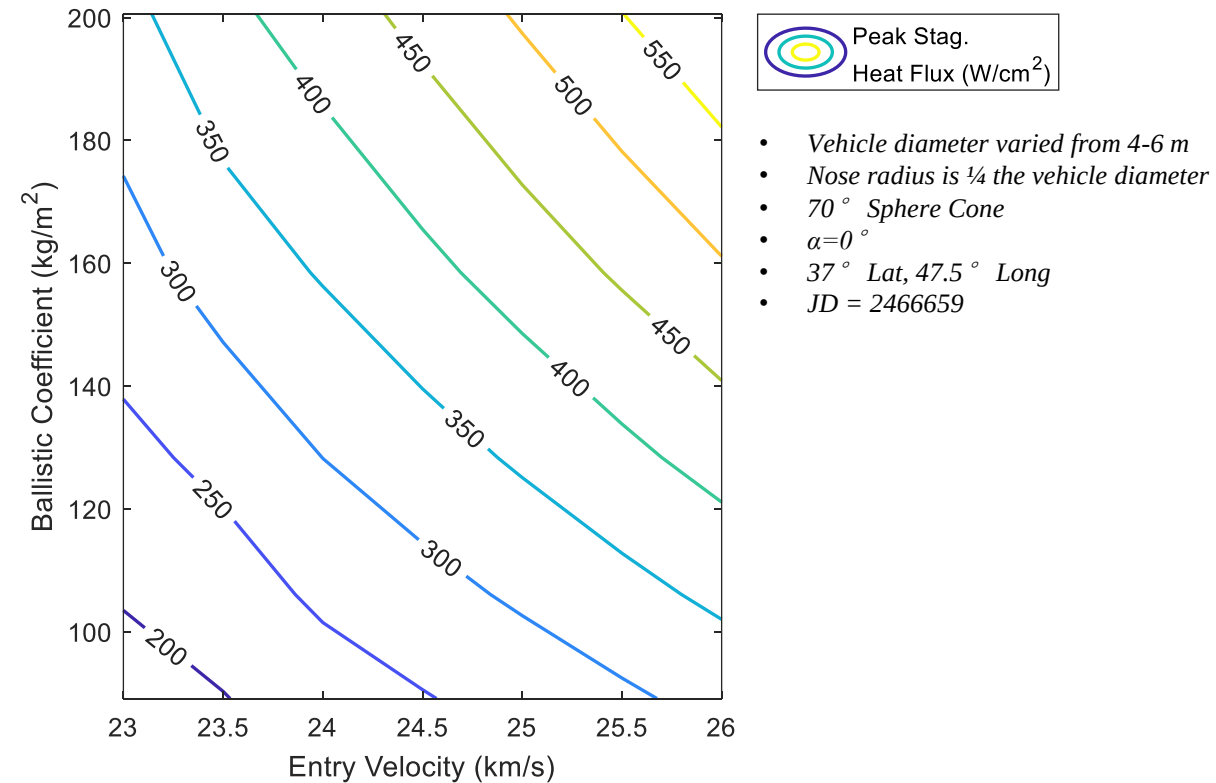


Uranus Aerocapture Analysis



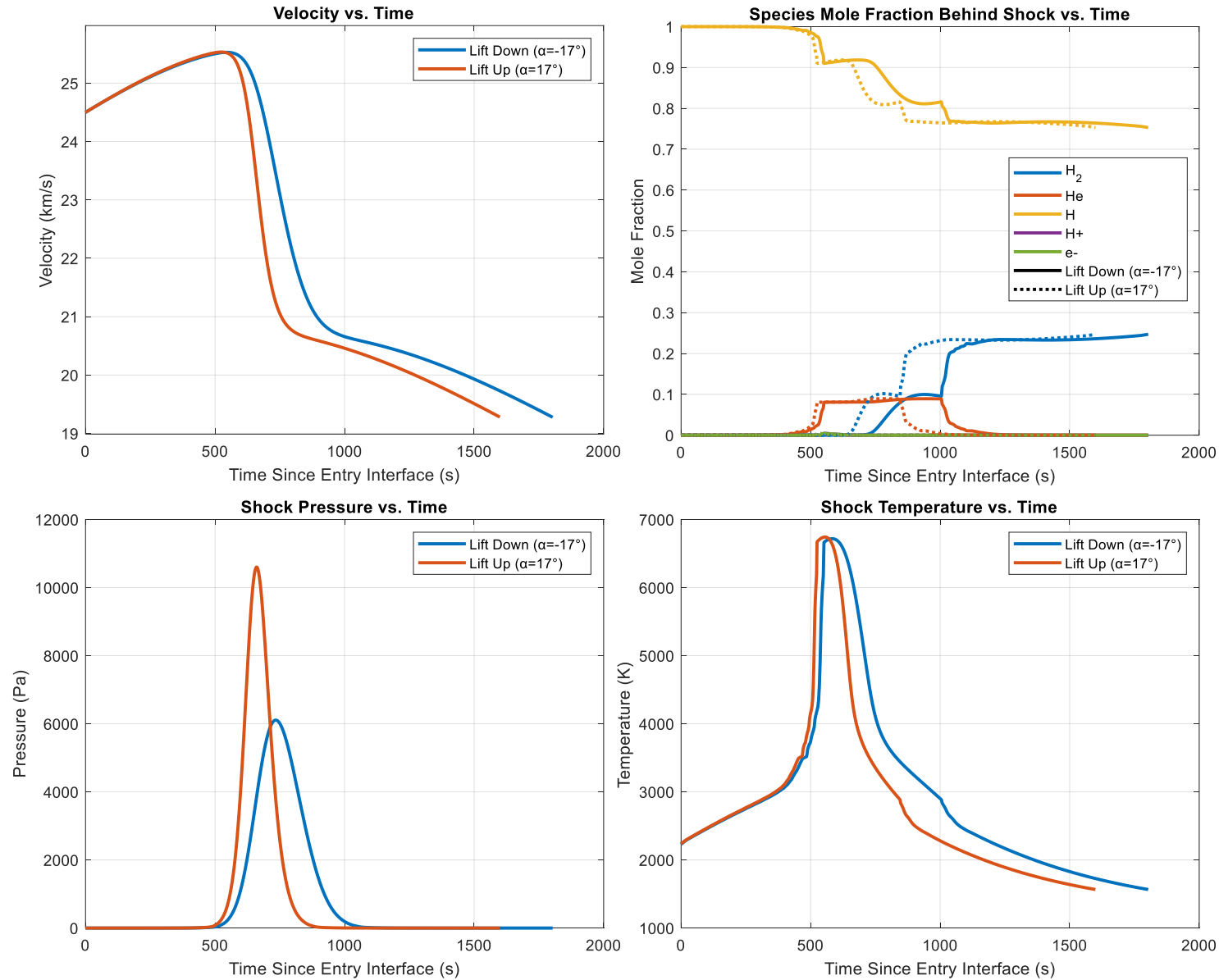
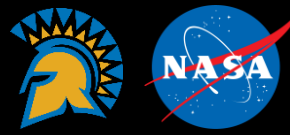
Spacecraft Input Parameters		Entry State	
Initial Mass	4064 kg	Inertial Velocity	24.5 km/s
Trim Angle of Attack (α)	17°/-17°	Lift up EFPA	-23.806°
C_D at $\alpha=17^\circ/-17^\circ$	1.436	Lift down EFPA	-23.509°
C_L at $\alpha=17^\circ/-17^\circ$	± 0.378	Planetocentric Altitude	4000 km
Diameter	5 m	Longitude (deg)	47.5°
Nose Radius	1.25 m	Azimuth (deg)	-90°
Sphere Cone Angle	70°	Planetocentric Latitude	37°
B_C at $\alpha=0^\circ$	128.4 kg/m ²	Target Apoapsis Altitude	500000 km
B_C at $\alpha=17^\circ/-17^\circ$	144.1 kg/m ²	Atmosphere Cutoff	4000 km

$\alpha=17^\circ$, alt = 310 km, Mach=26.93	Calorically Perfect ($\gamma=1.45$)	Chemical Equilibrium (H_2/He)
C_A	1.4836	1.5840
C_N	0.0587	0.0627
C_D	1.4360	1.5331
C_L	0.3776	0.4032



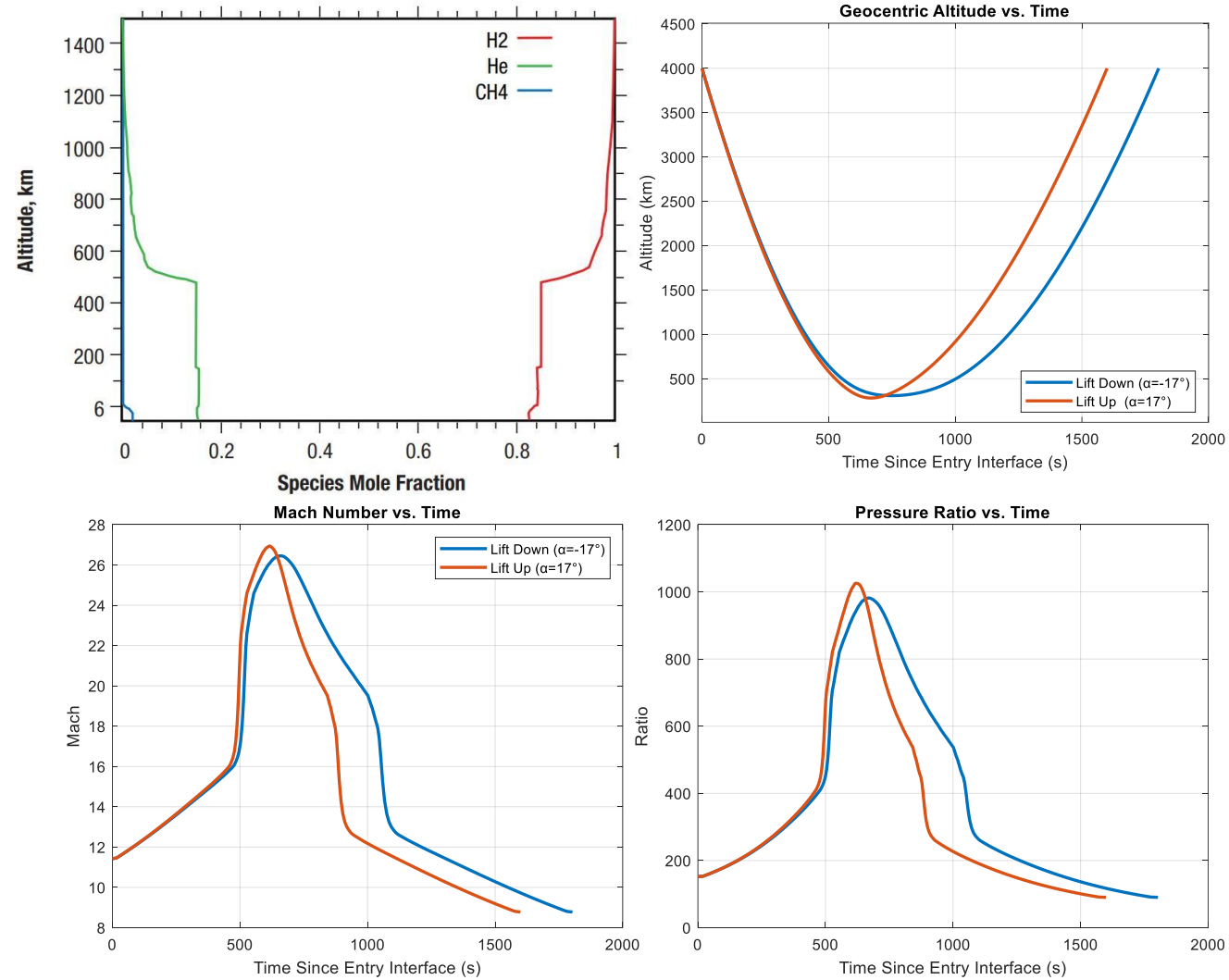
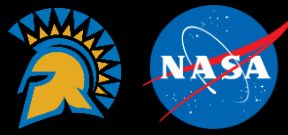


Uranus Aerocapture Analysis



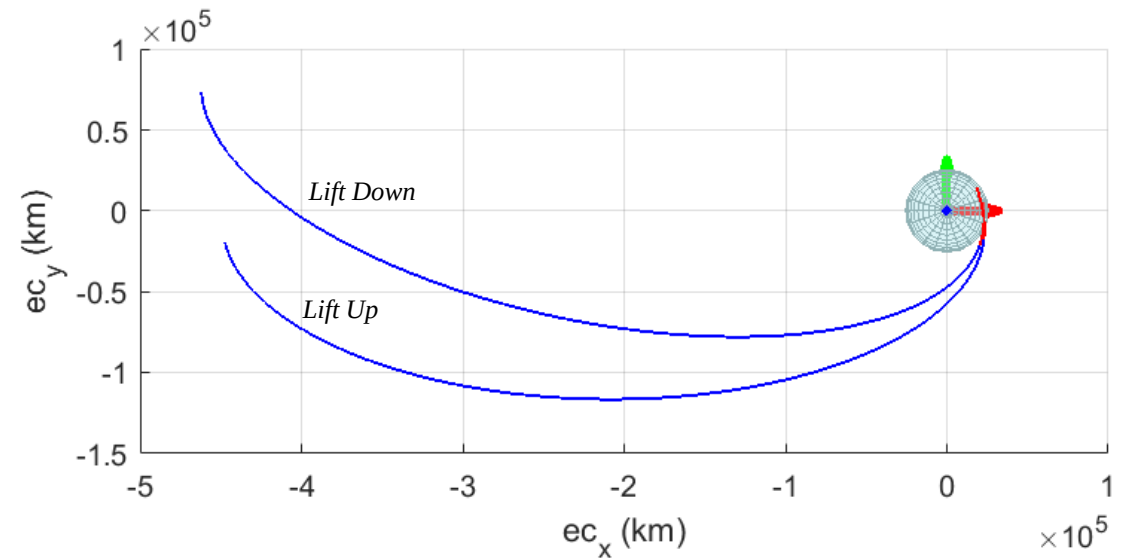
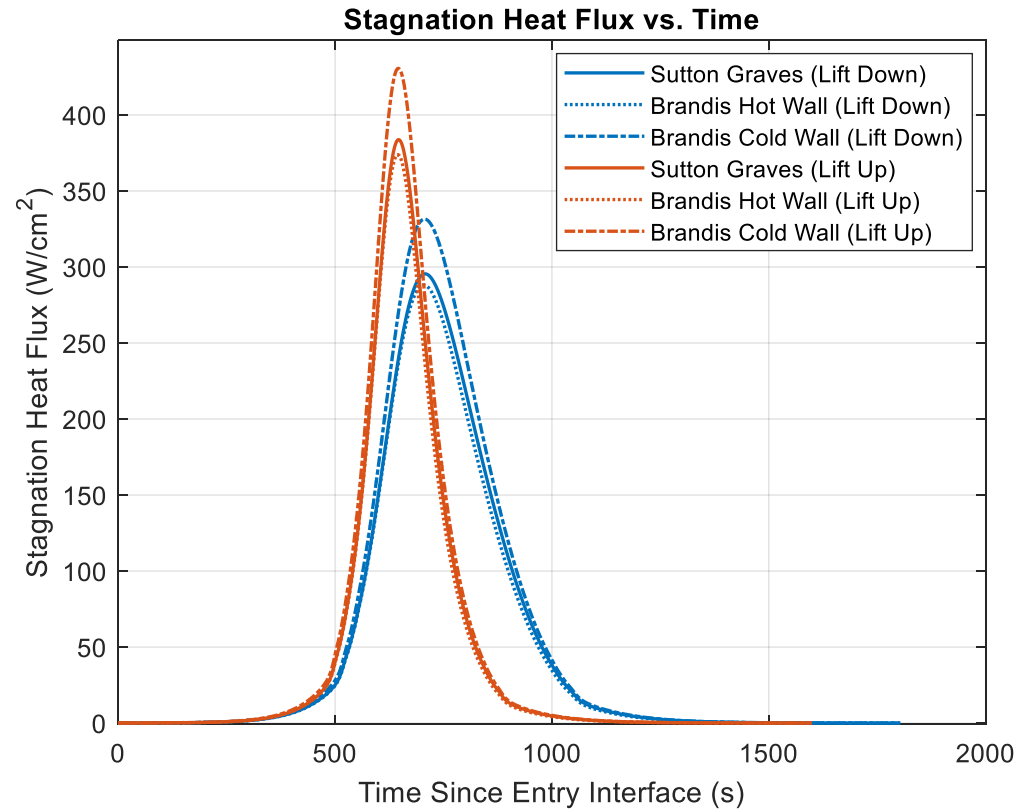
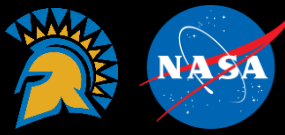


Uranus Aerocapture Analysis



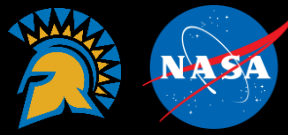


Uranus Aerocapture Analysis





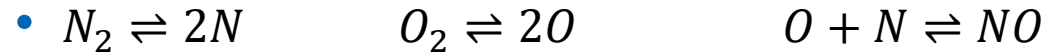
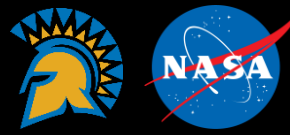
CEA Thermodynamic Lookups



- $\frac{c_P^o(T)}{\mathcal{R}} = a_1 T^{-2} + a_2 T^{-1} + a_3 + a_4 T + a_5 T^2 + a_6 T^3 + a_7 T^4$
- $\frac{H_T^o(T)}{\mathcal{R}T} = -a_1 T^{-2} + \frac{a_2 \ln T}{T} + a_3 + \frac{a_4}{2} T + \frac{a_5}{3} T^2 + \frac{a_6}{4} T^3 + \frac{a_7}{5} T^4 + \frac{b_1}{T}$
- $\frac{S_T^o(T)}{\mathcal{R}} = -\frac{a_1 T^{-2}}{2} - a_2 T^{-1} + a_3 \ln T + a_4 T + \frac{a_5}{2} T^2 + \frac{a_6}{3} T^3 + \frac{a_7}{4} T^4 + b_2$
- McBride, B. J., Zehe, M. J., and Gordon, S., “NASA Glenn coefficients for calculating thermodynamic properties of individual species,” NASA/TP—2002-211556, John H. Glenn Research Center at Lewis Field, 2002.



Chemistry Symbolic Math Overview (N2/O2)



- $K_{p1} = \frac{P_N^2}{P_{N_2}}$

- $K_{p2} = \frac{P_O^2}{P_{O_2}}$

- $K_{p3} = \frac{P_N P_O}{P_{NO}}$

- $p_t = P_{N_2} + P_{O_2} + P_N + P_O + P_{NO}$

- $MR = \frac{2P_{N_2} + P_N + P_{NO}}{2P_{O_2} + P_O + P_{NO}}$

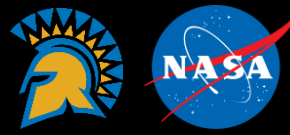
- $P_{N_2} = f(K_{p1}, P_N)$

- $P_{O_2} = f(K_{p2}, P_O)$

- $P_{NO} = f(K_{p3}, P_N, P_O)$



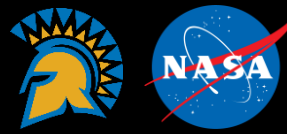
Chemistry Symbolic Math Overview (N2/O2)



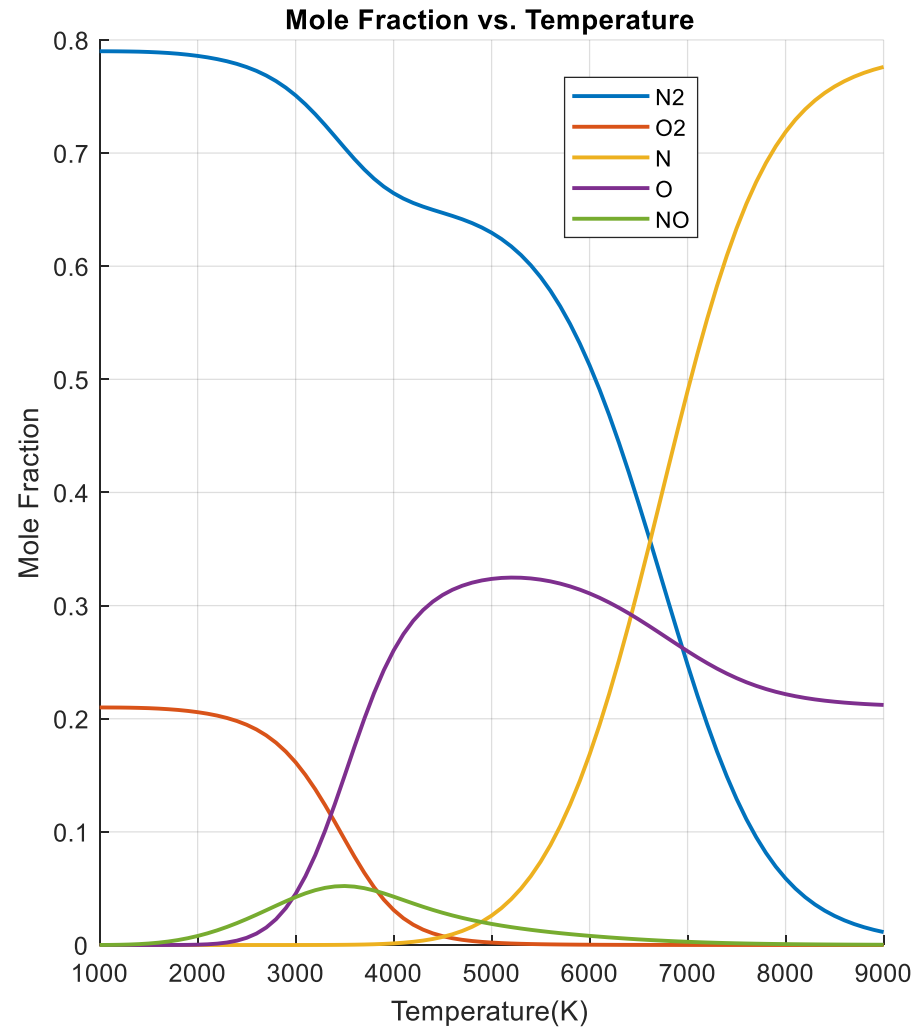
- $P_t = f(K_{p1}, K_{p2}, K_{p3}, P_N, P_O)$
- $P_O = f(K_{p1}, K_{p2}, K_{p3}, P_N, P_t)$
- $f_{P_N} = f(K_{p1}, K_{p2}, K_{p3}, P_N, P_t, MR)$ (solve iteratively for P_N)
- $\sum_{i=1}^{NG} X_i a_{ei} = 0$ (additional constraint for ionization)



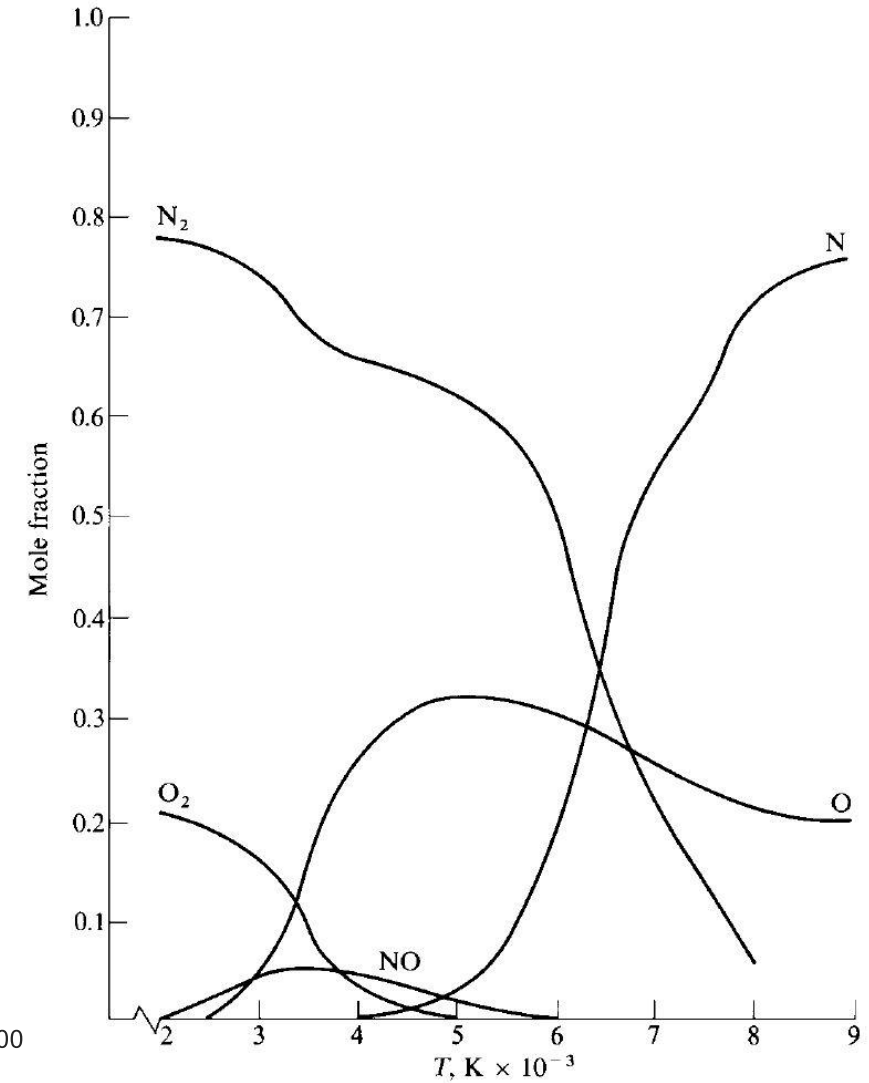
Chemistry Model Validation (N₂/O₂)



- Anderson, John D. Hypersonic and High Temperature Gas Dynamics. 2nd ed., American Institute of Aeronautics and Astronautics, 1801 Alexander Bell Drive, Reston, VA. 2006.

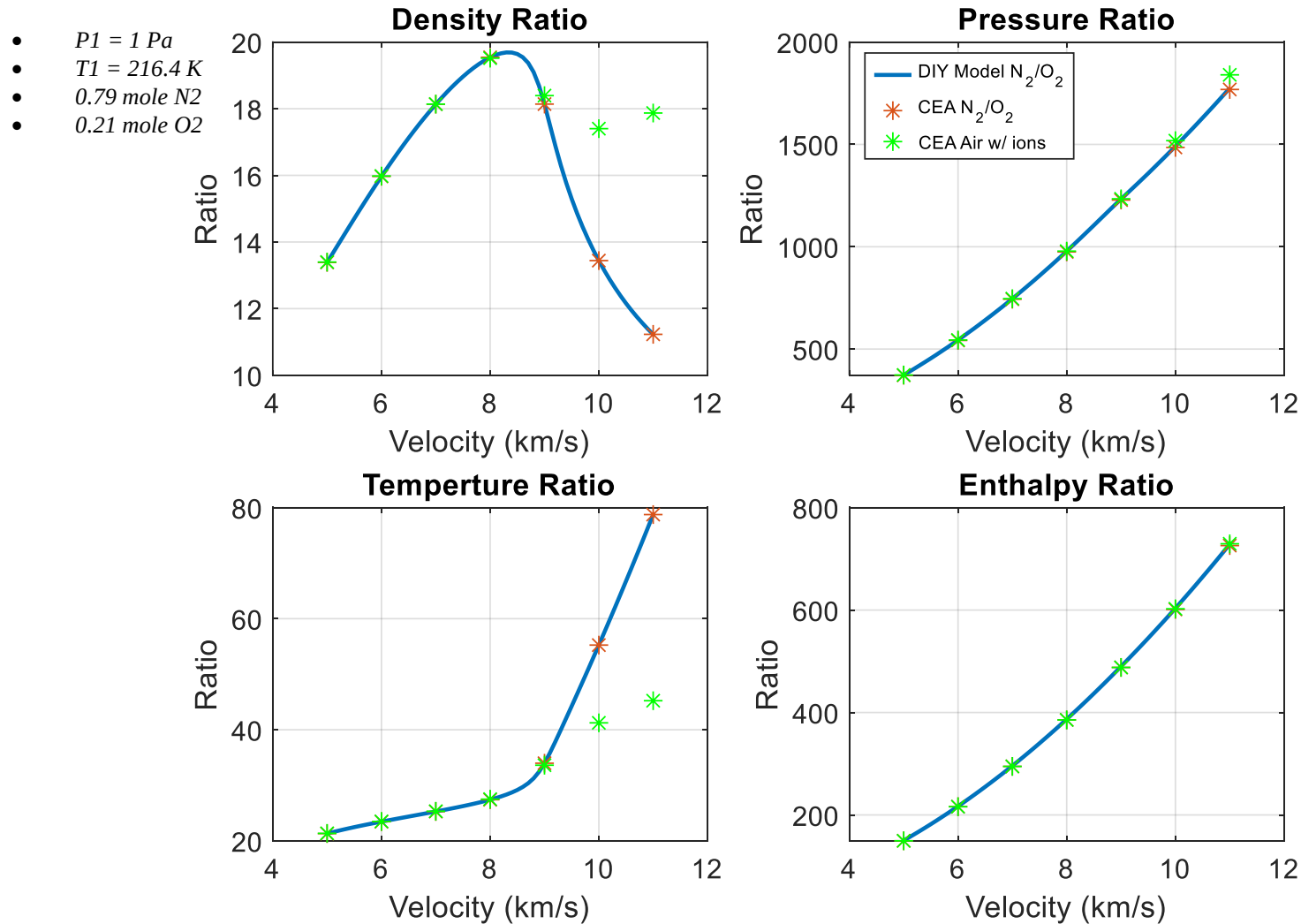
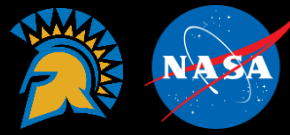


Conditions at 1 atm, Textbook: Fig 11.12, P.541)





Chemistry Model Validation with CEA



Conditions at 1 atm, Textbook: Fig 11.12, P.541)