

Wall-Cooling Effects on Hypersonic Boundary-Layer Instabilities and Transition

Nolan Little*, Mateus Schuabb[†], and Lian Duan[‡]
The Ohio State University, Columbus, Ohio, 43235, USA

Nicholas Bisek[§]
Air Force Research Laboratory, Wright-Patterson AFB, Ohio, 45433, USA

Anton Scholten[¶]
National Institute of Aerospace, Hampton, VA 23666, USA

Pedro Paredes^{||}
NASA's Langley Research Center, Hampton, VA 23681, USA

Linear stability analysis (LSA) and direct numerical simulations (DNS) with random inflow forcing are used to study the effects of wall cooling on boundary-layer instabilities and transition (BLT) over a sharp 7-degree half-angle cone at zero-degrees angle of attack in a Mach 6 flow representative of the Air Force Research Laboratory's Mach-6 Ludwig Tube. The study focuses on cones with two different isothermal wall temperatures: an uncooled warm-wall case with $T_w = 298$ K and a cooled cold-wall case with $T_w = 84$ K. For both wall temperatures, LSA based on the parabolized stability equations identifies Mack's second mode as the dominant instability mechanism. The planar Mack's second-mode waves are dominant for the cold-wall case, but the oblique Mack's second-mode waves are more amplified than the planar ones for a large extent of the warm-wall cone. At the measured transition location of $x = 0.2$ m, the most amplified waves have a disturbance azimuthal wavenumber of $m = 6$, which is used to define the azimuthal domain for the DNS to compute BLT in the warm-wall case. Complementary DNS with a sufficiently large azimuthal domain confirms the amplification of the oblique waves for the warm-wall case. The DNS of the cold-wall case validates many of the predictions made by linear PSE. The second-mode frequencies are higher due to the smaller boundary layer that forms over the cooled cone. The cold-wall DNS shows a downstream shift in transition location along with diminished oblique modes and a more dominant second Mack-mode instability. The study finds that with careful tuning of the forcing amplitude, a random inflow forcing method can produce very similar spectral content to the LSA predictions and the expected second-mode dominant transition at a location similar to that of a noisy wind tunnel experiment.

Nomenclature

C_f	Skin-friction coefficient
C_p	Specific heat at constant pressure [J/(kg·K)]
ds	Wall-parallel grid spacing [m]
dz_n	Wall-normal grid spacing [m]
δ	Boundary-layer height based on 99% recovery of u/u_e [m]
f	Frequency [Hz]
k	Turbulent kinetic energy [J/kg]
L	Cone length [m]

*Graduate Student, Department of Mechanical and Aerospace Engineering, Student Member, AIAA

[†]Graduate Student, Department of Mechanical and Aerospace Engineering, Student Member, AIAA

[‡]Professor, Department of Mechanical and Aerospace Engineering, Associate Fellow, AIAA

[§]Senior Research Aerospace Engineer, AFRL/RQHA, Associate Fellow, AIAA

[¶]Research Engineer, Member, AIAA

^{||}Aerospace Technologist, Computational AeroSciences Branch, Associate Fellow, AIAA

m	Azimuthal wavenumber
M	Mach number, $M \equiv u/a$
N_E	N -factor based on total disturbance energy
μ	Coefficient of dynamic viscosity [kg/(m·s)]
P	Pressure [Pa]
Pr	Prandtl number
Re_∞	Freestream unit Reynolds number, $Re_\infty \equiv \rho_\infty u_\infty / \mu_\infty$ [m ⁻¹]
q	Dynamic pressure, $q = \frac{1}{2} \rho u^2$ [Pa]
q_w	Wall heat flux [W/m ²]
St	Stanton number, $St = q_w / (c_p \rho_\infty u_\infty (T_0 - T_w))$
T	Temperature [K]
T_r	Recovery temperature, $T_r = T_\infty (1 + 0.89 \frac{\gamma-1}{2} M_\infty^2)$ [K]
τ_w	Wall shear stress [Pa]
u	Streamwise velocity [m/s]
u_τ	Friction velocity, $u_\tau \equiv \sqrt{\tau_w / \rho_w}$ [m/s]
x	Streamwise distance [m]
x_i	Inflow forcing location [m]
z	Radial distance [m]
z_τ	Viscous length, $z_\tau = \nu_w / u_\tau$ [m]
κ	Coefficient of thermal conductivity [W/(m·K)]
ρ	Density [kg/m ³]

Subscripts

0	Stagnation
∞	Freestream
$\infty, 2$	Post-shock freestream
n	Normal
r	Recovery
tr	Transition
w	Wall

Superscripts

+	Variable in wall units, $(\cdot) \equiv (\cdot)/z_\tau$
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I. Introduction

Extreme heating and high shear stress are two of the biggest challenges facing the design of hypersonic vehicles. Understanding boundary-layer transition (BLT) is crucial to predicting and controlling these loads. The most dominant method of transition for sharp cones at zero-degrees angle of attack (AoA) has been shown to be the amplification of Mack-mode (or second-mode) instabilities [1], and wall cooling has substantial qualitative and quantitative effects on the development of second-mode instabilities and subsequent transition processes in hypersonic boundary layers [2]. The Hypersonic International Flight Research Experimentation (HIFiRE) program has given an opportunity to compare flight test data to wind-tunnel and computational results. Tests conducted at the CUBRC facility included measurements at a wall-to-stagnation temperature ratio (T_w/T_0) of 0.13, which compares closely to the flight conditions of HIFiRE-5b, where $T_w/T_0 = 0.18$ [3, 4]. Although the effect of wall cooling has long been explored as a means of controlling BLT at hypersonic speeds, previous studies [5–10] have produced varying conclusions about its effect on the location of the boundary-layer transition. Some studies found that wall cooling increases the transition Reynolds number (i.e., the freestream unit Reynolds number multiplied by a characteristic length equal to the location on the model where transition occurs) [5, 6], while others reported no correlation between transition and wall cooling or even a decrease in the transition Reynolds number due to wall cooling [7–10].

In an attempt to explain the discrepancies in the literature on the effect of wall cooling on BLT, a series of experiments were recently performed in the Mach-6 Ludwig Tube (M6LT) of the Air Force Research Laboratory (AFRL) [11–14]. These experiments employed a sharp 7° half-angle cone model with a tip radius of 0.1 mm, a base diameter of 15.11 cm, and a length of 60.96 cm. The cone surface was either kept at room temperature or cooled by continuously flooding

the built-in internal cavity of the model with liquid nitrogen (LN2). Modern flow diagnostic techniques including high-speed schlieren and focused laser differential interferometry (FLDI) were used to determine the boundary-layer transition location and quantify the second-mode disturbance frequency and amplitude as a function of wall temperature. In particular, at a fixed unit Reynolds number of $Re_\infty = 15.0 \times 10^6/\text{m}$, Clingenpeel [14] reported that severe wall cooling with $T_w/T_e = 1.34$ shifts the transition front downstream by approximately 150 mm relative to that of a room-temperature cone with $T_w/T_e = 4.74$. Despite the difference in the transition onset, the fully-cooled and room-temperature cases showed similar second-mode frequencies and amplitudes at comparable locations with respect to transition onset. They hypothesized that the initial second-mode amplitude for the fully-cooled case is orders of magnitude smaller when compared to the room-temperature case, and the mechanism driving the aforementioned change in receptivity, and thereby the initial second-mode amplitude, is tied to a change from an unstable slow-mode branch to the fast-mode branch as a result of severe wall cooling. Further experimental and/or computational work is needed to determine the wall-temperature-dependent receptivity effect on the initial second-mode amplitude and quantify the growth rate and maximum amplitude of the second-mode waves to parse out their hypothesis. To facilitate computational studies of the receptivity process, it is important to account for the effect of tunnel noise by establishing a link between forcing at the inlet and modal disturbance amplification within the boundary layer, considering that almost all the aforementioned experimental observations have been made in conventional (i.e., “noisy”) hypersonic wind tunnels. This will help relate the computational results to experimentally observed phenomena such as reduced receptivity under cold-wall conditions in AFRL’s M6LT.

The current study performs linear stability analysis and direct numerical simulation characterization of the boundary-layer instability and transition process over a Mach 6 sharp-cone model at warm-wall and cold-wall temperature conditions. The objective is to shed light on the effect of wall temperature on the relevant instability modes that led to the transition delay observed in the experiments.

The rest of the paper is structured as follows. Section II introduces the flow conditions and cone geometry. Section III shows the linear stability analysis based on the parabolized stability equations (PSE) for both the warm- and cold-wall cones. Section IV discusses DNS results for the warm-wall and cold-wall configuration, making comparisons to PSE and direct comparisons of wall-cooling effects. A summary of the results and final conclusions is provided in Section V. The Appendix presents a short case study on the effect of a slightly reduced inflow forcing domain.

II. Flow Conditions and Cone Geometry

This study conducts linear stability analysis and DNS of Mach 6 flow over a 0.6-meter-long circular, axisymmetric cone with a half angle of 7 degrees and a 0.1-millimeter nose tip, with the freestream conditions and cone geometry representative of those of the experiments in the AFRL’s M6LT [12, 14]. Similar to the experiments of Clingenpeel [14], the current numerical study focuses on boundary-layer instability and transition over a sharp cone with and without wall cooling. The uncooled cone has a warm-wall temperature of $T_w = 298.0$ K, which corresponds to a wall-to-recovery temperature ratio of $T_w/T_r = 0.72$, while the cooled cone has a cold-wall temperature of $T_w = 84.0$ K, which corresponds to a wall-to-recovery temperature ratio of $T_w/T_r = 0.20$. To isolate the effect of wall cooling, the freestream conditions are fixed for both warm- and cold-wall configurations. A summary of the conditions used is given in Table 1.

Table 1 Freestream and wall-temperature conditions for the warm-wall and cold-wall configurations.

Case	T_w (K)	T_w/T_r	M_∞	Re_∞ ($\times 10^6 \text{ m}^{-1}$)	T_∞ (K)	u_∞ (m/s)	ρ_∞ (kg/m^3)	P_∞ (Pa)
Warm-wall	298.0	0.72	6.1	15.0	54.1	902.4	0.05875	909.6
Cold-wall	84.0	0.20						

III. Linear Stability Analysis

In this section, the PSE methodology is explained and the results from linear stability analysis based on the PSE for both the warm- and cold-wall cones are presented.

A. PSE Methodology

To perform a linear stability analysis, the laminar base flows are first calculated with the VULCAN-CFD solver for both the warm- and cold-wall cones. VULCAN-CFD is a second-order-accurate, finite-volume algorithm for the compressible Navier–Stokes equations [15]. The VULCAN-CFD solution is based on the axisymmetric Navier–Stokes equations and uses the solver’s built-in capability to iteratively adapt the computational grid to the bow shock and boundary-layer edge [16]. A grid of 2201×826 points (streamwise and wall-normal directions, respectively) is used for both configurations. This grid resolution provides converged stability results as demonstrated in previous work on a similar geometry [17]. Sutherland’s law for air is used to calculate the dynamic viscosity as a function of temperature, and the Prandtl number was set to be $Pr = 0.72$.

The base flow solutions are then used with the PSE to study the modal amplification of convectively unstable boundary-layer waves. The PSE computes the linear amplification of disturbances using quasi-parallel Linear Stability Theory (LST) results at the neutral location as the initial condition.

For this problem, an orthogonal, body-fitted coordinate system (ξ, η, ζ) is employed, representing the streamwise, wall-normal, and azimuthal directions, respectively. The corresponding velocity components are (u, v, w) , and density and temperature are denoted by ρ and T . The metric factors for the coordinate transformation are defined as

$$h_\xi = 1 + \kappa\eta, \quad (1)$$

$$h_\zeta = r_b + \eta \cos(\theta), \quad (2)$$

where κ denotes the streamwise curvature, r_b is the local radius, and θ is the local half-angle along the axisymmetric surface, i.e., $\sin(\theta) = dr_b/d\xi$. The Cartesian coordinates are represented by (x, y, z) . The perturbation variable vector is denoted by $\tilde{\mathbf{q}}(\xi, \eta, \zeta, t) = (\tilde{\rho}, \tilde{u}, \tilde{v}, \tilde{w}, \tilde{T})^T$, and the vector of disturbance functions is $\check{\mathbf{q}}(\xi, \eta, \zeta) = (\check{\rho}, \check{u}, \check{v}, \check{w}, \check{T})^T$. The vector of basic state variables is $\hat{\mathbf{q}}(\xi, \eta, \zeta) = (\hat{\rho}, \hat{u}, \hat{v}, \hat{w}, \hat{T})^T$. For axisymmetric geometries at zero degrees angle of attack, the basic state variables are independent of the azimuthal coordinate, and the linear perturbations can be assumed to be harmonic in time and azimuthal direction

$$\tilde{\mathbf{q}}(\xi, \eta, \zeta, t) = \check{\mathbf{q}}(\xi, \eta) \exp[i(m\zeta - \omega t)] + \text{c.c.}, \quad (3)$$

where the m is the azimuthal wavenumber and ω is the angular frequency.

The PSE approximation to the harmonic linearized Navier–Stokes equations (HLNSE) is based on isolating the rapid phase variations in the streamwise direction via the disturbance ansatz

$$\check{\mathbf{q}}(\xi, \eta) = \hat{\mathbf{q}}(\xi, \eta) \exp\left[i \int_{\xi_0}^{\xi} \alpha(\xi') d\xi'\right], \quad (4)$$

where $\hat{\mathbf{q}}(\xi, \eta, \zeta) = (\hat{\rho}, \hat{u}, \hat{v}, \hat{w}, \hat{T})^T$ represents amplitude functions that vary slowly in the streamwise direction compared to the phase term. Transition onset is characterized using the logarithmic amplification ratio, or N -factor, relative to the initial instability point ξ_I :

$$N_E = - \int_{\xi_I}^{\xi} \alpha_i(\xi') d\xi' + 1/2 \ln [\hat{E}(\xi)/\hat{E}(\xi_I)], \quad (5)$$

where $\hat{E}$ denotes the energy norm of $\hat{\mathbf{q}}$ as defined by [18]. Additional details on the PSE method can be found in Refs. [19, 20].

B. PSE Results

Figures 1a and 1b plot the logarithmic amplification factor, or N -factor, based on the total disturbance energy, N_E , of planar and oblique waves for the warm- and cold-wall cases, respectively, at the corresponding experimentally measured transition locations, $x_{tr, warm} = 0.2$ m and $x_{tr, cold} = 0.407$ m, for the selected flow condition [12]. The PSE analysis identifies both Mack’s first- and second-mode waves for the warm-wall to be amplified, with the oblique Mack’s second-mode waves ($f = 425$ kHz and $m = 6$) being the most amplified disturbance at $x = 0.2$ m. For the cold-wall cone, however, Mack’s first-mode waves are predicted to be stable, and only the high-frequency Mack’s second mode waves are amplified. The most amplified disturbance at $x = 0.407$ m corresponds to a planar wave ($m = 0$) with a frequency of $f = 405$ kHz.

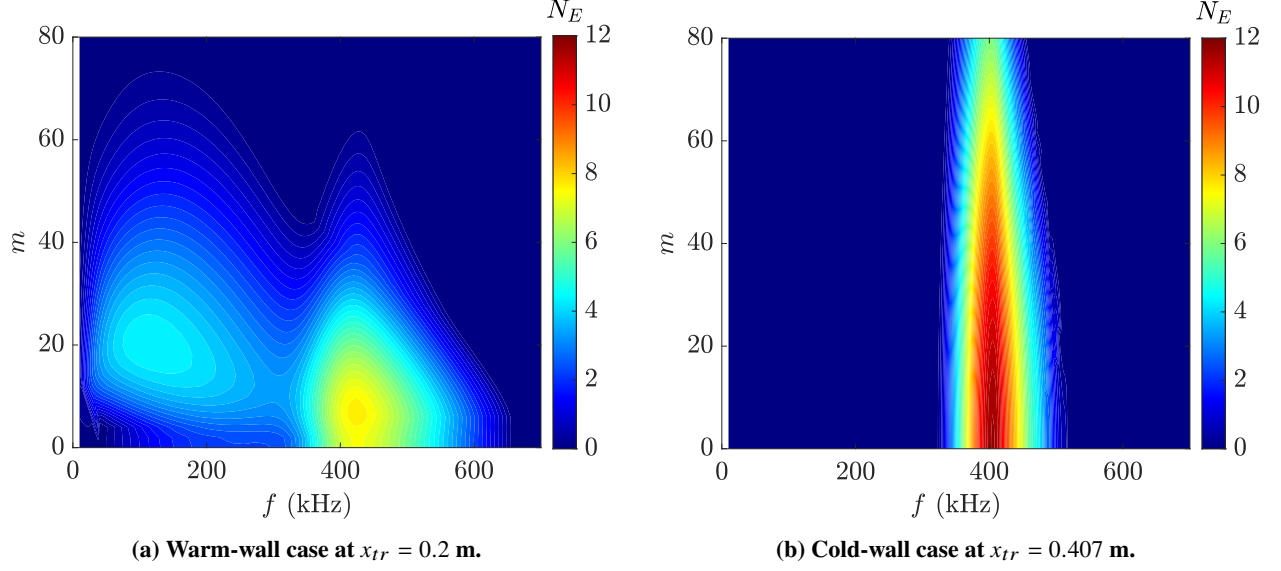


Fig. 1 Contours of N_E at the experimental transition location.

Figure 2 shows N -factor envelopes based on total disturbance energy, N_E , for convectively unstable planar and oblique waves. For the warm-wall case, the wavenumber associated with the most amplified disturbance increases with streamwise distance, switching from a planar component ($m = 0$) to an oblique component ($m > 0$). In contrast, for the cold-wall cone, the planar component of Mack's second mode is dominant for the whole streamwise extent.

The change from oblique to planar dominant instability waves at the colder wall temperature suggests that different transition mechanisms could dominate the transition process as a result of wall cooling, which leads to the downstream movement of the transition onset location with corresponding higher N -factor values for the cold-wall case.

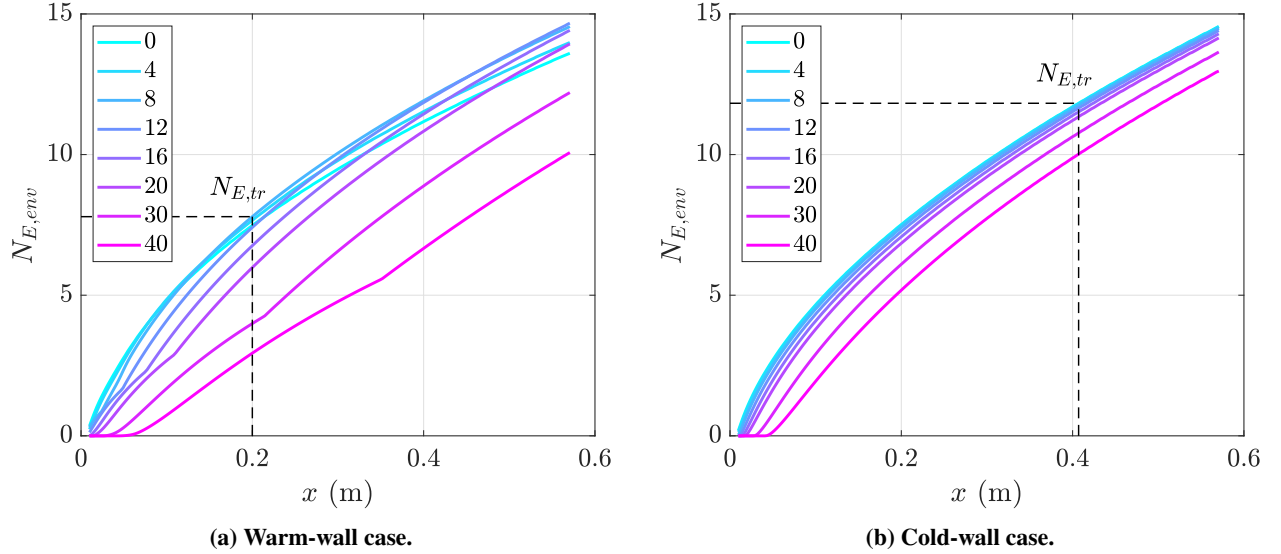


Fig. 2 Evolution of N_E envelopes for selected azimuthal wavenumbers.

The spectra of the N -factor maximized over the azimuthal wavenumbers along the length of the cone is shown in Figures 3a and 3b for the warm-wall and cold-wall cases, respectively. For both wall temperatures, the frequency associated with Mack's second mode decreases as the boundary layer thickens over the cone. As expected, wall cooling leads to a thinner boundary layer, increasing the frequency of the boundary-layer instabilities. Figure 3b further shows the apparent absence of the low-frequency Mack's first-mode instability throughout the cold-wall configuration.

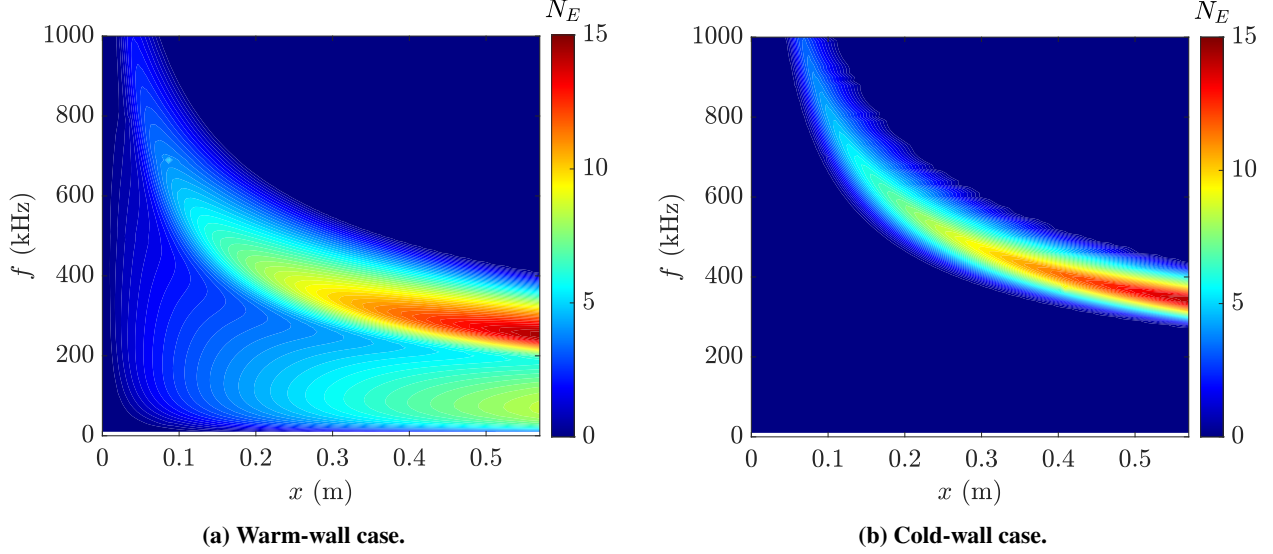


Fig. 3 Contours of N_E maximized over the azimuthal wavenumber along the cone length.

It should be mentioned that the present PSE results for the cold-wall case agree with the quasiparallel LST results for planar waves presented in Oddo et al. [12]. However, the inclusion of oblique waves and non-parallel effects in the present PSE analysis leads to a significantly higher amplification of the boundary-layer instabilities for the warm-wall case compared to the LST analysis of planar waves in Oddo et al. [12]. The following section presents DNS methodology and compares results for the primary warm-wall and cold-wall cases.

IV. Direct Numerical Simulations

In addition to linear stability analysis, DNS are performed to characterize the instabilities and transition in the boundary layer as a function of wall temperature. The methodology used for the DNS is presented first, with a description of the governing equations and numerical methods employed, baseflow comparisons against VULCAN-CFD, the multi-box approach for the DNS, and the inflow forcing method used. Following the methodology are the warm- and cold-wall DNS results, each subdivided into sections to cover the visualization of the boundary-layer transition, evolution of the mean flow, analysis of the turbulent spectra, and development of the turbulent flow.

A. DNS Methodology

1. Governing Equations and Numerical Methods

To simulate boundary-layer instability and transition on a circular cone, the full three-dimensional compressible Navier-Stokes equations in conservation form are solved numerically in cylindrical coordinates. The working fluid is air (with an ideal gas constant of $R = 287 \text{ J/(K}\cdot\text{kg)}$) treated as a perfect gas. The usual constitutive relations for a Newtonian fluid are used: the viscous stress tensor is linearly related to the rate-of-strain tensor, and the heat flux vector is linearly related to the temperature gradient through Fourier's law. The coefficient of viscosity μ is computed from Sutherland's law, and the coefficient of thermal conductivity κ is computed from $\kappa = \mu C_p / Pr$, with the molecular Prandtl number $Pr = 0.71$. The inviscid fluxes of the governing equations are computed using a fifth-order weighted essentially nonoscillatory (WENO) scheme [21]. The viscous fluxes are discretized using a fourth-order central difference scheme, and time integration is performed using a third-order low-storage Runge-Kutta scheme [22]. The DNS methodology has been successfully applied to similar cases that are documented in several of our previous papers [23–25].

For all DNS cases, a no-slip, isothermal wall condition is used on the wall. At the outlet boundaries, unsteady nonreflecting boundary conditions are imposed [26]. Dirichlet boundary conditions are used on the top boundary, where the flow variables are prescribed at the values from the converged laminar base flow. Periodic boundary conditions are used in the azimuthal direction.

2. Base Flow

Prior to the 3D unsteady DNS, a two-dimensional (2D) base flow is first computed. It includes the entire cone with the top domain boundary lying outside of the shock for the respective conditions. The 2D base flow simulation is fully converged before running the three-dimensional (3D) DNS that only includes the downstream portion of the cone and the top domain boundary of which is placed behind the shock. Figure 4 visualizes the computational domain of the base flow calculation of the warm-wall case, along with the global and wall-fitted coordinate systems.

To validate the laminar base flow computed by the DNS, Figure 5 shows the comparison of the wall-normal boundary layer profiles computed by both the DNS and VULCAN-CFD solvers at six streamwise locations along the cone ($x = 0.05, 0.1, 0.2, 0.3, 0.4$, and 0.5 meters). Excellent agreement is observed in the velocity, temperature, and density profiles between the two cases.

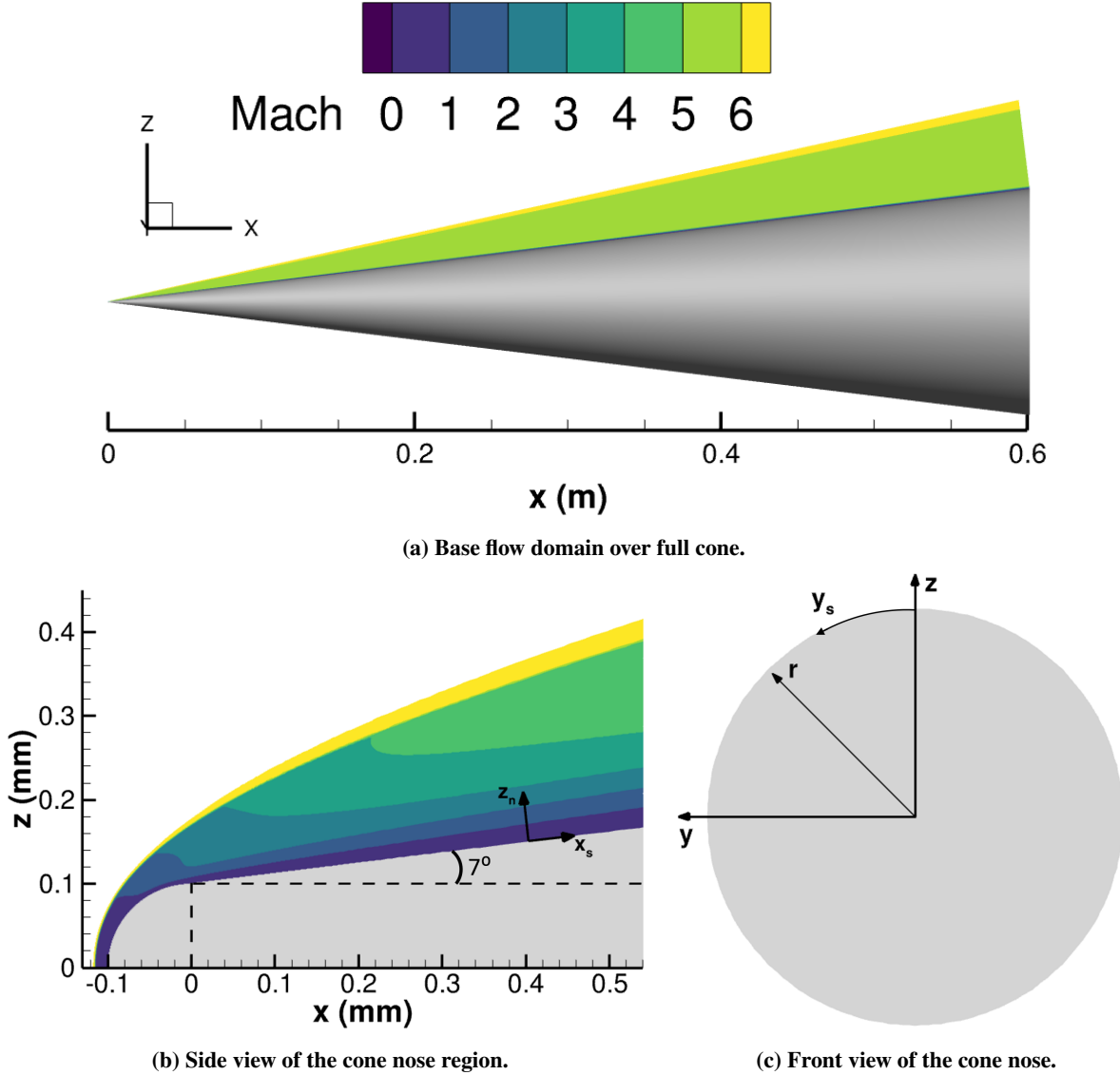


Fig. 4 Base flow domain and cone geometry for the warm-wall case ($T_w = 298$ K), including the coordinate systems and base flow Mach number contours.

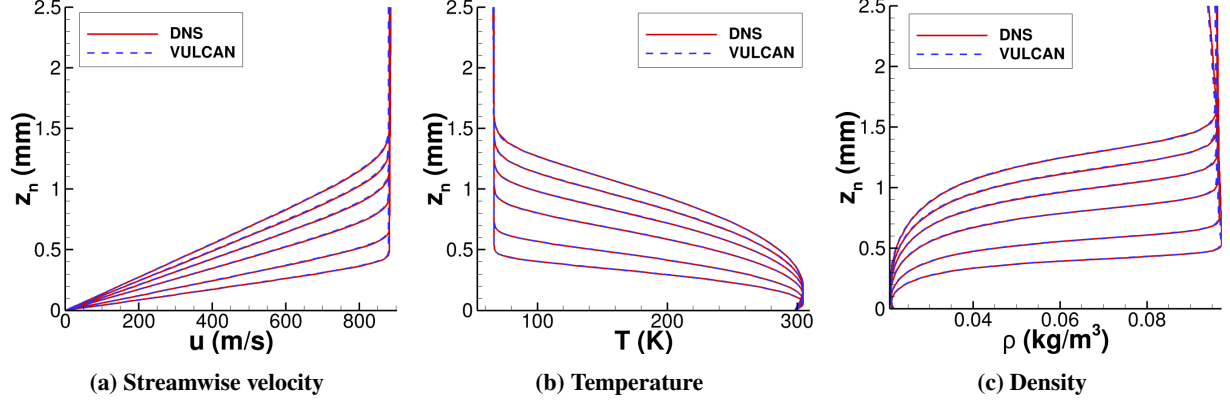


Fig. 5 Laminar-base-flow boundary-layer comparisons between DNS and VULCAN-CFD for the warm-wall case of $T_w = 298$ K. Wall normal profiles were taken at locations $x = 0.05, 0.1, 0.2, 0.3, 0.4$, and 0.5 meters.

3. 3D DNS Setup

The 3D DNS domain is constructed by first cutting the 2D base flow domain in the streamwise and wall-normal directions and then extruding the truncated domain in the azimuthal direction. The warm-wall 3D DNS run (referred to as Tw298) has an azimuthal domain size of $\phi = 60^\circ$, which corresponds to an azimuthal wavenumber of $m = 6$. Such a domain is wide enough to accommodate both planar second-mode waves and the most amplified oblique waves, as shown in Figure 1a. The cold-wall 3D DNS run (referred to as Tw84) uses a slightly smaller domain ($\phi = 38^\circ$) to match the ratio of spanwise width to boundary-layer height of the warm-wall case. A smaller boundary layer due to wall cooling allows for a reduced domain width, saving computational cost. The warm-wall (Figure 6a) and cold-wall (Figure 6b) 3D DNS domains are visualized along with the Mach contour of the 2D laminar base flow for reference.

The runs are conducted in multiple stages involving two overlapping streamwise domains as schematically shown in Figure 6. The first box of the 3D DNS runs starts slightly downstream of the nose tip at $x = 0.05$ m and ends at $x = 0.3$ m, with the top domain boundary lying just below the leading-edge shock. The second box starts at $x = 0.26$ m for the Tw298 case and $x = 0.28$ m for the Tw84 case, and these boxes extend to the end of the cone at $x = 0.6$ m. The exclusion of the nose tip significantly reduces computational cost, allowing for a significantly larger time step in the simulation. Box 1, therefore, starts with a relatively small time step, and Box 2 uses a larger time step to evolve the transitional or turbulent boundary layer towards the end of the cone, with the inflow conditions prescribed using the data saved from the upstream DNS box. A multidomain simulation approach of this type was previously applied to the DNS of the BLT over a cone [25, 27, 28].

Table 2 summarizes the setup of these cases, including the domain size and grid resolutions for each case. B1 and B2 stand for Box 1 and Box 2 respectively. Here, N_x , N_y , and N_z are the number of grid points while Δx_s^+ , Δy_s^+ , and Δz_n^+ represent the streamwise, azimuthal, and wall-normal resolutions respectively, normalized by the local viscous length scale.

Table 2 Summary of the domain sizes and the grid resolutions for all the warm-wall cases.

Case	x_{range} (m)	$N_x \times N_y \times N_z$	Δx_s^+	Δy_s^+	Δz_n^+
Tw298 B1	0.05 – 0.30	$1872 \times 500 \times 855$	5.20-8.99	0.58-4.36	0.16-1.12
Tw298 B2	0.26 – 0.60	$2512 \times 500 \times 855$	6.33-9.02	4.50-7.60	1.03-1.71
Tw84 B1	0.05 – 0.30	$8133 \times 330 \times 560$	5.55-13.28	3.96-24.84	0.24-1.44
Tw84 B2	0.28 – 0.60	$8080 \times 330 \times 560$	6.99-14.84	20.34-44.15	1.18-2.55

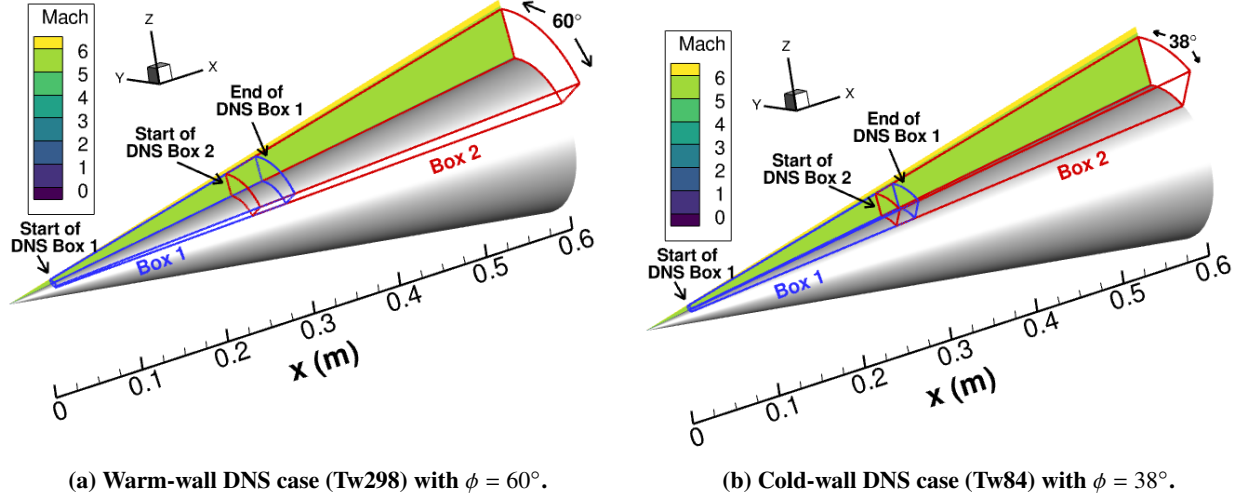


Fig. 6 Visualizations of the 3D computational domains for the DNS cases with the laminar base flow Mach contour for reference.

4. Inflow Forcing Methods

Previous computational studies of boundary-layer receptivity in the hypersonic regime had used random forcing with a broad range of frequencies to trigger the "natural" transition process [29–33]. Here, we use the broadband inflow forcing approach by Hader and Fasel [30] as a simple model to trigger convectively unstable planar and oblique waves, which accounts for the majority of the sources of wind-tunnel freestream disturbances in the experiment. This method has been shown to eliminate bias towards any particular breakdown mechanism and therefore does not require a selection of specific frequencies or wavenumbers needed in a “controlled” breakdown simulation [30]. The method uses the FORTRAN pseudo-random number generator to impose random pressure perturbation (p') into the flow at a user-specified domain as

$$p'_{i,j,k,n}/q_\infty = A(2r - 1), \text{ where } r = \text{random_number}(), \quad (6)$$

where i , j , and k are the grid points in the streamwise, azimuthal, and wall-normal directions, respectively, n represents the time step, r is the random number between zero and one, q_∞ is the freestream dynamic pressure ($q_\infty = \frac{1}{2}\rho_\infty u_\infty^2$), and A is the specified amplitude of the perturbations and is referred to in percentage of the dynamic pressure q_∞ in this paper. For the results presented here, the disturbances are applied over a singular streamwise plane, 6 grid points downstream from the inlet ($i=6$), and the amplitude A is altered to be large enough to lead to BLT within the DNS domain but without overly perturbing the flow, still allowing the skin-friction and other boundary-layer parameters to recover to those of the laminar base flow before transition occurs. Table 3 shows the inflow forcing location (x_i), forcing amplitude (A), and the corresponding transition location (x_{tr}) for both DNS cases.

For each case, the transition location is determined to be the x -location that the mean boundary-layer thickness δ of the 3D DNS surpasses that of the 2D laminar base flow. Here, δ was calculated at the wall-normal location where streamwise velocity reaches 99% of the post-shock streamwise velocity ($u = 0.99u_{\infty,2}$). A similar δ -based method for determining the location of BLT was used in the experiment by Clingenpeel [14].

Table 3 Summary of the random inflow forcing parameters for all the warm-wall DNS cases.

Case	T_w (K)	ϕ (deg)	x_i (m)	A (%)	x_{tr} (m)
Tw298	298	60.0	0.05	3.3	0.204
Tw84	84	38.0	0.05	3.3	0.238

The amplitude and seeding location were varied in an auxiliary study from previous work [34] to assess the applicability and limitations of the random inflow forcing method of Equation 6 in simulating natural BLT in a noisy

wind tunnel. The study found that the best seeding location and amplitude for disturbances were $x = 0.05$ m and $A = 3.3\%$, which were then used for both cases. This condition best matched the warm-wall transition location to experiments conducted by Clingenpeel [14].

In the cold-wall DNS, the first 40 points off the wall (a small portion of the boundary layer) were left unforced to ensure numerical stability. This was predicted not to significantly alter the flow since second-mode transition originates from the edge of the boundary layer, but a short study on the effects of forcing domain was conducted on a warm-wall auxiliary case to ensure this would not alter transition location. The results of this study show no difference in transition location and can be found in the Appendix.

B. Warm-Wall DNS Results

In this section, the results for the warm-wall DNS case (Tw298) along with comparison to PSE are presented. Inflow disturbances are seeded at $x = 0.05$ m, and a disturbance amplitude of $A = 3.3\%$ is used for an azimuthal domain of 60° . As mentioned earlier, the azimuthal domain corresponds to an azimuthal wavenumber of $m = 6$ and is wide enough to accommodate both planar second-mode waves and the most amplified oblique waves identified by the PSE.

1. Visualization of BLT Stages

Figure 7 shows numerical schlieren images based on the instantaneous density gradient over eight snapshots sampled at a rate of 83.33 kHz. The snapshots are of the centerline x - z plane. The figure shows the formation of rope-like structures that form inside the boundary layer, grow, and eventually break down into turbulence. These structures are indicative of second-mode instabilities [35]. The red arrow in each snapshot follows an individual wave packet as it grows and eventually breaks down. The structures travel at an approximate convection speed of 830 m/s ($\approx 0.94u_{\infty,2}$).

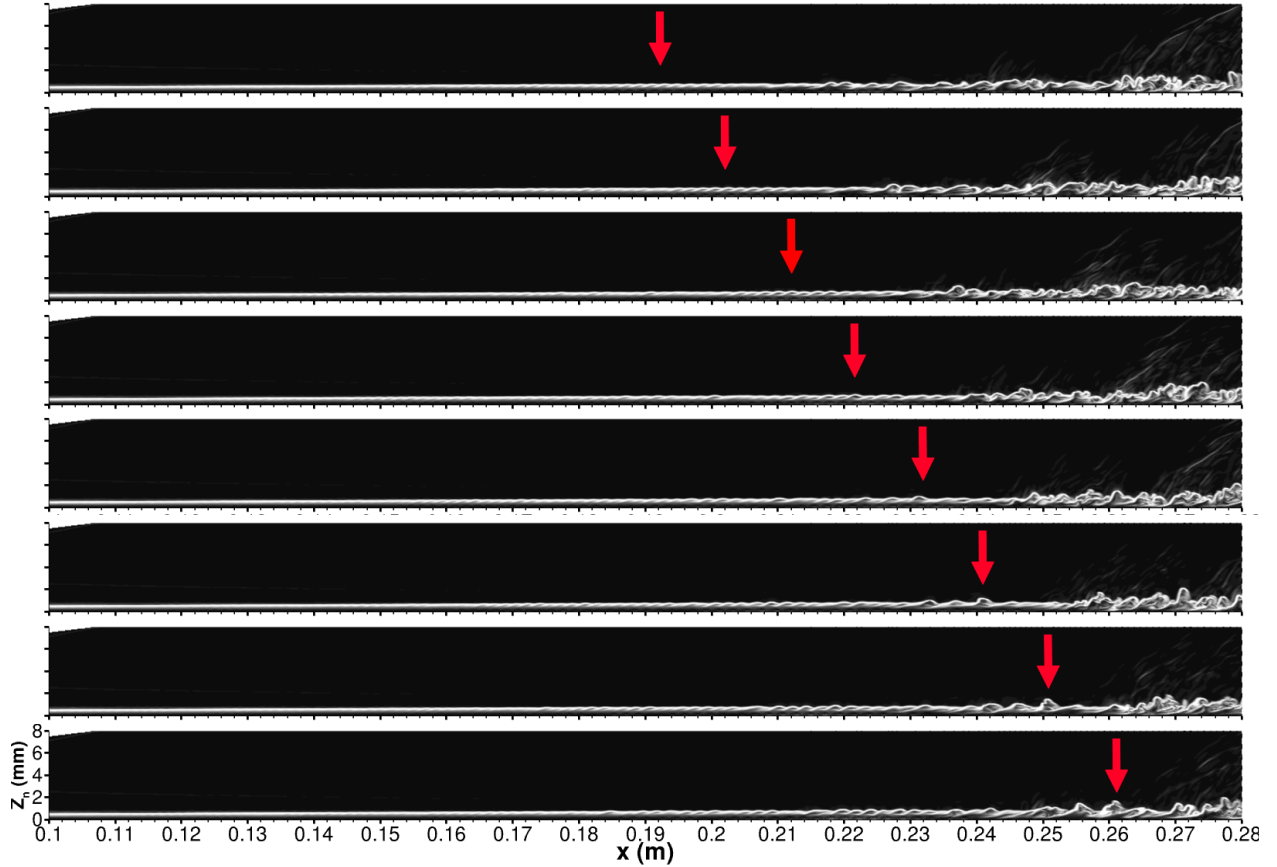


Fig. 7 Numerical schlieren for the warm-wall DNS case (Tw298) over a sequence of flow snapshots sampled at 83.33 kHz.

Figure 8 shows the instantaneous Stanton number for a snapshot of the warm-wall case at $t = 0.648$ ms. The figure shows the entire 60° domain for $0.1 \text{ m} < x < 0.3 \text{ m}$. The faint roller-like azimuthal structures visible at the beginning of the domain amplify and eventually break down by $x = 0.24 \text{ m}$, becoming more chaotic and marking the beginning of turbulent flow. The chaotic structures persist through the end of the domain. Similar instantaneous figures were observed in all scenarios. Collecting and processing this data allows for span and time averaging to compute typical quantities of interest.

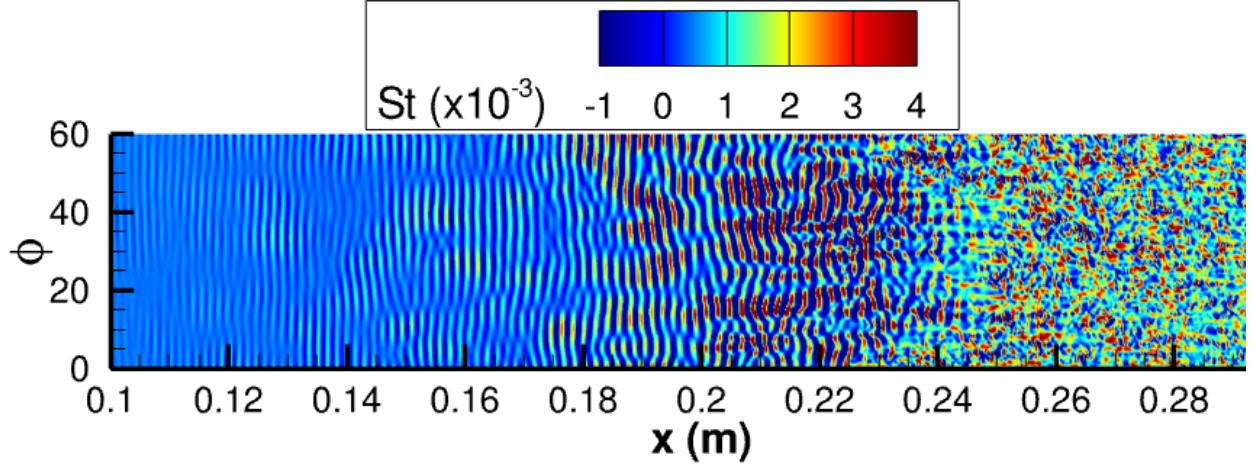


Fig. 8 Streamwise evolution of instantaneous surface Stanton number contours along the warm-wall cone (Tw298).

2. Evolution of Mean Flow Parameters

Figure 9 shows the spanwise and time-averaged skin-friction coefficient (C_f), Stanton number (St), boundary layer thickness (δ), and wall pressure (P_w) as a function of the streamwise coordinate in comparison with the 2D laminar base flow values, as well as the turbulent solutions from VULCAN-CFD using the SA-neg and Menter SST turbulence models. Due to the injection of random disturbances near the inlet, C_f , St and P_w rapidly diverge from the base flow values, while the boundary-layer thickness does not change considerably. After a small transient region, C_f and P_w recover to similar values from the base flow, while St needs a larger distance to recover its base flow values. At around $x \approx 0.18 \text{ m}$, the skin-friction and Stanton number coefficients rapidly overshoot turbulent predictions, reaching about three times the laminar base flow values at $x \approx 0.27 \text{ m}$. A similar overshoot was found by Franko and Lele [36] over a Mach 6 flat-plate boundary layer and by Schuabb et al. [37] over a cone at Mach 8. After the overshoot, the values gradually approach the VULCAN-CFD turbulent solutions and end up matching them well. On the other hand, the deviation of the boundary layer thickness starts slightly downstream, giving a transition location at $x = 0.204 \text{ m}$, which closely matches the experimental value by Clingenpeel [14].

3. Analysis of Instability Spectra

Additional analysis of the spectral properties of the warm-wall DNS case Tw298 is performed and visualized. Figures 10a and 10b plot the frequency power spectral density (PSD) of wall-pressure fluctuations at different streamwise locations across the simulated domain. Consistent with the previous linear stability analysis, the second-mode instability, which corresponds to the predominant hump near 500 kHz in the frequency PSD, is the dominant instability leading to BLT in the warm-wall cone. The frequency of the second-mode-wave hump decreases as the boundary layer thickens. In particular, at $x = 0.2 \text{ m}$, the peak frequency is at 410 kHz, which is in close agreement with the frequency of the maximum N -factor from the stability analysis results in Figure 1. Such a peak frequency also closely matches the experimental values reported by Clingenpeel [14] and Oddo et al. [12]. In addition to the second-mode-wave hump and its superharmonics, another lower-frequency hump is apparent near 100 kHz between $0.15 \text{ m} \lesssim x \lesssim 0.2 \text{ m}$ in Figure 10a. This lobe may correspond to the oblique first-mode instability as predicted by the linear PSE analysis in

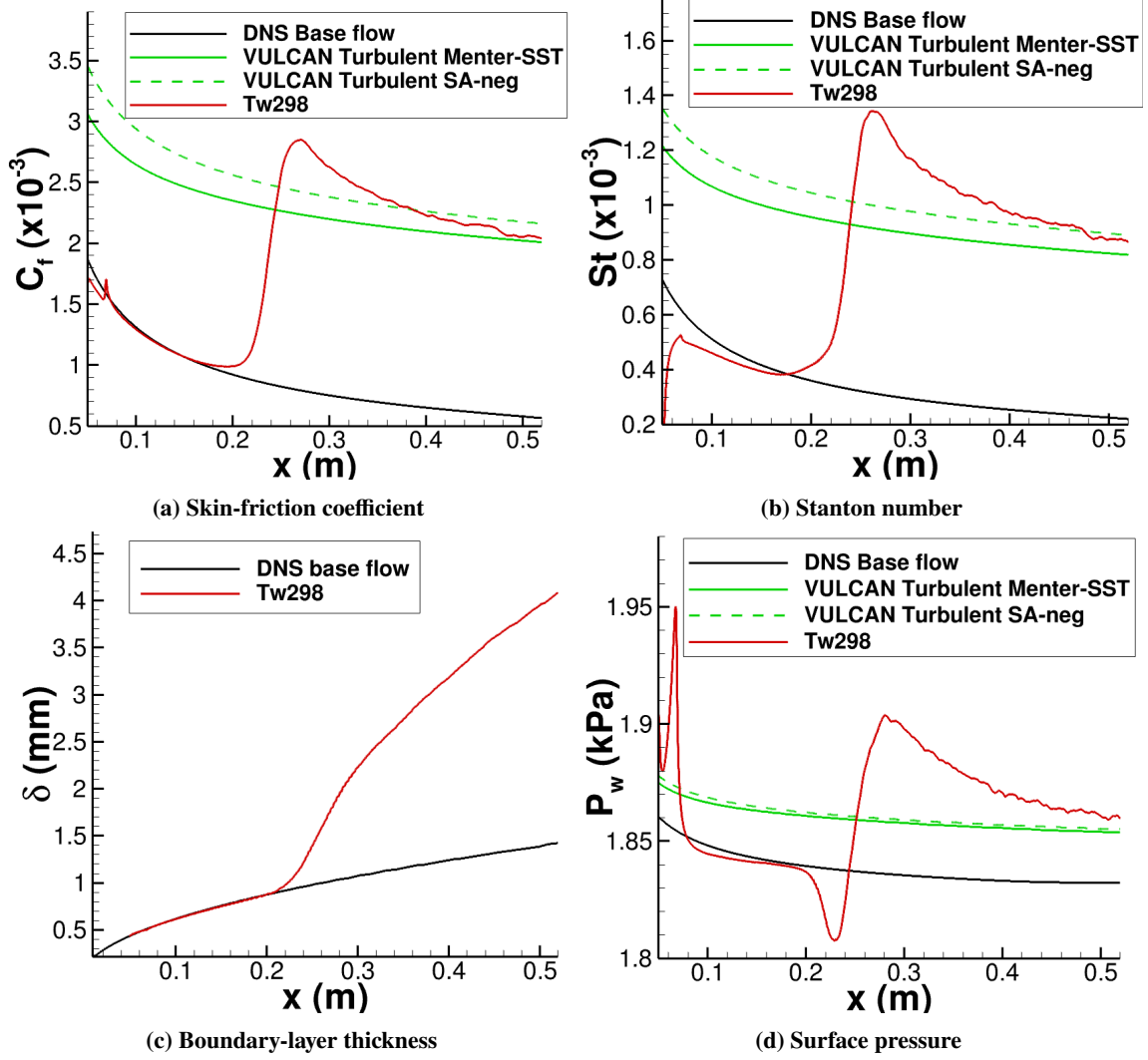


Fig. 9 Streamwise evolution of the skin-friction coefficient (C_f), Stanton number (C_h), boundary layer thickness (δ), and mean wall pressure (P_w) in comparison with those of the laminar base flow and VULCAN-CFD turbulent models for the warm-wall DNS case (Tw298).

Figure 1a. However, further study based on nonlinear PSE is needed to confirm the role this mode plays leading to earlier transition. At farther downstream locations ($x \gtrsim 0.25$ m), the spectral peaks are no longer visible, resulting in a broadband “turbulence-like” spectrum. Figures 10c and 10d show the azimuthal wavenumber spectrum in terms of azimuthal wavenumber m as a function of streamwise location x . Similarly to the frequency spectrum, the azimuthal wavenumber spectrum has an abrupt spectral broadening from $x_g \approx 0.2$, suggesting the emergence of higher azimuthal wavenumbers during laminar breakdown.

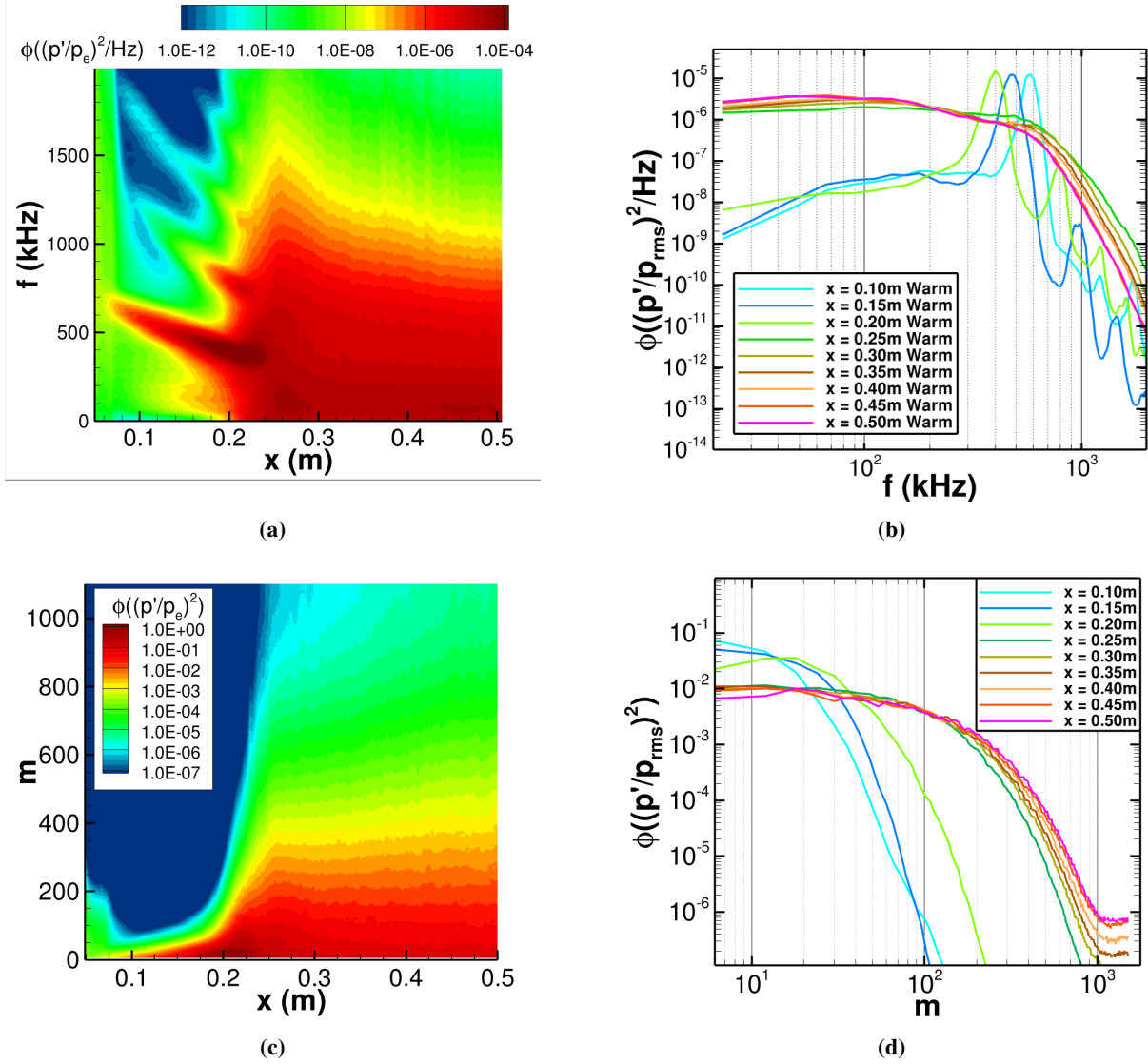


Fig. 10 (a, b) Frequency PSD and (c, d) azimuthal wavenumber spectra of the wall-pressure fluctuations as a function of streamwise location for the warm-wall DNS case Tw298.

Figures 11a and 11b further show the spectral amplitude of the wall-pressure fluctuations computed in the frequency and azimuthal wavenumber plane. The results at selected axial stations are Fourier transformed in both time and the azimuthal direction. The warm-wall N -factor based on wall pressure, N_{P_w} , contour from the PSE are also plotted in Figures 11c and 11d to give a side-by-side comparison to DNS. At $x = 0.15$ m and 0.20 m, the $f - m$ DNS spectrum peaks at around 475 kHz and 400 kHz respectively, and the spectral peak is for low wavenumbers, which is in agreement with results shown in Figures 10b, 11c, and 11d. A secondary concentration of energy for lower frequency and higher wavenumber around $(f - m) = (150 \text{ kHz}, 22)$ can be observed at $x = 0.15$ m, which is an indication of Mack’s first mode, which was also predicted by the PSE analysis (shown in Figure 11d). At $x = 0.2$ m, the DNS shows energy

concentration begins to spread out as the flow nears transition, agreeing with PSE results.

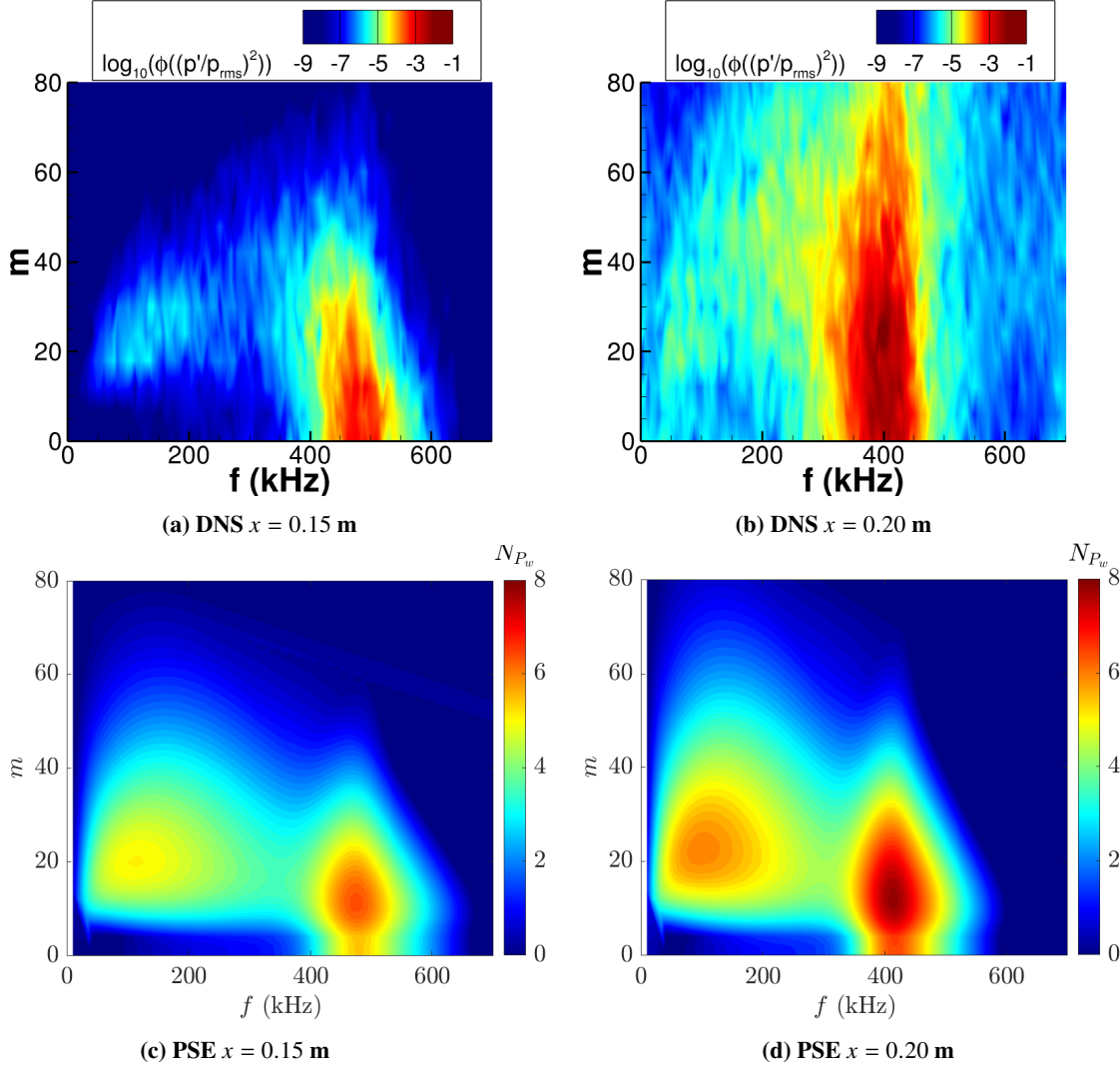


Fig. 11 (a,b) Frequency/azimuthal-wavenumber ($f - m$) spectrum of the wall-pressure fluctuations at different streamwise locations predicted by the warm-wall DNS case Tw298, and (c,d) the PSE N_{P_w} contours for direct comparison.

4. Warm-Wall Turbulent Flow Development

In this section, further analysis is done to characterize the fully turbulent flow development. It was already seen in Figure 9 that the flow becomes fully turbulent by the end of the computational domain. Figure 12 compares the wall-normal profiles of density-weighted turbulence intensity components for different streamwise distances along the cone. In the pretransitional region ($x \lesssim 0.2$ m), all components of turbulence intensity are very low as expected. These values rapidly increase by $x = 0.25$ m, and settle into a fully turbulent state by $x = 0.3$ m.

The same trend of a natural transition to turbulence is seen in the profiles of turbulent heat flux components as shown in Figure 13. Both the spanwise and streamwise components start very close to zero until the transitional region ($x \approx 0.25$), where they peak and saturate to fully turbulent values.

Figure 14a plots wall-normal profiles of the temperature-velocity correlation coefficient, defined as

$$R_{u''T''} = \frac{\overline{u''T''}}{\sqrt{\overline{T''^2}}\sqrt{\overline{u''^2}}}, \quad (7)$$

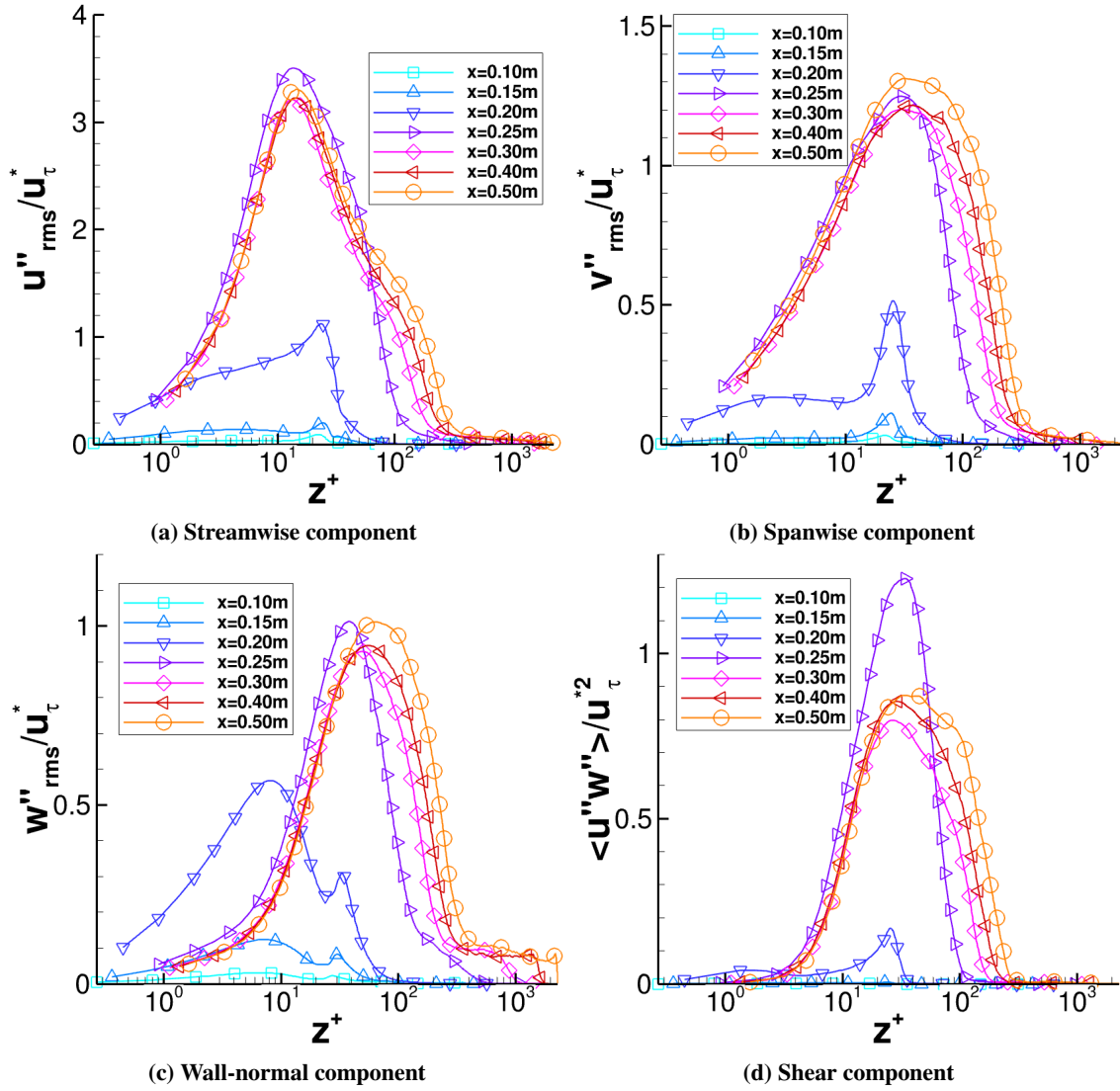


Fig. 12 Comparison of the density-weighted turbulence intensities at different wall-normal profiles. Case: Tw298.

where $\widetilde{(\cdot)}$ is the Favre-averaged variable. Figure 14b plots turbulent Prandtl number profiles at different streamwise locations and is defined as

$$Pr_t = \frac{[\overline{\rho u'' w''}](\partial \widetilde{T} / \partial z)}{[\overline{\rho T'' w''}](\partial \widetilde{u} / \partial z)}. \quad (8)$$

Figure 14 shows that profiles for $x \gtrsim 0.25$ m follow the expected trends of fully developed high-speed turbulence with temperature-velocity correlation settling at $-R_{u''T''} \approx 0.8$ and the turbulent Prandtl number leveling out at $Pr_t \approx 1$.

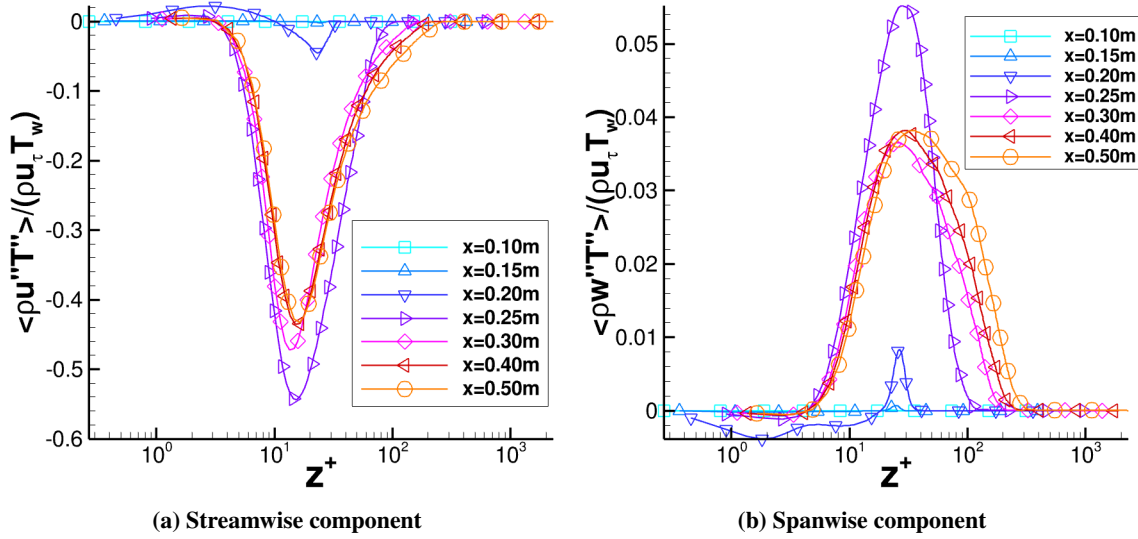


Fig. 13 Comparison of (a) streamwise and (b) wall-normal turbulent heat flux components at various wall-normal profiles. Case: Tw298.

C. Cold-wall DNS Results

Like the warm-wall case discussed in the previous section, the cold-wall case (Tw84) also uses inflow disturbances with $A = 3.3\%$ seeded at $x = 0.05$ m. This case uses an isothermal wall temperature $T_w = 84$ K for the cone. This section presents the results for the cold-wall DNS case Tw84 along with comparisons to corresponding PSE results.

1. Visualization of Cold-Wall BLT Stages

Figure 15 shows numerical schlieren images using the instantaneous density gradient over eight snapshots sampled at a rate of 83.33 kHz. The snapshots are of the centerline x - z plane. Rope-like structures are visible inside the boundary layer as they grow and eventually break down into turbulence. These structures are indicative of second-mode instabilities [35]. The red arrow in each snapshot follows an individual wave packet as it grows and eventually breaks down. The structures and their downstream velocities (approximately $0.94u_{\infty,2}$) are very similar to the warm-wall case.

Figure 16 shows the instantaneous Stanton number for a snapshot of the warm-wall case at $t = 0.762$ ms. The figure shows the entire 38° domain for $0.1 \text{ m} < x < 0.57 \text{ m}$. The faint roller-like azimuthal structures are visible at the beginning of the domain until around $x \approx 0.24$ m. Between 0.25 and 0.30 m, those roller-like structures amplify and wave trains with narrow azimuthal extent with larger amplitudes appear. More analysis is needed to see if wave trains originate from secondary instabilities. By $x = 0.30$ m, those waves eventually break down, becoming more chaotic and marking the beginning of turbulent flow. The chaotic structures persist through the end of the domain. Similar instantaneous figures were observed in all scenarios. Collecting and processing this data allows for span and time averaging to compute typical quantities of interest.

2. Evolution of Mean Flow Parameters for the Cold-Wall Case

Figure 17 shows the spanwise and time-averaged skin-friction coefficient (C_f), Stanton number (St), boundary layer thickness (δ), and wall pressure (P_w) as a function of the streamwise coordinate in comparison with the 2D

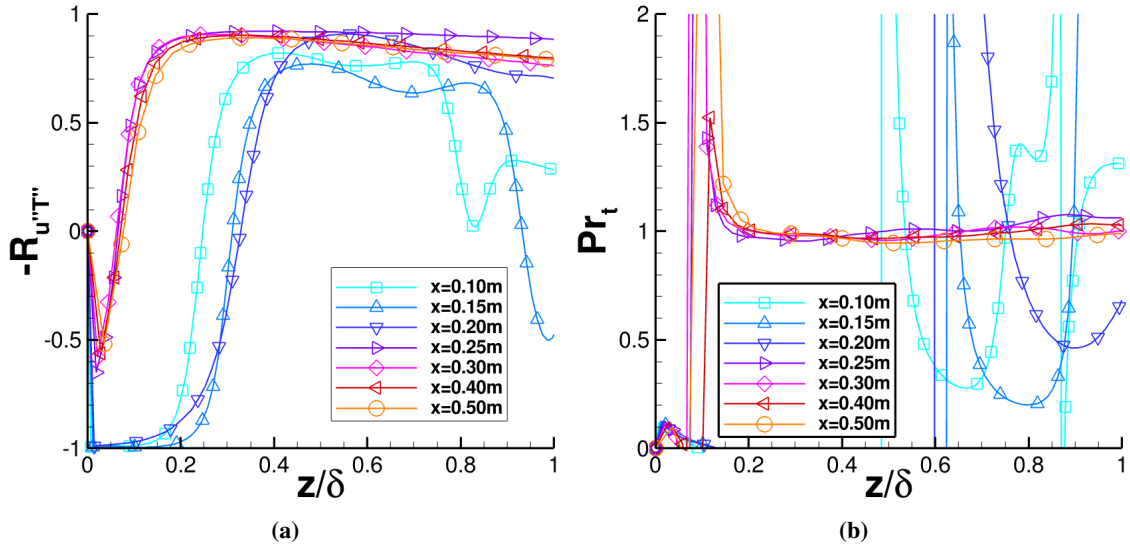


Fig. 14 Comparisons of the (a) temperature-velocity correlation coefficient ($R_{u''T''}$) and the (b) turbulent Prandtl number (Pr_t) at different wall-normal profiles. Case: Tw298.

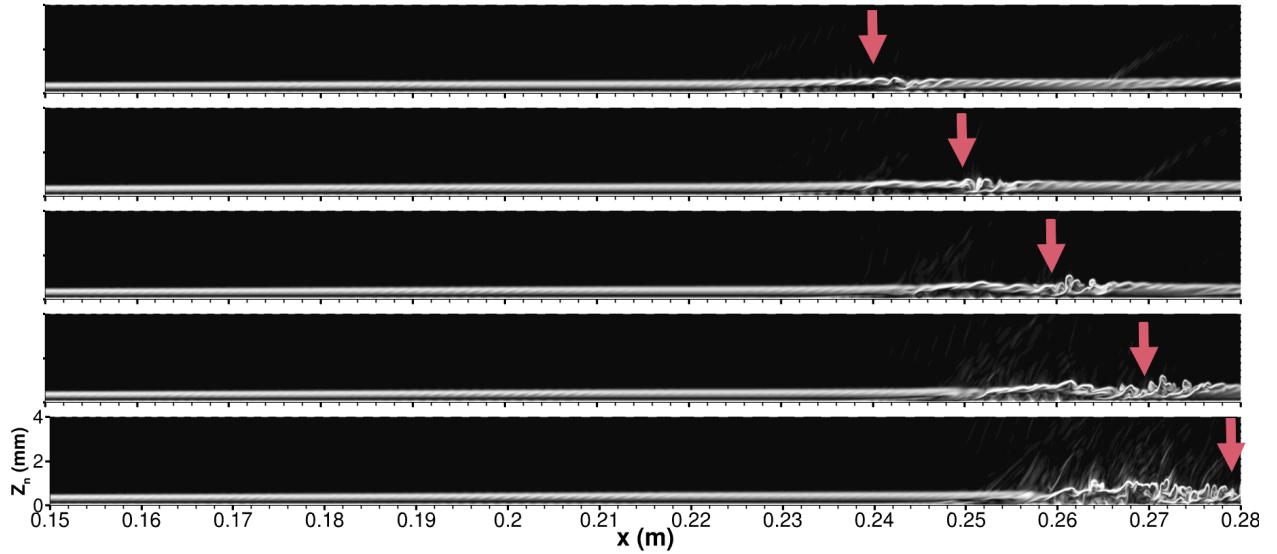


Fig. 15 Numerical schlieren for the cold-wall DNS case (Tw84) over a sequence of flow snapshots sampled at 83.33 kHz.

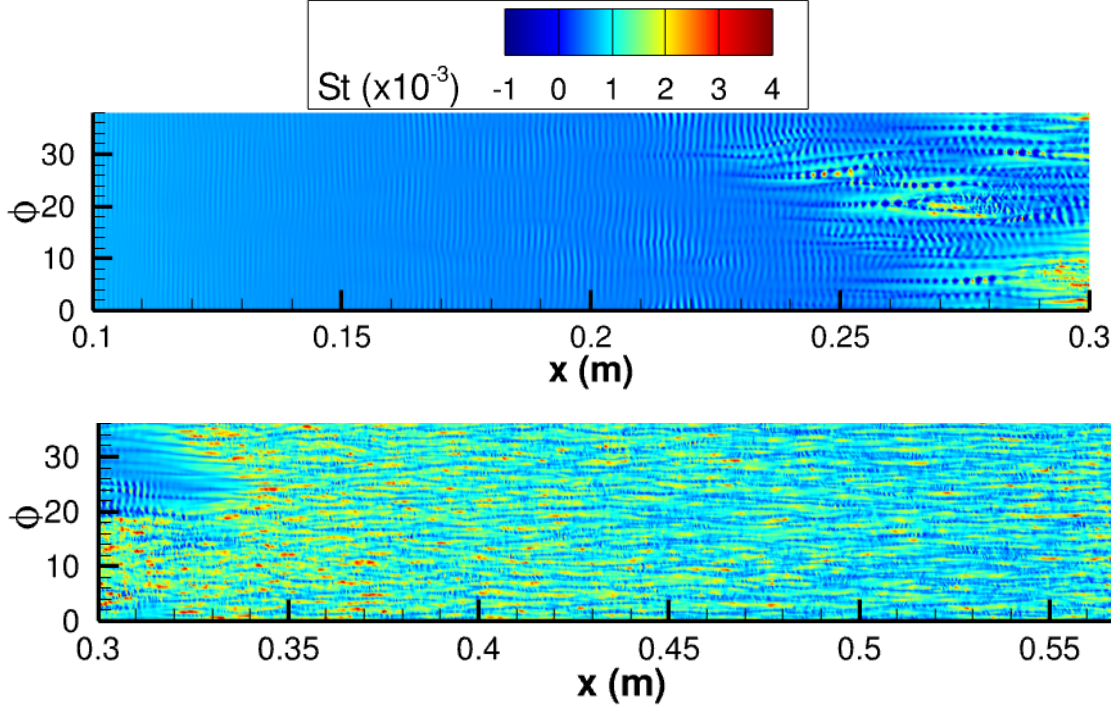


Fig. 16 Streamwise evolution of instantaneous surface heat flux contours along the cold-wall cone. Case: Tw84.

laminar base flow values and the VULCAN-CFD solutions obtained using the SA-neg and Menter SST turbulence models. Very similar trends are observed in the cold-wall case as in the warm-wall case. One difference, however, is that the VULCAN-CFD turbulent solutions overestimate Stanton number and the skin friction coefficient. The overshoot of these values does not surpass the turbulent values. This is not surprising as these turbulence models do not make corrections based on wall-temperature. The mean DNS boundary layer thickness deviates from the laminar base flow at $x = 0.238$ m, marking the transition location for this case.

3. Analysis of Cold-Wall Instability Spectra

Additional spectral analysis of the cold-wall DNS case Tw84 is performed and visualized. Figures 18a and 18b plot the frequency power spectral density (PSD) of wall-pressure fluctuations at different streamwise locations along the simulated domain. Like the warm-wall case, the second-mode instability is still dominant, with frequencies beginning at 1000 kHz and decreasing to ≈ 500 kHz as the boundary layer grows before transition. Since the frequency of the second-mode waves is inversely proportional to the boundary-layer thickness, this range of frequencies is higher than those of the warm-wall case due to the smaller boundary layer that forms over the cooled cone. The 500 kHz peak at $x = 0.238$ m matches the peak N-factor frequency at the same location in the PSE (Figure 3b).

Figures 18c and 18d show the streamwise variation of the azimuthal wavenumber spectrum. Similarly to the frequency spectrum, the azimuthal wavenumber spectrum begins abrupt spectral broadening from $x_g \approx 0.225$, suggesting the emergence of higher azimuthal wavenumbers during laminar breakdown.

Figures 19a and 19b further show the spectral amplitude of the wall-pressure fluctuations computed in the frequency and azimuthal wavenumber plane. The results at selected axial stations are Fourier transformed in both time and the azimuthal direction. The PSE-predicted N -factors based on the wall pressure (N_{P_w}) for the cold-wall case are also shown in Figure 19c and 19d to enable a side-by-side comparison with the DNS results. At $x = 0.15$ m and 0.20 m, the $f - m$ spectrum peaks at around 630 kHz and 590 kHz respectively, and the spectral peak is for low wavenumbers, in agreement with PSE results shown in Figures 19c, 19d, and 18b. Figure 19a shows only a strong narrow band of energy (corresponding to the second mode instability) just before transition as is predicted near transition by LSA in Figure 19d. When directly comparing DNS to PSE, the frequency at which wall pressure fluctuations are largest matches exactly

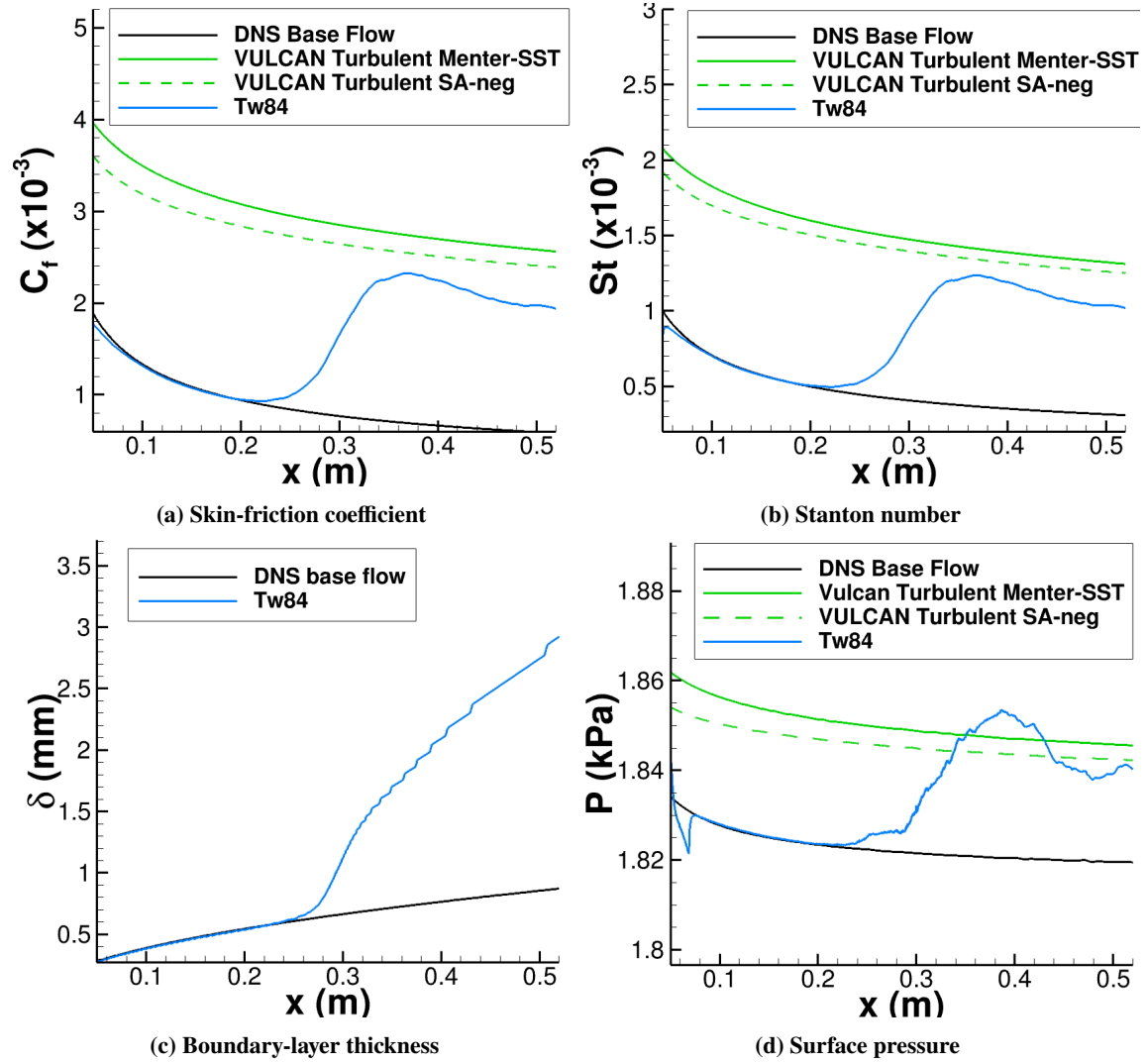


Fig. 17 Streamwise evolution of the skin-friction coefficient (C_f), Stanton number (St), boundary layer thickness (δ), and mean wall pressure (P_w) in comparison with those of the laminar base flow and VULCAN turbulent wall values for the cold-wall DNS case (Tw84).

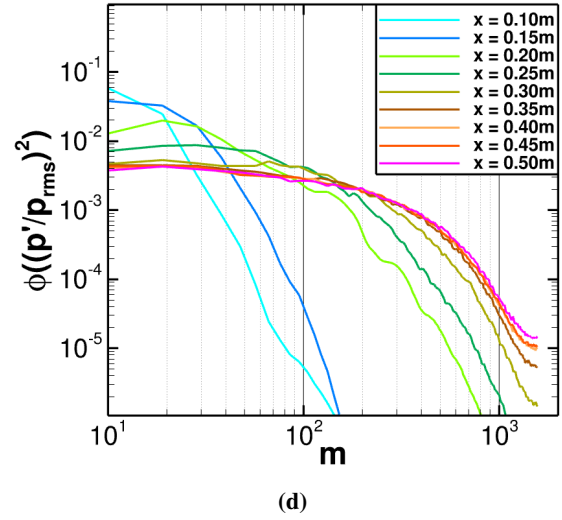
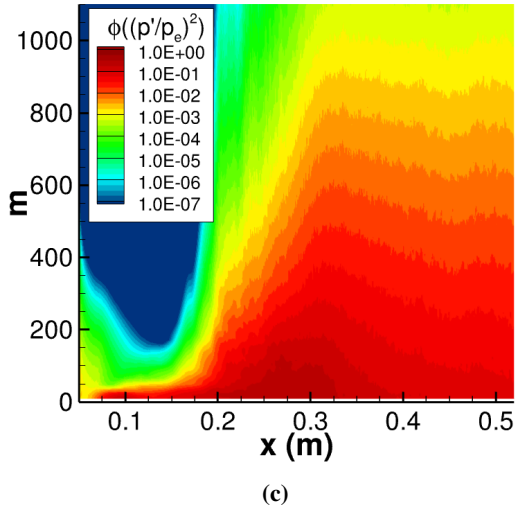
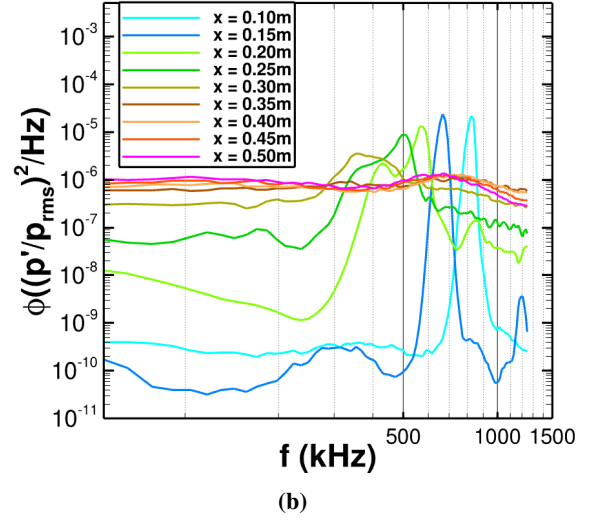
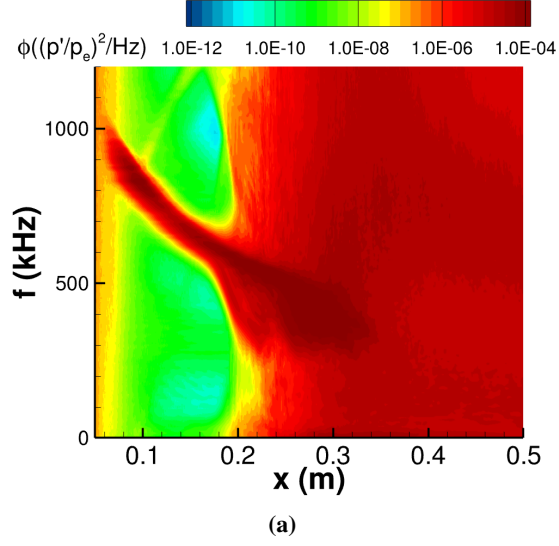


Fig. 18 (a, b) Frequency PSD and (c, d) azimuthal wavenumber spectra of the wall-pressure fluctuations as a function of streamwise location for the cold-wall DNS case Tw84.

with the frequency of maximum N_{P_w} . The contour shapes at this location also match well.

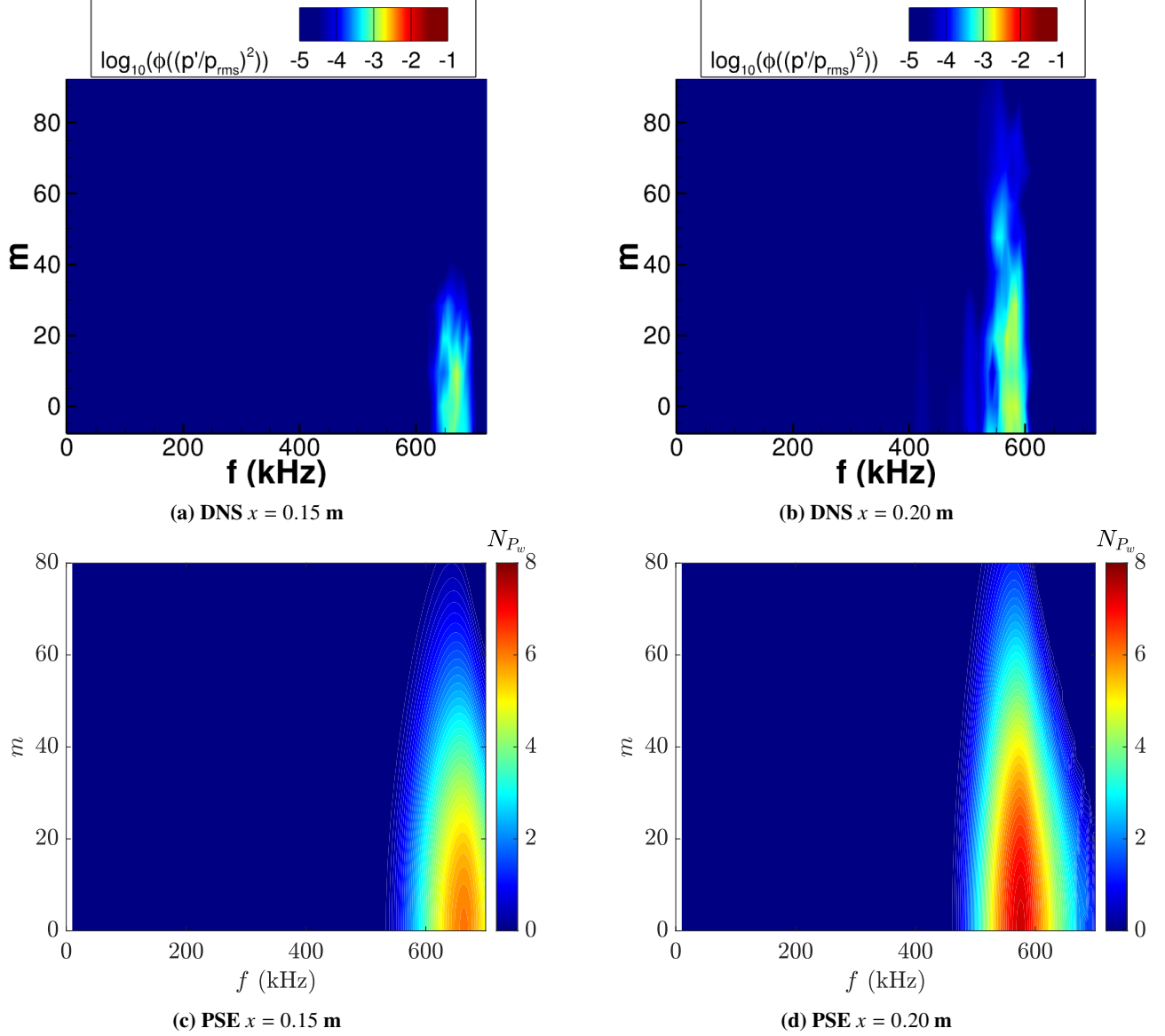


Fig. 19 (a,b) Frequency/azimuthal-wavenumber ($f - m$) spectrum of the wall-pressure fluctuations at different streamwise locations predicted by the cold-wall DNS case Tw84, and (c,d) the PSE N_{P_w} contours for direct comparison.

4. Cold-Wall Turbulent Flow Development

After further analyzing the development of a fully turbulent boundary layer subject to wall-cooling, we see the same trend in turbulence intensities in Figure 20 as was seen for the warm-wall case (Figure 12). As expected, full turbulence does not appear to develop until $x \gtrsim 0.3$ m, downstream of the warm-wall case.

Turbulent heat flux values for the cold-wall case show similar trends to the warm-wall case except with a slight delayed in the streamwise direction (Figure 21). At streamwise locations upstream of 0.3 m, there is very little intensity in the heat flux components. Fully developed turbulence is reached at $x = 0.3$ m. Figure 22a shows the expected trend is achieved in temperature-velocity correlation as the downstream profiles approach $-R_{u''T''} \approx 0.8$ as z/δ increases. Although the different turbulent Prandtl number profiles of figure 22b appear chaotic at first, the general trend of their

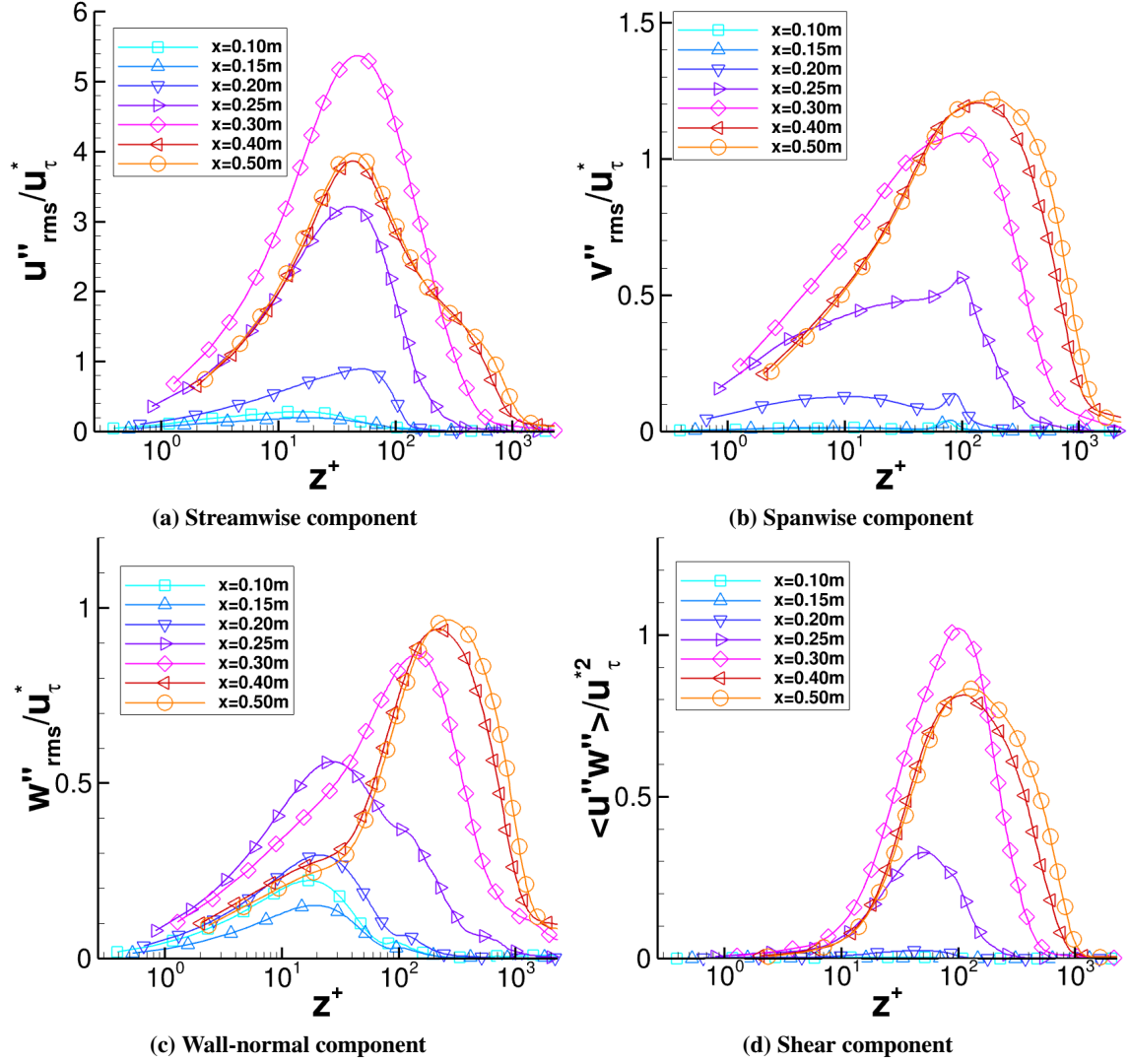


Fig. 20 Comparison of the density-weighted turbulence intensities at different wall-normal profiles. Case: Tw84.

values approaching 1 is still seen for the two most downstream profiles ($x = 0.4, 0.5$ m). Overall, this downstream transition is expected, and the domain is confirmed to be long enough in both cases to capture the full transition to turbulence and some of the fully turbulent boundary layer.

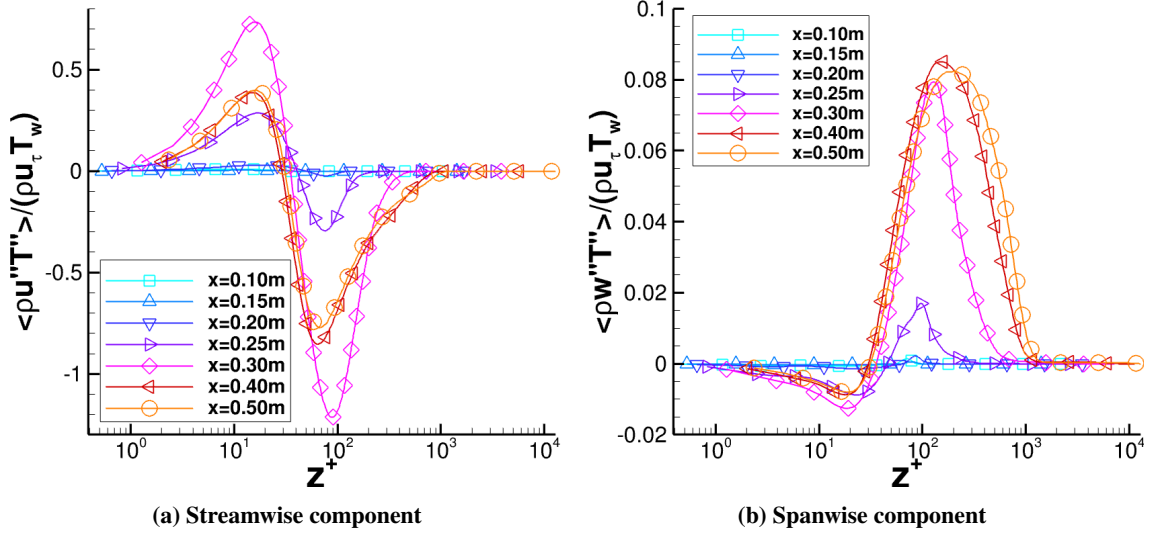


Fig. 21 Comparison of (a) streamwise and (b) wall-normal turbulent heat flux components at various wall normal profiles. Case: Tw84.

5. Effects of Wall Cooling

Figure 23 shows the difference in boundary-layer heights between the cold- and warm-wall cases. The determined transition location is given by the vertical dotted line for each case. The boundary layer thickness deviates from the laminar trend later ($x = 0.238$ m) for a cold wall compared to a warm one ($x = 0.204$ m). This confirms the transition delay that was expected from experiments and predictions from LSA. This downstream shift of the transition location shift by about 17% is not as significant as predicted by experiments ($\approx 74\%$ [14] and $\approx 100\%$ [12]). The overall boundary layer height of the cold wall is smaller than that of the warm one, as expected.

Figure 24 compares the frequency PSD of both cases. The second-mode frequencies are slightly higher for the cold-wall case than for the warm-wall case due to the thinner boundary layer. Cooling the wall seems to reduce the presence of a lower-frequency hump visible on Figure 24b, which was also predicted by the convective wave analysis provided by linear PSE. The PSD of wall-pressure fluctuations computed at $x = 0.15$ m are presented in Figure 25 for different wavenumbers using the same color scale between both cases. Again, a slight difference is seen in the dominant, second-mode lobes due to differences in boundary-layer height. A second lobe at lower frequency is observed for the warm-wall, corresponding to Mack's first-mode, and is absent in the cold-wall case. This indicates the stabilization of the first-mode due to a cooler wall, partially responsible for the transition delay observed. The planar ($m = 0$) component of Mack's second mode is strongly amplified in both cases. The larger wavenumber extent of Mack's second mode for the cold wall could indicate the oblique component being dominant, which would align with PSE predictions and suggest a different transition mechanism is dominating the transition process, thus delaying it.

Wall-normal profiles of the streamwise and spanwise components of the turbulent heat flux fluctuations are given in Figure 26. For each case, two profiles are compared. One is taken at the respective transition locations $x_{tr} = 0.20$ m and $x_{tr} = 0.24$ m for the warm-wall and cold-wall cases respectively. The second profile is taken at the location corresponding to the maximum overshoot in Stanton number; $x_{overshoot} = 0.27$ m and $x_{overshoot} = 0.37$ m for the warm- and cold-wall cases, respectively. For each profile, the magnitudes for the cold-wall are larger than those of the warm-wall. For the streamwise component of the cold wall, an initial increase in turbulent heat flux near the wall (at $z^+ \approx 20$) is observed before the quantity decreases to negative values and eventually settles around zero in the freestream. This initial increase to positive heat flux is not as prominent in the warm-wall case. Furthermore, a slight decrease in the spanwise component for the cold case is observed close to the wall, which is not as noticeable for the warm case.

Figure 27 shows the root mean square (rms) of wall pressure fluctuations normalized by the local wall pressure for

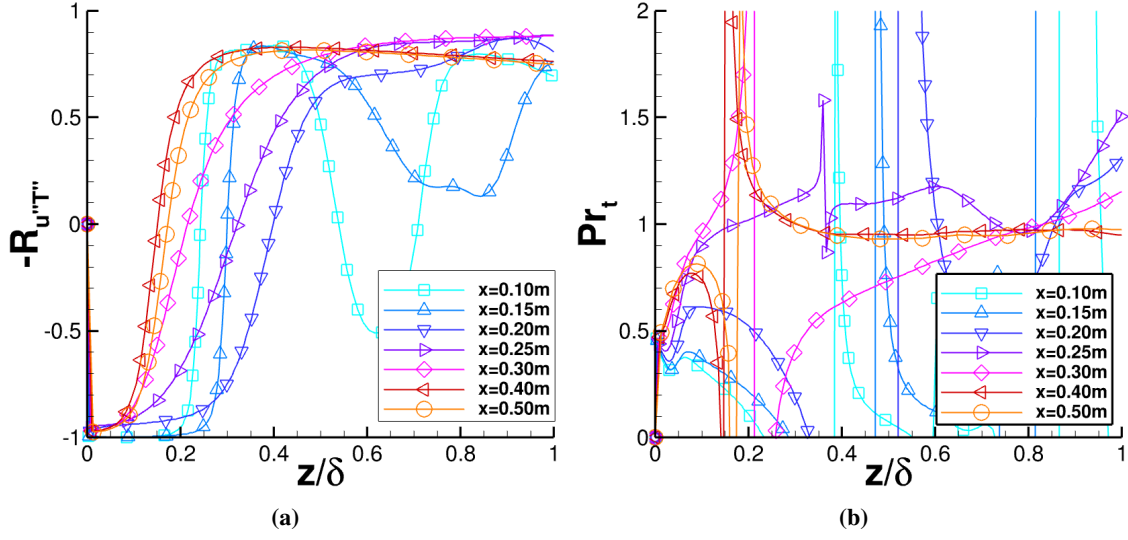


Fig. 22 Comparisons of the (a) temperature-velocity correlation coefficient ($R_{u''T''}$) and the (b) turbulent Prandtl number (Pr_t) at different wall-normal profiles. Case: Tw84.

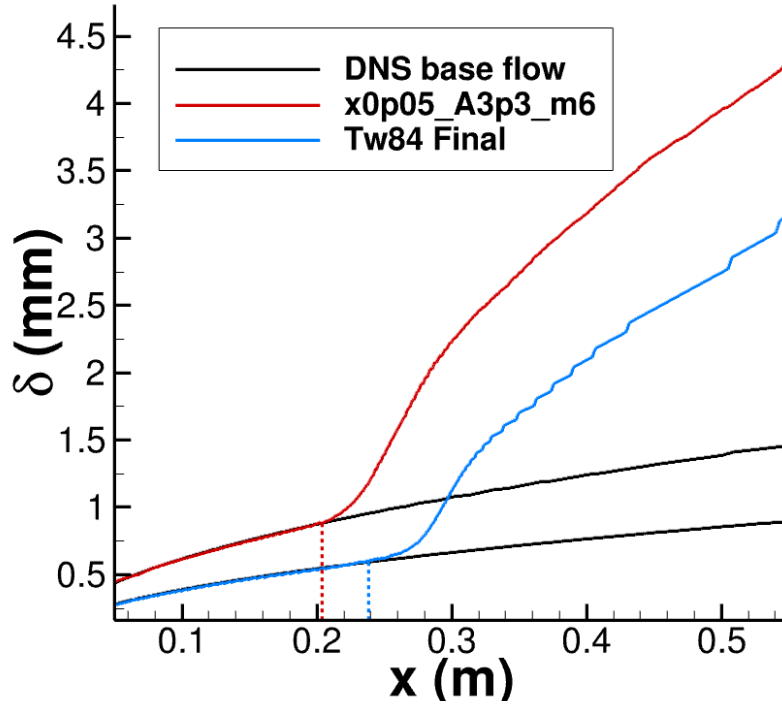


Fig. 23 Boundary layer thickness (δ) comparison between the warm-wall (Tw298) and cold-wall (Tw84) cases with respective laminar base flows. The dotted lines show the transition location of each scenario.

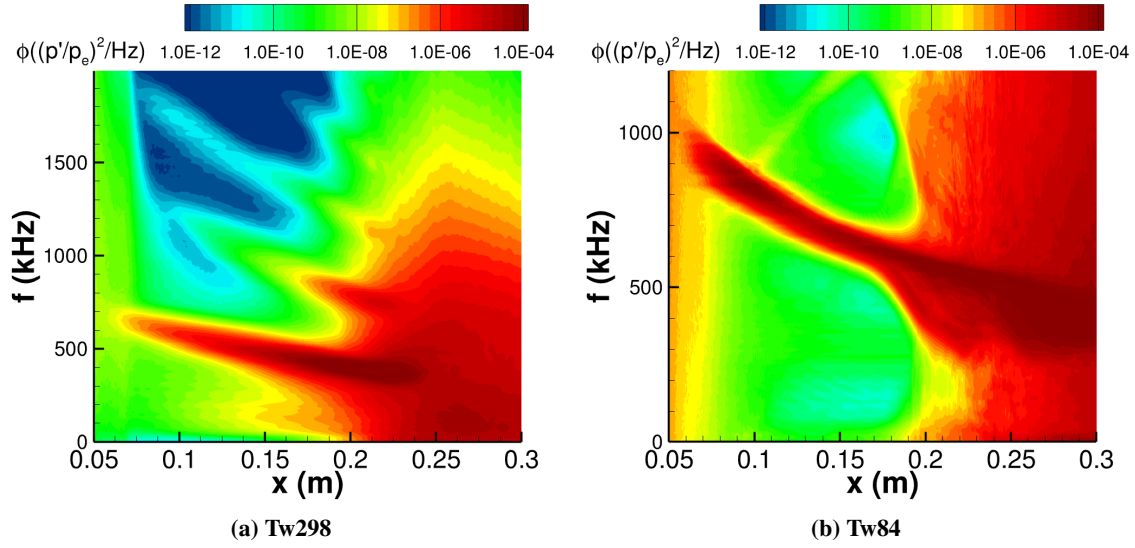


Fig. 24 Frequency PSD contour comparison between the warm-wall (Tw298) and cold-wall (Tw84) cases.

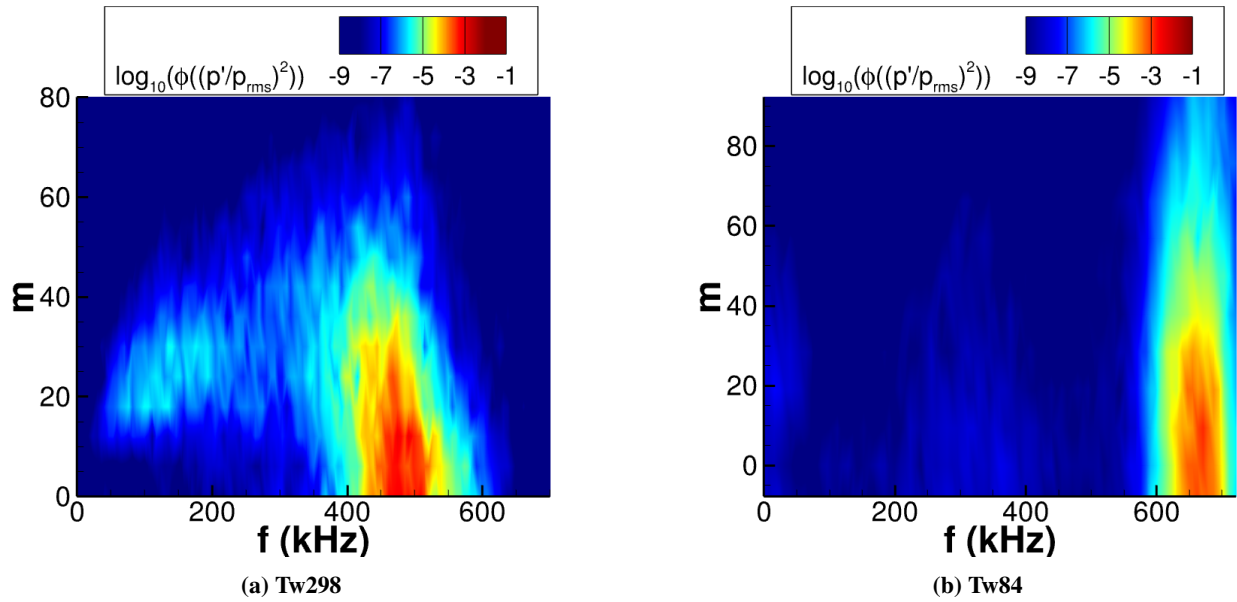


Fig. 25 Comparison of warm-wall and cold-wall DNS frequency/azimuthal-wavenumber ($f - m$) spectrum of the wall-pressure fluctuations at $x = 0.15$ m, slightly upstream of the measured transition locations. Note the logarithmic scale of the horizontal axis.

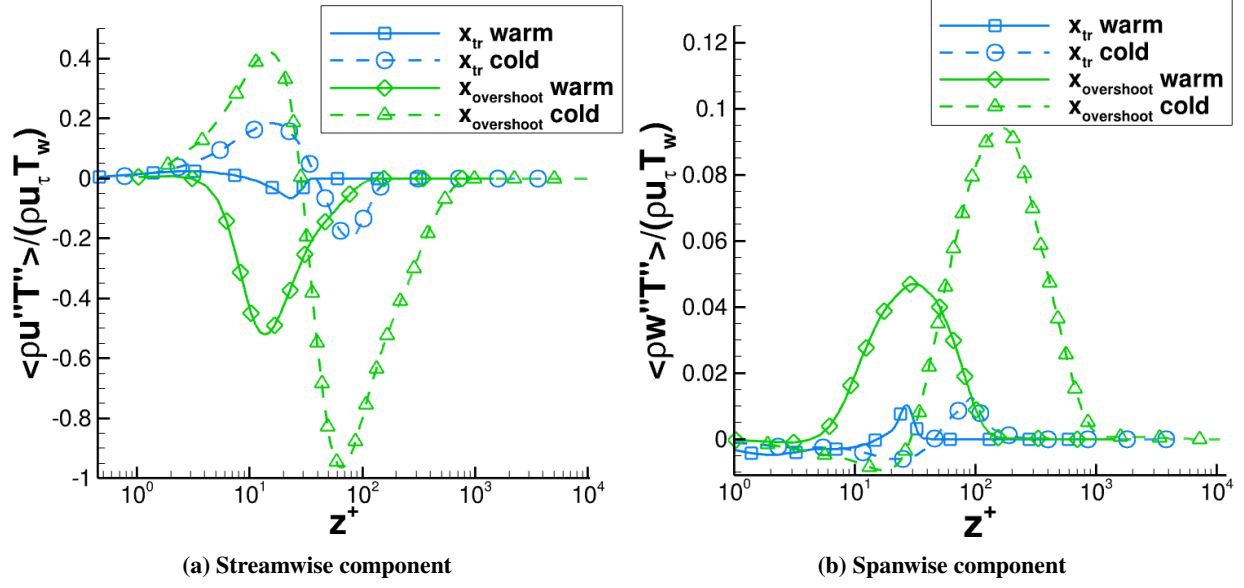


Fig. 26 Comparison of (a) streamwise and (b) wall-normal turbulent heat flux components at similar locations for the warm-wall (Tw298) and cold-wall (Tw84) cases.

each case. For the warm wall, an exponential increase in $(P'_w)_{rms}/P_w$ is seen until $x = 0.22$ m. The values then decrease and settle around $(P'_w)_{rms}/P_w = 0.069$. The cold wall shows a similar trend qualitatively, but with a notable peak in wall pressure fluctuations around $x = 0.10$ m followed by a secondary peak at around $x = 0.31$ ($\approx 40\%$ further downstream and with slightly higher amplitude than the warm wall). The cold-wall values stabilize at around $(P'_w)_{rms}/P_w = 0.10$, which is 45% higher than the warm wall.

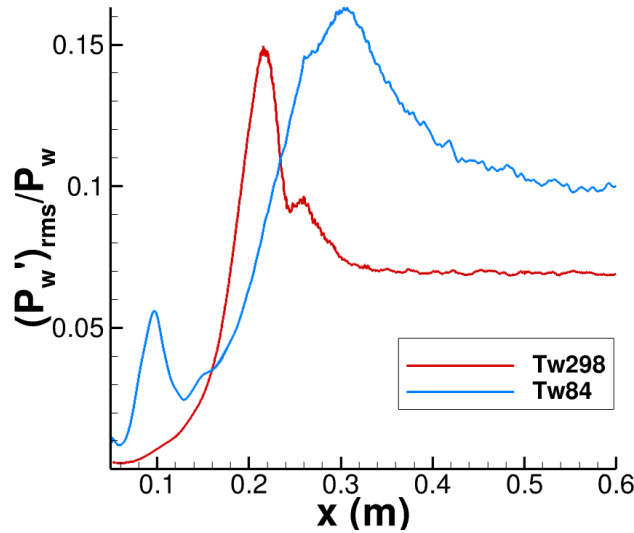


Fig. 27 Root-mean-square of the wall pressure fluctuations for the cold and warm walls.

Figure 28 shows the mean Stanton number (St) and skin friction coefficient (C_f) for the two wall temperatures. The VULCAN-CFD laminar and turbulent (using the Menter SST model) wall quantities are also plotted for reference. While both cases follow similar trends, the peak values are reached further downstream for the cold wall. The difference between Stanton number and skin friction coefficient curves obtained from the DNS simulation between both cases are much less than the differences as predicted by the RANS turbulence model. The post-transition Stanton number for the cold-wall case settles at a higher value than for the warm-wall case despite the smaller St overshoot. Skin friction

coefficient for the cold-wall has a lower peak and downstream value than for the warm-wall case despite the turbulence model predicting higher C_f values.

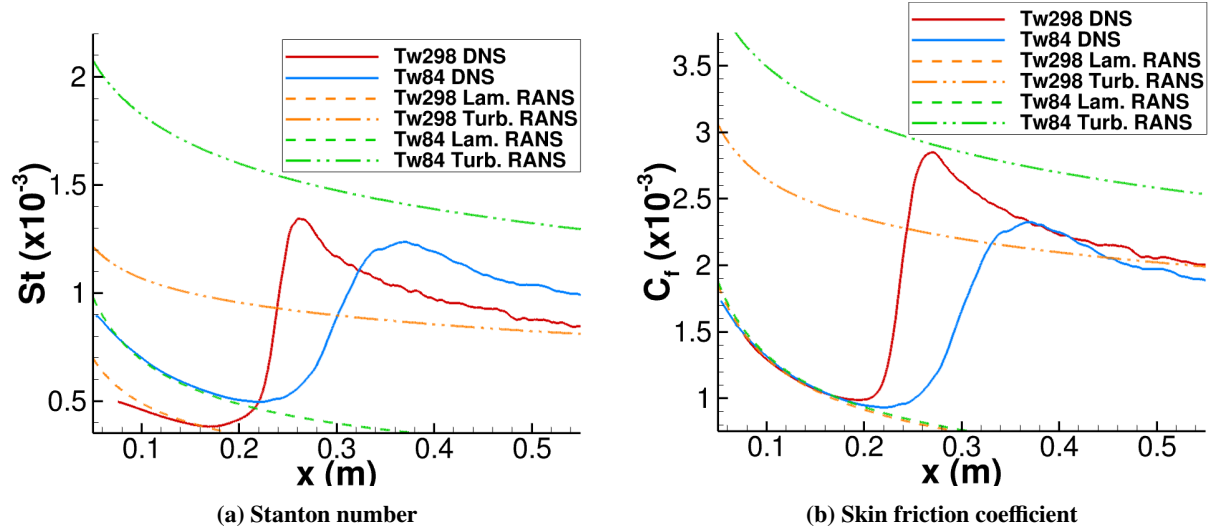


Fig. 28 Comparison of (a) Stanton number and (b) skin friction coefficient for the warm wall (Tw298) and cold-wall (Tw84) cases.

V. Conclusions

Linear parabolized stability equations (PSE) and direct numerical simulations (DNS) are applied to the Mach 6.1 flow over a sharp-cone model with both warm- and cold-wall temperatures to identify the relevant instability mechanisms that lead to the experimentally observed transition delay due to wall cooling. PSE results suggest a different transition breakdown mechanism may be present in each configuration due to the change from oblique to planar dominant instability waves. For the warm-wall case, both Mack's first- and second-mode waves are amplified, with the oblique Mack's second-mode waves being the most amplified disturbance at the transition location. However, for the cold-wall cone, Mack's first-mode waves are predicted to be stable and only the high-frequency Mack's second mode waves are amplified.

Complementary DNS with broadband random inflow forcing and sufficiently large spanwise domains confirm the amplification of oblique Mack's first-mode waves in the warm-wall case corresponding to those predicted by PSE, in addition to the dominant Mack's second-mode and its superharmonics. DNS of the cold-wall case also validates PSE predictions that Mack's first-mode is stabilized for a highly-cooled wall. The cold-wall DNS identifies higher second-mode frequencies due to the thinner boundary layer that forms over the cool wall. A 17% delay in transition location for the cold cone is observed from the DNS results, along with diminished oblique waves and a dominant second Mack-mode instability. The current linear PSE and DNS studies suggest that the inclusion of oblique waves and nonparallel effects leads to a significantly higher amplification of the boundary-layer instabilities for the warm-wall case compared to previous results using the quasiparallel Linear Stability Theory (LST) of just planar waves.

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Appendix

Reducing the forcing very near the wall was found to be necessary to feasibly run the cold-wall case at comparable conditions to the primary, warm-wall, case. In order to assure this reduction of the forcing domain would not significantly affect the flow's transition to turbulence an inexpensive test was set up on an auxiliary case denoted x0p05_A3p3 consisting of a smaller azimuthal domain than the main warm-wall (Tw298) case described in the rest of this paper. The new case, referred to as x0p05_A3p3_RF, has a reduced forcing domain near the wall by approximately the same percentage of the boundary layer as was done in the cold-wall case. The first 15 wall-normal points were omitted from the forcing domain while every other aspect between the two simulations remained the same. The study found that the small reduction in forcing domain had almost no impact on the flow. Figure 29 shows the boundary layer thickness in both forcing scenarios. There is no difference in transition location, and only a negligible difference in the overall boundary layer thickness over the first 0.3 m of the cone. In summary, the reduced forcing domain in the cold-wall case will effectively produce the same results as it would with forcing an entire streamwise plane while improving the stability of the DNS.

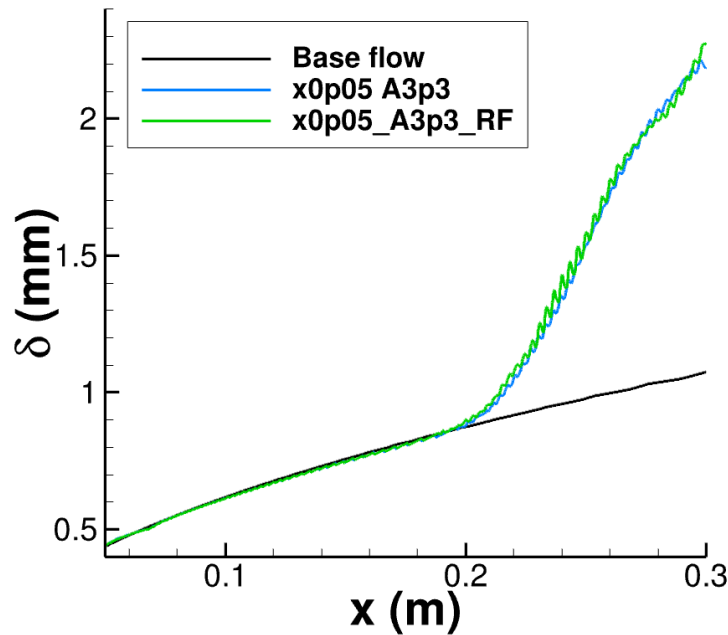


Fig. 29 Comparison of the boundary-layer thicknesses for the two scenarios.

References

- [1] Schneider, S. P., "Hypersonic Laminar-turbulent Transition On Circular Cones And Scramjet Forebodies," *Progress in Aerospace Sciences*, Vol. 40, No. 1-2, 2004. doi:10.1016/j.paerosci.2003.11.001.
- [2] Unnikrishnan, S., and Gaitonde, D. V., "Instabilities And Transition In Cooled Wall Hypersonic Boundary Layers," *Journal of Fluid Mechanics*, Vol. 915, 2021, p. A26. doi:10.1017/jfm.2021.84.
- [3] Holden, M., Wadhams, T., MacLean, M., and Mundy, E., "Reviews Of Studies Of Boundary Layer Transition In Hypersonic Flows Over Axisymmetric And Elliptic Cones Conducted In The CUBRC Shock Tunnels," 47th AIAA Aerospace Sciences Meeting Paper, 2009. doi:10.2514/6.2009-782.
- [4] Kimmel, R., Adamczak, D., Borg, M., Jewell, J., Juliano, T., Stanfield, S., and Berger, K., "First And Fifth Hypersonic International Flight Research Experimentation's Flight And Ground Tests," *Journal of Spacecraft and Rockets*, 2019. doi:10.2514/1.A34287.
- [5] Hameed, A., Parziale, N., Paquin, L., Butler, C., and Laurence, S., "Spectral Analysis Of A Hypersonic Boundary Layer On A Right, Circular Cone," *AIAA Journal*, Vol. 61, No. 2, 2022. doi:10.2514/1.J061846.

- [6] Major, M. M., "Effects Of Cone Tip Changes On Wall-Cooled Hypersonic Boundary Layer Transition And Turbulence," Master's thesis, Air Force Institute of Technology, Wright-Patterson AFB, OH, 2022. <https://scholar.afit.edu/etd/5441>.
- [7] Blanchard, A. E., and Selby, G. V., "An Experimental Investigation Of Wall-Cooling Effects On Hypersonic Boundary-Layer Stability In A Quiet Wind Tunnel," Ph.D. thesis, Old Dominion University, Norfolk, VA, 1996. https://digitalcommons.odu.edu/mae_etds/110.
- [8] Stetson, K., Thompson, E., Donaldson, J., and Siler, L., "Laminar Boundary Layer Stability Experiments On A Cone At Mach 8. V-Tests With A Cooled Model," *20th Fluid Dynamics, Plasma Dynamics and Lasers Conference*, 1989, p. 1895. doi:10.2514/6.1989-1895.
- [9] Paquin, L. A., Skinner, S., and Laurence, S. J., "Boundary-Layer Disturbances And Surface Heat-Flux Profiles On A Cooled Slender Cone," *AIAA SCITECH 2022 Forum*, 2022, p. 1709. doi:10.2514/6.2022-1709.
- [10] Siddiqui, F., Gragston, M., and Bowersox, R. D., "Measurements Of The Influence Of Wall Cooling On Boundary-Layer Transition On A Flared Cone At Mach 6 Using Focused Laser Differential Interferometry," *AIAA SCITECH 2022 Forum*, 2022, p. 1825. doi:10.2514/6.2022-1825.
- [11] Oddo, R., "High-Speed Schlieren Imaging Of Second Mode Disturbances In A Super-Cooled Hypersonic Boundary Layer," Master's thesis, Air Force Institute of Technology, Wright-Patterson AFB, OH, 2020.
- [12] Oddo, R., Hill, J. L., Reeder, M. F., Chin, D., Embrador, J., Komives, J., Tufts, M., Borg, M., and Jewell, J. S., "Effect Of Surface Cooling On Second-Mode Dominated Hypersonic Boundary Layer Transition," *Experiments in Fluids*, Vol. 62, No. 7, 2021, p. 144. doi:10.1007/s00348-021-03237-0.
- [13] Embrador, J., "Effects Of Highly-Cooled Walls On Hypersonic Boundary Layer Transition And Turbulence," Master's thesis, Air Force Institute of Technology, Wright-Patterson AFB, OH, 2021.
- [14] Clingenpeel, J. T., "Wall-Cooling Effect On Boundary-Layer Instability Growth And Transition On A Sharp Cone In Mach-6 Flow," Master's thesis, University of Dayton, 2023. https://ecommons.udayton.edu/graduate_theses/7341.
- [15] Baurle, R. A., and Edwards, J. R., "Hybrid Reynolds-Averaged/large-Eddy Simulations Of Hypersonic Turbulent Flows," *AIAA Journal*, Vol. 58, No. 6, 2020, pp. 2567–2580. doi:10.2514/1.J055339.
- [16] Scholten, A., "Modal And Nonmodal Stability Analysis Of Hypersonic Flows Over Canonical Geometries," Ph.D. thesis, North Carolina State University, Raleigh, NC, 2024. <https://www.lib.ncsu.edu/resolver/1840.20/44347>.
- [17] Davami, J., Juliano, T. J., Scholten, A., Paredes, P., Benitez, E. K., Running, C. L., Little, N., Duan, L., Dylewicz, K., Pezlar, V., Theofilis, V., Thiele, T., and Willems, S., "Separation And Transition On The ROTEX-T Cone-Flare," *AIAA Journal*, Vol. 63, No. 9, 2025, pp. 3939–3966. doi:10.2514/1.J064190.
- [18] Chu, B.-T., "On The Energy Transfer To Small Disturbances In Fluid Flow (Part I)," *Acta Mechanica*, 1965. doi:10.1007/BF01387235.
- [19] Herbert, T., "Parabolized Stability Equations," *Annual Review of Fluid Mechanics*, Vol. 29, 1997, pp. 245–283. doi:10.1146/annurev.fluid.29.1.245.
- [20] Paredes, P., Choudhari, M., and Li, F., "Instability Wave-Streak Interactions In A Supersonic Boundary Layer," *Journal of Fluid Mechanics*, Vol. 831, 2017, pp. 524–553. doi:10.1017/jfm.2017.630.
- [21] Jiang, G., and Shu, C., "Efficient Implementation Of Weighted ENO Schemes," *Journal of Computational Physics*, Vol. 126, No. 1, 1996. doi:10.1006/jcph.1996.0130.
- [22] Williamson, J., "Low-Storage Runge-Kutta Schemes," *Journal of Computational Physics*, Vol. 35, No. 1, 1980. doi:10.1016/0021-9991(80)90033-9.
- [23] Liu, Y., Schuabb, M., Duan, L., Paredes, P., and Choudhari, M., "Interaction Of A Tunnel-Like Acoustic Disturbance Field With A Blunt Cone Boundary Layer At Mach 8," *AIAA AVIATION 2022 Forum*, 2022. doi:10.2514/6.2022-3250.
- [24] Schuabb, M., Duan, L., Scholten, A., Paredes, P., and Choudhari, M. M., "Hypersonic Boundary-Layer Transition Over A Blunt Circular Cone In A Mach 8 Digital Wind Tunnel," *AIAA SCITECH 2024 Forum*, 2024. doi:10.2514/6.2024-2181.
- [25] Huang, J., Duan, L., Casper, K. M., Wagnild, R. M., and Bitter, N. P., "Transducer Resolution Effect On Pressure Fluctuations Beneath Hypersonic Turbulent Boundary Layers," *AIAA Journal*, Vol. 62, No. 3, 2024, pp. 882–895. <https://doi.org/10.2514/1.J062994>.

- [26] Thompson, K. W., "Time Dependent Boundary Conditions For Hyperbolic Systems," *Journal of Computational Physics*, Vol. 68, No. 1, 1987, pp. 1–24. [https://doi.org/10.1016/0021-9991\(87\)90041-6](https://doi.org/10.1016/0021-9991(87)90041-6).
- [27] Balakumar, P., and Kegerise, M. A., "Receptivity Of Hypersonic Boundary Layers Over Straight And Flared Cones," *AIAA Journal*, Vol. 53, No. 8, 2015, pp. 2097–2109. doi:10.2514/1.J053432.
- [28] Balakumar, P., and Chou, A., "Transition Prediction In Hypersonic Boundary Layers Using Receptivity And Freestream Spectra," 54th AIAA Aerospace Sciences Meeting, 2016. doi:10.2514/6.2016-0847.
- [29] Goparaju, H., Unnikrishnan, S., and Gaitonde, D. V., "Effects Of Nose Bluntness On Hypersonic Boundary-Layer Receptivity And Stability," *Journal of Spacecraft and Rockets*, 2021, pp. 1–17. doi:10.2514/1.A34829.
- [30] Hader, C., and Fasel, H. F., "Towards Simulating Natural Transition In Hypersonic Boundary Layers Via Random Inflow Disturbances," *Journal of Fluid Mechanics*, Vol. 847, 2018. doi:10.1017/jfm.2018.386.
- [31] Ran, W., Zare, A., Hack, M. P., and Jovanović, M. R., "Stochastic Receptivity Analysis Of Boundary Layer Flow," *Physical Review Fluids*, Vol. 4, No. 9, 2019, p. 093901. doi:10.1103/PhysRevFluids.4.093901.
- [32] Lugin, M., Beneddine, S., Leclercq, C., Garnier, E., and Bur, R., "Transition Scenario In Hypersonic Axisymmetrical Compression Ramp Flow," *Journal of Fluid Mechanics*, Vol. 907, 2021, p. A6. doi:10.1017/jfm.2020.833.
- [33] Cao, S., Hao, J., Guo, P., Wen, C.-Y., and Klioutchnikov, I., "Stability Of Hypersonic Flow Over A Curved Compression Ramp," *Journal of Fluid Mechanics*, Vol. 957, 2023, p. A8. doi:10.1017/jfm.2023.56.
- [34] Little, N., Duan, L., Schuabb, M., Bisek, N. J., Scholten, A., and Paredes, P., "Wall-Cooling Effects On Hypersonic Boundary-Layer Instabilities And Transition," AIAA AVIATION FORUM AND ASCEND 2025, 2025. doi:10.2514/6.2025-3451.
- [35] Casper, K. M., Beresh, S. J., Henfling, J. F., Spillers, R. W., and Pruett, B. O. M., "Hypersonic Wind-Tunnel Measurements Of Boundary-Layer Transition On A Slender Cone," *AIAA Journal*, Vol. 54, No. 4, 2016, pp. 1250–1263. <https://doi.org/10.2514/1.J054033>.
- [36] Franko, K. J., and Lele, S. K., "Breakdown Mechanisms And Heat Transfer Overshoot In Hypersonic Zero Pressure Gradient Boundary Layers," *Journal of Fluid Mechanics*, Vol. 730, 2013, pp. 491–532. <https://doi.org/10.1017/jfm.2013.350>.
- [37] Schuabb, M., Duan, L., Casper, K. M., Wagnild, R. M., Choudhari, M. M., and Paredes, P., "Hypersonic Boundary-Layer Transition Over A Circular Cone In A Mach 8 Digital Wind Tunnel," *Journal of Fluid Mechanics*, Vol. 1017, 2025, p. A33. doi:10.1017/jfm.2025.369.