

Delay of Crossflow Transition on a Yawed Cone at Mach 6

Pedro Paredes,* Meelan M. Choudhari[†] Fei Li[‡]
NASA's Langley Research Center, Hampton, VA 23681, USA

A combination of stability analysis methods are used to predict and control transition in a Mach 6 flow over a 0.5 m long, 7-degree half-angle cone with an angle of attack (or, equivalently, a yaw angle) of 6 degrees and freestream Reynolds number of 10.8 million per meter. The boundary layer on the yawed cone supports the amplification of both Mack's second mode (MM) and crossflow (CF) instabilities. While MM is moderately amplified on the windward side, stationary CF (SCF) vortices reach a quiet-flow transition N-factor as early as 0.33 m from the nose tip and at an azimuthal angle of 148 degrees from the windward side. A receptivity analysis is used to identify the effective CF initial amplitude as a function of the spanwise wavelength associated with the natural stochastic surface roughness. The inferred initial amplitudes are used to predict the nonlinear evolution of SCF vortices and their secondary instability characteristics, resulting in the identification of the SCF mode that is most likely to induce CF-driven transition. Control input in the form of subdominant CF vortices is superimposed on the naturally dominant SCF mode to delay transition by controlling the amplification of the target mode without causing premature transition due to excessive control input. Results show that, contrary to previous studies of low-speed swept-wing configurations that used shorter control mode wavelengths than the dominant CF vortex, the present yawed circular cone configuration shows potential for significant transition delay when using control modes with larger wavelengths. A potential transition delay of 41% is predicted for control modes with twice the wavelength of the dominant CF vortex.

Nomenclature

AoA	=	angle of attack [$^{\circ}$]
C_k	=	roughness receptivity coefficient
E	=	disturbance energy norm
h_R	=	roughness height distribution [m]
k	=	normalized maximum roughness height, $k = k_R/k_{ref}$
k_R	=	roughness height [m]
k_{ref}	=	reference roughness height [m]
L_R	=	roughness patch length [m]
M	=	Mach number
N	=	N-factor
$\bar{\mathbf{q}}$	=	vector of flow variables
$\tilde{\mathbf{q}}$	=	vector of perturbation variables
$\hat{\mathbf{q}}$	=	vector of amplitude function variables
Re_{∞}	=	unit Reynolds number [m^{-1}]
T	=	temperature [K]
(u, v, w)	=	streamwise, wall-normal, and spanwise velocity components [m s^{-1}]
u_I	=	magnitude of streamwise velocity perturbation at neutral location normalized by freestream velocity
(x, y, z)	=	streamwise, wall-normal, and spanwise body-oriented coordinates [m]
(X, Y, Z)	=	Cartesian coordinates [m]
(α, β)	=	disturbance streamwise and spanwise wavenumbers [m^{-1}]
α_R	=	streamwise wavenumber of roughness patch [m^{-1}]
ω	=	disturbance angular frequency [rad s^{-1}]

*Research Scientist, Computational AeroSciences Branch. AIAA Associate Fellow

[†]Research Scientist, Computational AeroSciences Branch. AIAA Fellow

[‡]Research Scientist, Computational AeroSciences Branch

ρ	=	density [kg m ⁻³]
(ξ, η, ζ)	=	streamwise, wall-normal, and spanwise computational coordinates [m]
λ	=	local spanwise wavelength [m]
ϕ	=	azimuthal coordinate [°]
Subscripts		
0	=	reference location
c	=	control mode
I	=	neutral location
p	=	primary instability
s	=	secondary instability
t	=	target mode
T	=	transition location
w	=	wall
∞	=	freestream values

I. Introduction

Laminar flow technology is recognized as a potential breakthrough for improved aerothermodynamic performance of hypersonic vehicles by the reduction of aerodynamic drag and aeroheating. This translates into reduced cost for access to space, increased range, reduced weight of hypersonic vehicles, and improved reliability because of increased safety margin against potential loss of vehicle due to a failure of thermal protection systems. Heat-transfer and wall-shear loads can increase by up to a factor of eight as the boundary-layer transitions from laminar to turbulent in attached flow over a hypersonic vehicle. The drop in aerothermodynamic performance of high-speed vehicles due to boundary-layer transition (BLT) has been reported to decrease the payload-to-weight ratio by up to three times compared to a fully laminar flow [1, 2].

Under low levels of background disturbances, transition is initiated by the exponential amplification of linearly unstable eigenmodes, i.e., modal instabilities of the laminar boundary layer. In two-dimensional boundary layers over sufficiently smooth aerodynamic surfaces, different types of instability mechanisms dominate the exponential growth phase depending on the flight speed. In the incompressible regime, the most amplified disturbances correspond to planar, i.e., two-dimensional, Tollmien-Schlichting (TS) waves, whereas Mack’s oblique first-mode instabilities are dominant in supersonic boundary layers. The hypersonic regime is again dominated by the growth of planar Mack’s second-mode waves [3]. In three-dimensional boundary layers, the transition process can also be initiated by stationary and/or traveling crossflow (CF) instabilities. Stationary CF vortices are efficiently excited by surface roughness, and a possible path to transition corresponds to rapid amplification of the high-frequency secondary instabilities of finite-amplitude stationary CF vortices [4].

The canonical configuration of a circular cone at angle of attack has been widely used in wind-tunnel experiments to study CF-induced transition in the context of both supersonic [e.g., 5–7] and hypersonic [e.g., 8–16] conditions. Computations of the nonlinear development of CF vortices and their secondary instability over a circular cone at angle of attack in a Mach 6 flow have been reported by Refs. [17–21]. These works predicted the measured frequencies and identified instability modes that originate from low-frequency traveling CF modes and from high-frequency Mack’s second mode instabilities of the unperturbed boundary layer. These disturbances get modulated and further destabilized by the presence of CF vortices. Furthermore, high-frequency shear-layer instability modes become unstable due to the shear layers that bound the large amplitude CF vortices.

The experimental study in a conventional hypersonic facility by Corke et al. [15] has demonstrated the sensitivity of CF-induced transition location to the parameters of an azimuthally periodic roughness array near the nose tip for a 7-degree half-angle cone at an angle of attack of 6 degrees. However, their study was limited to the comparison of transition locations between two arrays with different azimuthal spacings, and no measurements were reported regarding whether or not transition was delayed with respect to the baseline case with no roughness array. The two roughness arrays used for comparison included one array with the so called “critical” spacing that matched the azimuthal wavelength of the naturally most amplified CF modes and the other array with a “subcritical” spacing had an azimuthal spacing equal to 2/3 of the “critical” spacing. The measurements showed significant differences between the transition locations in the two cases, indicating that the initial amplitude of the CF disturbances can be manipulated via an appropriate set of roughness elements. However, in both cases, transition actually occurred earlier than in the unperturbed case wherein the boundary layer remained laminar throughout the range of tested flow conditions. The present computational study

employs discrete roughness elements (DRE) to generate subdominant CF modes, which reduce the growth of naturally dominant CF vortices. By mitigating these stationary vortices, the strategy weakens the amplification of secondary instabilities driving laminar breakdown, likely delaying the onset of transition in the flow over a circular cone at angle of attack.

The paper is organized as follows. Section II provides a summary of the linear and nonlinear stability analysis methods used in this paper. Section III presents the laminar flow solution over a 7-degree half-angle cone at 6 degrees angle of attack in a Mach 6 flow with a freestream Reynolds number of $10.8 \times 10^6 \text{ m}^{-1}$ corresponding to experiments conducted at the Purdue University Boeing/AFOSR Mach-6 Quiet Tunnel (BAM6QT) [12, 16, 22]. Furthermore, the linear instability characteristics of the boundary layer, the linear receptivity to roughness elements, and the secondary instability of dominant stationary CF vortex are investigated. Then, the explored transition control strategies and the results pertaining to the predicted transition delay with the selected control input are presented. Section IV provides a summary of this study and its results.

II. Theory

The onset of transition is usually estimated by using the conventional method of N-factor correlation based on the linear amplification of primary instability modes corresponding to SCF vortices. This linear transition criterion is expected to work well for predicting transition over aerodynamic surfaces with similar levels of surface finish (characterized by a similar wavenumber spectra of the roughness height distribution), but it does not account for either the effect of roughness variations on the baseline transition location or for the nonlinear interactions between the naturally excited CF vortices and the artificially seeded subdominant vortex modes that are at the heart of the transition control technique. Only a prediction criterion that accounts for the receptivity characteristics as well as for the simultaneous evolution of multiple CF disturbances and its effect on the laminar breakdown process is useful for investigating the various scenarios relevant to the optimization of the transition control via discrete roughness elements. Previous work [23] has shown that a transition criterion based on the N-factor envelope for the secondary instabilities of finite-amplitude CF vortices includes all of the critical processes inherent to CF transition. References [23–26] have reported partial progress toward utilizing more comprehensive modeling of the CF transition process to analyze DRE-based transition control on subsonic and supersonic wings. However, the present work represents the first attempt to integrate all aspects of the holistic transition criterion in the context of controlling CF transition. Therefore, we first outline the stability analysis methods used to identify the primary driver for transition in the baseline, i.e., uncontrolled case, and to assess various candidate strategies for transition control.

A. Stability Analysis

To predict the evolution of small-amplitude disturbances to a laminar basic state, we decompose the vector of flow variables, $\mathbf{q} = (\rho, u, v, w, T)^T$ into

$$\mathbf{q}(x, y, z, t) = \bar{\mathbf{q}}(x, y, z) + \tilde{\mathbf{q}}(x, y, z, t), \quad (1)$$

where the vector $\bar{\mathbf{q}} = (\bar{\rho}, \bar{u}, \bar{v}, \bar{w}, \bar{T})^T$ denotes the basic state variables and $\tilde{\mathbf{q}} = (\tilde{\rho}, \tilde{u}, \tilde{v}, \tilde{w}, \tilde{T})^T$ represents the vector of perturbation variables. The computational coordinates, (x, y, z) denote a curvilinear, orthogonal body-fitted coordinate system, where x denotes the tangent to the surface in the prescribed trajectory, y denotes the normal to the surface, and z represents the spanwise direction. The corresponding velocity components are (u, v, w) . Density and temperature are denoted by ρ and T , respectively. The azimuthal angle with origin at the windward side is denoted by ϕ . The Cartesian coordinates are represented by (X, Y, Z) . The metric factors are defined as

$$h_x = 1 + \kappa_x y, \quad (2)$$

$$h_z = (1 + \kappa_z y) \frac{\lambda}{\lambda_0}, \quad (3)$$

where κ_x denotes the streamwise curvature, κ_z denotes the azimuthal (i.e., spanwise) curvature, and λ/λ_0 denotes the prescribed evolution of the spanwise disturbance wavelength λ , normalized by the wavelength λ_0 at an arbitrary reference streamwise position, x_0 . See Fig. 1 for a schematic of circular cone at an angle of attack.

The onset of laminar-turbulent transition is estimated via the e^N methodology [27], [28], which correlates the transition location with the peak logarithmic amplification ratio, the so-called N -factor, relative to the neutral location x_I where the disturbance first becomes unstable,

$$N_\psi = \ln [\tilde{\psi}(x)/\tilde{\psi}(x_I)], \quad (4)$$

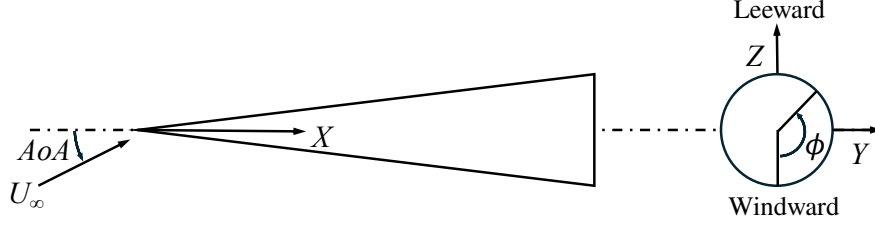


Fig. 1 Schematic of the yawed circular cone.

where ψ denotes a suitable amplitude norm of the perturbations at a given station, e.g., the local maximum of the streamwise velocity perturbation, $|\tilde{u}|_{max}$, or the square root of Chu's energy norm [29, 30], which is defined as

$$E(x) = \int_z \int_y \tilde{\mathbf{q}}^H \mathbf{M}_E \tilde{\mathbf{q}} h_x dy dz, \quad \mathbf{M}_E = \text{diag} \left[\frac{\bar{T}}{\gamma \bar{\rho} M^2}, \bar{\rho}, \bar{\rho}, \frac{\bar{\rho}}{\gamma(\gamma-1)\bar{T}M^2} \right]. \quad (5)$$

B. Primary Stability Analysis

To analyze the linear instability characteristics of three-dimensional boundary layer over a yawed cone, the linear, line-marching parabolized stability equations (PSE) are employed. These equations are integrated along inflection lines for stationary crossflow (SCF) vortices and along streamlines for Mack's second mode (MM) disturbances. The alignment of SCF disturbances with the inflection lines defined as vortex trajectories was verified in Refs. [31, 32] against direct numerical simulations [33] and experimental measurements [13]. Similar to Ref. [34], the variation of spanwise wavelength based on the diffuence of the inflection lines is included in the PSE analysis. The integration path for the MM disturbances is commonly selected along streamlines [35–37] because planar waves are more amplified than oblique waves in two-dimensional boundary layers [3].

The basic state variables are assumed to be homogeneous along the spanwise direction normal to the prescribed trajectory, $\tilde{\mathbf{q}}(x, y)$, and the conventional or line-marching PSE are used. In this context, the perturbations have the form

$$\tilde{\mathbf{q}}(x, y, z, t) = \hat{\mathbf{q}}(x, y) \exp \left[i \left(\int \alpha(x') dx' + \beta z - \omega t \right) \right], \quad (6)$$

where β denotes the spanwise wavenumber at the reference streamwise position, x_0 ($\beta = 2\pi/\lambda_0$), and ω is the angular frequency of the perturbation. The unknown, streamwise wavenumber $\alpha(x)$ is determined in the course of the solution by imposing an additional constraint

$$\int_y \hat{\mathbf{q}}^* \frac{\partial \hat{\mathbf{q}}}{\partial x} h_x dy = 0, \quad (7)$$

where the amplitude functions $\hat{\mathbf{q}}(x, y)$ vary slowly in the streamwise direction in comparison with the phase term $\exp [i \int \alpha(x') dx']$. The variation of the spanwise spacing between the integration trajectories defines the evolution of the spanwise disturbance wavelength λ from Eq. (3). The spanwise homogeneous assumption has been found to provide reasonable predictions of the amplification of SCF ($\omega = 0$) and planar MM ($\beta = 0$) disturbances along inflection lines and streamlines, respectively, over a three-dimensional configuration [31, 32, 34, 35, 37, 38].

The receptivity to small height surface roughness of various forms is computed by imposing a suitable form of inhomogeneous boundary conditions [39] on the harmonic linearized Navier-Stokes equations (HLNSEs) [40–42]. The HLNSE do not assume the separation of scales used in the PSE approach, and the perturbations have the following form for the present application with a two-dimensional basic state and zero disturbance frequency,

$$\tilde{\mathbf{q}}(x, y, z) = \hat{\mathbf{q}}(x, y) \exp [i\beta z]. \quad (8)$$

Choudhari [39] showed that, for a specified height of the surface roughness, the localized and wavy roughness distributions provide lower and upper bounds, respectively, for the effective initial amplitudes of the SCF vortices excited by the roughness. The inhomogeneous boundary condition models the roughness-induced forcing. For small-amplitude perturbations induced by suitably small roughness heights, the vector of perturbation variables at the wall can be set equal to

$$\tilde{\mathbf{q}}_w(x, z) = -h_R(x) \frac{\partial \tilde{\mathbf{q}}}{\partial y} \Big|_w \exp [i\beta z], \quad (9)$$

where $h_R(x)$ is the roughness height distribution defined as

$$h_R(x) = \frac{k_R}{2} \{ \operatorname{erf} [\sigma_x (x - x_R + L_R/2)] - \operatorname{erf} [\sigma_x (x - x_R - L_R/2)] \} \cos (\alpha_R (x - x_R)), \quad (10)$$

where k_R , x_R , L_R and α_R are the roughness patch height, center location, length, and streamwise wavenumber, respectively, $\sigma_x = \pi/(\Delta x)$ with $\Delta x = 0.1$ mm, and erf is the error function. The receptivity coefficient, $C_k = (u_I/k)_{\max}$, is defined as the maximum effective amplitude of induced streamwise disturbance velocity at the neutral location of the instability wave, $u_I = |\tilde{u}|_{\max}(x_I)/U_\infty$, scaled by $k = k_R/k_{ref}$, where $k_{ref} = 1 \mu\text{m}$ is a reference height value. For the calculation of the receptivity coefficient as a function of the disturbance wavelength, the most effective roughness shape is predicted by varying the roughness patch length (L_R) and streamwise wavenumber (α_R), with the center of the roughness patch held fixed at the neutral location of the selected instability wave $x_R = x_I$.

For disturbance calculations using the line-marching PSE and the two-dimensional HLNSE, a two-dimensional coordinate transformation of the form

$$\xi = \xi(x), \quad \eta = \eta(x, y, z) \quad (11)$$

is used to accommodate the boundary layer growth, such that

$$\partial(\bar{\mathbf{q}}, \hat{\mathbf{q}})/\partial\xi \ll \partial(\bar{\mathbf{q}}, \hat{\mathbf{q}})/\partial\eta, \quad (12)$$

where $\xi = x$ denotes the streamwise coordinate. The wall normal grid is scaled to account for streamwise variations in the boundary layer thickness, $y(\xi) = \eta\sqrt{\xi/\xi_0}$, where η represents the wall-normal grid at $\xi = \xi_0$, i.e., $\eta = \eta(\xi_0)$, which is designed to include the desired number of points within the boundary layer thickness.

C. Secondary Stability Analysis of Stationary Crossflow Instability

For the present study of crossflow-induced transition, the finite-amplitude SCF disturbances are superimposed on the unperturbed basic state along the inflection line trajectory. Therefore, the vector of fluid variables can be expressed as follows

$$\mathbf{q}(x, y, z, t) = \bar{\mathbf{q}}(x, y) + \tilde{\mathbf{q}}_p(x, y, z) + \tilde{\mathbf{q}}_s(x, y, z, t), \quad (13)$$

where the stationary, three-dimensional perturbation $\tilde{\mathbf{q}}_p$ corresponds to the finite-amplitude, crossflow vortex. and the traveling, three-dimensional perturbation $\tilde{\mathbf{q}}_s$ corresponds to the linear, secondary instability upon the perturbed basic state, $\bar{\mathbf{q}}(x, y) + \tilde{\mathbf{q}}_p(x, y, z)$. Similar to Refs. [43, 44], the plane-marching PSE are used to compute both the finite-amplitude perturbations $\tilde{\mathbf{q}}_p$ and their linear instabilities, $\tilde{\mathbf{q}}_s$.

In the plane-marching PSE, the perturbations are expressed in the form

$$\tilde{\mathbf{q}}(x, y, z, t) = \hat{\mathbf{q}}(x, y, z) \exp \left[i \left(\int \alpha(x') dx' - \omega t \right) \right], \quad (14)$$

where $\omega = 2\pi f$ is the angular frequency, f denotes the frequency of the perturbation, and α is a complex wavenumber along the x direction.

To compute the disturbance evolution using the plane-marching PSE, a three-dimensional coordinate transformation of the form

$$\xi = \xi(x), \quad \eta = \eta(x, y, z), \quad \zeta = \zeta(x, y, z) \quad (15)$$

is used to accommodate the boundary layer growth along the trajectory of the SCF vortex (approximated via the inflection line), such that

$$\partial(\bar{\mathbf{q}}, \hat{\mathbf{q}})/\partial\xi \ll \partial(\bar{\mathbf{q}}, \hat{\mathbf{q}})/\partial\eta, \partial(\bar{\mathbf{q}}, \hat{\mathbf{q}})/\partial\zeta. \quad (16)$$

Similar to the line-marching PSE analysis, the coordinate ξ corresponds to the streamwise coordinate x (i.e., $\xi = x$) and the wall-normal grid is scaled with the boundary layer thickness via $y(\xi) = \eta\sqrt{\xi/\xi_0}$. The spanwise domain at each streamwise location is translated to follow the vortex trajectory according to

$$z(\xi) = \zeta(\xi_0) + \int_{\xi_0}^{\xi} (\tan(\theta(\xi)) + \tan(\phi(\xi))) d\xi, \quad (17)$$

where $\zeta(\xi_0)$ is a uniform grid distribution with domain length corresponding to the crossflow wavelength λ_z , $\theta(\xi) = -\pi/2 + \arctan(\beta/\alpha(\xi))$, represents the angle between the chordwise and vortex directions as denoted in Fig. 1,

$\beta = 2\pi/\lambda_z$ is the spanwise wavenumber, $\alpha(\xi)$ is the streamwise wavenumber of the linear crossflow vortex as computed with linear, line-marching PSE, and $\phi(\xi)$ is calculated during the nonlinear computation to adjust for the nonlinear effects on the vortex trajectory, which is based on the spanwise location of the minimum wall shear as follows

$$\tan(\phi(\xi)) = \frac{d}{d\xi} \left(\frac{\int_{\lambda_z} \zeta \left. \frac{\partial u}{\partial y} \right|_w d\zeta}{\int_{\lambda_z} \left. \frac{\partial u}{\partial y} \right|_w d\zeta} \right), \quad (18)$$

where $\left. \frac{\partial u}{\partial y} \right|_w$ refers to the wall-normal gradient of the streamwise velocity ($u = \bar{u} + \tilde{u}_p$) at the wall ($y = 0$).

1. Nonlinear Evolution of Stationary Crossflow Vortices

To follow the development of finite-amplitude SCF disturbances, the nonlinear formulation of the plane-marching PSE is used. In this case, the frequency parameter $\omega = 0$ and the streamwise wavenumber α is also equal to zero because the integration path is aligned with the vortex trajectory. Therefore, the perturbation variables reduce to the amplitude functions from Eq. (14), i.e., $\tilde{\mathbf{q}}_p = \hat{\mathbf{q}}_p$, and only a single mode needs to be tracked by the governing equations. We use an implicit discretization of those equations to facilitate efficient convergence of the solution. A similar strategy was followed by Paredes et al. [43, 44] for the calculation of high amplitude streaks in a supersonic, two-dimensional, flat-plate boundary layer, where the spanwise wavelength was fixed along the integration trajectory.

2. Linear Secondary Instability Analysis

The secondary instability of the perturbed basic state with finite-amplitude, SCF vortices, $\tilde{\mathbf{q}}'(x, y, z) = \tilde{\mathbf{q}}(x, y) + \tilde{\mathbf{q}}_p(x, y, z)$, is analyzed with the linear form of the plane-marching PSE. In this case, the perturbations follow the form as given in Eq. (14), with the amplitude function depending on the three spatial directions, $\hat{\mathbf{q}}(x, y, z)$. For the calculation of the secondary instability disturbances with nonzero frequency ($\omega \neq 0$), the normalization condition introduced for the conventional, line-marching PSE in Eq. (7), is extended to a crossplane integral as

$$\int_z \int_y \hat{\mathbf{q}}^* \frac{\partial \hat{\mathbf{q}}}{\partial x} h_x h_z dy dz = 0. \quad (19)$$

D. Discretization and Boundary Conditions

High-order, finite-difference schemes [45, 46] of sixth order are used to discretize the stability equations on a nonuniform grid along the the wall-normal direction. For the results presented here, the wall-normal direction is discretized with $N_\eta = 201$, with the nodes clustered toward the wall. For the plane-marching PSE analysis, the spanwise direction is discretized with $N_\zeta = 64$ equispaced points per wavelength with a sixth-order central, finite-difference scheme with periodic boundary conditions. The discretized PSE are integrated along the streamwise coordinate by using second-order backward differentiation. The number of discretization points in the three spatial directions were varied in selected cases to ensure the numerical convergence of the results. No-slip, isothermal boundary conditions are used at the wall, i.e., $\hat{u} = \hat{v} = \hat{w} = \hat{T} = 0$. The amplitude functions are forced to decay at the farfield boundary by imposing the Dirichlet conditions $\hat{p} = \hat{u} = \hat{v} = \hat{T} = 0$.

The solution of the linear systems of equations arising in the line-marching PSE, the two-dimensional HLNSE, and the plane-marching PSE methods is obtained with the sparse direct solver MUMPS [47].

III. Results

First, the linear instability characteristics of the boundary layer flow over the yawed cone at selected freestream conditions are studied to identify the most amplified stationary crossflow vortices. We then compute the nonlinear evolution of the stationary crossflow vortices based on the receptivity characteristics to wall roughness and use the resulting basic state corresponding to the sum of the unperturbed boundary layer and the finite-amplitude crossflow vortex to analyze the linear amplification characteristics of secondary instability modes. The onset of transition is predicted based on N-factor methodology for the secondary instability. Finally, a subdominant crossflow vortex is introduced to modify the growth of the dominant crossflow vortices, and the secondary instability of the modified basic state with both vortices is analyzed to estimate the resulting delay in transition.

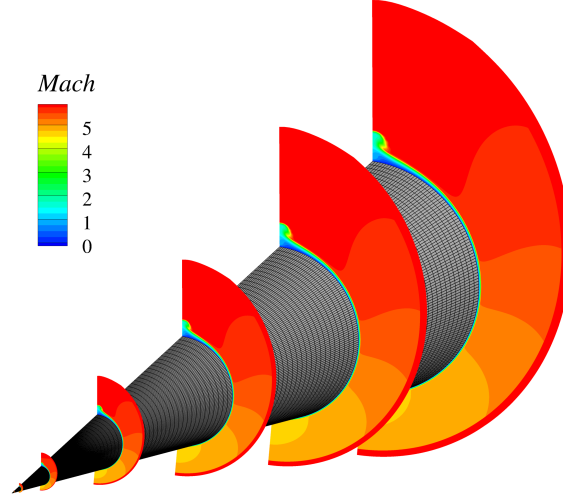


Fig. 2 Mach number contours and surface grid of laminar boundary-layer flow over circular cone at $AoA = 6^\circ$, $M_\infty = 6$ and $Re_\infty = 10.8 \times 10^6 \text{ m}^{-1}$.

A. Flow Configuration and Laminar Flow Solution

The flow configuration of interest is a 0.457-m-long (18 in.), nominally-sharp-nosed cone (modeled with a nose radius of 10^{-4} times the cone length) with a half angle of 7° and an angle of attack of $AoA = 6^\circ$ that was tested in the Boeing/AFOSR Mach-6 Quiet Tunnel (BAM6QT) at Purdue University [12, 16, 22]. The freestream conditions correspond to a Mach number of $M_\infty = 6$, unit Reynolds number $Re_\infty = 10.8 \times 10^6 \text{ m}^{-1}$, and temperature $T_\infty = 52.44 \text{ K}$. The temperature of the model surface T_w is set equal to 300 K. The length of the computational model is 0.5 m.

The laminar boundary-layer flow over the cone was computed by Li et al. [18] by using the viscous upwind algorithm for complex flow analysis (VULCAN-CFD) software [48]. The code solves the unsteady conservation equations appropriate for laminar or turbulent flow of calorically or thermally perfect gases with a spatially-second-order accurate cell-centered, finite-volume scheme. VULCAN-CFD solves the full Navier-Stokes equations with a built-in capability to iteratively adapt the computational grid to the shock. For the present calculations, the code solved the unsteady conservation equations for laminar flow of calorically perfect air. The inviscid fluxes are constructed with the Monotonic Upstream-centered Scheme for Conservation Laws (MUSCL) $\kappa = 0$ scheme, the van Albada gradient limiter [49], and the Low Dissipation Flux Split Scheme (LDFSS) of Edwards [50]. The working fluid is assumed to be a calorically and thermally perfect gas (air) with gas constant of $R = 287.0583 \text{ J/(Kg K)}$ and heat ratio of $\gamma = 1.4$. The coefficient of viscosity is computed from Sutherlands's law, and the coefficient of thermal conductivity is computed by assuming a constant Prandtl number $Pr = 0.72$. A grid of $273 \times 129 \times 353$ in the axial, azimuthal, and wall-normal (or radial) directions, respectively, is used. Exploiting the azimuthal symmetry of the boundary layer about the windward and leeward planes, only one half of the circular cone was modeled in the boundary layer computation. The solution was compared with finer grid solutions by Li et al. [18]. The Mach number contours, and the surface grid of the basic state solution are shown in Fig. 2.

B. Linear Amplification of MM and stationary CF Instabilities

To establish the instability characteristics for the selected configuration, line-marching PSE is used to predict the linear amplification of stationary CF (SCF) disturbances along 31 inflection lines and the evolution of the MM disturbances along 31 inviscid streamlines. Both sets of trajectories are numbered according to ascending azimuthal angle and are seeded at a uniform azimuthal interval of 6 degrees at a reference location of $X_0 = 0.5 \text{ m}$. The alignment of SCF disturbances with the inflection lines, which are defined such that the generalized inflection point of the CF velocity profile coincides with a zero of the same profile, was verified in Refs. [31, 32] against direct numerical simulations [33] and experimental measurements [13]. Similar to Ref. [34], the variation of spanwise wavelength based on the diffuence of the inflection lines is included in the PSE analysis. The integration path for the MM disturbances is

commonly selected along streamlines [35–37] because locally planar waves are more amplified than oblique waves in two-dimensional boundary layers [3].

Figure 3 shows the surface contours of the envelope N-factor based on $|\tilde{u}|_{max}$ for MM and SCF over the yawed cone in Figs. 3(a) and 3(b), respectively. Specifically, the N-factor contours are plotted in the $X - \phi$ plane where ϕ denotes the azimuthal coordinate with respect to the windward ray. The black, thin, solid lines correspond to the streamlines and inflection lines used for the PSE integration of the MM and SCF instabilities, respectively. Figure 3(a) shows that the MM instabilities reach $N \approx 5$ at the end of the model ($X = 0.5$ m) along the windward ray, and the N-factor values decrease toward the leeward side. However, the SCF disturbances are substantially more amplified along the acreage of the cone between the windward and leeward sides, reaching N-factors greater than 13 at $X = 0.5$ m. Therefore, the dominant mechanism for transition corresponds to the CF instability. In particular, the SCF vortices are preferentially excited in low-disturbance environments such as external flight configurations [39]. Accordingly, we focus on the SCF as the driver for boundary-layer transition. It may be noted from Fig. 2 that the laminar boundary-layer flow in the vicinity of the leeward ray feature a prominent streak. Such streaks can sustain multiple unstable modes [51, 52]; however, their influence remains localized in the vicinity of the leeward ray and does not affect the transition process away from the symmetry planes studied in this work. The measured Stanton number contours with distributed surface roughness created via sandblasting of the cone surface at the conditions of interest in the BAM6QT of Purdue University [22] are shown in Fig. 4. The crossflow vortices are visible as streaky patterns in the heat transfer contours, and the transition front is inferred where the Stanton number values abruptly increase towards the base of the cone. The comparison of the SCF N-factor contours with the experimental measurements reveal a transition front that resembles the $N_{SCF} = 10$ isoline. Furthermore, Tufts et al. [38] also found a good correlation between the measured locations of CF-induced transition on the HIFiRE-5 flight configuration with an envelope N-factor of $N_{SCF} = 10$ based on linear PSE analysis. Because of this, the predicted transition front is chosen to correspond to the locus of points along the cone surface where the envelope of the N-factor curves for individual vortex disturbances reaches $N_{SCF} = 10$, which is denoted by a black, thick, dashed line in Fig. 3(b). Also, the neutral location of the CF vortex that reaches $N = 10$ along each trajectory is indicated with large circles colored in gray scale on the basis of the corresponding value of the local spanwise wavelength. The range of local wavelengths over the highlighted neutral locations corresponds to $\lambda_I = [0.7, 1.0]$ mm, where the subscript I indicates that the wavelength is measured at the respective neutral locations. Surface roughness with length scales in this range would be particularly effective in exciting these dominant modes of SCF instability. The critical N-factor of $N_{SCF} = 10$ is first reached along the vortex trajectory #29 (marked with a black, thick, solid line in Fig. 3(b)) at $X_T = 0.33$ m and $\phi \approx 148^\circ$. Based on this finding, the primary target for transition delay must be this particular vortex trajectory and, therefore, all of the subsequent analysis is focused on vortex trajectory #29.

The development of disturbance wavelength is determined by the diffuence of the trajectories. Figure 5(a) shows the evolution of the disturbance wavelength normalized by its value at the reference location $X_0 = 0.5$ m, λ/λ_0 , for the 31 inflection lines used to calculate the crossflow N-factor contours of Fig. 3(b). The disturbance wavelength is nearly zero close to the nose tip and grows due to the geometrical expansion of the cone. The scaled wavelengths monotonically increase from the nose to the base of the cone for inflection lines #1 (closest to windward ray) to #24, but follow a nonmonotonic evolution with a maximum larger than unity, shifting upstream from inflection line #25 to #31 (closest to the leeward side). The N-factor curves associated with selected crossflow wavelengths for inflection line #29 are shown in Fig. 5(b). Due to the rapid increase in disturbance wavelength near the nose tip, the neutral location does not shift downstream as the wavelength increases, which is different from infinite-span swept-wing boundary layers (e.g., Malik et al. [53]) where a downstream shift occurs due to the growth in boundary layer thickness. The combined effect of spatial variations in the azimuthal wavelength and boundary layer thickness leads to a different scenario for the yawed cone. The neutral location of CF waves are displaced upstream as the reference wavelength is increased from $\lambda_0 = 1.8$ mm to 5.4 mm, and moves slightly downstream from $\lambda_0 = 5.4$ mm to $\lambda_0 = 8.1$ mm. The CF vortex that first reaches $N = 10$ corresponds to $\lambda_0 = 3$ mm ($\lambda_I = 0.82$ mm at $X_I = 0.071$ m).

C. Roughness Induced Receptivity of Stationary Crossflow Vortices

The receptivity to small height surface roughness of various forms is computed by imposing the inhomogeneous boundary conditions of Eq. (9) on the HLNSE with zero frequency, i.e., with $\omega = 0$. The roughness patch roughness is defined by the height distribution $h_R(x)$ of Eq. (10) along the selected inflection line trajectory. The roughness height distribution is characterized by the maximum height k_R , patch length, L_R , and wavenumber, α_R . The effective initial amplitudes of the dominant CF disturbances, denoted by the effective magnitude of streamwise (inflection line aligned) disturbance velocity at the neutral location normalized by the freestream velocity ($u_I = |\tilde{u}_I|/U_\infty$), are primarily

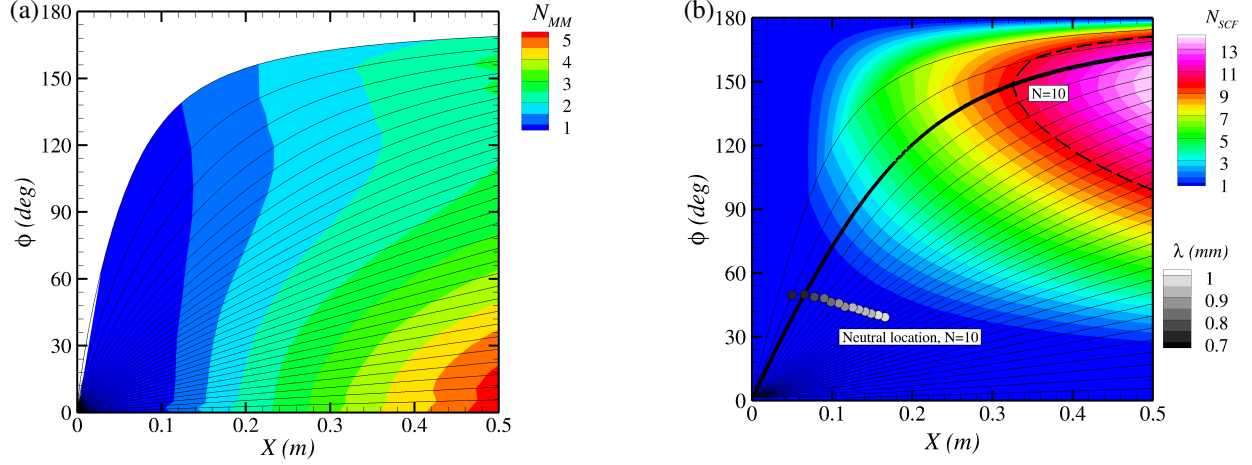


Fig. 3 N-factor envelope contours of (a) MM and (b) SCF disturbances. The predicted transition front for $N_{SCF} = 10$ is marked with a black, thick, dashed line. The neutral location for the CF waves that first reach $N = 10$ is marked with circles colored by the local spanwise wavelength. The (a) streamlines and (b) inflection lines used for PSE integration are marked with black, thin, solid lines. The inflection line #29 corresponding to the most upstream transition onset is denoted with a black, thick, solid line.

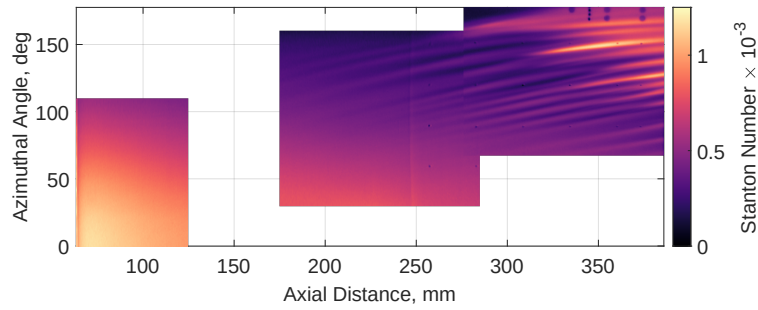


Fig. 4 Measured Stanton number contours over a 7° half-angle cone at $AoA = 6^\circ$ with distributed surface roughness at $Re_\infty = 11 \times 10^6 \text{ m}^{-1}$ in the BAM6QT at Purdue University [22].

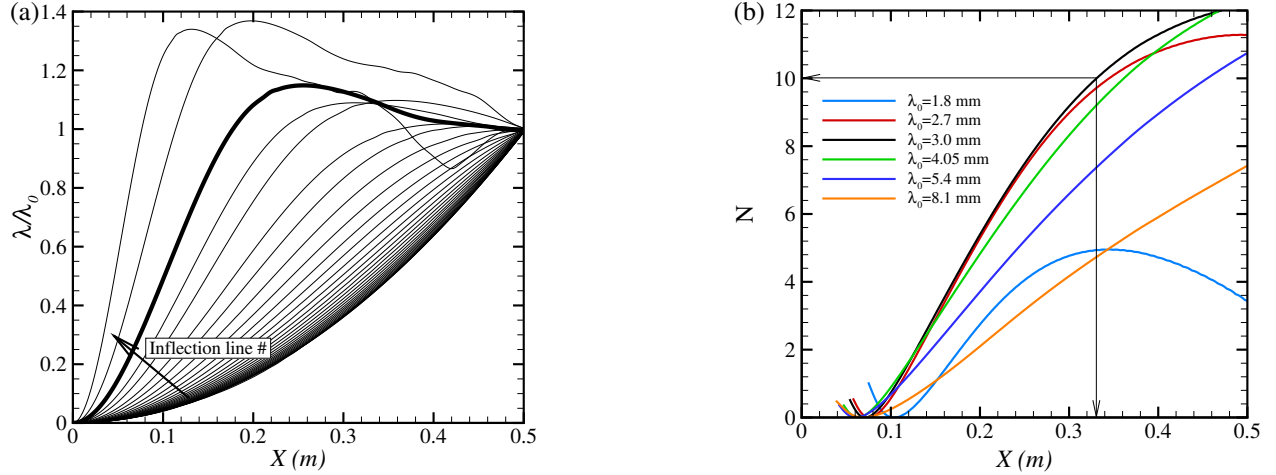


Fig. 5 (a) Evolution of disturbance wavelength normalized with the wavelength λ_0 at the reference location $X_0 = 0.5$ m for the 31 inflection lines used for crossflow instability analysis. Inflection line #29 is marked with a thick line. (b) N-factor evolution of linear CF waves for selected disturbance wavelengths along the inflection line #29.

determined by $\lambda_I = 2\pi/\beta_I$, i.e., the CF wavelength at the neutral station. However, in this paper, the SCF disturbances of interest are identified on the basis of their spanwise wavelength λ_0 at the reference location of $X_0 = 0.5$ m. Figures 6(a) and 6(b) show the effect of the roughness patch length (L_R) and streamwise wavenumber (α_R), respectively, on the induced u_I scaled by the normalized roughness height $k = k_R/k_{ref}$. The results show that the disturbance amplitude increases with the streamwise extent of the roughness patch, but the nearly linear trend flattens after the patch length has increased to approximately $L_R/\lambda_I = 35$. Figure 6(b) shows that the most effective roughness streamwise wavenumber is zero. Since calculations are performed along the inflection lines that closely follow the CF vortex path, so the streamwise wavenumber of the vortex is much smaller than the spanwise wavenumber and nearly zero. Therefore, the maximum receptivity magnitude corresponds to roughness distributions with nearly zero streamwise wavenumber that extend a certain distance upstream and downstream of the corresponding neutral location. This numerical finding is qualitatively consistent with the requirement of local phase synchronization identified via the theory of distributed receptivity [39]. In Figure 6(c), we plot the receptivity coefficient spectrum as a function of the vortex wavelength λ_0 . This initial amplitude spectrum is used to estimate the variation in effective CF amplitude associated with the natural stochastic surface roughness. Without any loss of generality, the receptivity coefficient in Figure 6(c) has been normalized with respect to its value for $\lambda_0 = 3$ mm, because this wavelength corresponds to the CF mode that first achieves $N = 10$ along the vortex trajectory #29 as shown in Figure 5(b). Although not shown, a similar variation with respect to the CF wavelength λ_0 was also obtained for local roughness elements with an assumed smooth shape.

D. Secondary Instability of Stationary Crossflow Vortices

The laminar breakdown process on the baseline configuration is initiated via the secondary instability of SCF vortices excited by the natural surface roughness. Thus, we first compute the nonlinear evolution of SCF modes along vortex trajectory #29 for each selected spanwise wavelength via a fully implicit solution of the stationary, nonlinear plane-marching PSE. In reality, one is likely to encounter a broadband spectrum of SCF vortices; however, given the large growth factors involved, it is reasonable to assume that the local vortex spectrum would be dominated by a narrow range of azimuthal wavelengths. Subsequently, the linear form of the plane-marching PSE is used to study the linear, non-parallel stability characteristics of the boundary layer flow distorted by each finite-amplitude SCF disturbance.

We assume a baseline, i.e., natural surface roughness that yields an initial SCF amplitude of $u_I = 10^{-3}$ at $\lambda_0 = 3$ mm. This initial amplitude selection is based on a parametric study varying u_I with a fixed $\lambda_0 = 3$ mm to reach a secondary N-factor of $N_E = 15$ in the vicinity of $X = 0.33$ mm where the primary SCF N-factor of $N_{SCF} = 10$ is reached for inflection line #29 as shown in Fig. 3(b). Then, the initial amplitudes at other azimuthal wavelengths can be estimated according to the receptivity spectrum in Fig. 6(c), i.e., $u_I(\lambda_0) = 10^{-3}C_k(\lambda_0)$. For instance, for the SCF mode with

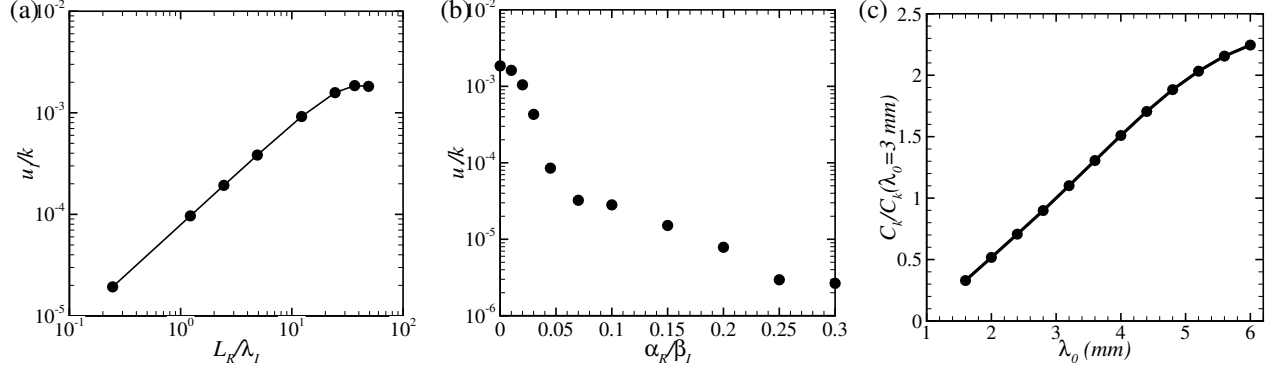


Fig. 6 Ratio of disturbance streamwise velocity normalized by the freestream velocity and roughness height, u_I/k , as a function of (a) the roughness length normalized by the CF spanwise wavelength at the neutral location, L_R/λ_I , with $\alpha_R = 0$, and (b) the roughness streamwise wavelength normalized by the CF spanwise wavenumber at the neutral location, α_R/β_I , with $L_R/\lambda_I = 36.71$. (c) Receptivity coefficient scaled with its value at $\lambda_0 = 3$ mm as a function of the wavelength λ_0 at the reference location $X_0 = 0.5$ m.

$\lambda_0 = 2.7$ mm, the initial amplitude is given by $u_I = 0.75 \times 10^{-3}$. The nonlinear solution for a SCF mode with $\lambda_0 = 2.7$ mm and an initial amplitude equal to $u_I = 0.75 \times 10^{-3}$ is shown in Fig. 7. Although not shown, similar computations are performed for a range of CF wavelengths while varying the initial amplitude according to the receptivity coefficient at a fixed roughness height as previously described in Fig. 6(c). Next, the secondary instability of the nonlinearly distorted basic states is analyzed, and the onset of transition is assumed to correlate with a secondary amplification of $N_E = 15$ (where the subscript E denotes the N-factor based on the amplification of disturbance energy of Eq. (5) corresponding to the secondary instability mode). The selected value of the critical N-factor is based on both the post-flight analysis of Mack mode transition during the HIFiRE-1 flight experiment [17] and, more important, on the basis of the measured onset of transition near the centerline of the HIFiRE-5b flight article due to stationary streaks that resemble the CF vortices in the present case [54]. Fig. 8 shows that the secondary N-factor criterion of $N_E = 15$ is first satisfied for a CF disturbance with $\lambda_0 = 2.7$ mm at $X = 0.324$ m, despite the lower value of the receptivity coefficient for shorter wavelengths in Fig. 6(c). Based on this higher fidelity transition criterion, the SCF mode with $\lambda_0 = 2.7$ mm and an initial amplitude of $u_I = 0.001C_k = 0.75 \times 10^{-3}$ is selected as the primary target for transition control via the seeding of subdominant CF vortices.

E. Control of Crossflow-Induced Transition

The current study seeks to delay transition by using an array of discrete roughness elements to introduce subdominant CF modes. These subdominant modes would reduce the growth of the naturally dominant CF vortices and, hence, delay the onset of laminar breakdown resulting from the secondary instabilities associated with the combined presence of both target and control modes. Here, the target and control modes refer to the naturally dominant CF vortex and the DRE-induced subdominant CF vortex, respectively, and are denoted by subscripts t and c . As previously described in Section III.D, the target CF mode corresponds to a spanwise wavelength of $\lambda_{t,0} = 2.7$ mm and an initial amplitude of $u_{t,I} = 0.75 \times 10^{-3}$ that is predicted to yield transition via secondary instability at $X_{t,T} = 0.33$ m. This transition location was determined by applying the N-factor criterion $N_E = 15$ from secondary instability analysis and based on the HiFiRE-5b flight data (effectively assuming the surface roughness characteristics to resemble those of that model). For the analysis of transition control, the inflow disturbance now includes both the target mode and a control input with a different (i.e., subdominant) wavelength and a suitably large initial amplitude in comparison with the target mode. The nonlinear interaction between the target and control disturbances is also computed with the nonlinear plane-marching PSE. Although not shown, it was found that the previously established strategy of using a control input with subcritical spacing, i.e., $\lambda_c < \lambda_t$, for low-speed swept-wing configurations [23, 55] would not be successful for the present hypersonic flow configuration. The ineffectiveness of the shorter control mode wavelengths in the present case is related to a fundamental difference in the linear growth characteristics of primary CF vortices in comparison with the swept-wing configurations. Specifically, the neutral locations of CF modes with smaller wavelengths relative to the target mode wavelength ($\lambda_{t,0} = 2.7$ mm) are further downstream with respect to the neutral location for the target mode

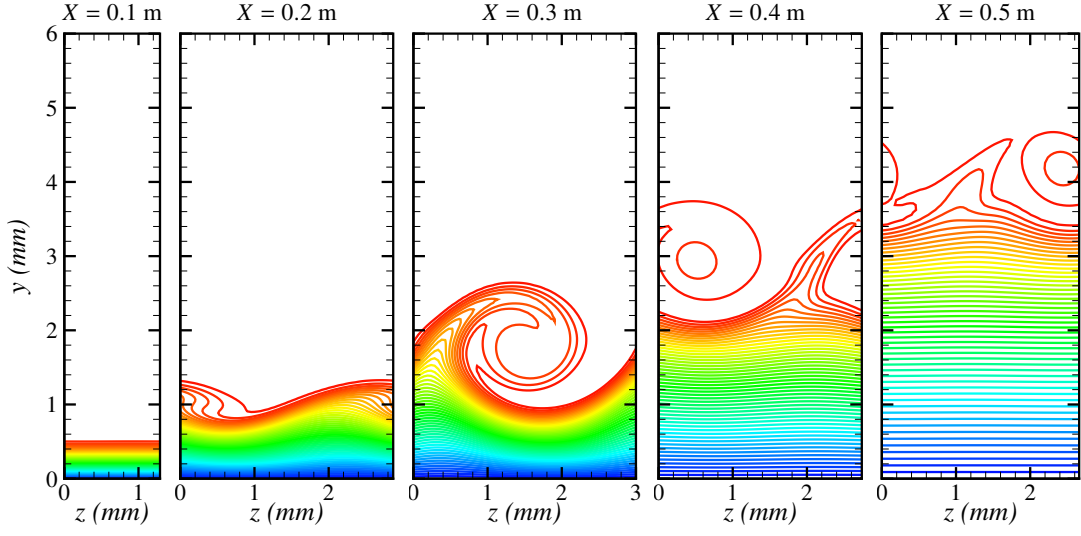


Fig. 7 Isolines of streamwise velocity normalized with freestream velocity $u/U_\infty = (0.02 : 0.02 : 0.98)$ at $X = 0.1, 0.2, 0.3, 0.4$, and 0.5 m for the case of SCF vortex with initial amplitude equal to $u_I = 0.75 \times 10^{-3}$ and reference wavelength of $\lambda_0 = 2.7$ m.

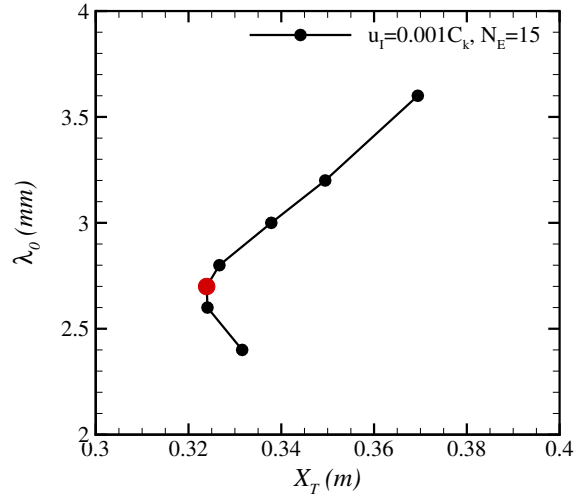


Fig. 8 Predicted transition locations based on secondary instability $N_E = 15$ for a baseline roughness level corresponding to an initial CF amplitude of $u_I(\lambda_0) = 10^{-3} C_k(\lambda_0)$.

as shown in Fig. 5(b). As a result, the initial amplitude of the control mode required to produce an appreciable effect on the target mode needs to be excessively large, and such large inputs are likely to induce boundary layer tripping in the vicinity of the roughness elements.

Conversely, computations demonstrated that selecting control modes with larger wavelengths than the target mode wavelength, i.e., $\lambda_c > \lambda_t$, has the potential to yield a significant transition delay. Figure 9 shows the nonlinear solution for the superposition of the target mode with $\lambda_{t,0} = 2.7$ mm and an initial amplitude equal to $u_{t,I} = 0.75 \times 10^{-3}$, and the control mode with $u_{c,I} = 13.7 \times 10^{-3}$ and a wavelength of twice that of the target mode, i.e., $\lambda_{c,0} = 2\lambda_{t,0} = 5.4$ mm. Due to the effective suppression of the target mode, the control CF vortex dominates the combined evolution throughout the streamwise domain. Figure 10 shows the predicted transition delay $X_{c,T}/X_{t,T}$ based on the secondary N-factor criterion applied to the joint nonlinear evolution of the interacting target and control modes. The estimated transition delay is plotted as a function of the ratio between the initial amplitudes of the control and target modes at their respective neutral locations, $u_{c,I}/u_{t,I}$, and the ratio of the control and target mode wavelengths, λ_c/λ_t . The threshold initial amplitude of the control mode that corresponds to 2% of the freestream velocity is marked with a horizontal dashed line, to indicate that control modes with higher initial amplitudes might lead to undesired effects due to disturbance nonlinearities in the vicinity of the roughness elements. The control modes with $\lambda_c/\lambda_t = 3/2$ indicate the potential for a maximum transition delay of up to 33%. The control modes with $\lambda_c/\lambda_t = 2$ are predicted to be even more effective in yielding a larger transition delay of 41% with a lower initial amplitude. In principle, the required control input could be achieved via multiple shapes of roughness elements. The control mode with $\lambda_c/\lambda_t = 3$ ($\lambda_{c,0} = 8.1$ mm) exhibits lower linear amplification, as observed in Fig. 5(b), which explains why further increasing the control mode wavelength reduces the effectiveness of the control input when the initial amplitude of the control mode is constrained to less than the threshold of 2% of the freestream velocity. Thus, the optimal transition delay is found for control CF modes with wavelengths of $\lambda_c/\lambda_t = 2$ and initial amplitude ratios of $u_{c,I}/u_{t,I} < 25$, (green curve from Fig. 10).

IV. Summary and Concluding Remarks

The present work investigates the potential of employing discrete roughness elements (DREs) to delay the onset of crossflow-induced laminar-turbulent transition in a high-speed boundary-layer flow over a 7° half-angle cone at 6° angle of attack, or, equivalently, angle of yaw. The freestream conditions with a mach number of $M_\infty = 6$, unit Reynolds number of $Re_\infty = 10.8 \times 10^6 \text{ m}^{-1}$, and temperature of $T_\infty = 52.44$ K, correspond to the experiments conducted in the BAM6QT at Purdue University [12, 16, 22]. The compressible Navier-Stokes equations are used to obtain the laminar flow solution of the three-dimensional boundary layer over the yawed cone. Linear instability analysis based on the parabolized stability equations (PSE) shows that even though the boundary layer supports the amplification of both Mack's second mode (MM) and crossflow (CF) instabilities, the transition process away from the windward and leeward rays is likely to be driven by CF instabilities. Furthermore, the CF N-factor contour of $N = 10$ compares well with the transition front inferred from experimental measurements.

The current study seeks to delay transition by using an array of DREs to introduce subdominant crossflow modes that would reduce the growth of the naturally dominant crossflow vortices and, hence, delay the onset of laminar breakdown via secondary instabilities associated with the combined presence of both target and control modes. To incorporate a comprehensive model of the transition process in the uncontrolled case as well as its delay via the DREs within a physics-based engineering model, we utilize a transition criterion that accounts for the receptivity characteristics as well as for the simultaneous evolution of multiple crossflow disturbances and their effects on the laminar breakdown process. Linear receptivity theory is used to estimate the initial crossflow amplitude spectrum excited via natural stochastic surface roughness. Specifically, the receptivity coefficient provides the initial amplitude as a function of the crossflow wavelength, which allows one to calculate the nonlinear evolution of individual crossflow vortex modes along the selected vortex trajectory by using the fully implicit nonlinear plane-marching PSE. Subsequently, the linear form of the plane-marching PSE is used to study the secondary instability of the boundary-layer flows distorted by the crossflow vortices. The N-factor framework based on the secondary instability allows one to estimate the onset of transition in the baseline case as well as the primary CF mode that plays a dominant role in the transition process and, hence, becomes the target of the DRE-based control. The DREs seed subdominant CF modes that reduce the growth of the target mode via nonlinear interactions in such a way that the overall growth of secondary instabilities is reduced and transition can be delayed.

Previous studies with low-speed swept-wing configurations had used control modes with smaller wavelengths than the dominant CF vortex. In contrast, for the present yawed circular cone, control modes with larger wavelengths demonstrate the potential to yield a significant transition delay. Shorter (i.e., subcritical) wavelength control modes are

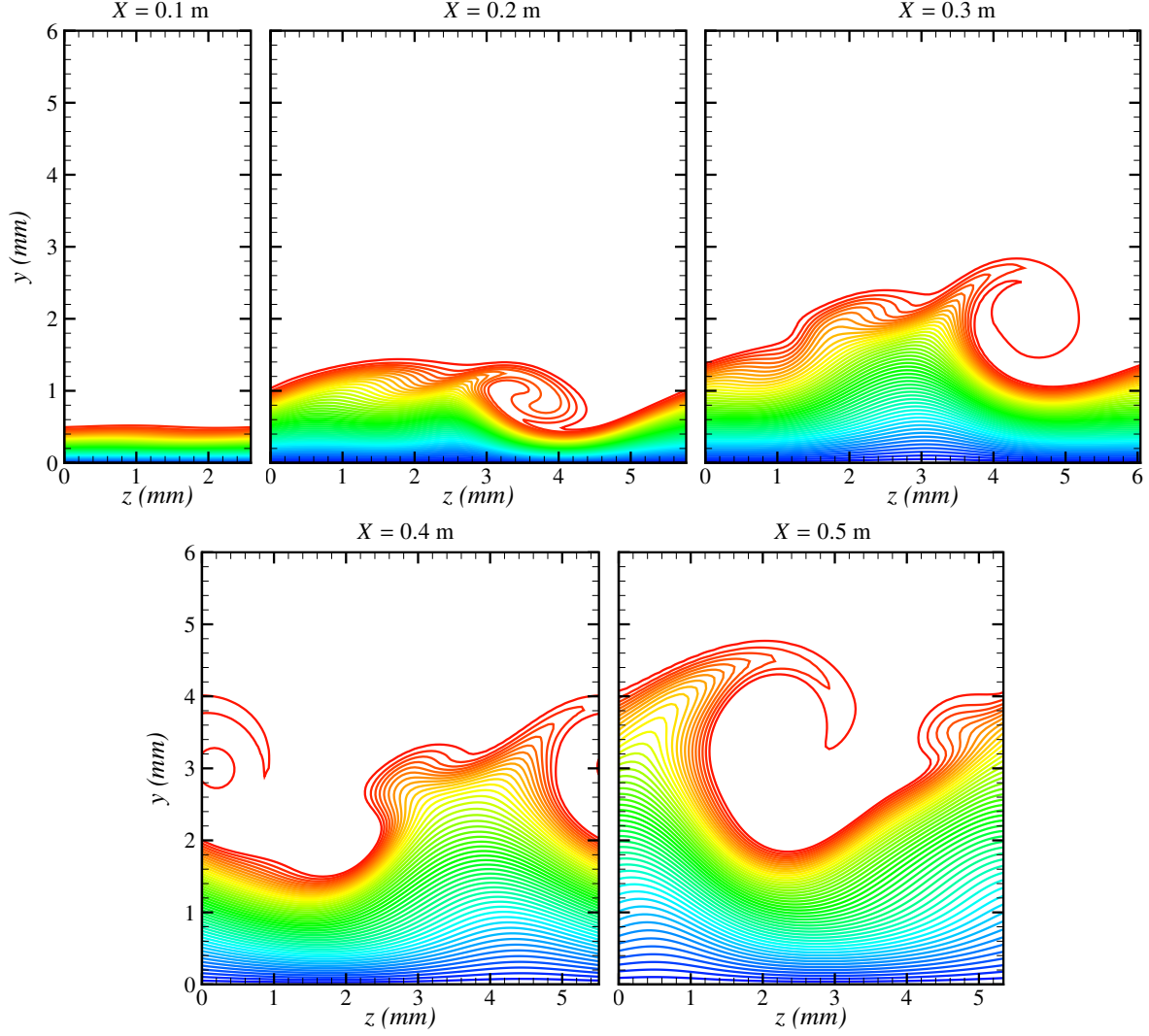


Fig. 9 Isolines of streamwise velocity normalized with freestream velocity $u/U_\infty = (0.02 : 0.02 : 0.98)$ at $X = 0.1, 0.2, 0.3, 0.4$, and 0.5 m for the combined solution with the target SCF vortex ($u_{t,I} = 0.75 \times 10^{-3}$ and $\lambda_{t,0} = 2.7$ m) and a control CF vortex with $u_{c,I}/u_{t,I} = 18.3$ and $\lambda_c/\lambda_t = 2$.

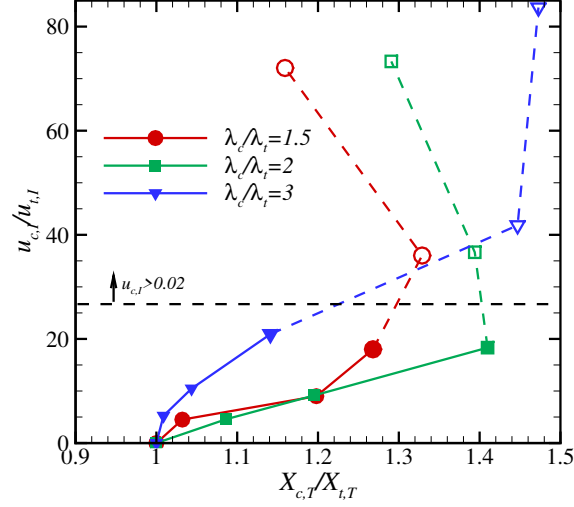


Fig. 10 Predicted transition delay as a function of the ratios of control to target mode wavelengths, λ_c/λ_t , and amplitudes, $u_{c,I}/u_{t,I}$. The horizontal dashed line denotes the threshold for control wave initial amplitude equal to 2% of the freestream velocity.

ineffective because the combined effects of the conical model and the associated variations in boundary layer thickness reduce their linear growth potential. Specifically, the neutral locations of CF modes with smaller wavelengths with respect to the target wavelength are further downstream and consequently, the initial amplitude of the control mode to affect the target mode evolution becomes excessively large. With the initial control mode amplitude restricted to less than 2% of the freestream velocity, the superposition of target and control modes predicted a transition delay of 33%, 41%, and 14% for control modes with 1.5, 2 and 3 times the target CF vortex wavelength, respectively. The restriction on the control mode amplitude was designed to prevent unintentional tripping of the boundary layer.

Detailed laboratory experiments and numerical simulations are required to establish the effect of DREs on the generation of the control CF vortices and the interaction with the CF vortices induced by natural surface roughness that drive the transition in the baseline case. Furthermore, the analysis of the nonlinear breakdown for the baseline and controlled flows, different shapes of the DREs, or the inclusion of multiple rows of DREs, are all important considerations, and they should be considered in the future development of this transition control concept.

Acknowledgments

This research is sponsored by the NASA High-Speed Flight (HSF) project under the Aeronautics Research Mission Directorate (ARMD). The computational resources supporting this work were provided by the K cluster at NASA's Langley Research Center and by the NASA High-End Computing (HEC) Program through the NASA Advanced Supercomputing (NAS) Division at the Ames Research Center. The authors would like to acknowledge Ms. Cassandra Butler and Drs. Joseph Jewell, Joshua Edelman, and Steven Schneider for their help.

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