

A Model of Turbulence in the Langley Transonic Dynamics Tunnel for Aeroservoelastic Analysis

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A general formulation to generate a model of the turbulence field is proposed which is suitable for integration into aeroservoelastic (ASE) models. The proposed formulation uses the more accurate von Kármán spectra, is valid for small turbulence length scales, such as inside of a wind tunnel, and does not require assumptions about the configuration. The method generates the appropriate turbulence velocity components and frequency spectra that can be integrated into an ASE model. Measurements of turbulence in the Transonic Dynamics Tunnel (TDT) at NASA's Langley Research Center were used to define the parameters of the von Kármán spectrum. The formulation was applied to the Integrated Adaptive Wing Technology Maturation (IAWTM) test article in the TDT. The turbulence model in the TDT shows significant differences from a typical uniform turbulence. The proposed formulation was shown to replicate the existing standard turbulence models with assumptions such as large turbulence length scales.

Nomenclature

b	wingspan
$\bar{c}$	mean aerodynamic chord
$H_{tt}(k)$	turbulence field principal transfer function
$\hat{k}$	turbulence reduced frequency
k	reduced frequency
$K_n()$	modified Bessel function of the second kind
L	scale length
L_{ni}	gust sniffer wake characteristic length
n	spectrum denominator exponent
n_1	gust sniffer wake first numerator coefficient
n_2	gust sniffer wake second numerator coefficient
$P_{jj}(k)$	turbulence correlation function
V_T	true airspeed
x_{jj}	streamwise distance between aerodynamic points
α	gust sniffer angle
β	spectrum numerator parameter
γ	spectrum denominator parameter
$\Gamma()$	gamma function
δ_{jj}	distance between aerodynamic points
$\hat{\delta}_{jj}(k)$	nondimensional correlation distance
Λ_{jt}	turbulence field principal components
ϕ	turbulence spectrum
φ	expanded turbulence spectrum
σ	turbulence intensity
ω	dimensional frequency, rad/s
ζ	gust sniffer wake damping

Subscripts

j	set of aerodynamic reference points
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t simplified turbulence components

Superscripts

- measured turbulence parameter
 $\sim$ estimated turbulence parameter

Symbols

$\oslash$ Hadamard (element-wise) matrix division
 $\triangleq$ equal by definition

I. Introduction

THE effect of turbulence on an aircraft is of significant concern for its structural integrity, flying qualities and performance. The two standard models for describing atmospheric turbulence were created by Dryden [1] and von Kármán [2]. These turbulence models are effective at describing the velocity variations at a point in space due to turbulence, with the von Kármán model generally being considered more accurate. However, additional assumptions and approximations are required to make these turbulence spectra suitable for use in loads or flying qualities analyses.

The industry standards for loads due to continuous turbulence are defined in JSSG-2006 [3] for military aircraft and 14 C.F.R. §25.341[4] for transport aircraft. Both standards use a very similar specification of the turbulence field, and both suggest the von Kármán turbulence spectra. The standards specify length scales for the turbulence of 2500 ft at cruise down to 500 ft at low altitudes. Although studies suggest a wide range of values for turbulence in the atmosphere, the length scale values are typically greater than 500 ft [5, 6].

The load calculations typically use a turbulence that is uniform across the wingspan [6, 7]. This assumption is generally considered accurate when the length scale of the turbulence is significantly larger than the wingspan of the aircraft. Additionally, the assumption of uniform spanwise turbulence is generally conservative for loads analyses since the spanwise variations generally reduce the critical loads. However, the loads standard does not provide further guidance or detail of how much larger the turbulence length scale needs to be. For vehicles with very large wingspans [8–10] it does become necessary to consider the spanwise variations.

The other typical analysis with these turbulence models is in the application of flying qualities described in MIL-STD-1797 [11, 12]. There is no flying quality standard for transport aircraft. The turbulence analysis in the current MIL-STD-1797 is adapted from the earlier MIL-F-8785 [13]. The flying qualities standards also suggest the use of the von Kármán turbulence spectra, explicitly for consistency with the loads analysis. The rationale is that the most severe turbulence simulated would correspond to the limit of the aircraft's structure [14].

In contrast to the loads analysis, flying qualities analyses [14] assume turbulence that varies linearly across the span to create an effective roll rate. The von Kármán spectra is used for the uniform component, but the von Kármán spectra was determined to be too complex to derive the spanwise varying turbulence. Instead, the exact value of these spectra for the Dryden turbulence model with a linearly spanwise varying turbulence are derived by Etkin [15]. The standards MIL-F-8785 and MIL-STD-1797 use a curve fit of these Dryden spectra for the spanwise varying portion of both the Dryden and von Kármán turbulence models. In addition to using a different turbulence spectrum, the derivation of the Etkin turbulence model [15] also assumes an unswept wing. The unswept wing implies that there is no delay in the turbulence encountering the wing based on the span due to convection of the freestream.

The turbulence spectra in MIL-STD-1797 are specified slightly differently, but the length scales are consistent with the values used in the loads standards. However, unlike the loads standards, the flying qualities standards do provide guidelines of when the assumption of a spanwise varying turbulence is sufficient. The simplest guideline is the length scale is $L \geq 10b$ [16]. The standard suggests a guideline to neglect the spanwise variations in Eq. (1) based on the lateral stability derivatives of roll damping, C_{l_p} , and dihedral effect, C_{l_β} .

$$L \geq \frac{b}{2} \left(\frac{C_{l_p}}{C_{l_\beta}} \right)^2 \quad (1)$$

These turbulence models used in flying qualities have significant simplifying assumptions and have been frequently noted to not accurately represent the response of the aircraft to turbulence [17, 18]. These turbulence models are effective in flying qualities analysis because they replicate the pilot workload. However, these models are not directly applicable when trying to correlate the vehicle response to test data.

These implementations of turbulence models have been effective for the certification of aircraft. However, a more general approach is sometimes needed when considering unusual configurations, when considering model-scale tests in a wind tunnel, or when correlating to test data. To create a more general turbulence model, Section II derives an approach to calculating the turbulence field shapes for arbitrary turbulence parameters. The Integrated Adaptive Wing Technology Maturation (IAWTM) model, a transport configuration test article, and a method of estimating turbulence parameters from the testing is described in Section III. In Section IV, the new turbulence method has been applied to the IAWTM model to reproduce the standard flight approaches and show the difference for wind tunnel turbulence parameters.

II. Model Definition

While multiple models of atmospheric turbulence exist, the von Kármán turbulence model [2, 5] is widely considered to be the most accurate of these models [6]. The spectrum of the von Kármán turbulence is shown in Eqs. (2a) and (2b).

$$\phi(k) = \frac{\sigma^2 L}{\pi} \frac{1 + \frac{8}{3} \hat{k}^2}{[1 + \hat{k}^2]^{11/6}} \quad (2a)$$

$$\hat{k} = 1.339 \frac{2L}{\bar{c}} k \quad (2b)$$

The spectrum represents the variations in the velocity with respect to time, or equivalently in frequency, at a single point. The parameter σ^2 represents the variance of the velocity and therefore by Parseval's theorem the spectrum is required to satisfy the criterion of Eq. (3).

$$\sigma^2 \triangleq \frac{1}{2\pi} \int_0^\infty \phi(k) dk \quad (3)$$

The correlation function for von Kármán turbulence [19], Eqs. (4a) and (4b), results in a matrix that describes the relationship between two aerodynamic panels. The relationship is a function of the distance between the panels and introduces the spatial component to the turbulence model.

$$\mathbf{P}_{jj}(k) = \frac{2^{2/3}}{\Gamma(1/3)} \left[\hat{\delta}_{jj}^{1/3} K_{1/3}(\hat{\delta}_{jj}) - \frac{1}{2} \hat{\delta}_{jj}^{4/3} K_{2/3}(\hat{\delta}_{jj}) \right] e^{-ik \frac{x_{jj}}{\bar{c}}} \quad (4a)$$

$$\hat{\delta}_{jj}(k) = \frac{\delta_{jj}}{1.339L} \sqrt{1 + \left(1.339k \frac{2L}{\bar{c}} \right)^2} \quad (4b)$$

The correlation matrix in Eq. (4a) is equal in rank to the number of aerodynamic reference points and is much too large for practical use. To simplify the problem, for each frequency, the correlation matrix is assumed to be a linear combination of some small set of principal components, Eq. (5).

$$\mathbf{P}_{jj}(k) \approx \mathbf{\Lambda}_{jt} \mathbf{H}_{tt}(k) \mathbf{\Lambda}_{jt}^T \quad (5)$$

These principal components are sometimes also called modes, but they are distinct from the structural modes. Standard industry practice [4, 11, 12] is to use an assumed shape for these principal components. These are typically based on rigid body modes even in flexible aircraft. However, the validity of the approach, discussed in Section I, requires assumptions about the configuration of the vehicle which can be frequently inaccurate.

For the present work, the low rank representation is instead based on principal component analysis (PCA) [20]. The use of PCA requires no assumptions about the configuration. Typically, in PCA the matrix $\mathbf{H}_{tt}$ would be a diagonal matrix. However, for these turbulence fields there is not a single matrix $\mathbf{\Lambda}_{jt}$ that will diagonalize the correlation matrix, $\mathbf{P}_{jj}$, for all frequencies. Instead, a matrix is found that will make the diagonal dominant, and the off-diagonal terms are then neglected. To simplify the turbulence field, first the correlation matrices for each frequency, Eq. (4a), are stacked as in Eq. (6). This stacked correlation matrix, $\mathcal{P}$, is then decomposed in Eq. (7) into the matrices $\mathbf{U}_{jp}$, $\mathbf{S}_{pp}$, and $\mathbf{V}_{jp}$ using the singular value decomposition (SVD).

$$\mathcal{P} \triangleq \begin{bmatrix} \mathbf{P}_{jj}(k_1) & \mathbf{P}_{jj}(k_2) & \cdots & \mathbf{P}_{jj}(k_n) \end{bmatrix} \quad (6)$$

$$\mathcal{P} = \mathbf{U}_{jp} \mathbf{S}_{pp} \mathbf{V}_{jp}^T \quad (7)$$

The use of singular value decomposition sorts these turbulence components into the shapes that maximize the total variation in the velocity field.

Selecting the dominant principal components, $\mathbf{U}_{jt}$, the basis for the turbulence field and the power spectral density (PSD) of the turbulence field are calculated from Eqs. (8a) and (8b). The PSD, Eq. (8b), is scaled such that the area under the curve is 2π , consistent with the criteria in Eq (3).

$$\mathbf{A}_{jt} = \frac{1}{\sqrt{2\pi}} \mathbf{U}_{jt} \left(\sum_{i=1}^n w_i \text{diag} \left[\mathbf{U}_{jt}^{-1} \mathbf{P}_{jj} (k_i) \mathbf{U}_{jt}^{-T} \right] \right) \quad (8a)$$

$$\mathbf{H}_{tt} (k) = 2\pi \left[\mathbf{U}_{jt}^{-1} \mathbf{P}_{jj} (k_i) \mathbf{U}_{jt}^{-T} \right] \oslash \text{diag} \left[\sum_{i=1}^n w_i \mathbf{U}_{jt}^{-1} \mathbf{P}_{jj} (k_i) \mathbf{U}_{jt}^{-T} \right] \quad (8b)$$

A. Turbulence in the Transonic Dynamics Tunnel

The Transonic Dynamics Tunnel (TDT) at the NASA Langley Research Center (LaRC) in Hampton, VA is a wind tunnel designed for studying and understanding aeroelastic phenomena. The TDT has a 16 ft square test section with cropped corners. The tunnel can run with either air or R-134a heavy gas as a test medium. The pressure inside the tunnel, the wind off pressure, can be reduced to change the dynamic pressure independent of the Mach number.

Rough estimates of the length scale of internal flows, such as a wind tunnel, range from 4% to 13% of the diameter [6]. For the 16 ft wide slotted test section of TDT the turbulence length scale would be expected to be on the order of 0.6 – 2 ft, up to three orders of magnitude smaller than the turbulence length scales in the standards [4, 11, 12, 21]. Although wind tunnel models are smaller than the full scale aircraft, the difference is much smaller than the difference in turbulence length scales.

The turbulence velocity components in the TDT were measured by Sleeper et. al. [22], where a probe was used to measure the lateral and vertical velocity components of the free-stream turbulence. These measurements were performed in air and at relatively high wind off pressure. These results have been normalized, discussed further in Appendix A, to be consistent with the current results. A similar experiment was performed by Wiesman and Sleeper [23] in the R-134a heavy gas. However, the heavy gas measurements reported only pressures and were not calibrated to give velocities. The pressures do show the bandwidth of the turbulence and can give insight to the length scales of the turbulence. The heavy gas measurements confirm that the length scales of the turbulence do not change significantly with the gas used in the tunnel.

III. Experimental Description

The turbulence methodology has been applied to the IAWTM model, shown mounted in the TDT in Fig. 1. The IAWTM is a semispan model representative of a modern transport with an aspect ratio 13.5 wing [24]. The model has a wing semispan of 12.7 ft, to represent a transport with a span of 236 ft, and a sweep of 35 deg. The model has been tested up to Mach 0.8 and dynamic pressure of 200 psf. Due to the limitations in the previous TDT turbulence characterization, additional estimation of the tunnel turbulence was desired.

During the second wind tunnel entry of the IAWTM testing, a gust sniffer, shown in Fig. 2, was installed in the TDT. The gust sniffer is designed to measure the local angle of attack in the wind tunnel due to the gust generation system installed in the tunnel [25, 26]. While the gust generator is off, the gust sniffer can be used to characterize that free stream turbulence.

Figure 3 shows the vane mounted on the front of the gust sniffer that is used to measure the flow angularity. The gust sniffer is mounted approximately 1 ft below the tunnel centerline so that its wake will not interfere with the model. The angle measurement of the gust sniffer is calibrated. No downwash correction is generated for the gust sniffer. However, given the geometry of the gust sniffer the downwash is likely small.



Fig. 1 The IAWTM wind tunnel model (credit: NASA / Mark Knopp).

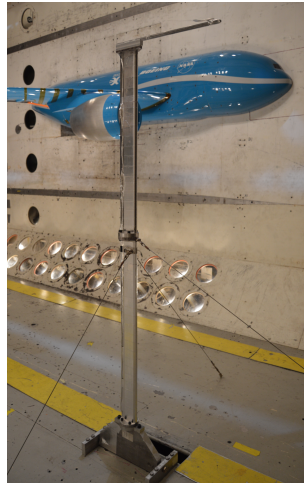


Fig. 2 The gust sniffer installed upstream from the IAWTM wind tunnel model (credit: NASA).

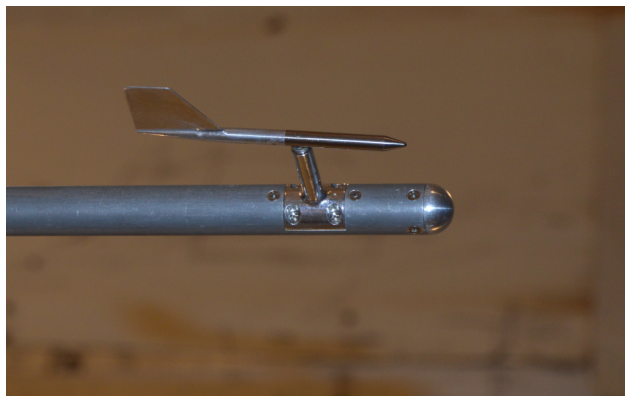


Fig. 3 Close up of the vane on the gust sniffer (credit: NASA).

The vertical velocity component of the turbulence is calculated from the measured sniffer angle and tunnel freestream velocity in Eq. (9).

$$w = V_T \sin \alpha \quad (9)$$

The spectra measured by the gust sniffer for multiple flow conditions are shown in Fig. 4. At airspeeds above 100 ft/s the spectra show a very clear peak at frequencies around 10 Hz to 50 Hz. The sharp peak is a significantly different characteristic from the typical von Kármán spectrum or the spectra observed in previous characterizations of the TDT turbulence [22, 23].

To better understand why the spectra in the IAWTM tests were different from previous tests, we begin by looking at how the spectra change with airspeed. The peak in Fig. 4a does show a significant change in the dimensional frequency with the flow condition. However, when looking at the reduced frequency, Fig. 4b, the peak shows a very consistent reduced frequency. The consistency in the reduced frequency suggests that this is a characteristic of the flow, and not a structural mode of the model or the gust sniffer.

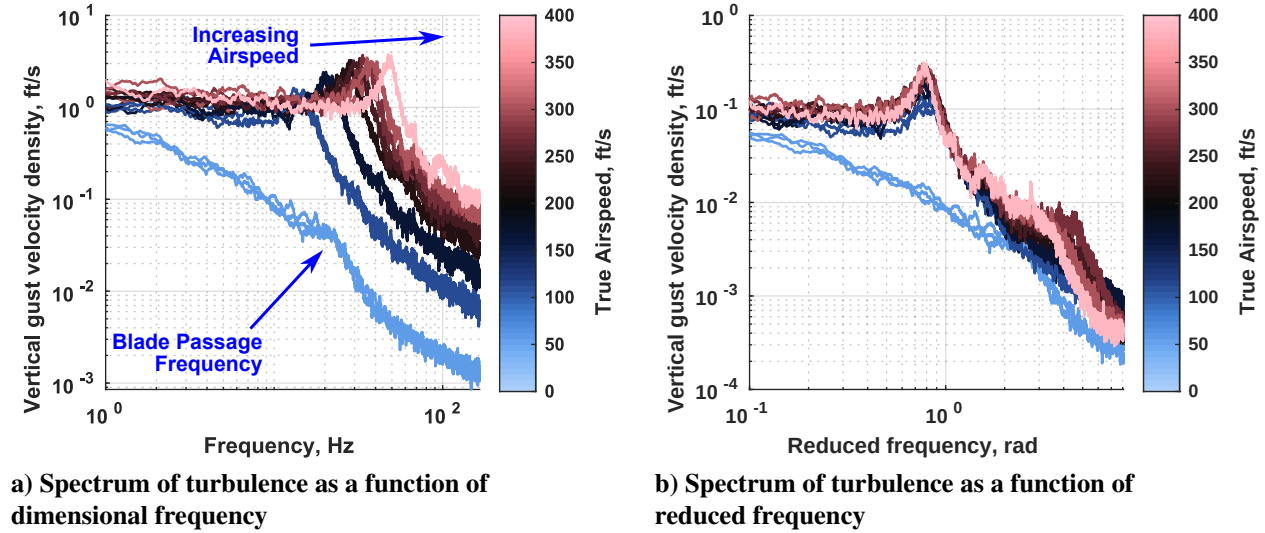


Fig. 4 Spectrum of turbulence as measured by gust sniffer.

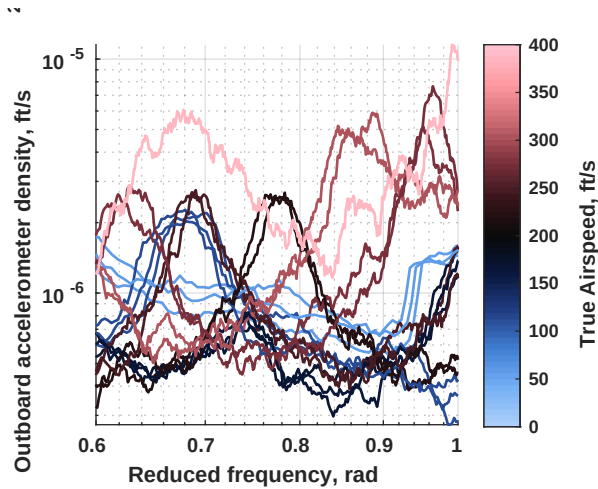


Fig. 5 Excitation of model accelerometer by turbulence.

The tunnel blade passing frequency was also considered, but the blade passing frequency is a higher frequency than this peak. The peak due to the blade passing frequency can be seen in some cases, but is very small and does not require further correction in the present analysis.

The PSD near the reduced frequency of the peak in Fig. 4b for an accelerometer located at the wingtip of the model

is shown in Fig. 5. This figure includes all of the test conditions being considered. All of the peaks in the model excitation correlate to structural modes. The model accelerometer does not show a consistent peak at the same frequency as seen in Fig. 4b. Some test conditions show a peak where a structural mode is at that frequency. However, the peak does not consistently appear and is significantly smaller than the gust sniffer measurements. The large peak in the gust sniffer measurements are not passed to the model. The peak may be a wake from the sniffer reaching the vane, but because it is not impacting the model, these disturbances will be ignored in the turbulence modeling.

Therefore, we can conclude that this peak is not a characteristic of the tunnel flow, and is only a characteristic of the gust sniffer data. The peak represents some type of additional noise on the signal that must be considered effect when estimating the turbulence parameters.

A. Estimation of Turbulence Parameters

To estimate the gust parameters, an expanded spectrum is defined in Eq. (10). The normalization constant in Eq. (10) is slightly different from Eq. (2a) such that the integral of the spectra with respect to the dimensional frequency is equal to the gust intensity. To handle the extra peak in the spectra, a bandpass filter is multiplied by the gust spectra. The bandpass parameters are not used in the turbulence modeling.

$$\varphi(\omega) \triangleq \frac{\tilde{\sigma}^2 \tilde{L} V_T}{2\pi} \frac{1 + \frac{8}{3} \left(1.339 \frac{\tilde{L}\omega}{V_T}\right)^2}{\left[1 + \left(1.339 \frac{\tilde{L}\omega}{V_T}\right)^2\right]^{11/6}} \left| \frac{(i\omega\tilde{n}_1 + 1)(i\omega\tilde{n}_2 + 1)}{1 + 2i\tilde{\zeta} \left(\frac{\tilde{L}_n\omega}{V_T}\right) - \left(\frac{\tilde{L}_n\omega}{V_T}\right)^2} \right|^2 \quad (10)$$

The estimated parameters are selected using a nonlinear least square algorithm [27] to minimize the square error as defined in Eq. (11). The measured spectrum term, $\sqrt{\tilde{\varphi}}(\omega_i)$, is the magnitude of the Fourier transform of the data.

$$\text{Error} \triangleq \sum_i \left[\sqrt{\tilde{\varphi}}(\omega_i) - \sqrt{\varphi}(\omega_i) \right]^2 \quad (11)$$

IV. Results

This section presents the results of our study in three progressive stages. First, we discuss the estimated turbulence parameters for the TDT. Where available these are compared against previous studies. Next, the proposed turbulence model is applied to the IAWTM with the corresponding simplifying assumptions used in loads and flying qualities standards. The turbulence component shapes, i.e., the columns of the matrix $\mathbf{U}_{jp}$ in Eq. (7), are compared against the assumed shapes used to create the standards. Finally, the applicability of the proposed turbulence model is demonstrated using the estimated turbulence parameters for the TDT.

A. Turbulence Parameters

The estimated turbulence length scales in Fig. 6 resulting from the fit of Eq. (10) show very similar results to the length scales for air testing in Ref. [22]. The differences in test media and wind off pressures appears to have little effect on the length scales of the turbulence.

The estimated length scales at low airspeeds suggest a large length scale. The length scales decrease rapidly from 0 – 150 ft/s. However, there is also a large variability in the estimates for repeated conditions which suggests that the method may not be getting a good estimate. For moderate airspeeds the length scale is has a weak dependence on airspeed. Above 150 ft/s the length scale shows a slightly increasing trend. Although the length scale does show a small trend with airspeed, the remaining analysis will use a fixed length scale of 4.13 ft.

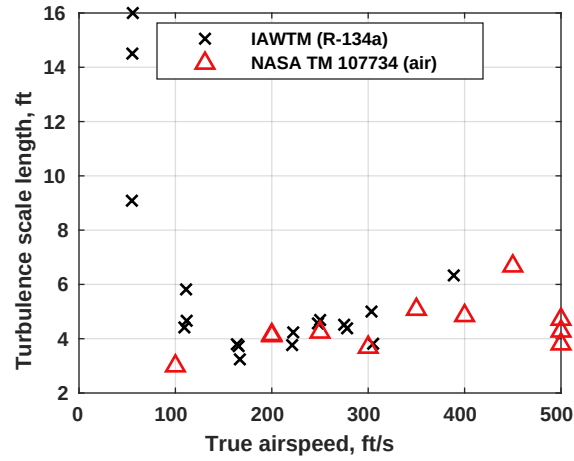


Fig. 6 Estimated turbulence length scale for TDT.

The turbulence intensity in Fig. 7 is shown as a percentage of the tunnel freestream velocity. The percent intensity shows a more consistent value with respect to the tunnel condition. The raw root mean square (RMS) is the intensity without a correction for the additional turbulence seen by the sniffer. Because there is no additional excitation at low airspeeds, the raw intensity matches very closely to the corrected value. As airspeeds increase, the raw value shows a significant error in the turbulence intensity. The turbulence intensities show a significant difference between the testing in air and heavy gas (R-134a). The survey of turbulence in various wind tunnels in Ref. [28] suggested that the heavy gas test medium in TDT may result in a lower turbulence level.

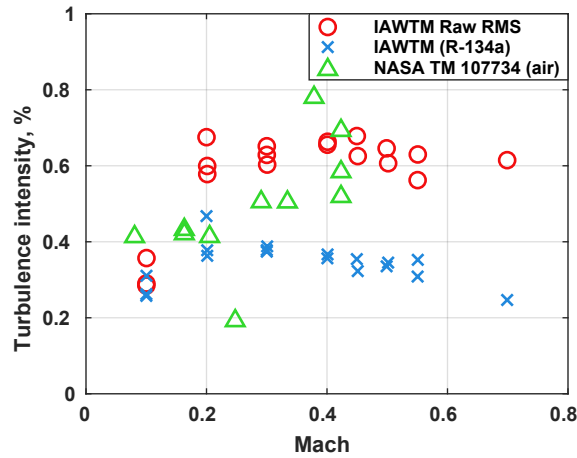


Fig. 7 Estimated turbulence intensity as a percentage of true airspeed for TDT.

The estimated parameters for the bandpass filter for the gust sniffer are shown in Figs. 8 and 9. The estimates do show a reasonable trend and are similar for repeated test conditions which occurred on different days of the test. The difference in parameters are consistent for the different days which suggests some of these differences are systematic errors due to variability of tunnel parameters, e.g., heavy gas purity and temperature, and not variability in the estimation.

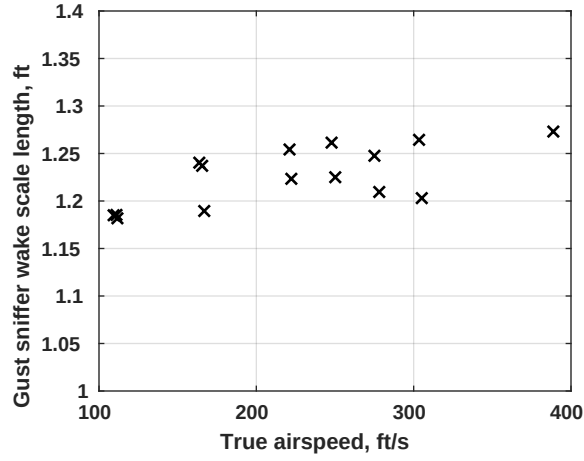


Fig. 8 Estimated length scale of filter for correcting sniffer data.

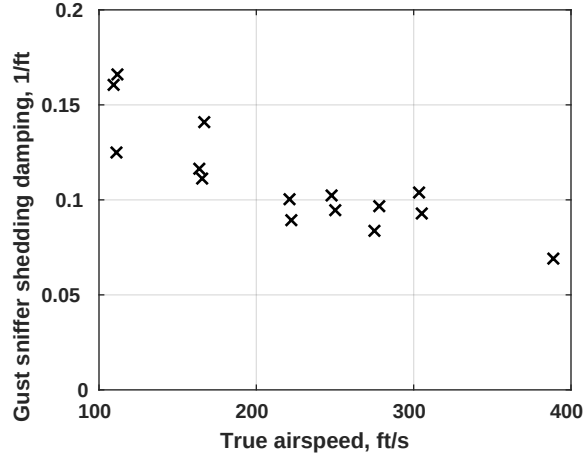


Fig. 9 Estimated damping ratio of filter for correcting sniffer data.

B. Replicating the Loads Turbulence Model

Using the present approach to turbulence modeling, it is possible to replicate the model of turbulence used in typical loads analysis that was described in Section I. Directly applying the turbulence spectrum parameters from [4, 21] is sufficient to replicate the uniform spanwise turbulence. To illustrate the conservatism in the assumption of a spanwise turbulence, the turbulence length scale is further reduced to $L = 10b$ for the present comparisons.

The first turbulence component for the loads turbulence model, Fig. 10, is uniform with no perceptible variations along the span. Some variation in the streamwise direction could be expected due to the finite velocity of the freestream. However, these streamwise phase variations are very small due to the high aspect ratio wing of IAWTM (with no tail) and the range of reduced frequencies used in the analysis.

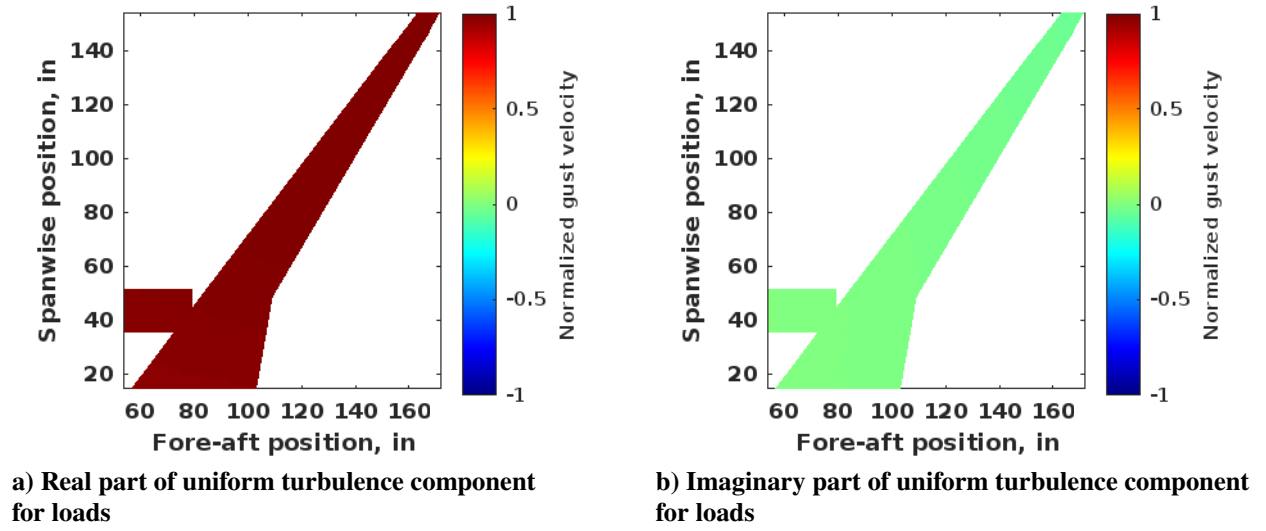


Fig. 10 First (uniform) turbulence component for loads.

The portion of the total velocity variations, i.e., the diagonal of $\mathbf{S}_{pp}$ in Eq. (7), for the turbulence field with the loads model, a scree plot, is shown in Fig. 11. The scree plot for the loads model shows that that for the IAWTM model, the first turbulence component captures over 96% of the velocity variations. Even for the high aspect ratio wing, a uniform turbulence model would be very effective in replicating the total turbulence field in flight.

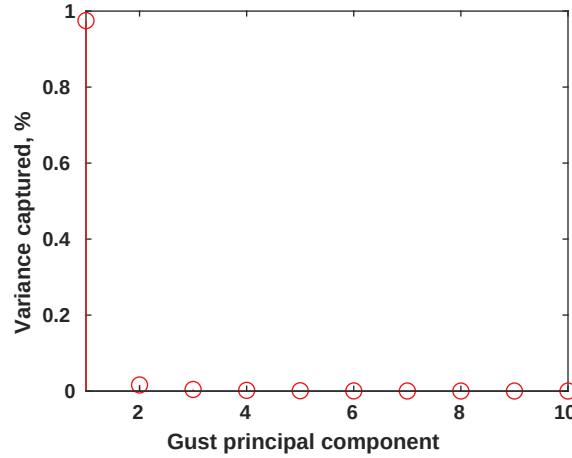


Fig. 11 Turbulence field velocity variations due to each turbulence component for loads.

The real part of the second turbulence component for the loads turbulence model, Fig. 12a, is varying in a spanwise direction like a roll variation. However, it is not a perfect roll mode because the shape is not uniform along the streamwise direction. The imaginary part of the second turbulence component for the loads turbulence model, Fig. 12b, does show some variations at the wing tip.

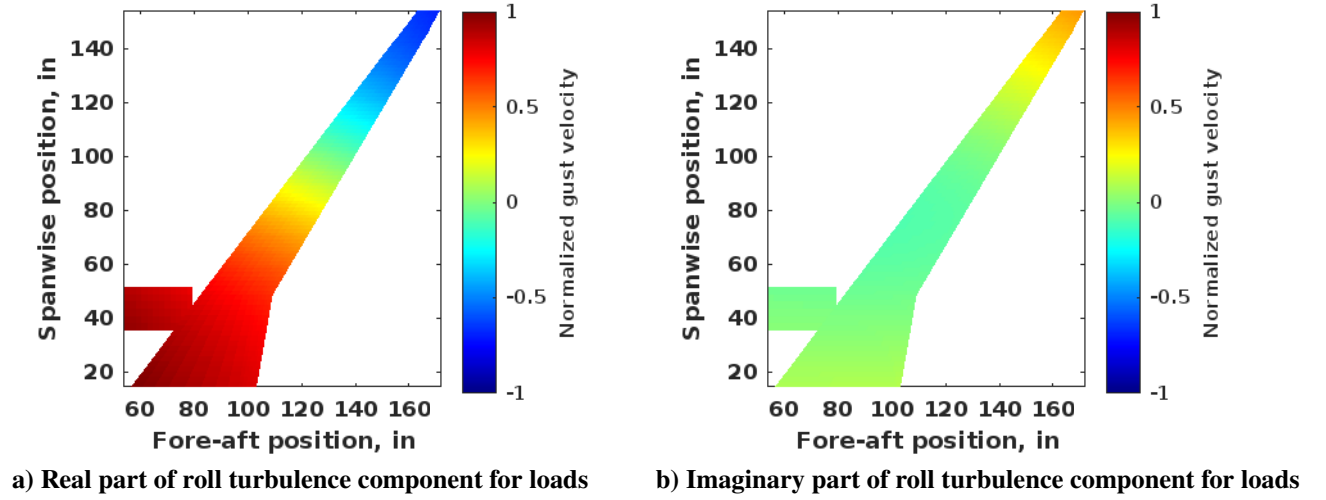


Fig. 12 Second (roll) turbulence component for loads.

C. Replicating the Flying Qualities Turbulence Model

Similarly, it is possible to replicate the turbulence models used in flying qualities analysis that were described in Section I. The same turbulence length scale from the loads model, $L = 10b$, is used for replicating the flying qualities model. The assumption of an unswept wing in Refs. [11–13], means that $\mathbf{x}_{jj} = 0$ and implies that the exponential term in Eq. (4a) is negligible.

The first turbulence component for the flying qualities turbulence model, Fig. 13, is nearly identical to the uniform turbulence component from the loads model, Fig. 10.

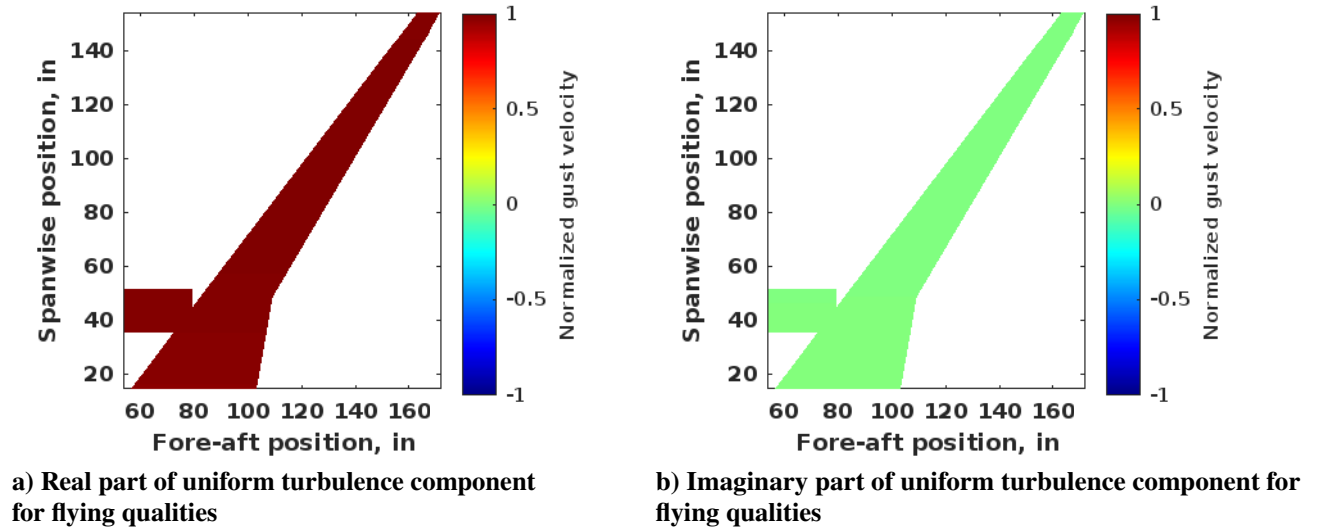


Fig. 13 First (uniform) turbulence component for flying qualities.

As with the loads model, the second turbulence component for the flying qualities turbulence model, Fig. 14, is varying in a spanwise direction like a roll variation. Unlike the loads model, because the lag is neglected, the turbulence component does not vary in the streamwise direction and is exactly consistent with rolling motion. Furthermore, neglecting the lag means that there is no variation in the imaginary part, Fig. 14b. The turbulence components from the

loads model, Figs. 10 and 12, are likely more accurate, but the simplifications used in generating the flying qualities model are reasonable, at least for the present transport configuration, and significantly simplify the implementation of the simulations needed for analysis.

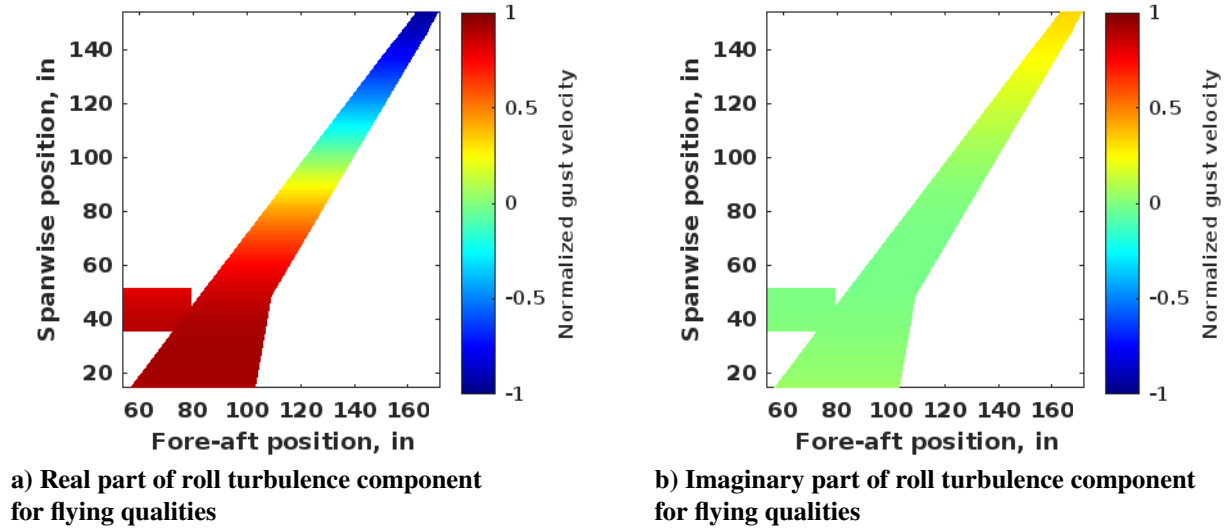


Fig. 14 Second (roll) turbulence component for flying qualities.

For the flying qualities turbulence model, the scree plot shows very similar results to the loads model. Even a single uniform turbulence will effectively capture the velocity variations. However, the second component may be significant for capturing the roll dynamics, which is a metric not considered in the present analysis.

D. Turbulence in the Transonic Dynamics Tunnel

Due to the shorter turbulence length scales, the contribution of each turbulence component to the in Fig. 15 show more components with a significant magnitude than was observed in Fig. 11. The scree plot does not show a clear drop-off that would give a clear indication of how many turbulence components to include. For the present analysis, four components will be used.

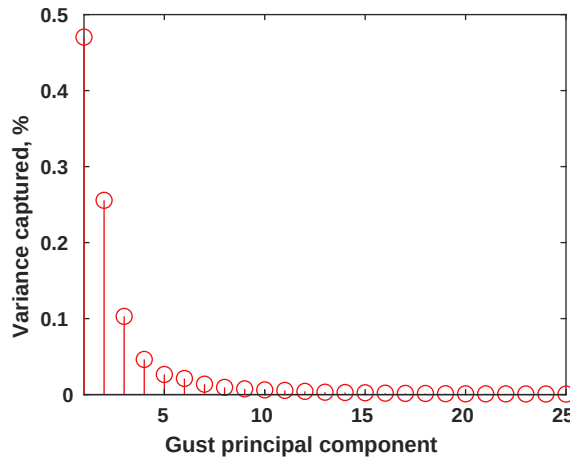


Fig. 15 Turbulence field velocity variations due to each turbulence component for TDT.

Even the first turbulence component for the TDT turbulence model, Fig. 16, shows significant variations in the

turbulence field along the wing. Furthermore, the imaginary part in Fig. 16b shows a much larger variation than seen with larger turbulence length scales. These variations in the turbulence field could produce a significantly different loading and excitation of the structure than would be observed by a uniform turbulence field. Additionally, these variations in the turbulence field are significantly more complex and would be very challenging to produce analytically. The complexity of these turbulence fields also make labeling or classifying these shapes challenging as well.

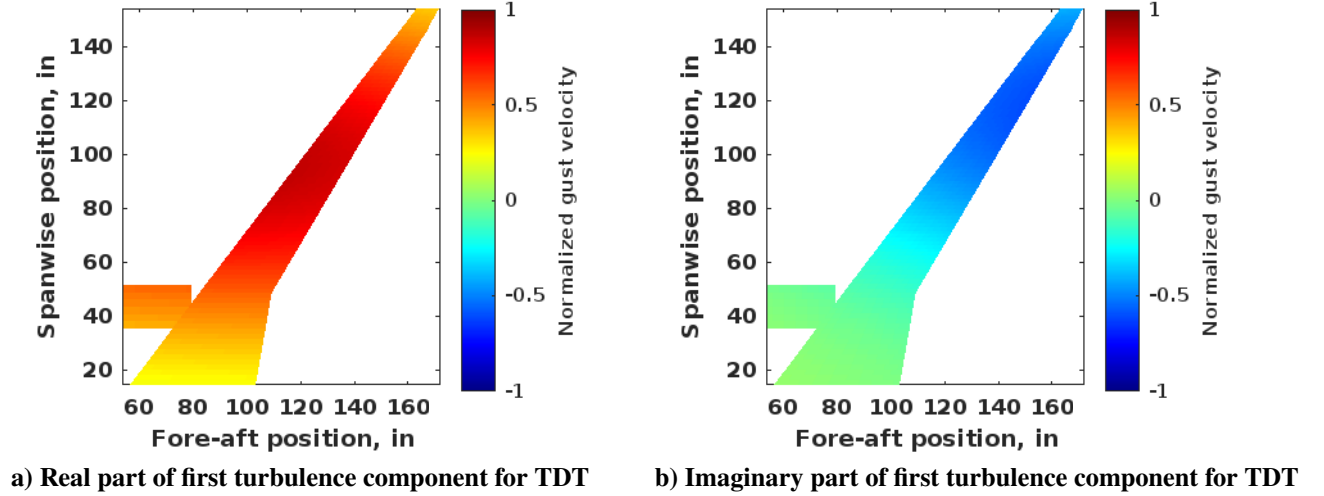


Fig. 16 First turbulence component for TDT.

The fourth turbulence component for the TDT turbulence model, Fig. 17, shows high order variations in the turbulence velocity along the span. Additionally, one can see noticeable chordwise variations in the imaginary parts, Fig. 17b.

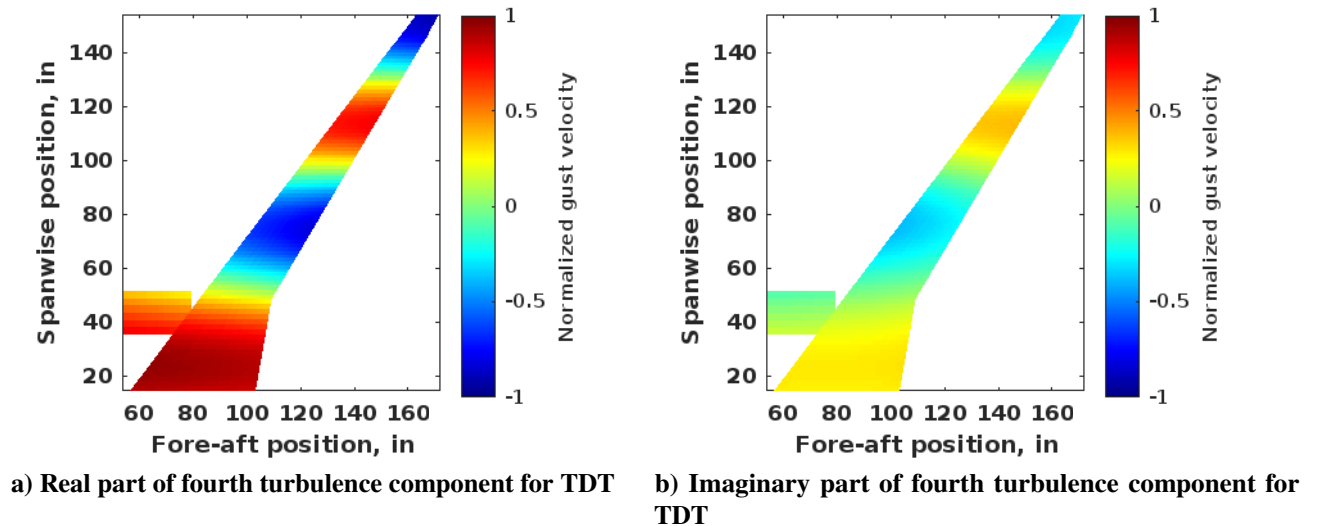


Fig. 17 Fourth turbulence component for TDT.

The PSD for all these turbulence components in Fig. 18 shows the typical knee shape observed with turbulence spectra. The higher components tend to show a slight increase in bandwidth and a slower roll off. Because these PSDs are normalized, the higher bandwidth and slower roll off mean that the low frequency has a lower magnitude. However,

the trend is not universal as the first component does not match this trend. A transfer function can be fit to these PSDs to create the appropriate coloring filter for an ASE model.

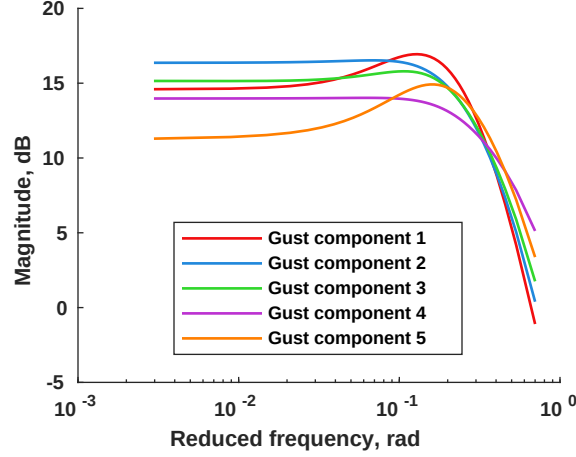


Fig. 18 PSD of turbulence components for TDT.

V. Conclusion

A method of generating a simplified turbulence model for aeroservoelastic analysis has been developed. The method was demonstrated with the high aspect ratio IAWTM transport configuration wind tunnel model. This method worked for arbitrary turbulence scales, including the very short turbulence length scales present in a wind tunnel such as TDT. By adjusting the turbulence scale, the method also effectively replicates the uniform turbulence shape used in most turbulence load analyses. The analysis also shows that a single uniform turbulence mode is a very reasonable approximation for a transport aircraft. Flying qualities analysis makes further simplifying assumptions, but by replicating these assumptions the current method was able to replicate the assumed turbulence field used in simulations for flying qualities assessment. The present model remains a relatively simple model and is likely of limited fidelity. However, for short turbulence length scales the proposed approach offers significant improvements in fidelity with a small increase in complexity. The improved turbulence model will enable better evaluation of the disturbance rejection the control laws being tested with IAWTM and correlating the process noise when updating the model parameters.

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Appendix A: Previous Measurements of Turbulence Velocity

In Sleeper et. al. [22] the model of Eqs. (12) and (13) was fit to each of these velocity components measured in air.

$$\tilde{\phi} = \frac{\tilde{\sigma}^2 2\tilde{L}}{\pi} \frac{1 + \tilde{\beta}\tilde{k}^2}{[1 + \tilde{\gamma}\tilde{k}^2]^{\tilde{n}}} \quad (12)$$

$$\tilde{k} \triangleq \frac{\omega\tilde{L}}{V_T} \quad (13)$$

As Eqs. (12) and (13) are implemented, the length parameter, $\tilde{L}$ is redundant. The standard von Kármán turbulence spectrum has a fixed nondimensional break frequency of 1.345 when normalized by the gust length scale. The characteristic lengths are corrected in Eq. (14) to remove the redundant parameter and make nondimensional break frequency consistent with the nondimensional break frequency of the von Kármán turbulence spectrum.

$$L = \frac{\tilde{L}}{1.3452} \left(\frac{\tilde{\beta}}{\tilde{\gamma}^{\tilde{n}}} \right)^{\frac{1}{2(\tilde{n}-1)}} \quad (14)$$

The adjusted turbulence length scales are shown in Table 1. The length scale does increase slightly with dynamic pressure. However, the change in length scale is significantly smaller than the uncertainty in the length scale. Therefore, the change in turbulence length scale is assumed negligible. The length of recorded time histories of turbulence in [22] are too short to clearly show the low frequency behavior. Therefore, the fit parameters of the numerator, which primarily effect the low frequency behavior, are not examined further.

Table 1 Adjusted TDT turbulence length scales from [22].

Velocity, ft/s	Dynamic Pressure, psf	Length Scale, ft
100	11.4	3.00
200	44.8	4.11
200	44.5	4.15
250	69.4	4.24
300	98.7	3.68
350	131.2	5.08
400	167.6	4.85
450	207.3	6.68
500	248.8	4.71
500	248.5	4.28
500	249.6	3.80
Average		4.42
Median		4.24
Confidence Interval		2.110

References

- [1] Dryden, H. L., “A Review of the Statistical Theory of Turbulence,” *Quarterly of Applied Mathematics*, Vol. 1, No. 1, 1943, pp. 7–42.
<https://doi.org/https://doi.org/10.1090/qam/8209>.
- [2] de Karman, T., and Howarth, L., “On the statistical theory of isotropic turbulence,” *Proceedings of the Royal Society of London*, Vol. 164, No. 917, 1938, pp. 192–215.
<https://doi.org/https://doi.org/10.1098/rspa.1938.0013>.
- [3] “Joint Service Specification Guide, Vehicle Control and Management System (VCMS),” JSSG-2008, Oct. 1998.
- [4] “Airworthiness Standards: Transport Category Airplanes,” 14 CFR 25.341, 2021.
- [5] Houbolt, J. C., Steiner, R., and Pratt, K. G., “Dynamic response of airplanes to atmospheric turbulence including flight data on input and response,” NASA TR R–199, Jun. 1964.
- [6] Houbolt, J., “Atmospheric turbulence,” *AIAA Journal*, Vol. 11, No. 4, 1973, pp. 421–437.
<https://doi.org/10.2514/6.1972-219>.
- [7] Hoblit, F. M., *Gust Loads on Aircraft: Concepts and Applications*, AIAA, 1988.

- [8] Patil, M. J., and Taylor, D. J., “Gust Response of Highly Flexible Aircraft,” AIAA 2006–1638, Newport, RI, Jun. 2006.
<https://doi.org/10.2514/6.2006-1638>.
- [9] Patil, M. J., “Nonlinear Gust Response of Highly Flexible Aircraft,” AIAA 2007–2103, Honolulu, HI, Apr. 2007.
<https://doi.org/10.2514/6.2007-2103>.
- [10] Danowsky, B. P., Lisoski, D. L., Mukherjee, J. S., and Taylor, D., “A Structural Design Methodology for Highly Flexible HAPS Aircraft,” AIAA 2024–1733, Orlando, FL, Jan. 2024.
<https://doi.org/10.2514/6.2024-1733>.
- [11] “Flying Qualities of Piloted Aircraft,” MIL-HDBK-1797, Dec. 1997.
- [12] “Flying Qualities of Piloted Aircraft,” MIL-STD-1797B, Feb. 2006.
- [13] “Flying Qualities of Piloted Airplanes,” MIL-F-8785C, Nov. 1980.
- [14] Chalk, C. R., Neal, T. P., Harris, T. M., Pritchard, F. E., and Woodcock, R. J., “Background Information and User Guide for MIL-F-8785B (ASG), ‘Military Specification-Flying Qualities of Piloted Airplanes’,” AFFDL TR 69–72, Aug. 1969.
- [15] Etkin, B., “Theory of the Flight of Airplanes in Isotropic Turbulence – Review and Extension,” AGARD Report 372, Apr. 1961.
- [16] Etkin, B., “A Theory of the Response of Airplanes to Random Atmospheric Turbulence,” *Journal of the Aerospace Sciences*, Vol. 26, No. 7, 1959, pp. 409–420.
<https://doi.org/10.2514/8.8127>.
- [17] Houbolt, J. C., “Survey on Effect of Surface Winds on Aircraft Design and Operation and Recommendations for Needed Wind Research,” NASA CR–2360, Dec. 1973.
- [18] Leggett, D., Moorhouse, D., and Zeh, J., “Simulating turbulence and gusts for handling qualities evaluation,” AIAA 1990–2845, Portland, OR, Aug. 1990.
<https://doi.org/10.2514/6.1990-2845>.
- [19] Houbolt, J. C., and Sen, A., “Cross-spectral functions based on Von Karman’s spectral equation,” NASA CR–2011, Mar. 1972.
- [20] Johnson, R. A., and Wichern, D. W., *Applied Multivariate Statistical Analysis*, Pearson Prentice Hall, 2007.
- [21] “Joint Service Specification Guide, Aircraft Structures,” JSSG–2006, Oct. 1998.
- [22] Sleeper, R. K., Keller, D. F., Perry III, B., and Sandford, M. C., “Characteristics of vertical and lateral tunnel turbulence measured in air in the Langley Transonic Dynamics Tunnel,” NASA TM–107734, Mar. 1993.
- [23] Wiesman, C. D., and Sleeper, R. K., “Measurements of Flow Turbulence in the NASA Langley Transonic Dynamics Tunnel,” NASA TM–2005–213529, Feb. 2005.
- [24] Heaney, P. S., and Quindlen, J. F., “Overview of the Integrated Adaptive Wing Technology Maturation Wind-Tunnel Test Objectives,” AIAA 2024–2509, 2024.
<https://doi.org/10.2514/6.2024-2509>.
- [25] Scott, R. C., Vetter, T. K., Penning, K. B., Coulson, D. A., and Heeg, J., “Aeroservoelastic Testing of Free Flying Wind Tunnel Models Part 1: A Sidewall Supported Semispan Model Tested for Gust Load Alleviation and Flutter Suppression,” NASA TP–2013–218051, 2013.
- [26] Scott, R. C., Vetter, T. K., Penning, K. B., Coulson, D. A., and Heeg, J., “Aeroservoelastic Testing of Free Flying Wind Tunnel Models: Part 2: A Centerline Supported Fullspan Model Tested for Gust Load Alleviation,” NASA TP–2014–218170, 2014.
- [27] Moré, J. J., “The Levenberg-Marquardt algorithm: implementation and theory,” Argonne National Lab, 1977.
- [28] Whitfield, J. D., and Dougherty, N. S., “A Survey of Transition Research at AEDC,” AEDC TR–77–52, 1976.