

# Boundary-Layer Transition over a Smooth Circular Cone at Non-Zero Angle of Attack in a Mach 8 Digital Wind Tunnel

Mateus Schuabb\* and Lian Duan†  
*The Ohio State University, Columbus, OH 43210, USA*

Anton Scholten‡  
*National Institute of Aerospace, Hampton, VA 23666, USA*

Pedro Paredes§ and Meelan M. Choudhari¶  
*NASA's Langley Research Center, Hampton, VA 23681, USA*

Douglas W. Carter||  
*Illinois Institute of Technology, Chicago IL 60477, USA*

Katya M. Casper\*\* and Ross M. Wagnild††  
*Sandia National Laboratories, Albuquerque, New Mexico 87185, USA*

**This paper presents a three-dimensional direct numerical simulation (DNS) of hypersonic boundary-layer transition over a sharp, 7-degree half-angle cone at an 8-degree angle of attack subjected to tunnel-like freestream acoustic disturbances. The flow conditions and geometry match those of experiments conducted in the Sandia Hypersonic Wind Tunnel at Mach 8 (Sandia HWT-8). The DNS captures all stages of the transition process driven by tunnel noise, including the propagation of broadband acoustic fluctuations through the bow shock; boundary-layer receptivity; the generation and amplification of Mack's second-mode and crossflow waves and the unsteady shear-layer instability of the leeward vortex; the nonlinear growth and breakdown of disturbances; and the subsequent development of a fully turbulent boundary layer. The inflow acoustic disturbances were reconstructed from acoustic radiation generated by the turbulent boundary layer over the Sandia nozzle wall, a methodology previously validated in DNS of the same cone at zero degrees angle of attack. The present study extends this framework to a nonzero angle of attack and provides a characterization of the resulting asymmetric transition on the cone in a Mach 8 “noisy” environment. The results reveal a strongly asymmetric transition pattern induced by the angle of attack. On the leeward side, the mushroom-shaped vortex experiences early distortion and breakdown, leading to localized shocklet formation and transition to turbulence upstream of the windward side. In contrast, the windward side remains laminar over a longer streamwise distance, with transition dominated by second-mode instabilities that develop later along the cone. Analysis of (synthetic) Schlieren, vorticity, and divergence fields indicates that the interactions with freestream acoustic disturbances play a key role in the complex three-dimensional transition process. The wall-pressure spectra across all azimuthal locations in the turbulent regions of the cone compared well with those measured in the Sandia HWT-8.**

---

\*Graduate Student, Department of Mechanical and Aerospace Engineering, Student Member, AIAA

†Professor, Department of Mechanical and Aerospace Engineering, Associate Fellow, AIAA

‡Research Engineer, Member, AIAA

§Aerospace Technologist, Computational AeroSciences Branch, Associate Fellow, AIAA

¶Aerospace Technologist, Computational AeroSciences Branch, Fellow, AIAA

||Assistant Professor, Department of Mechanical, Materials, and Aerospace Engineering, Member, AIAA

\*\*Distinguished Member of the Technical Staff, Associate Fellow, AIAA

††Principal Member of the Technical Staff, Associate Fellow AIAA.

## Nomenclature

|              |   |                                                                                                               |
|--------------|---|---------------------------------------------------------------------------------------------------------------|
| AoA          | = | angle of attack, degree                                                                                       |
| $C_p$        | = | heat capacity at constant pressure, J/(K·kg)                                                                  |
| $f$          | = | frequency, Hz                                                                                                 |
| $L_s$        | = | Cone length from the experiments, m                                                                           |
| $M$          | = | Mach number, dimensionless                                                                                    |
| $Pr$         | = | Prandtl number, $Pr = 0.71$ , dimensionless                                                                   |
| $R$          | = | ideal gas constant, $R = 287$ , J/(K·kg)                                                                      |
| $Re_\infty$  | = | freestream unit Reynolds number, $Re_\infty = \frac{\rho_{\infty,1} U_{\infty,1}}{\mu_{\infty,1}}$ , $m^{-1}$ |
| rms          | = | root mean square                                                                                              |
| $T$          | = | temperature, K                                                                                                |
| $T_r$        | = | recovery temperature, $T_r = T_{\infty,1} (1 + 0.9 * \frac{\gamma-1}{2} M_{\infty,1}^2)$ , K                  |
| $U_\infty$   | = | freestream velocity, m/s                                                                                      |
| $p$          | = | pressure, Pa                                                                                                  |
| $r$          | = | radial coordinate                                                                                             |
| $u$          | = | streamwise velocity, m/s                                                                                      |
| $u_\tau$     | = | friction velocity, $u_\tau \equiv \tau_w / \bar{\rho}_w$ , m/s                                                |
| $v$          | = | spanwise velocity, m/s                                                                                        |
| $w$          | = | wall-normal velocity, m/s                                                                                     |
| $x_g$        | = | streamwise direction of the right hand Cartesian coordinate                                                   |
| $y_g$        | = | spanwise direction of the right hand Cartesian coordinate                                                     |
| $z_g$        | = | wall-normal direction of the right hand Cartesian coordinate                                                  |
| $z_\tau$     | = | viscous length, $z_\tau = \nu_w / u_\tau$ , m                                                                 |
| $\gamma$     | = | specific heat ratio, $\gamma = C_p / C_v$ , dimensionless                                                     |
| $\delta$     | = | boundary layer thickness (based on 99.5% of the total enthalpy), m                                            |
| $\kappa$     | = | thermal conductivity, $\kappa = \mu C_p / Pr$ , W/(m·K)                                                       |
| $\mu$        | = | dynamic viscosity, kg/(m·s)                                                                                   |
| $\rho$       | = | density, kg/m <sup>3</sup>                                                                                    |
| $\tau_w$     | = | wall shear stress, Pa                                                                                         |
| Subscripts   |   |                                                                                                               |
| w            | = | wall variables                                                                                                |
| $\infty$     | = | freestream variables                                                                                          |
| Superscripts |   |                                                                                                               |
| $+$          | = | inner wall units                                                                                              |
| $(\cdot)$    | = | statistically averaged variables                                                                              |
| $(\cdot)'$   | = | perturbation from averaged variable                                                                           |

## I. Introduction

Boundary layer transition (BLT) accounts for a major source of uncertainty in the heating loads on high-speed vehicles. Therefore, the prediction and control of transition onset and the associated variation in aerothermodynamic parameters in high-speed flows are critical factors in the design of next-generation aerospace vehicles. However, our ability to predict transition during hypersonic flight is hindered by the high levels of acoustic noise in most hypersonic ground-test facilities [1, 2]. There are few existing quiet hypersonic wind tunnels, and these tunnels are limited to Mach 6, moderate Reynolds numbers, low freestream enthalpy, and small test sections only accommodating subscale models [3]. Thus, an analysis with tunnel noise is invaluable in order to properly interpret the data from the nation's hypersonic wind tunnel which are mostly noisy tunnels.

In a low-enthalpy hypersonic wind tunnel with adequate flow conditioning, the freestream disturbance environment is dominated by acoustic radiation from tunnel-wall turbulent boundary layers (TBL) [4–8]. Tunnel noise can impact not only the transition location, but possibly the transition mechanism as well. The effect of tunnel noise on transition cannot be quantified by a single metric that corresponds to the root-mean-square amplitude. For the acoustic second-mode instability in particular, its space and time scales can coincide with those of typical acoustic waves present in the

free stream [9]. Thus, hypersonic BLT is particularly sensitive to the detailed spectrum of freestream acoustic noise. Experiments in the Purdue quiet tunnel have shown that tunnel noise has a dramatic effect on the nonlinear breakdown of second-mode waves [10, 11]. Specifically, there is a large discrepancy in the breakdown amplitude of second-mode waves between the data obtained in a quiet tunnel versus a conventional one [11]. Such an influence is hard to quantify because the Reynolds number limitation of quiet tunnels often prevents a fully turbulent flow in experiments.

To elucidate the role of freestream disturbances and nonmodal growth of traveling disturbances within the entropy layer during the transition process, Liu et al. [12] performed DNS and nonmodal stability analysis to study boundary-layer receptivity to broadband freestream acoustic disturbances from the nozzle wall of a digital conventional wind tunnel at Mach 8. In their DNS, a “tunnel-like” acoustic disturbance field, that reproduces both the frequency-wavenumber spectra and the temporal evolution of the broadband tunnel noise radiated from the nozzle wall, was input upstream of the bow shock of a 7-degree half-angle cone with a nose radius of  $R_n = 5.2$  mm. The DNS showed prominent entropy layer disturbances resembling those obtained by nonmodal theory [13]. The DNS-predicted spectra of wall-pressure and heat-transfer fluctuations were shown to recover the signature of the axisymmetric waves predicted by the nonmodal analysis, and the numerical schlieren contours of the DNS showed similar inclined structures commonly observed in blunt cone experiments.

Similarly, Schuabb et al. [14] also performed DNS of boundary-layer instability and transition subject to the same freestream conditions as Liu et al. [12], but over a nominally sharp  $7^\circ$  half-angle circular cone with a nose radius of  $R_n = 0.5$  mm at zero degrees angle of attack. Their DNS captured all the stages of the transition to turbulence caused by tunnel noise, including the passage of broadband freestream noise through the shock wave, the receptivity process leading to the generation of Mack’s second-mode waves, their nonlinear growth to saturation, the laminar breakdown to turbulence, and the post-transitional, fully turbulent flow. The computed skin friction and Stanton number distributions in the initial breakdown region showed an overshoot compared to the turbulent predictions by the van Driest II theory [15]. The current paper extends the pioneering DNS of [14] by considering an  $8^\circ$  angle of attack to provide further information regarding the downstream evolution of the stationary crossflow disturbances and the eventual breakdown to turbulence.

The flow conditions, numerical methods, and the numerical grid for the circular cone at nonzero degrees angle of attack are outlined in section II. Section III presents DNS results for the interaction of a tunnel-like acoustic disturbance field with a  $7^\circ$  half-angle cone and 0.5 mm nose radius at Mach 8 at an 8-degree angle of attack. Lastly, results are summarized and future work is outlined in Section IV.

## II. DNS Methodology

The freestream conditions are given first, followed by a description of the governing equations, numerical methods, and domain used for the DNS simulation.

### A. Flow Condition

DNS of boundary-layer instabilities and transition is conducted over a 0.60-meter-long,  $7^\circ$  half-angle circular cone with a 0.5 mm nose radius at an angle of attack (AoA) of  $8^\circ$ . To provide an accurate “tunnel-like” inflow environment, a precursor DNS of the full-scale axisymmetric Sandia HWT-8 nozzle was first performed at its nominal operating conditions, with total pressure  $P_0 \approx 4692$  kPa and total temperature  $T_0 \approx 617$  K. The instantaneous flow field (including unsteady freestream acoustic fluctuations) near the centerline of the nozzle exit was extracted, modeled, and imposed as the freestream boundary condition for the simulation of the cone. More details on the precursor digital wind tunnel DNS and the extraction procedure of the freestream acoustic disturbances can be found in Liu et al. [12] and Schuabb et al. [14]. The wall temperature of the cone is fixed at  $T_w = 298$  K, corresponding to a wall-to-recovery temperature ratio of  $T_w/T_r \approx 0.537$ . Table 1 summarizes the extracted mean freestream conditions of the digital wind tunnel used for the present cone DNS. The subscript  $(\cdot)_{\infty,1}$  denotes freestream tunnel quantities measured upstream of the cone’s leading-edge shock. Therefore,  $U_{1,\infty}$  represents the streamwise velocity aligned with the tunnel axis, which differs from the cone axis. The coordinate transformation used for the cone are discussed in Section II.C.

**Table 1 Preshock freestream conditions and wall temperature for the DNS of the Mach 8 cone at  $8^\circ$  AoA.**

| $M_{1,\infty}$ | $U_{1,\infty}$ (m/s) | $p_{1,\infty}$ (Pa) | $T_{1,\infty}$ (K) | $T_w$ (K) | $T_w/T_r$ | $Re_\infty$ (1/m)  |
|----------------|----------------------|---------------------|--------------------|-----------|-----------|--------------------|
| 8.08           | 1091.82              | 448.80              | 43.90              | 298       | 0.537     | $12.2 \times 10^6$ |

## B. Governing Equations and Numerical Methods

The full three-dimensional compressible Navier-Stokes equations in conservation form are solved in generalized cylindrical coordinates. Nitrogen is used as the working fluid, modeled as a perfect gas with the gas constant set to  $R = 296.7 \text{ J/(K}\cdot\text{kg)}$ . The Newtonian constitutive relations are employed such that the viscous stress tensor is proportional to the rate-of-strain tensor and the heat conduction follows Fourier’s law. The molecular viscosity  $\mu$  is evaluated using Keyes’ law [16] and the thermal conductivity is given by  $\kappa = \mu C_p / Pr$  with a constant molecular Prandtl number  $Pr = 0.71$ .

The inviscid fluxes are discretized using a weighted essentially nonoscillatory (WENO) scheme [17] of fifth or seventh order, depending on the local flow region. To capture the imposed freestream acoustic disturbances and the instability waves within the pre-transitional boundary layer, the optimal WENO stencil is used throughout most of the freestream and in the laminar/pre-transitional boundary-layer region. WENO adaptation is activated only in the vicinity of the leading-edge shock and within the breakdown and turbulent regions to maintain numerical stability while minimizing dissipation. Viscous fluxes are discretized using a fourth-order central-difference scheme, and temporal integration is performed using the third-order low-storage Runge-Kutta method [18].

The coordinate singularity at the centerline ( $r = 0$ ) is handled using the pole-shifting approach of Mohseni and Colonius [19], where radial grid points are shifted by half a grid spacing to avoid one-sided finite differences by mapping the flow domain from  $(0, R) \times (0, 2\pi)$  to  $(-R, R) \times (0, \pi)$ , where  $R$  is the radius of the computational domain. Near the centerline, the azimuthal grid spacing becomes increasingly small, leading to severe time-step constraints. Following the work of Bogey et al. [20], this limitation is addressed by dropping every few azimuthal grid points in the computation of azimuthal derivatives as  $r \rightarrow 0$ , thereby increasing the effective spacing and relaxing the stability constraint.

To simulate boundary-layer receptivity to tunnel noise, the inflow boundary of the computational domain is placed outside the leading-edge shock of the cone. “Tunnel-like” freestream disturbances obtained from the precursor DNS of an empty digital wind tunnel are extracted, reconstructed, and imposed at the outer boundary of the present DNS; see Liu et al. [12] for details. At the cone surface, no-slip conditions are applied to all three velocity components and an isothermal boundary condition is used for temperature. Periodic boundary conditions are applied in the azimuthal direction, with the computational domain covering the full  $360^\circ$  geometry.

## C. DNS Domain, Grid, and Coordinate System

The DNS domain is designed to accurately resolve both the small-scale flow features near the nose and the downstream evolution of the boundary layer. A primary challenge is that the azimuthal domain size grows substantially with downstream distance. Both the azimuthal and wall-normal extents of the domain increase, and maintaining comparable resolution in wall units requires a larger number of points in the downstream region than near the nose. Using the same downstream resolution in the nose region would therefore lead to significant over-resolution and impose an unnecessarily small time step. To address this, an overlapping multibox strategy is employed. The streamwise domain is divided into several overlapping boxes, each with point counts and spacings tailored to the local flow scales. The nose box contains fewer azimuthal and wall-normal points, while downstream boxes contain progressively more points in these directions to preserve resolution as the domain widens. This approach enables larger time steps in downstream boxes and avoids over resolution of the nose region, while maintaining consistency across box interfaces through streamwise overlap. Figure 1 illustrates the DNS configuration employed for the present study of boundary-layer transition over a  $7^\circ$  half-angle cone with a 0.5 mm nose radius at  $AoA = 8^\circ$  and Mach 8. A similar multidomain simulation approach of this type was previously applied to the DNS of the BLT over a cone [12, 14, 21, 22].

A second major consideration is the variation of the shock standoff distance around the cone due to angle of attack. On the windward side, the shock is much closer to the surface, while on the leeward side it is substantially farther away. Thus, the physically meaningful wall-normal domain height is not uniform in the azimuthal direction. Unlike prior zero-angle-of-attack studies that used a constant wall-normal domain across the span [14], the present work allows the wall-normal extent of each DNS box to follow the local shock shape. To construct this shock-conforming wall-normal domain, a steady, disturbance-free laminar flow is first computed using the VULCAN-CFD solver [23]. The shock surface is identified on the VULCAN-CFD grid and then mapped onto the DNS grid using a ray-tracing procedure. For each azimuthal location, a wall-normal ray is cast from the surface of the cone; its intersection with the shock is computed using a ray-surface intersection algorithm and the intersection coordinates are interpolated bilinearly. This procedure yields a smooth, azimuthally varying wall-normal domain height that tracks the shock around the entire circumference. With the shock-based wall-normal extent defined, the DNS grid in the  $r$ -direction is generated using a piecewise hyperbolic-tangent distribution. The grid is strongly clustered near the wall to resolve viscous layers and

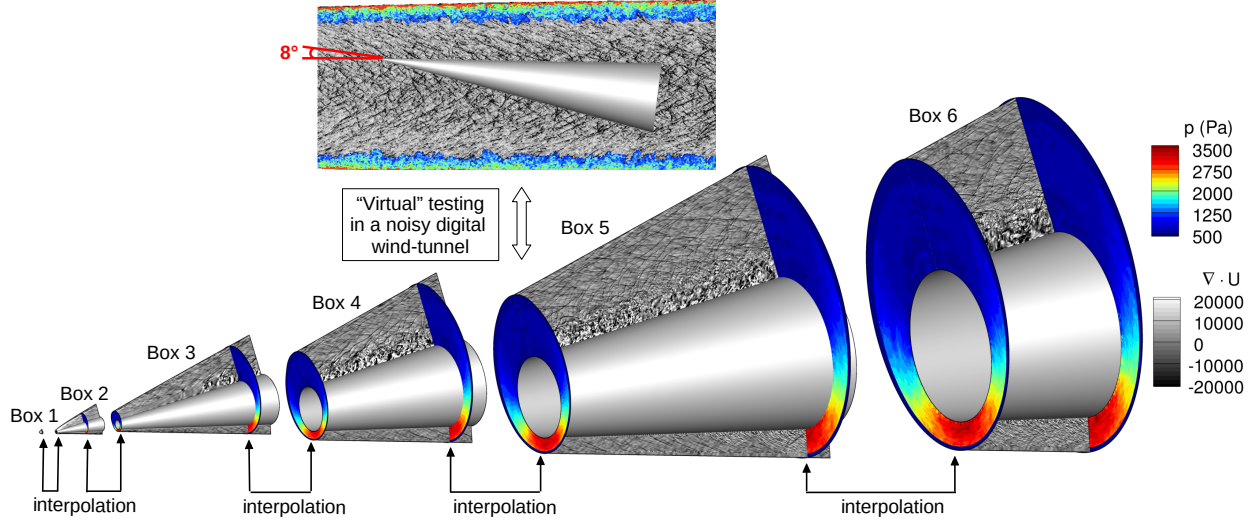


**Fig. 1** Simulation setup for DNS of the Mach 8 flow over a  $7^\circ$  half-angle circular cone with a nose radius of  $R_n = 0.5$  mm at 8 degrees angle of attack. The contour plots are generated by overlaying the divergence of the velocity field (grayscale) and pressure (colored). The image at the top of the Figure shows the cone model relative to the tunnel's test section and at 8 degrees angle of attack relative to the nozzle axis (the model was not included in the precursor wind-tunnel simulation).

transitions to a nearly uniform spacing across the boundary layer. It is then refined smoothly in the vicinity of the shock and becomes uniform again outside the shock. The streamwise grid is generated using a similar hyperbolic-tangent distribution: it is highly refined near the nose to capture geometric curvature and steep gradients, and nearly uniform farther downstream. Figure 2 shows the grid distribution in a streamwise plane and in two longitudinal planes on the leeward and windward sides. The red lines represent the bow-shock location computed from the VULCAN-CFD laminar flow.

For clarity and consistency, the DNS employs a coordinate system aligned with the cone's axis rather than with the tunnel axis. The angle of attack is therefore imposed by rotating the incoming freestream velocity vector (upstream of the shock) by  $8^\circ$  relative to the cone-axis coordinate system. This ensures that the imposed disturbances and the mean flow entering the DNS domain correctly represent the physical inflow conditions associated with a cone at nonzero angle of attack. Figure 3 illustrates the Cartesian coordinate system on the surface of the sharp  $7^\circ$  half-angle cone. The upper portion of the figure shows the global and body-fitted coordinate systems, wherein the global coordinate system is denoted by subscript  $g$ . The lower portion has red, green, blue, and pink lines representing  $\phi = 0^\circ, 90^\circ, 180^\circ$ , and  $270^\circ$ , respectively. The angle  $\phi$  is defined on the projection of the cone's base, shown on the downstream plane. The grid dimensions, domain extents, and viscous-scaled resolutions for each DNS box are summarized in Table 2. Here,  $N_x$ ,  $N_\theta$ , and  $N_r$  denote the number of grid points in the streamwise, azimuthal, and radial directions, respectively. The quantity  $x_g$  specifies the streamwise extent of each box, while  $\Delta x^+$  represents the wall-parallel grid spacing at both the upstream and downstream ends of each domain. Similarly,  $(r \Delta \theta)^+$  and  $\Delta z^+$  denote the azimuthal and wall-normal spacings at the wall. The cone's leading edge is located at  $x = -0.5$  mm, and all viscous-scaled quantities (superscript “+”) are normalized by the local viscous length scale  $z_\tau$  computed from the VULCAN-CFD laminar flow. In this paper, the viscous length is defined as

$$z_\tau = \frac{\nu_w}{u_\tau}, \quad (1)$$

where  $\nu_w$  is the kinematic viscosity at the wall, and  $u_\tau$  is the friction velocity,

$$u_\tau = \sqrt{\frac{\|\tau_w\|}{\rho_w}}, \quad (2)$$

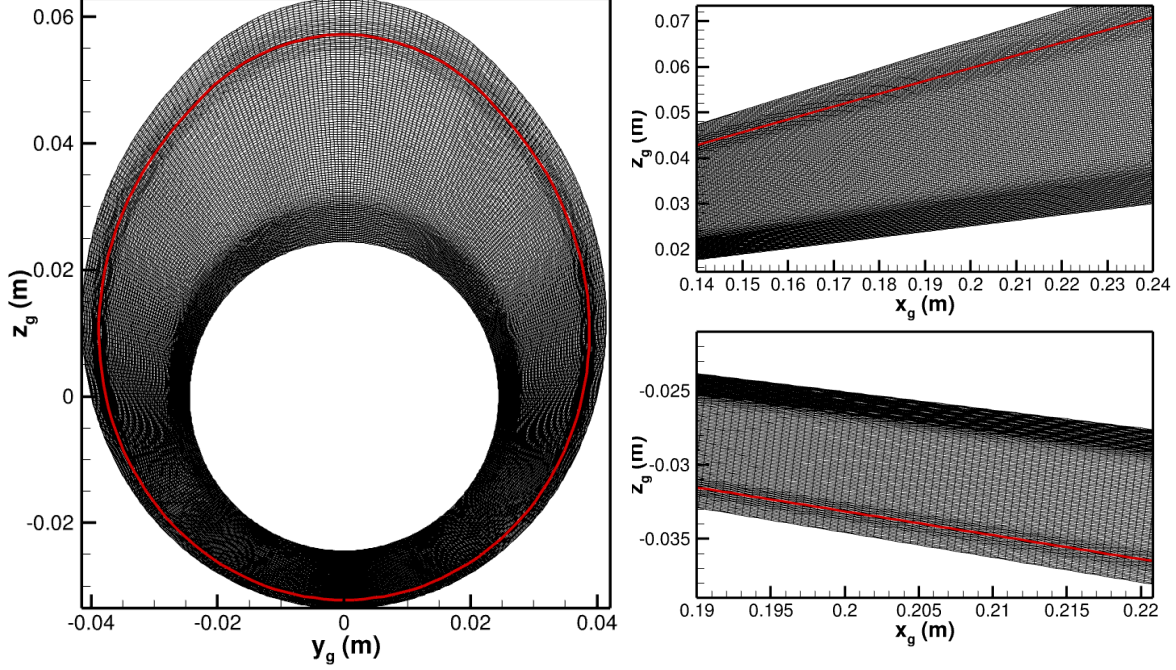


**Fig. 2** Grid distribution in a  $x$ - $z$  plane (left) and in a  $y$ - $z$  plane for leeward (right top) and windward side (right bottom) side. For illustrative purposes, the grid was coarsened to use only every fifth point in each direction. Red lines represent the shock location of the laminar flow.

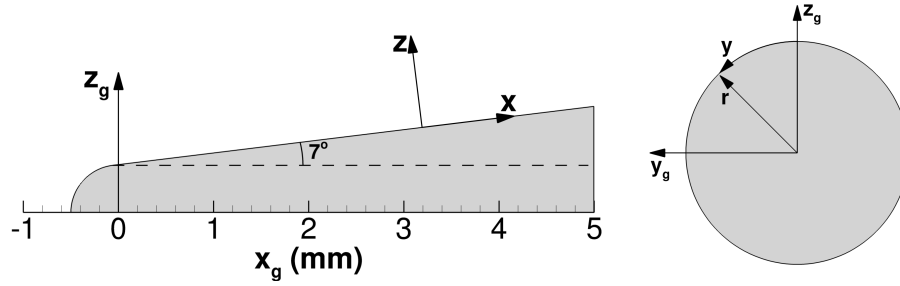
with  $\|\tau_w\|$  denoting the magnitude of the total wall shear stress,

$$\|\tau_w\| = \sqrt{\tau_{w,i}^2 + \tau_{w,j}^2}, \quad (3)$$

where  $\tau_{w,i}$  and  $\tau_{w,j}$  are the streamwise and spanwise components. Note that the total wall shear stress includes both streamline-aligned and crossflow components.

**Table 2** Domain size and grid resolution for the DNS runs.  $\Delta x$ ,  $r\Delta\theta$ , and  $\Delta z$  represent the grid spacing in the local streamwise, azimuthal, and wall-normal directions, respectively. The superscript “+” denotes normalization by the local viscous length  $z_\tau$  of the VULCAN-CFD laminar flow.

| Case  | $N_x \times N_\theta \times N_r$ | $x_g$ (m)        | $\Delta x^+$  | $(r\Delta\theta)^+$ | $\Delta z^+$  | $\Delta t$ (s)         |
|-------|----------------------------------|------------------|---------------|---------------------|---------------|------------------------|
| Box 1 | $600 \times 128 \times 400$      | -0.0005 - 0.0022 | [0.30 - 5.03] | [0.08 - 18.0]       | [0.01 - 0.18] | $2.50 \times 10^{-11}$ |
| Box 2 | $1200 \times 256 \times 400$     | 0.0012 - 0.042   | [0.26 - 10.1] | [0.92 - 16.4]       | [0.12 - 0.41] | $2.50 \times 10^{-10}$ |
| Box 3 | $1400 \times 1024 \times 400$    | 0.027 - 0.16     | [0.80 - 10.4] | [0.35 - 10.5]       | [0.16 - 0.99] | $1.25 \times 10^{-9}$  |
| Box 4 | $1400 \times 2048 \times 600$    | 0.14 - 0.28      | [0.65 - 8.8]  | [0.44 - 7.9]        | [0.22 - 0.98] | $5.00 \times 10^{-9}$  |
| Box 5 | $2500 \times 4096 \times 800$    | 0.26 - 0.51      | [0.53 - 7.6]  | [0.33 - 6.2]        | [0.23 - 1.34] | $6.25 \times 10^{-9}$  |
| Box 6 | $1100 \times 4096 \times 800$    | 0.49 - 0.60      | [0.51 - 6.5]  | [0.49 - 6.9]        | [0.33 - 1.54] | $6.25 \times 10^{-9}$  |



(a) Global Cartesian and local body-fitted coordinate systems

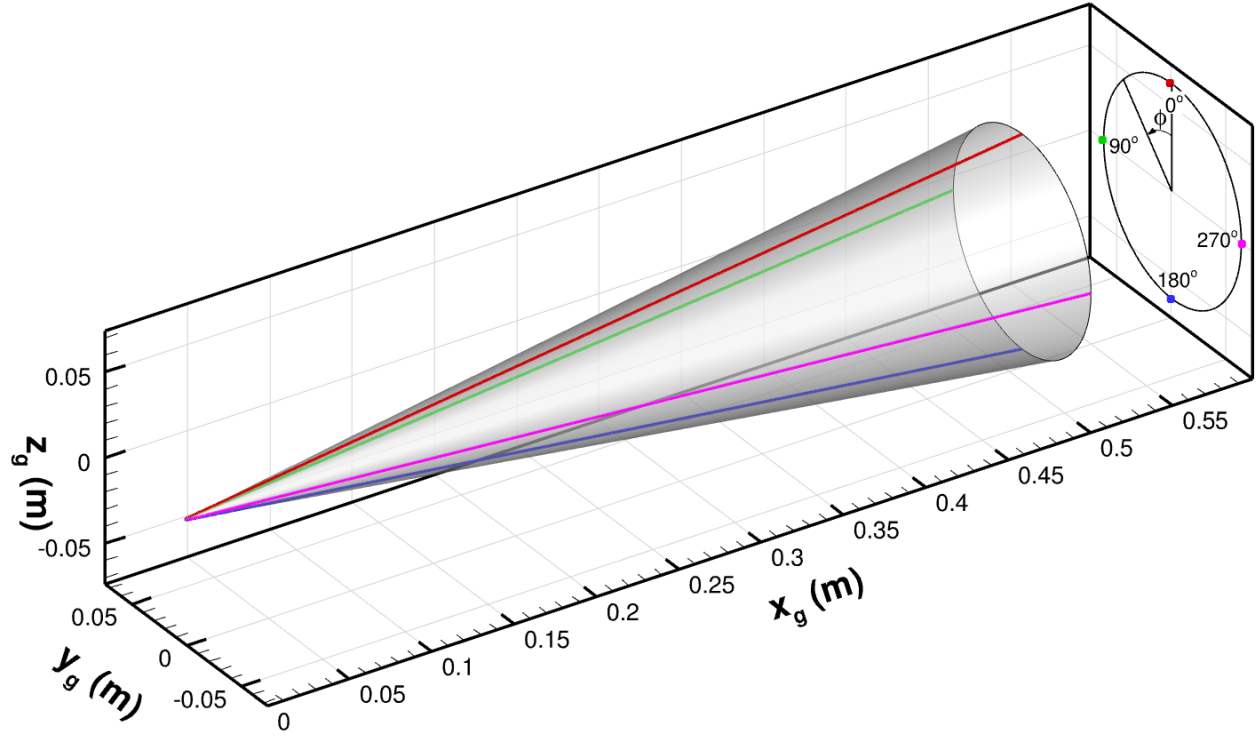


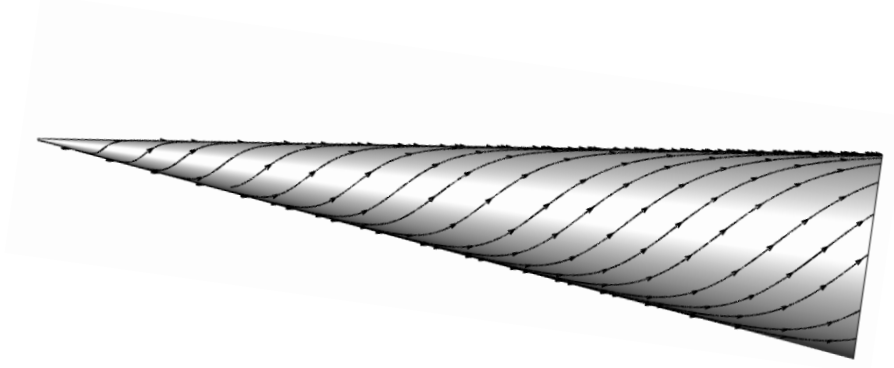
**Fig. 3 Computational coordinate system.** The red, green, blue, and pink lines represent, respectively,  $\phi = 0^\circ$  (leeward),  $90^\circ$ ,  $180^\circ$  (windward), and  $270^\circ$  and the are also illustrate as points in the project circumference at the rightmost plane.

### III. Results

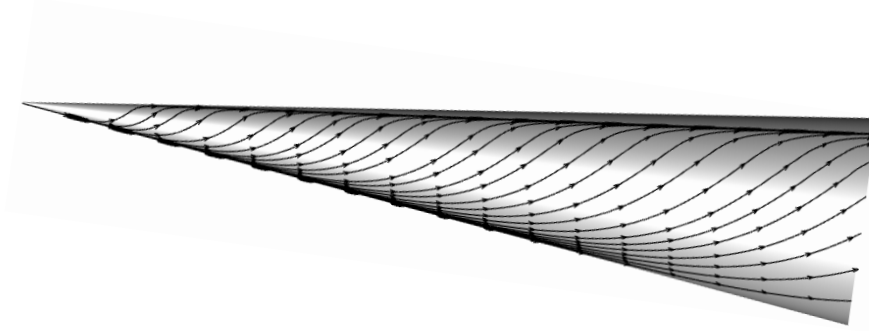
Results of the three-dimensional DNS of the Mach 8 flow over the cone with  $R_n = 0.5$  mm at 8 degrees angle of attack are discussed next. As previously mentioned in Section II, the leading edge of the cone corresponds to  $x_g = -0.5$  mm, and the reconstructed, tunnel-like freestream acoustic disturbances described in Liu et al. [12], Schuabb et al. [14] are used as the inflow forcing for the freestream boundary of the computational grid upstream of the bow shock. The full azimuthal domain of  $2\pi$  radians is included in the DNS. Figure 4 shows the streamlines obtained by the DNS against the laminar ones computed by VULCAN-CFD and those from oil-flow visualization [24]. The Oil flow visualization was performed in the Sandia Hypersonic Wind Tunnel (HWT) at Mach 8 for comparison of surface streamlines. The facility is a blowdown-to-vacuum design with pure nitrogen as the test gas. The axisymmetric nozzle of the HWT expands into a test section with a fixed diameter of 35.56 cm. Visualization data was collected with a field of view near the base of the seven-degree half-angle cone (base diameter of 12.7 cm) at a unit Reynolds number of  $13.4 \times 10^6 / \text{m}$ . The zygloil oil used for visualization was excited with water-cooled ultraviolet illumination and a yellow-band low-pass optical filter was placed in front of the 16-bit 4 megapixel CMOS sensor for imaging. During the tunnel runs, the cone was initially at zero angle of attack to allow the tunnel to start successfully and subsequently pitched to eight degrees angle of attack. The three sets of streamlines have an overall behavior that is quite similar: they originate on the windward side and gradually sweep, with a very similar curvature, toward the leeward side.

Figure 5 compares the experimental and DNS power spectral density (PSD) of wall-pressure fluctuations normalized by the local root-mean-square (rms) pressure. Pressure measurements were taken using PCB pressure sensors (PCB Piezotronics 132B38, magenta arrows,  $d_{probe} = 0.98$  mm diameter) located at  $x/L = 0.815$  ( $x \approx 0.421$  m) and  $x/L = 0.9614$  ( $x \approx 0.497$  m), using a Precision Filter, Inc 28000 signal conditioner and an NI PXIe-6396 data acquisition system with 18-bit A/D conversion. The experimental data was collected in separate runs with the sensors along the windward, yaw, and leeward rays at a nominal unit Reynolds number of  $17.06 \times 10^6 / \text{m}$  (or  $5.2 \times 10^6 / \text{ft}$ ). To gauge the influence of the finite-sensor size of the PCB-132 sensors in the experiment, the DNS wall-pressure spectrum at the same locations as the experiments was computed with and without spatial averaging. Here, the “raw” wall-pressure PSD predicted by DNS, without spatial averaging, is referred to as Case “DNS,  $d_{probe} = 0$  mm”, and the spatial averaged spectra (with the selected diameter of  $d_{probe} = 0.98$  mm that matches the sensing area of the PCB132 transducer) is referred to as Case “DNS,  $d_{probe} = 0.98$  mm”. At both axial locations, the PSD computed from the pressure signals measured by PCB132 shows a significantly faster spectral roll-off at high frequencies than those of the DNS without spatial averaging. Such a trend is especially significant along the windward ray given the thinner boundary layer thickness that leads to higher frequencies. Across all azimuthal locations, a much better comparison is achieved between the experimentally measured PSD and those computed from the spatially averaged DNS data, confirming the validity of the current DNS. A similar spatial-average approach to study the influence of sensor size had been used by Huang et al. [25] and Schuabb et al. [14] for a Mach 8 cone at zero angle of attack.

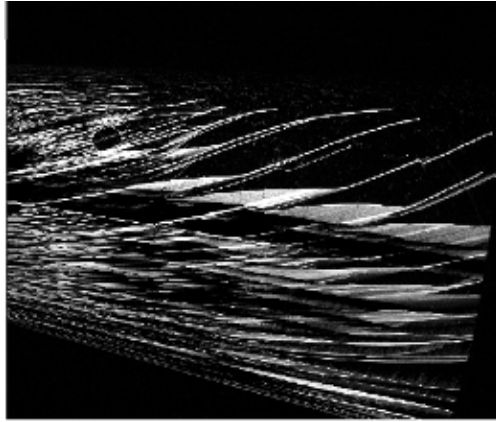
Figure 6 presents the frequency power spectral density (PSD) of wall-pressure fluctuations at five streamwise locations spanning the transitional and breakdown regions of the cone. As documented by Schuabb et al. [14], for sharp cones at Mach 8 and zero angle of attack with a small nose bluntness ( $R_n = 0.5$  mm), the dominant boundary-layer instability is the second mode. This instability behaves as a trapped acoustic wave within the boundary layer, with a characteristic frequency that scales inversely with the boundary-layer thickness,  $f \simeq U_{\infty,2} / (2\delta)$  [7, 26]. At a nonzero angle of attack, the transition mechanisms differ between the windward and leeward sides. On the windward side, beginning at  $x_g = 0.150$  m, the PSD shows a clear spectral hump centered near 900 kHz. This peak increases in amplitude downstream and gradually shifts toward lower frequencies, consistent with the growth of the boundary-layer thickness. This behavior is characteristic of second-mode waves. At  $x_g = 0.250$  m and 0.325 m, the dominant peaks occur near 800 kHz and 600 kHz, respectively. These peak frequencies are higher than those observed at zero angle of attack around 300 kHz as previously reported by Schuabb et al. [14], which is consistent with the thinner boundary layer on the windward side. Farther downstream, at  $x_g \approx 0.497$  m, the second-mode spectral hump and its superharmonics disappear, giving way to a broadband, turbulence-like spectrum that closely resembles the experimental observations shown in Fig. 5. In contrast, no distinct second-mode spectral hump is observed on the leeward side at any of the sampled locations. The PSD retains a similar shape along the streamwise direction, although the most upstream location exhibits a slightly narrower spectral bandwidth. The physical mechanisms responsible for this behavior on the leeward side remain under investigation. The yaw sides exhibit trends similar to those on the windward side, but with lower second-mode amplitudes and a downstream shift in the onset of the spectral hump. In these regions, the dominant peaks appear farther downstream than on the windward side, indicating a delayed amplification of the instability. In addition, crossflow instabilities are expected to be important on the yaw side (see, for example, Craig and Saric [27], Edelman et al. [28], and Edelman and Schneider [29]). These instabilities are expected at lower frequencies in the tens of kHz and



(a) Streamlines from DNS mean flow



(b) Streamlines from VULCAN-CFD laminar flow



(c) Experimental streamlines

**Fig. 4** Surface streamlines computed from (a) the DNS mean flow and (b) the VULCAN-CFD laminar flow over the entire cone; (c) streamlines at the end of the cone from oil-flow visualization [24].

therefore likely require a more converged DNS solution before they can be fully probed. This will be a focus of future efforts.

Figure 7 compares the time-averaged wall pressure, total wall shear stress, and wall heat flux from the DNS with the corresponding laminar flow results obtained with the VULCAN-CFD solver [23]. For reference, results from Schuabb et al. [14] for the same cone at zero angle of attack and identical freestream conditions are also included. Streamwise profiles of each quantity are shown at four values of the azimuthal angle  $\phi$ , as indicated in Fig. 3. The wall pressure follows the laminar flow distribution remarkably well along most of the cone, regardless of azimuthal position, and only begins to slightly rise above the laminar values near the downstream end of the domain. In contrast, the wall shear stress and wall heat flux have much more variation. On the leeward side ( $\phi = 0^\circ$ ), both quantities remain close to their

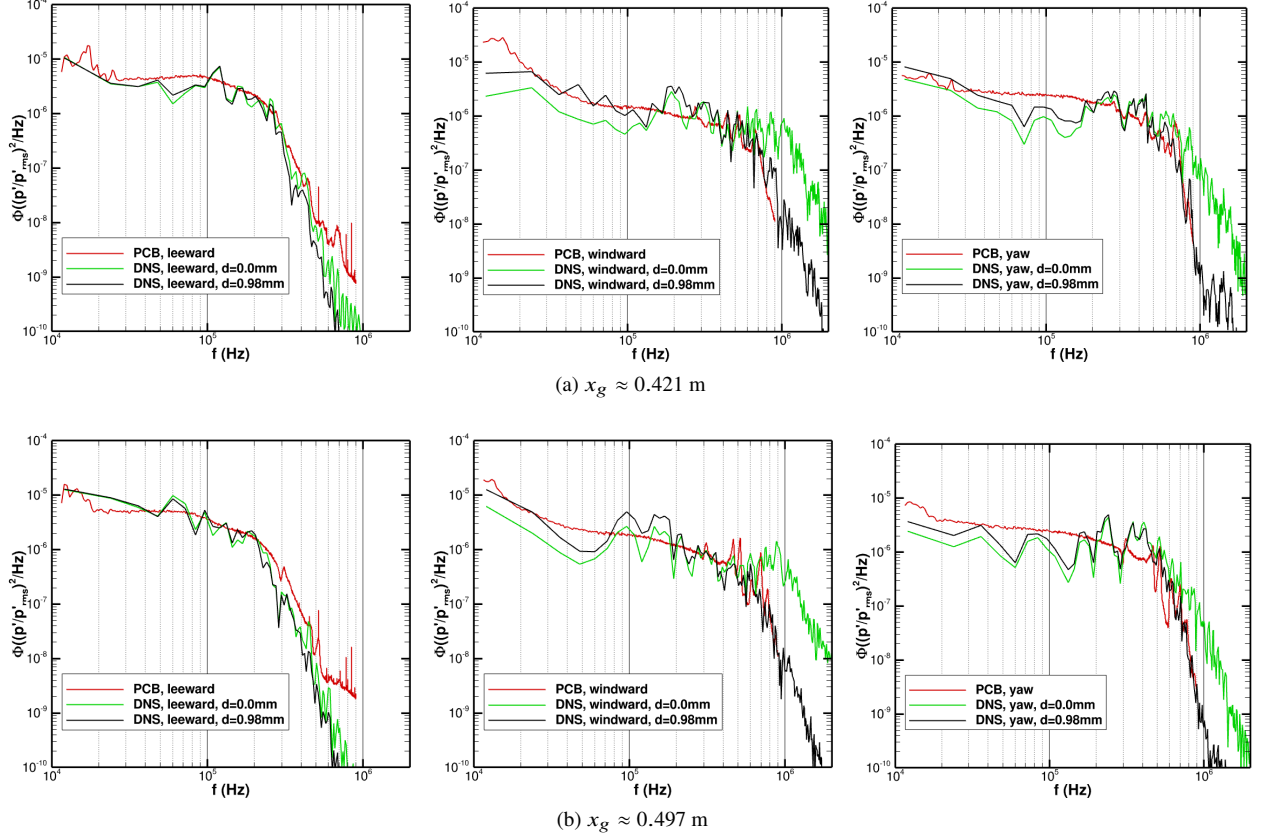


**Fig. 5** Comparison of the power spectrum density (PSD) between the DNS and experiments at two streamwise locations (a)  $x \approx 0.421$  m and (b)  $x \approx 0.497$  m at the leeward (left column), windward (middle column) and yaw side (right column). DNS results with and without spatial averaging are plotted to gauge the influence of the finite-sensor size of the PCB-132 sensors.

corresponding laminar values up to  $x_g \approx 0.10$  m. Then, between  $x_g = 0.10$  and  $0.20$  m, the values drop below the laminar ones and eventually rise until a local maximum is reached near  $x_g \approx 0.25$  m. Finally, both the wall shear-stress and heat flux decrease to their laminar levels and remain within their vicinity. On the windward side ( $\phi = 180^\circ$ ), the agreement with the laminar flow extends farther downstream, up to  $x_g \approx 0.28$  m. Downstream of this location, both the shear stress and heat flux rise rapidly, peaking around  $0.42$  m before decreasing. Similar overshoot in  $\tau_w$  and  $q_w$  can be seen for the zero angle of attack case computed by Schuabb et al. [14], but with a more gradual increase. Similar overshoot was observed by Franko and Lele [30] for a Mach 6 flat-plate boundary layer. The fluctuations in the mean quantities indicate that the time averaging window used is probably not long enough to reach full statistical convergence. At the yaw-side locations ( $\phi = 90^\circ$  and  $270^\circ$ ), the overall trend looks closer to that of the windward side, although the point at which the curves diverge from the laminar flow occur slightly farther downstream.

Figures 8, 9, and 10 show time-averaged (left column) and instantaneous (right column) contours of wall pressure, total wall shear stress, and wall heat flux, respectively. Each figure provides three different views that emphasize the leeward (top), yaw (middle), and windward (bottom) sides, respectively. The mean surface pressures in Fig. 8a confirm the expected asymmetry between the windward and leeward sides, with significantly higher pressure on the windward side and a smooth progression to the lower pressure on the leeward side. The instantaneous pressure contours from Fig. 8b indicate localized high-pressure peaks on the leeward side, whereas the yaw side and the windward side clearly show wavepackets that are reminiscent of second mode instabilities. Notably, in the yaw side view, the second-mode wavepackets appear inclined, propagating diagonally from the windward toward the leeward side, consistent with propagation along a local streamline direction. Near the end of the cone, small-scale fluctuations emerge, indicating the onset of turbulence. Similar second-mode wavepackets followed by small-scale pressure fluctuations were also reported in the transition study over a cone at zero angle of attack of Schuabb et al. [14]. Instantaneous plots of the

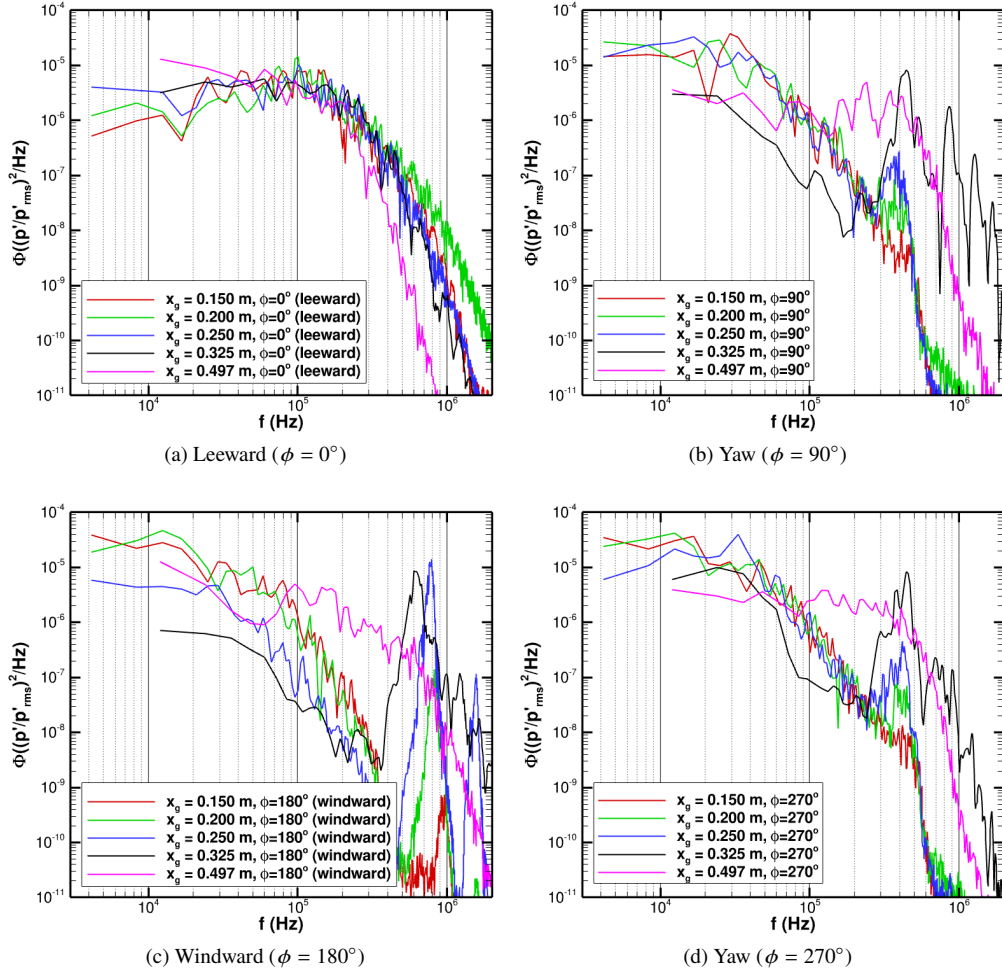


**Fig. 6 Power spectrum density (PSD) of wall pressure fluctuation from the DNS at five streamwise locations at the leeward, windward and both yaw sides.**

wall shear-stress (Fig. 9b) and wall heat-flux (Fig. 10b) reveal trends that are consistent with those observed in the wall pressure fields in Fig. 8b. On the leeward side, the localized peaks in  $\tau_w$  and  $q_w$  are more clearly distinguished, forming irregular, spatially scattered structures around  $\phi = 0^\circ$ . In contrast, the windward side is dominated by strong second-mode disturbances, followed downstream by the intermittent appearance of “hot islands” of high shear stress and heat flux, indicative of the initial stages of laminar breakdown. Farther downstream, these hot spots fragment into even smaller scales, marking the transition to a fully turbulent boundary layer. It is clear that the transition on the windward side is dominated by second-mode disturbances. In contrast, the dominant instability on the leeward side is not easily identifiable. Similarly, further analysis is necessary to identify the potential presence of secondary instabilities on either side and fully characterize the transition mechanisms in these regions. The first noticeable increase in time-averaged fields of the wall shear stress and heat flux is noted on the leeward side, although the magnitude of this change is small. In contrast, the windward side shows a much stronger rise in both quantities in the downstream region, and this growth gradually spreads in the yaw direction, consistent with the previously seen trends in Fig. 7. Along the yaw lines, long streaks develop and tilt toward the leeward side.

Figure 11 presents contours of streamwise velocity and temperature on several axial ( $y_g$ - $z_g$ ) planes of the cone, with the nose tip pointing out of the page. For visualization purposes, only regions where the streamwise velocity satisfies  $u < 1010$  m/s are shown, approximately corresponding to the boundary-layer edge velocity. Consistent with the analysis of surface quantities in the previous plots, the axial planes clearly indicate an earlier transition to turbulence along the leeward side when compared to the windward or yaw sides. Near the nosetip, the mushroom-shaped vortex structure

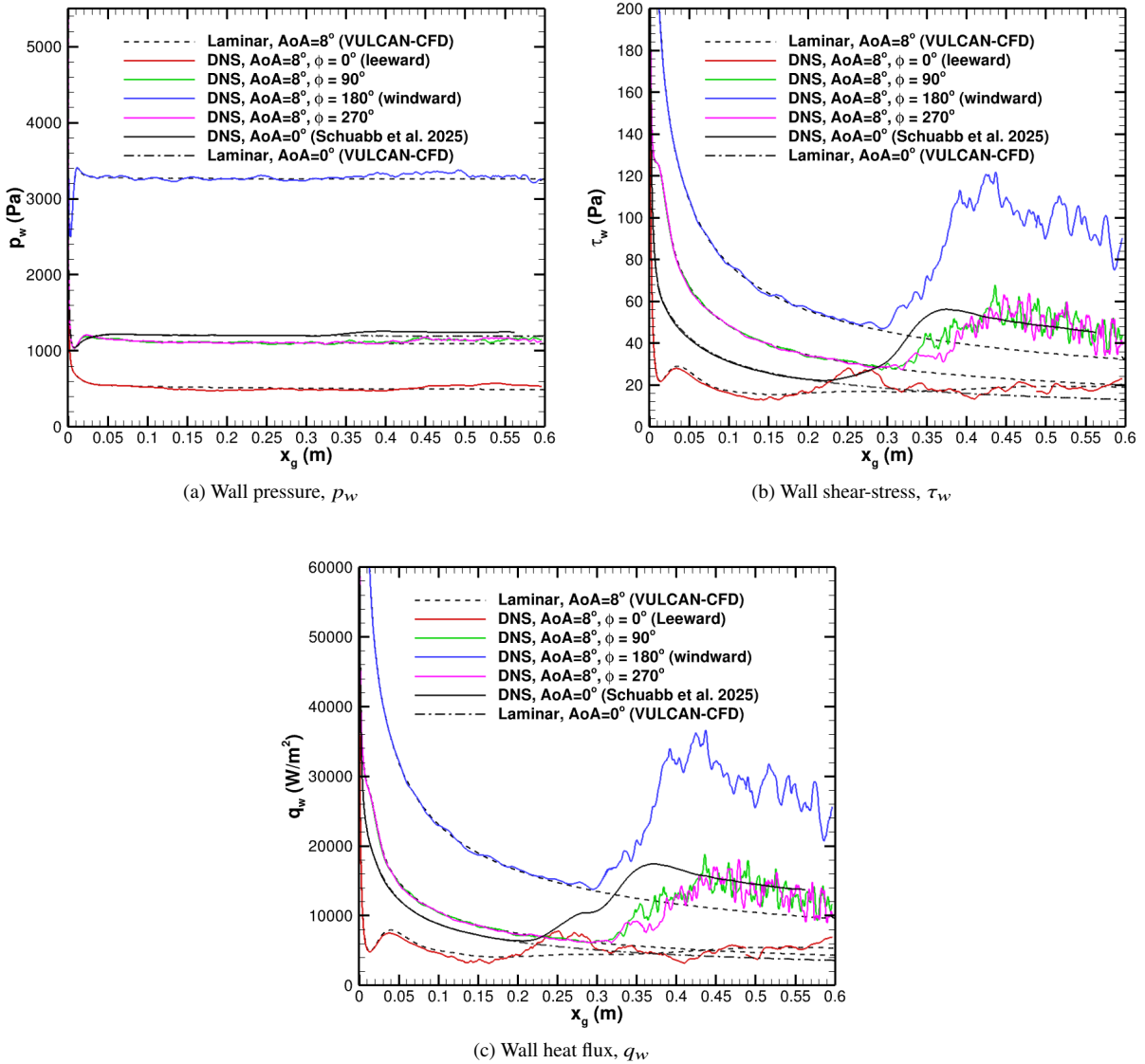


**Fig. 7 Time-averaged (a) wall pressure, (b) wall shear-stress, and (b) wall heat flux over the cone.**

remains laminar, although it is visibly perturbed by the imposed freestream acoustic disturbances. Farther downstream, the boundary layer thickens and becomes increasingly disorganized, particularly along the yaw and the windward sides.

Figure 12 shows the divergence of the velocity field on a sequence of axial planes at increasing downstream distance. In all planes, the freestream acoustic disturbances are clearly visible both upstream of the shock and within the post-shock region. At upstream locations ( $x_g \leq 0.100$  m), the vortex structure on the leeward side interacts with the acoustic disturbances, resulting in noticeable perturbations, including slight asymmetries and deformations of the vortex core. Farther downstream ( $0.150 \leq x_g \leq 0.250$  m), the canonical leeward vortex structure is no longer clearly identifiable, although the boundary layer on the yaw sides, as well as on the windward side, remains predominantly laminar. At intermediate locations ( $x_g = 0.200, 0.250, 0.300$ , and  $0.350$  m), localized shocklets begin to appear on the leeward side as a consequence of boundary-layer breakdown and the onset of turbulence. Farther downstream, similar shocklets, now more concentrated, also emerge on the windward side. These structures gradually spread laterally with increasing  $x_g$ , indicating that once the windward side transitions, the turbulent activity progressively expands toward both yaw sides of the cone.

Figure 13 shows the streamwise vorticity on the leeward ( $\phi = 0^\circ$ ) and windward ( $\phi = 180^\circ$ ) sides, along with a

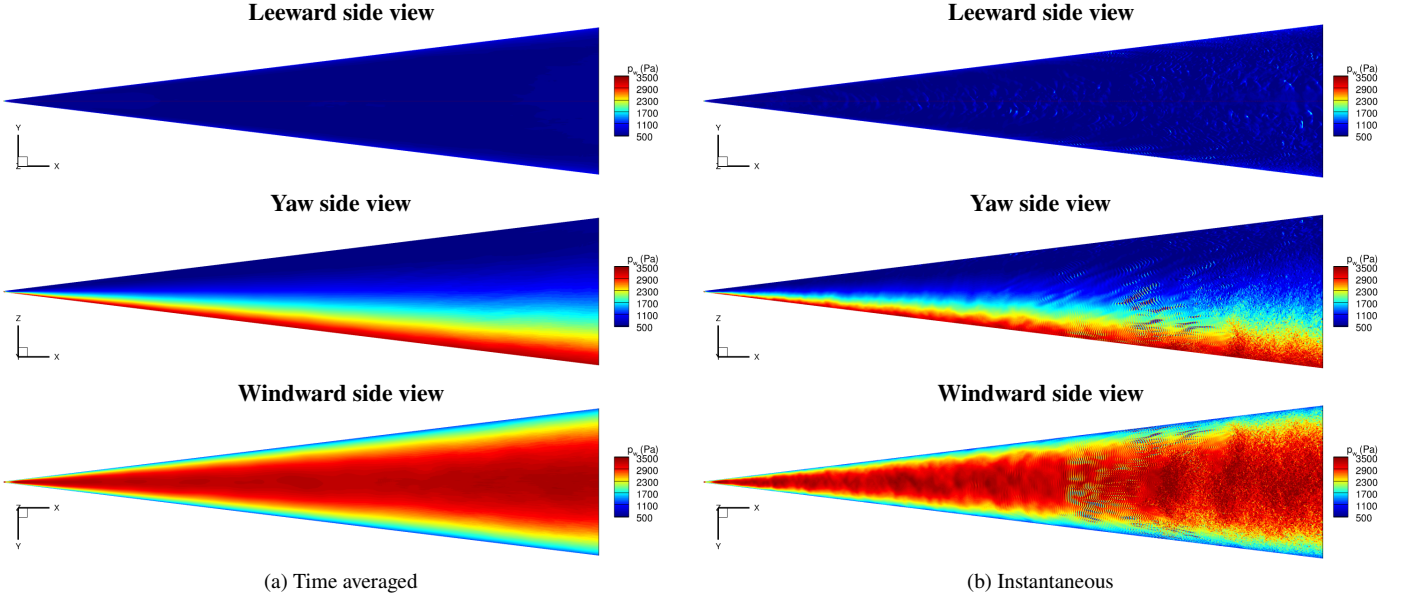


**Fig. 8** Wall-pressure,  $p_w$ , contours on the cone surface (up to  $x_g \approx 0.51$  m).

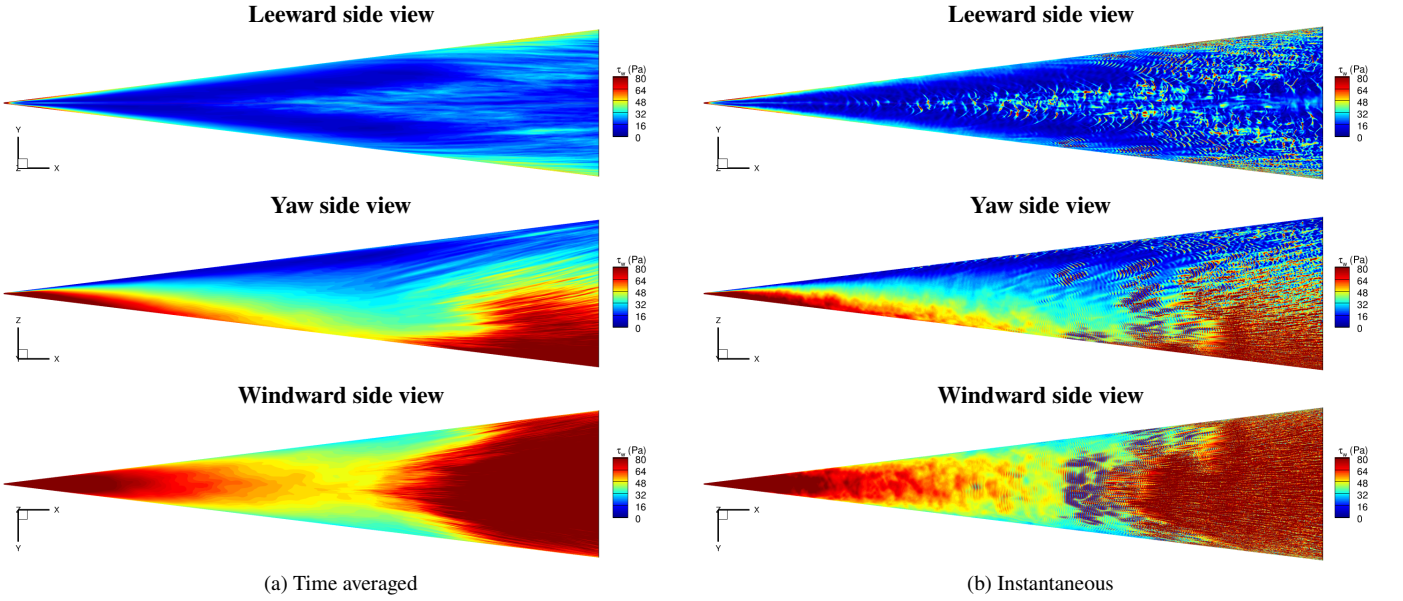


**Fig. 9** Wall shear stress,  $\tau_w$ , contours on the cone surface (up to  $x_g \approx 0.51$  m).

sequence of axial ( $y_g$ – $z_g$ ) planes at increasing downstream locations. These views highlight the evolution of vortical structures inside the boundary layer in both the streamwise and spanwise directions, while also illustrating that vortical disturbances in the freestream are negligible compared to the imposed acoustic disturbances. Consistent with the divergence fields, the leeward vortex remains laminar in the upstream region. This is evident in the  $x_g$ – $z_g$  plane on the leeward side, where the vorticity level stays low (predominantly green). This low level is expected because, at the symmetry plane of an undisturbed (laminar) flow, the streamwise vorticity should ideally vanish in the absence of disturbances. Farther downstream, the interaction with the freestream acoustic disturbances gradually distorts the vortex, breaking its symmetry and causing it to oscillate, as indicated by alternating signs of streamwise vorticity (blue–red–blue). Around  $x_g \approx 0.20$  m, the large coherent vortex associated with the mushroom-shaped leeward structure

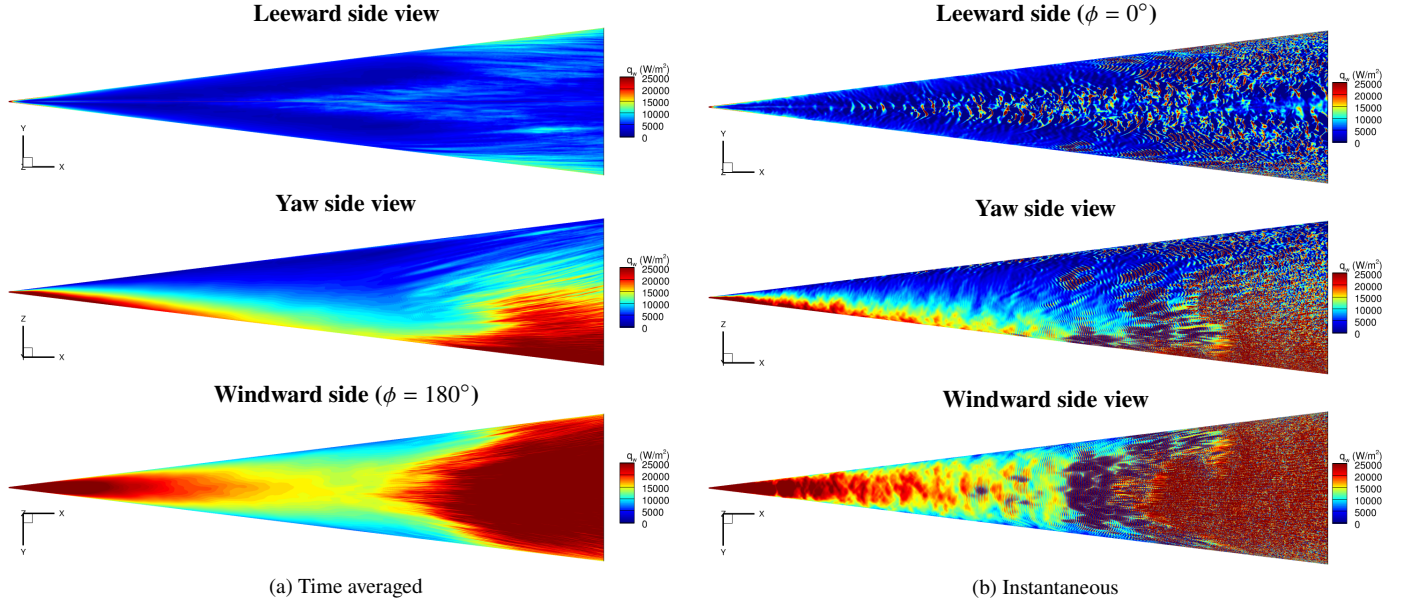


**Fig. 10** Wall heat flux,  $q_w$ , contours on the cone surface (up to  $x_g \approx 0.51$  m).

breaks down and smaller vortical eddies become dominant. Further downstream, similar small-scale vortical structures emerge on the yaw and windward sides. Toward the end of the domain, when the boundary layer is fully turbulent, fine-scale vortical eddies populate the entire circumference of the cone.

Lastly, Fig. 14 presents numerical schlieren images based on the density gradient, following the same visualization style used previously for the streamwise vorticity. In these schlieren fields, regions of high density gradient are emphasized by bright contours. Consistent with the earlier observations, the upstream portion of the boundary layer remains laminar, although the leeward vortex appears slightly distorted by freestream acoustic waves. This distortion is evident from the well-defined leeward structure outlined by the white contours in Fig. 14b and in the upstream region of the cone on the leeward side in Fig. 14a. Between the streamwise positions of  $x = 0.10$  m and  $x = 0.20$  m, Fig. 14a shows that the leeward vortex begins to break down. At the locations where this breakdown initiates, small shocklets form and propagate into the freestream region below the bow shock. These shocklets are also visible in Fig. 14c, indicating that their origin is closely associated with the core of the leeward vortex. Farther downstream, the leeward-side boundary layer becomes fully turbulent, and broadband acoustic radiation is emitted into the freestream region beneath the shock. On the windward side, the flow remains laminar up to approximately  $x \approx 0.40$  m. Beyond this location, the boundary layer transitions to turbulence and continues to radiate acoustically away from the cone's surface. Moreover, the magnitude of the radiated fluctuations on the windward side is noticeably higher than that observed on the leeward side due to the higher gradients associated with a thinner boundary layer over the windward side.

#### IV. Summary

The interaction of a tunnel-like acoustic disturbance field with a Mach 8 laminar boundary layer over a  $7^\circ$  half-angle cone with a nose radius of  $R_n = 0.5$  mm at 8 degrees angle of attack has been investigated using DNS. The imposed freestream disturbances in the DNS are planar acoustic waves modeled based on a digital wind tunnel to mimic the acoustic radiation from turbulent boundary layers present on the walls of a Mach 8 conventional wind tunnel.

The DNS results reveal a strongly asymmetric transition process driven by freestream acoustic disturbances and also influenced by geometry-induced crossflow effects. Comparisons of wall pressure, shear stress, and heat flux against laminar computations from VULCAN-CFD demonstrates that the leeward side undergoes an earlier and more irregular transition, while the windward side exhibits more organized structures dominated by second-mode instabilities. On the windward and yaw sides, clear second-mode wavepackets start forming near  $x_g \approx 0.23$ – $0.31$  m; they rapidly increase in amplitude, and eventually break down into small-scale structures that mark the onset of turbulence. In contrast, the leeward side shows only small fluctuations in the upstream region; here, the dominant instability mechanism is

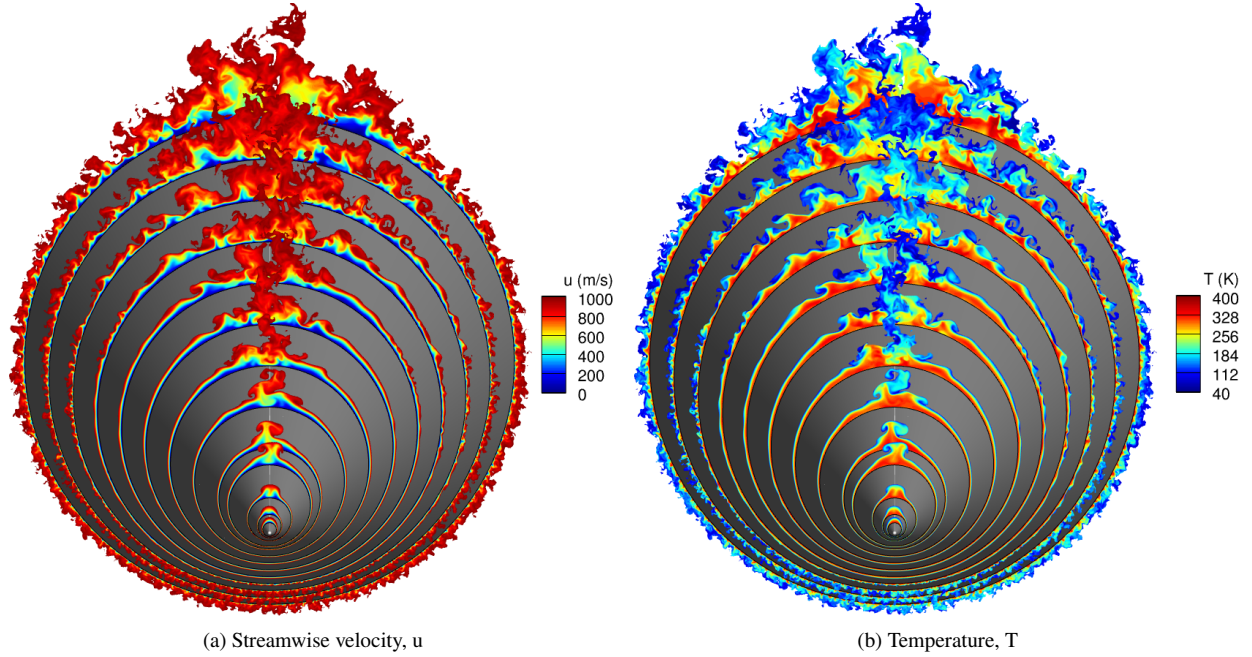


**Fig. 11** Contours on several axial planes of the cone (nose tip pointing out of the page), showing (a) streamwise velocity and (b) temperature, for regions where  $u < 1010$  m/s. Streamwise locations:  $x=0.010, 0.020, 0.040, 0.080, 0.100, 0.150, 0.200, 0.250, 0.300, 0.350, 0.400, 0.450, 0.500$  m

not immediately identifiable. Instead of a well-defined wavepacket, the leeward boundary layer develops localized high-pressure, high-shear, and high-heat-flux peaks, which precede an irregular transition to turbulence.

Axial-plane visualizations of velocity, temperature, and divergence further illustrate the transition behavior on the smooth cone. The canonical leeward vortex remains laminar near the leading edge but is gradually distorted by the freestream acoustic disturbances. An eventual breakdown of the leeward structure is observed between  $x_g = 0.10$  and  $0.20$  m, where small shocklets begin to appear and propagate beneath the bow shock. Farther downstream, the turbulent boundary layer on both sides emits acoustic radiation into the post-shock region, with particularly strong radiation originating from the windward side. Streamwise-vorticity fields confirm that, in the freestream region, vortical disturbances remain negligible compared to the imposed acoustic forcing. The onset of turbulence in the boundary layer is characterized by the rapid replacement of large, coherent vortices by fine-scale eddies that ultimately populate the entire cone circumference.

Overall, the DNS results reveal that transition on the windward side of the smooth cone is primarily driven by second-mode amplification, whereas the leeward side undergoes a more irregular breakdown associated with the deformation and collapse of the leeward vortex and the resulting formation of shocklets. The combined influence of angle of attack, freestream acoustics, and crossflow leads to a highly three-dimensional transition scenario with distinct mechanisms operating on different sides of the cone. The computed spectra of turbulent pressure fluctuations across all azimuthal locations at the cone surface were in excellent agreement with the measured data in the Sandia facility, after accounting for the finite size of the pressure transducers used in the experiment, which provides further validity to the current DNS approach.

Future work will focus on a detailed statistical analysis of the boundary-layer transition as well as exploring crossflow induced transition on the yaw side of the cone. Additional experiments at Sandia HWT-8 will be performed to collect wall-pressure data in the transitional region of the cone for comparing with the DNS. Pressure traces, spectral content, two-point correlations, coherence, and higher-order statistics will be extracted from the DNS along the cone surface to provide quantitative comparisons with the experimental measurements obtained on the same cone in the Sandia Hypersonic Wind Tunnel. These analyses will help further elucidate the spatial and temporal development of instabilities, validate the DNS predictions, and provide deeper insight into the mechanisms governing asymmetric transition in high-speed flows.

## Acknowledgments

The work by Mateus Schuabb, Lian Duan, and Anton Scholten is related to the Department of Navy award N00014-23-1-2456 issued by the Office of Naval Research under a subaward from the National Institute of Aerospace. Partial financial support for Mateus Schuabb and Lian Duan is provided by the Sandia National Laboratories. Pedro Paredes and Meelan Choudhari were supported by the Hypersonic Technology Project (HTP) under the NASA Aeronautics Research Mission Directorate (ARMD). Computational resources were provided by the DoD High-Performance Computing Modernization Program and the Ohio Supercomputer Center. Sandia National Laboratories is a multimission laboratory managed and operated by National Technology and Engineering Solutions of Sandia, LLC., a wholly owned subsidiary of Honeywell International, Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Institute of Aerospace, the Office of Naval Research, or the U.S. Government.

## References

- [1] Beckwith, I. E., and Miller, C. G., "Aerothermodynamics and Transition in High-Speed Wind Tunnels at NASA Langley," *Annu. Rev. Fluid Mech.*, Vol. 22, 1991, pp. 419–439. doi:10.1146/annurev.fl.22.010190.002223.
- [2] Schneider, S. P., "Effects of High-Speed Tunnel Noise on Laminar-Turbulent Transition," *Journal of Spacecraft and Rockets*, Vol. 38, No. 3, 2001, pp. 323–333. doi:10.2514/2.3705.
- [3] Schneider, S. P., "Development of Hypersonic Quiet Tunnels," *Journal of Spacecraft and Rockets*, Vol. 45, No. 4, 2008, pp. 641–664. doi:10.2514/1.34489.
- [4] Laufer, J., "Aerodynamic Noise in Supersonic Wind Tunnels," *J. Aerosp. Sci.*, Vol. 28, No. 9, 1961, pp. 685–692. doi:10.2514/8.9150.
- [5] Laufer, J., "Some Statistical Properties of the Pressure Field Radiated by a Turbulent Boundary Layer," *Physics of Fluids*, Vol. 7, No. 8, 1964, pp. 1191–1197. doi:10.1063/1.1711360.
- [6] Pate, S. R., "Dominance of Radiated Aerodynamic Noise on Boundary-Layer Transition in Supersonic-Hypersonic Wind Tunnels," Tech. rep., AEDC-TR-77-107, Arnold Engineering Development Center, 1978.
- [7] Stetson, K. F., "Nosetip Bluntness Effects on Cone Frustum Boundary-Layer Transition in Hypersonic Flow," AIAA Paper 1983-1763, 1983. doi:10.2514/6.1983-1763.
- [8] Bushnell, D. M., "Notes on Initial Disturbance Fields for the Transition Problem," *Instability and Transition*, edited by M. Y. Hussaini and R. G. Voigt, Springer-Verlag, Berlin, Vol. 1, 1990, pp. 217–232. doi:10.1007/978-1-4612-3430-2\_28.
- [9] Fedorov, A. V., "Transition and Stability of High-Speed Boundary Layers," *Annual Review of Fluid Mechanics*, Vol. 43, 2011, pp. 79–95. doi:10.1146/annurev-fluid-122109-160750.
- [10] Schneider, S. P., "Developing Mechanism-Based Methods for Estimating Hypersonic Boundary-Layer Transition in Flight: The Role of Quiet Tunnels," *Progress in Aerospace Sciences*, Vol. 72, 2015, pp. 17–29. doi:10.1016/j.paerosci.2014.09.008.
- [11] Edelman, J. B., Chynoweth, B. C., McKiernan, G. R., Sweeney, C. J., and Schneider, S. P., "Instability Measurements in the Boeing/AFOSR Mach-6 Quiet Tunnel," *AIAA Paper 2016-3343*, 2016. doi:10.2514/6.2016-3343.
- [12] Liu, Y., Schuabb, M., Duan, L., Paredes, P., and Choudhari, M., "Interaction of a Tunnel-Like Acoustic Disturbance Field With a Blunt Cone Boundary Layer at Mach 8," AIAA 2022-3250, 2022. doi:10.2514/6.2022-3250.
- [13] Paredes, P., Choudhari, M., and Li, F., "Mechanism for Frustum Transition over Blunt Cones at Hypersonic Speeds," *Journal of Fluid Mechanics*, Vol. 894, 2020, p. A22. doi:10.1017/jfm.2020.261.
- [14] Schuabb, M., Duan, L., Casper, K. M., Wagnild, R. M., Choudhari, M. M., and Paredes, P., "Hypersonic Boundary-Layer Transition over a Circular Cone in a Mach 8 Digital Wind Tunnel," *Journal of Fluid Mechanics*, Vol. 1017, 2025, p. A33. doi:10.1017/jfm.2025.369.
- [15] Driest, E. V., "The Problem of Aerodynamic Heating," *Aeronaut. Eng. Rev.*, Vol. 15, No. 10, 1956, pp. 26–41.
- [16] Keyes, F. G., "A Summary of Viscosity and Heat-Conduction Data for HE, A,  $H_2$ ,  $O_2$ ,  $CO$ ,  $CO_2$ ,  $H_2O$ , and Air," *Transactions of the American Society of Mechanical Engineers*, Vol. 73, 1951, pp. 589–596. doi:10.1115/1.4016339.

- [17] Jiang, G. S., and Shu, C. W., "Efficient Implementation of Weighted ENO Schemes," *Journal of Computational Physics*, Vol. 126, No. 1, 1996, pp. 202–228. doi:10.1006/jcph.1996.0130.
- [18] Williamson, J., "Low-Storage Runge-Kutta Schemes," *Journal of Computational Physics*, Vol. 35, No. 1, 1980, pp. 48–56. doi:10.1016/0021-9991(80)90033-9.
- [19] Mohseni, K., and Colonius, T., "Numerical Treatment of Polar Coordinate Singularities," *Journal of Computational Physics*, Vol. 157, 2000, pp. 787–795. doi:10.1006/jcph.1999.6382.
- [20] Bogey, C., de Cacqueray, N., and Bailly, C., "Finite Differences for Coarse Azimuthal Discretization and for Reduction of Effective Resolution Near Origin of Cylindrical Flow Equations," *Journal of Computational Physics*, Vol. 230, 2011, pp. 1134–1146. doi:10.1016/j.jcp.2010.10.031.
- [21] Balakumar, P., and Kegerise, M. A., "Receptivity of Hypersonic Boundary Layer over Straight and Flared Cones," *AIAA Journal*, Vol. 53, No. 8, 2015, pp. 2097–2109. doi:10.2514/1.J053432.
- [22] Balakumar, P., and Chou, A., "Transition Prediction in Hypersonic Boundary Layers using Receptivity and Freestream Spectra," *AIAA Paper 2016-0847*, 2016. doi:10.2514/6.2016-0847.
- [23] Litton, D., Edwards, J., and White, J., "Algorithmic Enhancements to the VULCAN Navier-Stokes Solver," *AIAA Paper 2003-3979*, 2003. doi:10.2514/6.2003-3979.
- [24] Carter, D. W., Stack, C., DeChant, L. J., Robbins, B., Coffin, P., Barone, M., De Zetter, M., Spillers, R., and Casper, K. M., "Turbulent Pressure Fluctuation Coherence Analysis on a Slender Cone at Angle of Attack," *AIAA Paper 2024-2124*, 2024. doi:10.2514/6.2024-2124.
- [25] Huang, J., Duan, L., Casper, K. M., Wagnild, R. M., and Bitter, N. P., "Transducer Resolution Effect on Pressure Fluctuations Beneath Hypersonic Turbulent Boundary Layers," *AIAA Journal*, Vol. 62, No. 3, 2024, pp. 882–895. doi:10.2514/1.J062994.
- [26] Casper, K. M., Beresh, S. J., Henfling, J. F., Spillers, R. W., and Pruett, B. O. M., "Hypersonic Wind-Tunnel Measurements of Boundary-Layer Transition on a Slender Cone," *AIAA Journal*, Vol. 54, No. 4, 2016, pp. 1250–1263. doi:10.2514/1.J054033.
- [27] Craig, S. A., and Saric, W. S., "Crossflow Instability in a Hypersonic Boundary Layer," *Journal of Fluid Mechanics*, Vol. 808, 2016, pp. 224–244.
- [28] Edelman, J., Casper, K. M., Henfling, J., Spillers, R., and Schneider, S. P., "Crossflow Transition on a Pitched Cone at Mach 8," *47th AIAA Fluid Dynamics Conference*, 2017, p. 4299.
- [29] Edelman, J. B., and Schneider, S. P., "Secondary Instabilities of Hypersonic Stationary Crossflow Waves," *AIAA Journal*, Vol. 56, No. 1, 2018, pp. 182–192.
- [30] Franko, K. J., and Lele, S. K., "Breakdown Mechanisms and Heat Transfer Overshoot in Hypersonic Zero Pressure Gradient Boundary Layers," *Journal of Fluid Mechanics*, Vol. 730, 2013, pp. 491–532.

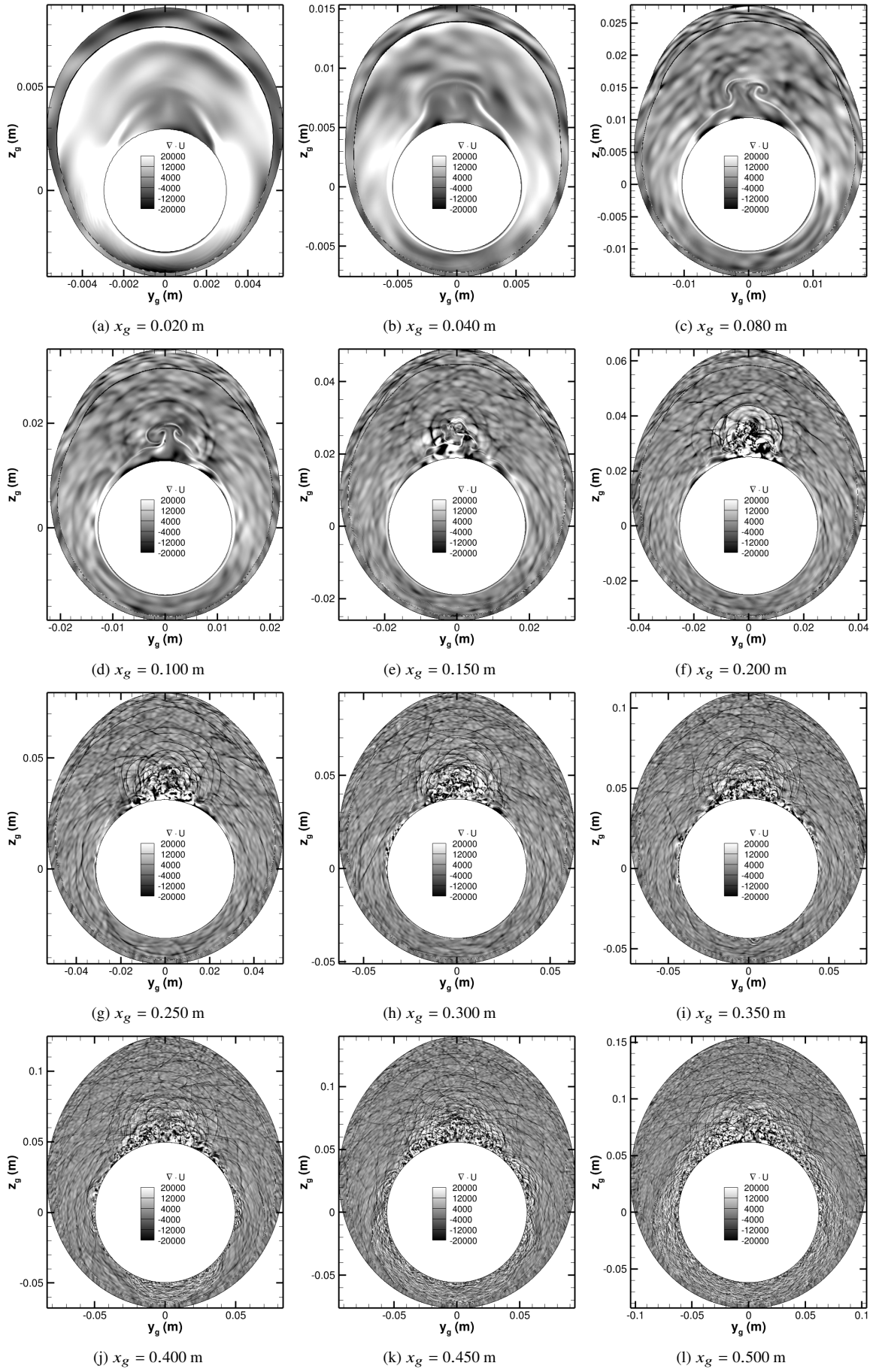
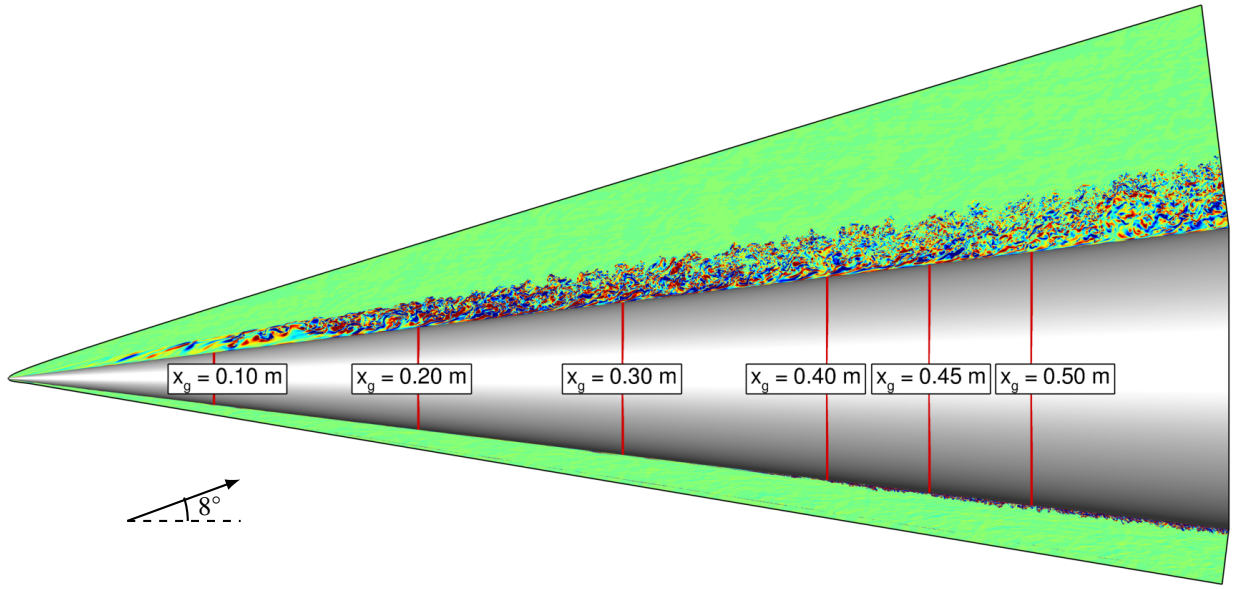
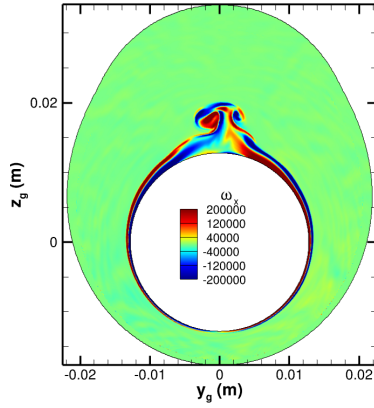


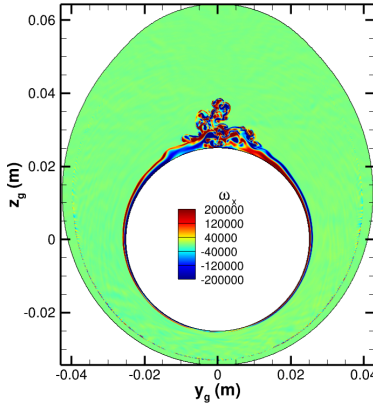
**Fig. 12** Instantaneous divergence of the velocity field shown across different  $y_g$ - $z_g$  planes distributed along the streamwise direction.



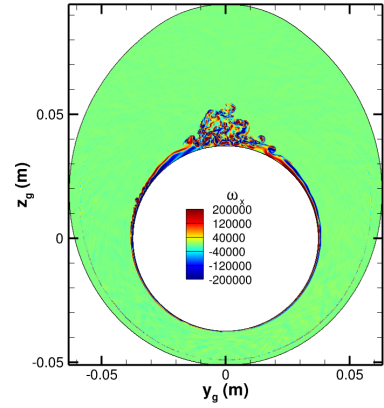
(a)  $x_g$ - $z_g$  plane



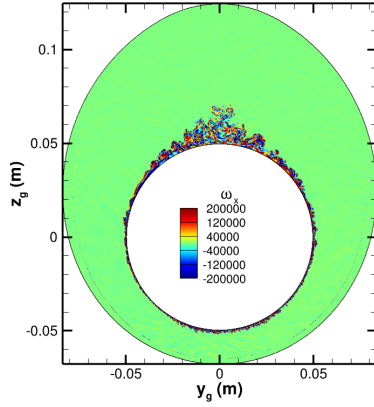
(b)  $x_g = 0.100$  m



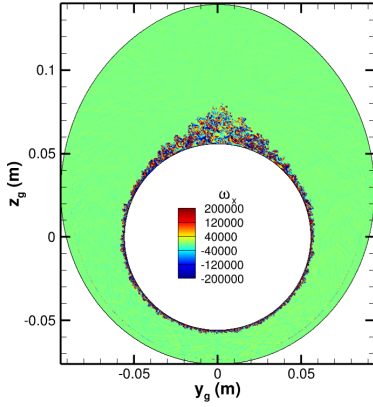
(c)  $x_g = 0.200$  m



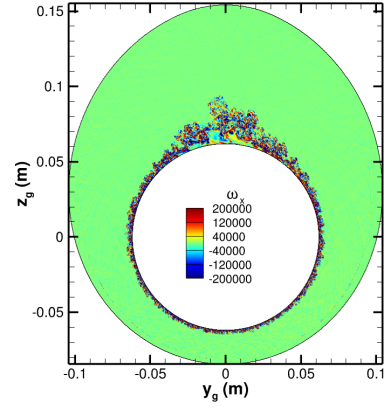
(d)  $x_g = 0.300$  m



(e)  $x_g = 0.400$  m

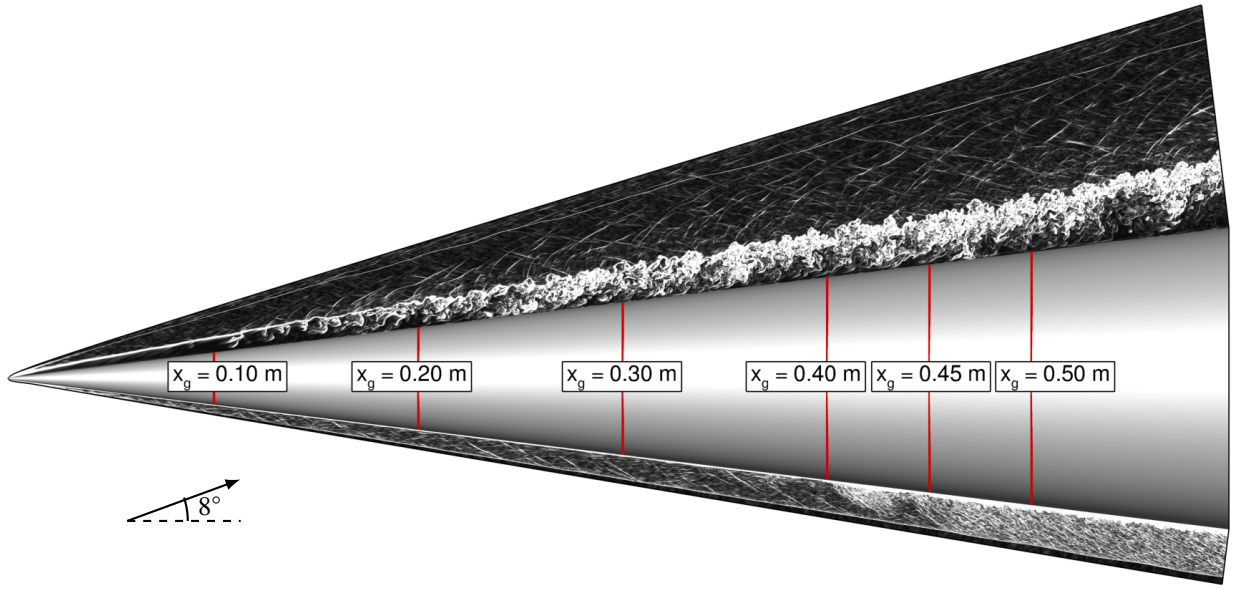


(f)  $x_g = 0.450$  m

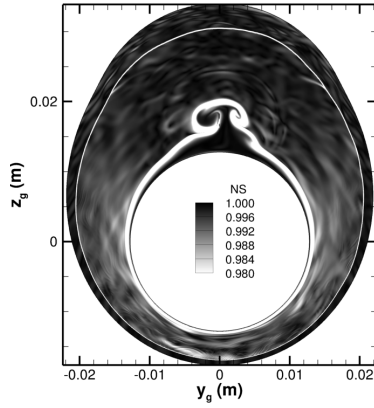


(g)  $x_g = 0.500$  m

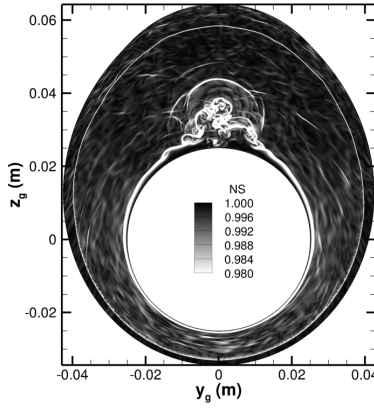
**Fig. 13** Streamwise vorticity,  $\omega_x$ , highlighting the development of vortical structures along the cone. The upper panel shows the global  $x_g$ - $z_g$  streamwise vorticity, with red markers denoting the streamwise positions of the extracted cross-sections. The lower panels present vorticity contours in different  $y$ - $z$  planes, revealing the small vortical structures from the breakdown and turbulent boundary layer.



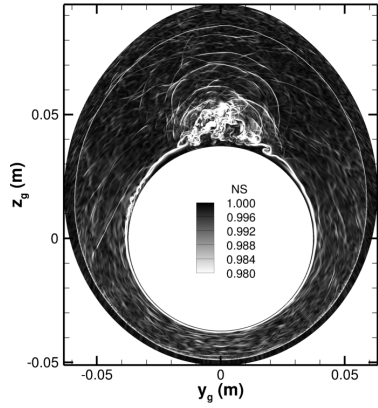
(a)  $x_g$ - $z_g$  plane



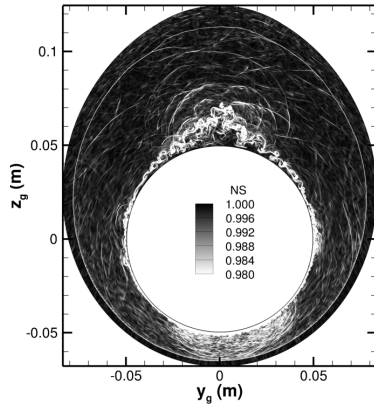
(b)  $x_g = 0.100$  m



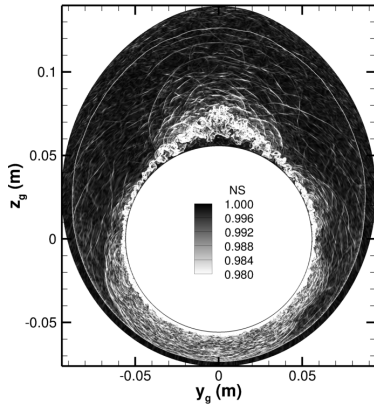
(c)  $x_g = 0.200$  m



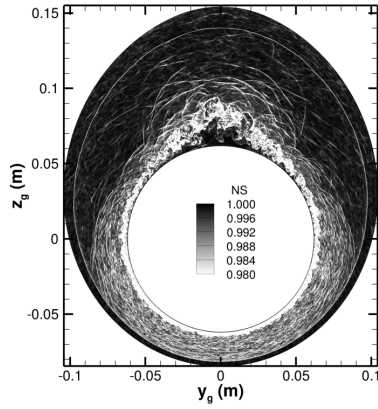
(d)  $x_g = 0.300$  m



(e)  $x_g = 0.400$  m



(f)  $x_g = 0.450$  m



(g)  $x_g = 0.500$  m

**Fig. 14** Numerical schlieren based on the density gradient highlighting the development of flow features along the cone. The upper panel shows the global  $x_g$ - $z_g$  schlieren distribution, with red markers denoting the streamwise positions of the extracted cross-sections. The lower panels present Schlieren contours in different  $y$ - $z$  planes, revealing the acoustic radiation from the breakdown and turbulent boundary layer.