

Design and Experimental Validation of Hamiltonian Control for Gust Load Alleviation and Flutter Suppression of a High Aspect Ratio Flexible Transport Wing

Christopher J. Forte*

KBR Wyle, Inc., Moffett Field, CA 94035

Nhan T. Nguyen†

NASA Ames Research Center, Moffett Field, CA 94035

This paper presents the design, implementation, and experimental evaluation of a Hamiltonian-based control strategy for gust load alleviation (GLA) and flutter suppression on a high-aspect-ratio transport wing model. The wing, based on a NASA Common Research Model variant (CRM13), was instrumented with distributed control surfaces and an array of sensors, including strain gauges and accelerometers, and tested in the NASA Langley Transonic Dynamics Tunnel. A full-order aeroservoelastic model was reduced for control design under the assumption sinusoidal steady-state response at a single frequency. The Hamiltonian framework provides a systematic approach for deriving state-feedback gains that guarantee dissipativity, with additional integral action incorporated through a linear-quadratic integral (LQI) control law. State estimation is performed using an extended Kalman filter for unmeasured structural and disturbance states. Experimental results demonstrate effective attenuation of wing root strain for multiple gust frequencies, and suppression of lightly damped flutter modes up to Mach 0.72. The study highlights the practical application of Hamiltonian control in aeroservoelastic systems and validates its effectiveness for both gust load alleviation and flutter suppression in a wind-tunnel environment.

I. Introduction

THE Integrated Adaptive Wing Technology Maturation (IAWTM) program, a collaborative effort between NASA and Boeing, investigates advanced high-aspect-ratio wing technologies for transport aircraft. As part of this program, a 13.5 aspect ratio variant of the NASA Common Research Model (CRM) was developed, increasing from the nominal aspect ratio of 9. This configuration, termed "CRM13," exemplifies the ongoing trend toward longer, lighter wings. Full scale simulation models and control laws for CRM13 were developed [1, 2], culminating in wind-tunnel testing at NASA Langley's Transonic Dynamics Tunnel (TDT) in 2024-2025 using a dynamically scaled, half span model equipped with distributed control surfaces, and an array of sensors. This test article in the TDT test section is shown in Fig. 1.

High aspectratio wings exhibit significant flexibility, making them susceptible to gust-induced loads and dynamic instabilities such as flutter. Two controller objectives are examined in this study: gust load alleviation (GLA) to reduce wing root strain due to gust disturbance, and flutter suppression to stabilize the unstable flutter mode. GLA controllers leverage predictive gust inputs from surrogate LIDAR sensors and onboard feedback to minimize strain, while flutter suppression relies on accelerometers to stabilize critical modes. Recent advances incorporate forward-looking LIDAR sensors [3, 4] to detect wind disturbances upstream of the aircraft. These sensors enable predictive GLA controllers [3–6].

The Hamiltonian control approach developed by Nguyen [7] is founded on Hamilton's principle for the distributed control of flexible structures modeled as infinite-dimensional Lagrangian systems. In this framework, operator theory is used to represent the structural dynamics through linear differential operators and modal expansions, allowing the system's total energy to be expressed through a Hamiltonian function. This formulation enables analysis of the closed-loop aeroservoelastic response under aerodynamic loads, where the generalized mass and stiffness matrices describe the dominant structural modes. Within this setting, the Hamiltonian framework provides dissipativity conditions that guide the design of gust load alleviation and flutter suppression controllers.

* Aerospace Engineer, Intelligent System Division, Member AIAA.

† Senior Research Scientist, Technical Group Lead, Intelligent Systems Division, Associate Fellow AIAA.

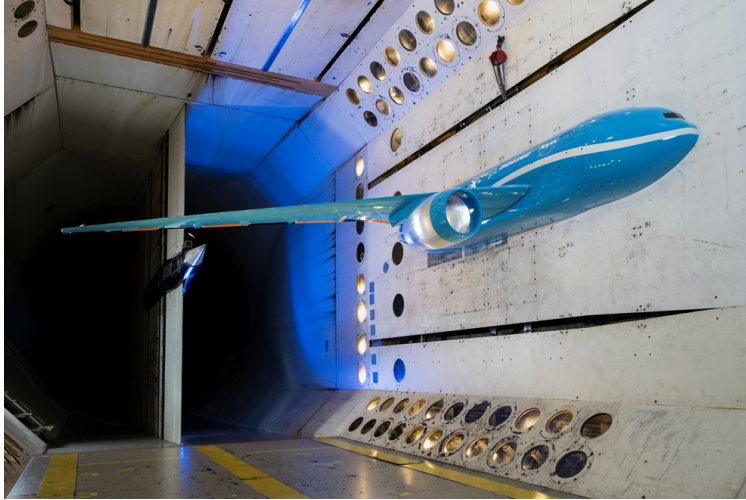


Fig. 1 IAWTM wind tunnel model in NASA Langley TDT.

II. Wind-Tunnel Test Overview

The wind tunnel model is a half-span, dynamically scaled representation of the NASA CRM13, with a shortened fuselage to fit the physical constraints of the tunnel and a flow-through nacelle as shown in Fig. 2. The IAWTM model in Fig. 2 has 10 control surfaces. Three control surfaces are high speed hydraulically actuated ailerons designated AILA-AILC and are intended for gust load alleviation and active flutter suppression controls. The 7 remaining control surfaces are slower acting electric actuators, called mini plain flaps (MPFs), and are designated MPFA-MPFG. The MPFs and hydraulic ailerons are used in conjunction for the drag minimization and maneuver load alleviation controls. Structural sensing is provided by 16 accelerometers and 10 strain gauges. For flutter suppression testing, a ballast is attached to the wing tip as shown in Fig. 2 to modify the aeroelastic stability of the model. This effectively lowers the flutter boundary for control law testing. The ballast is not used for GLA testing.

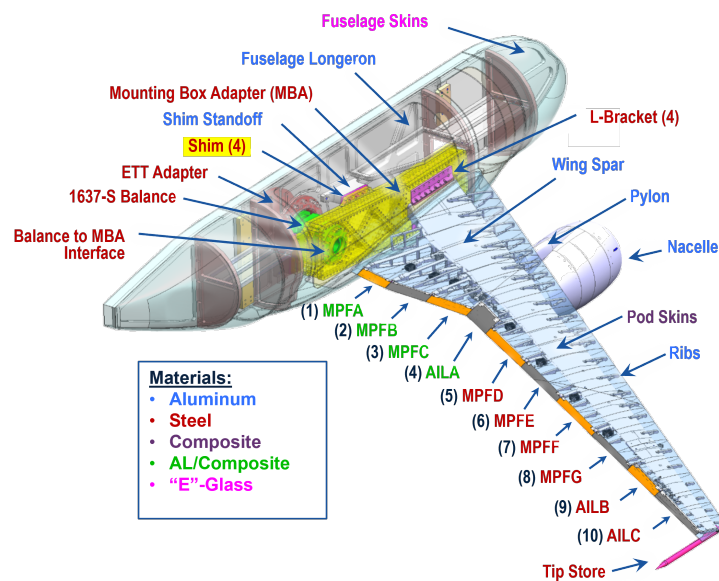


Fig. 2 IAWTM Test Article

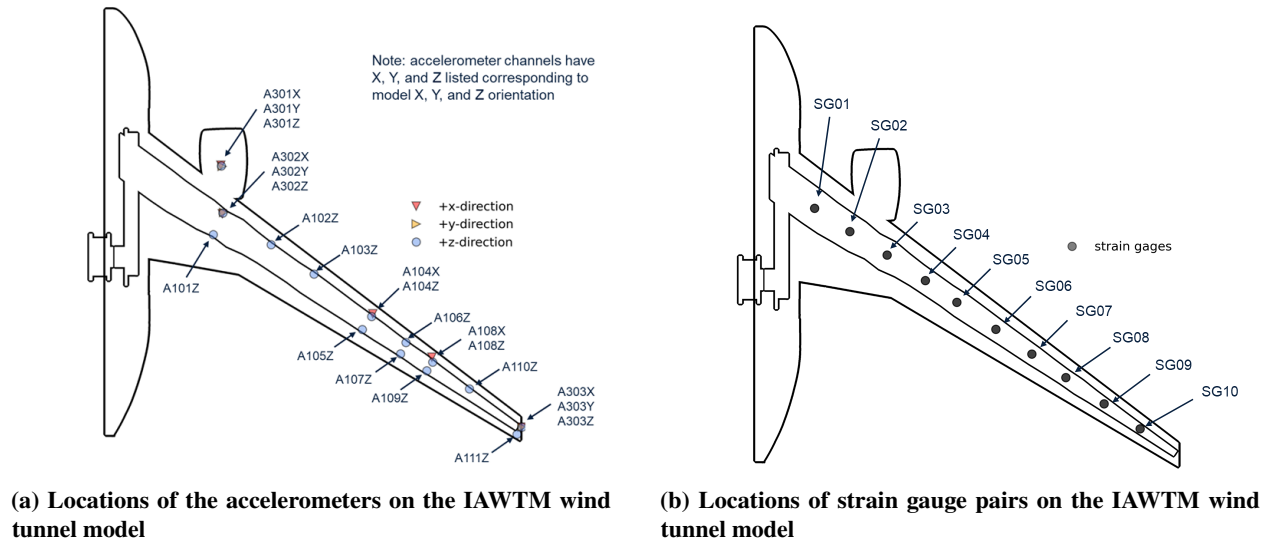


Fig. 3 Sensor layout on the IAWTM wind tunnel model.

Testing occurs in NASA Langley's 16 × 16 ft Transonic Dynamics Tunnel (TDT), which uses air or heavy gas (R134a) to replicate full-scale Mach and dynamic pressure conditions, shown in Figure 4. The tunnel includes an airstream oscillation system (AOS) with four gust vanes generating sinusoidal disturbances; for GLA testing, all vanes move in phase. A forward looking gust probe sits upstream of the model, emulating LIDAR to provide preview information for predictive GLA control, as seen in Figure 5. The tunnel also incorporates a bypass system that enables rapid reduction of dynamic pressure in the event of instability or other conditions requiring immediate intervention. Detailed descriptions of the wind-tunnel model and test setup are provided by Heaney and Quindlen [1, 8].

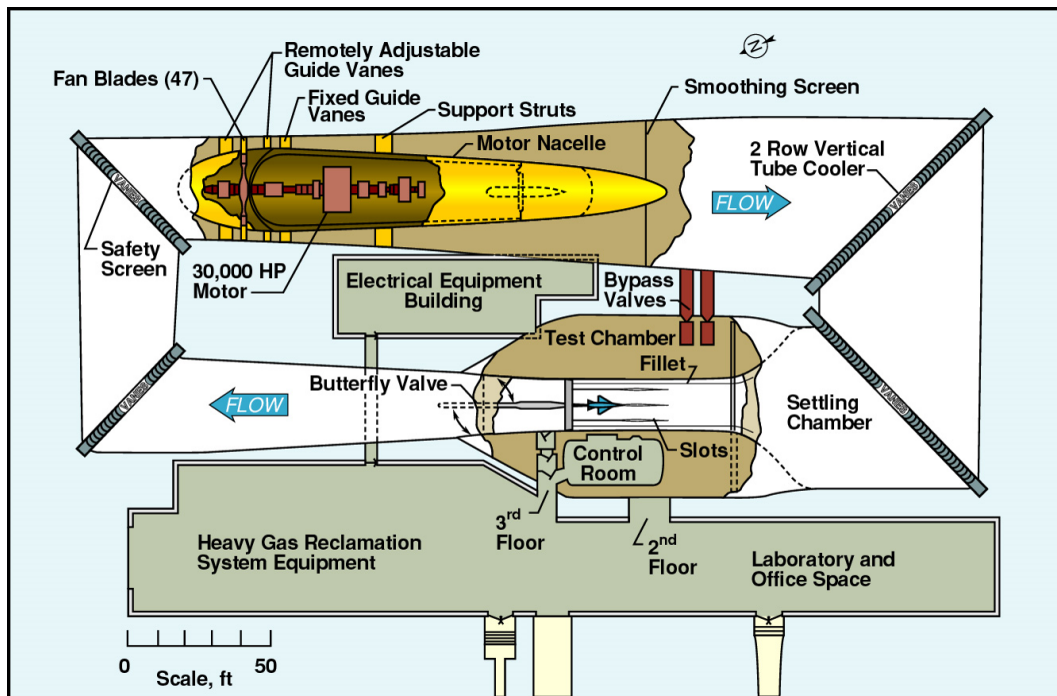


Fig. 4 Top-view of the Transonic Dynamics Tunnel.

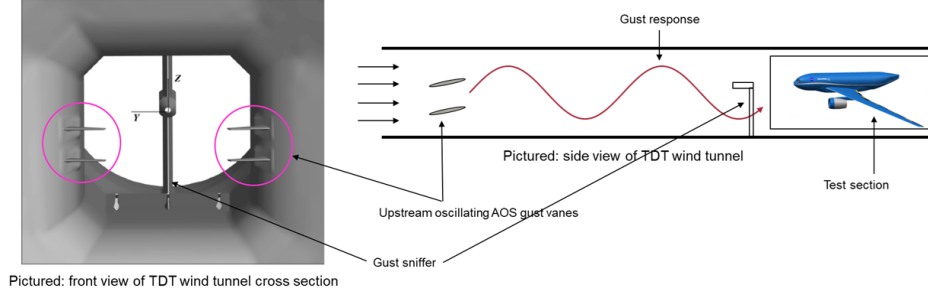


Fig. 5 Positioning of the AOS vanes in relation to the gust sniffer and IAWTM model.

III. Control Law Design

A. Aeroservoelastic Model

Characterization tests are conducted to improve the structural models, including four ground vibration tests. Following each test, the FEM structural models are updated and calibrated to match the measured data. Each structural model update is used to generate the corresponding set of aeroservoelastic models in ZAERO [9].

A total of 468 linear state-space aeroservoelastic models are developed per model configuration to span combinations of Mach number and dynamic pressure. There are a total of three model configurations. The first configuration, shims-off ballast-off (S0B0), is used for drag minimization and maneuver load alleviation testing. In this configuration, a five-component balance is employed to provide aerodynamic force and moment measurements. For flutter suppression and gust load alleviation (GLA) testing, shims are installed to bypass load transmission to the balance and protect it during higher risk test conditions. The GLA configuration, shims-on ballast-off (S1B0), includes shims without the installation of the tip ballast. For flutter suppression testing, a tip ballast is installed to reduce the dynamic pressure at which flutter occurs, enable controller evaluation. This configuration is referred to as shims-on ballast-on (S1B1). The model schedule is shown in Fig. 6 for S1B1, where green indicates open-loop stability and red indicates open-loop instability. Along the flutter test trajectory, the model is expected to encounter instability near Mach 0.75 at a dynamic pressure of 185 psf.

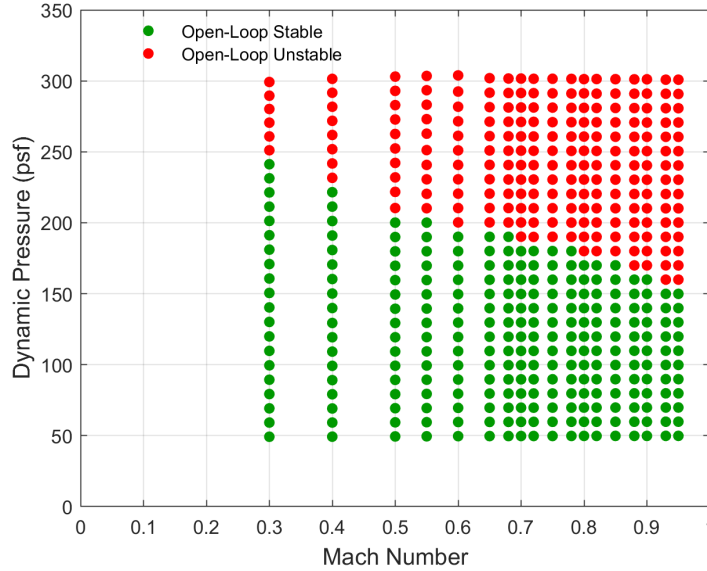


Fig. 6 ASE model schedule for S1B1 configuration

B. Aeroservoelastic Model Reduction

The ASE model comprises hundreds of states, which is excessive for practical control design and motivates model reduction. During GLA testing, the model is excited by gust vanes oscillating sinusoidally at a single frequency, ω_g . Model reduction is performed under the assumption that the steady-state response is sinusoidal at a single frequency. For GLA control, the reduced model assumes a steady-state response at the gust frequency, ω_g . For flutter suppression control, the gust vanes are removed, eliminating the gust terms from the model, and the reduction assumes a steady-state response at the flutter mode frequency, ω_f . The ASE model is given by:

$$\begin{bmatrix} \dot{x}_s \\ \dot{x}_l \\ \dot{x}_f \end{bmatrix} = \begin{bmatrix} A_{ss} & A_{sl} & 0 \\ A_{ls} & A_{ll} & 0 \\ A_{fs} & A_{fl} & A_{ff} \end{bmatrix} \begin{bmatrix} x_s \\ x_l \\ x_f \end{bmatrix} + \begin{bmatrix} B_{sa} \\ B_{la} \\ B_{fa} \end{bmatrix} x_a + \begin{bmatrix} E_{sw} \\ E_{lw} \\ E_{fw} \end{bmatrix} w \quad (1)$$

$$y = \begin{bmatrix} C_s & C_l & C_f \end{bmatrix} \begin{bmatrix} x_s \\ x_l \\ x_f \end{bmatrix} + D_a x_a + F_w w \quad (2)$$

$$x_s = \begin{bmatrix} q \\ \dot{q} \end{bmatrix} \quad (3)$$

$$x_a = \begin{bmatrix} \delta \\ \dot{\delta} \\ \ddot{\delta} \end{bmatrix} \quad (4)$$

$$w = \begin{bmatrix} \theta_g \\ \dot{\theta}_g \end{bmatrix} \quad (5)$$

The ASE model consists of the structural state vector $x_s(t)$ which includes the generalized structural displacement vector $q(t)$ and generalized rate vector $\dot{q}(t)$ for the first 20 structural modes, the aerodynamic lag state vector $x_l(t)$ which includes 12 lag states per structural mode, the anti-aliasing filter state vector $x_f(t)$ which includes 4 states per output, and gust disturbance vector $w(t)$. The actuator state vector $x_a(t)$, defined by Eq. 4, includes the actuator position $\delta(t)$, the actuator rate $\dot{\delta}(t)$, and actuator acceleration $\ddot{\delta}$. The lag state vector is removed first, with the lag state dynamics given by:

$$\dot{x}_l = A_{ll}x_l + A_{ls}x_s + B_{la}x_a + E_{lw}w \quad (6)$$

Upon expansion the following is obtained:

$$\dot{x}_l = A_{ll}x_l + A_{lq}q + A_{l\dot{q}}\dot{q} + B_{l\delta}\delta + B_{l\dot{\delta}}\dot{\delta} + B_{l\ddot{\delta}}\ddot{\delta} + E_{l\theta_g}\theta_g + E_{l\dot{\theta}_g}\dot{\theta}_g \quad (7)$$

Using the single frequency sinusoidal steady-state response these quantities can be written as:

$$x_l = a_1 \sin \omega_g t + a_2 \cos \omega_g t \quad (8)$$

$$q = b_1 \sin \omega_g t + b_2 \cos \omega_g t \quad (9)$$

$$\delta = c_1 \sin \omega_g t + c_2 \cos \omega_g t \quad (10)$$

$$\theta_g = d_1 \sin \omega_g t + d_2 \cos \omega_g t \quad (11)$$

Substituting, Eqs. 8-11 into 7 gives:

$$\begin{aligned} \dot{x}_l(t) &= a_1 \omega_g \cos \omega_g t - a_2 \omega_g \sin \omega_g t \\ &= A_{ll}(a_1 \sin \omega_g t + a_2 \cos \omega_g t) + A_{lq}(b_1 \sin \omega_g t + b_2 \cos \omega_g t) \\ &\quad + A_{l\dot{q}}(b_1 \omega_g \cos \omega_g t - b_2 \omega_g \sin \omega_g t) + B_{l\delta}(c_1 \sin \omega_g t + c_2 \cos \omega_g t) \\ &\quad + B_{l\dot{\delta}}(c_1 \omega_g \cos \omega_g t - c_2 \omega_g \sin \omega_g t) - B_{l\ddot{\delta}}(c_1 \omega_g^2 \sin \omega_g t + c_2 \omega_g^2 \cos \omega_g t) \\ &\quad + E_{l\theta_g}(d_1 \sin \omega_g t + d_2 \cos \omega_g t) + E_{l\dot{\theta}_g}(d_1 \omega_g \cos \omega_g t - d_2 \omega_g \sin \omega_g t) \end{aligned} \quad (12)$$

From which the following is written by comparing like terms:

$$-A_{ll}a_1 - a_2\omega_g = A_{lq}b_1 - A_{l\dot{q}}b_2\omega_g + B_{l\delta}c_1 - B_{l\delta}c_2\omega_g - B_{l\delta}c_1\omega_g^2 + E_{l\theta_g}d_1 - E_{l\dot{\theta}_g}d_2\omega_g \quad (13)$$

$$a_1\omega_g - A_{ll}a_2 = A_{lq}b_2 + A_{l\dot{q}}b_1\omega_g + B_{l\delta}c_2 + B_{l\delta}c_1\omega_g - B_{l\delta}c_2\omega_g^2 + E_{l\theta_g}d_2 + E_{l\dot{\theta}_g}d_1\omega_g \quad (14)$$

Then, a_1 and a_2 can be computed from this system of equations:

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} -A_{ll} & -\omega_g I \\ \omega_g I & -A_{ll} \end{bmatrix}^{-1} \begin{bmatrix} A_{lq}b_1 - A_{l\dot{q}}b_2\omega_g + B_{l\delta}c_1 - B_{l\delta}c_2\omega_g - B_{l\delta}c_1\omega_g^2 + E_{l\theta_g}d_1 - E_{l\dot{\theta}_g}d_2\omega_g \\ A_{lq}b_2 + A_{l\dot{q}}b_1\omega_g + B_{l\delta}c_2 + B_{l\delta}c_1\omega_g - B_{l\delta}c_2\omega_g^2 + E_{l\theta_g}d_2 + E_{l\dot{\theta}_g}d_1\omega_g \end{bmatrix} \quad (15)$$

This is rewritten as:

$$\begin{aligned} a_1 &= \alpha_{11}(A_{lq}b_1 - A_{l\dot{q}}b_2\omega_g + B_{l\delta}c_1 - B_{l\delta}c_2\omega_g - B_{l\delta}c_1\omega_g^2 + E_{l\theta_g}d_1 - E_{l\dot{\theta}_g}d_2\omega_g) \\ &\quad + \alpha_{12}(A_{lq}b_2 + A_{l\dot{q}}b_1\omega_g + B_{l\delta}c_2 + B_{l\delta}c_1\omega_g - B_{l\delta}c_2\omega_g^2 + E_{l\theta_g}d_2 + E_{l\dot{\theta}_g}d_1\omega_g) \end{aligned} \quad (16)$$

$$\begin{aligned} a_2 &= \alpha_{21}(A_{lq}b_1 - A_{l\dot{q}}b_2\omega_g + B_{l\delta}c_1 - B_{l\delta}c_2\omega_g - B_{l\delta}c_1\omega_g^2 + E_{l\theta_g}d_1 - E_{l\dot{\theta}_g}d_2\omega_g) \\ &\quad + \alpha_{22}(A_{lq}b_2 + A_{l\dot{q}}b_1\omega_g + B_{l\delta}c_2 + B_{l\delta}c_1\omega_g - B_{l\delta}c_2\omega_g^2 + E_{l\theta_g}d_2 + E_{l\dot{\theta}_g}d_1\omega_g) \end{aligned} \quad (17)$$

where

$$\begin{bmatrix} -A_{ll} & -\omega_g I \\ \omega_g I & -A_{ll} \end{bmatrix}^{-1} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \quad (18)$$

For a matrix of this form $\alpha_{11} = \alpha_{22}$ and $\alpha_{12} = -\alpha_{21}$. Then a_1 and a_2 are rewritten as:

$$\begin{aligned} a_1 &= \alpha_{11}(A_{lq}b_1 - A_{l\dot{q}}b_2\omega_g + B_{l\delta}c_1 - B_{l\delta}c_2\omega_g - B_{l\delta}c_1\omega_g^2 + E_{l\theta_g}d_1 - E_{l\dot{\theta}_g}d_2\omega_g) \\ &\quad + \alpha_{12}(A_{lq}b_2 + A_{l\dot{q}}b_1\omega_g + B_{l\delta}c_2 + B_{l\delta}c_1\omega_g - B_{l\delta}c_2\omega_g^2 + E_{l\theta_g}d_2 + E_{l\dot{\theta}_g}d_1\omega_g) \end{aligned} \quad (19)$$

$$\begin{aligned} a_2 &= -\alpha_{12}(A_{lq}b_1 - A_{l\dot{q}}b_2\omega_g + B_{l\delta}c_1 - B_{l\delta}c_2\omega_g - B_{l\delta}c_1\omega_g^2 + E_{l\theta_g}d_1 - E_{l\dot{\theta}_g}d_2\omega_g) \\ &\quad + \alpha_{11}(A_{lq}b_2 + A_{l\dot{q}}b_1\omega_g + B_{l\delta}c_2 + B_{l\delta}c_1\omega_g - B_{l\delta}c_2\omega_g^2 + E_{l\theta_g}d_2 + E_{l\dot{\theta}_g}d_1\omega_g) \end{aligned} \quad (20)$$

Then, Eqs. 19 and 20 are substituted into Eq. 8. Examining the terms containing b_1 and b_2 gives:

$$\begin{aligned} x_l &= \dots + \alpha_{11}A_{lq}b_1 \sin \omega_g t + \alpha_{11}A_{lq}b_2 \cos \omega_g t + \alpha_{11}A_{l\dot{q}}b_1\omega_g \cos \omega_g t - \alpha_{11}A_{l\dot{q}}b_2\omega_g \sin \omega_g t \\ &\quad - \alpha_{12}A_{lq}b_1 \cos \omega_g t + \alpha_{12}A_{lq}b_2 \sin \omega_g t + \alpha_{12}A_{l\dot{q}}b_1\omega_g \sin \omega_g t + \alpha_{12}A_{l\dot{q}}b_2\omega_g \cos \omega_g t + \dots \\ &= \dots + \alpha_1 b_1 \sin \omega_g t + \alpha_1 b_2 \cos \omega_g t - \alpha_2 b_1 \omega_g \cos \omega_g t + \alpha_2 b_2 \omega_g \sin \omega_g t \\ &\quad - \alpha_3 b_1 \cos \omega_g t + \alpha_3 b_2 \sin \omega_g t + \alpha_4 \omega_g b_1 \sin \omega_g t + \alpha_4 \omega_g b_2 \cos \omega_g t + \dots \\ &= \dots + \alpha_1 q + \alpha_2 \dot{q} + \alpha_3 \dot{q} + \alpha_4 q + \dots \end{aligned} \quad (21)$$

where

$$\alpha_1 = \alpha_{11}A_{lq} \quad (22)$$

$$\alpha_2 = -\alpha_{11}A_{l\dot{q}} \quad (23)$$

$$\alpha_3 = \alpha_{12}A_{lq} \quad (24)$$

$$\alpha_4 = -\alpha_{12}A_{l\dot{q}} \quad (25)$$

The final statement can be rewritten as:

$$\begin{aligned} \alpha_1 q + \alpha_2 \dot{q} + \alpha_3 \dot{q} + \alpha_4 q &= \bar{\alpha}_1 q + \bar{\alpha}_2 \dot{q} \\ &= K_1 x_s \end{aligned} \quad (26)$$

where $K_1 = \begin{bmatrix} \bar{\alpha}_1 & \bar{\alpha}_2 \end{bmatrix}$. Differentiating this term gives:

$$\frac{d}{dt}(K_1 x_s) = \bar{\alpha}_1 \dot{q} + \bar{\alpha}_2 \ddot{q} \quad (27)$$

Utilizing $\ddot{q} = -\omega_g^2 q$, which assumes the steady-state response of the model is sinusoidal, Eq. 27 becomes:

$$\begin{aligned} \frac{d}{dt}(K_1 x_s) &= \bar{\alpha}_1 \dot{q} - \bar{\alpha}_2 \omega_g^2 q \\ &= \begin{bmatrix} \bar{\alpha}_1 & \bar{\alpha}_2 \end{bmatrix} \begin{bmatrix} 0 & I \\ -\omega_g^2 I & 0 \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} \\ &= K_1 K_g x_s \end{aligned} \quad (28)$$

where

$$K_g = \begin{bmatrix} 0 & I \\ -\omega_g^2 I & 0 \end{bmatrix} \quad (29)$$

The actuator terms differ slightly. Examining the actuator terms gives:

$$x_l = \dots + \bar{\alpha}_3 \delta + \bar{\alpha}_4 \dot{\delta} + \bar{\alpha}_5 \ddot{\delta} \dots = \dots + K_2 \bar{\delta} + \dots \quad (30)$$

where

$$K_2 = \begin{bmatrix} \bar{\alpha}_3 & \bar{\alpha}_4 & \bar{\alpha}_5 \end{bmatrix} \quad (31)$$

Differentiating this term gives:

$$\frac{d}{dt}(K_2 x_a) = \bar{\alpha}_3 \dot{\delta} + \bar{\alpha}_4 \ddot{\delta} + \bar{\alpha}_5 \ddot{\delta} \quad (32)$$

Utilizing $\ddot{\delta} = -\omega_g^2 \delta$, this becomes:

$$\frac{d}{dt}(K_2 x_a) = \bar{\alpha}_3 \dot{\delta} + \bar{\alpha}_4 \ddot{\delta} - \omega_g^2 \bar{\alpha}_5 \delta = \begin{bmatrix} \bar{\alpha}_3 & \bar{\alpha}_4 & \bar{\alpha}_5 \end{bmatrix} \begin{bmatrix} 0 & I & 0 \\ 0 & 0 & I \\ 0 & -\omega_g^2 I & 0 \end{bmatrix} \begin{bmatrix} \delta \\ \dot{\delta} \\ \ddot{\delta} \end{bmatrix} = K_2 K_{g_a} x_a \quad (33)$$

The gust state vector is handled similarly. Eq. 21 therefore reduces to:

$$x_l = K_1 x_s + K_2 x_a + K_3 w \quad (34)$$

and its time derivative is given by:

$$\dot{x}_l = K_1 K_g x_s + K_2 K_{g_a} x_a + K_3 K_{g_w} w \quad (35)$$

where

$$K_3 = \begin{bmatrix} \bar{\alpha}_6 & \bar{\alpha}_7 \end{bmatrix} \quad (36)$$

$$K_{g_w} = \begin{bmatrix} 0 & I \\ -\omega_g^2 I & 0 \end{bmatrix} \quad (37)$$

Eqs. 34 and 35 are then substituted into the lag state equation given by Eq. 6, yielding:

$$\dot{x}_l = K_1 K_g x_s + K_2 K_{g_a} x_a + K_3 K_{g_w} w = A_{ll}(K_1 x_s + K_2 x_a + K_3 w) + A_{ls} x_s + B_{la} x_a + E_{lw} w \quad (38)$$

By comparing like terms, the resulting Sylvester equations are obtained from which K_1 , K_2 , and K_3 can be solved:

$$K_1 K_g - A_{ll} K_1 = A_{ls} \quad (39)$$

$$K_2 K_{g_a} - A_{ll} K_2 = B_{la} \quad (40)$$

$$K_3 K_{g_w} - A_{ll} K_3 = E_{lw} \quad (41)$$

Eq. 34 is substituted into the ASE system in Eqs. 1 and 2, thereby removing the lag states. The reduced system is given by:

$$\begin{bmatrix} \dot{x}_s \\ \dot{x}_f \end{bmatrix} = \begin{bmatrix} A_{ss} + A_{sl}K_1 & 0 \\ A_{fs} + A_{fl}K_1 & A_{ff} \end{bmatrix} \begin{bmatrix} x_s \\ x_f \end{bmatrix} + \begin{bmatrix} B_{sa} + A_{sl}K_2 \\ B_{fa} + A_{fl}K_2 \end{bmatrix} x_a + \begin{bmatrix} E_{sw} + A_{sl}K_3 \\ E_{fw} + A_{fl}K_3 \end{bmatrix} w \quad (42)$$

$$y = \begin{bmatrix} C_s + C_lK_1 & C_f \end{bmatrix} \begin{bmatrix} x_s \\ x_f \end{bmatrix} + (D_a + C_lK_2)x_a + (F_w + C_lK_3)w \quad (43)$$

To remove the anti-aliasing filter states, the same process is followed using the reduced system given by Eqs. 42 and 43. The anti-aliasing filter state vector is written as:

$$x_f = K_4x_s + K_5x_a + K_6w \quad (44)$$

and

$$\dot{x}_f = K_4K_gx_s + K_5K_{ga}x_a + K_6K_{gw}w \quad (45)$$

The anti-aliasing filter dynamic equation is written as:

$$\dot{x}_f = \bar{A}_{fs}x_s + \bar{B}_{fa}x_a + \bar{E}_{fw}w + A_{ff}x_f \quad (46)$$

where,

$$\bar{A}_{fs} = A_{fs} + A_{fl}K_1 \quad (47)$$

$$\bar{B}_{fa} = B_{fa} + A_{fl}K_2 \quad (48)$$

$$\bar{E}_{fw} = E_{fw} + A_{fl}K_3 \quad (49)$$

Substituting Eqs. 44 and 45 into Eq. 46 gives:

$$K_4K_gx_s + K_5K_{ga}x_a + K_6K_{gw}w = \bar{A}_{fs}x_s + \bar{B}_{fa}x_a + \bar{E}_{fw}w + A_{ff}(K_4x_s + K_5x_a + K_6w) \quad (50)$$

By comparing like terms, the resulting Sylvester equations are obtained from which K_4 , K_5 , and K_6 can be solved:

$$K_4K_g - A_{ff}K_4 = \bar{A}_{fs} \quad (51)$$

$$K_5K_{ga} - A_{ff}K_5 = \bar{B}_{fa} \quad (52)$$

$$K_6K_{gw} - A_{ff}K_6 = \bar{E}_{fw} \quad (53)$$

Substituting Eq. 44 into Eqs. 42 and 43 yields the following system with the anti-aliasing filter state vector removed:

$$\dot{x}_s = (A_{ss} + A_{sl}K_1)x_s + (B_{sa} + A_{sl}K_2)x_a + (E_{sw} + A_{sl}K_3)w \quad (54)$$

$$y = (C_s + C_lK_1 + C_fK_4)x_s + (D_a + C_lK_2 + C_fK_5)x_a + (F_w + C_fK_3 + C_fK_6)w \quad (55)$$

Furthermore, the state x_s in the system defined by Eqs. 54 and 55 is partitioned into the x_b which is the state vector for the first 10 structural modes and x_h which is the state vector for the remaining 10 structural modes, enabling the reduction of higher frequency modes that are outside the bandwidth of the actuators. The system is written as:

$$\begin{bmatrix} \dot{x}_b \\ \dot{x}_h \end{bmatrix} = \begin{bmatrix} A_{bb} & A_{bh} \\ A_{hb} & A_{hh} \end{bmatrix} \begin{bmatrix} x_b \\ x_h \end{bmatrix} + \begin{bmatrix} B_b \\ B_h \end{bmatrix} x_a + \begin{bmatrix} E_b \\ E_h \end{bmatrix} w \quad (56)$$

$$y = \begin{bmatrix} C_b & C_h \end{bmatrix} \begin{bmatrix} x_b \\ x_h \end{bmatrix} + (D_a + C_l K_2 + C_f K_5)x_a + (F_w + C_f K_3 + C_f K_6)w \quad (57)$$

Using the same method as before the high frequency structural state vector is written as:

$$x_h = K_7 x_b + K_8 x_a + K_9 w \quad (58)$$

and

$$\dot{x}_h = K_7 K_{g_b} x_b + K_8 K_{g_a} x_a + K_9 K_{g_w} w \quad (59)$$

The high frequency state dynamics are given by:

$$\dot{x}_h = A_{hb} x_b + A_{hh} x_h + B_h x_a + E_h w \quad (60)$$

Substituting Eqs. 58 and 59 into Eq. 60 gives:

$$K_7 K_{g_b} x_b + K_8 K_{g_a} x_a + K_9 K_{g_w} w = A_{hb} x_b + B_h x_a + E_h w + A_{hh} (K_7 x_b + K_8 x_a + K_9 w) \quad (61)$$

By comparing like terms, the resulting Sylvester equations are obtained from which K_7 , K_8 , and K_9 can be solved:

$$K_7 K_{g_b} - A_{hh} K_7 = A_{hb} \quad (62)$$

$$K_8 K_{g_a} - A_{hh} K_8 = B_h \quad (63)$$

$$K_9 K_{g_w} - A_{hh} K_9 = E_h \quad (64)$$

Eq. 58 is then substituted into 56 and 57 there by reducing the high frequency structural states, resulting in the reduced system:

$$\dot{x}_b = (A_{bb} + A_{bh} K_7) x_b + (B_b + B_h K_8) x_a + (E_b + E_h K_9) w \quad (65)$$

$$y = (C_b + C_h K_7) x_b + (D_a + C_l K_2 + C_f K_5 + C_h K_8) x_a + (F_w + C_f K_3 + C_f K_6 + C_h K_9) w \quad (66)$$

C. Actuator Model

A second order Butterworth low-pass filter with a cutoff frequency of 10 Hz is applied to the control command to eliminate high frequency content in the control signal. The 10 Hz cutoff frequency is chosen because the ASE model accuracy decreases at frequencies beyond this bandwidth. The control command low-pass filter dynamics are given by:

$$\dot{x}_{f_l} = A_{f_l} x_{f_l} + B_{f_l} u \quad (67)$$

$$u_f = C_{f_l} x_{f_l} \quad (68)$$

The actuator dynamics are given by the following state-space system:

$$\dot{x}_{act} = A_{act} x_{act} + B_{act} u_f \quad (69)$$

$$x_a = C_{act} x_{act} + D_{act} u_f \quad (70)$$

The actuator dynamics with low-pass filter are given by:

$$\begin{bmatrix} \dot{x}_{act} \\ \dot{x}_{f_l} \end{bmatrix} = \begin{bmatrix} A_{act} & B_{act} C_{f_l} \\ 0 & A_{f_l} \end{bmatrix} \begin{bmatrix} x_{act} \\ x_{f_l} \end{bmatrix} + \begin{bmatrix} 0 \\ B_{f_l} \end{bmatrix} u \quad (71)$$

$$x_a = \begin{bmatrix} C_{act} & D_{act} C_{f_l} \end{bmatrix} \begin{bmatrix} x_{act} \\ x_{f_l} \end{bmatrix} \quad (72)$$

Combining the actuator dynamics with low-pass filter and the structural dynamics yields the following state-space system utilized for control law design:

$$\dot{x} = Ax + Bu + Ew \quad (73)$$

$$y = Cx + Fw \quad (74)$$

where

$$x = \begin{bmatrix} x_r \\ x_{act} \\ x_{fi} \end{bmatrix} \quad (75)$$

$$A = \begin{bmatrix} (A_{bb} + A_{bh}K_7) & (B_b + B_hK_8)C_{act} & (B_b + B_hK_8)D_{act}C_{fi} \\ 0 & A_{act} & B_{act}C_{fi} \\ 0 & 0 & A_{fi} \end{bmatrix} \quad (76)$$

$$B = \begin{bmatrix} 0 \\ 0 \\ B_{fi} \end{bmatrix} \quad (77)$$

$$C = \begin{bmatrix} (C_r + C_hK_7) & (D_a + C_lK_2 + C_fK_5 + C_hK_8)C_{act} & (F_w + C_fK_3 + C_fK_6 + C_hK_9)D_{act}C_{fi} \end{bmatrix} \quad (78)$$

$$E = \begin{bmatrix} (E_r + E_hK_9) \\ 0 \\ 0 \end{bmatrix} \quad (79)$$

$$F = (F_w + C_hK_9) \quad (80)$$

D. Gust Disturbance Modeling

The gust vane dynamics are modeled as follows:

$$\dot{x}_v(t) = A_v x_v(t) + B_v u_v(t) \quad (81)$$

where $u_v(t)$ is the commanded gust vane position, $x_v(t)$ is the vector containing the gust vane position $\theta_v(t)$, the gust vane velocity $\dot{\theta}_v(t)$ and the gust vane acceleration $\ddot{\theta}_v(t)$ given by:

$$x_v(t) = \begin{bmatrix} \theta_v(t) \\ \dot{\theta}_v(t) \\ \ddot{\theta}_v(t) \end{bmatrix} \quad (82)$$

This can be expressed in transfer function form:

$$X_v(s) = G_1(s)U_v(s) \quad (83)$$

$$G_1(s) = (sI - A_v)^{-1}B_v \quad (84)$$

The output of the gust vane model contains the gust vane position and velocity and is expressed as:

$$y_v(t) = C_v x_v(t) \quad (85)$$

where

$$C_v = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad (86)$$

The disturbance state $w(t)$ represents the gust vane position and velocity delayed by the convective time delay between the gust vanes and the model which is given by:

$$w(t) = y_v(t - t_d) \quad (87)$$

The gust vanes move sinusoidally at a known frequency ω_g and amplitude θ_0 . Thus, $\theta_g(t)$ is written as:

$$\theta_g(t) = \theta_0 \sin(\omega_g t + \phi) \quad (88)$$

The dynamics for gust disturbance $w(t)$, seen in Eq. 5, can therefore be written as:

$$\dot{w}(t) = A_w w(t) \quad (89)$$

where

$$A_w = \begin{bmatrix} 0 & 1 \\ -\omega_g^2 & 0 \end{bmatrix} \quad (90)$$

E. Hamiltonian Control

The Hamiltonian controller developed by Nguyen [7] is based on Hamilton's principle for distributed control of infinite-dimensional Lagrangian systems governed by partial differential equations. This control framework employs operator theory to model Lagrangian systems. The structural dynamics of a flexible structure can be expressed as the following Lagrangian system in linear operator form:

$$m(x) \frac{\partial^2 w(x, t)}{\partial t^2} + Lw(x, t) = f \left(w(x, t), \frac{\partial w(x, t)}{\partial t}, u(x, t) \right) + e(x)g(t) \quad (91)$$

The distributed mass is $m(x)$ and L is a self-adjoint linear operator given by:

$$L = m^{-1}(x) \sum_{i=0}^m (-1)^i \frac{\partial^i}{\partial x^i} \left(k_i(x) \frac{\partial^i}{\partial x^i} \right) \quad (92)$$

The distributed displacement is $w(x, t)$ and is expressed as the product between the eigenfunction $\phi(x)$ and the generalized displacement $q(t)$:

$$w(x, t) = \phi(x)q(t) \quad (93)$$

The Hamiltonian function then provides the framework for representing the total energy of a flexible structure and for analyzing its dynamic behavior under control and aerodynamic forces. It therefore can be used to analyze the closed-loop structural dynamics under aerodynamic forces [7, 10, 11]. In this case, the ASE system is cast as a structural dynamic model, where the generalized mass and stiffness matrices capture the dominant structural modes. Here, the Hamiltonian approach is used to derive the gust load alleviation control design, providing conditions for dissipativity that inform gain selection and closed-loop performance. The structural dynamics of the system is expressed in the general form as:

$$M\ddot{q}(t) + Kq(t) = F_q q(t) + F_{\dot{q}} \dot{q}(t) + F_{\delta} \delta(t) + F_{\dot{\delta}} \dot{\delta}(t) + F_{\ddot{\delta}} \ddot{\delta}(t) + F_w w(t) \quad (94)$$

The right-hand side of the equation represents the generalized forces acting on the structure. Assuming that the control input $\delta(t)$ exhibits harmonic behavior at a single frequency ω_g , the acceleration term of the control input can be approximated as $\ddot{\delta}(t) \approx -\omega_g^2 \delta(t)$, which reduces the structural dynamics to

$$M\ddot{q}(t) + Kq(t) = F_q q(t) + F_{\dot{q}} \dot{q}(t) + (F_{\delta} - F_{\ddot{\delta}} \omega_g^2) \delta(t) + F_{\dot{\delta}} \dot{\delta}(t) + F_w w(t) \quad (95)$$

The control command is defined by a state-feedback law:

$$\delta_c(t) = -K_q q(t) - K_{\dot{q}} \dot{q}(t) - K_w w(t) \quad (96)$$

whose time derivative is given by:

$$\dot{\delta}_c(t) = -K_q \dot{q}(t) - K_{\dot{q}} \ddot{q}(t) - K_w \dot{w}(t) \quad (97)$$

The disturbance evolves according to $\dot{w}(t) = A_w w(t)$, thus

$$\dot{\delta}_c(t) = -K_q \dot{q}(t) - K_{\dot{q}} \ddot{q}(t) - K_w A_w w(t) \quad (98)$$

To assess system stability, the Hamiltonian is defined as:

$$H = \frac{1}{2} \dot{q}^\top(t) M \dot{q}(t) + \frac{1}{2} q^\top(t) K q(t) \quad (99)$$

whose time derivative is:

$$\dot{H} = \dot{q}^\top(t) M \ddot{q}(t) + \dot{q}^\top(t) K q(t) \quad (100)$$

$$= \dot{q}^\top(t) [M \ddot{q}(t) + K q(t)] \quad (101)$$

$$= \dot{q}^\top(t) [F_q q(t) + F_{\dot{q}} \dot{q}(t) + (F_\delta - F_{\delta} \omega_g^2) \delta_c(t) + F_{\delta} \dot{\delta}_c(t) + F_w w(t)] \quad (102)$$

The conditions for dissipativity are imposed as:

$$\Delta \dot{H}_q = \dot{q}^\top(t) [F_q - (F_\delta - F_{\delta} \omega_g^2) K_q] q(t) \leq 0 \quad (103)$$

$$\Delta \dot{H}_{\dot{q}} = \dot{q}^\top(t) [F_{\dot{q}} - (F_\delta - F_{\delta} \omega_g^2) K_{\dot{q}} - F_{\delta} K_q] \dot{q}(t) \leq 0 \quad (104)$$

$$\Delta \dot{H}_{\ddot{q}} = \dot{q}^\top(t) [-F_{\delta} K_{\ddot{q}}] \ddot{q}(t) \leq 0 \quad (105)$$

These inequalities ensure that the rate change of stored system energy remains non-positive, guaranteeing dissipativity. The gain K_q is determined by setting $\Delta \dot{H}_q$ to zero which gives:

$$F_q - (F_\delta - F_{\delta} \omega_g^2) K_q = 0 \quad (106)$$

The pseudo-inverse method is used to solve for K_q which results in:

$$K_q = [(F_\delta - F_{\delta} \omega_g^2)^\top (F_\delta - F_{\delta} \omega_g^2)]^{-1} (F_\delta - F_{\delta} \omega_g^2)^\top F_q \quad (107)$$

To determine $K_{\dot{q}}$, $\Delta \dot{H}_{\dot{q}}$ is set to a negative definite quantity which is chosen to be $-2\zeta\omega_n$ which gives:

$$F_{\dot{q}} - (F_\delta - F_{\delta} \omega_g^2) K_{\dot{q}} - F_{\delta} K_q = -2\zeta\omega_n \quad (108)$$

where ω_n are the modal frequencies and ζ are the prescribed damping ratios. The pseudo-inverse method is used to solve for $K_{\dot{q}}$ which results in:

$$K_{\dot{q}} = [(F_\delta - F_{\delta} \omega_g^2)^\top (F_\delta - F_{\delta} \omega_g^2)]^{-1} (F_\delta - F_{\delta} \omega_g^2)^\top (F_{\dot{q}} - F_{\delta} K_q + 2\zeta\omega_n) \quad (109)$$

A damping ratio of $1/\sqrt{2}$ is chosen as an initial damping ratio and most active modes are tuned between damping ratios of 0.28 and 0.71, with focus on the most active modes. Eq. 105 represents the apparent mass contribution to the system. This means the $K_{\ddot{q}}$ must be sufficiently small so the apparent mass remains positive definite.

To determine the gain K_w , the control command in Eq. 96 is substituted into Eq. 95. Isolating the terms involving the disturbance $w(t)$ setting them to zero yields:

$$-(F_\delta - F_{\delta} \omega_g^2) K_w w(t) - F_{\delta} K_w \dot{w}(t) + F_w w(t) = 0 \quad (110)$$

Since the disturbance evolves according to Eq. 89, therefore:

$$-(F_\delta - F_{\delta} \omega_g^2) K_w w(t) - F_{\delta} K_w A_w w(t) + F_w w(t) = 0 \quad (111)$$

Thus, the following Sylvester equation is obtained from which K_w is obtained:

$$-(F_\delta - F_{\delta} \omega_g^2) K_w - F_{\delta} K_w A_w + F_w = 0 \quad (112)$$

The feedback gains K_q and $K_{\dot{q}}$ can be expressed as:

$$\delta_c(t) = -K_q q(t) - K_{\dot{q}} \dot{q}(t) - K_w w(t) \quad (113)$$

$$= -K_{x_b} x_b(t) - K_w w(t) \quad (114)$$

where

$$K_{x_b} = \begin{bmatrix} K_q & K_{\dot{q}} \end{bmatrix} \quad (115)$$

The Hamiltonian gains provide a control command $\delta_c(t)$ but not the actuator input $u(t)$ directly. To achieve command tracking, a linear quadratic integral (LQI) control strategy is implemented. The control position $\delta(t)$ is given by:

$$\delta(t) = C_\delta x(t) \quad (116)$$

The integral error is defined by:

$$\begin{aligned} e(t) &= \int_0^t [\delta_c(\tau) - \delta(\tau)] d\tau \\ &= \int_0^t [\delta_c(\tau) - C_\delta x(\tau)] d\tau \end{aligned} \quad (117)$$

The time derivative of the integral error is given by:

$$\dot{e}(t) = \delta_c(t) - C_\delta x(t) \quad (118)$$

The system is extended to include the integral error which results in:

$$\dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \bar{B}u(t) + \bar{E}w(t) + V\delta_c(t) \quad (119)$$

$$y(t) = \bar{C}\bar{x}(t) + Fw(t) \quad (120)$$

where

$$\bar{x}(t) = \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} \quad (121)$$

$$\bar{A} = \begin{bmatrix} A & 0 \\ -C_\delta & 0 \end{bmatrix} \quad (122)$$

$$\bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix} \quad (123)$$

$$\bar{C} = \begin{bmatrix} C & 0 \end{bmatrix} \quad (124)$$

$$V = \begin{bmatrix} 0 \\ I \end{bmatrix} \quad (125)$$

The control command is then expressed in terms of $\bar{x}(t)$:

$$\delta_c(t) = -K_{x_b} x_b(t) - K_w w(t) \quad (126)$$

$$= -K_{\bar{x}} \bar{x}(t) - K_w w(t) \quad (127)$$

where

$$K_{\bar{x}} = \begin{bmatrix} K_{x_b} & 0 \end{bmatrix} \quad (128)$$

The LQI control minimizes the cost function given by:

$$J = \frac{1}{2} \int_0^\infty [\bar{x}^\top(t) Q \bar{x}(t) + u^\top(t) R u(t)] dt \quad (129)$$

The Hamiltonian is established as:

$$H = \frac{1}{2} \bar{x}^\top Q \bar{x} + \frac{1}{2} u^\top R u + \lambda^\top (\bar{A} \bar{x} + \bar{B} u + E w + V \delta_c) \quad (130)$$

$$= \frac{1}{2} \bar{x}^\top Q \bar{x} + \frac{1}{2} u^\top R u + \lambda^\top (\bar{A} \bar{x} + \bar{B} u + E w + V [-K_{\bar{x}} \bar{x} - K_w w]) \quad (131)$$

The necessary conditions for optimality are given by:

$$\dot{\lambda} = -\frac{\partial H^\top}{\partial \bar{x}} = -Q \bar{x} - (\bar{A} - V K_{\bar{x}})^\top \lambda \quad (132)$$

$$\frac{\partial H^\top}{\partial u} = -R u + \bar{B}^\top \lambda \quad (133)$$

The optimal control is obtained as:

$$u = R^{-1} \bar{B}^\top \lambda \quad (134)$$

Let $\lambda(t) = S(t)\bar{x}(t) + T(t)w(t)$. Then, differentiating $\lambda(t)$ yields:

$$\dot{S}\bar{x} + S\tilde{A}\bar{x} + \tilde{A}^\top S\bar{x} - S\bar{B}R^{-1}\bar{B}^\top S\bar{x} + Q\bar{x} + (\tilde{A}^\top - S\bar{B}R^{-1}\bar{B}^\top)Tw + T\dot{w} + \dot{T}w = SVK_w w \quad (135)$$

where

$$\tilde{A} = \bar{A} - VK_{\bar{x}} \quad (136)$$

For the infinite time-horizon optimal control, $\dot{S} = 0$ and $\dot{T} = 0$. Therefore, the modified Riccati equation is obtained:

$$S\tilde{A} + \tilde{A}^\top - S\bar{B}R^{-1}\bar{B}^\top S + Q = 0 \quad (137)$$

along with:

$$(\tilde{A}^\top - S\bar{B}R^{-1}\bar{B}^\top)Tw + T\dot{w} = SVK_w w \quad (138)$$

The disturbance evolves according to Eq. 89, which yields the Sylvester equation from which T can be solved:

$$(\tilde{A}^\top - S\bar{B}R^{-1}\bar{B}^\top)T + TA_w = SVK_w \quad (139)$$

The optimal control is expressed as:

$$u = -\tilde{K}_{\bar{x}}\bar{x} + \tilde{K}_w w \quad (140)$$

where

$$\tilde{K}_{\bar{x}} = R^{-1}\bar{B}^\top S \quad (141)$$

$$\tilde{K}_w = -R^{-1}\bar{B}^\top T \quad (142)$$

Note that $x(t)$ and $w(t)$ are not available and therefore estimated via Kalman filter. The control law is then designed as:

$$u = -\tilde{K}_{\bar{x}}\hat{x} + \tilde{K}_w\hat{w} \quad (143)$$

In the flutter-suppression application, the gust-related term is omitted because the wind-tunnel configuration does not include upstream gust vanes and no exogenous gust state is available or required. Consequently, the controller is tuned solely for structural mode stabilization, with prescribed damping ratios applied to the dominant flutter modes and adjusted to ensure adequate modal energy dissipation without excessively penalizing control authority. The Hamiltonian control command is given by:

$$\delta_c(t) = -K_{x_b}\hat{x}_b(t) \quad (144)$$

and the LQI tracking control command is given by:

$$u = -\tilde{K}_{\bar{x}}\hat{x} \quad (145)$$

Hamiltonian gains are computed for each available ASE model across the test envelope, and the resulting controllers are scheduled with Mach number and dynamic pressure to account for the strong aeroelastic dependence on operating condition.

IV. State and Gust Estimation

A. Process Noise Modeling

To compute the Kalman filter gain, L_k , both the measurement noise covariance and the process noise covariance must be specified. Determining the measurement noise covariance is straightforward: ambient measurements from all relevant sensors are recorded, and the covariance for each sensor is computed directly. In contrast, identifying an appropriate process noise covariance is less direct. In this application, the process noise originates from the unsteady aerodynamic forces acting on the model due to wind-tunnel turbulence. A process noise model is therefore constructed using the unsteady lift, pitching-moment, and drag measurements.

The measured aerodynamic data contain both unsteady aerodynamic content and structural response. To isolate the unsteady aerodynamic contribution, notch filtering is applied to remove dominant structural modes. The notch filters used here are based on a second-order band-stop form:

$$G_n(s) = \frac{s^2 + 2(1 - \alpha)\omega_c s + \omega_c^2}{(s + \omega_c)^2}, \quad (146)$$

where ω_c is the notch frequency and α is the notch-depth parameter. The parameters ω_c and α are selected to suppress the structural peaks observed in the FFTs of the aerodynamic test data. Figure 7 compares the unfiltered (red) and notch-filtered (black) FFTs of the lift measurement. The same notch filtering procedure is applied to the lift, drag, and pitching-moment test measurements.

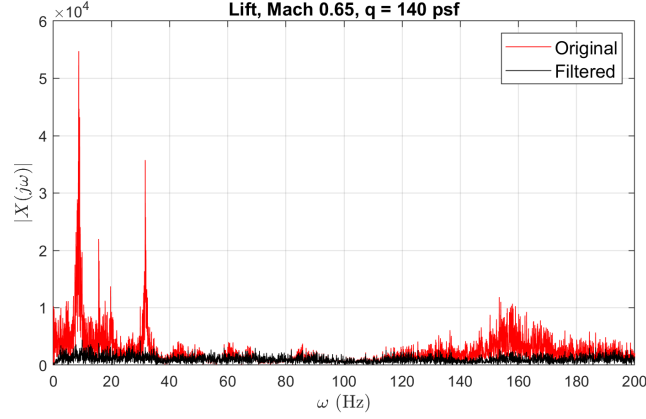


Fig. 7 Lift FFT showing original (red) and notch-filtered (black) spectra.

A nominal elliptical lift profile is assumed such that the spanwise integral matches the total mean lift in the test data:

$$\int_0^{b/2} l(y) dy = \bar{L}. \quad (147)$$

Random perturbations are then applied at each wing station to generate perturbed lift profiles $l_n(y)$, as shown in Fig. 8. This procedure is repeated for a large number of samples, matching the size of the test dataset. The spanwise lift is integrated for each sample, and the magnitude of the applied noise is scaled so that the variation in integrated lift matches that observed in the filtered test data.

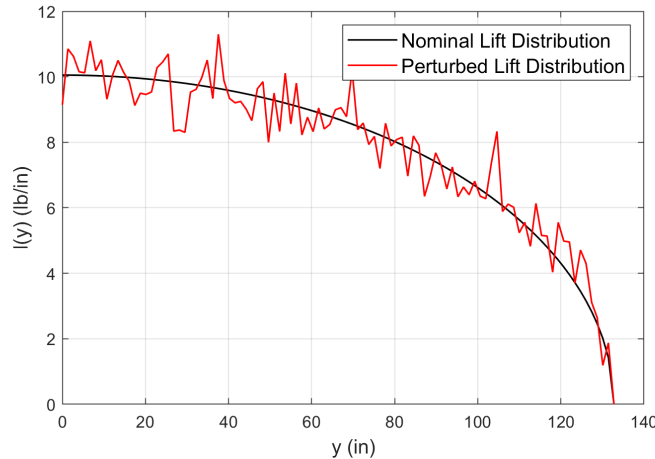


Fig. 8 Nominal and perturbed lift distributions along the wing span.

The drag distribution is modeled as a parabolic function along the semispan:

$$d(y) = k l(y)^2, \quad (148)$$

where the coefficient k is determined from the measured total lift-to-drag ratio. The total drag is expressed as:

$$D_{\text{tot}} = \frac{\bar{L}}{L/D} \quad (149)$$

$$= k \int_0^{b/2} l(y)^2 dy \quad (150)$$

from which

$$k = \frac{\bar{L}}{(L/D) \int_0^{b/2} l(y)^2 dy}. \quad (151)$$

Random perturbations are then applied at each wing station to generate perturbed drag profiles, $d_n(y)$, for a large number of samples. The magnitude of the noise is scaled such that the variation in integrated drag matches that observed in the filtered test data.

The pitching-moment distribution along the span is computed as

$$m(y) = l(y) [x_{bc} - x_{qc}(y)] + m_{qc}(y), \quad (152)$$

where x_{bc} is the x-location of the balance center, $x_{qc}(y)$ is the x-location of the local quarter chord, and $m_{qc}(y)$ is the aerodynamic moment about the quarter chord at each spanwise station.

Because the local quarter-chord moments $m_{qc}(y)$ are not directly measured, they are assumed to be constant along the span. The constant value is then determined by enforcing the total measured pitching moment from the wind-tunnel balance:

$$\int_0^{b/2} m(y) dy = \bar{M}_y, \quad (153)$$

where $\bar{M}_y$ is the mean measured pitching moment. This ensures that the modeled spanwise pitching-moment distribution is consistent with the experimentally measured total moment, while maintaining a physically reasonable spanwise variation.

As with lift and drag, random perturbations are applied at each wing station to generate perturbed pitching-moment profiles, $m_n(y)$, for a large number of samples. The magnitude of the noise is scaled such that the variation in integrated pitching moment matches that observed in the filtered test data.

The generalized force for the i -th mode is computed by projecting the distributed forces and moments onto the mode shapes along the elastic axis:

$$Q_i = \int_0^{b/2} \left[l_N(y) \phi_{T_3,i}(y) + d_N(y) \phi_{T_1,i}(y) + m_N(y) \phi_{R_2,i}(y) \right] dy \quad (154)$$

where

$$l_N(y) = l_n(y) - l(y) \quad (155)$$

$$d_N(y) = d_n(y) - d(y) \quad (156)$$

$$m_N(y) = m_n(y) - m(y) \quad (157)$$

The z-direction (translational) mode shape is represented by $\phi_{T_3,i}(y)$, the x-direction (translational) mode shape by $\phi_{T_1,i}(y)$, and the y-rotation mode shape by $\phi_{R_2,i}(y)$. These mode shapes are obtained from the NASTRAN finite element model.

Generalized forces are computed for all perturbed aerodynamic profiles across all structural modes. The variance of each mode is then calculated and used to assemble the covariance matrix for the Kalman filter noise model.

B. Estimation

An extended-state Kalman filter is utilized for state and gust estimation for the GLA variant of the Hamiltonian controller. The states in Eqs. 119 and 120 are extended using the disturbance dynamics given in Eq. 89,

$$\dot{x}_e(t) = A_e x_e(t) + B_e u(t) \quad (158)$$

$$y(t) = C_e x_e(t) \quad (159)$$

where

$$x_e(t) = \begin{bmatrix} \bar{x}(t) \\ w(t) \end{bmatrix} \quad (160)$$

$$A_e = \begin{bmatrix} \bar{A} - VK_{\bar{x}} & \bar{E} - VK_w \\ 0 & A_w \end{bmatrix} \quad (161)$$

$$B_e = \begin{bmatrix} \bar{B} \\ 0 \end{bmatrix} \quad (162)$$

$$C_e = \begin{bmatrix} \bar{C} & F \end{bmatrix} \quad (163)$$

The Kalman filter gain L_k is computed utilizing the sensor noise covariance information and a process noise model. The equation for the extended-state Kalman filter is given by,

$$\dot{\hat{x}}_e(t) = A_e \hat{x}_e(t) + B_e u(t) + L_k [y(t) - C_e \hat{x}_e(t)] \quad (164)$$

For the flutter-suppression controller variant, state augmentation is unnecessary, and the estimator reduces to

$$\dot{\hat{x}}(t) = \bar{A}_k \hat{x}(t) + \bar{B}u(t) + L_k [y(t) - \bar{C}\hat{x}(t)] \quad (165)$$

where $\bar{A}_k = \bar{A} - VK_{\bar{x}}$.

C. Stability Analysis and Sensor Selection

To evaluate sensing configurations and quantify closed-loop robustness, the full-order plant, estimator, and control formulations must first be expressed in a unified framework. This section summarizes the equations governing the system dynamics, state estimation, and feedback law, and then develops the closed-loop representation used for gain and phase-margin analysis. The full-order plant model, denoted by the f subscript, is:

$$\dot{x}_f = A_f x_f + B_f u + E_f w, \quad (166)$$

$$y = C_f x_f + F_f w. \quad (167)$$

The Kalman filter estimator evolves as:

$$\dot{\hat{x}}_e = A_e \hat{x}_e + B_e u + L_k (y - C_e \hat{x}_e). \quad (168)$$

The control law is:

$$u = -\tilde{K}_{\bar{x}} \hat{\bar{x}} + \tilde{K}_w \hat{w} \quad (169)$$

$$= -K_{x_e} \hat{x}_e, \quad (170)$$

where

$$K_{x_e} = \begin{bmatrix} \tilde{K}_{\bar{x}} & -\tilde{K}_w \end{bmatrix}. \quad (171)$$

The resulting closed-loop system is:

$$\begin{bmatrix} \dot{\hat{x}}_f \\ \dot{\hat{x}}_e \end{bmatrix} = \begin{bmatrix} A_f & -B_f K_{x_e} \\ L_k C_f & A_e - L_k C_e - B_e K_{x_e} \end{bmatrix} \begin{bmatrix} \hat{x}_f \\ \hat{x}_e \end{bmatrix} + \begin{bmatrix} E_f \\ F_f \end{bmatrix} w. \quad (172)$$

Closed-loop stability margins are computed by injecting gain and phase variations at the plant input. The perturbed closed-loop system matrix is:

$$A_{CL} = \begin{bmatrix} A_f & -k_m B_f K_{x_e} e^{-j\phi_m} \\ L_k C_f & A_e - L_k C_e - B_e K_{x_e} \end{bmatrix}, \quad (173)$$

where k_m is the additional input gain and ϕ_m is the applied phase lag. The perturbed closed-loop system matrix for the flutter suppression controller is given by:

$$A_{CL}^{FS} = \begin{bmatrix} A_f & -k_m B_f \tilde{K}_{\bar{x}} e^{-j\phi_m} \\ L_k C_f & \bar{A}_k - L_k \bar{C} - \bar{B} \tilde{K}_{\bar{x}} \end{bmatrix}, \quad (174)$$

Gain margin is evaluated by setting $\phi_m = 0$ and increasing k_m until an eigenvalue of A_{CL} crosses into the unstable region. Phase margin is evaluated by setting $k_m = 1$ and increasing ϕ_m until instability occurs.

This procedure is used to select a robust sensor set. For GLA, candidate sensors include the five inboard strain gauges and the z -axis accelerometers A104z–A111z (see Fig. 3a), with SG02 removed due to malfunction. A combinatorial search is performed over sensor sets of size four and five, with SG01 required in all sets. The most robust set for GLA is determined to be SG01, SG04, A104z, and A111z. For all GLA configurations, a minimum stability margin of 45° phase margin and 6 dB gain margin is ensured. For comparison, a second set consisting of SG01, SG03, SG04, and SG05 is also tested.

For flutter suppression, the estimator is limited to AC-coupled accelerometers. Accelerometers A111z and A303z were initially used; however, both exhibited anomalous behavior due to flutter ballast and were subsequently replaced with A109z and A110z. The resulting phase and gain margins for the flutter suppression controller across all ASE models are shown in Fig. 9. The red line indicates the approximate flutter suppression test trajectory.

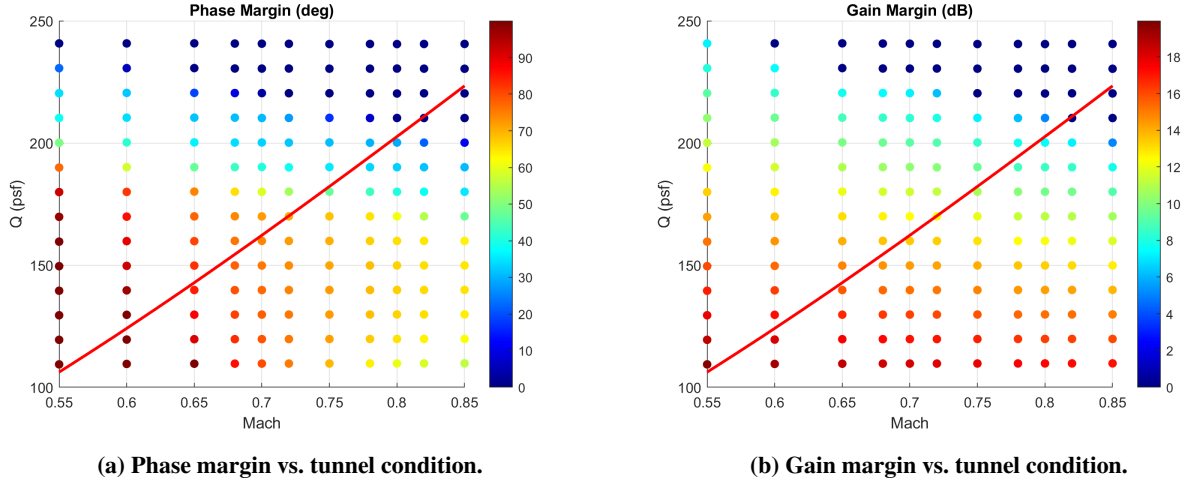


Fig. 9 Closed-loop stability margins for flutter suppression controller

V. Test Results

A. GLA Results

Closed-loop GLA tests were conducted to evaluate the performance of the Hamiltonian controller. Controllers were tested at a tunnel condition of Mach 0.55 and a dynamic pressure of $\bar{q}_\infty = 110$ psf. For the GLA, all three hydraulic ailerons were utilized. The gust vane was driven at frequencies of 0.5 Hz, 1 Hz, 2 Hz, and 3 Hz. The objective of this test series was to assess the controller's effectiveness in attenuating structural response, particularly root strain, due to gust disturbances, and to evaluate the performance of different gust estimation schemes.

Figures 10-12 present representative results for the Hamiltonian controller at 0.5 Hz. This controller also employed the extended-state Kalman filter estimation scheme, with a reduced ramp time for control engagement. Attenuation of the response is evident in both the strain gauge and tip laser displacement signals. The RMS levels of SG01 and laser displacement were reduced by 66.0% and 55.2%, respectively, while control effort remained reasonable.

Two sensor sets were tested: sensor set 1 includes SG01, SG04, A104z, and A111z; sensor set 2 includes SG01, SG03, SG04, and SG05. While results were comparable, sensor set 1 outperformed sensor set 2. A summary of the test results across all gust frequencies and controller configurations is provided in Table 1.

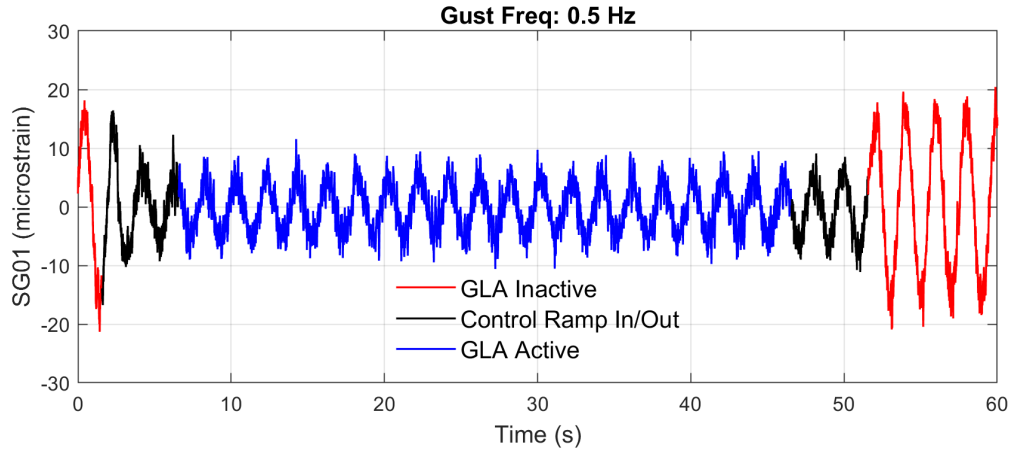


Fig. 10 Root Strain Gauge Measurement for Hamiltonian GLA Controller

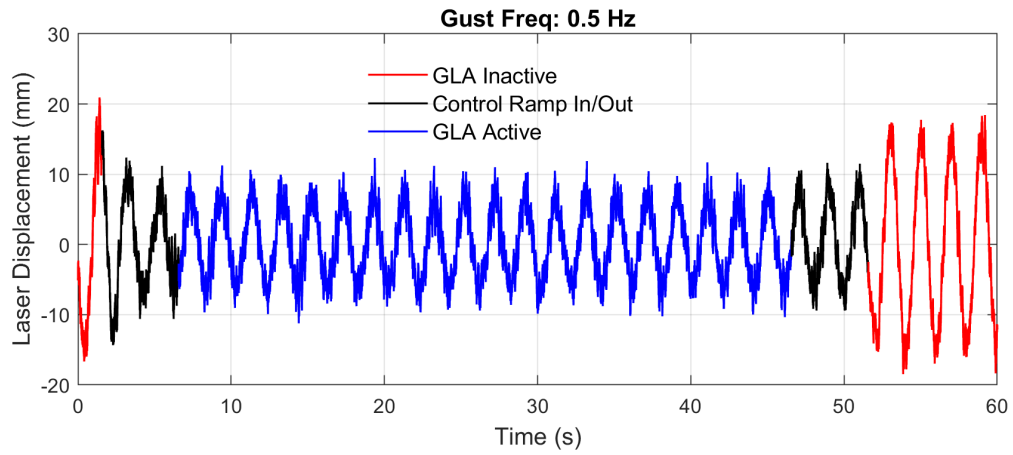


Fig. 11 Tip Laser Displacement Measurement for Hamiltonian GLA Controller

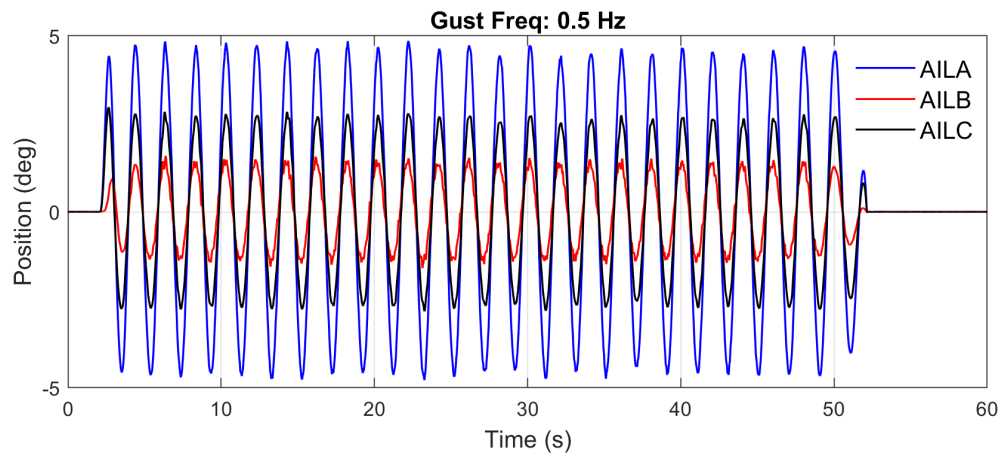


Fig. 12 Control Surface Positions for Hamiltonian GLA Controller

Table 1 GLA Performance Summary Across Gust Frequencies and Sensor Sets

Gust Frequency (Hz)	Sensor Set	%SG01 Reduction	% Laser Disp. Reduction
0.5	Set 1	66.0	55.2
1	Set 1	66.3	56.3
2	Set 1	48.5	40.3
3	Set 1	57.3	55.1
0.5	Set 2	61.8	52.6
1	Set 2	62.9	53.6
2	Set 2	46.4	38.5
3	Set 2	55.1	46.7

B. Flutter Suppression Test Results

Flutter testing began at $M = 0.30$ and proceeded in increments of 0.05 up to $M = 0.65$. Mach 0.65 was the highest condition that remained open-loop stable in the flutter configuration (with tip ballast). Above $M = 0.65$, smaller Mach increments were used to maintain safety margins. At each Mach set point, a doublet input was commanded on aileron B, an otherwise inactive control surface, to excite the flutter mode. Doublet amplitudes of 2° , 3° , and 4° were applied.

Figure 13 shows the tip accelerometer (A111z) response at $M = 0.45$, and Figure 14 shows the corresponding FFT. Moderate attenuation of the accelerometer response is observed in closed loop, particularly in the first mode near 3 Hz. The associated control command is shown in Figure 15.

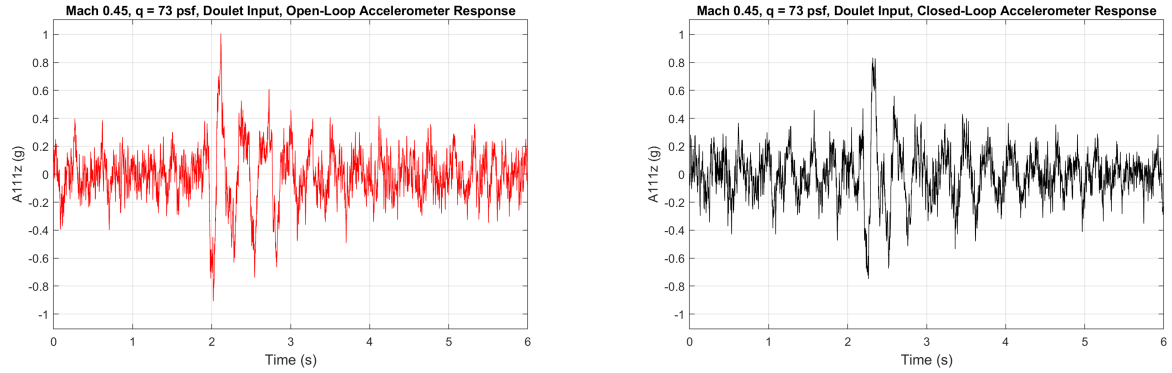


Fig. 13 Tip Accelerometer Response Comparison for Mach 0.45, $q = 73$ psf

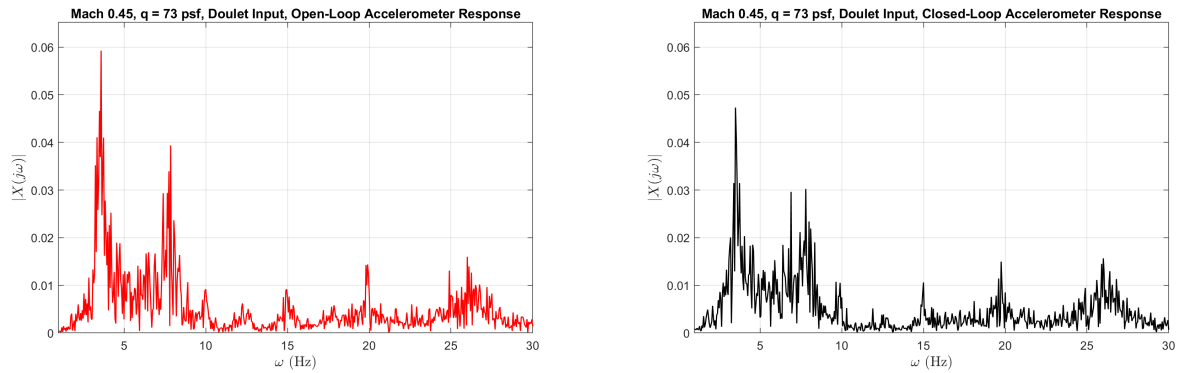


Fig. 14 Tip Accelerometer FFT Comparison for Mach 0.45, $q = 73$ psf

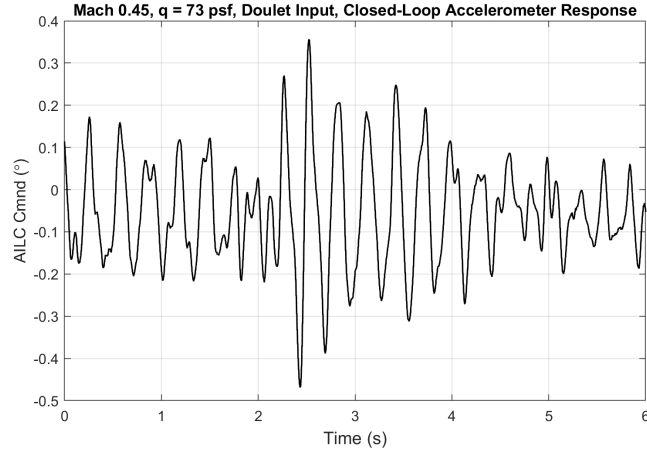


Fig. 15 Control Command at Mach 0.45, $q = 73$ psf

Figure 16 shows the A111z response at $M = 0.55$, with the FFT in Figure 17. Little attenuation is evident, although a slight reduction is visible near the 7.5 Hz mode.

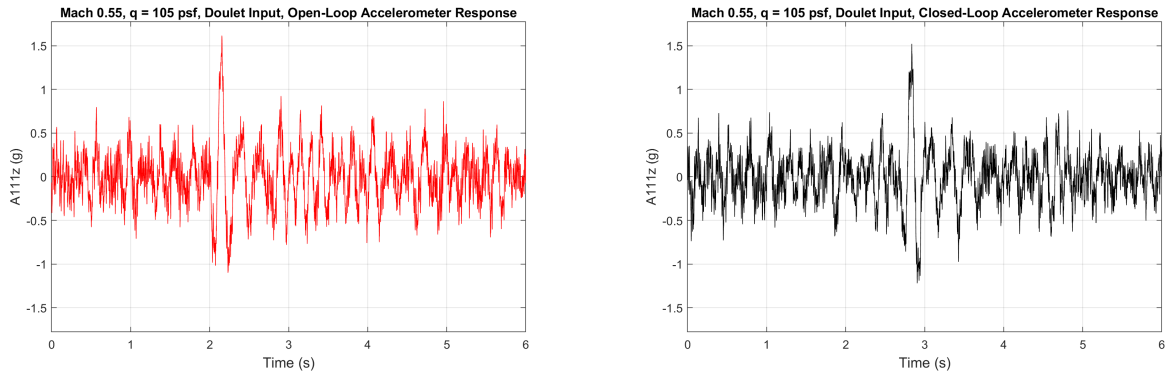


Fig. 16 Tip Accelerometer Response Comparison for Mach 0.55, $q = 105$ psf

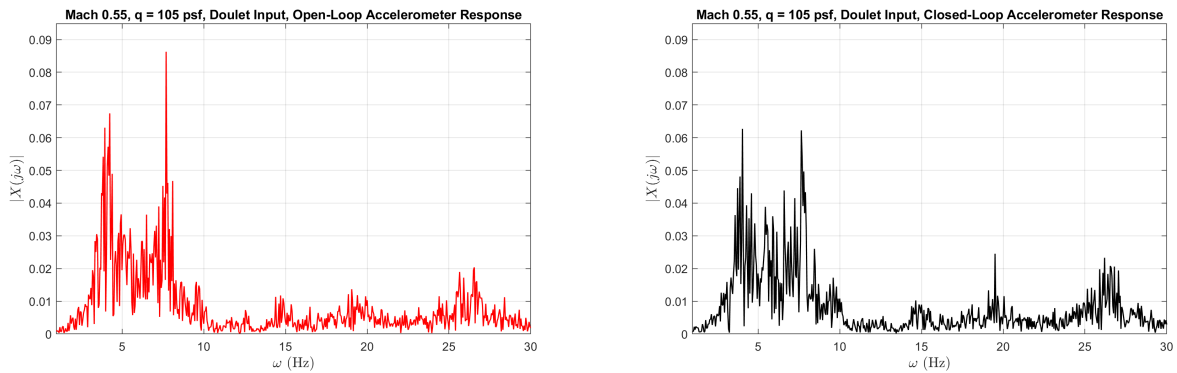


Fig. 17 Tip Accelerometer FFT Comparison for Mach 0.55, $q = 105$ psf

Figure 18 presents the A111z response at $M = 0.65$, with the FFT in Figure 19. Closed-loop attenuation is apparent, most notably in the first mode near 3 Hz. The control command is shown in Figure 20.

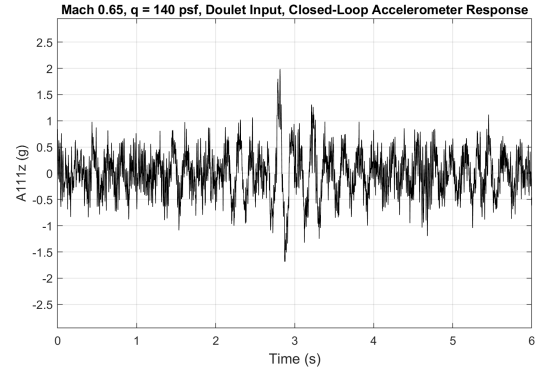
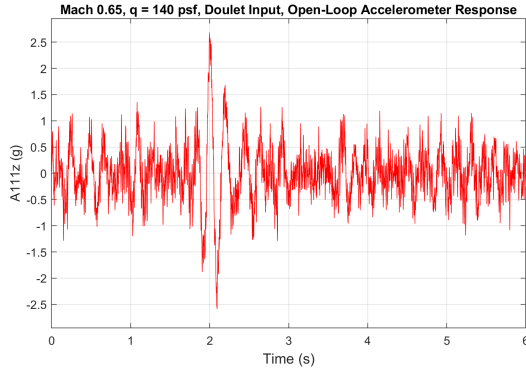


Fig. 18 Tip Accelerometer Response Comparison for Mach 0.65, $q = 140$ psf

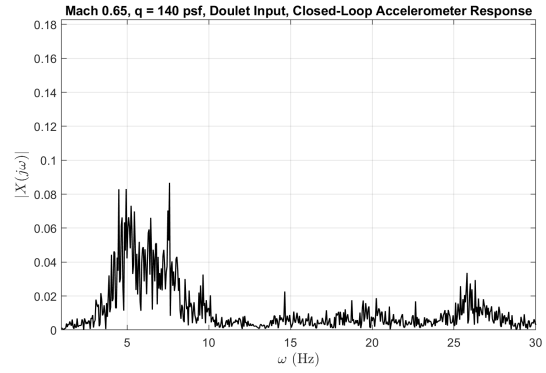
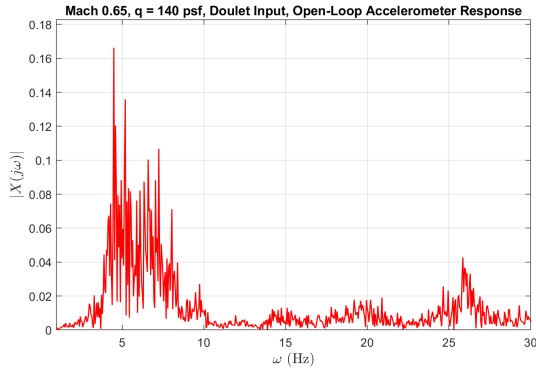


Fig. 19 Tip Accelerometer FFT Comparison for Mach 0.65, $q = 140$ psf

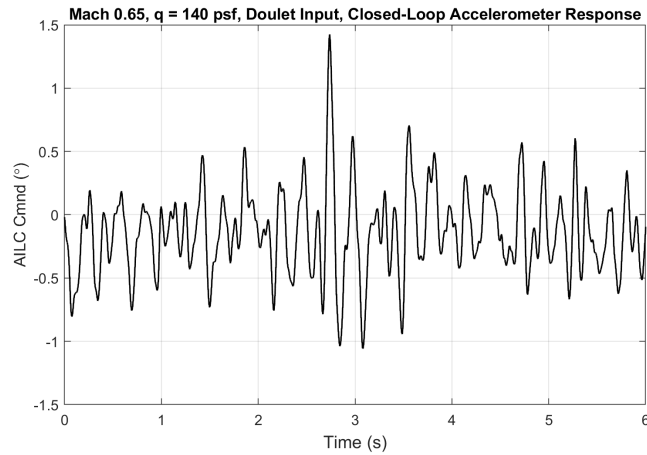
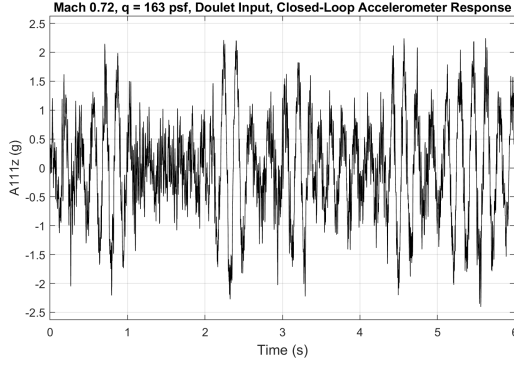
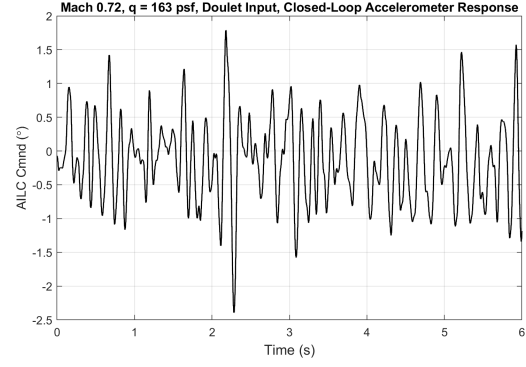


Fig. 20 Control Command at Mach 0.65, $q = 140$ psf

Figure 21 shows the closed-loop accelerometer response and control command at $M = 0.72$, with the FFT shown in Figure 22. Open-loop data were not obtained, but the response shows a marked increase in amplitude, suggesting proximity to instability. Testing was halted to preserve model safety.



(a) Tip Accelerometer Response



(b) Control Command

Fig. 21 Closed-loop flutter suppression results at Mach 0.72, $q = 163$ psf.

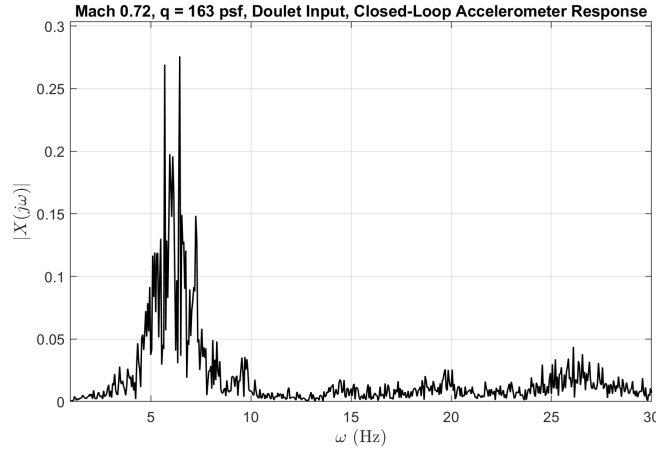


Fig. 22 Tip Accelerometer FFT at Mach 0.72, $q = 163$ psf

VI. Conclusion

This study demonstrates the effectiveness of the Hamiltonian control framework for both gust load alleviation (GLA) and flutter suppression of a flexible aeroelastic model. For GLA, the extended-state Kalman filter combined with strategically selected sensor sets—including strain gauges and z -axis accelerometers—enabled significant attenuation of structural responses to gust disturbances, with RMS reductions of up to 66% in root strain and 55% in tip displacement. Sensor selection was performed via a combinatorial search, ensuring robustness across all gust frequencies and maintaining a minimum stability margin of 45° phase and 6 dB gain margin.

For flutter suppression, control was limited to accelerometers due to their DC-coupled nature. Initial sensors exhibited anomalous behavior due to flutter ballast and were replaced with more reliable alternatives, resulting in clear attenuation of critical flutter modes in closed loop. The controlled system remained stable up to Mach 0.72, demonstrating robust suppression across the test envelope. Controllers were designed for all available ASE models and gain-scheduled with Mach number and dynamic pressure, ensuring stability across all conditions tested.

Overall, the Hamiltonian control approach effectively decouples structural mode suppression from gust load alleviation, providing a systematic framework for designing robust controllers that achieve both noise attenuation and stability across a wide range of aerodynamic conditions. These results highlight the potential of Hamiltonian-based methods for advanced aeroelastic control in both experimental and operational settings.

VII. Acknowledgments

This work was supported by the NASA Advanced Air Transport Technology (AATT) Project and the Advanced Air Vehicles Program (AAVP) as part of a Cooperative Agreement with Boeing. The authors would like to acknowledge significant contributions to this project and the work presented in this paper over the course of many years by team members from Boeing, NextGen Aeronautics, and NASA.

References

- [1] Heaney, P. S., and Quindlen, J. F., "Overview of the Integrated Adaptive Wing Technology Maturation Wind-Tunnel Test Objectives," *AIAA SciTech Forum*, Orlando, FL, 2024. <https://doi.org/https://doi.org/10.2514/6.2024-2509>.
- [2] Nguyen, N., and Xiong, J., "Real-Time Drag Optimization of Aspect Ratio 13.5 Common Research Model with Distributed Flap System," *AIAA SciTech Forum*, 2021. <https://doi.org/https://doi.org/10.2514/6.2021-0069>.
- [3] Rabadan, G. J., Schmitt, N. P., Pistner, T., and Rehm, W., "Airborne Lidar for Automatic Feedforward Control of Turbulent In-Flight Phenomena," *Journal of Aircraft*, Vol. 47, No. 2, 2010. <https://doi.org/https://doi.org/10.2514/1.44950>.
- [4] Zeng, J., Moulin, B., de Callafon, R., and Brenner, M. J., "Adaptive Feedforward Control for Gust Load Alleviation," *Journal of Guidance, Control, and Dynamics*, Vol. 33, No. 3, 2010. <https://doi.org/https://doi.org/10.2514/1.46091>.
- [5] Fezans, N., and Joos, H.-D., "Combined Feedback and LIDAR-Based Feedforward Active Load Alleviation," *AIAA AVIATION Forum*, Denver, CO, 2017. <https://doi.org/https://doi.org/10.2514/6.2017-3548>.
- [6] Giessler, H.-G., Kopf, M., Faulwasser, T., Varutti, P., and Findeisen, R., "Model Predictive Control for Gust Load Alleviation," *4th IFAC Nonlinear Model Predictive Control Conference*, Noordwijkerhout, NL, 2012.
- [7] Nguyen, N. T., Kelley E. Hashemi, E. A., and Yucelen, T., "Model-Reference Adaptive Control of Distributed Lagrangian Infinite-Dimensional Systems Using Hamilton's Principle," *AIAA SciTech Forum*, 2020. <https://doi.org/https://doi.org/10.2514/6.2020-0840>.
- [8] Quindlen, J. F., Ortega, D. M., Clark, G. J., Hussain, H. S., Forte, C. J., Nguyen, N. T., Xiong, J., Heaney, P. S., Bentley, P. J., Coulson, D. A., Giles, I. M., McHugh, G. R., Pinkerton, J. L., Ramey, J. M., and Stevens, W. L., "Summary of the Integrated Adaptive Wing Technology Maturation Wind Tunnel Tests of a High Aspect Ratio Commercial Transport Aircraft," *AIAA SciTech Forum*, Orlando, FL, 2026.
- [9] Waite, J., Bartels, R. E., and Stanford, B., "roelastic Model Development for the Integrated Adaptive Wing Technology Maturation Project Wind-Tunnel Test," *AIAA Aviation Forum*, 2020. <https://doi.org/https://doi.org/https://doi.org/10.2514/6.2020-2717>.
- [10] Nguyen, N., "Distributed Hamiltonian Control of Flexible Aircraft with Adaptive Gust Rejection," *AIAA Scitech Forum*, Orlando, FL, 2024. <https://doi.org/https://doi.org/10.2514/6.2024-1784>.
- [11] Nguyen, N., "Distributed Hamiltonian Adaptive Disturbance Rejection Control with Actuator Dynamics," *AIAA SciTech Forum*, Orlando, FL, 2025. <https://doi.org/https://doi.org/10.2514/6.2025-2801>.