

Design and Experimental Validation of Multi-Objective Gust Load Alleviation Optimal Control for a High Aspect Ratio Flexible Transport Wing

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This paper presents the design and experimental validation of a multi-objective optimal control approach for gust load alleviation on a high-aspect-ratio flexible transport wing. The controller extends a standard LQG framework by directly penalizing wing-root strain in the performance index and is evaluated using a dynamically scaled CRM13 half-span model tested in the NASA Langley Transonic Dynamics Tunnel. State and gust estimation are performed using an extended-state Kalman filter, with additional evaluation of preview-based estimation using an upstream gust sniffer. Wind-tunnel results demonstrate significant attenuation of gust-induced structural response, with wing-root strain and wingtip displacement reductions exceeding 50% and 40%, respectively, across multiple gust frequencies. The onboard-only estimation approach provides the most consistent performance, validating the practicality and effectiveness of multi-objective optimal control for gust load alleviation on highly flexible wings.

I. Introduction

THE Integrated Adaptive Wing Technology Maturation (IAWTM) program represents a multi-year collaboration between Boeing and NASA focused on advancing high-aspect-ratio commercial transport aircraft. As part of this effort, a higher-aspect-ratio version of the NASA Common Research Model (CRM) was developed, increasing the nominal aspect ratio from 9 to 13.5. This configuration, named *CRM13*, continues the trend toward longer, lighter wings. Prior work has established simulation models and control laws for a full-scale CRM13 [1, 2]. The program culminates in wind-tunnel testing at NASA Langley’s Transonic Dynamics Tunnel (TDT) in 2024 and 2025, using a dynamically-scaled, half-span model outfitted with sensors and distributed control surfaces (Fig. 1).

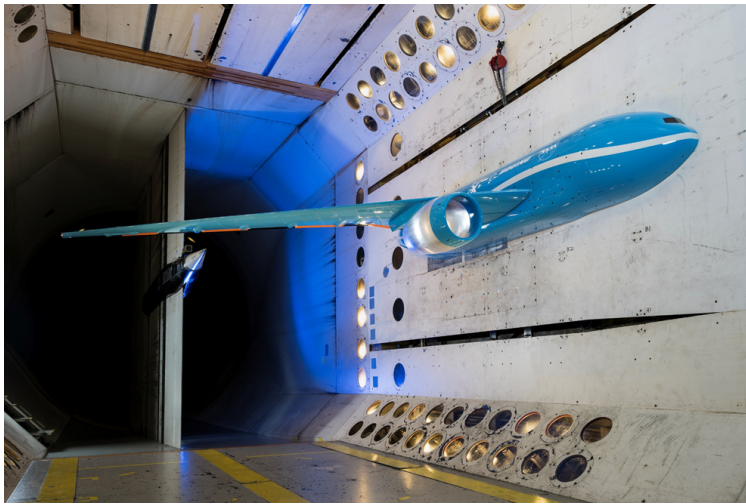


Fig. 1 IAWTM wind-tunnel model in NASA Langley TDT.

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Designing high-aspect-ratio flexible wings requires careful consideration of aeroservoelastic loads, as maneuver and gust loads primarily drive wing structural sizing. While active maneuver load alleviation can reduce maneuver loads, gust loads increasingly limit wing design. Gust load alleviation (GLA) uses active control to mitigate gust-induced loads, lowering structural weight, reducing fatigue, and improving ride comfort. Numerous GLA control strategies have been proposed and studied [3–5].

Recent advances incorporate forward-looking LIDAR sensors that detect upstream wind disturbances, enabling predictive GLA controllers [6–9].

Within IAWTM, Boeing and NASA developed multiple GLA controllers, tested in the TDT during the second entry in 2025. A surrogate LIDAR system in the tunnel supported predictive control. Tested controllers included: a feedforward controller that minimizes inboard wing strains using the surrogate LIDAR, a multi-objective LQG controller combining upstream and onboard feedback sensors, and a Hamiltonian-based GLA controller developed by NASA. All controllers demonstrated reductions in wing root strain (Figs. 6a and 6b).

This paper details the design of a multi-objective optimal control approach for gust load alleviation. The multi-objective approach offers strong potential for structural load reduction [10–12]. In this study, the LQG controller provides a practical evaluation of this concept, using a cost function that extends the standard LQR formulation by adding a weighting on the wing root strain output, combined with a Kalman filter for state estimation.

II. Wind-Tunnel Test Overview

This section briefly describes the NASA TDT and the IAWTM wind-tunnel model. Detailed information is available in [1, 13].

A. Transonic Dynamics Tunnel

The TDT is a $16 \times 16 \text{ ft}^2$ closed-circuit wind tunnel capable of using heavy gas (R134a) to simulate full-scale flight Mach and dynamic pressure conditions. A top view of the tunnel layout is shown in Fig. 2.

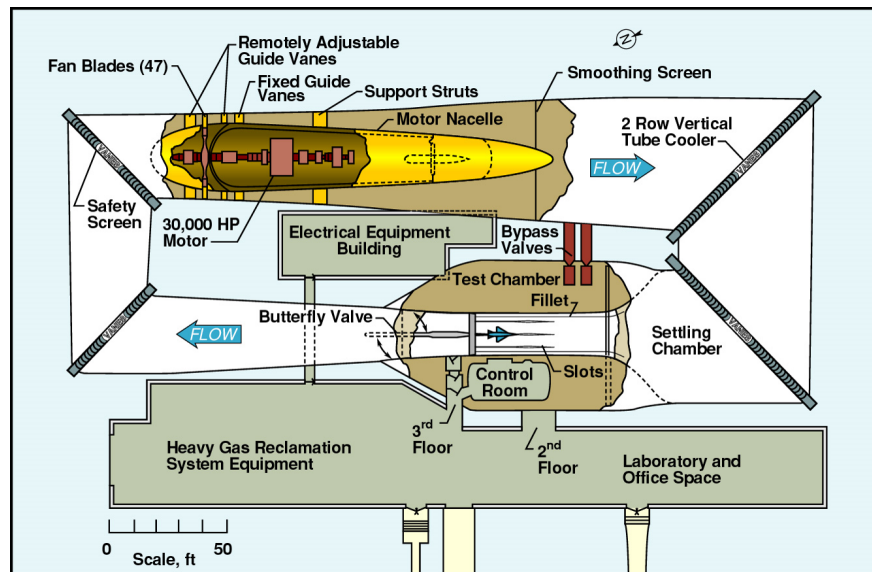


Fig. 2 Top view of NASA TDT.

Critical to GLA testing, the upstream airstream oscillation system (AOS), shown in Fig. 3, uses four vanes to generate controlled flow disturbances. Each pair of vanes can oscillate either in-phase or out-of-phase. For GLA testing, all vanes are operated in-phase. A forward-looking gust probe called the gust sniffer, or *gust sniffer*, is positioned mid-test section to emulate a LIDAR sensor, enabling predictive GLA control. A diagram of the test setup showing the positioning of the AOS vanes, gust sniffer and test article is shown in Fig. 4. Additional sensors measure Mach number, dynamic pressure, and angle-of-attack, and a laser displacement system monitors wingtip motion, providing supplementary performance metrics.

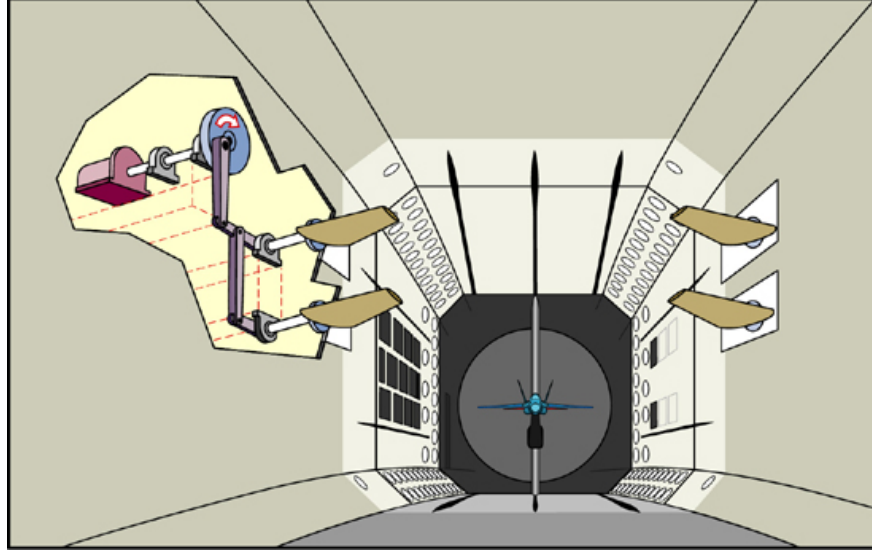


Fig. 3 TDT AOS vanes.

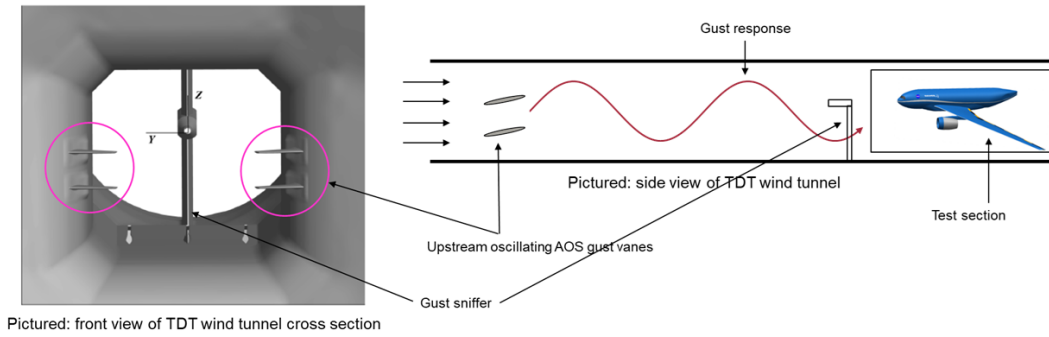


Fig. 4 Position of AOS vanes relative to the gust sniffer and wind-tunnel model.

B. Wind-Tunnel Model

The IAWTM model is a half-span, dynamically-scaled representation of CRM13. The fuselage is shortened to fit the tunnel, and the wing features a flow-through nacelle to model structural inertia effects. The model and its major components are shown in Fig. 5.

The model has 10 control surfaces. Three high-speed, hydraulically actuated ailerons (AILA–AILC) are intended for gust load alleviation and active flutter suppression, while the seven remaining slower-acting electric mini plain flaps (MPFs, MPFA–MPFG) are used in combination with the ailerons for drag minimization and maneuver load alleviation control tasks. The distribution of accelerometers and strain-gages used for performance evaluation is shown in Figs. 6a and 6b. SG01, located near the wing root, serves as the primary GLA performance metric.



Fig. 5 Wind-tunnel model control surfaces and major components.

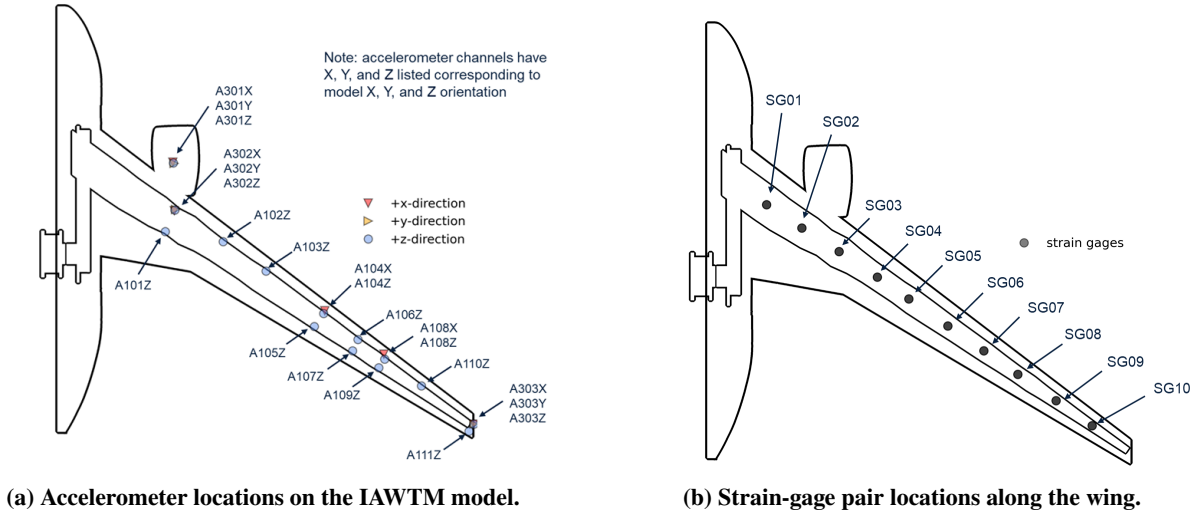


Fig. 6 Sensor locations on the IAWTM model: (a) accelerometers, (b) strain-gages.

III. Controller Design

A. Aeroservoelastic Model

Characterization tests, including four ground vibration tests, are conducted to improve structural models. Following each test, FEM structural models are updated and calibrated to match measured data. Each structural model update is then used to generate a corresponding set of aeroservoelastic (ASE) models in ZAERO [?].

A total of 468 linear state-space ASE models are developed per configuration to cover combinations of Mach number and dynamic pressure. Three model configurations are considered.

The first configuration, shims-off ballast-off (SOB0), is used for drag minimization and maneuver load alleviation testing. In this configuration, a five-component balance provides aerodynamic force and moment measurements.

For flutter suppression and GLA testing, shims are installed to bypass load transmission to the balance and protect it during higher-risk test conditions. The GLA configuration, shims-on ballast-off (S1B0), includes shims without the tip ballast. The tip ballast is installed in the shims-on ballast-on (S1B1) configuration to reduce the dynamic pressure at which flutter occurs and to enable controller evaluation. ASE models from the S0B0 configuration are used for controller design and simulation.

B. Aeroservoelastic Model Reduction

The ASE model comprises hundreds of states, which is excessive for practical control design and motivates model reduction. During GLA testing, the model is excited by gust vanes oscillating sinusoidally at a single frequency, ω_g . Model reduction is performed under the assumption that the steady-state response is sinusoidal at a single frequency. The ASE model is given by:

$$\begin{bmatrix} \dot{x}_s \\ \dot{x}_l \\ \dot{x}_f \end{bmatrix} = \begin{bmatrix} A_{ss} & A_{sl} & 0 \\ A_{ls} & A_{ll} & 0 \\ A_{fs} & A_{fl} & A_{ff} \end{bmatrix} \begin{bmatrix} x_s \\ x_l \\ x_f \end{bmatrix} + \begin{bmatrix} B_{sa} \\ B_{la} \\ B_{fa} \end{bmatrix} x_a + \begin{bmatrix} E_{sw} \\ E_{lw} \\ E_{fw} \end{bmatrix} w \quad (1)$$

$$y = \begin{bmatrix} C_s & C_l & C_f \end{bmatrix} \begin{bmatrix} x_s \\ x_l \\ x_f \end{bmatrix} + D_a x_a + F_w w \quad (2)$$

$$x_s = \begin{bmatrix} q \\ \dot{q} \end{bmatrix} \quad (3)$$

$$x_a = \begin{bmatrix} \delta \\ \dot{\delta} \\ \ddot{\delta} \end{bmatrix} \quad (4)$$

$$w = \begin{bmatrix} \theta_g \\ \dot{\theta}_g \end{bmatrix} \quad (5)$$

The ASE model consists of the structural state vector $x_s(t)$ which includes the generalized structural displacement vector $q(t)$ and generalized rate vector $\dot{q}(t)$ for the first 20 structural modes, the aerodynamic lag state vector $x_l(t)$ which includes 12 lag states per structural mode, the anti-aliasing filter state vector $x_f(t)$, which includes 4 states per output, and gust disturbance vector $w(t)$. The actuator state vector $x_a(t)$, defined by Eq. 4, includes the actuator position $\delta(t)$, the actuator rate $\dot{\delta}(t)$, and actuator acceleration $\ddot{\delta}$. The lag state vector is removed first, with the lag state dynamics given by:

$$\dot{x}_l = A_{ll}x_l + A_{ls}x_s + B_{la}x_a + E_{lw}w \quad (6)$$

Expanding Eq. 6 yields:

$$\dot{x}_l = A_{ll}x_l + A_{lq}q + A_{l\dot{q}}\dot{q} + B_{l\delta}\delta + B_{l\dot{\delta}}\dot{\delta} + B_{l\ddot{\delta}}\ddot{\delta} + E_{l\theta_g}\theta_g + E_{l\dot{\theta}_g}\dot{\theta}_g \quad (7)$$

Using the single frequency sinusoidal steady-state response, these quantities can be written as:

$$x_l = a_1 \sin \omega_g t + a_2 \cos \omega_g t \quad (8)$$

$$q = b_1 \sin \omega_g t + b_2 \cos \omega_g t \quad (9)$$

$$\delta = c_1 \sin \omega_g t + c_2 \cos \omega_g t \quad (10)$$

$$\theta_g = d_1 \sin \omega_g t + d_2 \cos \omega_g t \quad (11)$$

Substituting, Eqs. 8-11 into 7 gives:

$$\begin{aligned}
\dot{x}_l(t) &= a_1 \omega_g \cos \omega_g t - a_2 \omega_g \sin \omega_g t \\
&= A_{ll}(a_1 \sin \omega_g t + a_2 \cos \omega_g t) + A_{lq}(b_1 \sin \omega_g t + b_2 \cos \omega_g t) \\
&\quad + A_{l\dot{q}}(b_1 \omega_g \cos \omega_g t - b_2 \omega_g \sin \omega_g t) + B_{l\delta}(c_1 \sin \omega_g t + c_2 \cos \omega_g t) \\
&\quad + B_{l\dot{\delta}}(c_1 \omega_g \cos \omega_g t - c_2 \omega_g \sin \omega_g t) - B_{l\ddot{\delta}}(c_1 \omega_g^2 \sin \omega_g t + c_2 \omega_g^2 \cos \omega_g t) \\
&\quad + E_{l\theta_g}(d_1 \sin \omega_g t + d_2 \cos \omega_g t) + E_{l\dot{\theta}_g}(d_1 \omega_g \cos \omega_g t - d_2 \omega_g \sin \omega_g t)
\end{aligned} \tag{12}$$

From which the following is written by comparing like terms:

$$-A_{ll}a_1 - a_2 \omega_g = A_{lq}b_1 - A_{l\dot{q}}b_2 \omega_g + B_{l\delta}c_1 - B_{l\dot{\delta}}c_2 \omega_g - B_{l\ddot{\delta}}c_1 \omega_g^2 + E_{l\theta_g}d_1 - E_{l\dot{\theta}_g}d_2 \omega_g \tag{13}$$

$$a_1 \omega_g - A_{ll}a_2 = A_{lq}b_2 + A_{l\dot{q}}b_1 \omega_g + B_{l\delta}c_2 + B_{l\dot{\delta}}c_1 \omega_g - B_{l\ddot{\delta}}c_2 \omega_g^2 + E_{l\theta_g}d_2 + E_{l\dot{\theta}_g}d_1 \omega_g \tag{14}$$

Then, a_1 and a_2 can be computed from this system of equations:

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} -A_{ll} & -\omega_g I \\ \omega_g I & -A_{ll} \end{bmatrix}^{-1} \begin{bmatrix} A_{lq}b_1 - A_{l\dot{q}}b_2 \omega_g + B_{l\delta}c_1 - B_{l\dot{\delta}}c_2 \omega_g - B_{l\ddot{\delta}}c_1 \omega_g^2 + E_{l\theta_g}d_1 - E_{l\dot{\theta}_g}d_2 \omega_g \\ A_{lq}b_2 + A_{l\dot{q}}b_1 \omega_g + B_{l\delta}c_2 + B_{l\dot{\delta}}c_1 \omega_g - B_{l\ddot{\delta}}c_2 \omega_g^2 + E_{l\theta_g}d_2 + E_{l\dot{\theta}_g}d_1 \omega_g \end{bmatrix} \tag{15}$$

This is rewritten as:

$$\begin{aligned}
a_1 &= \alpha_{11}(A_{lq}b_1 - A_{l\dot{q}}b_2 \omega_g + B_{l\delta}c_1 - B_{l\dot{\delta}}c_2 \omega_g - B_{l\ddot{\delta}}c_1 \omega_g^2 + E_{l\theta_g}d_1 - E_{l\dot{\theta}_g}d_2 \omega_g) \\
&\quad + \alpha_{12}(A_{lq}b_2 + A_{l\dot{q}}b_1 \omega_g + B_{l\delta}c_2 + B_{l\dot{\delta}}c_1 \omega_g - B_{l\ddot{\delta}}c_2 \omega_g^2 + E_{l\theta_g}d_2 + E_{l\dot{\theta}_g}d_1 \omega_g)
\end{aligned} \tag{16}$$

$$\begin{aligned}
a_2 &= \alpha_{21}(A_{lq}b_1 - A_{l\dot{q}}b_2 \omega_g + B_{l\delta}c_1 - B_{l\dot{\delta}}c_2 \omega_g - B_{l\ddot{\delta}}c_1 \omega_g^2 + E_{l\theta_g}d_1 - E_{l\dot{\theta}_g}d_2 \omega_g) \\
&\quad + \alpha_{22}(A_{lq}b_2 + A_{l\dot{q}}b_1 \omega_g + B_{l\delta}c_2 + B_{l\dot{\delta}}c_1 \omega_g - B_{l\ddot{\delta}}c_2 \omega_g^2 + E_{l\theta_g}d_2 + E_{l\dot{\theta}_g}d_1 \omega_g)
\end{aligned} \tag{17}$$

where

$$\begin{bmatrix} -A_{ll} & -\omega_g I \\ \omega_g I & -A_{ll} \end{bmatrix}^{-1} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \tag{18}$$

For a matrix of this form $\alpha_{11} = \alpha_{22}$ and $\alpha_{12} = -\alpha_{21}$. Then a_1 and a_2 are rewritten as:

$$\begin{aligned}
a_1 &= \alpha_{11}(A_{lq}b_1 - A_{l\dot{q}}b_2 \omega_g + B_{l\delta}c_1 - B_{l\dot{\delta}}c_2 \omega_g - B_{l\ddot{\delta}}c_1 \omega_g^2 + E_{l\theta_g}d_1 - E_{l\dot{\theta}_g}d_2 \omega_g) \\
&\quad + \alpha_{12}(A_{lq}b_2 + A_{l\dot{q}}b_1 \omega_g + B_{l\delta}c_2 + B_{l\dot{\delta}}c_1 \omega_g - B_{l\ddot{\delta}}c_2 \omega_g^2 + E_{l\theta_g}d_2 + E_{l\dot{\theta}_g}d_1 \omega_g)
\end{aligned} \tag{19}$$

$$\begin{aligned}
a_2 &= -\alpha_{12}(A_{lq}b_1 - A_{l\dot{q}}b_2 \omega_g + B_{l\delta}c_1 - B_{l\dot{\delta}}c_2 \omega_g - B_{l\ddot{\delta}}c_1 \omega_g^2 + E_{l\theta_g}d_1 - E_{l\dot{\theta}_g}d_2 \omega_g) \\
&\quad + \alpha_{11}(A_{lq}b_2 + A_{l\dot{q}}b_1 \omega_g + B_{l\delta}c_2 + B_{l\dot{\delta}}c_1 \omega_g - B_{l\ddot{\delta}}c_2 \omega_g^2 + E_{l\theta_g}d_2 + E_{l\dot{\theta}_g}d_1 \omega_g)
\end{aligned} \tag{20}$$

Then, Eqs. 19 and 20 are substituted into Eq. 8. For the purposes of this derivation, only the terms containing containing b_1 and b_2 are examined; other terms are omitted for clarity:

$$\begin{aligned}
x_l &= \dots + \alpha_{11}A_{lq}b_1 \sin \omega_g t + \alpha_{11}A_{lq}b_2 \cos \omega_g t + \alpha_{11}A_{l\dot{q}}b_1 \omega_g \cos \omega_g t - \alpha_{11}A_{l\dot{q}}b_2 \omega_g \sin \omega_g t \\
&\quad - \alpha_{12}A_{lq}b_1 \cos \omega_g t + \alpha_{12}A_{lq}b_2 \sin \omega_g t + \alpha_{12}A_{l\dot{q}}b_1 \omega_g \sin \omega_g t + \alpha_{12}A_{l\dot{q}}b_2 \omega_g \cos \omega_g t + \dots \\
&= \dots + \alpha_1 b_1 \sin \omega_g t + \alpha_1 b_2 \cos \omega_g t - \alpha_2 b_1 \omega_g \cos \omega_g t + \alpha_2 b_2 \omega_g \sin \omega_g t \\
&\quad - \alpha_3 b_1 \cos \omega_g t + \alpha_3 b_2 \sin \omega_g t + \alpha_4 \omega_g b_1 \sin \omega_g t + \alpha_4 \omega_g b_2 \cos \omega_g t + \dots \\
&= \dots + \alpha_1 q + \alpha_2 \dot{q} + \alpha_3 \ddot{q} + \alpha_4 q + \dots
\end{aligned} \tag{21}$$

where

$$\alpha_1 = \alpha_{11}A_{lq} \tag{22}$$

$$\alpha_2 = -\alpha_{11}A_{l\dot{q}} \tag{23}$$

$$\alpha_3 = \alpha_{12}A_{lq} \quad (24)$$

$$\alpha_4 = -\alpha_{12}A_{l\dot{q}} \quad (25)$$

The final statement can be rewritten as:

$$\begin{aligned} \alpha_1 q + \alpha_2 \dot{q} + \alpha_3 \dot{q} + \alpha_4 q &= \bar{\alpha}_1 q + \bar{\alpha}_2 \dot{q} \\ &= K_1 x_s \end{aligned} \quad (26)$$

where $K_1 = \begin{bmatrix} \bar{\alpha}_1 & \bar{\alpha}_2 \end{bmatrix}$. Differentiating this term gives:

$$\frac{d}{dt}(K_1 x_s) = \bar{\alpha}_1 \dot{q} + \bar{\alpha}_2 \ddot{q} \quad (27)$$

Utilizing $\ddot{q} = -\omega_g^2 q$, which assumes the steady-state response of the model is sinusoidal, Eq. 27 becomes:

$$\begin{aligned} \frac{d}{dt}(K_1 x_s) &= \bar{\alpha}_1 \dot{q} - \bar{\alpha}_2 \omega_g^2 q \\ &= \begin{bmatrix} \bar{\alpha}_1 & \bar{\alpha}_2 \end{bmatrix} \begin{bmatrix} 0 & I \\ -\omega_g^2 I & 0 \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} \\ &= K_1 K_g x_s \end{aligned} \quad (28)$$

where

$$K_g = \begin{bmatrix} 0 & I \\ -\omega_g^2 I & 0 \end{bmatrix} \quad (29)$$

The actuator terms differ slightly. Examining the actuator terms gives:

$$x_l = \dots + \bar{\alpha}_3 \delta + \bar{\alpha}_4 \dot{\delta} + \bar{\alpha}_5 \ddot{\delta} \dots = \dots + K_2 \bar{\delta} + \dots \quad (30)$$

where

$$K_2 = \begin{bmatrix} \bar{\alpha}_3 & \bar{\alpha}_4 & \bar{\alpha}_5 \end{bmatrix} \quad (31)$$

Differentiating this term gives:

$$\frac{d}{dt}(K_2 x_a) = \bar{\alpha}_3 \dot{\delta} + \bar{\alpha}_4 \ddot{\delta} + \bar{\alpha}_5 \ddot{\delta} \quad (32)$$

Utilizing $\ddot{\delta} = -\omega_g^2 \delta$, this becomes:

$$\frac{d}{dt}(K_2 x_a) = \bar{\alpha}_3 \dot{\delta} + \bar{\alpha}_4 \ddot{\delta} - \omega_g^2 \bar{\alpha}_5 \delta = \begin{bmatrix} \bar{\alpha}_3 & \bar{\alpha}_4 & \bar{\alpha}_5 \end{bmatrix} \begin{bmatrix} 0 & I & 0 \\ 0 & 0 & I \\ 0 & -\omega_g^2 I & 0 \end{bmatrix} \begin{bmatrix} \delta \\ \dot{\delta} \\ \ddot{\delta} \end{bmatrix} = K_2 K_{g_a} x_a \quad (33)$$

The gust state vector is handled similarly. Eq. 21 therefore reduces to:

$$x_l = K_1 x_s + K_2 x_a + K_3 w \quad (34)$$

and its time derivative is given by:

$$\dot{x}_l = K_1 K_g x_s + K_2 K_{g_a} x_a + K_3 K_{g_w} w \quad (35)$$

where

$$K_3 = \begin{bmatrix} \bar{\alpha}_6 & \bar{\alpha}_7 \end{bmatrix} \quad (36)$$

$$K_{g_w} = \begin{bmatrix} 0 & I \\ -\omega_g^2 I & 0 \end{bmatrix} \quad (37)$$

Eqs. 34 and 35 are then substituted into the lag state equation given by Eq. 6, yielding:

$$\dot{x}_l = K_1 K_g x_s + K_2 K_{ga} x_a + K_3 K_{gw} w = A_{ll}(K_1 x_s + K_2 x_a + K_3 w) + A_{ls} x_s + B_{la} x_a + E_{lw} w \quad (38)$$

By comparing like terms, the resulting Sylvester equations are obtained from which K_1 , K_2 , and K_3 can be solved:

$$K_1 K_g - A_{ll} K_1 = A_{ls} \quad (39)$$

$$K_2 K_{ga} - A_{ll} K_2 = B_{la} \quad (40)$$

$$K_3 K_{gw} - A_{ll} K_3 = E_{lw} \quad (41)$$

Eq. 34 is substituted into the ASE system in Eqs. 1 and 2, thereby removing the lag states. The reduced system is given by:

$$\begin{bmatrix} \dot{x}_s \\ \dot{x}_f \end{bmatrix} = \begin{bmatrix} A_{ss} + A_{sl} K_1 & 0 \\ A_{fs} + A_{fl} K_1 & A_{ff} \end{bmatrix} \begin{bmatrix} x_s \\ x_f \end{bmatrix} + \begin{bmatrix} B_{sa} + A_{sl} K_2 \\ B_{fa} + A_{fl} K_2 \end{bmatrix} x_a + \begin{bmatrix} E_{sw} + A_{sl} K_3 \\ E_{fw} + A_{fl} K_3 \end{bmatrix} w \quad (42)$$

$$y = \begin{bmatrix} C_s + C_l K_1 & C_f \end{bmatrix} \begin{bmatrix} x_s \\ x_f \end{bmatrix} + (D_a + C_l K_2) x_a + (F_w + C_l K_3) w \quad (43)$$

To remove the anti-aliasing filter states, the same process is followed using the reduced system given by Eqs. 42 and 43. The anti-aliasing filter state vector is written as:

$$x_f = K_4 x_s + K_5 x_a + K_6 w \quad (44)$$

and

$$\dot{x}_f = K_4 K_g x_s + K_5 K_{ga} x_a + K_6 K_{gw} w \quad (45)$$

The anti-aliasing filter dynamic equation is written as:

$$\dot{x}_f = \bar{A}_{fs} x_s + \bar{B}_{fa} x_a + \bar{E}_{fw} w + A_{ff} x_f \quad (46)$$

where,

$$\bar{A}_{fs} = A_{fs} + A_{fl} K_1 \quad (47)$$

$$\bar{B}_{fa} = B_{fa} + A_{fl} K_2 \quad (48)$$

$$\bar{E}_{fw} = E_{fw} + A_{fl} K_3 \quad (49)$$

Substituting Eqs. 44 and 45 into Eq. 46 gives:

$$K_4 K_g x_s + K_5 K_{ga} x_a + K_6 K_{gw} w = \bar{A}_{fs} x_s + \bar{B}_{fa} x_a + \bar{E}_{fw} w + A_{ff} (K_4 x_s + K_5 x_a + K_6 w) \quad (50)$$

By comparing like terms, the resulting Sylvester equations are obtained from which K_4 , K_5 , and K_6 can be solved:

$$K_4 K_g - A_{ff} K_4 = \bar{A}_{fs} \quad (51)$$

$$K_5 K_{ga} - A_{ff} K_5 = \bar{B}_{fa} \quad (52)$$

$$K_6 K_{gw} - A_{ff} K_6 = \bar{E}_{fw} \quad (53)$$

Substituting Eq. 44 into Eqs. 42 and 43 yields the following system with the anti-aliasing filter state vector removed:

$$\dot{x}_s = (A_{ss} + A_{sl} K_1) x_s + (B_{sa} + A_{sl} K_2) x_a + (E_{sw} + A_{sl} K_3) w \quad (54)$$

$$y = (C_s + C_l K_1 + C_f K_4)x_s + (D_a + C_l K_2 + C_f K_5)x_a + (F_w + C_f K_3 + C_f K_6)w \quad (55)$$

Furthermore, the state x_s in the system defined by Eqs. 54 and 55 is partitioned into the x_b which is the state vector for the first 10 structural modes and x_h which is the state vector for the remaining 10 structural modes, enabling the reduction of higher frequency modes that are outside the bandwidth of the actuators. The system is written as:

$$\begin{bmatrix} \dot{x}_b \\ \dot{x}_h \end{bmatrix} = \begin{bmatrix} A_{bb} & A_{bh} \\ A_{hb} & A_{hh} \end{bmatrix} \begin{bmatrix} x_b \\ x_h \end{bmatrix} + \begin{bmatrix} B_b \\ B_h \end{bmatrix} x_a + \begin{bmatrix} E_b \\ E_h \end{bmatrix} w \quad (56)$$

$$y = \begin{bmatrix} C_b & C_h \end{bmatrix} \begin{bmatrix} x_b \\ x_h \end{bmatrix} + (D_a + C_l K_2 + C_f K_5)x_a + (F_w + C_f K_3 + C_f K_6)w \quad (57)$$

Using the same method as before the high frequency structural state vector is written as:

$$x_h = K_7 x_b + K_8 x_a + K_9 w \quad (58)$$

and

$$\dot{x}_h = K_7 K_{gb} x_b + K_8 K_{ga} x_a + K_9 K_{gw} w \quad (59)$$

The high frequency state dynamics are given by:

$$\dot{x}_h = A_{hb} x_b + A_{hh} x_h + B_h x_a + E_h w \quad (60)$$

Substituting Eqs. 58 and 59 into Eq. 60 gives:

$$K_7 K_{gb} x_b + K_8 K_{ga} x_a + K_9 K_{gw} w = A_{hb} x_b + B_h x_a + E_h w + A_{hh} (K_7 x_b + K_8 x_a + K_9 w) \quad (61)$$

By comparing like terms, the resulting Sylvester equations are obtained from which K_7 , K_8 , and K_9 can be solved:

$$K_7 K_{gb} - A_{hh} K_7 = A_{hb} \quad (62)$$

$$K_8 K_{ga} - A_{hh} K_8 = B_h \quad (63)$$

$$K_9 K_{gw} - A_{hh} K_9 = E_h \quad (64)$$

Eq. 58 is then substituted into 56 and 57 thereby removing the high frequency structural states, resulting in the reduced system:

$$\dot{x}_b = (A_{bb} + A_{bh} K_7) x_b + (B_b + B_h K_8) x_a + (E_b + E_h K_9) w \quad (65)$$

$$y = (C_b + C_h K_7) x_b + (D_a + C_l K_2 + C_f K_5 + C_h K_8) x_a + (F_w + C_f K_3 + C_f K_6 + C_h K_9) w \quad (66)$$

C. Actuator Model

A second order Butterworth low-pass filter with a cutoff frequency of 10 Hz is applied to the control command to eliminate high frequency content in the control signal. The 10 Hz cutoff frequency is chosen because the ASE model accuracy decreases at frequencies beyond this bandwidth. The control command low-pass filter dynamics are given by:

$$\dot{x}_{f_l} = A_{f_l} x_{f_l} + B_{f_l} u \quad (67)$$

$$u_f = C_{f_l} x_{f_l} \quad (68)$$

The actuator dynamics are given by the following state-space system:

$$\dot{x}_{act} = A_{act} x_{act} + B_{act} u_f \quad (69)$$

$$x_a = C_{act} x_{act} + D_{act} u_f \quad (70)$$

The actuator dynamics with low-pass filter are given by:

$$\begin{bmatrix} \dot{x}_{act} \\ \dot{x}_{fi} \end{bmatrix} = \begin{bmatrix} A_{act} & B_{act}C_{fi} \\ 0 & A_{fi} \end{bmatrix} \begin{bmatrix} x_{act} \\ x_{fi} \end{bmatrix} + \begin{bmatrix} 0 \\ B_{fi} \end{bmatrix} u \quad (71)$$

$$x_a = \begin{bmatrix} C_{act} & D_{act}C_{fi} \end{bmatrix} \begin{bmatrix} x_{act} \\ x_{fi} \end{bmatrix} \quad (72)$$

Combining the actuator dynamics with low-pass filter and the structural dynamics yields the following state-space system utilized for control law design:

$$\dot{x} = Ax + Bu + Ew \quad (73)$$

$$y = Cx + Fw \quad (74)$$

where

$$x = \begin{bmatrix} x_r \\ x_{act} \\ x_{fi} \end{bmatrix} \quad (75)$$

$$A = \begin{bmatrix} (A_{bb} + A_{bh}K_7) & (B_b + B_hK_8)C_{act} & (B_b + B_hK_8)D_{act}C_{fi} \\ 0 & A_{act} & B_{act}C_{fi} \\ 0 & 0 & A_{fi} \end{bmatrix} \quad (76)$$

$$B = \begin{bmatrix} 0 \\ 0 \\ B_{fi} \end{bmatrix} \quad (77)$$

$$C = \begin{bmatrix} (C_r + C_hK_7) & (D_a + C_lK_2 + C_fK_5 + C_hK_8)C_{act} & (F_w + C_fK_3 + C_fK_6 + C_hK_9)D_{act}C_{fi} \end{bmatrix} \quad (78)$$

$$E = \begin{bmatrix} (E_r + E_hK_9) \\ 0 \\ 0 \end{bmatrix} \quad (79)$$

$$F = (F_w + C_hK_9) \quad (80)$$

D. Gust Disturbance Modeling

The gust vane dynamics are modeled as follows:

$$\dot{x}_v(t) = A_v x_v(t) + B_v u_v(t) \quad (81)$$

where $u_v(t)$ is the commanded gust vane position and $x_v(t)$ is the vector containing the gust vane position $\theta_v(t)$, the gust vane velocity $\dot{\theta}_v(t)$, and the gust vane acceleration $\ddot{\theta}_v(t)$ which is given by:

$$x_v(t) = \begin{bmatrix} \theta_v(t) \\ \dot{\theta}_v(t) \\ \ddot{\theta}_v(t) \end{bmatrix} \quad (82)$$

This can be expressed in transfer function form:

$$X_v(s) = G_1(s)U_v(s) \quad (83)$$

$$G_1(s) = (sI - A_v)^{-1}B_v \quad (84)$$

The output of the gust vane model contains the gust vane position and velocity and is expressed as:

$$y_v(t) = C_v x_v(t) \quad (85)$$

where

$$C_v = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad (86)$$

The disturbance state $w(t)$ represents the gust vane position and velocity delayed by the convective time delay between the gust vanes and the model which is given by:

$$w(t) = y_v(t - t_d) \quad (87)$$

The gust vanes move sinusoidally at a known frequency ω_g and amplitude θ_0 . Thus, $\theta_g(t)$ written as:

$$\theta_g(t) = \theta_0 \sin(\omega_g t + \phi) \quad (88)$$

The dynamics for gust disturbance $w(t)$ can therefore be written as:

$$\dot{w}(t) = A_w w(t) \quad (89)$$

where

$$A_w = \begin{bmatrix} 0 & 1 \\ -\omega_g^2 & 0 \end{bmatrix} \quad (90)$$

E. Multi-Objective Optimal Control

The control is derived using the following multi-objective cost function:

$$J = \lim_{t_f \rightarrow \infty} \frac{1}{2} \int_0^{t_f} [x^\top(t) Q x(t) + u^\top(t) R u(t) + M^\top(t) Q_M M(t)] dt \quad (91)$$

The last term in the integral is an addition to the standard LQR cost function and penalizes the response in the wing-root strain sensor. The wing-root strain sensor output is given by

$$M(t) = M_x x(t) + M_w w(t) \quad (92)$$

The Hamiltonian is formed:

$$H(t) = \frac{1}{2} x^\top(t) Q x(t) + \frac{1}{2} u^\top(t) R u(t) + \frac{1}{2} (x^\top(t) M_x^\top + w^\top(t) M_w^\top) Q_M (M_x x(t) + M_w w(t)) - \lambda^\top(t) [A(t) x(t) + B u(t) + E w(t)] \quad (93)$$

The conditions for optimality are:

$$\dot{\lambda}(t) = -\frac{\partial H(t)}{\partial x(t)}^\top = -Q x(t) - M_x^\top Q_M (M_x x(t) + M_w w(t)) - A^\top \lambda(t) \quad (94)$$

$$\frac{\partial H(t)}{\partial u(t)}^\top = R u(t) + B^\top \lambda(t) = 0 \quad (95)$$

The optimal control is given by

$$u(t) = -R^{-1} B^\top \lambda(t) \quad (96)$$

Using sweep method to solve for the optimal control, the adjoint solution is given by

$$\lambda(t) = S x(t) + P w(t) \quad (97)$$

Differentiating gives,

$$\dot{\lambda}(t) = S \dot{x}(t) + P \dot{w}(t) = -Q x(t) - M_x^\top Q_M (M_x x(t) + M_w w(t)) - A^\top \lambda(t) \quad (98)$$

Substituting the plant dynamics, control and adjoint equation gives,

$$S[Ax(t) - BR^{-1}B^\top(Sx(t) + Pw(t)) + Ew(t)] + P\dot{w}(t) = -Qx(t) - M_x^\top Q_M [M_x x(t) + M_w w(t)] - A^\top(Sx(t) + Pw(t)) \quad (99)$$

This yields the modified Riccati equation

$$SA + A^\top S - SBR^{-1}B^\top S + \bar{Q} = 0 \quad (100)$$

and

$$(A^\top - SBR^{-1}B^\top)Pw(t) + P\dot{w}(t) + Q_w w(t) = 0 \quad (101)$$

where

$$\bar{Q} = Q + M_x^\top Q_M M_x \quad (102)$$

$$Q_w = M_x^\top Q_M M_w + SE \quad (103)$$

However, $\dot{w} = A_w w$ is substituted into Eq. 101, which gives the Sylvester equation:

$$(A^\top - SBR^{-1}B^\top)P + PA_w + Q_w = 0 \quad (104)$$

S and P are solved from Riccati equation 100 and Sylvester equation 104. The optimal control is given by

$$u(t) = -R^{-1}B^\top(Sx(t) + Pw(t)) = -K_x x(t) + K_w w(t) \quad (105)$$

where

$$K_x = R^{-1}B^\top S \quad (106)$$

$$K_w = -R^{-1}B^\top P \quad (107)$$

IV. Estimation

A. Process Noise Modeling

The Kalman filter gain, L_k , requires specification of both measurement and process noise covariances. While the measurement noise covariance is determined directly from ambient sensor data, estimating the process noise covariance is more involved. In this work, the process noise arises from unsteady aerodynamic forces induced by wind-tunnel turbulence. Accordingly, a process noise model is constructed using unsteady lift, pitching-moment, and drag measurements.

The measured aerodynamic data include both structural responses and unsteady aerodynamic content. To isolate the latter, notch filters are applied to remove dominant structural modes. A second-order band-stop filter is used:

$$G_n(s) = \frac{s^2 + 2(1 - \alpha)\omega_c s + \omega_c^2}{(s + \omega_c)^2}, \quad (108)$$

where ω_c is the cutoff frequency and α defines the notch depth. These parameters are chosen to suppress the structural peaks identified in the FFT of the test data. Figure 7 illustrates the effect of notch filtering on the lift FFT; the same procedure is applied to drag and pitching-moment data.

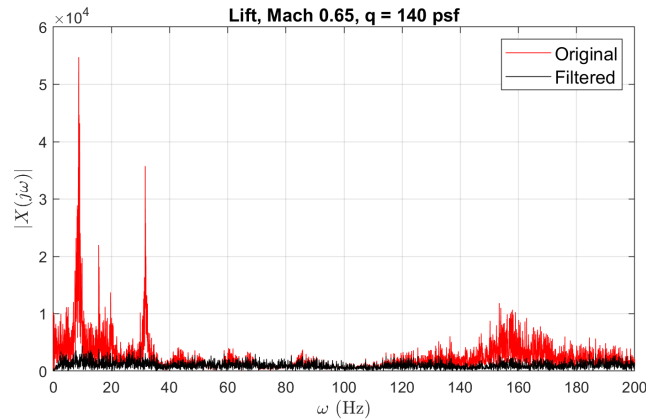


Fig. 7 Lift FFT showing original (red) and notch-filtered (black) spectra.

A nominal elliptical lift distribution is defined such that its spanwise integral matches the measured mean lift:

$$\int_0^{b/2} l(y) dy = \bar{L}. \quad (109)$$

Random perturbations are then applied at each spanwise station to generate a set of perturbed lift profiles $l_n(y)$ (Fig. 8). The magnitude of these perturbations is scaled so that the variation in integrated lift matches the filtered test data.

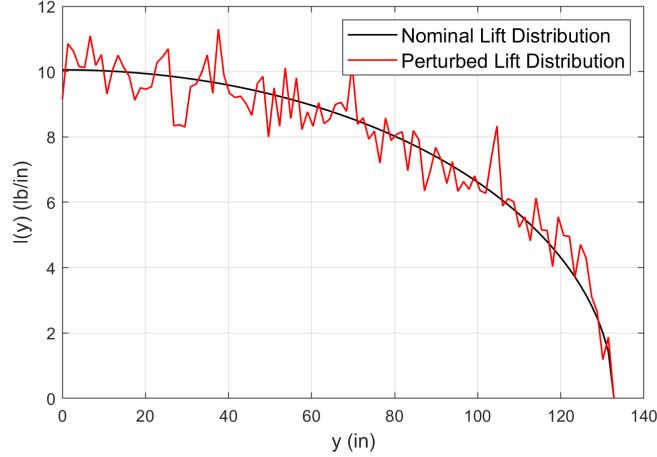


Fig. 8 Nominal and perturbed lift distributions along the wing span.

The drag distribution is modeled as a parabolic function along the semispan:

$$d(y) = k L(y)^2, \quad (110)$$

with the coefficient k determined from the measured lift-to-drag ratio:

$$k = \frac{L_{\text{tot}}}{(L/D) \int_0^{b/2} L(y)^2 dy}. \quad (111)$$

Perturbed drag profiles $d_n(y)$ are generated similarly, with noise scaled to match the observed variation in total drag.

The pitching-moment distribution along the span is expressed as:

$$m(y) = l(y) [x_{bc} - x_{qc}(y)] + m_{qc}(y), \quad (112)$$

where x_{bc} is the balance center location, $x_{qc}(y)$ the local quarter-chord, and $m_{qc}(y)$ the local aerodynamic moment about the quarter chord. Since $m_{qc}(y)$ is not directly measured, it is assumed constant and determined by enforcing the total measured pitching moment:

$$\int_0^{b/2} m(y) dy = \bar{M}_y. \quad (113)$$

Perturbed pitching-moment profiles $m_n(y)$ are generated similarly, with noise scaled to match the measured variation.

For each structural mode, the generalized force is obtained by projecting the perturbed aerodynamic distributions onto the mode shapes along the elastic axis:

$$Q_i = \int_0^{b/2} \left[l_N(y) \phi_{T_3,i}(y) + d_N(y) \phi_{T_1,i}(y) + m_N(y) \phi_{R_2,i}(y) \right] dy, \quad (114)$$

where

$$l_N(y) = l_n(y) - l(y), \quad (115)$$

$$d_N(y) = d_n(y) - d(y), \quad (116)$$

$$m_N(y) = m_n(y) - m(y). \quad (117)$$

Here, $\phi_{T_{3,i}}(y)$, $\phi_{T_{1,i}}(y)$, and $\phi_{R_{2,i}}(y)$ are the z-translation, x-translation, and y-rotation mode shapes, respectively, obtained from the NASTRAN finite element model. The generalized forces are computed for all perturbed profiles and structural modes, and their variances are used to assemble the process noise covariance matrix for the Kalman filter.

B. State and Gust Estimation

Two approaches are utilized for state and gust estimation. The first employs an extended-state Kalman filter and the second utilizes the upstream gust sniffer to estimate the disturbance. The state vector in Eqs. 73 and 74 are extended using the disturbance dynamics given in Eq. 89:

$$\dot{x}_e(t) = A_e x_e(t) + B_e u(t) \quad (118)$$

$$y(t) = C_e x_e(t) \quad (119)$$

where

$$x_e(t) = \begin{bmatrix} x(t) \\ w(t) \end{bmatrix} \quad (120)$$

$$A_e = \begin{bmatrix} A & E \\ 0 & A_w \end{bmatrix} \quad (121)$$

$$B_e = \begin{bmatrix} B \\ 0 \end{bmatrix} \quad (122)$$

$$C_e = \begin{bmatrix} C & F \end{bmatrix} \quad (123)$$

The Kalman filter gain L_k is computed utilizing the sensor noise covariance information and a process noise model[14]. The equation for the extended-state Kalman filter is given by:

$$\dot{\hat{x}}_e = A_e \hat{x}_e(t) + B_e u(t) + L_k (y(t) - C_e \hat{x}_e(t)) \quad (124)$$

Note that $\hat{x}_e$ contains both state and gust estimates that feed into the control law. This method uses onboard accelerometers and strain gauges (without the gust sniffer). Inboard strain gauges and outboard accelerometers are considered for the Kalman filter sensor set.

In contrast, the second approach incorporates the upstream gust sniffer as a preview measurement to support reconstruction of the gust state. The gust sniffer is mounted upstream of the model and measures the local flow angle. Its dynamics are modeled as:

$$\dot{x}_{sn}(t) = A_{sn} x_{sn}(t) + B_{sn} x_v(t - t_g), \quad (125)$$

$$y_{sn}(t) = C_{sn} x_{sn}(t), \quad (126)$$

or equivalently in the Laplace domain:

$$Y_{sn}(s) = G_2(s) X_v(s) e^{-st_g}, \quad (127)$$

where

$$G_2(s) = C_{sn}(sI - A_{sn})^{-1} B_{sn}. \quad (128)$$

Because the gust sniffer measurement contain sensor noise, two filtering techniques are employed. The first uses a Kalman filter based on a sinusoidal model of the flow angle measured by the gust sniffer at the gust frequency ω_g :

$$\dot{x}_{snf}(t) = A_{snf} x_{snf}(t) \quad (129)$$

$$y_{sn}(t) = C_{snf} x_{snf}(t) \quad (130)$$

where

$$A_{snf} = \begin{bmatrix} 0 & 1 \\ -\omega_g^2 & 0 \end{bmatrix}, \quad (131)$$

$$x_{snf}(t) = \begin{bmatrix} \theta_{sn}(t) \\ \dot{\theta}_{sn}(t) \end{bmatrix} \quad (132)$$

$$C_{snf} = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad (133)$$

Let L_{snf} denote the Kalman filter gain. The resulting estimator dynamics are:

$$\dot{\hat{x}}_{snf}(t) = A_{snf}\hat{x}_{snf}(t) + L_{snf}(y_{sn}(t) - C_{snf}\hat{x}_{snf}(t)) \quad (134)$$

with transfer-function representation:

$$\hat{Y}_{sn,f}(s) = G_{3k}(s)Y_{sn}(s) \quad (135)$$

where

$$G_{3k}(s) = C_{snf}(sI - A_{snf} + L_{snf}C_{snf})^{-1}L_{snf} \quad (136)$$

A conventional low-pass filter is also used for noise reduction. An eight-pole Butterworth low-pass filter with a 4.5 Hz cutoff frequency is employed. The cutoff is chosen to be marginally higher than the maximum planned gust-excitation frequency of 4 Hz, thereby attenuating noise while retaining the relevant gust content. Its dynamics are expressed as:

$$Y_{sn,f}(s) = G_{3lp}(s)Y_{sn}(s). \quad (137)$$

The overall relationship between the gust vane command $U_v(s)$ and the filtered sensor output $\hat{Y}_{sn,f}(s)$ is:

$$\begin{aligned} \hat{Y}_{sn,f}(s) &= G_3(s)G_2(s)G_1(s)U_v(s)e^{-st_g} \\ &= G(s)U_v(s)e^{-st_g} \end{aligned} \quad (138)$$

where $G(s) = G_3(s)G_2(s)G_1(s)$ represents the transfer function of the combined system without the delay. Note that when utilizing the Kalman filter for gust sniffer noise filtering $G_3(s) = G_{3k}(s)$ and $G_3(s) = G_{3lp}(s)$ when the low-pass filter is used. For a sinusoidal vane command $u_v(t) = u_v \cos \omega_g t$ the filtered sniffer output is:

$$\hat{y}_{sn,f}(t) = |G(j\omega_g)|u_v \left(t + \frac{\angle G(j\omega_g)}{\omega_g} - t_g \right) \quad (139)$$

Hence the gust vane position command can be recovered as:

$$u_v(t) = \frac{1}{|G(j\omega_g)|} \hat{y}_{sn,f} \left(t - \frac{\angle G(j\omega_g)}{\omega_g} + t_g \right) \quad (140)$$

With $u_v(t)$ it is then possible to compute $x_v(t)$ using Eq. 81. Then the $x_v(t)$ is delayed by t_d which is the convective time delay between the gust vanes and the model which provides $w(t)$. Because the gust vane states are not directly measured but reconstructed from the filtered sensor output using the known transfer functions and delay compensations, the resulting signals represent the estimated gust quantities. A Kalman filter is also utilized in the second scheme for state estimation, although the state is not extended and the gust estimate from the sniffer estimation is treated as an known input to the Kalman filter. This system is given by:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L[y(t) - (C\hat{x}(t) + F\hat{w}(t))] + E\hat{w}(t) \quad (141)$$

C. Stability Margin Analysis and Sensor Selection

To evaluate sensing configurations and quantify closed-loop robustness, the full-order plant, estimator, and control formulations must first be expressed in a unified framework. This section summarizes the equations governing the system dynamics, state estimation, and feedback law, and then develops the closed-loop representation used for gain and phase-margin analysis. The full-order plant model, denoted by the f subscript, is:

$$\dot{x}_f = A_f x_f + B_f u + E_f w, \quad (142)$$

$$y = C_f x_f + F_f w. \quad (143)$$

The extended-state Kalman filter estimator evolves as:

$$\dot{\hat{x}}_e = A_e \hat{x}_e + B_e u + L_k (y - C_e \hat{x}_e). \quad (144)$$

The control law is:

$$u = -K_x \hat{x} + K_w w \quad (145)$$

$$= -K_{x_e} x_e, \quad (146)$$

where

$$K_{x_e} = \begin{bmatrix} K_x & -K_w \end{bmatrix}. \quad (147)$$

The resulting closed-loop system is:

$$\begin{bmatrix} \dot{x}_f \\ \dot{\hat{x}}_e \end{bmatrix} = \begin{bmatrix} A_f & -B_f K_x \\ L_k C_f & A - L_k C - B K_x \end{bmatrix} \begin{bmatrix} x_f \\ \hat{x}_e \end{bmatrix} + \begin{bmatrix} E_f \\ F_f \end{bmatrix} w. \quad (148)$$

Closed-loop stability margins are computed by injecting gain and phase variations at the plant input. The perturbed closed-loop matrix is:

$$A_{CL} = \begin{bmatrix} A_f & -k_m B_f K_{x_e} e^{-j\phi_m} \\ L_k C_f & A_e - L_k C_e - B_e K_{x_e} \end{bmatrix}, \quad (149)$$

where k_m is the additional input gain and ϕ_m is the applied phase lag.

For the gust sniffer estimation, the Kalman filter is given by Eq. 141. Note the state is not extended for this configuration and the closed loop stability is no longer tied to the estimation of the gust $w(t)$. The perturbed closed-loop matrix is given by:

$$A_{CL}^{gs} = \begin{bmatrix} A_f & -k_m B_f \tilde{K}_{\tilde{x}} e^{-j\phi_m} \\ L_k C_f & \tilde{A}_k - L_k \tilde{C} - \tilde{B} \tilde{K}_{\tilde{x}} \end{bmatrix}, \quad (150)$$

Gain margin is evaluated by setting $\phi_m = 0$ and increasing k_m until an eigenvalue of A_{CL} crosses into the unstable region. Phase margin is evaluated by setting $k_m = 0$ and increasing ϕ_m until instability occurs.

This procedure is used to select a robust sensor set. For GLA, candidate sensors include the five inboard strain gauges and the z -axis accelerometers A104z–A111z (see Fig. 6a), with SG02 removed due to malfunction. A combinatorial search is performed over sensor sets of size four and five, with SG01 required in all sets. The most robust set for GLA is determined to be SG01, SG04, A104z, and A111z. For all GLA configurations, a minimum stability margin of 45° phase margin and 6 dB gain margin is ensured.

V. Wind Tunnel Test Results

Closed-loop GLA tests were conducted to evaluate the performance of the Linear Quadratic Gaussian (LQG) controller. The controllers were tested at the tunnel condition of Mach 0.55 and a dynamic pressure of $\bar{q}_\infty = 110$ psf. The gust vane was driven at frequencies of 0.5 Hz, 1 Hz, 2 Hz and 3 Hz. The objective of this test series was to assess the controller effectiveness in attenuating structural response, particularly root strain, due to the gust disturbance, as well as to evaluate the performance of different gust estimation schemes.

Fig. 9 presents the performance of the gust estimation schemes when the controller was inactive but the estimation was active. Three estimation configurations were examined. The first, labeled Kalman Filter Only, utilized an extended-state Kalman filter (ESKF) based solely on onboard sensor measurements, without input from the gust sniffer. The second, labeled Gust Sniffer Est. 1, incorporated the gust sniffer signal described in Section IV.B, with a Kalman filter applied to reduce the effect of measurement noise in the gust sniffer sensor. The third, Gust Sniffer Est. 2, used the same gust sniffer signal but applied a Butterworth low-pass filter instead of a Kalman filter for noise reduction in the gust sniffer sensor. All three estimation schemes were able to track the gust vane displacement effectively; however, the onboard-only estimation performed slightly better, while the Gust Sniffer Est. 2 approach appeared more susceptible to sensor noise.

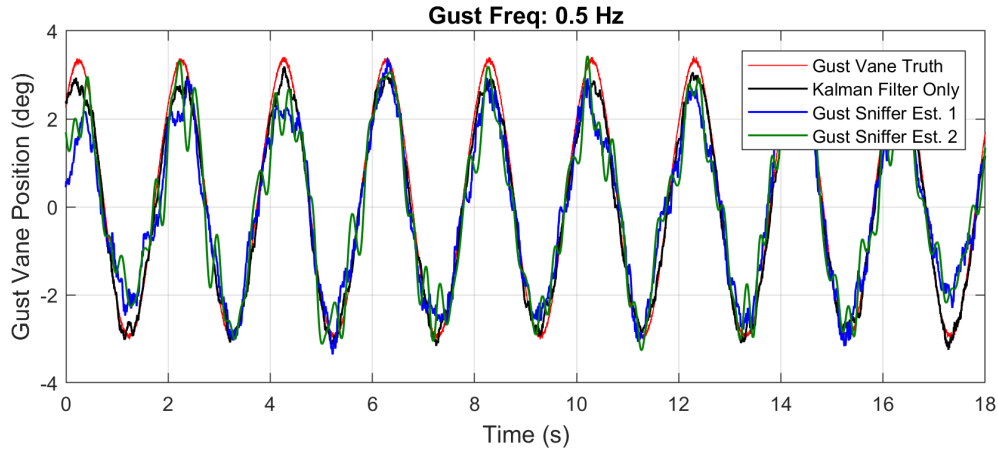


Fig. 9 Gust Estimation Performance for 0.5 Hz AOA Perturbation

Figs. 10-12 show representative test results for the LQG controller at 0.5 Hz gust excitation. The plots include the root strain gauge (SG01) measurement, laser displacement measurement, and control surface position. The controller employed the extended-state Kalman filter estimation scheme. During the test, the controller engagement was ramped in and out over a 20-second period to minimize transient loads during initial closed-loop testing. Subsequent tests utilized shorter ramp times once confidence in system stability increased. For this case, the response attenuation is visible in both SG01 and laser displacement measurements. The root-mean-square (RMS) values of SG01 and laser displacement were reduced by 53.5% and 41.0%, respectively, while the control surface activity remained within acceptable limits.

Fig. 13 shows the strain-gauge signal estimation for the sensors used by the Kalman filter. The measured sensor output is shown in red, while the corresponding state estimate is shown in black. The estimate for SG04 closely tracks the measured response, whereas the estimation for SG01 underestimates the magnitude of the strain response. Fig. 14 presents the estimation results for the accelerometers included in the Kalman filter. These measurements are subject to substantial noise, which obscures clear response trends. Nevertheless, the Kalman filter provides noticeable noise attenuation when compared to the raw measurements.

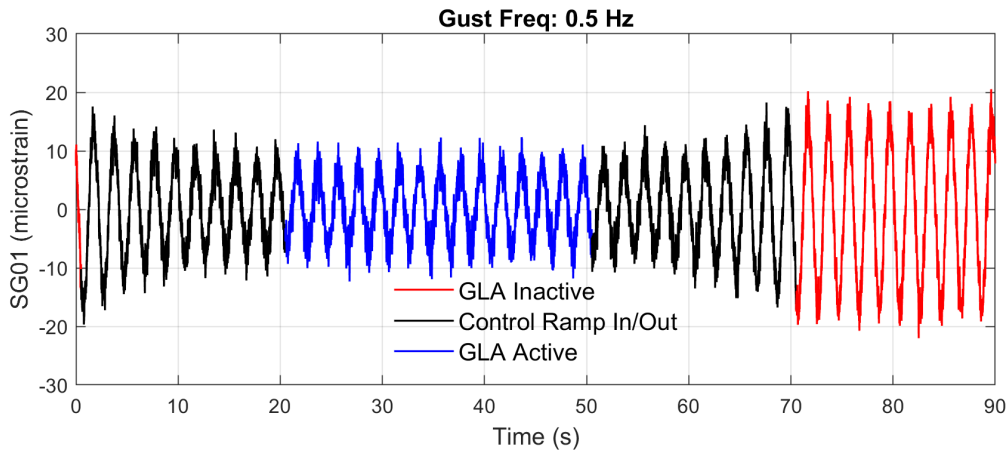


Fig. 10 Root Strain Gauge Measurement for LQG GLA Controller

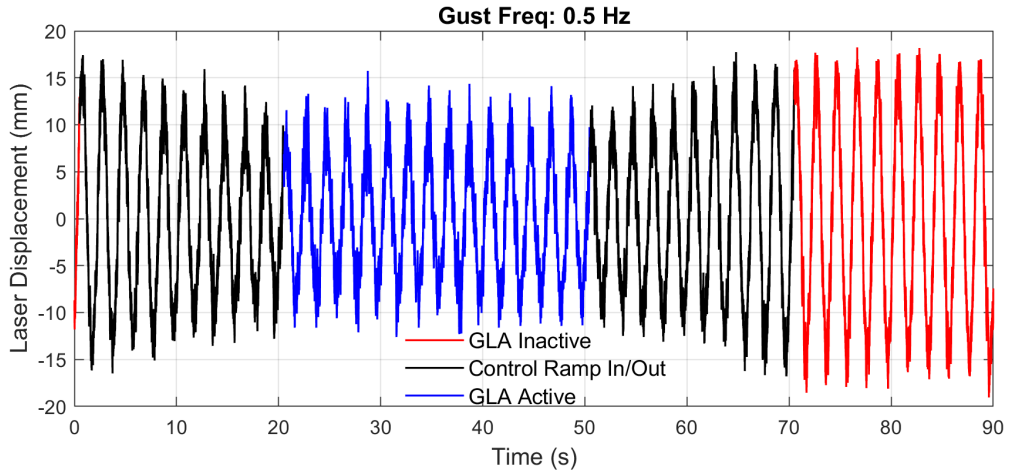


Fig. 11 Tip Laser Displacement Measurement for LQG GLA Controller

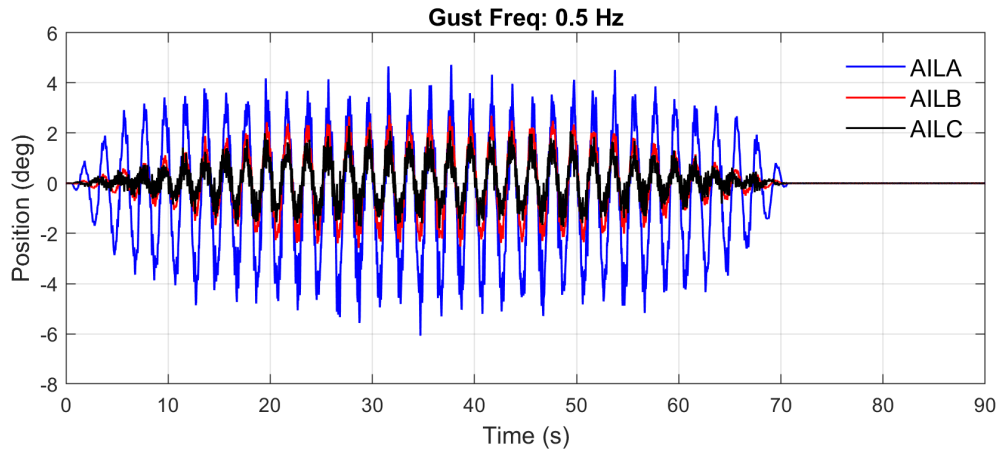


Fig. 12 Control Surface Positions for LQG GLA Controller

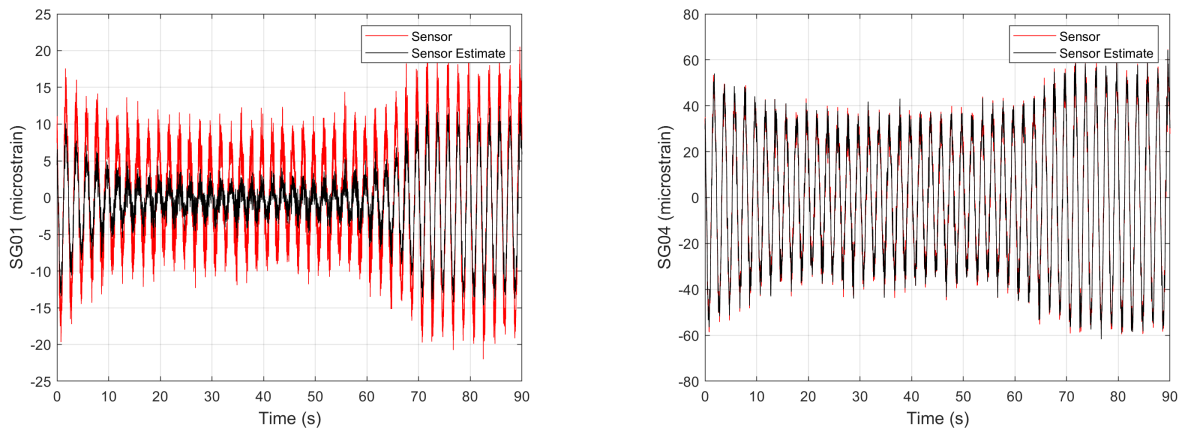


Fig. 13 Estimation Performance for Strain Gauges

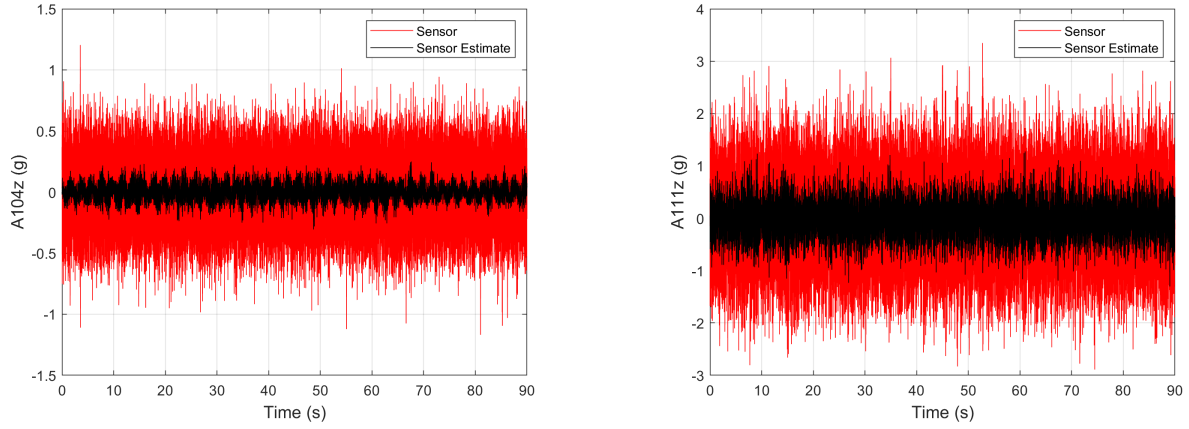


Fig. 14 Estimation Performance for Accelerometers

A summary of the test results across all gust frequencies and estimation schemes is provided in Table 1. Overall, the extended-state Kalman filter demonstrated the most consistent estimation performance across all test cases. While the gust sniffer configurations were functional, their performance was generally lower than that of the onboard-only estimation approach.

Additional tests were conducted utilizing AILA and AILC only. This controller was prepared because AILB exhibited overall inconsistent behavior throughout testing. Despite this, the controllers without AILB performed comparably to the original control law and slightly outperformed the configuration using all three ailerons. This suggests that the inclusion of the AILB may have a low or even negative impact on closed-loop performance under the tested conditions.

Table 1 GLA Performance Summary

Controller	Gust Frequency (Hz)	Estimation Scheme	%SG01 Reduction	% Laser Disp. Reduction
LQG	0.5	Extended-State KF	53.5	41.0
LQG	0.5	Gust Sniffer Est. 1	37.0	20.0
LQG	0.5	Gust Sniffer Est. 2	32.7	15.9
LQG	1	Extended-State KF	53.2	41.7
LQG	1	Gust Sniffer Est. 1	36.7	19.1
LQG	1	Gust Sniffer Est. 2	34.7	17.1
LQG	2	Extended-State KF	52.6	43.8
LQG	2	Gust Sniffer Est. 1	39.5	24.0
LQG	2	Gust Sniffer Est. 2	35.5	18.2
LQG	3	Extended-State KF	55.2	52.9
LQG	3	Gust Sniffer Est. 1	44.2	35.5
LQG	3	Gust Sniffer Est. 2	43.1	31.9
LQG (No AILB)	3	Extended-State KF	55.2	46.2
LQG (No AILB)	3	Gust Sniffer Est. 1	46.4	32.7
LQG (No AILB)	3	Gust Sniffer Est. 2	44.5	30.1

VI. Conclusions

This paper presented the design and experimental validation of a multi-objective optimal control approach for gust load alleviation on a high-aspect-ratio flexible transport wing. The controller was developed within the context of the

NASA–Boeing Integrated Adaptive Wing Technology Maturation (IAWTM) program and evaluated experimentally using a dynamically scaled CRM13 half-span model in the NASA Langley Transonic Dynamics Tunnel.

A multi-objective LQG control framework was formulated by extending the standard LQR cost function to directly penalize wing-root strain while maintaining control effort constraints. The resulting control law was combined with state and gust estimation schemes, including an extended-state Kalman filter and a preview-based approach utilizing an upstream gust sniffer. Detailed modeling of gust vane dynamics, gust propagation, and unsteady aerodynamic disturbances enabled systematic evaluation of controller and estimator performance.

Wind-tunnel test results demonstrated that the proposed multi-objective LQG controller achieved significant reductions in structural response due to gust excitation, with root strain and wingtip displacement reductions up to 54% and 41%, respectively. Additional testing without the inboard aileron (AILB) revealed that this surface exhibited inconsistent behavior and provided limited benefit, suggesting that its inclusion may have a negligible or negative impact on closed-loop performance for gust load alleviation.

Overall, the results validate the effectiveness of multi-objective optimal control for gust load alleviation on highly flexible wings and demonstrate its practical applicability in an experimental environment. The findings provide valuable insight into sensor selection, estimator design, and control surface utilization for future high-aspect-ratio transport aircraft, and support the continued development of advanced aeroservoelastic control strategies for next-generation flexible wing designs.

VII. Acknowledgments

This work was supported by the NASA Advanced Air Transport Technology (AATT) Project and the Advanced Air Vehicles Program (AAVP) as part of a Cooperative Agreement with Boeing. The authors would like to acknowledge significant contributions to this project and the work presented in this paper over the course of many years by team members from Boeing, NextGen Aeronautics, and NASA.

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