

Design and Testing of Active Flutter Suppression Control Laws for the IAWTM Wind Tunnel Test

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Active flutter suppression (AFS) control law designs for the Integrated Adaptive Wing Technology Maturation (IAWTM) wind tunnel tests are discussed. The test article is a half-span version of a transport aircraft model with an extended wingspan that introduced significant flexibility and numerous challenges for AFS design. The article was tested in the Transonic Dynamics Tunnel (TDT) at NASA Langley Research Center in Hampton, VA. The control laws commanded up to three hydraulically-actuated trailing-edge control surfaces based on measurements from accelerometers, strain gauges, and fiber-optic strain sensor (FOSS) cables distributed along the wing. Control law designs used classical, linear quadratic Gaussian (LQG), generalized predictive control (GPC), robust control (H_∞ and μ -synthesis), Hamiltonian control, and modal control design techniques. These control laws were generally based on linear aeroelastic models of the system that were updated using experimental test data and results from computational fluid dynamics (CFD) analyses. Designs and test results are summarized and discussed.

Nomenclature

j = imaginary number, $= \sqrt{-1}$
 s = Laplace variable
 t = time, s
 ω = frequency, rad/s

Superscripts

T = transpose
 $\cdot$ = time derivative

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I. Introduction

THE NASA Advanced Air Transport Technology (AATT) project aims to develop concepts and technologies that improve the energy efficiency and decrease the adverse environmental impacts of fixed-wing transport aircraft [1]. One current effort within AATT is the Integrated Adaptive Wing Technology Maturation (IAWTM) sub-project, which is a collaboration involving multiple NASA centers and Boeing [2, 3]. Goals of the project include developing an ultra-high aspect ratio wing with a cantilever design and mature adaptive wing technologies for real-time drag reduction, gust and maneuver load alleviation, and active flutter suppression (AFS).

The associated wind-tunnel test article is a side wall-mounted test article with a 12.75 ft half span. The geometry is based on the NASA Common Research Model (CRM) [4] but the aspect ratio has been increased to 13.5. More details on the test article are given in Refs. [2, 3]. Wind-tunnel tests were conducted in the Transonic Dynamics Tunnel (TDT) at NASA Langley Research Center (LaRC) in Hampton, VA. Flutter testing was performed over the course of two days during July 2025, which was during the second entry of the test article into the wind tunnel.

This paper summarizes the efforts to develop and test the AFS control laws over the course of the project. As noted in the state of the art given in Ref. [5], a wide spectrum of flutter controllers are popular in the open literature. Here, classical control was applied, as well as modern control and robust control, and new experimental approaches to control law design. Of the several controllers pursued, three control laws were tested over the course of two days, and these experimental results are discussed.

The rest of the paper is organized as follows. Section II presents the test article and wind tunnel. Section III describes the mathematical models used for control design. Section IV summarizes the different AFS control law designs. Section V presents some results from the wind-tunnel tests. Section VI concludes the paper.

II. Test Article and Wind Tunnel

Figure 1 shows the IAWTM test article mounted in the test section of the Transonic Dynamics Tunnel (TDT) at NASA Langley Research Center. The test article is a wall-mounted, half-span model of a transport configuration airplane. The design was based on the NASA Common Research Model (CRM) [4] but with an increased wing aspect ratio, shortened fuselage, and a flow-through engine nacelle. There is no tail section and no landing gear. It weighs about 1800 lbf and has a 13.3 ft fuselage length, 12.75 ft wing (half) span distance, 2.04 ft mean aerodynamic chord, 24.09 ft² (half) wing reference area, and 13.5 wing aspect ratio.

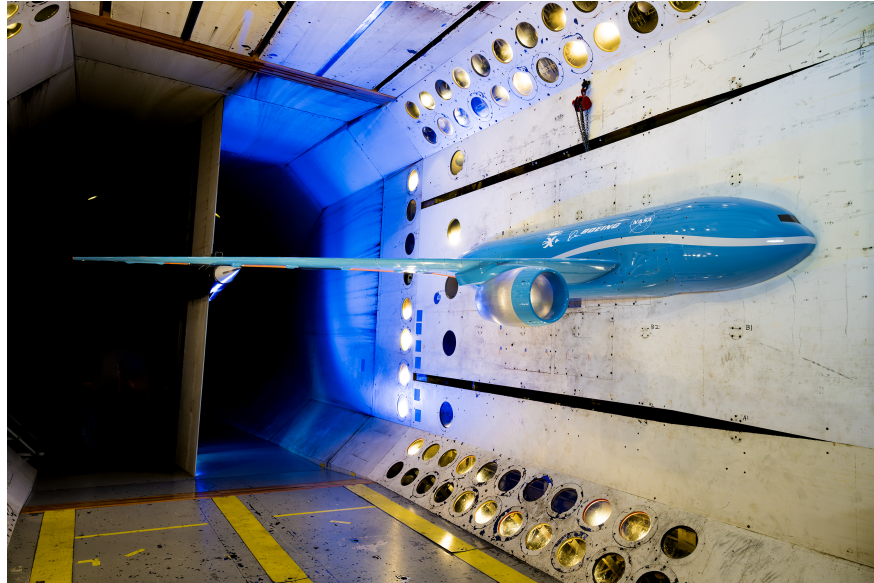


Fig. 1 The IAWTM wind-tunnel test article (NASA / Mark Knopp).

Figure 2 shows a cut-away drawing of the test article with annotations. There are 10 control surfaces. The three ailerons, which are labeled AILA, AILB, and AILC, are located behind the engine and near the wing tip. These surfaces are powered by hydraulic actuators and have higher dynamic bandwidths and hinge moment limits, which made them

attractive for AFS control design. The remaining seven surfaces are miniature plain flaps (MPFs) and labeled MPFA through MPFG from the wing root towards the wing tip. These surfaces are powered by electric servo motors and have lower bandwidths and hinge moment limits, and were not considered for AFS design. The sign convention used for all surfaces is that trailing-edge down (TED) is a positive deflection. Servo output angles were recorded, and calibrations were applied to obtain control surface deflection angles.

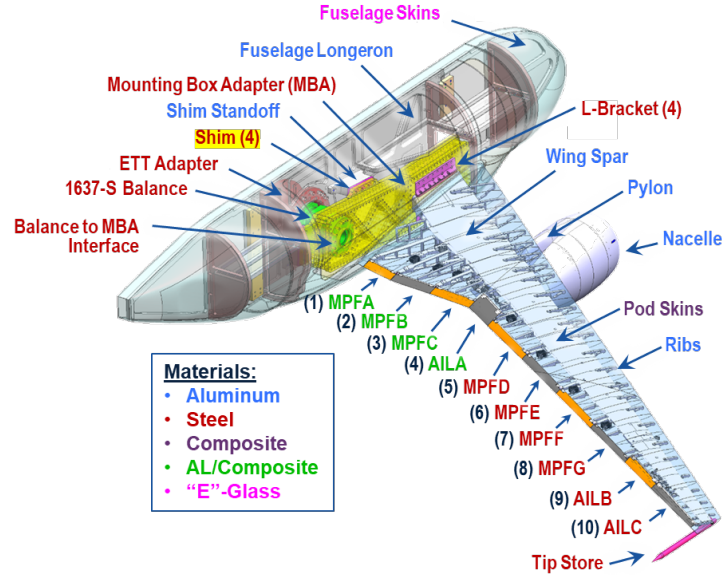


Fig. 2 Test article drawing showing control surfaces and numbering.

Figure 3 shows planform drawings with the primary sensors available for AFS control design. Figure 3(a) shows the locations of 22 accelerometers. Most of these accelerometers are single-axis sensors aligned in the vertical direction, although there are also some single-axis sensors aligned in the forward direction and some tri-axial sensors. The first number in the annotations indicates single or triple axis, and the last letter is the direction of sensitivity. The directions are relative to a body axis in that x points out the nose, y points out the right wing, and z points down. Figure 3(b) shows the locations of 10 full-bridge strain gauges. Also available but not shown in these diagrams are three fiber-optic strain sensor (FOSS) cables with a total of 117 gratings.

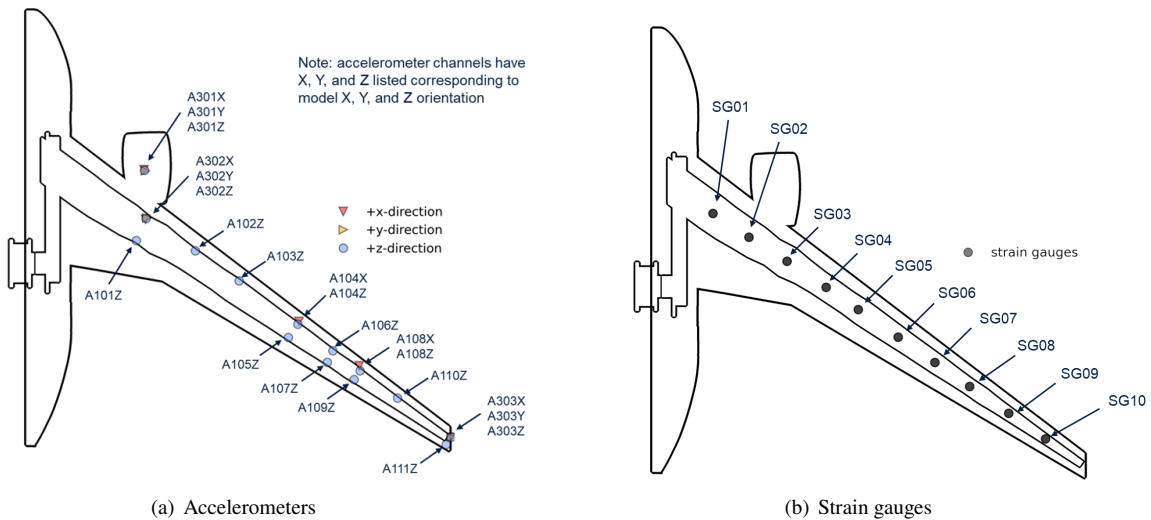


Fig. 3 Primary output measurements available for active flutter suppression feedback control design.

Also present in Fig. 3 is a tip store / flutter ballast mounted at the wing tip. The original test article would not flutter within the test parameters of the TDT, which prohibited flutter testing. The tip store was therefore designed and added to the test article to decrease the Mach numbers and dynamic pressures at which the test article flutters. The ballast is aerodynamically shaped, weighs 7.7 lbf, has a 24 in length, and can travel 7 in relative to the spar elastic axis. All modeling and testing was done with the tip store in the full aft position, which induced flutter at the lowest conditions.

Tests were performed in the TDT at NASA LaRC in Hampton, VA. The first wind tunnel entry was during July–August 2024 and was primarily for model characterization. The second entry was during June–July 2025 for closed-loop testing. Two days were allotted during the end of the second entry for flutter testing, in part due to the risks inherently involved in flutter testing. Figure 1 shows the test article mounted in the TDT test section, which has a cross-section of 16 ft by 16 ft with cropped corners. The tunnel can be filled with heavy gas R-134a (as was done during these tests), and can run up to Mach number 1.2 and a Reynolds number of 10×10^6 . More information on the tunnel and capabilities can be found in Refs. [6, 7].

III. Mathematical Design Model

The mathematical models used for control design were developed in state-space format. These were scheduled for 18 different Mach numbers between 0.3 and 0.95, and for 26 different dynamic pressures between 50 and 300 lbf/ft², totaling 468 different models. The state-space models had 15 inputs that included commands for 10 control surfaces, four upstream gust vanes, and the model angle of attack. The state-space models had 152 outputs that included accelerometers, strain gauges, FOSS gratings, wind-tunnel balance measurements, and a gust sniffer angle of attack.

The description of the design model is parsed into three sections below. Section III.A describes the structural dynamics, Section III.B presents the aerodynamics, and Section III.C discusses additional subsystem dynamics (i.e., actuators, sensors, and measurement hardware). Afterwards, Section III.D shows a system survey of the plant model and Section III.E discusses unique difficulties for the IAWTM AFS control design.

A. Structural Dynamics

The structural dynamics were modeled using a finite element model (FEM) in MSC Nastran[®]*. A total of 38,667 grid points were used that consisted of rigid beam, flexible beam, flat plate, shell, and solid hexahedral elements. The model retained 20 vibration modes, which were modeled using 40 states, with modes spanning from about 3 to 62 Hz. The FEM was tuned to ground vibration test (GVT) data obtained while the model was mounted in the wind tunnel. A particular effort was made to model the supporting mount hardware outside of the test section, and to account for sections of tape along the wing that affected the wing torsion.

The first four structural modes modeled in the FEM are listed in Table 1 and illustrated in Fig. 4. The values for damping ratio were taken from the GVT analysis results. Note that all of these modes generally contain significant wing bending. This observation was exploited in some control law designs.

Table 1 First four structural modes included in the finite element model

Index	ID	Frequency, Hz	Damping Ratio, %	Description
1	W1VB	2.69	0.51	Wing 1st vertical bending
2	W2VB-BP1	8.04	2.12	Wing 2nd vertical bending, ballast pitch
3	W1FA	8.21	3.33	Wing 1st fore-aft bending, vehicle yaw
4	W2VB-BP2	10.20	3.26	Wing 2nd vertical bending, ballast pitch, vehicle pitch

*The use of trademarks or names of manufacturers in this report is for accurate reporting and does not constitute an official endorsement, either expressed or implied, of such products or manufacturers by the National Aeronautics and Space Administration.

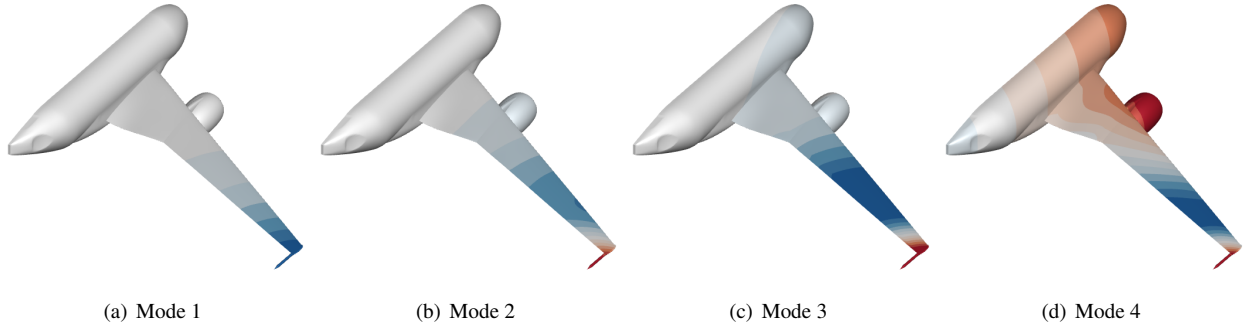


Fig. 4 First four structural mode shapes showing vertical deflection (red is upward deflection, blue is downward).

B. Aerodynamics

The aerodynamics were modeled using ZAERO™, by ZONA Technology Inc., which employs panel methods and assumes potential flow [8]. Flat plate elements were used for the wing and pylon, and body elements were used for the fuselage and engine nacelle. Roger's method [8] was used to fit rational function approximations (RFAs) with 12 lag states per mode to each of the 20 generalized aerodynamic forces (GAFs), since there are 20 structural modes modeled, using 39 reduced frequencies from 0 to 0.7. The aerodynamics model contributed 240 states to the model.

Other studies were done to model the aerodynamics at the higher Mach numbers of interest using computational fluid dynamics (CFD) solvers [9–11]. In some investigations, excitations were used on the structural mode states and the control surface states, which then allowed for the GAFs to be approximated by RFAs in a system identification analysis [11]. These models required weeks of computation and analysis, and were primarily used for discrete checks, whereas the linear approaches using panel methods could be turned around much quicker and were primarily used for control design.

During model characterization experiments from the first test entry into the TDT, noticeable differences between the modeled aileron control power from the linear panel codes and the overall gain of the corresponding frequency responses were observed. The reason for the discrepancy is not yet understood, but was not attributed to control surface reversal (which is discussed in Section III.D) because it was present at low Mach numbers and dynamic pressures. To accommodate for this difference in the control design, knock-down factors of about 0.6 and 0.5 were applied to AILB and AILC, respectively, based on simple fittings to the experimental frequency response data.

C. Actuators, Sensors, and Measurement Hardware

Several tests were performed to identify actuator models at various wind tunnel conditions. During these tests, orthogonal phase-optimized multisines [12] were injected into the command path to excite all 10 control surfaces simultaneously but with different harmonic frequencies up to 50 Hz. Frequency responses were then computed from this data [13], and models were determined and fit [13, 14].

It was determined that over this frequency range, the hydraulic actuators were well modeled by the transfer function

$$\frac{\delta(s)}{\delta_c(s)} = K \left(\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \right) e^{-\tau s} \quad (1)$$

where K is the gain, ζ is the damping ratio, ω_n is the natural frequency, and τ is the time delay. Identified values of these parameters, given in Table 2, were found not to significantly change for the hydraulic actuators over the tunnel conditions. The aileron deflections were generally limited to ± 10 deg, although there was ± 20 deg of range available. During testing, control surface rates greater than 400 deg/s were achieved on each aileron, and rate limiting behavior was not experienced. The hydraulic actuators contributed 6 states to the model. When combined with the other control effectors, the total actuator model contributed 30 states.

During the actuator testing, problems with AILB were noticed. Nonlinearities in the system and quantization of the servo position measurement resulted in slow performance with small-displacement excitations. At higher amplitudes and with higher dynamic pressures, however, AILB behaved more like AILC.

Data sheets were investigated for the accelerometers and strain sensors. The frequency responses for these measurements were at least a decade above the frequencies of interest, and were therefore not included in the model.

Table 2 Actuator transfer function parameters

Actuator	K	ζ	ω_n , rad/s	τ , s
AILA	0.97	0.53	152	0.005
AILB	1.09	0.55	63	0.006
AILC	1.02	0.34	195	0.006

The anti-aliasing filters in the data acquisition process were modeled. These were fourth-order filters with a cutoff frequency of 200 Hz. Although the cutoff frequency was high, these filters were important to include in the model because the phase response contributed about 5 deg of lag near the 40 rad/s flutter frequency.

Measurements were digitized and recorded at 1000 Hz. An additional 1 frame pure time delay was modeled to account for time lags in the digitization process. This delay contributed another 3 deg of phase lag at the flutter frequency.

D. System Survey

The combination of the structural dynamics in Section III.A and aerodynamics in Section III.B constitute the aeroelastic model. In this paper, the aeroelastic model is sometimes also called the aeroservoelastic (ASE) model, although it may or may not also include subsystem dynamics and/or the control law.

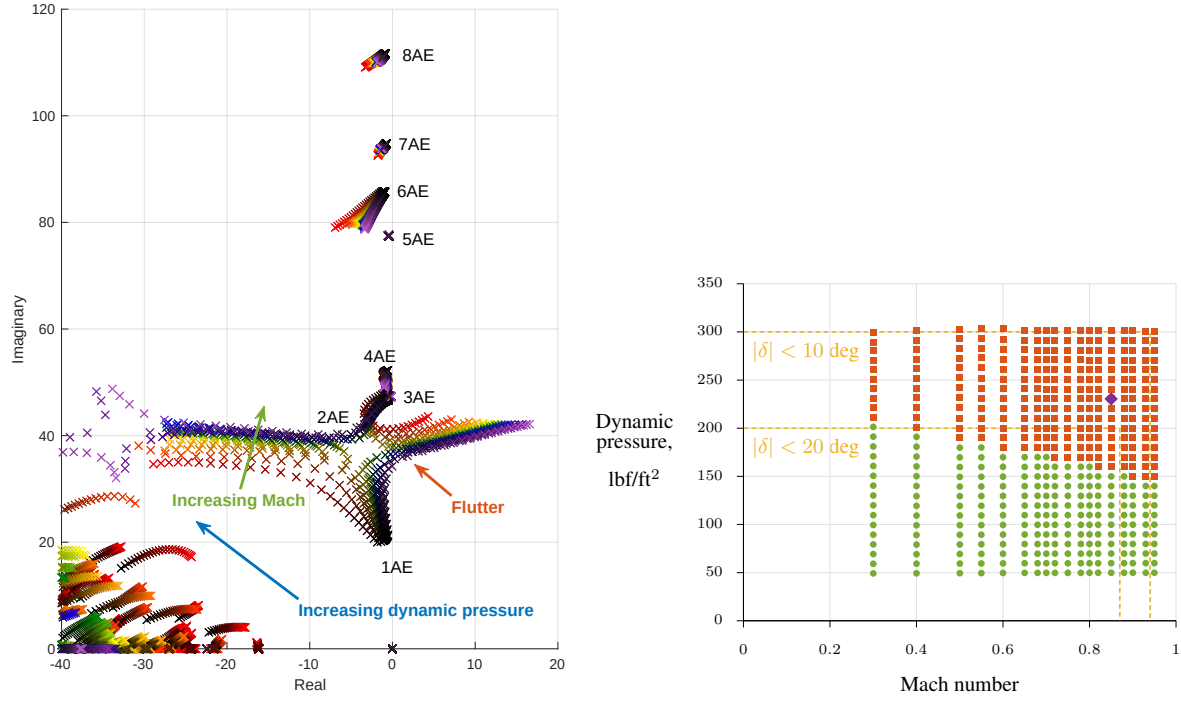
Figure 5 surveys the ASE model dynamics. Figure 5(a) shows some of the model poles on the complex plane. The color of the markers, moving from red through the rainbow to violet, indicates the Mach number. The saturation of the markers, moving from dark to light, indicates the dynamic pressure. The first 8 aeroelastic modes are seen near the positive imaginary axis and are annotated with the aeroelastic mode number. In the lower-left corner near the negative real axis are some of the aerodynamic lag states.

As dynamic pressure increases, most of the aeroelastic modes either remain constant or increase in damping while decreasing in natural frequency. However, the first two aeroelastic modes coalesce and then form the flutter instability with increasing dynamic pressure. At Mach numbers below 0.65, the first aeroelastic mode goes unstable. Conversely, above Mach 0.65, the second aeroelastic mode goes unstable. Again, the flutter frequency is about 40 rad/s or 6.4 Hz. The instability is considered a hard flutter that progresses quickly into the right-half plane with increases in dynamic pressure.

Figure 5(b) shows a marker for each of the 468 linear models as Mach number and dynamic pressure is varied. The green circles indicate points where the open-loop plant model is stable, whereas the red squares show unstable plant models, which is where the test article is expected to flutter. The purple diamond indicates the design point (i.e., nominal cruise condition) at Mach 0.85 and dynamic pressure 230 lbf/ft². In Ref. [9], the higher-fidelity CFD analyses predicted the flutter boundary at about 50 lbf/ft² less than the panel code solution for a given Mach number.

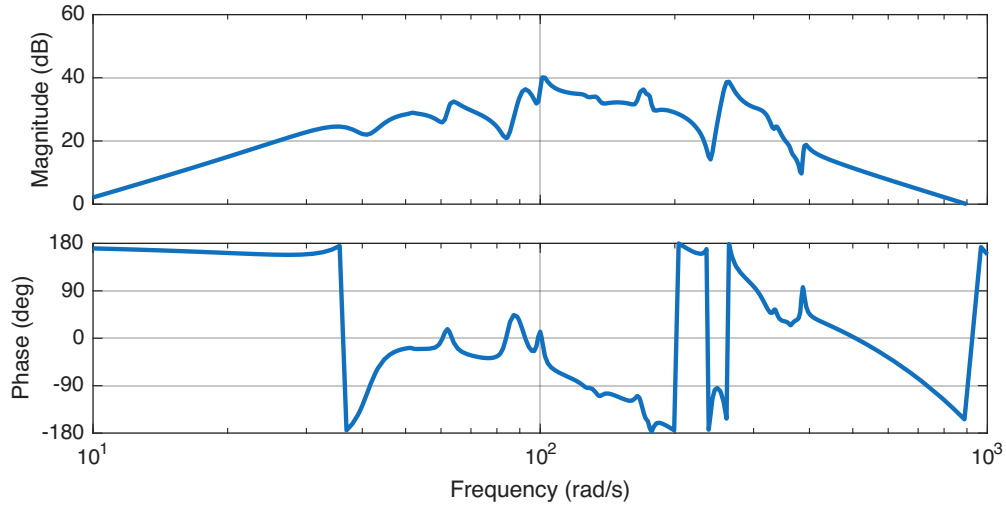
Figure 5(c) shows a representative Bode plot between the AILC command input and the A110Z vertical accelerometer output at the design point of Mach 0.85 and dynamic pressure 230 lbf/ft². Most of the bending-type modes that contribute to the flutter mechanism are observable with this input-output pair. The model has content over two decades of frequency, from 10 to 1000 rad/s. The 20 modes are closely spaced with little frequency separation between the modes. The flutter instability can be seen near 40 rad/s where there is a phase increase near two coalescing modes, which have migrated from the air-off natural frequencies of 16.9 and 50.5 rad/s.

Another phenomenon that was captured in the design plant model is control surface reversal. As the dynamic pressure increases, deflection of a control surface twists the wing, which reduces the control power and eventually creates lift in the opposite sign compared to lower dynamic pressures. For example, a positive control surface deflection creates a trailing-edge down deflection, and an increase in local lift. However, due to flexibility in the wing, the control surface deflection can also create a nose-down twist in the wing, which works to diminish the local lift created by the control surface. Figure 6 shows the areas where reversal occurs (red squares) as opposed to normal operation (green circles). No shading is provided to indicate the degradation of control power. Figure 6(c), in comparison to Fig. 5(b), shows that AILC was expected to experience control reversal well below the flutter boundary. Similarly, AILB was expected to experience this very close to the flutter boundary, and AILA was not expected to experience control reversal.



(a) Root locus over Mach (color) and dynamic pressure (saturation)

(b) Stable (green) and unstable (red) open-loop conditions



(c) Bode plot from AILC command to A110Z at the design point (Mach 0.85 and dynamic pressure 230 lb/ft²)

Fig. 5 System survey of the design plant model.

E. Challenges for AFS Design

Active flutter control is a notoriously challenging problem, with numerous difficulties and no clear standard solution [5]. Specifically for the IAWTM test article, the AFS problem is made challenging by a variety of factors:

- The IAWTM test article is extremely flexible and has numerous coupled and closely-spaced aeroelastic modes at low frequencies and within the bandwidth of the actuators. Many modes need to be modeled to achieve modal convergence and accurately represent the model dynamics.
- The test article is a half-span model. As such, there is no decoupling of symmetric and anti-symmetric modes that can simplify the model, analysis, and control design.
- Due to the locations along the wing span, AILC and AILB were more effective than AILA for AFS. However,

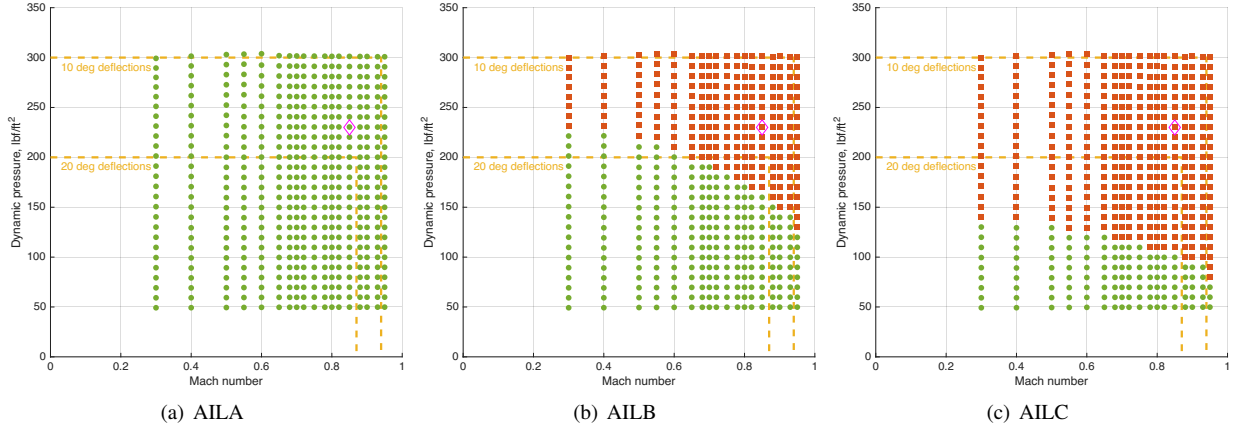


Fig. 6 Predicted regions where aileron deflection creates the intended direction of lift (green) vs. opposite direction of lift (red), which indicates aileron control reversal.

quantization problems affect the reliability of AILB, and control reversal diminishes the power of AILC at higher Mach numbers and dynamic pressures, and before the predicted flutter boundary.

- Models are based on linear techniques, which degrade as the transonic region is approached. The CFD studies have found evidence of nonlinear behavior at transonic Mach numbers [11].
- The flutter mechanism for the IAWTM in this configuration corresponds to a hard flutter crossing, where the flutter mode damping rapidly changes with a small change in dynamic pressure.
- There is a relatively large amount of uncertainty that stems from the complicated interactions of structural dynamics, aerodynamics, and measurement processes.
- The system undergoes large changes in dynamics (i.e., poles and zeros) as the Mach number and dynamic pressure change. For example, the poles shown in Fig. 5(a) migrate significantly over the test envelope.

IV. Control Law Designs

This section describes the main AFS control design approaches pursued. Section IV.A describes the Hamiltonian approach. Section IV.B describes the generalized predictive control approach. Section IV.C presents a robust control design based on μ -synthesis. Section IV.D describes an approach to reconstruct structural states from strain and accelerometers for feedback. Lastly, Section IV.E describes a collocation approach for control using classical design concepts. Other efforts to design controllers using linear quadratic Gaussian (LQG) control are not discussed in this paper.

There were no requirements on performance or robustness established for the control laws by the subproject. The general approach was therefore to attempt to minimize excess control usage and maintain stability margins (nominally 6 dB gain margin and 45 deg phase margin) as the tunnel test conditions increased and the test article destabilized, and to demonstrate this beforehand using the linear simulation models.

A. Hamiltonian Control

The Hamiltonian controller developed by Nguyen [15] is based on Hamilton's principle for distributed control of infinite-dimensional Lagrangian systems governed by partial differential equations. This control framework employs operator theory to model Lagrangian systems. The distributed displacement is $w(x, t)$ and is assumed to be the product of the eigenfunction $\phi(x)$, the generalized displacement $q(t)$, where x is the spanwise location along the elastic axis:

$$w(x, t) = \phi(x)q(t) \quad (2)$$

The structural dynamics of a flexible structure can be expressed as the following Lagrangian system in linear operator form:

$$m(x) \frac{\partial^2 w(x, t)}{\partial t^2} + Lw(x, t) = f \left(w(x, t), \frac{\partial w(x, t)}{\partial t}, u(x, t) \right) \quad (3)$$

The distributed mass is $m(x)$ and L is a self-adjoint linear operator given by:

$$L = m^{-1}(x) \sum_{i=0}^n (-1)^i \frac{\partial^i}{\partial x^i} \left(k_i(x) \frac{\partial^i}{\partial x^i} \right) \quad (4)$$

The first term in Eq. 3 represents the structural mass and the second term represents the structural stiffness. The right-hand side includes the aerodynamic force and control force.

The Hamiltonian function then provides the framework for representing the total energy of a flexible structure and for analyzing its dynamic behavior under control and aerodynamic forces. It therefore can be used to analyze the closed-loop structural dynamics under aerodynamic forces [15–17]. In this case, the ASE system is cast as a structural dynamics model, where the generalized mass and stiffness matrices capture the dominant structural modes. Here, the Hamiltonian approach is used to derive the flutter suppression control design, providing conditions for dissipativity that inform gain selection and closed-loop performance. Full details on the Hamiltonian control approach can be found in Ref. [18]. The structural dynamics of the system are expressed in the general form as:

$$M\ddot{q}(t) + Kq(t) = F_q q(t) + F_{\dot{q}} \dot{q}(t) + F_{\delta} \delta(t) + F_{\dot{\delta}} \dot{\delta}(t) + F_{\ddot{\delta}} \ddot{\delta}(t) \quad (5)$$

The right-hand side of the equation represents the generalized forces acting on the structure. Assuming that the control input $\delta(t)$ exhibits harmonic behavior at the frequency of the flutter mode ω_f , the acceleration term of the control input can be approximated as $\ddot{\delta}(t) = -\omega_f^2 \delta(t)$, which reduces the structural dynamics to:

$$M\ddot{q}(t) + Kq(t) = F_q q(t) + F_{\dot{q}} \dot{q}(t) + (F_{\delta} - F_{\ddot{\delta}} \omega_f^2) \delta(t) + F_{\dot{\delta}} \dot{\delta}(t) \quad (6)$$

The control law is defined by:

$$\delta_c(t) = -K_q q(t) - K_{\dot{q}} \dot{q}(t) \quad (7)$$

whose time derivative is given by:

$$\dot{\delta}_c(t) = -K_q \dot{q}(t) - K_{\dot{q}} \ddot{q}(t) \quad (8)$$

To assess system stability, the Hamiltonian $H(t)$ is defined as:

$$H(t) = \frac{1}{2} \dot{q}^T(t) M \dot{q}(t) + \frac{1}{2} q^T(t) K q(t) \quad (9)$$

whose time derivative is:

$$\dot{H}(t) = \dot{q}^T(t) [M \ddot{q}(t) + K q(t)] \quad (10)$$

$$= \dot{q}^T(t) \left[F_q q(t) + F_{\dot{q}} \dot{q}(t) + (F_{\delta} - F_{\ddot{\delta}} \omega_f^2) \delta_c(t) + F_{\dot{\delta}} \dot{\delta}_c(t) \right] \quad (11)$$

The conditions for dissipativity are imposed as:

$$\Delta \dot{H}_q = \dot{q}^T(t) [F_q - (F_{\delta} - F_{\ddot{\delta}} \omega_f^2) K_q] q(t) \leq 0 \quad (12)$$

$$\Delta \dot{H}_{\dot{q}} = \dot{q}^T(t) [F_{\dot{q}} - (F_{\delta} - F_{\ddot{\delta}} \omega_f^2) K_{\dot{q}} - F_{\dot{\delta}} K_q] \dot{q}(t) \leq 0 \quad (13)$$

$$\Delta \dot{H}_{\ddot{q}} = \dot{q}^T(t) [-F_{\ddot{\delta}} K_{\ddot{q}}] \ddot{q}(t) \leq 0 \quad (14)$$

The gains K_q and $K_{\dot{q}}$ are chosen such that the resulting functions are negative semi-definite, ensuring dissipative closed-loop behavior. The gain K_q is determined by setting $\Delta \dot{H}_q$ to zero, which gives:

$$F_q - (F_{\delta} - F_{\ddot{\delta}} \omega_f^2) K_q = 0 \quad (15)$$

The pseudo-inverse method is used to solve for K_q , which results in:

$$K_q = [(F_{\delta} - F_{\ddot{\delta}} \omega_f^2)^T (F_{\delta} - F_{\ddot{\delta}} \omega_f^2)]^{-1} (F_{\delta} - F_{\ddot{\delta}} \omega_f^2)^T F_q \quad (16)$$

To determine $K_{\dot{q}}$, $\Delta \dot{H}_{\dot{q}}$ is set to a negative definite quantity $-2\zeta\omega_n$, which gives:

$$F_{\dot{q}} - (F_{\delta} - F_{\ddot{\delta}} \omega_f^2) K_{\dot{q}} - F_{\dot{\delta}} K_q = -2\zeta\omega_n \quad (17)$$

where ω_n is the vector of modal natural frequencies and ζ is a diagonal matrix of prescribed damping ratios. The control law is tuned by selecting ζ to enforce the desired closed-loop modal damping. The pseudo-inverse method is used to solve for K_q , which results in:

$$K_q = [(F_\delta - F_{\delta}\omega_f^2)^T (F_\delta - F_{\delta}\omega_f^2)]^{-1} (F_\delta - F_{\delta}\omega_f^2)^T (F_q - F_{\delta}K_q + 2\zeta\omega_n) \quad (18)$$

An initial damping ratio of $1/\sqrt{2}$ is selected for each mode and subsequently tuned between 0.28 and 0.71, with emphasis on the most active modes. The term in Eq. 14 represents the apparent mass contribution to the system. This means K_q must be sufficiently small so the apparent mass remains positive definite.

The Hamiltonian control formulation provides a generalized control command, $\delta_c(t)$, but does not directly account for actuator dynamics or command tracking. To address this, a linear quadratic integral (LQI) controller is employed to track the Hamiltonian-derived command by augmenting the system state with an integral of the command tracking error. Since the full system state is not directly measurable, a Kalman filter is used to estimate the augmented state based on available sensor measurements and a process noise model. The resulting control and estimation gains are computed for each aeroelastic model and scheduled as functions of Mach number and dynamic pressure to account for the strong dependence of the aeroelastic dynamics on operating condition. Additional details on the augmented formulation, LQI synthesis, and state estimation approach are provided in Ref. [18].

B. Generalized Predictive Control

The concept of generalized predictive control (GPC) was originally introduced by Clarke et al. in Ref. [19] as a potential adaptive control algorithm. The basic idea is to identify a model from data, and then devise an appropriate control law. Reference [19] proposed the concepts of a prediction horizon, h_p , a control horizon, h_c , after which control commands become zero, and a least-squares observer parameter estimator, p , referred to as the order of the model. However, no details are presented in Ref. [19] regarding the parameter computation or system identification. Juang and Phan in Ref. [20] expanded this effort and explored various system identification processes of Autoregressive Moving Average model with exogenous inputs (ARMAX) models with multiple input multiple output (MIMO) systems and enabled unknown disturbance rejection. Building upon the ARMAX foundations of Ref. [20] and the GPC introduction of Ref. [19], Kvaternik et al. in Ref. [21] developed an objective function similar to optimal control theory and turned GPC into a practical and powerful MIMO controller. Reference [21] is a foundational, albeit informal, paper describing the state-of-the-art of GPC. Reference [22] demonstrated the use of GPC experimentally with a tiltrotor aircraft and Ref. [23] further explored the use of GPC as a tracker for rigid-body control of a flight vehicle.

The derivation of the equations used in the modern GPC controller was presented in Ref. [24] and is further expanded upon in Ref. [25]. For clarity, the governing equation is presented here as

$$\mathbf{y}_{h_p}(k) = \mathbf{T}\mathbf{u}_{h_c}(k) + \mathbf{A}\mathbf{y}_p(k-p) + \mathbf{B}\mathbf{u}_p(k-p) \quad (19)$$

where $\mathbf{y}$ is the response output, $\mathbf{u}$ is the control input, and k is the discrete time step index. Past input and output values are denoted as $\mathbf{u}_p$ and $\mathbf{y}_p$, respectively, and contain p values of $\mathbf{u}$ and $\mathbf{y}$. The matrices $\mathbf{T}$, $\mathbf{A}$, and $\mathbf{B}$ are described in Refs. [24–26] and represent the Observer Markov Parameters (OMP). A system identification process, also described in Refs. [24–26], is used to determine the OMP matrices. In this section, “system identification” refers to a successfully computed set of OMP matrices with associated weight values rather than the process of identifying those matrices. In Eq. (19), the subscripts h_p , h_c , and p denote the size of the respective vectors, whereas the parenthetical values represent the starting indices.

Kvaternik et al. in Ref. [21] present the objective function

$$J = \boldsymbol{\epsilon}^T \mathbf{R} \boldsymbol{\epsilon} + \mathbf{u}_{h_c}^T \mathbf{Q} \mathbf{u}_{h_c} \quad (20)$$

where $\boldsymbol{\epsilon}$ is the error between the target response $\mathbf{y}_T$ and the predicted response $\boldsymbol{\epsilon} = \mathbf{y}_T - \mathbf{y}_{h_p}$, $\mathbf{R}$ is the weighting matrix for error, and $\mathbf{Q}$ is the weighting matrix for the control input. Controls $\mathbf{u}$ are then determined from minimization of this function as described in Refs. [21, 24, 25].

Traditional GPC is a very effective controller that has been used in many dynamic systems [21–26] and often outperforms other control options with regard to successful dynamic suppression and control robustness [25]. However, the performance of traditional GPC can degrade as conditions change from those used to derive the system identification. To correct the controller for changing flight or vehicle conditions, the concept of advanced generalized predictive control (AGPC) was introduced and demonstrated in Refs. [24–26].

Reference [24] introduces AGPC and the definition of a system error term ξ that can be numerically determined from measured data using

$$\xi = \mathbf{y}_{h_p \text{ actual}} - \mathbf{y}_{h_p \text{ predicted}} + \phi \quad (21a)$$

$$= \mathbf{y}_{h_p \text{ actual}} - \left[\mathbf{T} \mathbf{u}_{h_c \text{ actual}}(k) + \mathbf{A} \mathbf{y}_p(k-p) + \mathbf{B} \mathbf{u}_p(k-p) \right] + \phi \quad (21b)$$

after the system described by Eq. (19) progresses at least h_p time steps beyond k , where ϕ represents noise and external disturbances. The significance of this development is that a working controller can be improved or updated by changes to the OMP matrices done by passive observation of the input-output relationships during controller operation.

The subscript *actual* in Eq. (21) indicates the actual measurements of response and control inputs after an additional h_p time steps after beyond k . It is assumed ϕ is a random process. If we continue to an additional s time steps, then one would have $s - h_p$ computations of ξ to minimize the effect of ϕ . If ϕ is minimized and all the values of $\mathbf{y}$ and $\mathbf{u}$ shown in Eq. (21) are now measured, then the only uncertainty that should remain is the error in the observer matrices $\mathbf{T}$, $\mathbf{A}$, and $\mathbf{B}$. After these s time steps, Eq. (21) can be expanded into the matrix form

$$\xi = \begin{bmatrix} \Delta \mathbf{T} & \Delta \mathbf{A} & \Delta \mathbf{B} \end{bmatrix} \begin{bmatrix} \mathbf{u}_{h_c \text{ actual}} \\ \mathbf{y}_p(k-p) \\ \mathbf{u}_p(k-p) \end{bmatrix} \quad (22)$$

where the Δ indicates the error in the observer $\mathbf{T}$, $\mathbf{A}$, and $\mathbf{B}$ matrices where the control and state matrices are of width $s - h_p$. The Δ matrix values can then be determined from Eq. (22) using the Moore-Penrose pseudoinverse, and the matrices are updated according to

$$\mathbf{T}_{\text{new}} = \mathbf{T}_{\text{old}} + \Delta \mathbf{T} \quad (23a)$$

$$\mathbf{A}_{\text{new}} = \mathbf{A}_{\text{old}} + \Delta \mathbf{A} \quad (23b)$$

$$\mathbf{B}_{\text{new}} = \mathbf{B}_{\text{old}} + \Delta \mathbf{B} \quad (23c)$$

$$(23d)$$

C. μ -Synthesis

Robust control design techniques were also applied to the IAWTM models to contend with the model structure error and disturbances expected in flutter testing. A brief summary is given in this section. For more details, see Ref. [27].

The robust H_∞ control design was developed using μ -synthesis (also known as DK-iteration) [28, 29]. Several projects have previously applied μ -synthesis to the problem of AFS with varying levels of performance and robustness margins in the final controllers [5, 30–33]. The μ -synthesis framework represents the uncertainty set as a bounded-input bounded-output (BIBO) system scaled to a maximum of unity gain. This uncertainty set is then placed in feedback with the plant and controller and the control synthesis attempts to find a controller to meet performance specifications for all possible uncertainties in this set [34, 35].

The design of a robust controller is based on the same general principles as optimal control: the response of a performance signal is optimized in response to exogenous inputs. The robust control problem augments this with the requirement that this optimization is done for all possible uncertainties contained within the uncertainty set simultaneously. This requires designing weighting filters to inform the control synthesis of what signals are of concern at what frequencies and how the dynamic uncertainty block interacts with the system.

The exogenous inputs in this control design are sensor noise and disturbance forces acting on the wing. The performance signals are composed of the wing-tip vibrations and aileron control usage, with the optimization attempting to minimize both simultaneously. The aileron uses cost filters or weighting filters that are designed to lightly penalize frequencies below the flutter band and heavily penalize usage above the actuator rate limits, while minimizing the cost within the predicted flutter frequency range. Due to performance limitations with AILB, the cost associated with using this actuator was increased by a factor of 5 relative to the other two ailerons. Similarly, vibrations above and below the flutter frequency band are only minimally penalized, while vibrations within the flutter band are heavily penalized. The vertical accelerometers along the wing are used to measure the vibration, with increased emphasis placed on the sensors closer to the wing tip.

As there is no convenient way to accurately model the aerodynamic disturbance forces on the wing, random perturbations of the control surfaces are used as a stand-in. The disturbance weight filters are designed such that the

control surface perturbations have an amplitude of 1 deg deflection within the flutter frequency range and roll off by 20 dB/dec away from the flutter range.

The sensor noise is modeled to have constant magnitude across all frequencies, as between the control use, vibration cost, and disturbance weighting filters the synthesis is sufficiently focused on the system response within the flutter band. Early testing in the control design process found that more realistic noise models did not meaningfully affect the performance or robustness of the controller despite increasing the order of the generalized plant and resulting controller.

The uncertainty set accounted for in the robust control synthesis was a combination of the wind tunnel operating at conditions that do not perfectly match the design condition, inaccuracies between the aeroelastic model and the test article, and model reduction leading to information loss. Input multiplicative uncertainty (IMU) was used as testing early in the development of this controller, and it was found that the IMU framework yielded a tighter bound on the uncertainty with lower order filters than additive or output multiplicative frameworks. The uncertainty weights were used to scale the influence of the uncertainty block from unity gain to over-bound the magnitude of the dynamic uncertainty. The uncertainty weights were designed using the MATLAB[®] function `ucover.m`, so that the maximum singular value of the nominal model with worst-case uncertainty over-bounded all models for the adjacent conditions. The surrounding models are the set of aeroelastic models designed for conditions with dynamic pressure changed by 10 lbf/ft² or Mach number changed by 0.05 from the control design point. This design of the uncertainty set ensures that the resulting controller is robust against changes in the aerodynamics from the nominal model, due to the wind tunnel not operating at the exact conditions used in modeling or discrepancies between the real and modeled aeroelastic dynamics.

The combination of the aeroelastic wing model, actuator model, and assorted frequency weighting filters were collectively large enough dimension to make running μ -synthesis using a standard workstation computer infeasible. As such, performing μ -synthesis with this framework necessitated significant model order reduction and limiting the controller to using 8 out of the approximately 150 sensors on the model. Ultimately, this control scheme uses the 3 hydraulic ailerons to suppress wing-tip vibration using acceleration measurements from 8 vertical-axis accelerometers. While numerous methods for model reduction were investigated, the aerodynamic lag states were truncated from the aeroelastic model. This ultimately reduced the size of the generalized plant by approximately half and allowed for the now unmodeled aerodynamics to be treated as a disturbance acting on the wing to be rejected. The uncertainty set was adjusted to include robustness against model reduction by using the reduced model at the design condition as the nominal model, while using the full models at the surrounding conditions as the sampled representation of the uncertainty set.

The minimum robust H_∞ performance achieved through μ -synthesis for this design was approximately 30, while accounting for 1/30 of the modeled uncertainty. This result does not guarantee stability of the closed-loop system with an arbitrary realization of the dynamic uncertainty. The multi-loop disk margin [36] with independent variations in all input and output channels was assessed with the controller as a secondary metric to determine the stability margins. The disk margin was found to be 0.0014, indicating that the system was stabilized by the controller, however the margin was deemed insufficient to safely test the controller on the model in the wind tunnel.

D. Modal Control

One of the difficulties of aeroelastic control is that the structural states of the model are not directly measured by traditional sensors, but instead are measured in linear weighted combinations. For example, strain measurements give a summation of the modal displacements, weighted by the strain mode shapes. Similarly, accelerometers measure a summation of the modal accelerations, weighted by the displacement mode shapes.

Another difficulty is that because the traditional sensor measurements couple into several structural modes, it becomes difficult to decouple the closed-loop response and improve the characteristics of some aeroelastic modes without degrading characteristics of other aeroelastic modes. As such, it would be beneficial to target specific modes, but this requires measurements of the structural states.

One solution to this problem is LQG control, where a dynamic model of the ASE plant is used with an observer to estimate the system states, which are then used with a linear quadratic regulator (LQR) controller. This can be difficult to use with ASE models because of inaccuracies in the system model, the large order of the model (which may require simplification to implement), and issues with controllability and/or observability.

Another well-known solution is to apply a modal filter, where numerous strain measurements are combined with the known strain mode shapes in a least-squares estimator to produce estimates of the modal displacements at each time step. This approach suffers from a number of potential drawbacks [37, 38], including that numerous measurements are needed to estimate a large number of structural states to achieve modal convergence and accurate state displacement

estimates. Using FOSS is promising in obviating this problem [39].

The modal approach here relies on the fact that many strain measurements are measured for the IAWTM using the traditional strain sensors and FOSS cables. In addition, the same idea is applied to the numerous accelerometers to estimate modal accelerations. Furthermore, these two sets of modal state estimates are combined in a Kalman filter to produce enhanced estimates of the modal displacements and rates for feedback control. This modal reconstruction approach has been applied to the X-56A airplane for real-time system identification [40], and reworked for an AFS controller with the IAWTM [41].

Figure 7 shows a block diagram of the system model, where the dashed line encloses the control law. The noisy strain and accelerometer measurements are denoted ϵ and a , respectively. These are fed into separate least-squares estimators, which use the measurements and the respective mode shapes to estimate the modal displacements η or the modal accelerations $\ddot{\eta}$. Note that for the IAWTM configuration, the mode shapes are constants, so most of the least-squares solution can be pre-computed, leaving only a single matrix multiplication to be performed in real time.

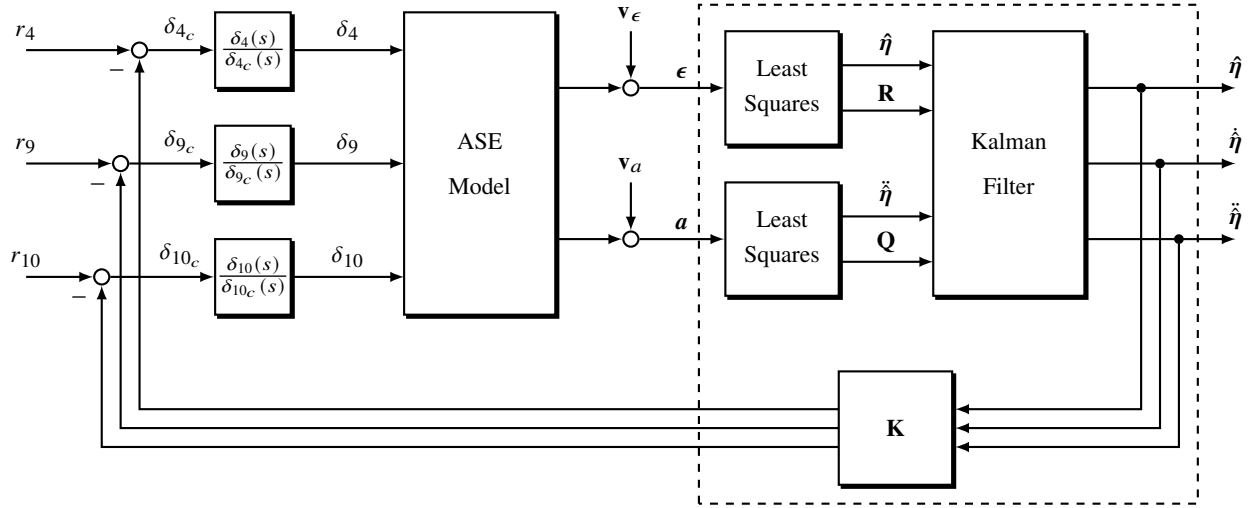


Fig. 7 Block diagram for the modal controller.

The number of modes to be estimated is limited by the accuracy of the mode shapes and the number of sensors available. It was found with simulation that estimating 18 structural modes gave a good balance between modal convergence and noise rejection. This was possible because 10 strain gauges, 117 FOSS gratings, and 22 accelerometers were available.

These estimates are then input to a Kalman filter. Because the outputs are kinematics involving only derivatives, the model equations used in the Kalman filter is a set of double integrators for each structural mode included. Unlike most Kalman filters, this filter in theory does not need tuning because the least-squares estimators produce uncertainties of the estimated modal displacements and accelerations, which become the measurement noise and process noise covariance matrices, respectively.

The outputs of the Kalman filter can then be used for feedback. One approach is to use classical control methods to add damping and/or stiffness to individual modes, such as the flutter mode(s). Another approach would be to use full-state feedback with modes deemed important and accurately estimated. This part of the control law design would require the full ASE model for gain tuning.

E. Collocation

The simplest control law pursued was based on the concept of collocation [42]. For aircraft aeroelastic problems, this approach has been called identically-located accelerometer and force (ILAF) [43–45]. The concept states that if a velocity is fed back to a collocated force input, damping can be increased for all observable and controllable modes. This idea is usually implemented by integrating an accelerometer and feeding it back to a nearby control surface. The collocation concept is abused for aircraft control problems because pure collocation is seldom achieved, and because control surfaces do not just generate forces, but also moments and generalized forces.

The original ILAF design for the IAWTM was documented in Ref. [46]. This used the three ailerons for control. Because accelerometers were not collocated with control surfaces, nearby accelerometers were blended to produce a composite measurement near the local mean aerodynamic quarter chord. For example, the feedback signal for AILC was computed as

$$a_{zc} = (2 \cdot A110Z + 1 \cdot A303Z + 1 \cdot A111Z) / 4 \quad (24)$$

This blending also had the effect of averaging measurement noise and providing some redundancy in case of accelerometer drop-outs or failures.

In addition, a lag filter was used instead of a pure integrator to obtain velocity from measured acceleration. This simplified the implementation and obviated the need for integrator wind-up protection [44]. A washout filter was also included to roll off the controller at low frequencies. This element reduced controller sensitivity to bias errors in the accelerometers and changing tunnel conditions (e.g., creating different static-elastic deformations), in addition to facilitating the operation of other lower-frequency control laws such as gust or maneuver load alleviation.

An initial set of feedback gains were selected using the technique of sequential loop closures, first closing the AILC loop, then the AILB loop, and lastly the AILA loop. The root locus method was used in this process. Afterwards, LQR with output feedback [47] was used to optimize the gains. This was a numerical optimization, and the sequential loop closure solution was used as starting values for the gains.

As the control law developed, it was found that AILA was not effective for AFS, and that AILB was unreliable as a control input. The decision was made to only use AILC for flutter control. This increased the feedback gain and the control usage for AILC, and also decreased the robustness of the closed-loop system.

Figure 8 shows the root locus using only AILC for control. This feedback could successfully stabilize the unstable aeroelastic mode. However, a large gain was necessary, and this caused another higher-frequency mode to destabilize in the process because perfect collocation was not achieved and subsystem dynamics were included. After many attempts to stabilize the system with lead control and applying additional roll off, an inverted notch filter [48] was added to cancel the stabilizing zero for the unstable aeroelastic mode, and replace it further in the left-half plane. This had the effect of stabilizing the unstable mode with a lower gain while keeping the other higher-frequency modes stable. Care was taken in this design to make sure the new zero did not affect the expected trajectories of other closed-loop poles.

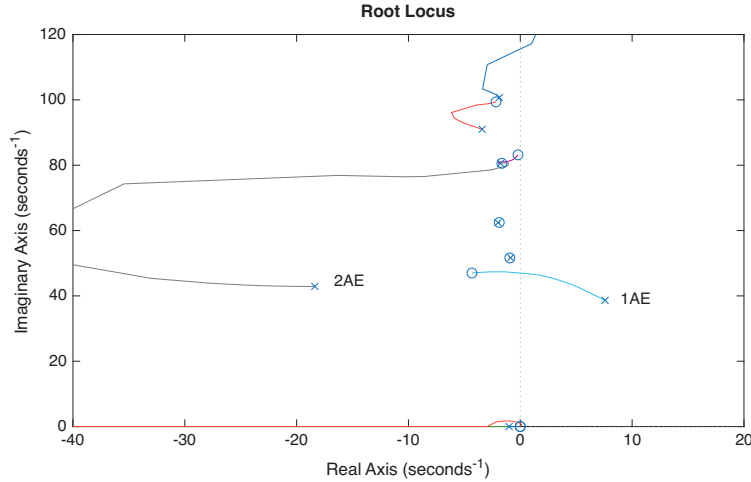


Fig. 8 Root locus of $a_{zc} \rightarrow$ AILC at the design point (Mach 0.85 and dynamic pressure 230 lbf/ft²).

After the first day of flutter testing, it was also found that the twisting of the wing tip created a case pressure on the accelerometers A303Z and A111Z near the wing tip, causing those signals to drop out intermittently during testing. The decision was made to remove these measurements from the feedback, and only use the sensor A110Z. This not only reduced the noise averaging of the blended feedback, but also degraded the collocation and produced less-favorable root loci diagrams. Despite this, the feedback gain was selected at each tunnel condition to maximize the robustness.

The final transfer function for this control law design was

$$\frac{AILC(s)}{A110Z(s)} = \left(\frac{s}{s+1} \right) \left(\frac{K}{s+1} \right) \left[\frac{s^2 + 2\zeta_n\omega_n s + \omega_n^2}{s^2 + 2\zeta_d\omega_d s + \omega_d^2} \right] \quad (25)$$

The second-order poles for the inverse notch filter (determined by ζ_d and ω_d) were scheduled to exactly cancel the zero attracting the unstable pole. The numerator parameters, which define the new zero location, were set to $\zeta_n = 0.06$ and $\omega_n = 48$ rad/s, for all tunnel conditions. The gains K were optimized to minimize the peak of the sensitivity transfer function, which maximizes the distance of the open-loop frequency response from the critical point and maximizes robustness in the classical sense.

The feedback gain and inverse notch zero location were tabulated for all 468 linear design models. Linear interpolation was performed for these parameters as a function of the tunnel Mach number and dynamic pressure, which were also low-pass filtered with a cutoff frequency of 1 rad/s.

V. Wind-Tunnel Test Results

A. Control Law Implementation and Testing Procedure

The control laws were implemented with Simulink[®] software, version 2017b, and dSPACE[®] hardware. Figure 9 shows a screenshot of the top level of the Simulink. The blue box near the center of the figure contains the control laws. These models were made in discrete time and ran at 200 Hz.

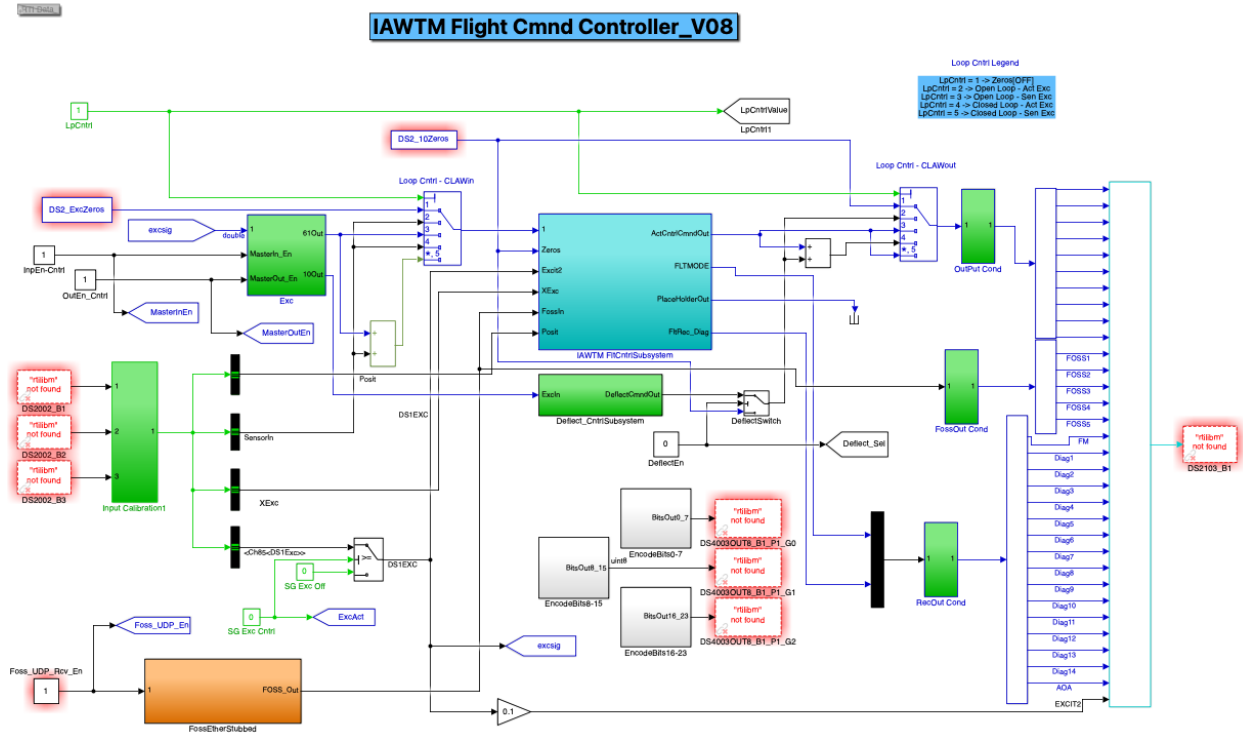


Fig. 9 Software implementation of control laws in Simulink.

The flutter testing procedure was to bring the tunnel up to Mach 0.3 and about 33 lbf/ft² of dynamic pressure. Note that Mach 0.55 was safely achieved in a previous open-loop test without control laws active. The accelerometers were zeroed out to account for the static elastic deformations at this condition. A control law was then turned on and ran continuously until that test run was completed. A doublet was then injected on AILB with a 1-pulse length of 0.08 s to perturb the system and excite the 40 rad/s flutter frequency. If the response appeared sufficiently damped to the test engineers, a multisine input was applied to compute stability margins, as discussed in Section V.D. Afterwards, the tunnel speed was increased, going up the total pressure line of 700 lbf/ft², and this process repeated. Figure 10 shows representative test points planned prior to testing.

During testing, several test engineers were monitoring several parameters including model vibrations, strain gauge loads and limits, accelerometer outputs, control signal time histories, etc. Three test engineers had “chicken switches” in hand to open bypass valves and very quickly drop the dynamic pressure inside the test section to save the model if it

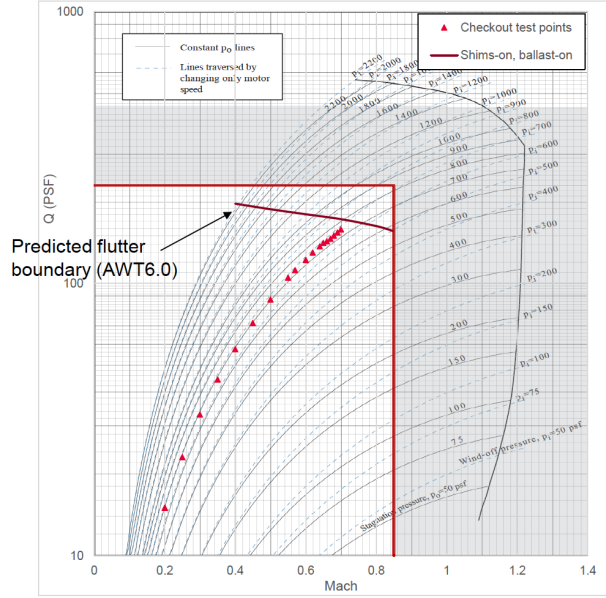


Fig. 10 Planned test points for flutter testing.

began to flutter. Testing procedures and safety features of the TDT are discussed more in Refs. [6, 7].

For various reasons, only three control law designs were actually tested during the two days allocated for flutter testing during the second entry into the wind tunnel. The first was the Hamiltonian controller discussed in Section IV.A. The second was the ILAF controller from Section IV.E. And the third was the GPC controller from Section IV.B.

During testing, it was observed that the two vertical accelerometers at the wing tip, A303Z and A111Z, were producing intermittent drop outs. This was due to the structure of the wing twisting, putting pressure on the accelerometer cases, and leading to degraded sensor outputs. In addition, it was determined that AILA was ineffective for AFS and the behavior of AILB was not reliable. As such, the Hamiltonian and ILAF controllers were quickly reconfigured to not use these sensors and actuators. Similarly, the GPC controller switched from using accelerometer feedback to strain gauge feedback, and weighted AILB less in the control commands.

B. Hamiltonian Controller Results

Flutter testing began at Mach 0.30 and the tunnel condition was increased in increments of 0.05 up to Mach 0.65. Above Mach 0.65, smaller Mach number increments were used. Mach 0.65 was the highest condition that was cleared during open-loop testing in the flutter suppression test configuration. During the closed-loop controller tests a doublet input was commanded on AILB to excite the dominant flutter mode at each Mach set point. Doublet amplitudes of 2, 3, and 4 deg were used.

Figure 11 shows the tip accelerometer A111Z response to the AILB doublet at Mach 0.45. The red trace (left) corresponds to the open-loop response with the Hamiltonian AFS controller disabled; the black trace (right) shows the closed-loop response with the controller active. Figure 12 presents the fast Fourier transforms (FFTs) of these signals, and Fig. 13 shows the corresponding control command. Moderate attenuation of the accelerometer response is observed during closed-loop operation, particularly in the first mode near 3 Hz, which is clearly reduced in the FFT.

Figure 14 similarly shows the A111Z response but at Mach 0.55, along with the open- and closed-loop responses plotted as before. The FFTs are given in Fig. 15, along with the associated control command in Fig. 16. Little attenuation is evident in this case as compared to the previous test point, although a slight reduction is visible near the 7.5 Hz mode. The lower-frequency modal response also occurs at a higher frequency than at Mach 0.45.

Figure 17 presents the A111Z response at Mach 0.65, again comparing the open-loop and closed-loop behavior. The corresponding FFTs are shown in Fig. 18. Closed-loop attenuation is again apparent, most notably in the first mode near 3 Hz, which is clearly diminished. The two modal responses were beginning to coalesce near about 6 Hz. Figure 19 shows the corresponding control signal.

Figure 20 shows the accelerometer response and control command at Mach 0.72, and Fig. 21 provides the FFT.

Open-loop data were not obtained at this Mach number, but the closed-loop response exhibits a marked increase in amplitude, suggesting proximity to instability. Testing was halted at this point due to large modal responses to preserve test article safety.

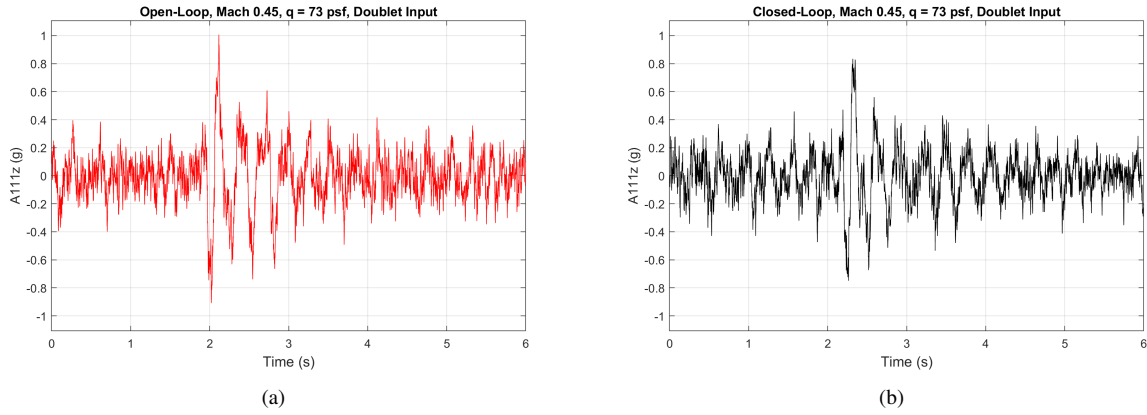


Fig. 11 Tip Accelerometer A111Z response comparison at Mach 0.45, $q = 73 \text{ lbf/ft}^2$.

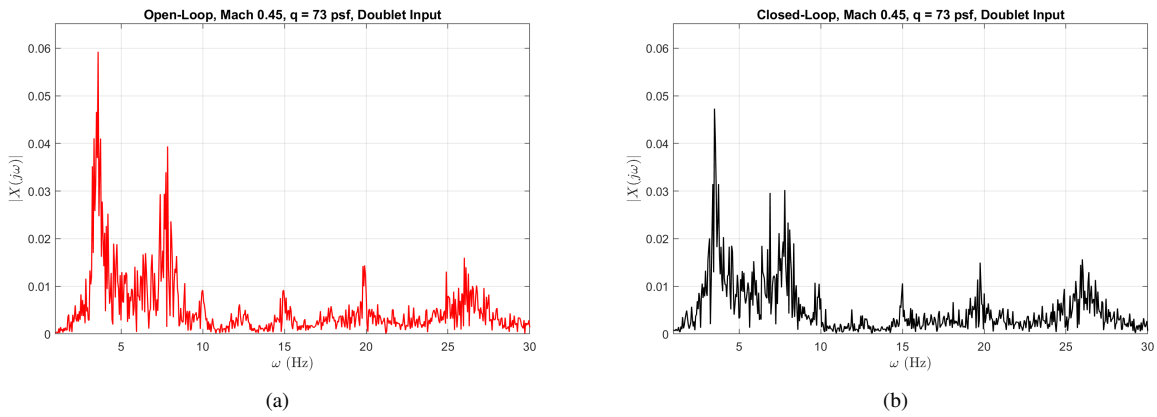


Fig. 12 Tip accelerometer A111Z FFT comparison at Mach 0.45, $q = 73 \text{ lbf/ft}^2$.

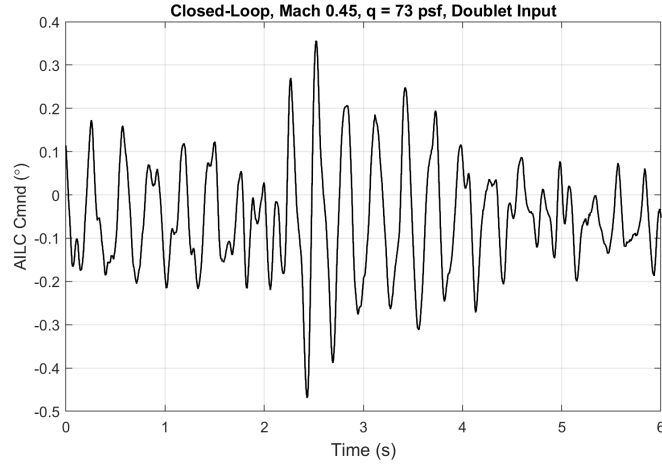
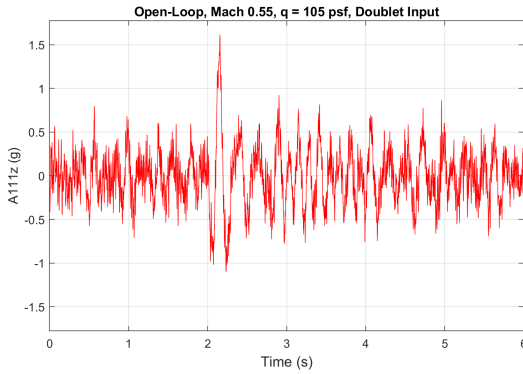
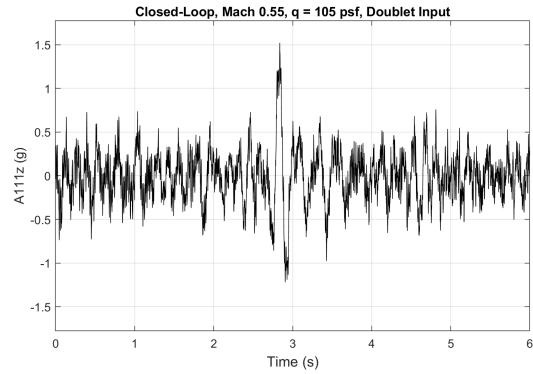


Fig. 13 Control command at Mach 0.45, $q = 73 \text{ lbf/ft}^2$.

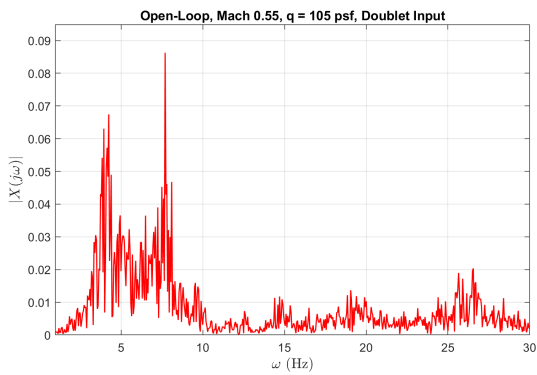


(a)

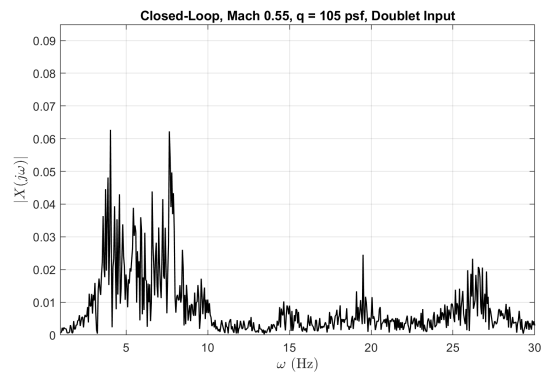


(b)

Fig. 14 Tip accelerometer response comparison at Mach 0.55, $q = 105 \text{ lbf/ft}^2$.



(a)



(b)

Fig. 15 Tip accelerometer FFT comparison at Mach 0.55, $q = 105 \text{ lbf/ft}^2$.

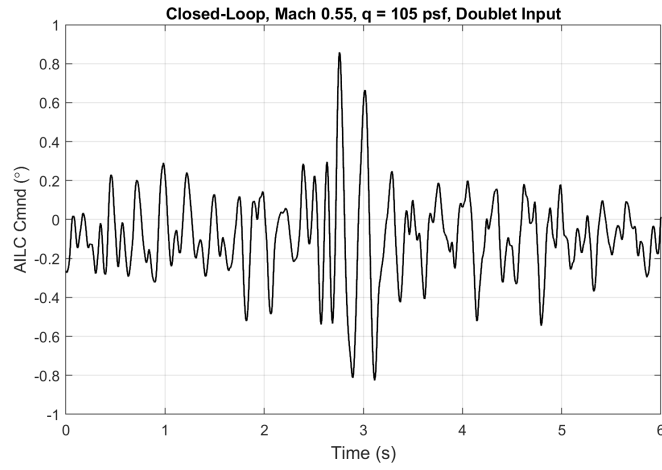
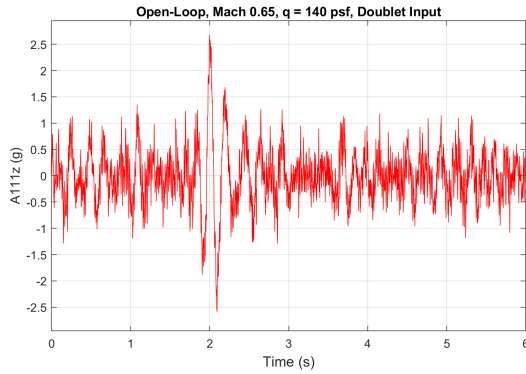
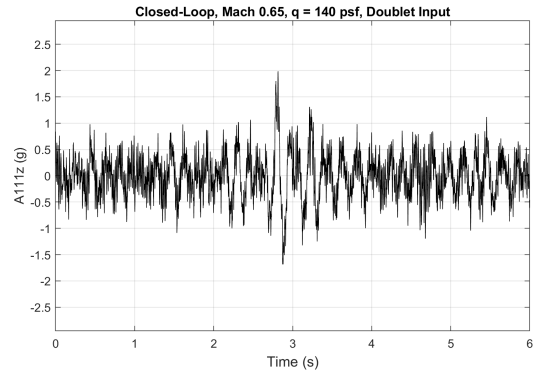


Fig. 16 Control command at Mach 0.55, $q = 105 \text{ lbf/ft}^2$.

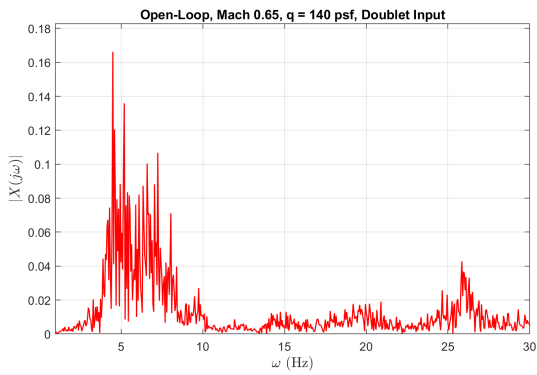


(a)

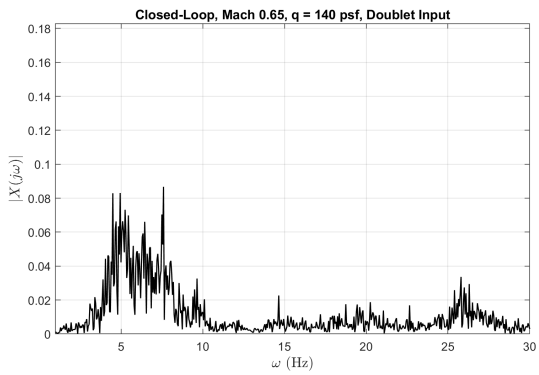


(b)

Fig. 17 Tip accelerometer response comparison at Mach 0.65, $q = 140 \text{ lbf/ft}^2$.



(a)



(b)

Fig. 18 Tip accelerometer FFT comparison at Mach 0.65, $q = 140 \text{ lbf/ft}^2$.

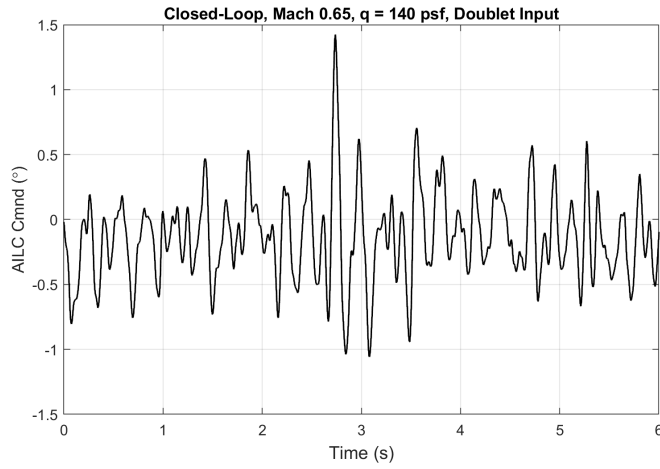
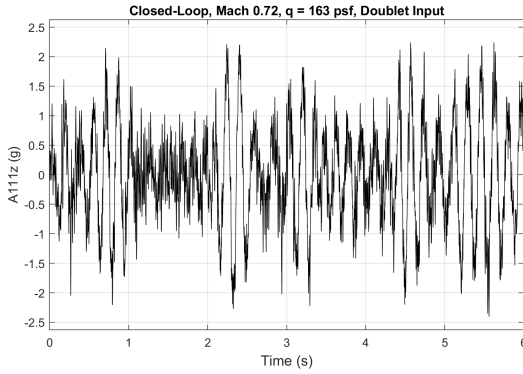
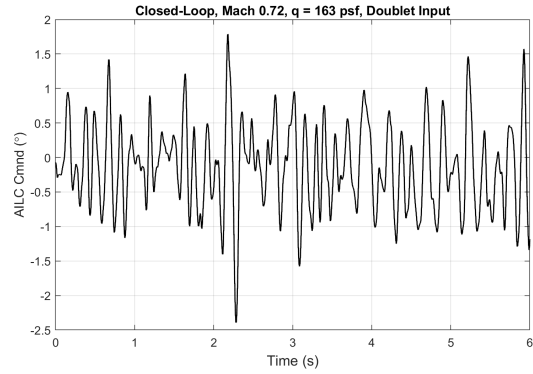


Fig. 19 Control command at Mach 0.65, $q = 140 \text{ lbf/ft}^2$.



(a) Tip accelerometer response



(b) Control command

Fig. 20 Hamiltonian flutter suppression response at Mach 0.72, $q = 163 \text{ lbf/ft}^2$.

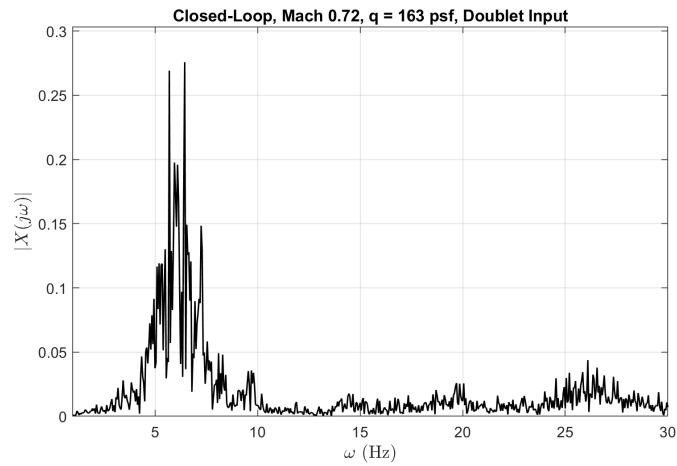


Fig. 21 Tip accelerometer FFT at Mach 0.72, $q = 163 \text{ lbf/ft}^2$.

C. ILAF Controller Results

The ILAF controller was turned on at Mach 0.3 without incident. The tunnel speed was increased and several test points were taken at Mach 0.3 and 0.4 as the overall controller gain was increased and assessed based on the visual response of the wing and the control command signal.

Beyond those conditions, the controller did not perform as expected. Figure 22 shows the controller frequency response at Mach 0.60 and dynamic pressure 121 lbf/ft². From Eq. (25), the controller basically acts as a gain and integrator above 1 rad/s, except for the inverted notch filter resonance at a higher frequency. Below 1 rad/s, the controller is rolled off due to the washout filter and lag filter approximation of the integrator.

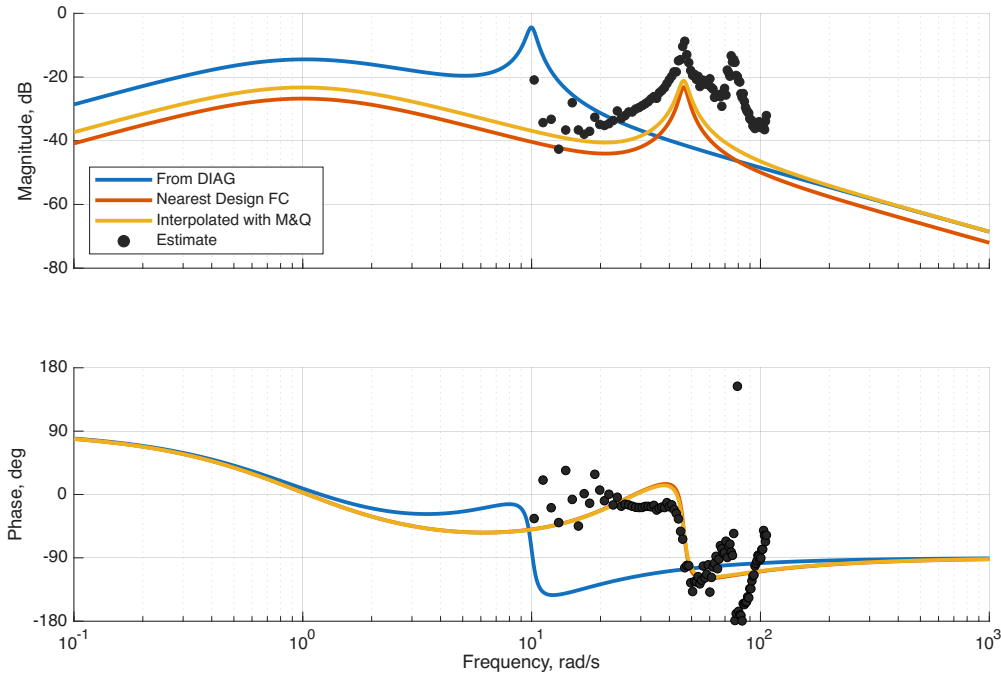


Fig. 22 ILAF controller frequency responses (Mach 0.60 and dynamic pressure 121 lbf/ft²).

The blue line in Fig. 22 is the control law reconstructed from diagnostic signals (i.e., the controller output variables “DIAG” created for debugging) outputting the scheduled controller parameters. The red and orange lines are the intended controllers based on the nearest flight condition (FC), which was computed using the measured tunnel Mach number and dynamic pressure, and linear interpolation. The black dots show frequency response estimates at the frequencies in the multisine excitation. Based on the observed scatter and engineering experience, the experimental frequency response estimates are expected to be accurate from about 20 to 100 rad/s.

Although the interpolation appears to be working properly, there are discrepancies between the diagnostic signals and the experimental frequency response estimates. It is believed that these differences are due to the controller implementation and inexperience with the dSPACE system. Despite these differences and a lack in expected performance, the modified ILAF control law (using only control effector AILC and sensor A110Z) was able to stabilize the system up to Mach 0.68 and dynamic pressure 151 lbf/ft². Testing was halted at this point to preserve model safety.

D. Stability Margin Estimation

An effort was made to estimate the open-loop frequency responses of the system during testing to evaluate classical robustness metrics. This procedure can be performed in real time to help improve the safety and efficiency of experimental testing [49, 50] and could be implemented in future testing. However, only results from post-test analysis are shown here.

During testing of the Hamiltonian and ILAF control laws, a multisine input [12] was injected to the command path

between the control law and actuator. Three multisines were designed, one for each aileron, which all had the form

$$\delta_j(t) = \sum_{k=1}^M a_k \sin\left(\frac{2\pi k}{T}t + \phi_k\right) \quad (26)$$

for $j = 4, 9$, and 10 , corresponding to the ailerons in Fig. 2. The duration of the input was chosen as $T = 65.536$ s, which is typical of excitations used for aeroelastic modeling, to provide sufficient frequency resolution and to have a period equal to a power of 2 for other spectral analyses. The waveform was repeated, for a total duration of about 131 s, to improve data quality. The harmonics k between 105 and 2084 were used, which corresponded to frequencies between about 1.6 and 31.8 Hz. The harmonics, which produce orthogonal sinusoids, are distributed to the different ailerons in an alternating manner so they can be run simultaneously during one maneuver without correlating the input data. Every 10th harmonic was used for each input (so that each multisine had 198 excitation frequencies), which left enough harmonics to also design additional multisine excitations for the MPFs, if desired. The phase angles ϕ_k were optimized for minimum peak factor for each multisine. The amplitudes a_k were set to a uniform spectrum, and the excitation was scaled to unit amplitude. Figure 23 shows the three multisines for a 1 deg normalized amplitude. During testing, a 2 deg amplitude was most often used.

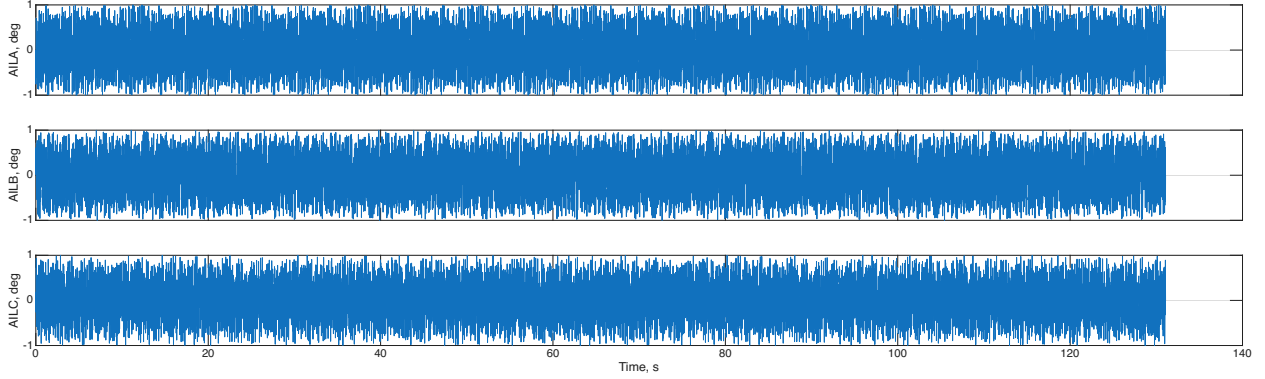


Fig. 23 Multisine excitations for open-loop frequency response estimation.

The control law output y , and commands into the actuators u (after the multisine excitations) were measured during testing. From these data, the frequency response for the corresponding open-loop system, at the excitation frequencies ω_k , was computed as

$$L(j\omega_k) = -\frac{y(j\omega_k)}{u(j\omega_k)} \quad (27)$$

where $y(j\omega)$ and $u(j\omega)$ are the Fourier transforms of the output and input using a chirp-z transform. The negative sign in Eq. (27) reflects the standard convention for negative feedback. Classical robustness metrics were then computed from the experimental frequency response data.

Figure 24 shows robustness metrics for the Hamiltonian controller. The left side shows the gain margin, phase margin, and disk margin (distance to the critical point in the complex plane), whereas the associated crossover frequencies are shown on the right side. Positive margins are shown in blue, and negative margins are shown in red; both markers are computed from experimental data.

The results show that as the tunnel speed increased, the controller experienced a slow and steady decline in robustness. The appearance of negative gain margins above Mach 0.7 indicates the open-loop system was unstable at those test points and that flutter suppression was achieved.

Figure 25 shows robustness metrics for the ILAF controller. There are multiple values for some Mach numbers because the gains were changed in real time and tests were repeated with the same tunnel conditions. In general, as the tunnel speed increased, robustness metrics decreased. Where they exist, the expected metrics are shown as the diamond markers. The experimental results are significantly below the expected results, again indicating an implementation error, as mentioned in Section V.C, or model errors.

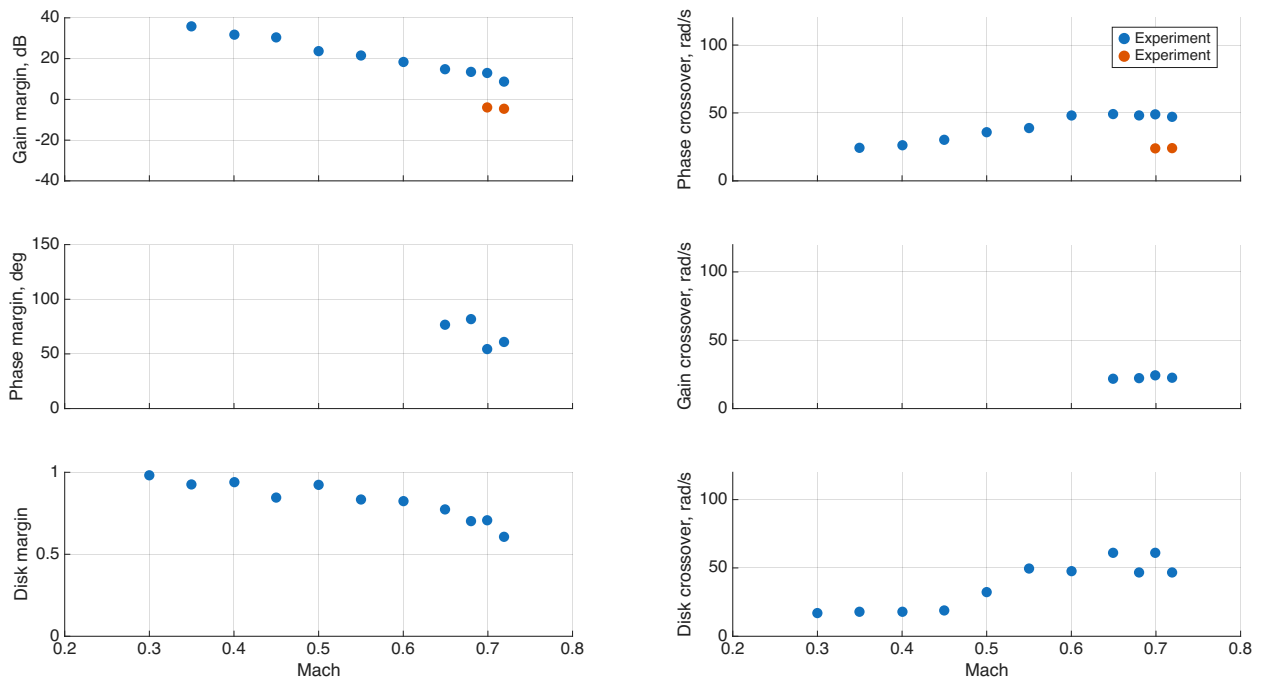


Fig. 24 Estimated robustness metrics, Hamiltonian controller.

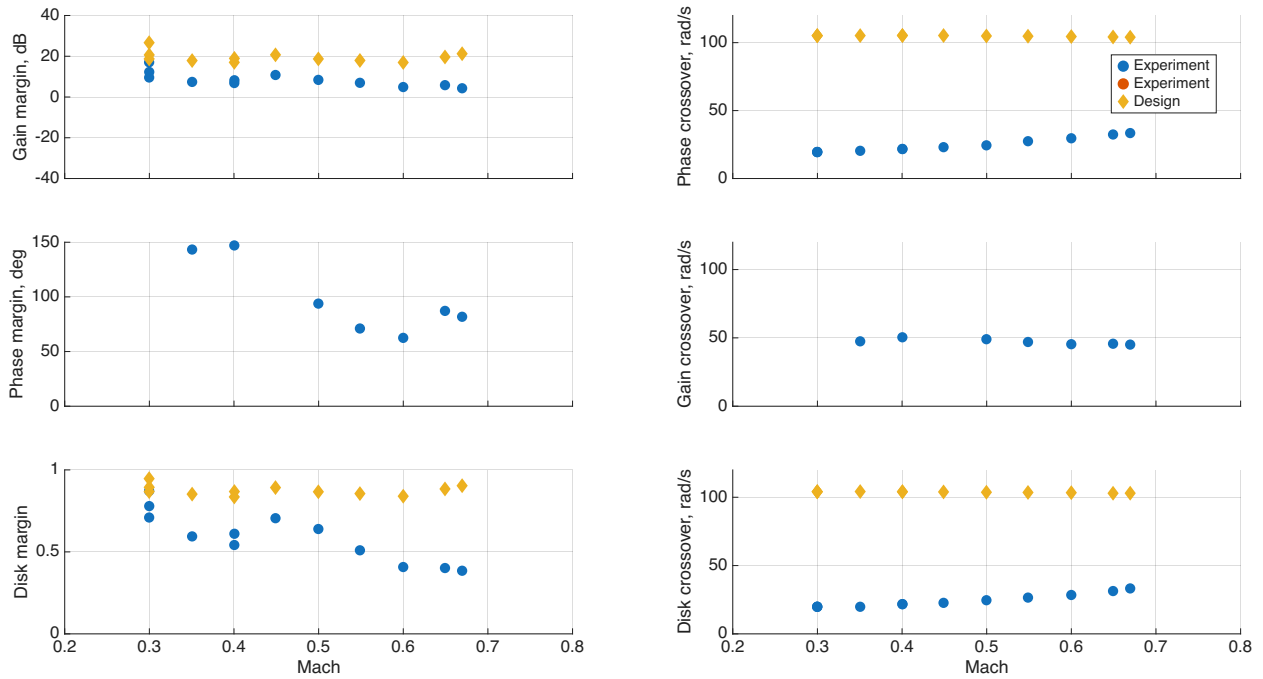


Fig. 25 Estimated robustness metrics, ILAF controller.

E. GPC Controller Results

Only moderate demonstration of GPC system operation was achieved during the IAWTM wind-tunnel test due to (1) pretest preparation and operational constraints and (2) challenges with on-the-fly iterative controller adjustments. Since no pretest analytical controller performance simulations were conducted, the GPC testing was only planned to be conducted well within the verified open-loop stable region of the test envelope to mitigate risk of disabling controller if an instability was observed. In addition, as described below, the GPC controller was not implemented on the primary active control system but rather on an external system; this system typology restricted the available signal flow paths and operational challenges with estimating closed-loop stability using the externally input multisines as described in Section V.D. To mitigate these preparation and operational constraints, for future testing (1) performance of the GPC controller should be analytically simulated and verified prior to testing to increase confidence in the robustness of the controller so that testing could be conducted in less stable regions of the test envelope, and (2) the signal flow between primary and secondary active control systems should be tested and validated prior to testing or the GPC active controller should be implemented within the primary active control system.

During testing, on-the-fly adjustments of the input sensor set and weighting matrices were conducted to improve performance of the controller. Pretest analysis using the ASE models may have yielded a more optimal initial sensor set, but the final sensor set chosen heuristically based on known model dynamics appeared to be adequate for the controller. As new system identifications were completed at each test condition, the weighting matrices were iteratively modified to improve their qualitative goodness. Due to the requirement to operate within the open-loop stable region, the closed-loop performance of the controller for suppressing vibration and flutter was challenging to demonstrate using each new set of weighting matrices. One interesting observation during this phase of testing, however, was that controllers developed using a system identification acquired at a lower tunnel condition (lower in M and q) were closed-loop unstable when turned on at higher tunnel conditions (increases in M and q). This will be one of the primary challenges with using GPC for an aeroelastic system with a hard flutter mechanism, such as this IAWTM test article: conducting the system identification at an open-loop stable test condition always yielded a closed-loop unstable controller as the tunnel condition was increased. This is a different behavior than that previously demonstrated for the TRAST model, where the whirl flutter mechanism is quite different in character. Work is ongoing to implement more realtime adjustments in the system identifications such that the GPC controller can be adjusted while still active, potentially enabling its use in testing where the flutter boundary is approached and the system dynamics are rapidly changing.

The GPC controller was configured to use all three ailerons for control effectors, and three inputs. The controller system was used previous tiltrotor stability research [51] and applied to the IAWTM in the same form. Due to practical time limitations, AGPC was not evaluated and GPC was not as thoroughly investigated in the present IAWTM research as it was for that previous tiltrotor stability research. However, this is the first known application of GPC to a fixed-wing aircraft for stability or vibration reduction, making this effort novel. The IAWTM model exhibited high modal damping at subcritical conditions making evaluation of the working controller difficult. Unlike previous tiltrotor work, the flutter boundary for IAWTM has a sharp drop in modal damping known as a hard flutter crossing. Therefore, evaluating the increase to modal damping as the boundary is approached incurs greater risk.

Even with the large damping of the IAWTM model, the ability of GPC to add modal suppression was demonstrated. The time history traces of open-loop and closed-loop responses are compared in Fig. 26. In this figure, the wing was excited in resonance with the ailerons, and then the excitation was stopped at approximately 10.75 seconds. The free response decayed rapidly due to the high structural damping of the model. Also included in Fig. 26 is the response when GPC is activated following the excitation using the three ailerons. For clarity, Fig. 26(b) shows a subset of the data records highlighting the time following excitation. Two vertical lines indicate the time that the excitation is halted and the time that GPC is subsequently activated. In Fig. 26, the time history trace with GPC active suppressed the vibration slightly faster than the open-loop free response.

Another demonstration of the controller effectiveness was shown by the ability to suppress resonant excitation of the wing. The time history of AILA command position and the response of strain gage SG10 is shown in Fig. 27. For the first part of the data record, the wing is excited in resonance. At approximately 10 seconds into the record, GPC was enabled, but the excitation command was left on and mixed with the AILB GPC command. In Fig. 27(a), the excitation was still large after GPC activation. However, in Fig. 27(b), the response of the wing was substantially reduced.

It should be noted that the commanded excitation was visible to the controller. This is because the GPC controller utilized a separate dSPACE system borrowed from recent tiltrotor testing and operated externally to the other systems. To implement GPC, the signal was routed through the external excitation channel in the nominal dSPACE controller. Therefore, an excitation external to the GPC controller was not possible due to the test configuration setup. A better comparison would have been an excitation that the GPC controller was not observing.

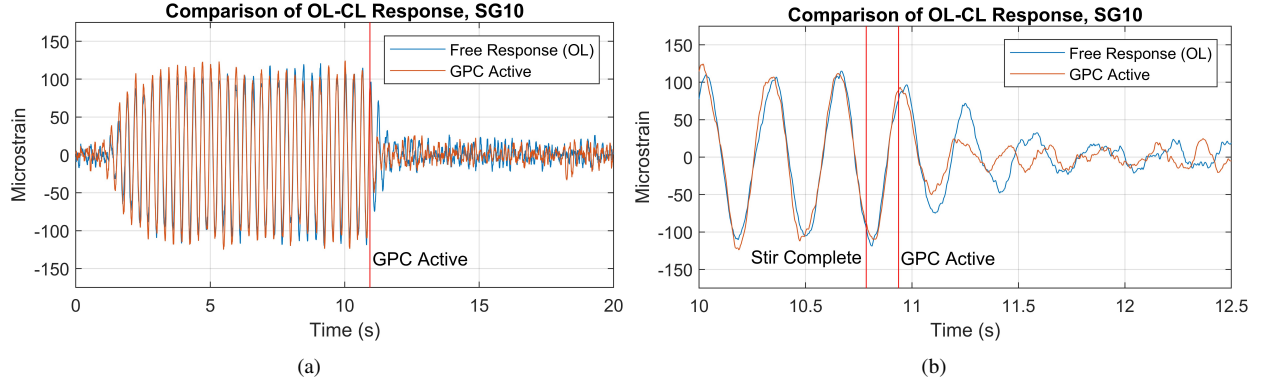


Fig. 26 Overlay of open-loop free response and GPC suppression (Mach 0.4 and dynamic pressure 58.5 lbf/ft²).

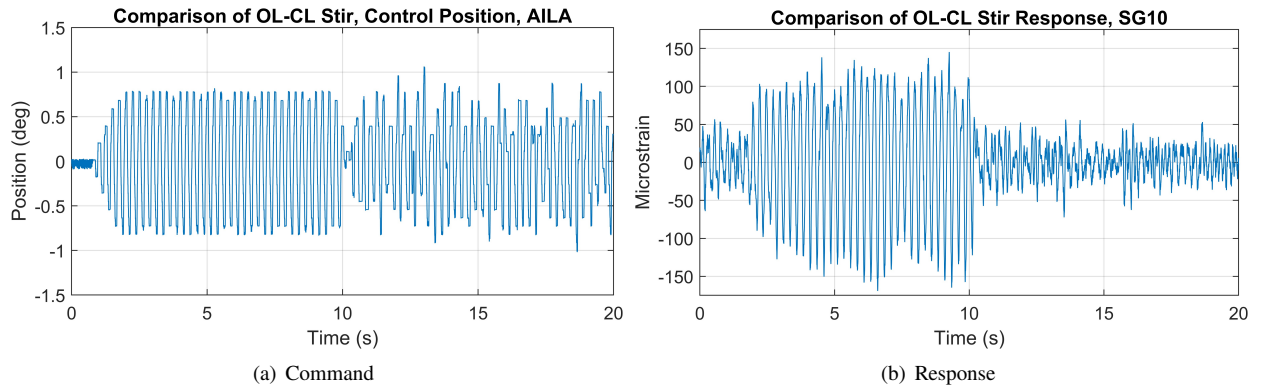


Fig. 27 Command and response, constant stir, GPC engaged mid-record (Mach 0.55 and dynamic pressure 105 lbf/ft²).

VI. Conclusions

This paper summarizes the efforts to design and implement active flutter suppression control laws for the IAWTM half-span, side wall-mounted, wind-tunnel test article. The paper presents all the control designs. Afterwards, wind-tunnel test results are discussed for three of the control laws.

Findings of this paper can be noted as the following:

- The active flutter control design for the IAWTM was difficult, both due to both the physics involved (e.g., many densely-spaced and coupled structural modes with the actuator bandwidth, control surface reversal, hard flutter mechanism, etc.) and practical considerations (e.g., control surfaces AILA and AILB, and vertical accelerometers A303Z and A111Z, were not reliable for feedback control). Practical limitations on the most effective sensors and actuators for controlling flutter limited the performance of designed control laws.
- A wide range of flutter controllers were designed, ranging from classical to modern to robust control laws, as well as experimental new controllers.
- Three flutter controllers were ultimately turned on over only two days of testing: Hamiltonian control, classical collocation / ILAF, and generalized predictive control.
- The Hamiltonian controller was able to stabilize the unstable model up to Mach 0.72 and a dynamic pressure of 163 lbf/ft².
- Although not available for real-time analysis in this test, the ability to inject excitations and compute open-loop frequency responses and classical robustness metrics (e.g., gain and phase margins) could be useful for future flutter tests to predict the onset of instability.

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