

High-Fidelity Flutter Prediction of Aspect Ratio 13.5 Flexible Wing Wind Tunnel Model

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This paper presents a high-fidelity computational approach for flutter prediction of an aspect ratio 13.5 Common Research Model wind tunnel model tested in the NASA Langley Research Center Transonic Dynamics Tunnel. A computationally efficient approach is proposed using a multi-sine, multi-mode, frequency-shifted bowed square wave train excitation to efficiently excite multiple structural dynamic modes over a wide frequency range with a single unsteady Reynolds-averaged Navier-Stokes computational fluid dynamics simulation in FUN3D. The generalized aerodynamic forces are extracted as a function of the reduced frequency. Reduced-order modeling is performed via Roger's rational function approximation to obtain a frequency-domain approximation of the generalized aerodynamic forces computed by FUN3D. By converting the frequency-domain approximation into a time-domain state-space realization, an eigenvalue analysis is conducted to compute the frequencies and damping ratios as functions of the tunnel dynamic pressure. The predicted flutter boundary is used as a guide to determine an allowable test envelope for the ensuing flutter suppression control experiment. This method aims to provide a more efficient and accurate prediction of flutter characteristics by capturing complex mode interactions. The predicted flutter boundary is compared to that predicted by NASTRANTM flutter solution. Comparison of the flutter prediction to limited wind tunnel test data at Mach 0.65 is made to validate the flutter solution.

I. Introduction

The current trend in the modern transport aircraft design is increasingly focused on eco-efficient technologies such as lightweight materials and high aspect ratio thin wing structures for improved energy efficiency. In the recent decades, the wing aspect ratios of transport aircraft continue to trend upward with some current aircraft configurations already employing wing aspect ratios greater than 10. The aircraft industry continues to push toward even greater wing aspect ratios. The Boeing Transonic Truss-Braced Wing (TTBW) employs a braced thin wing structure with an aspect ratio of 19.565,^{1,2} almost doubling a typical wing aspect ratio in the current aircraft design. The TTBW design alleviates the structural loads by the two main struts that brace the wing at the mid-span. Comparing a conventional cantilever wing to a truss-braced wing, each wing design affords certain advantages in terms of design complexity, aerodynamic efficiency, and operational considerations. A natural question may arise as to what is the maximum wing aspect ratio that a conventional cantilever wing design can safely support. The answer to this question requires a multidisciplinary approach in the analysis of the effects of materials and structures, aerodynamics, aeroelasticity, stability and control, as well as other multidisciplinary interactions on the overall aerodynamic efficiency, structural efficiency, flight safety, and passenger comfort.

As the wing aspect ratio continues to increase, the wing structural flexibility interacts significantly with the wing aerodynamics. The wing spanwise loading, hence wing aerolastic twist, can vary substantially during cruise flight.

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This results in a drag penalty that can offset the aerodynamic efficiency gain at the design point. Structural considerations also need to be taken into account. Extreme structural load excursions during gust encounters or during maneuvering flights can impose safety limits on the structural design which could offset the benefits afforded by a thin wing structural design. Flutter clearance is a certification challenge for ultra-high aspect ratio cantilever wing design. Other safety considerations include increased dynamic response of a highly flexible wing and aeroelastic interactions with the rigid aircraft flight dynamic motion, both of which could exacerbate stability and control as well as passenger ride qualities.

A study in 2010 identified a potential adaptive wing solution for flexible wing aircraft with a proposed distributed flap system called the variable camber continuous trailing edge flap (VCCTEF).³⁻⁵ The study also proposed a multi-objective wing shaping control to adaptively retwist the wing shape for drag reduction as well as maintaining flight control tasks. NASA and Boeing began to collaborate further on the development of the adaptive wing VCCTEF concept and conducted a series of proof-of-concept wind tunnel tests in collaboration with University of Washington to demonstrate drag reduction, high-lift, and gust load alleviation capabilities.⁶⁻⁹ This collaboration effort was followed by another program called the Integrated Adaptive Wing Technology Maturation (IAWTM) funded by the NASA Advanced Air Transport Technology (AATT) project in 2014. The IAWTM program is a joint Boeing-NASA collaborative effort that seeks to develop an ultra-high aspect ratio cantilever wing design and mature adaptive wing technologies for real-time drag reduction, gust and maneuver load alleviation, and active flutter suppression.^{10,11}

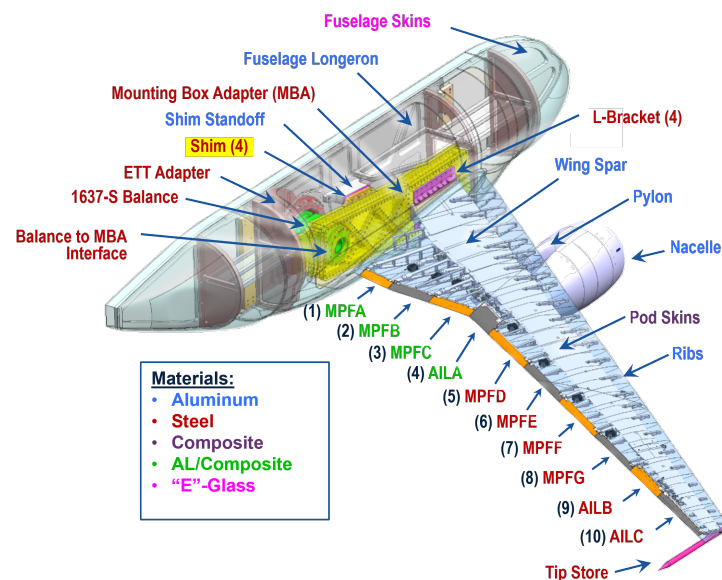


Figure 1. IAWTM Wind Tunnel Model of Aspect Ratio 13.5 CRM Wing Equipped with Ten Distributed Control Surfaces.

An aspect ratio 13.5 wing design has been developed by Boeing for demonstration of the high aspect ratio thin wing technology. The wing design is based on the Common Research Model (CRM) with the original wing aspect ratio of 9. Aerodynamic modifications of the baseline CRM design are made to increase the wing aspect ratio to 13.5. A distributed flap system that spans the entire wing is designed to incorporate functionalities to enable adaptive wing capabilities. A 10.8%-scale wind tunnel model is fabricated for active aeroservoelastic control validation. The wind tunnel model is equipped with ten distributed control surfaces and a wide range of onboard sensors designed for real-time drag optimization, gust and maneuver load alleviation, and active flutter suppression.^{10,11} A schematic diagram of the IAWTM wind tunnel model is illustrated in Figure 1 showing a cutaway view of the test article and the ten control surfaces with their corresponding designations. Three of these control surfaces, designated as AILA-AILC, are fast acting control surfaces driven by hydraulic actuators. These control surfaces are intended for the gust load alleviation and active flutter suppression controls. The other seven control surfaces, designated as MPFA-MPFG, are slower acting control surfaces driven by electric actuators. These control surfaces are used in conjunction with the fast acting control surfaces for real-time drag optimization and maneuver load alleviation controls. The wind tunnel model also includes a tip store design with a ballast mass to reduce the natural frequencies for the active flutter suppression experiment.

The wind tunnel model underwent a series of wind tunnel tests in NASA Langley Research Center Transonic Dynamics Tunnel (TDT) during the first test entry in 2024 and the second test entry in 2025.^{10,11} Figure 2 shows the IAWTM wind tunnel model installed in the TDT test section. The TDT is a closed-circuit, continuous-flow, variable-pressure wind tunnel capable of using either air or heavy gas R-134a for high Reynolds number testing up to 10 million per foot at Mach number up to 1.2. The TDT test section is 16 ft by 16 ft with four cropped corners. The wing semi-span of the test article is 12.75 ft which also includes a 1.25-inch stand-off spacer between the fuselage mid-plane and the test section wall to account for the wall boundary layer in the test section. The IAWTM wind tunnel tests were conducted with the heavy gas in the transonic Mach range up to 0.8.

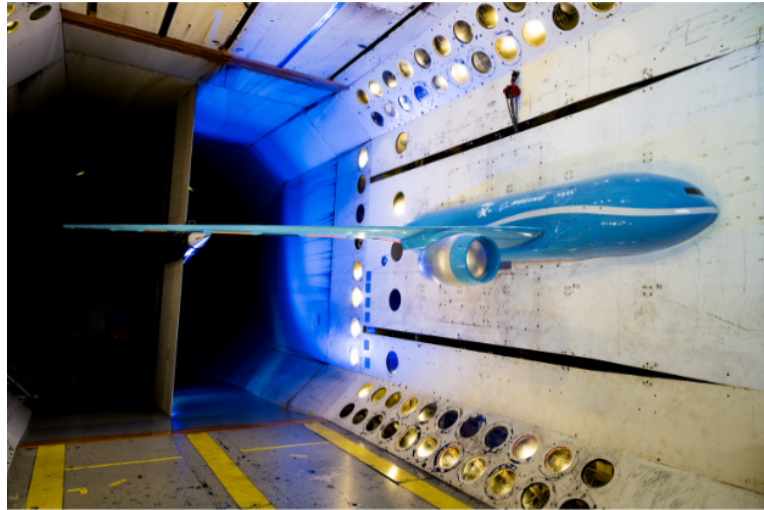


Figure 2. IAWTM Wind Tunnel Model Installed in Transonic Dynamics Tunnel Test Section (Credit: NASA / Mark Knopp).

During the first test entry, extensive aerodynamic and structural dynamic measurements were made to characterize the test article. The data collected are used to update the aerodynamic and aeroservoelastic (ASE) models of the test article. During the second test entry, closed-loop control laws were tested and the wind tunnel test objectives were successfully completed.

This paper reports a flutter analysis study for the IAWTM wind tunnel model using high-fidelity computational fluid dynamics (CFD). The paper also describes a recent development of a reduced-order modeling method using a multi-sine, multi-mode, frequency-shifted bowed square wave train excitation for obtaining the generalized aerodynamic forces using CFD. The high-fidelity flutter analysis is conducted to verify the initial flutter prediction performed in NASTRANTM. The predicted flutter boundary is used to establish safe limits for the active flutter suppression control experiment during the second test entry.

II. High-Fidelity Static Aeroelastic Analysis

A static aeroelastic analysis is initially performed to establish the static aeroelastic deformation of the test article. A NASTRANTM finite-element model (FEM) is first developed by Boeing. NASA and Boeing conduct numerous ground vibration tests (GVT) during the course of the program in an effort to validate the FEM.¹⁰ The final validated FEM, designated as AWT06.0, is delivered to NASA for use in the development of the ASE model and flutter analysis of the test article. The NASTRANTM structural mesh and mode shapes, shown in Figures 3 and 4, respectively, are prepared as inputs for the static aeroelastic analysis in FUN3D CFD. The FUN3D solver solves the steady-state Reynolds-averaged Navier-Stokes (RANS) equations on unstructured grids using a finite-volume method.^{12,13} The Roe flux difference splitting scheme is used in the current study. The Venkatakrishnan limiter is used in the current study to limit the reconstructed flux values. The one-equation model of Spalart-Allmaras (S-A) is used to model turbulence in this study. A second-order time-accurate solution is obtained by a semi-implicit backward Euler finite-difference time integration scheme. At each time step, the system of equations is approximately solved with either a multi-color point-implicit procedure or an implicit-line relaxation scheme. Local time-step scaling is employed to accelerate the solution convergence.

Figure 5 shows the surface mesh of the IAWTM test article geometry with a tip store design for the active flutter suppression wind tunnel experiment. The tip store includes a ballast mass designed to reduce the natural frequencies of the test article and promote modal coupling in order to move the flutter boundary inside the test envelope. To prevent damage to the wind tunnel balance, the test article is disconnected from the load transmission to the wind tunnel balance. This configuration is referred to as shims-on ballast-on configuration. A standoff spacer with a thickness of 1.25 inches is modeled to account for the wind tunnel wall boundary layer effects. Two CFD meshes are created to model the test article alone and the test article inside the TDT test section to account for the test section wall interference effect. Due to the test article size which spans 80% of the test section width, wall interference effect is one of the concerns. As a rule-of-thumb, the test article size should be limited to no greater than three quarter of the test section width in order to minimize the wall interference effect. A previous study has been conducted to investigate the test section wall interference effect on the IAWTM wind tunnel model.¹⁴ The study shows noticeable effects on the aerodynamic characteristics of the test article.

The CFD mesh for the test article alone has a total of 50 million grid points. The CFD mesh for the test article modeled with the TDT test section has a total of 70 million grid points. The volume mesh is comprised of tetrahedral elements and a prism layer near the wall. The prism layer is used to resolve the turbulent boundary layer. The y^+ value of the first cell from the wall is less than 1. Figure 6 shows the CFD mesh of the test article inside the test section of the TDT. The CFD model also includes the test section slots with the East wall slots closed, the plenum surrounding the test section, and the re-entry flaps that allow the air from the plenum to re-enter the wind tunnel downstream of the test section.

The local pressure on the CFD mesh is interpolated at the nodes of the NASTRANTM structural mesh. The generalized aerodynamic force (GAF) is computed for each mode shape. The generalized displacements are computed by the direct stiffness method in FUN3D. The static aeroelastic deformation is computed next by the Galerkin's method from the generalized displacements. A mesh deformation is then performed in FUN3D for the flow solution at the next iteration. This process is repeated until the solution converges.

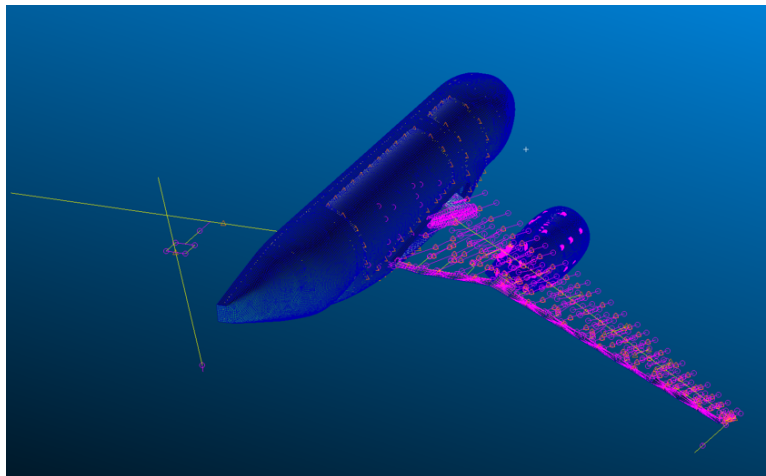


Figure 3. NASTRANTM Model of IAWTM Wind Tunnel Model.

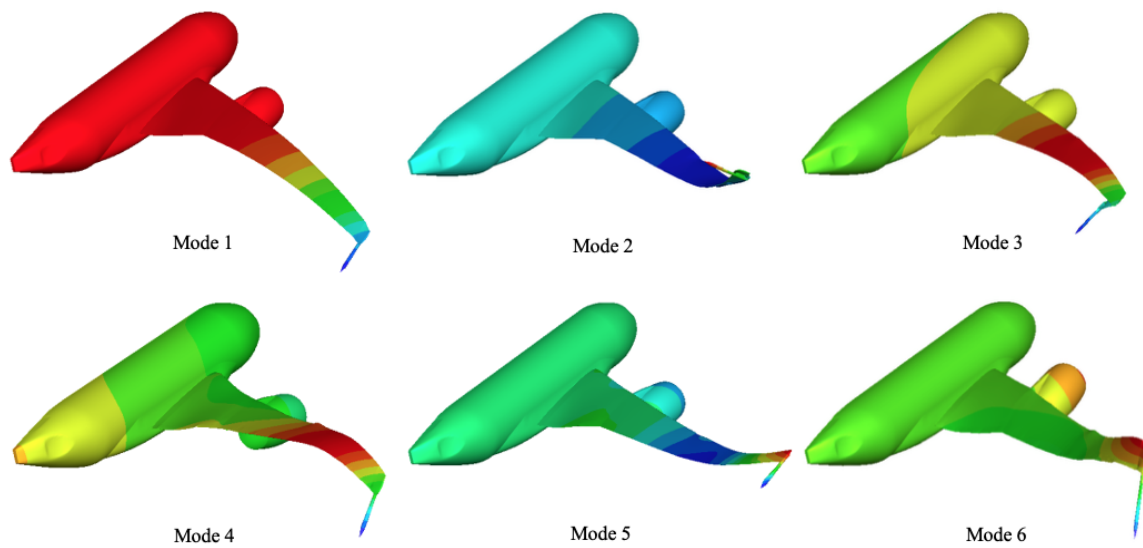


Figure 4. NASTRAN™ Mode Shapes Superimposed on FUN3D CFD Model.

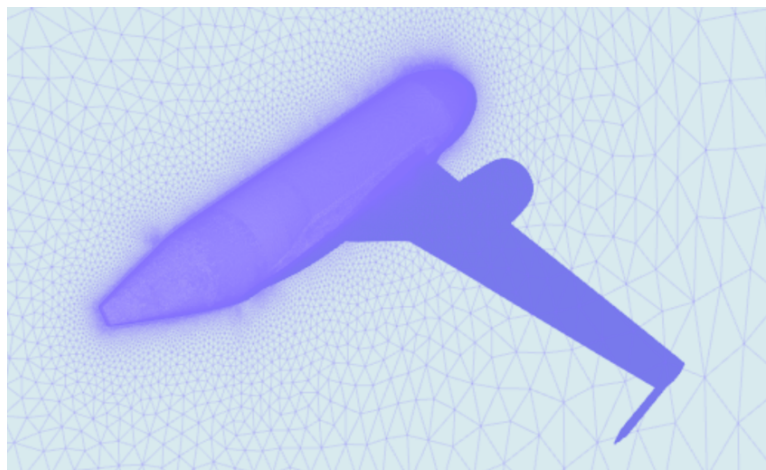


Figure 5. CFD Mesh of IAWTM Wind Tunnel Model Alone.

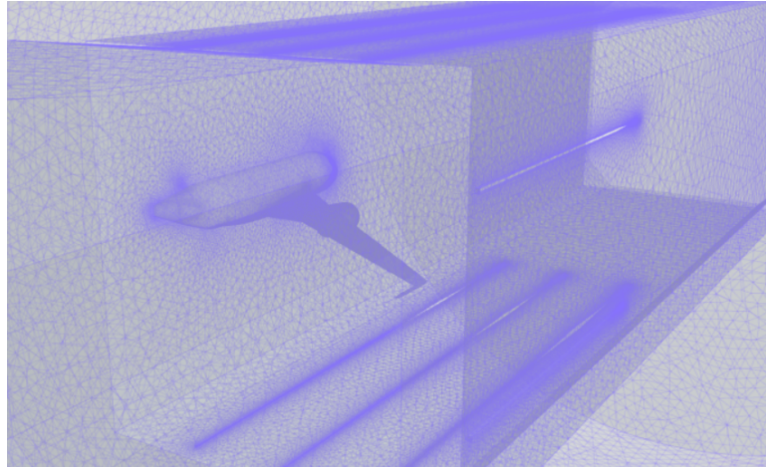


Figure 6. CFD Mesh of IAWTM Wind Tunnel Model Inside TDT Test Section.

Figures 7 and 8 show the vertical bending deflection and torsional twist about the elastic axis at zero degree angle of attack, respectively, computed by FUN3D for the IAWTM wind tunnel model alone without the test section wall interference. The wing tip deflects 15.14 inches and twists -5.65° about the elastic axis at Mach 0.8 and the tunnel dynamic pressure of 202.6 psf. This wing tip deflection is about 9.9% of the wing semi-span which approaches a rule-of-thumb value of 10% where structural nonlinearity may become significant.

A grid sensitivity study is conducted with a refined CFD mesh with 100 million grid points for the IAWTM wind tunnel model in free air. The static aeroelastic deformation is compared to determine the grid sensitivity. Figures 7 and 8 also show the comparison of the vertical bending deflection and torsional twist about the elastic axis at Mach 0.8 computed by the two mesh sizes. The percentage differences are 0.3% in the bending deflection and 0.6% in the torsional twist at the wing tip. Based on the static aeroelastic solutions, the 50-million grid mesh size is deemed adequate for this study.

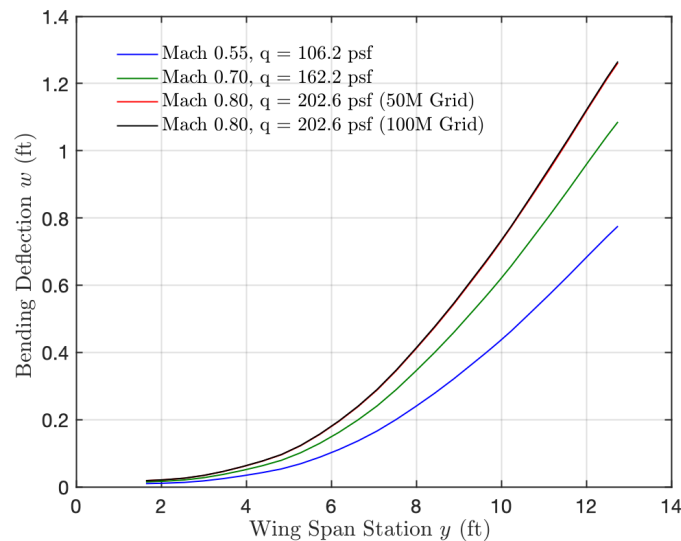


Figure 7. Wing Vertical Bending Deflection at Zero Degree Angle of Attack.

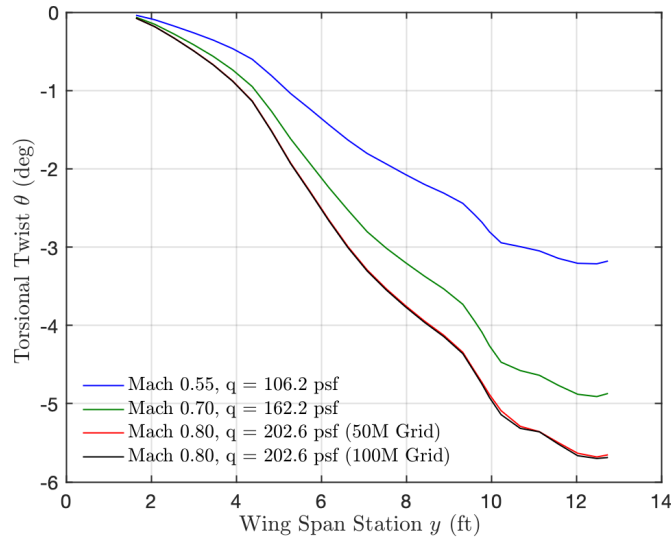


Figure 8. Wing Torsional Twist about Elastic Axis at Zero Degree Angle of Attack.

III. High-Fidelity Generalized Aerodynamic Force Analysis

A flutter analysis is performed initially in NASTRANTM to predict the flutter boundary of the IAWTM wind tunnel model in the shims-on ballast-on configuration for the active flutter suppression wind tunnel experiment. For the flutter analysis, the first 11 generalized stiffness values computed by NASTRANTM for the normalized generalized mass are replaced by the experimental values obtained from the natural frequency measurements from the GVT. In addition, a baseline of 2% structural damping or 1% viscous damping is used for the modal damping, except for the first 11 modes for which the experimental viscous damping ratios determined from the GVT are used. The flutter analysis is conducted in NASTRANTM using the doublet lattice method.

Mach	Flutter Dynamic Pressure (psf)
0.40	224.68
0.50	211.74
0.60	200.65
0.70	190.34
0.80	177.95
0.85	170.08

Table 1. NASTRANTM-Predicted Flutter Dynamic Pressures of IAWTM Wind Tunnel Model in TDT.

In addition to the NASTRANTM flutter analysis, an effort to refine the flutter analysis using FUN3D CFD is pursued. The NASTRANTM doublet lattice flutter solution generally cannot capture the transonic unsteady aerodynamics. A concern is raised in that the transonic effect could be significant at the high-Mach test conditions for the active flutter suppression control experiment. Figure shows the critical pressure coefficient for sonic flow condition and the pressure coefficient on the upper surface of the test article predicted by FUN3D static aeroelastic solution. The pressure coefficient on the test article reaches the critical pressure coefficient at Mach 0.59. Another concern is identified in that the effect of the test section wall interference on flutter is unknown, given that the test article spans 80% of the test section width. As a result, the NASTRANTM flutter prediction could potentially be inaccurate. These concerns are to be resolved by using a high fidelity CFD approach.

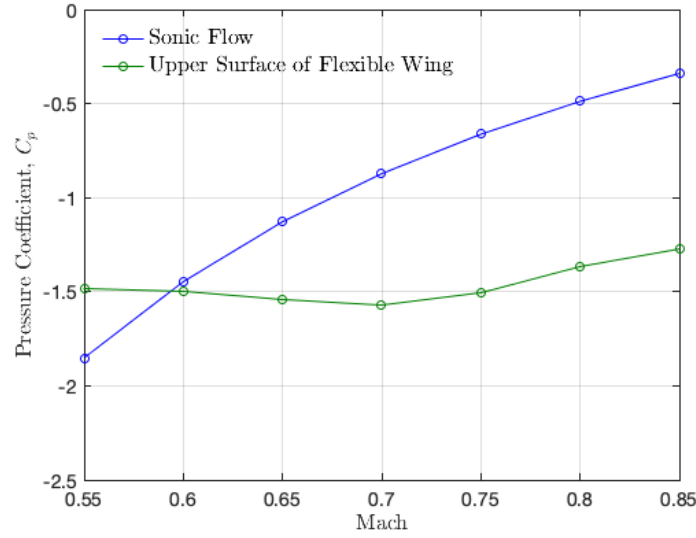


Figure 9. Pressure Coefficient of IAWTM Wind Tunnel Model versus Mach Number.

A high-fidelity flutter analysis is performed using the FUN3D CFD solver which solves the full unsteady Reynolds-averaged Navier-Stokes (uRANS) equations. The unsteady GAFs are computed by FUN3D by means of a moving mesh capability that dynamically deforms the mode shapes in a prescribed manner. The moving mode shapes are implemented in FUN3D relative to the static aeroelastically deformed geometry of the test article computed by the FUN3D static aeroelastic solution. A preliminary CFD simulation is performed to apply a sinusoidal oscillation of the second mode shape in order to provide an estimate on the execution time. In order to achieve sufficient convergence, three periods of oscillation are computed and the execution time is about 23 hours. There are twenty four mode shapes. In order to obtain a full matrix of GAFs over a wide range of frequencies, it would require $24 \times N_f$ CFD runs, where N_f is the number of discrete frequency points. The computational cost of doing these CFD runs would be prohibitive.

Multi-sine multi-mode excitation methods for flutter analysis have been used to reduce the computational cost for flutter analysis. One such method is based on the orthogonal input Walsh function.¹⁵ The Walsh function consists of trains of square pulses. The square pulse trains satisfy certain orthogonality properties. A proper orthogonal decomposition (POD) technique also has been used for flutter analysis.¹⁶ In this study, we propose two frequency-shifted wave train methods that can prescribe multiple frequencies to all the modes simultaneously. To allow all the modes to be excited simultaneously, the prescribed frequencies for all the modes are designed in a manner that they do not coincide or overlap. Since the prescribed frequencies are non-overlapping, the proposed wave train functions satisfy the orthogonality property. The first method is described by a fundamental square wave train that prescribes the generalized displacement of the j -th mode according to

$$q_j = q_0 \sum_{n=1,3,\dots}^{N_h} \frac{1}{n} \sin(n\omega_0 + \Delta\omega_j)t \quad (1)$$

where N_h is the number of odd-valued harmonics and $j = 1, \dots, N$ is the number of modes with $N = \frac{N_h-1}{2} + 1$.

To minimize any nonlinear response, the amplitude q_0 is selected to be a small-valued quantity. In this particular study, $q_0 = 0.01$ is selected. The fundamental frequency is selected to be $\omega_0 = 6\pi$ rad/sec. To prevent the GAF from having overlapping frequency components, a frequency shift $\Delta\omega_j$ for each mode specified by Eq. (2) is prescribed according to

$$\Delta\omega_j = \frac{2(j-1)\omega_0}{N} \quad (2)$$

The fundamental generalized displacement corresponding to $j = 1$ with $\Delta\omega_1 = 0$ produces a square wave train.

Figure 10 illustrates the first four generalized displacements produced by the frequency-shifted square wave train method for $N_h \rightarrow \infty$. The generalized displacement q_1 is a square wave train, but all other generalized displacements

appear as frequency-shifted amplitude-modulated wave trains. These wave trains are orthogonal due to the property

$$\frac{1}{T} \int_0^T q_i q_j dt = \frac{q_0^2}{2} \delta_{ij} \sum_{n=1,3,\dots}^{N_h} \frac{1}{n^2} \quad (3)$$

where δ_{ij} is the Kronecker delta.

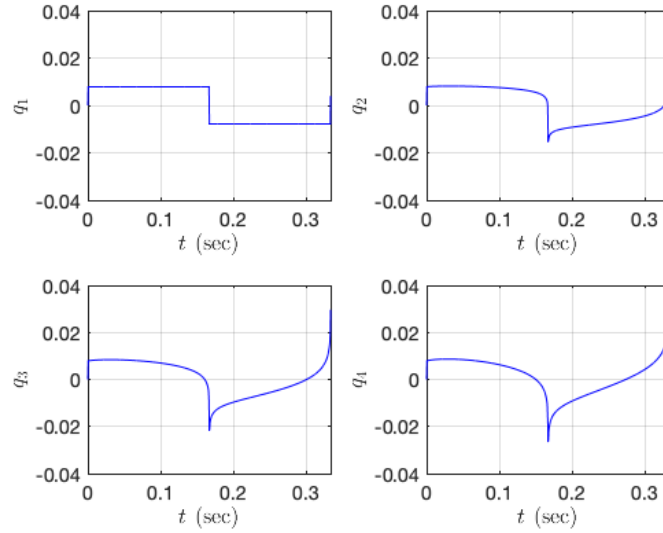


Figure 10. Generalized Displacements of Frequency-Shifted Square Wave Train Method.

The second excitation method is proposed to address potential shortcomings with the frequency-shifted square wave method. This method is referred to as the frequency-shifted 'bowed' square wave where the amplitudes of the individual sine waves decrease more slowly than those for the frequency-shifted square wave method. The prescribed generalized displacement of the j -th mode shape is specified by

$$q_j = q_0 \sum_{n=1,3,\dots}^{N_h} \frac{1}{\sqrt{n}} \sin(n\omega_0 + \Delta\omega_j)t \quad (4)$$

Figure 11 shows the first four generalized displacements produced by the frequency-shifted bowed square wave train method. The fundamental square wave for q_1 provides a flat amplitude in comparison to q_1 corresponding to the bowed square wave train which exhibits a convex or concave shape followed by sharp pulses at the zero crossover points. The frequency-shifted bowed square wave train is also orthogonal with the property

$$\frac{1}{T} \int_0^T q_i q_j dt = \frac{q_0^2}{2} \delta_{ij} \sum_{n=1,3,\dots}^{N_h} \frac{1}{n} \quad (5)$$

The energy of the signal is the integral square $\frac{1}{T} \int_0^T q_i^2 dt$. Thus, the bowed square wave train contains higher energy content than the square wave train. The higher energy improves the excitation amplitude and the GAF response. As N_h increases, the energy of the frequency-shifted square wave train excitation tends to a limit

$$\lim_{N_h \rightarrow \infty} \frac{1}{T} \int_0^T q_i^2 dt = \left(\frac{\pi q_0}{4} \right)^2 \quad (6)$$

This implies that as the number of harmonics N_h increases beyond some practical limit, the frequency-shifted square wave excitation provides little additional energy for excitation. On the other hand, the frequency-shifted bowed square wave excitation continues to add energy to the excitation as the number of harmonics increases.

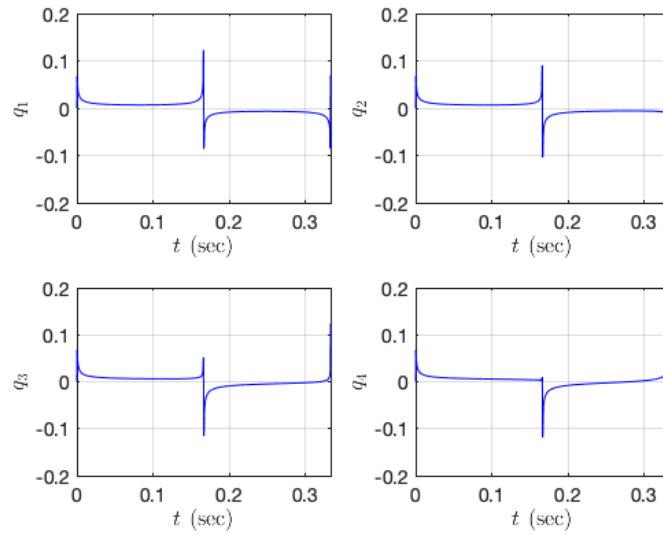


Figure 11. Generalized Displacements for Frequency-Shifted Bowed Square Wave Train Method.

Comparing to the frequency-shifted square wave excitation for which the amplitude of each harmonic varies as $\frac{1}{n}$, the amplitude of the frequency-shifted bowed square wave excitation which varies as $\frac{1}{\sqrt{n}}$ decreases much more slowly. The amplitude of the highest harmonic for the bowed square wave method is $\sqrt{N_h}$ times that for the square wave method. This is shown in Figure 12. Thus, the frequency-shifted bowed square wave method provides a larger amplitude of excitation as the frequency increases. As a result, improved accuracy in the GAF analysis can be better attained with the frequency-shifted bowed square wave method than with the frequency-shifted square wave method.

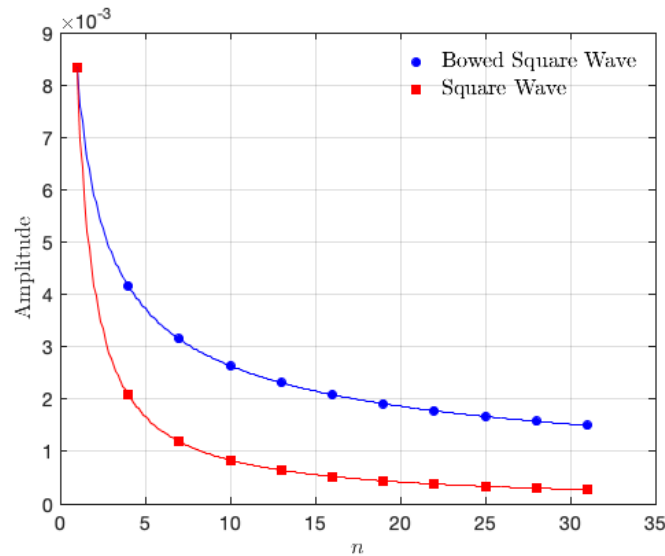


Figure 12. Generalized Displacement of Mode 1 for Frequency-Shifted Square Wave and Bowed Square Wave Methods.

In terms of the computational cost for this particular application, a sine wave excitation of each mode shape at a discrete frequency takes approximately 23 hours to complete on a 1100-core Linux cluster. On the other hand, the frequency-shifted bowed square excitation of 24 mode shapes at 16 frequencies simultaneously takes 182 hours to complete. Thus, if the discrete sine excitation is used, it would have required a wall clock time of 8832 hours, or 48

times that for the bowed square wave method.

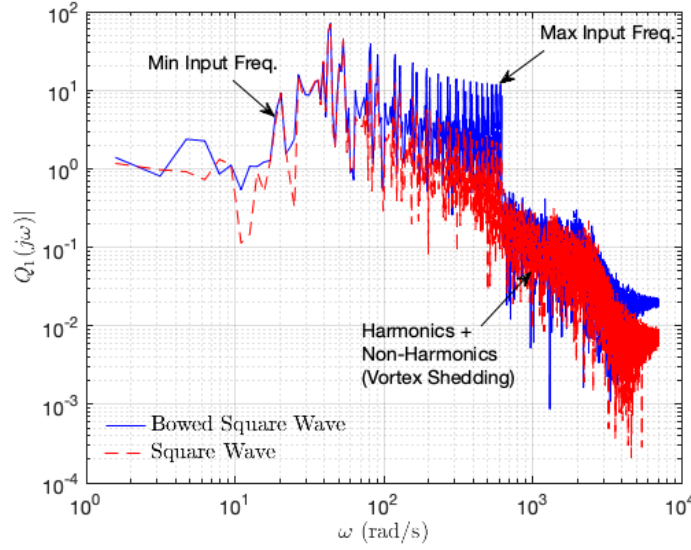


Figure 13. Fourier Transforms of Generalized Aerodynamic Force of Mode 1.

The comparison of the Fourier transforms of the GAF of mode 1 at Mach 0.8 for the frequency-shifted bowed square wave and square wave methods is shown in Figure 13. The Fourier transforms of the GAF of mode 1 at Mach 0.8 for both methods exhibits high frequency contents above the maximum input frequency. These high frequency contents include higher-order harmonics of the input frequencies which indicate a nonlinear behavior and non-harmonic frequencies due to potential vortex shedding. The frequency response for the frequency-shifted bowed square wave method is substantially higher than that for the frequency-shifted square wave method below the maximum input frequency. Above the maximum input frequency, the frequency responses for both methods appear to be similar up to about 500 rad/sec. This indicates that the nonlinear behavior and vortex shedding have a relatively stronger influence on the frequency response for the square wave method.

A reduced-order modeling (ROM) process is performed to model the GAF computed by CFD by means of numerical approximation. The GAF is modeled by a frequency response transfer function as

$$G_{ij}(k) = \frac{\partial Q_i}{\partial q_j} = \frac{1}{2} \rho V^2 A_{ij}(k) \quad (7)$$

where $A_{ij}(k)$ is an aerodynamic influence coefficient (AIC) of the GAF Q_i of the i -th mode due to the generalized displacement q_j of the j -th mode and $k = \frac{\omega \bar{c}}{2V}$ is the reduced frequency with $\bar{c}$ denoting the mean aerodynamic chord.

The AIC matrix in the ROM is modeled by the Rogers' rational function approximation method¹⁷ as

$$A_{ij}(\bar{s}) = a_0 + a_1 \bar{s} + a_2 \bar{s}^2 + \sum_{n=1}^4 \frac{a_{n+2} \bar{s}}{\bar{s} - p_n} \quad (8)$$

where $\bar{s} = ik = \frac{i\omega \bar{c}}{2V}$.

The underlying procedure in the construction of a ROM is the system identification method which is performed to identify the parameters of the ROM by a frequency-domain regression technique. The poles p_n are to be chosen to be negative real such that $-p_n < \max k$ so as to ensure stable unsteady aerodynamic response.

The element of the matrix of AIC $A_{ij}(\bar{s})$ is expressed as

$$A_{ij}(\bar{s}) = \frac{N(\bar{s})}{D(\bar{s})} = \frac{\sum_{n=0}^6 c_n \bar{s}^n}{\bar{s}^4 + \sum_{n=0}^3 b_n \bar{s}^n} \quad (9)$$

where the coefficients b_n are obtained from the poles p_n . Note that the identification of the ROM parameters is described in the appendix.

Figure 14 shows the AIC of the generalized force of mode 1 due to the generalized displacement of mode 1 for Mach 0.55 generated by the bowed square wave. The highly correlated data gives a very good approximation of the AIC A_{11} .

The reconstructed GAF from the ROM is expressed as

$$\hat{Q}_i = \sum_{j=1}^{N_m} G_{ij}(k) q_j \quad (10)$$

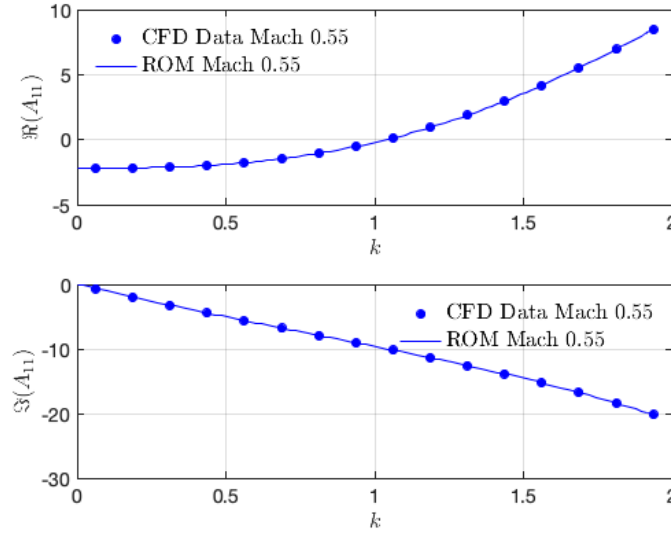


Figure 14. Aerodynamic Influence Coefficient of Generalized Aerodynamic Force of Mode 1 Due to Generalized Displacement of Mode 1.

The AIC A_{11} for Mach 0.55 for the frequency-shifted bowed square wave and the square wave methods are compared in Figure 15. Both the frequency-shifted bowed square wave and square wave excitation methods produce an excellent match in the AIC A_{11} . However, the same comparison at Mach 0.8 leads to a different conclusion. Figure 16 shows the AIC A_{11} for Mach 0.80 produced by the frequency-shifted bowed square wave and square wave methods. It is observed that the AIC A_{11} for Mach 0.8 is not as well correlated as that for Mach 0.55 for the bowed square wave as there is a noticeable degree of scatter in the CFD data. This scatter is likely due to the influence of the nonlinear response whereby the frequency response at an input frequency is not only due to the linear contribution but also the nonlinear contribution. For example, a cubic nonlinear behavior due to $\sin^3 \omega t$ contains not only $\sin \omega t$ which contributes to the linear frequency response, but also $\sin 3\omega t$ which appears as the third-order harmonic frequency response. The AIC A_{11} for the frequency-shifted square wave method exhibits a much greater degree of scatter. As a result, the ROM for the frequency-shifted square wave method at Mach 0.8 is poorly correlated, thereby resulting in a large error in the approximation and hence flutter prediction. To verify that the frequency-shifted bowed square wave method produces more accurate GAF, a series of discrete sine excitation of mode 1 at frequencies of 3 Hz, 39 Hz, 45 Hz, and 93 Hz are simulated in FUN3D. The values of the AIC A_{11} at these discrete frequencies are also plotted in Figure 16. These values are in very good agreement with the AIC A_{11} produced by the frequency-shifted bowed square wave method. On the other hand, the AIC A_{11} for Mach 0.8 produced by the frequency-shifted square wave method shows larger discrepancies compared to the AIC A_{11} produced by the discrete sine and the frequency-shifted bowed square wave methods. The analysis confirms the accuracy of the frequency-shifted bowed square wave method for computational efficiency in computing the GAF for flutter analysis using CFD.

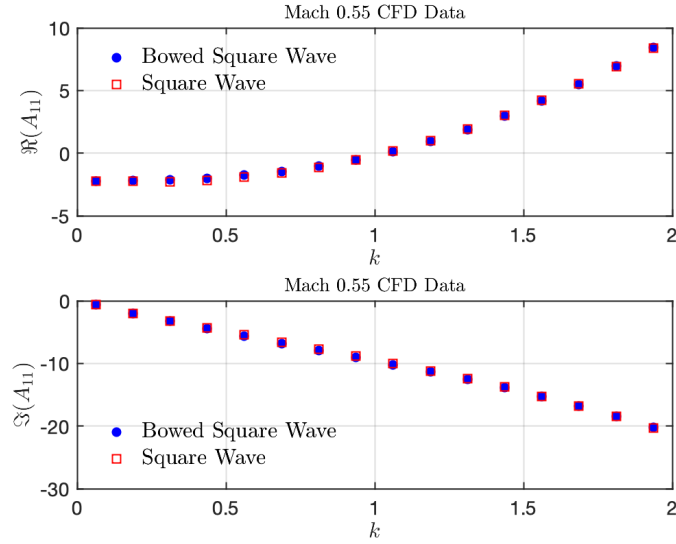


Figure 15. Comparison of Aerodynamic Influence Coefficient of Generalized Aerodynamic Force of Mode 1 Due to Generalized Displacement of Mode 1 at Mach 0.55 Between Frequency-Shifted Bowed Square Wave and Square Wave Methods.

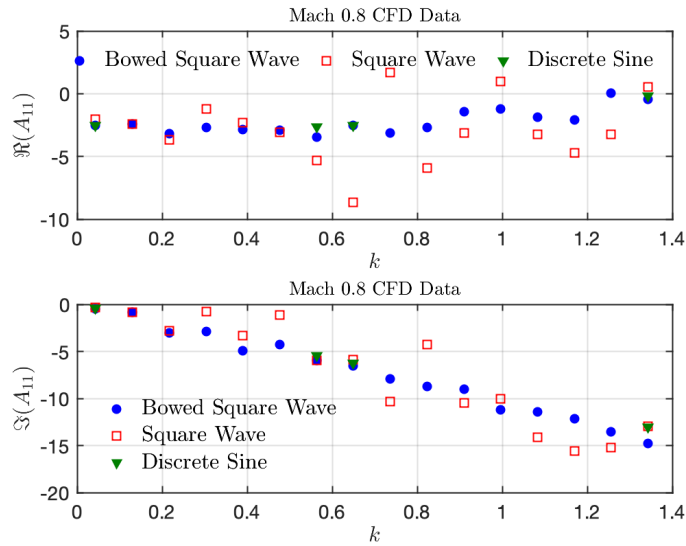


Figure 16. Comparison of Aerodynamic Influence Coefficient of Generalized Aerodynamic Force of Mode 1 Due to Generalized Displacement of Mode 1 at Mach 0.8 Between Frequency-Shifted Bowed Square Wave and Square Wave Methods.

At high Mach conditions, nonlinear transonic aerodynamics can affect the GAF analysis. Figure 17 illustrates this point. The Fourier transforms of the GAF Q_1 at Mach 0.8 due to the discrete sine excitation at 3 Hz, 39 Hz, 45 Hz, and 93 Hz are shown. For the 3 Hz and 39 Hz excitations, the frequency response is entirely due to the input frequency. At 45 Hz and 93 Hz, the frequency response contains both the input frequency and the third-order harmonic which suggests a cubic nonlinear frequency response. The nonlinear behavior is frequency-dependent and can affect the accuracy of the flutter prediction.

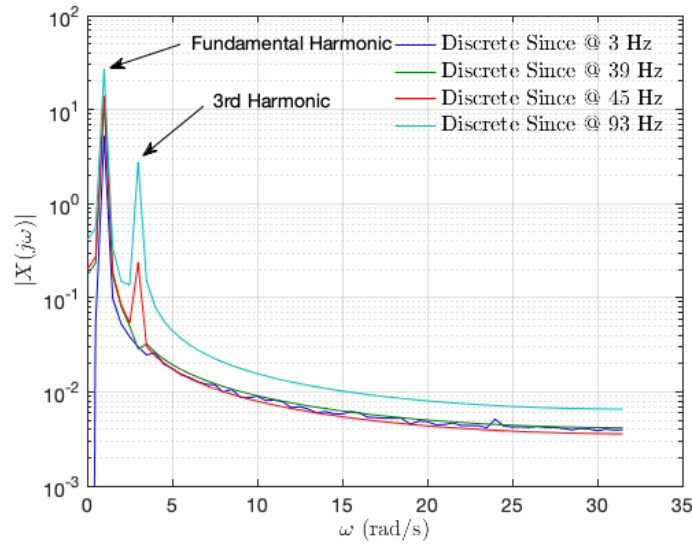


Figure 17. Nonlinear Frequency Response to Discrete Sine Excitation

Figure 18 shows the FUN3D-computed GAF Q_1 of mode 1 for Mach 0.8 produced by the frequency-shifted bowed square wave method and its reconstructed $\hat{Q}_1$ by the ROM. The agreement between the CFD-computed and the reconstructed GAF is excellent. For comparison, Figure 19 shows the CFD-computed GAF Q_1 produced by the frequency-shifted square wave method and its ROM. The accuracy of the reconstructed GAF is poorer with the frequency-shifted square wave method than with the bowed square wave method.

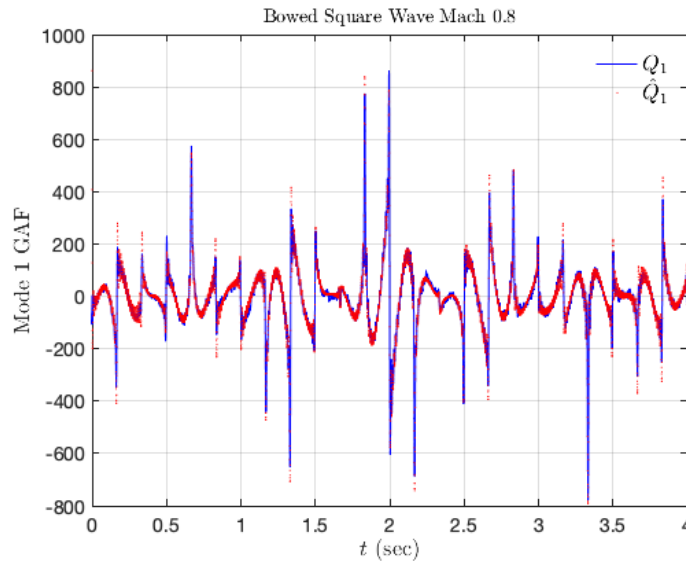


Figure 18. Generalized Aerodynamic Force of Mode 1 for Mach 0.8 by Frequency-Shifted Bowed Square Wave Method

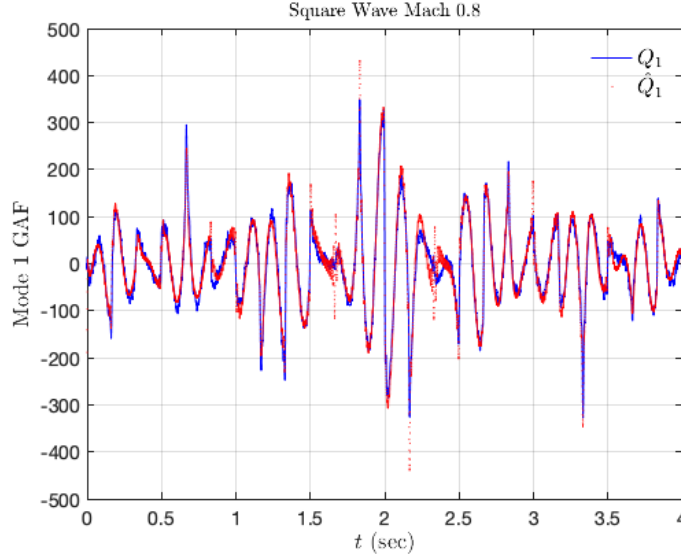


Figure 19. Generalized Aerodynamic Force of Mode 1 for Mach 0.8 by Frequency-Shifted Square Wave Method

IV. Flutter Analysis

The time-domain representation of the reconstructed GAF is expressed in general as

$$\hat{Q}_i = \sum_{j=1}^N G_{ij} q_j = \frac{1}{2} \rho V^2 \sum_{j=1}^N \left(a_{0,ij} q_j + a_{1,ij} \frac{\dot{q} \bar{c}}{2V} + a_{2,ij} \frac{\ddot{q} \bar{c}^2}{4V^2} + \sum_{n=1}^4 b_{n,ij} r_{n,ij} \right) \quad (11)$$

We introduce the unsteady aerodynamic state variable $r_{n,ij}(t) : [0, \infty) \rightarrow \mathbb{R}^{4N^2}$, $n = 1, \dots, 4$, in this expression. The aerodynamic state variable is generated by the dynamic equation of the unsteady aerodynamic response given by

$$\dot{r}_{n,ij} = \frac{2V}{\bar{c}} p_{n,ij} r_{n,ij} + \dot{q}_j \quad (12)$$

We write the reconstructed GAF more compactly in a matrix form as

$$\frac{\mathbf{Q}}{\frac{1}{2} \rho V^2} = \mathbf{A}_0 \mathbf{q} + \frac{\bar{c}}{2V} \mathbf{A}_1 \dot{\mathbf{q}} + \frac{\bar{c}^2}{4V^2} \mathbf{A}_2 \ddot{\mathbf{q}} + \sum_{n=1}^4 \mathbf{B}_n \mathbf{r}_n \quad (13)$$

$$\dot{\mathbf{r}}_n = \mathbf{C}_n \mathbf{r}_n + \mathbf{D} \dot{\mathbf{q}} \quad (14)$$

where $\mathbf{Q} = [\hat{Q}_1 \quad \dots \quad \hat{Q}_N]^\top$, $\mathbf{q} = [q_1 \quad \dots \quad q_N]^\top$, $\mathbf{r}_n = [r_{n,11} \quad \dots \quad r_{n,1N} \quad \dots \quad r_{n,N1} \quad \dots \quad r_{n,NN}]^\top$, and

$$\mathbf{A}_0 = \begin{bmatrix} a_{0,11} & \dots & a_{0,1N} \\ \vdots & \ddots & \vdots \\ a_{0,N1} & \dots & a_{0,NN} \end{bmatrix} \quad (15)$$

$$\mathbf{A}_1 = \frac{\bar{c}}{2V} \begin{bmatrix} a_{1,11} & \dots & a_{1,1N} \\ \vdots & \ddots & \vdots \\ a_{1,N1} & \dots & a_{1,NN} \end{bmatrix} \quad (16)$$

$$\mathbf{A}_2 = \frac{\bar{c}^2}{4V^2} \begin{bmatrix} a_{2,11} & \dots & a_{2,1N} \\ \vdots & \ddots & \vdots \\ a_{2,N1} & \dots & a_{2,NN} \end{bmatrix} \quad (17)$$

$$\mathbf{B}_n = \begin{bmatrix} a_{n+2,11} & \cdots & a_{n+2,1N} & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & a_{n+2,N1} & \cdots & a_{n+2,NN} \end{bmatrix} \quad (18)$$

$$\mathbf{C}_n = \frac{2V}{\bar{c}} \begin{bmatrix} p_{n,11} & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & p_{n,1N} & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & p_{n,N1} & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & p_{n,NN} \end{bmatrix} \quad (19)$$

$$\mathbf{D} = \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix} \quad (20)$$

We further express the GAF in the form

$$\mathbf{Q} = -\mathbf{K}_a \mathbf{q} - \mathbf{C}_a \dot{\mathbf{q}} - \mathbf{M}_a \ddot{\mathbf{q}} + \mathbf{Q}_r \mathbf{r} \quad (21)$$

$$\dot{\mathbf{r}} = \mathbf{A}_r \mathbf{r} + \mathbf{B}_r \dot{\mathbf{q}} \quad (22)$$

where $\mathbf{M}_a$, $\mathbf{C}_a$, and $\mathbf{K}_a$ are the generalized aerodynamic mass, damping, and stiffness matrices defined as

$$\mathbf{K}_a = -\frac{1}{2} \rho V^2 \mathbf{A}_0 \quad (23)$$

$$\mathbf{M}_a = -\frac{1}{2} \rho V^2 \frac{\bar{c}^2}{4V^2} \mathbf{A}_2 \quad (24)$$

$$\mathbf{C}_a = -\frac{1}{2} \rho V^2 \frac{\bar{c}}{2V} \mathbf{A}_1 \quad (25)$$

and $\mathbf{r} = \begin{bmatrix} \mathbf{r}_1^\top & \mathbf{r}_2^\top & \mathbf{r}_3^\top & \mathbf{r}_4^\top \end{bmatrix}^\top$ along with

$$\mathbf{Q}_r = \frac{1}{2} \rho V^2 \begin{bmatrix} \mathbf{B}_1 & \mathbf{B}_2 & \mathbf{B}_3 & \mathbf{B}_4 \end{bmatrix} \quad (26)$$

$$\mathbf{A}_r = \begin{bmatrix} \mathbf{C}_1 & 0 & 0 & 0 \\ 0 & \mathbf{C}_2 & 0 & 0 \\ 0 & 0 & \mathbf{C}_3 & 0 \\ 0 & 0 & 0 & \mathbf{C}_4 \end{bmatrix} \quad (27)$$

$$\mathbf{B}_r = \begin{bmatrix} \mathbf{D} \\ \mathbf{D} \\ \mathbf{D} \\ \mathbf{D} \end{bmatrix} \quad (28)$$

Finally, the combined aeroelastic equation is expressed as

$$\begin{bmatrix} \dot{\mathbf{q}} \\ \ddot{\mathbf{q}} \\ \dot{\mathbf{r}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} \\ -(\mathbf{M} + \mathbf{M}_a)^{-1}(\mathbf{K} + \mathbf{K}_a) & -(\mathbf{M} + \mathbf{M}_a)^{-1}(\mathbf{C} + \mathbf{C}_a) & (\mathbf{M} + \mathbf{M}_a)^{-1}\mathbf{Q}_r \\ \mathbf{0} & \mathbf{B}_r & \mathbf{A}_r \end{bmatrix} \begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \\ \mathbf{r} \end{bmatrix} \quad (29)$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the generalized structural mass, damping, and stiffness matrices.

This equation is written succinctly as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} \quad (30)$$

where $\mathbf{x} = \begin{bmatrix} \mathbf{q}^\top & \dot{\mathbf{q}}^\top & \mathbf{r}^\top \end{bmatrix}^\top$.

The flutter solution at a given Mach number is obtained by varying the dynamic pressure until the real part of one of the eigenvalues of the $\mathbf{A}$ matrix crosses zero. By varying the Mach number in the GAF analysis, a flutter boundary is obtained. Figures 20 through 23 show the $q_\infty - \omega$ and $q_\infty - g$ flutter diagrams for Mach 0.55, Mach 0.65, Mach 0.7, and Mach 0.8, respectively, using the frequency-shifted bowed square wave method. The flutter dynamic pressures are determined by the zero crossovers in the $q_\infty - g$ flutter diagrams as shown in Table 2. The flutter mode is mode 2 at Mach 0.55 and mode 1 at Mach 0.65, Mach 0.7, and Mach 0.8. It should be noted that these results correspond to the IAWTM wind tunnel model alone. Results for the IAWTM wind tunnel model inside the TDT test section will be reported at a future date.

Mach	Flutter Dynamic Pressure (psf)	Flutter Mode		Flutter Frequency (Hz)
0.55	204.84	2	204.60	6.4072
0.65	193.57	1	192.24	6.2101
0.70	186.93	1		6.0954
0.80	174.56	1		6.0749

Table 2. FUN3D-Predicted Flutter Dynamic Pressures of IAWTM Wind Tunnel Model in TDT

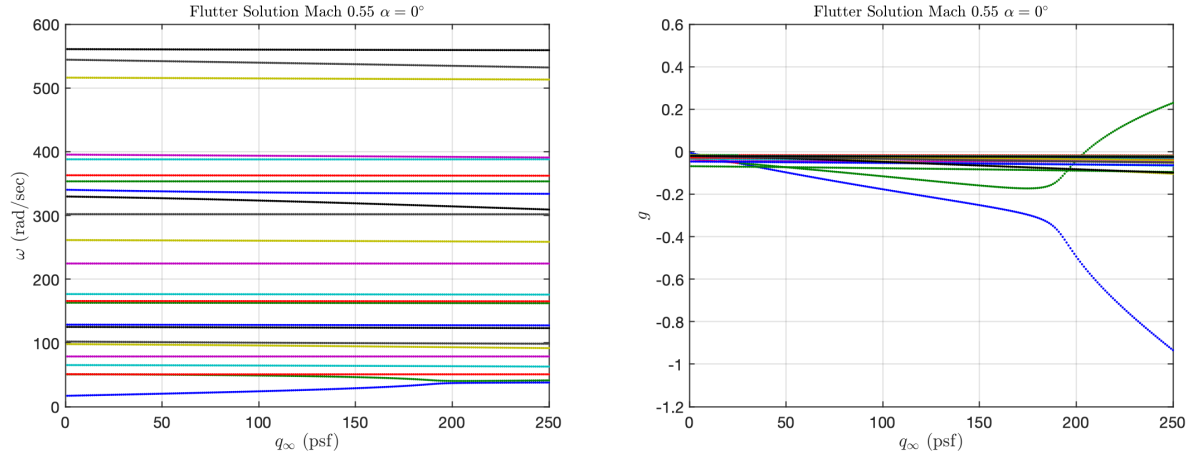


Figure 20. Flutter Diagrams at Mach 0.55

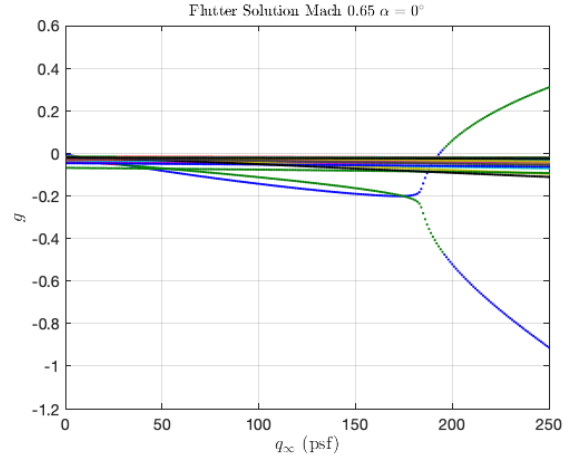
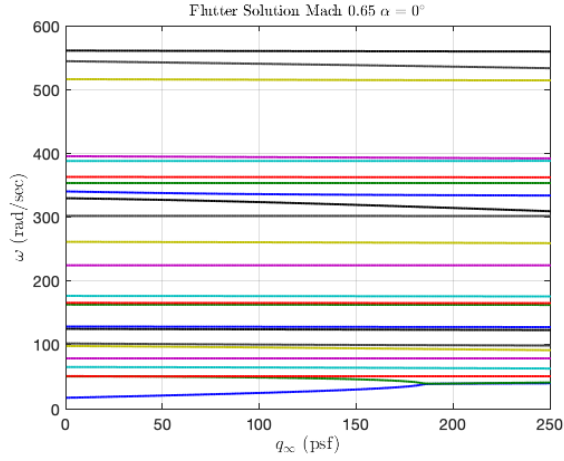


Figure 21. Flutter Diagrams at Mach 0.65

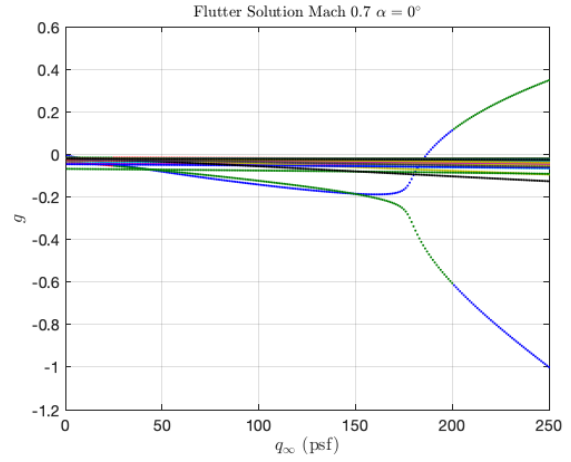
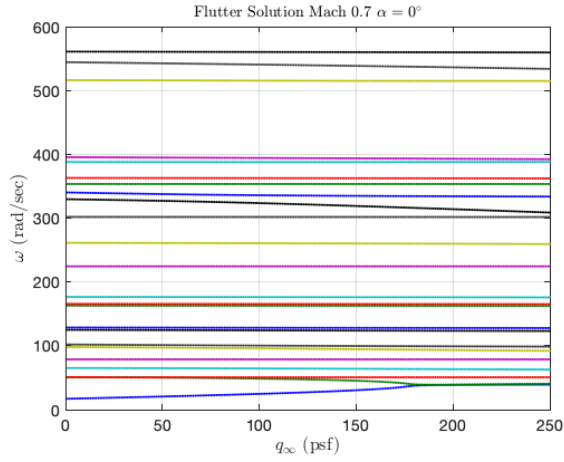


Figure 22. Flutter Diagrams at Mach 0.7

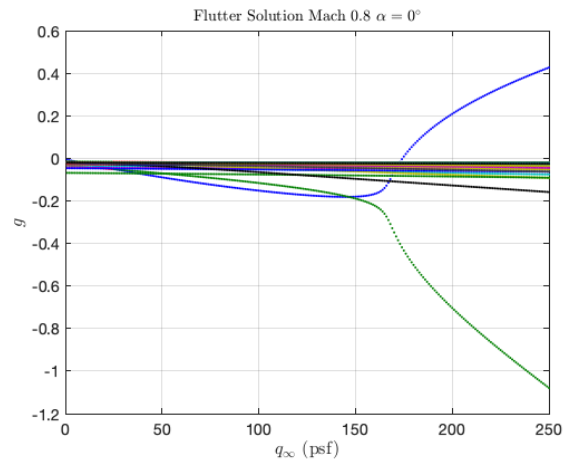
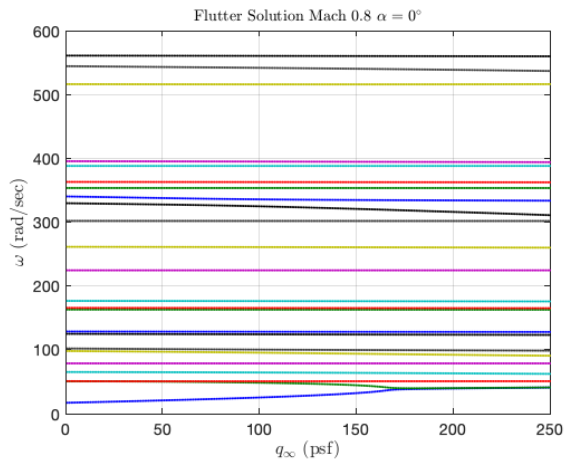


Figure 23. Flutter Diagrams at Mach 0.8

During the active flutter suppression control experiment, limited accelerometer data were acquired in the open-loop mode. These accelerometer data can provide insight into the accuracy of the flutter prediction. In particular, the frequency response data of one of the accelerometers located on the upper surface near the wing tip, labeled as A111z, at Mach 0.65 and a dynamic pressure of 137 psf is shown in Figure 24. The two largest peaks are associated with mode 1 and mode 2. The center frequencies of mode 1 and mode 2 are 4.60 Hz and 7.65 Hz, respectively. For comparison, the predicted corresponding frequencies for these two modes are 4.51 Hz and 7.48 Hz obtained from the $q_\infty - \omega$ diagram for Mach 0.65 in Figure 21. The relative error in the frequency prediction is about 2% for both the modes. The accuracy of the frequency prediction is considered to be excellent. This is due to the significant effort in the validation of the NASTRANTM model to ensure good modal correlation with the GVT data.

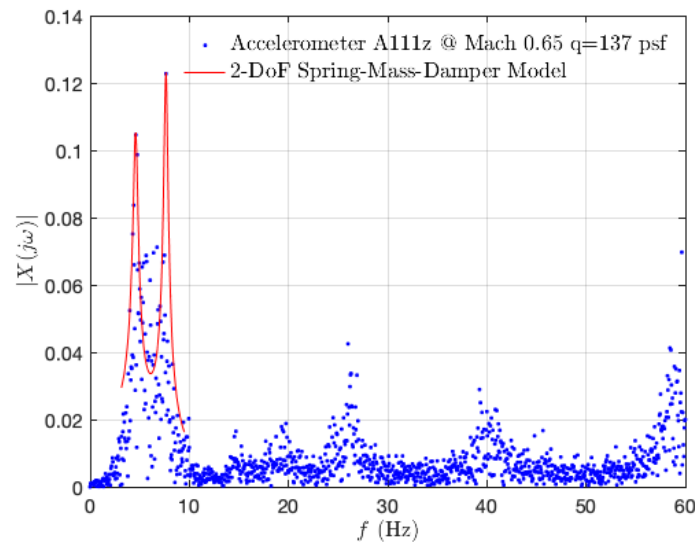


Figure 24. Frequency Response of Accelerometer A111z at Mach 0.65 and Dynamic Pressure of 137 psf.

From the measured frequency response of the accelerometer A111z, damping estimates of mode 1 and mode 2 can be computed using a 2-degree-of-freedom (DoF) spring-mass-damper model. The critical damping ratios ζ for mode 1 and mode 2 at Mach 0.65 and a dynamic pressure of 137 psf are estimated to be about 6% and 3%, respectively. For comparison, the corresponding wind-off critical damping ratios measured from the GVT and subsequently used in the flutter analysis are 0.26% and 1.67%, respectively. The predicted critical damping ratios $\zeta = \frac{g}{2}$ of mode 1 and mode 2 at Mach 0.65 and a dynamic pressure of 137 psf obtained from the $q_\infty - g$ diagram for Mach 0.65 in Figure 21 are 9% and 7.5%, respectively. These critical damping ratios are larger than those estimated from the frequency response of the accelerometer. The discrepancy could be due to many factors such as experimental uncertainty and computational uncertainty. Moreover, damping estimation from frequency response test data is generally subject to large uncertainty due to noise in the test data.

Figure 25 shows the flutter boundary of the wind tunnel model computed by the NASTRANTM and FUN3D flutter solutions. The flutter dynamic pressure computed by FUN3D is quite similar to that computed by NASTRANTM, with the largest percentage difference of 1.94% between the two solutions for Mach 0.8. As the Mach number increases, the differences grow.

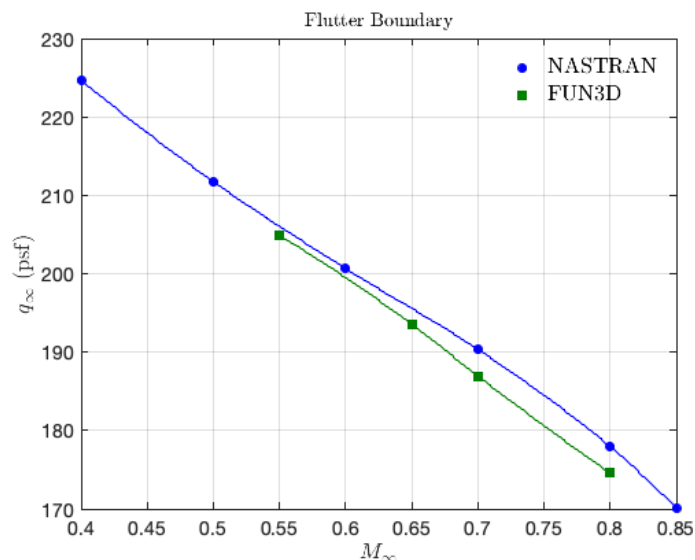


Figure 25. Flutter Boundary Computed by NASTRAN™ and FUN3D

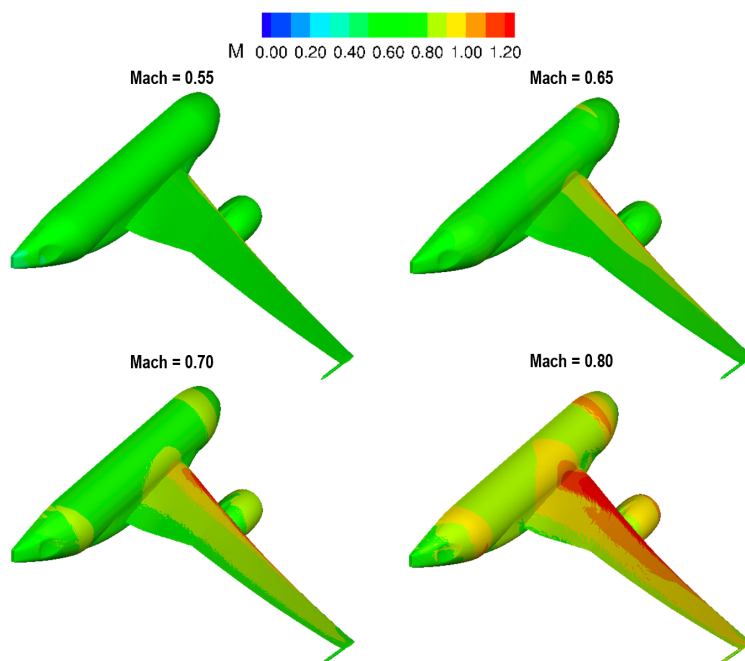


Figure 26. Mach Contours on Test Article

Figure 26 shows the Mach number contours of the test article at the four Mach numbers. The flow at Mach 0.55 is entirely subsonic. The flow at Mach 0.65 exhibits a small region of supersonic flow in the leading edge area near the wing root station. As the Mach number increases, the supersonic flow region on the wing root leading edge area grows and progressively moves outboard. At Mach 0.8, there also appears to be weak supersonic flow areas near the nose of the fuselage and the engine nacelle leading edge. The small difference in the flutter dynamic pressure at Mach 0.55 between NASTRAN™ and FUN3D could be due to various factors such as the viscous effect which is not captured by the doublet lattice method and possibly numerical uncertainty. As the Mach number increases, the difference in the flutter boundary predicted by NASTRAN™ and FUN3D could be attributed to the transonic effect. However, the

transonic effect is not dominant and the transonic dip phenomenon¹⁸ is not observed up to Mach 0.8.

V. Conclusion

This paper presents a high-fidelity flutter analysis of a wind tunnel model of an aspect ratio 13.5 flexible wing designed for aeroservoelastic control wind tunnel experiments. The flutter analysis is conducted using the FUN3D CFD solver. Two multi-sine multi-mode excitation methods are proposed in this study: a frequency-shifted square wave train and a frequency-shifted bowed square wave train. These excitation methods enable the generalized aerodynamic forces over a wide frequency range to be computed from a single CFD run. As a result, a significant reduction in the computational cost is achieved in comparison to a discrete sine excitation. The study examines the advantages and disadvantages of these two excitation methods. The analysis results indicate that the frequency-shifted bowed square wave train excitation produces much more accurate generalized aerodynamic forces than the frequency-shifted square wave train method in comparison to the discrete sine excitation. A reduced-order modeling of the generalized aerodynamic forces is conducted using a rational function approximation in the frequency domain. A state-space realization is constructed from the reduced-order model for the flutter prediction. A flutter boundary for the wind tunnel model is obtained. The CFD-predicted flutter boundary is similar to that predicted by NASTRANTM with the largest error of about 2% at Mach 0.8. A comparison of the frequencies and critical damping ratios of the first two modes to the frequency response test data of an accelerometer at one specific test condition is made. The comparison shows a very good agreement in the frequency prediction with an error of 2%. The estimates of the critical damping ratio from the frequency response test data appear to be smaller than those values computed by the flutter analysis.

Appendix

The system identification to estimate the unknown coefficients of the frequency response transfer function of the GAF is described herein.

The approximation error between the GAF model and the CFD-generated GAF data is defined as

$$\varepsilon(k) = N(k) - D(k)[F(k) + iG(k)] \quad (31)$$

where F and G are the Fourier sine and cosine series coefficients evaluated for the bowed square wave method as

$$F = \frac{2q_0}{T\sqrt{n}} \int_0^T Q_i \sin(n\omega_0 + \Delta\omega_j) t dt \quad (32)$$

$$G = \frac{2q_0}{T\sqrt{n}} \int_0^T Q_i \cos(n\omega_0 + \Delta\omega_j) t dt \quad (33)$$

We write the approximation error as

$$\varepsilon(k) = \varepsilon_R(k) + i\varepsilon_I(k) \quad (34)$$

where

$$\varepsilon_R(k) = \Phi_R^\top(k) \Theta - \Delta_R(k) \quad (35)$$

$$\varepsilon_I(k) = \Phi_I^\top(k) \Theta - \Delta_I(k) \quad (36)$$

$$\Theta = \begin{bmatrix} c_6 \\ c_5 \\ c_4 \\ c_3 \\ c_2 \\ c_1 \\ c_0 \end{bmatrix} \quad \Phi_R = \begin{bmatrix} -k^6 \\ 0 \\ k^4 \\ 0 \\ -k^2 \\ 0 \\ 1 \end{bmatrix} \quad \Phi_I = \begin{bmatrix} 0 \\ k^5 \\ 0 \\ -k^3 \\ 0 \\ k \\ 0 \end{bmatrix} \quad (37)$$

$$\Delta_R = Fk^4 + b_3Gk^3 - b_2Fk^2 - b_1Gk + b_0F \quad (38)$$

$$\Delta_I = Gk^4 - b_3Fk^3 - b_2Gk^2 + b_1Fk + b_0G \quad (39)$$

The parameter vector Θ is computed by a linear regression posed as a dual loop minimization of the cost function

$$J = \min_{\{p_j\}_{j=1,\dots,4}} \min_{\Theta} \frac{1}{2} \sum_{i=1}^{N_k} \varepsilon^2(k_i) = \min_{\{p_j\}_{j=1,\dots,4}} \min_{\Theta} \frac{1}{2} \sum_{i=1}^{N_k} \left\{ \left[\Phi_R^\top(k_i) \Theta - \Delta_R(k_i) \right]^2 + \left[\Phi_I^\top(k_i) \Theta - \Delta_I(k_i) \right]^2 \right\}$$

The inner loop minimization is a least-squares optimization with respect to Θ for a given set of preselected poles p_j , $j = 1, \dots, 4$. The outer loop minimization is a random search for the poles p_j that minimizes the cost function.

The least-squares optimization is established by the stationary condition of the gradient of the cost function given by

$$\frac{\partial J}{\partial \Theta} = \sum_{i=1}^{N_k} \left\{ \Phi_R(k_i) \left[\Phi_R^\top(k_i) \Theta - \Delta_R(k_i) \right] + \Phi_I(k_i) \left[\Phi_I^\top(k_i) \Theta - \Delta_I(k_i) \right] \right\} = 0 \quad (40)$$

where $N_k = \frac{N_h-1}{2} + 1$ is the number of the frequency response data points generated by FUN3D.

We obtain the matrix equation

$$A_\Theta \Theta = B_\Theta \quad (41)$$

where

$$A_\Theta = \sum_{i=1}^{N_k} \left[\Phi_R(k_i) \Phi_R^\top(k_i) + \Phi_I(k_i) \Phi_I^\top(k_i) \right] \quad (42)$$

$$B_\Theta = \sum_{i=1}^{N_k} \left[\Phi_R(k_i) \Delta_R(k_i) + \Phi_I(k_i) \Delta_I(k_i) \right] \quad (43)$$

The parameter Θ is solved as

$$\Theta = A_\Theta^{-1} B_\Theta \quad (44)$$

The coefficients a_i can be computed from the coefficients b_i and c_i with

$$a_0 = A_{ij}(0) \quad (45)$$

$$a_1 = \frac{A_{ij}(1) - A_{ij}(-1)}{2} - \sum_{n=1}^4 \frac{a_{n+2} p_n}{1 - p_n^2} \quad (46)$$

$$a_2 = \frac{A_{ij}(1) + A_{ij}(-1)}{2} - a_0 - \sum_{n=1}^4 \frac{a_{n+2}}{1 - p_n^2} \quad (47)$$

$$a_{n+2} = \frac{N(p_n)}{p_n \prod_{m=1, n \neq m}^4 (p_n - p_m)} \quad (48)$$

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