



Development, Characterization, and Validation of Elevated-Temperature Constitutive Models: Deformation and Damage

*Steven M. Arnold and Bradley A. Lerch
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Space Administration

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Four equations were corrected:

- (1) Page 6, Table I, GVIPS row, top line: Equation was corrected to change a plus sign to a minus sign and to add if $q_{ij}^{(a)}(\sigma_{ij} - \alpha_{ij}) \geq 0$.

Original:
$$\dot{q}_{ij}^{(a)} = M_{ijkl}^{(a)}(\dot{\epsilon}_{kl} - \dot{\epsilon}_{kl}^I) + M_{ijkl}^{(a)}\eta_{klrs}^{(a)-1}q_{rs}^{(a)}$$

Corrected:
$$\dot{q}_{ij}^{(a)} = M_{ijkl}^{(a)}(\dot{\epsilon}_{kl} - \dot{\epsilon}_{kl}^I) - M_{ijkl}^{(a)}\eta_{klrs}^{(a)-1}q_{rs}^{(a)} \quad \text{if } q_{ij}^{(a)}(\sigma_{ij} - \alpha_{ij}) \geq 0$$

- (2) Page 19: Equation (36) was corrected to add a dot on top of the sigma.

Original:
$$\sigma_{ij} = E_{ijkl}(\dot{\epsilon}_{kl} - \dot{\epsilon}_{kl}^I) + \dot{q}_{ij}$$

Corrected:
$$\dot{\sigma}_{ij} = (1 - \psi)E_{ijkl}(\dot{\epsilon}_{kl} - \dot{\epsilon}_{kl}^I) + \dot{q}_{ij}$$

- (3) Page 19: Equation (37) was corrected to add a dot on top of the q and to change the plus sign to a minus sign.

Original:
$$q_{ij}^{(a)} = M_{ijkl}^{(a)}(\epsilon_{kl} - \epsilon_{kl}^I) + M_{ijkl}^{(a)}\eta_{klrs}^{(a)-1}q_{rs}^{(a)} - (\Psi - 1)\sigma_{ij}$$

Corrected:
$$\dot{q}_{ij}^{(a)} = (1 - \psi)M_{ijkl}^{(a)}(\dot{\epsilon}_{kl} - \dot{\epsilon}_{kl}^I) - M_{ijkl}^{(a)}\eta_{klmn}^{(a)-1}q_{mn}^{(a)} - \frac{\dot{\Psi}}{1 - \Psi}q_{ij}^{(a)}$$

- (4) Page 19: Equation (41) was corrected to change superscript m to β in denominator.

Original:
$$\dot{a}_{ij}^{(b)} = Q_{ijkl}^{(b)} \left\{ \dot{\epsilon}_{kl}^I - \frac{R_{(b)} [G^{(b)}]^{m_{(b)}}}{H_{(b)} \langle 1 - \sqrt{G^{(b)}} \rangle^{m_{(b)}}} \pi_{kl}^{(b)} - (\theta^{(b)} - 1) \pi_{kl}^{(b)} \right\}$$

for $\pi_{kl}^{(b)} \cdot (\sigma_{kl} - a_{kl}) \geq 0$;
= 0 otherwise

Corrected:
$$\dot{\alpha}_{ij}^{(b)} = Q_{ijkl}^{(b)} \left\{ \dot{\epsilon}_{kl}^I - \frac{\left\{ R_b [G^{(b)}]^{m_{(b)}} (1 - \theta) \right\}}{H_{(b)} \langle 1 - \sqrt{G^{(b)}} \rangle^{\beta_{(b)}}} \pi_{kl}^{(b)} \right\}$$

for $\pi_{kl}^{(b)} \cdot (\sigma_{kl} - \alpha_{kl}) \geq 0$;
= 0 otherwise

- (4) Page 19: The footnote was corrected by adding neither and $\alpha_b = 2\sqrt{(3)}k_{(b)}/(2 + \beta)$.

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Summary

This report provides a brief review of experimentally observed hereditary and nonhereditary material behavior along with background information on standard as well as advanced internal state variable constitutive modeling at elevated temperature. A description of exploratory, characterization, and validation testing is presented along with a detailed outline of what constitutes “sufficient” data content (i.e., quality and quantity) for developing or enhancing, characterizing, and validating a particular sophisticated nonlinear time- and history-dependent (hereditary) class of constitutive models known as GVIPS (generalized viscoplasticity with potential structure). The tests described are necessary to reveal a material’s behavior in the reversible (or viscoelastic) and irreversible (or viscoplastic) regimes, for the identification of both deformation and damage model parameters. Results presented are primarily for metallic materials. In all cases, both uniaxial and multiaxial tests are described, and the linkage between the specific tests and the parameters within the model that can be characterized from the results of these tests are also discussed. Discussion is also provided relative to the role information management must play relative to material data collection, analysis, maintenance, and dissemination. The need for such an information system is particularly important as both analyst and designer move toward utilizations of sophisticated, nonlinear time- and history-dependent (hereditary) constitutive models. Lastly, the concept of state space and its utility in understanding and establishing constitutive models is addressed throughout.

The intent behind this document is to help both the modeler and experimentalist understand each other’s specific points of view and provide guidance for both model development and characterization. Emphasis has been placed on providing the mechanician with information regarding how tests are performed and what issues to be aware of when interpreting results. It is hoped that experimentalists will take away a new perspective on the types of information that modelers and analysts are looking for from them.

Primary Nomenclature

Invariants

$\Omega; \Omega_R; \Omega_{IR}$	complementary dissipation potential; R is reversible, IR is irreversible
$\Phi; \Phi_R; \Phi_{IR}$	Gibb’s complementary potential; R is reversible, IR is irreversible
F	Bingham-Prager threshold function
$G^{(b)}$	normalized second invariant function
H_b	invariant material hardening function

Stresses

σ_{ij}	applied Cauchy stress tensor
$(\sigma_{ij})_s$	viscoelastic equilibrium stress tensor
$\sigma_{ij} - \alpha_{ij}$	viscoplastic equilibrium (effective) stress tensor
$a_{ij}^{(b)}$	viscoplastic nonequilibrium stress tensor per mechanism
κ	threshold stress separating reversible and irreversible response regions
$k_{(b)}$	either a normalizing stress or hardening threshold stress per mechanism
$q_{ij}, q_{ij}^{(a)}$	viscoelastic nonequilibrium internal stress tensor

Strains

$\varepsilon_{ij}, e_{ij}, \varepsilon_{ij}^R, \varepsilon_{ij}^I, \varepsilon_{ij}^{th}$	total, reversible mechanical, reversible (viscoelastic), irreversible (viscoplastic), and thermal strain tensors, respectively
p_{ij}	internal reversible strain tensor (displacementlike)
γ_{ij}	internal irreversible strain tensor

*Currently retired.

Material Parameters

C_{ijkl} or E_{ijkl} , $M_{ijkl}^{(a)}$, $\eta_{ij}^{(a)}$	reversible elastic stiffness tensors, viscoelastic stiffness tensor, and viscosity modulus tensor for each a^{th} mechanism, respectively
ν	Poisson's ratio
E_s , $E_m^{(a)}$	elastic stiffness and Maxwell spring stiffness per a^{th} mechanism, respectively
E_D	dynamic modulus
μ	material parameter associated with irreversible viscosity
$r((G)^{(b)}); h((G)^{(b)}); f(F)$	material functions: thermal recovery, hardening, and flow, respectively
$H_{(b)}$, $R_{(b)}$	hardening and thermal recovery material parameter per b^{th} mechanism
n	flow law exponent
$\beta_{(b)}$, $m_{(b)}$	hardening and recovery material exponent per b^{th} mechanism
ψ	stiffness reduction parameter
c_e , $\bar{Y}_0$, μ_e , n_e	stiffness degradation parameters
$c_d^{(i)}$, $Y_0^{(i)}$, $\mu_d^{(i)}$, $n_d^{(i)}$	strength degradation parameters for each i^{th} viscoplastic mechanism

Miscellaneous

Λ_{ijkl}	deviatoric operator (isotropic or anisotropic)
N_{ijkl}	isotropic directionality tensor
δ_{ij}	Kronecker delta function
$\langle \rangle$	Macauly bracket
$H_v[\dots]$	Heaviside unit function
$(\cdot)$	time derivative (or rate) notation
M	number of reversible mechanisms
N	number of irreversible mechanisms
T	temperature
T_m	melting temperature

1.0 Introduction

The solution of a solid mechanics problem involves the establishment of a statically admissible field (satisfies equilibrium internally and traction boundary conditions (BCs)) and a kinematically admissible field (satisfies strain-displacement relations and displacement BCs) along with the satisfaction of material constitutive laws. Constitutive theory (laws) concerns

the mathematical modeling of the physical response (output) of a material to a given stimulus (input); that input can be a generalized force or displacement. The importance of accurate constitutive relationships is illustrated in Figure 1, as they form the primary link between stress (σ_{ij}) and strain (ϵ_{ij}) components at any point within a body. These relations may be simple (as in the case of linear elasticity) or extremely complex (as in the case of viscoplasticity), depending upon the material of the body and the conditions to which the body is subjected (e.g., thermal, mechanical, or thermomechanical loading; rate of loading; and extreme environment). Constitutive relations for a particular material are determined experimentally, and they may involve both physically (directly) measurable quantities (e.g., strain, temperature, time) and not directly measurable internal parameters, often referred to as “internal state variables.” It is important to realize that for mathematical convenience, materials are often classified into specific idealized groups (e.g., linear elastic, plastic, viscoelastic, and viscoplastic), yet nothing can compel real materials to behave according to these idealized models.

In aeropropulsion applications, like aircraft engine hot-section components, many structural components involve temperature levels, thermal transients, and complex multiaxial loads severe enough to result in significant inelastic deformation. For example, although a change of a fraction of a percent in the throat profile of a rocket engine liner may not produce a change in thrust, it could develop or relieve thermally induced stresses in that section, shortening or lengthening the life of the liner. Likewise, applications with tight tolerances may require time-dependent analyses if they are loaded or subject to thermally induced stresses. Bolted parts may deform with time and lose their required clamping force. This has implications for the inspection and maintenance of the liner, injector, and other parts made from metallic alloys that see significant time-dependent deformations. Structural analysis and life prediction of these hot-section components depend

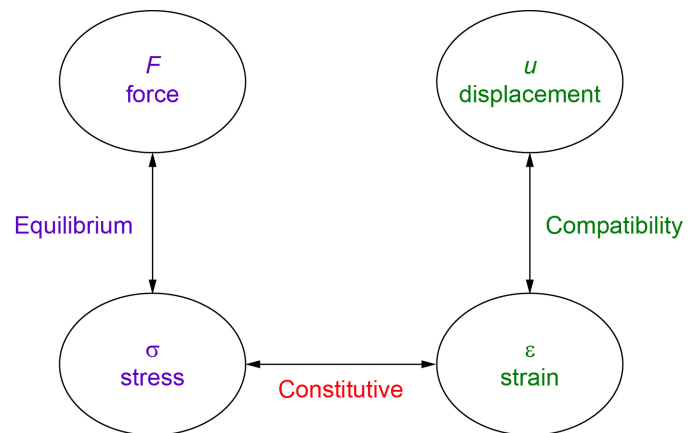


Figure 1.—Key aspects of mechanics problem.

strongly on accurately understanding the nonlinear interaction between stress, strain, and temperature fields (i.e., constitutive behavior) at elevated temperature. To be generally applicable, constitutive equations must be expressed in multiaxial form and address all modes of mechanical and thermal loading expected to be experienced by hot-section components and materials (e.g., sustained, cyclic, nonisothermal, nonproportional, etc.).

Typically, when dealing with actual complex material behavior as represented in Figure 2, it is not sufficient to describe this behavior with simple single-parameter relationships or pointwise values (e.g., Young's modulus E , Poisson's ratio ν , yield stress σ_y , hardening modulus H , etc.). Consequently, it is essential that one capture the complete response spectrum (i.e., the entire uniaxial and/or multiaxial response curve and not just model-specific pointwise parameter values) for a given material for subsequent analysis and modeling. Furthermore, although specification of a typical test matrix involves, either explicitly or implicitly, a priori selection of a modeling approach, to maximize the value of any given experimental program (for both present as well as yet-to-be-developed models), one must attempt to capture and document as much material response information as possible—without prejudice to any specific model (idealization). Lastly, obviously, the more sophisticated the constitutive model, the greater the demand on any informatic infrastructure or experimental program.

The response of real materials to thermal and mechanical stimuli can vary greatly, depending upon the magnitude and multiaxiality of loading and the magnitude of the homologous temperature ($T_H = T/T_m$), where T_m is the melting point of the material. For example, at room temperature, material response is typically time independent (where time is represented by t in Figure 2) and either reversible (i.e., linear elastic) or irreversible (inelastic), depending upon whether or not the yield (threshold) stress of the material has been exceeded. Alternatively, when $T_H > 0.25$, time-dependent behavior—both reversible and irreversible—can be commonly observed as illustrated schematically in Figure 2, wherein strain-rate-sensitive, creep, relaxation, thermal recovery, dynamic recovery, and creep-plasticity interaction response behaviors are all shown. Other complex time- and path-dependent behaviors such as cyclic ratcheting (see Figure 3), creep-fatigue interaction, and thermomechanical fatigue are also often observed, depending upon the magnitude and type of loading (e.g., thermal or mechanical) being applied. In Figure 3, one can clearly see how specific material behavior will manifest itself differently (e.g., shakedown or ratcheting) depending upon the mode of load control that is applied, such as stress or strain control.

The purpose of this document is to identify and articulate what constitutes sufficient data content (i.e., quality and quantity) for primarily enhancing, characterizing, and validating a particular

sophisticated nonlinear time- and history-dependent (hereditary) class of constitutive models known as GVIPS (generalized viscoplasticity with potential structure). However, this report will begin by providing background on constitutive modeling in general to provide insight into what types of experiments are needed to assess the appropriate functional forms of a given model, identify state (external or internal) variables, and determine the associated model parameters. Then the discussion will focus on the summarization of the GVIPS deformation and damage model formulation (Refs. 1 to 4) and the associated software for estimating the associated constitutive model parameters (i.e., COMPARE) (see Refs. 5 and 6). That is followed by a detailed discussion of the tests that need to be performed to characterize (i.e., specify the specific model parameters for) the material deformation behavior in the reversible (or viscoelastic) regime, followed by those required in the irreversible (or viscoplastic) regime. Next is a discussion of those tests

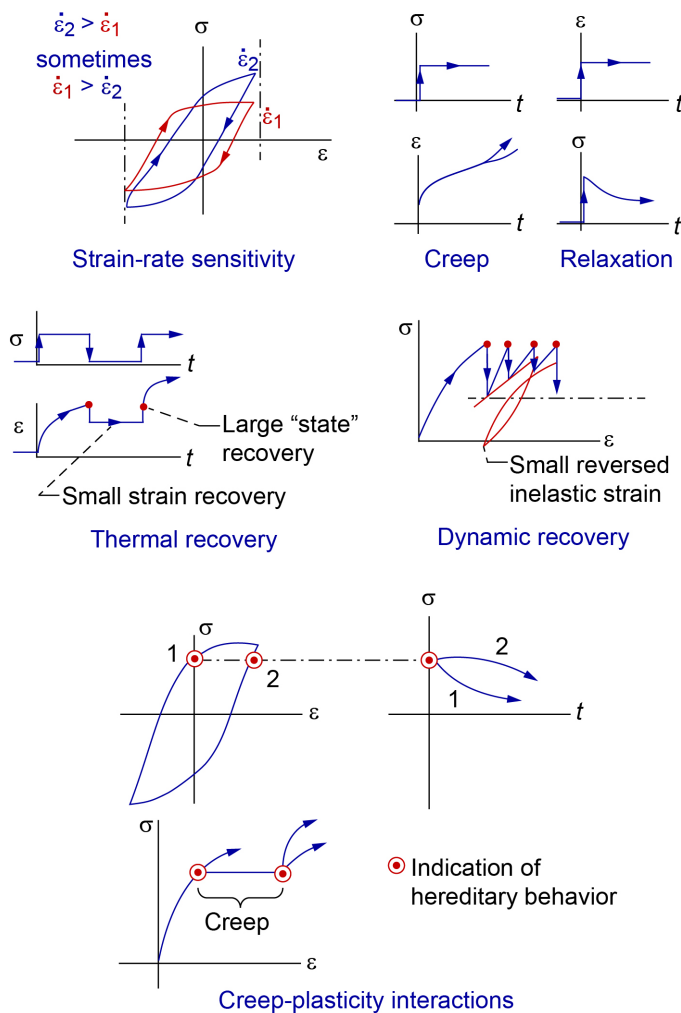
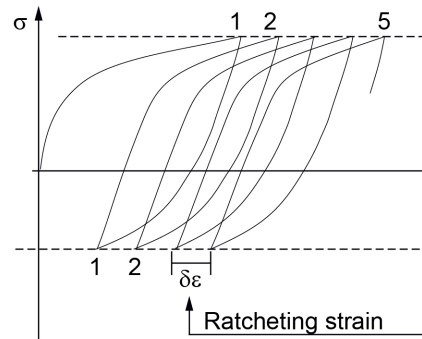
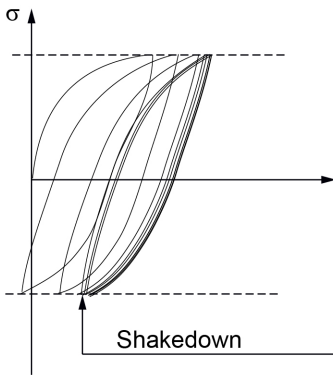
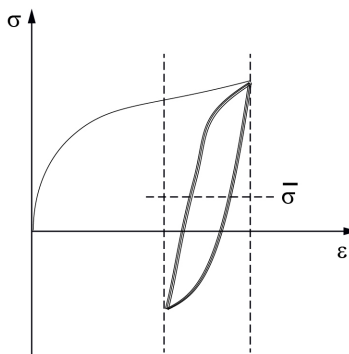


Figure 2.—Representative hereditary material behaviors at elevated temperature ($T_H > 0.25$).

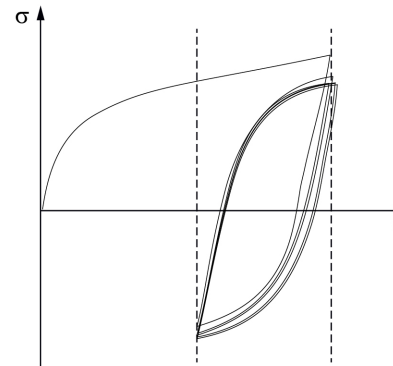
Stress controlled



Strain controlled



Nonrelaxation of mean stress



Relaxation of mean stress

Shakedown
behavior

Ratcheting
behavior

Figure 3.—Cyclic behavior of metals at high T_H (>0.25).

required to identify, characterize, and validate the softening material behavior; that is, behavior that will be exhibited because of some type of damage occurring within the material, whether a type of stiffness degradation or strength degradation. In all cases, both uniaxial and multiaxial tests will be described, and the linkage between the specific tests and the parameters within the model that can be characterized from the results of these tests will be discussed. From these discussions, a generic sample test matrix, which could be applied to any material, will be constructed. Although this content is meant to be agnostic relative to materials, models, and experimental approaches, the examples shown in this report deal exclusively with metallic materials.

2.0 Background: Constitutive Modeling

A prerequisite for meaningful assessment of component durability and life and, consequently, the design of structural components, is the ability to accurately predict the stress and strains

occurring within a loaded structure composed of a given material. Constitutive material models (whether simple or complex) provide the required mathematical link between stress and strain, and this necessitates the development and characterization of an appropriate constitutive behavior model for any material before that material can be certified for use by a designer. Thus, constitutive models with varying levels of idealizations have been proposed and utilized, each with its own advantages and shortcomings. For the purpose of applications, material-system response is typically classified into idealized groups (e.g., elasticity, plasticity, viscoelastic, and viscoplastic) relative to time scales of observations (see Figure 4 and Refs. 7 to 11 for more details). In Figure 4, four classic one-dimensional (1D) rheological models are shown with the following material parameters: E_1 and E_2 are linear spring constants (i.e., stiffness moduli), η_1 is linear viscous modulus, and η_2 and η_3 are nonlinear viscous moduli; m and n are exponents of nonlinearity σ is applied stress, σ_s is equilibrium stress, and α is nonequilibrium stress.

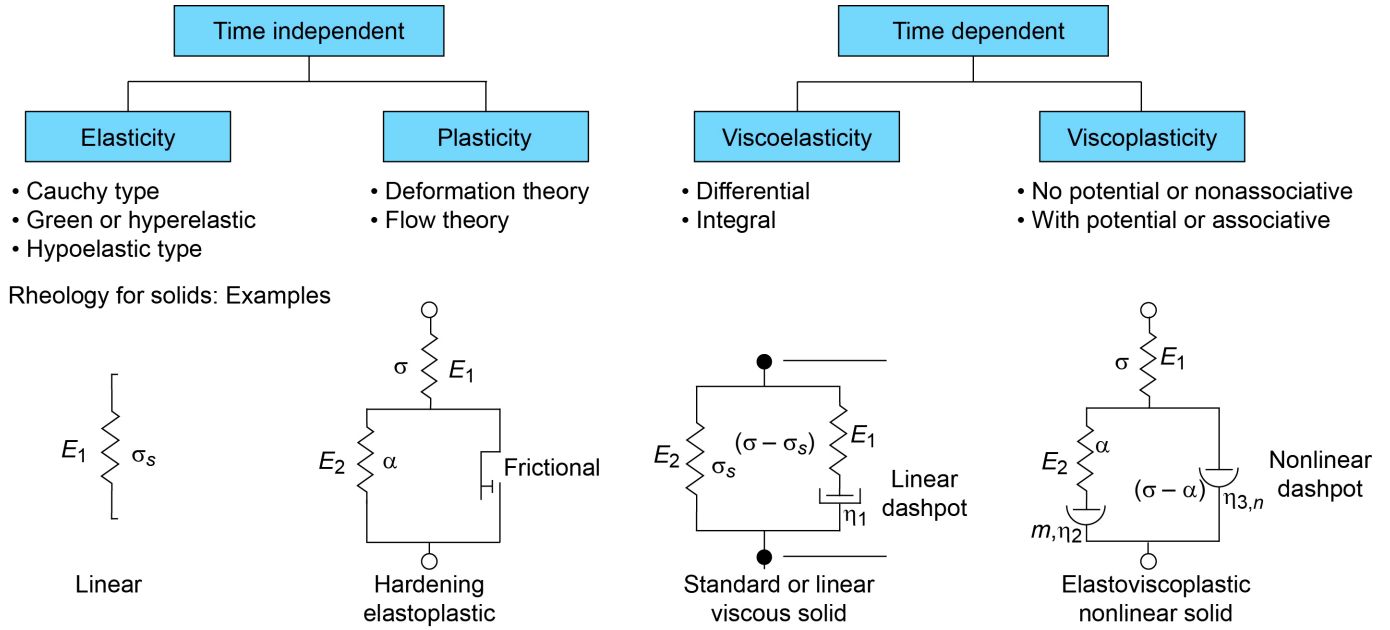


Figure 4.—Different classifications of constitutive model idealizations.

The most well-known and widely used constitutive model is Hooke's law (Ref. 11),

$$\sigma_{ij} = C_{ijkl} e_{kl} \quad (1)$$

which describes time-independent, linear (proportional) reversible material behavior.

Extension into the irreversible regime (e.g., plastic) has been accomplished by assuming an additive decomposition (small-strain assumption) of the total strain tensor, ε_{ij} ,

$$\varepsilon_{ij} = e_{ij} + \varepsilon_{ij}^I + \varepsilon_{ij}^{th} \quad (2)$$

or

$$e_{ij} = \varepsilon_{ij} - \varepsilon_{ij}^I - \varepsilon_{ij}^{th} \quad (3)$$

into three strain components: a reversible mechanical, e_{ij} (i.e., elastic or viscoelastic); irreversible, ε_{ij}^I (i.e., inelastic or viscoplastic); and reversible thermal ε_{ij}^{th} . After substituting expression (3) into Equation (1), a generalized stress-strain relation (i.e., generalized Hooke's law) results that incorporates irreversible strains as well as reversible ones:

$$\sigma_{ij} = C_{ijkl} (\varepsilon_{kl} - \varepsilon_{kl}^I - \varepsilon_{kl}^{th}) \quad (4)$$

whereby numerous models describing the evolution of the inelastic strain have been proposed in the literature (Refs. 7 to 11).

Note the preceding stress measure is the Cauchy stress tensor, which under the small-strain assumption is equivalent to engineering stress. Table I contains a few representative model examples with a brief description of their required material parameters.

Figure 5 illustrates the material response under monotonic loading, wherein engineering (nominal) stress σ (which is defined by applied load, P , divided by initial cross-sectional area A_0) and engineering strain ε (defined as change in length, ΔL , divided by original length, L_0) are illustrated, as are several important features throughout the response (e.g., yield stress, ultimate tensile stress (UTS), and failure stress and strain. Several important ideas need to be remembered when interpreting experimental response:

- Strain measurements are average responses over the gage length L_0 , so it is essential to be aware if gradients (temperature, strain, or stress) are present. Thus, there is an implicit volume of material ($A_0 \times L_0$) associated with these measurements.
- Typical engineering applications involve deformations in the small-strain region (i.e., $\varepsilon < 2$ to 3 percent). Consequently, once one goes beyond this strain limit, true stress and strain measures need to be assessed.
- It is important to use strain control during testing instead of displacement control; this is especially true at elevated temperature ($T_H \geq 0.25$) since everything becomes rate or time dependent (e.g., apparent yield, apparent elastic limit, etc.).

TABLE I.—EXAMPLES OF VARIOUS CONSTITUTIVE MODELS AND THEIR REQUIRED MATERIAL PARAMETERS

Model	Idealized material behavior	Mathematical representation	Required isotropic material parameters
Ramberg-Osgood (Ref. 7)	Elastic plastic	$\varepsilon^I = \left(\frac{\sigma}{H} \right)^{1/n}$	H, n
Norton-Bailey (Ref. 8)	Uniaxial creep	$\dot{\varepsilon}^I = A\sigma^n$	A, n
Odqvist (Ref. 9)	Multiaxial creep	$\dot{\varepsilon}_{ij}^I = B(J_2')^m S_{ij}$ where $m = \frac{1}{2}(n-1)$ and $B = \frac{1}{2} \left(3^{(n+1)/2} \right) A$ $J_2' = \frac{1}{2} S_{ij} S_{ij}$ with $S_{ij} = \sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij}$	A, n
GVIPS (Refs. 1 to 4, 12, and 13)	Viscoelastoplastic	$\dot{q}_{ij}^{(a)} = M_{ijkl}^{(a)} (\dot{\varepsilon}_{kl} - \dot{\varepsilon}_{kl}^I) - M_{ijkl}^{(a)} \eta_{klrs}^{(a)-1} q_{rs}^{(a)}$ if $q_{ij}^{(a)} (\sigma_{ij} - \alpha_{ij}) \geq 0$ $\dot{\varepsilon}_{ij}^I = \begin{cases} f(F) \Gamma_{ij} & \text{if } F \geq 0 \\ 0 & \text{if } F < 0 \end{cases}; f(F) = \frac{F^n}{2\mu};$ $g(G^{(b)}) = H_{(b)} \left(1 - \sqrt{G^{(b)}} \right)^{\beta_{(b)}}$ $\dot{\alpha}_{ij}^{(b)} = Q_{ijkl}^{(b)} \left[\dot{\varepsilon}_{kl}^I - \frac{R_{(b)} [G^{(b)}]^{m_{(b)}}}{H_{(b)} \left(1 - \sqrt{G^{(b)}} \right)^{\beta_{(b)}}} \pi_{kl}^{(b)} \right]$ if $\pi_{kl}^{(b)} (\sigma_{kl} - \alpha_{kl}) \geq 0$ where $q_{ij} = \sum_{a=1}^M q_{ij}^{(a)}, \quad \alpha_{ij} = \sum_{b=1}^N \alpha_{ij}^{(b)}$ $\Gamma_{ij} = \Lambda_{ijkl} (\sigma_{kl} - \alpha_{kl}); \quad \pi_{kl}^{(b)} = \Lambda_{kl ij} \alpha_{ij}^{(b)}$ $F = \frac{1}{2\kappa^2} (\sigma_{ij} - \alpha_{ij}) \Lambda_{ijkl} (\sigma_{kl} - \alpha_{kl}) - 1; \quad G^{(b)} = \frac{1}{2k_{(b)}^2} \alpha_{kl}^{(b)} \Lambda_{kl ij} \alpha_{ij}^{(b)}$ and all three fourth-order tensors, E_{ijkl} , M_{ijkl} , and $\eta_{ijkl}^{(a)}$, are taken to be coaxial; that is, $E_{ijkl} = E_s N_{ijkl}$, $M_{ijkl}^{(a)} = M_{(a)} N_{ijkl}$, and $\eta_{ijkl}^{(a)} = \rho_{(a)} M_{ijkl}^{(a)}$ where $N_{ijkl} = \left\{ \frac{\nu}{(1+\nu)(1-2\nu)} \delta_{ij} \delta_{kl} + \frac{1}{(1+\nu)} (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) \right\}$ and E_s , $M_{(a)}$, and ρ_a are the corresponding uniaxial moduli. Finally, $Q_{ijkl}^{(b)} = H_{(b)} \left(1 - \sqrt{G^{(b)}} \right)^{\beta_{(b)}} \left[Z_m - \frac{\beta}{2k_{(b)}^2 \sqrt{G^{(b)}} [1 - \{1-\beta\} \sqrt{G^{(b)}}]} \alpha_{ij}^{(b)} \alpha_{kl}^{(b)} \right]$ Note: Z_m is the generalized inverse of Λ_{ijkl} , which in the case of isotropy, is a deviatoric operator (see Eq. (45)); see Saleeb and Wilt (Ref. 13) for further elaboration on this.	2+2M reversible constants: $E_s, M_{(a)}, \nu, \rho_{(a)}$ and 3+5N irreversible constants: κ, n, μ $H_{(b)}, R_{(b)}, m_{(b)}, k_{(b)}, \beta_{(b)}$ where N defines the number of viscoplastic mechanisms and M, the number of viscoelastic

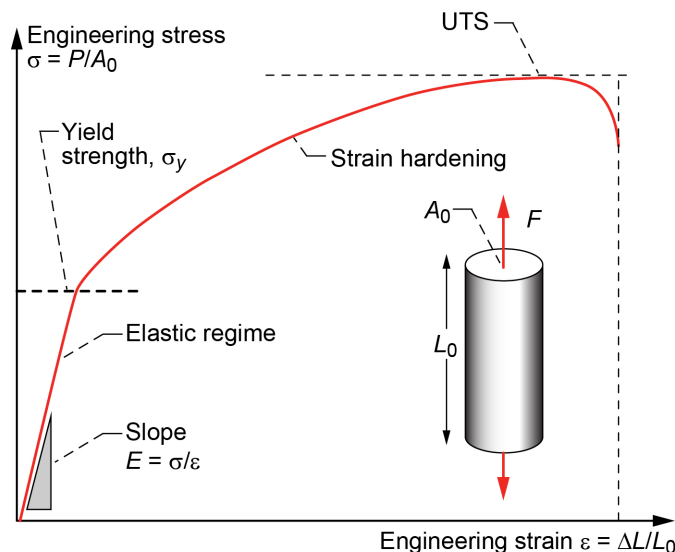


Figure 5.—Typical monotonic tensile behavior at room temperature.

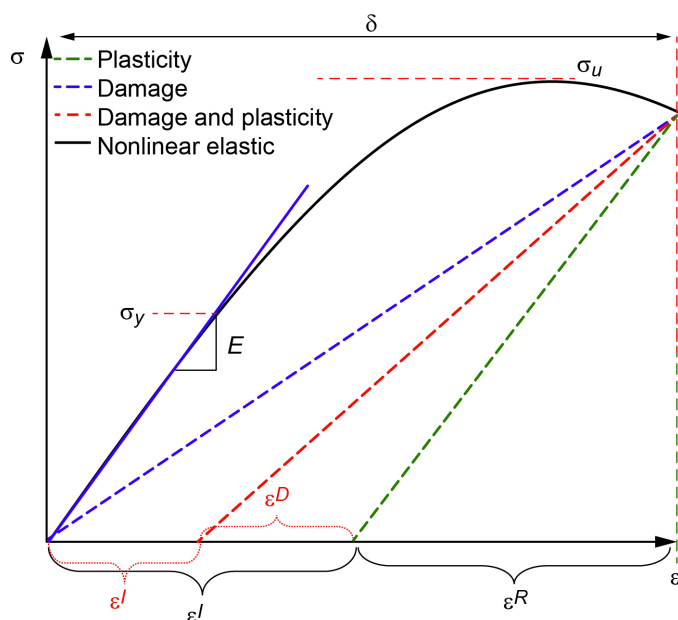


Figure 6.—Illustrative tensile response with unloading.

Monotonic loading provides very little information regarding the underlying mechanisms of deformation; therefore, it is critical to perform unloading (particularly when dealing with elevated temperature response) before assessing the cause of any nonlinearity in the curve, whether the response is inelastic (e.g., plastic) and/or damage (e.g., of the stiffness degradation type). The importance of this is illustrated in Figure 6, where in one case (Plasticity line), unloading would indicate that the nonlinearity (irreversible deformation) is due to, for example, plastic flow because the unloading modulus is identical to the

loading modulus; whereas the Damage and plasticity line illustrates both inelastic strain (ϵ^I) and stiffness degradation (ϵ^D) at the same time. If the unloading response had gone through the origin (Damage line in Figure 6), then the nonlinear response would be attributed to only damage (e.g., stiffness degradation) or if the unloading had followed the exact path of loading (Nonlinear elastic line), then the material response would have been classified as nonlinear elastic behavior.

Another important concept at the core of continuum-based constitutive modeling is the concept of a homogeneous volume element of material, over which the conjugate fields (e.g., stress and total strain) are treated as uniform and whose properties and response can be experimentally measured. In other words, the material point does not explicitly account for any internal details (microstructure) within the material, such as the presence of inclusions, grains, or molecular arrangement. This is particularly pertinent to anisotropic and effective (macrolevel) composite material constitutive, damage, and failure models, wherein the macroscopic anisotropy is a byproduct of the presence and interaction of different phases or inclusions at a lower length scale. For example, in the case of composite (heterogenous) materials, the various directional dependence associated with a macro-mechanics model must be characterized through extensive composite testing because such models do not, by definition, consider what is happening in each constituent (e.g., fiber or inclusion, matrix) at the appropriate physical scale. Macromechanics models have proven to be well suited to predict deformation response of composites because the intrinsic, history-dependent, interactive effects of the composite constituents are implicitly embedded in the experimental results, and thus their in situ behavior is automatically captured during the calibration of the specific model parameters. However, their ability to predict damage evolution and life remains problematic, particularly at elevated temperature where rate-dependent and nonlinear effects dominate. This is understandable because failure mechanisms (i.e., fiber/matrix debonding, fiber breakage, matrix cracking) are influenced to a greater extent by underlying features that are on a lower length scale (which are ignored and/or homogenized in macromechanics).

Clearly, an obvious question to pose is: When is it appropriate to idealize a material as homogeneous? To answer this question, consider a generic structure (Figure 7) that is anchored along a portion of its bounding surface, S_u , and loaded with applied surface tractions along S_σ . A minimum characteristic dimension (length) of the structure is denoted by L , the nominal temperature of the structure by T_o , and a characteristic temperature (T) gradient in the body by $\text{grad}(T)$. D is the minimum length associated with the material volume that is being idealized as homogenous. The largest size of our continuum volume element, D , can be determined (upon temporarily ignoring the internal structure of the material) by choosing a size that provides

essentially a homogeneous state over the volume element, that satisfies the following inequalities:

$$\frac{D}{L} \ll 1 \quad (5)$$

$$\frac{D}{T_o} \text{grad}(T) \ll 1 \quad (6)$$

wherein the specific choice of D relative to the characteristic body dimensions and gradients is a matter of how much less than unity is implied in these inequalities, Equations (5) and (6), which is associated with the level of precision one wants to maintain.

Once a reasonable dimension D is chosen, this must be compared to d , which is the maximum length associated with microstructural features (e.g., grain, phase, or fiber) present within the material volume (see Figure 7). This is to say that at the length scale being observed, the material appears to be homogeneous, with no visible microstructure. This will be true only if d/D is small, that is,

$$\frac{d}{D} \ll 1 \quad (7)$$

For example, if one magnifies a digital image, the pixels become visible. However, if the observation length is sufficiently large, no pixels are distinguishable. Similarly, in a metallic material, if d/D is small, the material will appear to be homogeneous, and the constituents (phases) will be indistinguishable. Although this can never be truly satisfied, as D becomes small compared to L (the minimum characteristic length of the structure), or $D/L \ll 1$, and temperature gradients diminish, the approximation of the homogeneous material volume as a continuum point will become better.

Figure 8 provides an example of a tubular, polycrystalline, metallic test specimen in which the material volume element size, D/L ratio longitudinally is $700/25,400 = 0.027 < 1$, and transverse is $515/12,700 = 0.04 < 1$, thus indicating that there are enough volume elements within the gage length to be relatively homogenous and there are enough grains within the volume element, $d/D < 0.1$, thus enabling the concept of continuum volume element to be utilized.¹

The entire purpose of providing this background constitutive modeling section is to briefly introduce or remind the reader of the important foundational aspects of constitutive modeling and

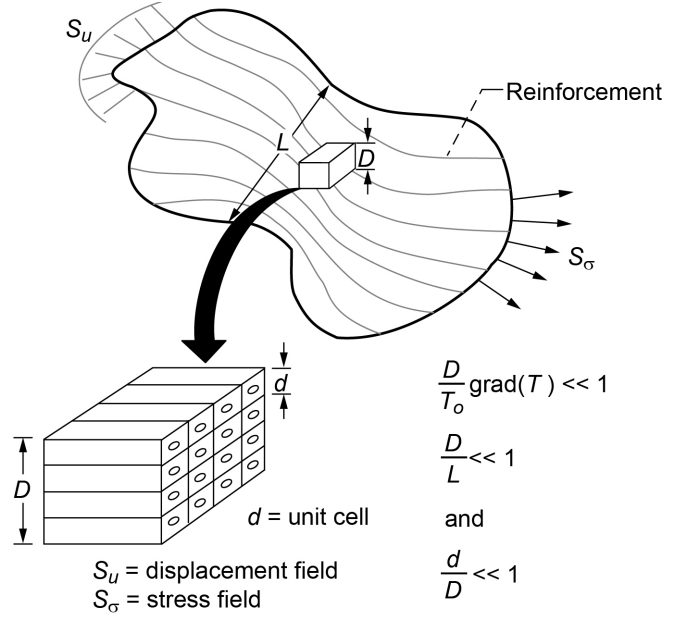


Figure 7.—Relationship between scale of representative volume element and structure.

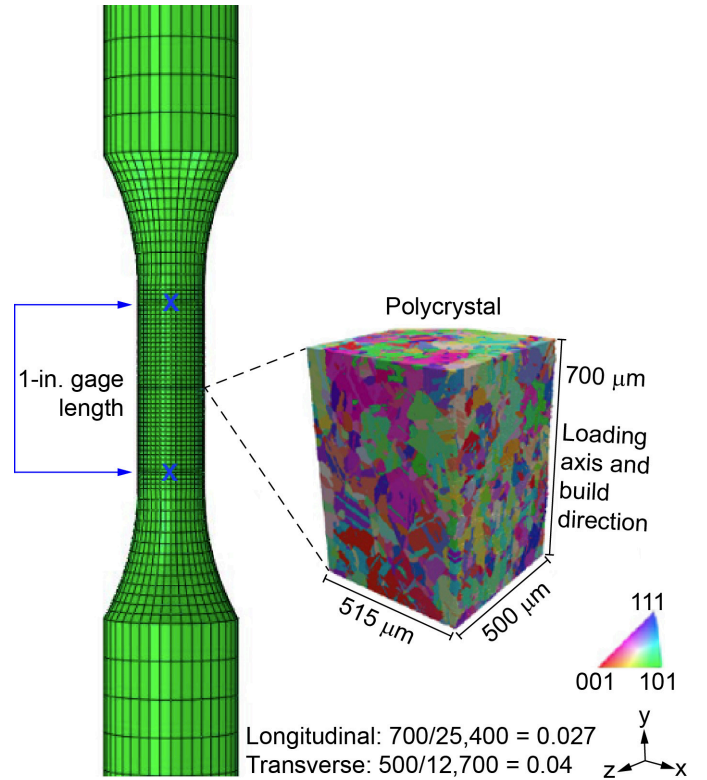


Figure 8.—Tubular test specimen of 0.5-in. (12,700-μm) diameter and 1-in. (25,400-μm) gage length.

¹Be aware, given the density of the finite element mesh (36 elements in the vertical direction (705 μm each) and 34 elements in the horizontal direction (373 μm each)), the ratio of D /(element size) is 0.993

longitudinal and 1.37 transverse, and the volume element is equal to or greater than the element size. This can be problematic because an integration point could, in reality, represent only a few grains.

level set readers' thinking in the hope that they can make connections between experimentation and modeling. It is extremely important that experimentalists and modelers work hand in hand when developing a given test matrix, as quantities and parameters that a given model requires often can be hard—if not impossible—to measure. Furthermore, it is imperative that modelers strive to understand exactly how a given experiment was conducted to ensure proper interpretation of the results (see Ref. 14, Appendix D, for an example) and to maintain idealization consistency. Many interdependent factors are comprised within the concept of idealization consistency and can be classified into three main categories: theoretical, mechanistic, and numerical consistencies: (1) Theoretical consistency requires preserving the mathematical aspects (e.g., theory, model dimensionality, functional form, etc.) used throughout characterization to prediction; (2) Mechanistic consistency would be associated with assumed measures and definitions of damage and modes of failure and their inter-actions, which should be accounted for not only in the final prediction but also during characterization; (3) Length scale (e.g., material volume element) and mesh accuracy are both examples that would fall within the numerical consistency category; therefore, the preservation of these accuracies from characterization to prediction is important. It is also important to remember that these categories are typically not mutually exclusive, and thus the influence of one on another is often difficult to determine explicitly. This is particularly true in the case of multiscale analysis, wherein all factors can inter-act aggressively as one traverses various length scales (Ref. 15).

2.1 Formulation Requirement

For theoretical soundness of a constitutive model, one must consider the following for the solution of any initial boundary value problem (IBVP): It must obey (see Ref. 11):

1. Thermodynamics
2. Frame indifference (objectivity)
3. Stability (Drucker's stability postulates)
4. Existence (uniqueness); conditions for uniqueness: normality, convexity, and associativity

Herein the concept of thermodynamic internal state variable theory utilizing a fully associative potential structure will be utilized. Such a general three-dimensional (3D) viscoelastoplastic model (of the state-variable type; i.e., no convolution/ hereditary integrals) will automatically satisfy the dissipation inequality of thermodynamics. It also eliminates the ad hoc nature of model development in that only the functional formulation of the potential is open to interpretation because the mathematics involving equations of state and flow and evolution equations are fully

determined. Therefore, the starting point is a careful partitioning of energy into stored energy, Φ , (e.g., hardening) and dissipation energy, Ω , (e.g., recovery and inelastic flow), leading to conjugate or dual-state variables (stress and strainlike) connected by equations of state (from Φ) that describe the present internal (microstructural) state of the material; for example, dislocation networks, cells, and/or subgrains in metals:

$$y_i = \frac{\partial \Phi(x_i)}{\partial x_i} \quad (8)$$

and flow and evolution equations (from Ω) that describe the velocity or rate of change of the evolving microstructure (e.g., dislocations mobility or diffusional rates of vacancies, masses, point defects, etc.) in metals:

$$\dot{y}_i = \frac{\partial \Omega(y_i)}{\partial y_i} \quad (9)$$

This thermodynamic potential formulation (Refs. 1, 2, and 12) provides a consistent framework for deformation and damage modeling, which lends itself to robust numerical implementation, constitutes the cornerstone of regularity and bounding (or limit) theorems in plasticity and viscoplasticity, and whose nonisothermal and/or anisotropic extension is straightforward and whose nonproportional loading histories are automatic because the formulation starts with a 3D representation and reduces to 1D and/or two dimensional 2D for characterization purposes.

2.1.1 Thermodynamics

The First Law of Thermodynamics states that energy cannot be created or destroyed; it can only be converted from one form to another. Therefore, conservation of energy (the first law) requires that the work done by stresses must either be stored as recoverable internal energy in the solid or be dissipated as heat (or a combination of both). As such, the energy equation must be solved pointwise:

$$\rho \dot{u} = \sigma_{ij} \dot{\epsilon}_{ij} + r - q_{i,i} \quad (10)$$

where ρ is the density of the material, $\dot{u}$ is the rate of internal energy, $\sigma_{ij} \dot{\epsilon}_{ij}$ is the external work, r is the volumetric density of internal heat production, and $\mathbf{q}$ is the heat flux vector. The second law requires that if a sample of material is subjected to a cycle of deformation that starts and ends with an identical strain and internal energy (at constant temperature or without heat exchange with the surroundings), the total work done must be positive (i.e., nonnegative energy dissipation). The Clausius-Duhem inequality (Ref. 8),

$$\int \left(\rho \dot{s} + \frac{q_{i,i}}{T} - \frac{r}{T} \right) dV \geq 0 \quad (11)$$

combined with the previous conservation of energy term (via r), and introducing a new term known as specific free energy, Ψ , (where $\Psi = u - Ts$, such that T is the temperature and s is specific entropy per unit mass) will provide the following after some manipulation:

$$\sigma_{ij} \dot{\epsilon}_{ij} - \rho \left(\dot{\Psi} + s \dot{T} \right) - q_i \frac{T_{,i}}{T} \geq 0 \quad (12)$$

2.1.2 Frame Indifference (Objectivity)

The principle of frame indifference states that any material response (model) should be independent of the chosen frame of reference, or equivalently, it should be independent of any super-imposed rigid-body motion (or change of observer). Stated mathematically, any tensorial quantity must obey the transformation laws, depending on its tensorial order and character (e.g., given a second-order tensor $\sigma_{ij}^{(1)} = R_{ik} \sigma_{kl}^{(0)} R_{lj}$, where R_{ik} is a rotation tensor). Consequently, any material model must only include objective tensors (e.g., invariants) in its functional forms. Furthermore, the theory of algebraic invariants (see Ref. 16) states that a function, depending on the number and type or character of tensors, can be expressed in terms of its integrity basis. For example, in the case of a potential function composed of a single second-order tensor like stress (σ_{ij}), the function would take on the form

$$F(\sigma_{ij}) = f(I_1, I_2, I_3) \text{ or } F(\sigma_{ij}) = f(I_1, J_2, J_3) \quad (13)$$

or any linear combination thereof (I_1, I_2, I_3, J_2 , or J_3) where

Total stress	Deviatoric stress
$I_1 = \sigma_{kk}$	$J_1 = s_{kk} = 0$
$I_2 = \frac{1}{2} \sigma_{ij} \sigma_{ij}$	$J_2 = \frac{1}{2} s_{km} s_{km} = \frac{1}{3} (I_1^2 - 3I_2)$
$I_3 = \frac{1}{3} \sigma_{ij} \sigma_{jk} \sigma_{ki}$	$J_3 = \frac{1}{3} s_{ij} s_{jk} s_{ki} = \frac{1}{27} (2I_1^3 - 9I_1 I_2 + 27I_3)$

and the deviatoric stress $s_{ij} = \sigma_{ij} - \delta_{ij} \sigma_{kk}$ (i.e., pure shear) where δ_{ij} is the Kronecker delta function, which yields either 1 or 0, depending on the two indices i and j . The preceding notation is called Einstein indicial notation (Refs. 9 and 11) where, for

example, $\sigma_{kk} = \sigma_{11} + \sigma_{22} + \sigma_{33}$. Note, it is very important to understand the implications of each invariant, in that

I_1 = hydrostatic stress dependence² (so if one wants pressure dependence in the material response, I_1 must be included in the functional form)

J_2 = shear response

J_3 = asymmetry in tension and compression captured in material response.

Therefore, in the case of pressure-insensitive materials (e.g., for metals, see Ref. 19) the general functional form would be

$$F = F(J_2^m, J_3^n) \quad (14)$$

with n = even if tension equals compression response

= odd if material response is different in tension and compression

m = any value because J_2 is always positive

The most popular form of plastic flow potential is the classic von Mises (established in 1913, see Ref. 20), which assumes only J_2 dependence ($m = 1/2$ and $n = 0$) as defined previously:

$$F(J_2) = \sqrt{3J_2} - k^2 \quad (15)$$

which immediately implies that the material is initially fully isotropic, is insensitive to hydrostatic stress, exhibits the same behavior in tension and compression, and in the case of classical plasticity is rate insensitive. Note k is the yield stress in shear. A flow functional (J_2, J_3) that enables better multiaxial predictions but with the same behavior in both tension and compression was put forth by Robinson and Ellis (Ref. 21):

$$F[J_2(\Sigma_{ij}), J_3(\Sigma_{ij})] = \frac{(J_2^3 + cJ_3^2)^{1/3}}{k^2} - 1 \quad (16)$$

where $\Sigma_{ij} = s_{ij} - a_{ij}$ (effective stress, which is the difference between deviatoric stress ($s_{ij} = \sigma_{ij} - \delta_{ij} \sigma_{kk}$) and deviatoric internal stress ($a_{ij} = \alpha_{ij} - \delta_{ij} \alpha_{kk}$)) and c (the strength of J_3) must be determined from non-uniaxial testing (e.g., tension-torsion tests). See Figure 9 for the influence of the magnitude of c .

The simplest extension of the surface to enable asymmetry in stress response (i.e., difference in tension and compression) would be a form in which $m = 1/2$ and $n = 1$:

$$F(J_2, J_3) = \sqrt{3J_2} + \alpha J_3 - \beta \quad (17)$$

²Typical materials that exhibit pressure dependence are polymers (see Ref. 17 and the references in Ref. 18), ceramics (see references in Ref. 18), and concrete materials (Ref. 11).

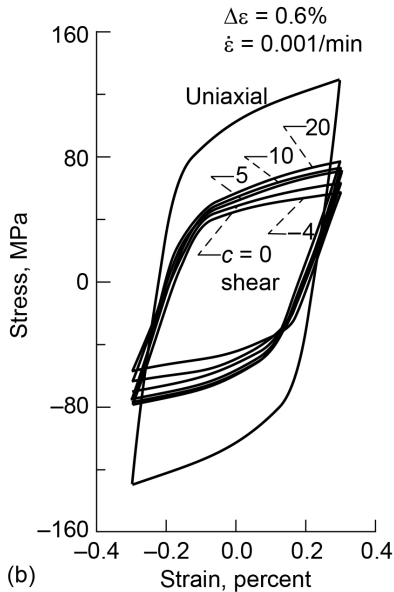
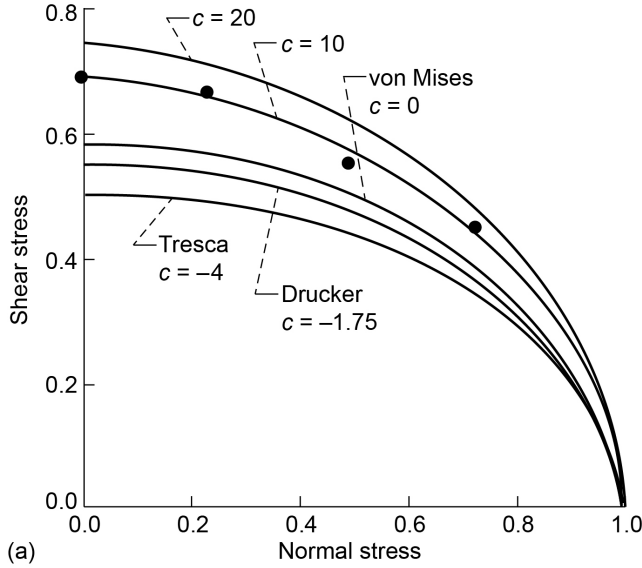


Figure 9.—Response of flow functional (J_2 , J_3) as function of material parameter c . (a) Constant flow surface, $F = \text{constant}$; points are experimental data. (b) Saturated hysteresis loops.

Here, α indicates the amount of asymmetry and β is the yield stress.

Finally, in the case of pressure-sensitive materials (e.g., polymers, ceramics, concrete), the general functional form would be

$$F = F(I_1^p, J_2^m, J_3^n) \quad (18)$$

where these forms range from two-parameter to five-parameter models; for example, the case of $p = 1$, $m = 1/2$, and $n = 0$ gives the two-parameter Drucker-Prager model (1953) (see Ref. 22),

which is the simplest extension to include linear pressure dependence:

$$F(I_1, J_2) = \sqrt{3J_2} + \alpha I_1 + \beta \quad (19)$$

In this case, α indicates the amount of pressure sensitivity and β is the yield stress. Alternatively, $p = 0, 1$, or 2 ; $m = 1/2$; and $n = 1$ gives the five-parameter William-Warnke model (Ref. 23; see Chen and Saleeb (Ref. 11, Secs. 5.3 to 5.5) for more details).

In summary, it is precisely these types of questions, relative to the foundational behavior of a given material and thus mathematical functional dependence, that must be examined and established during the exploratory testing phase (see Section 2.3, Classification of Testing: Exploratory, Characterization, and Validation).

2.1.3 Stability

For a material to be stable according to Drucker, the external work done over the period of stress must be positive (Refs. 8, 11, and 24)

$$\dot{W} = (\sigma_{ij} - \sigma_{ij}^*) \dot{\epsilon}_{ij}^I + \dot{\sigma}_{ij} \dot{\epsilon}_{ij}^I > 0 \quad (20)$$

Drucker's stability postulates are

- In the small (incremental rate): $\dot{\sigma}_{ij} \dot{\epsilon}_{ij}^I \geq 0$; this is equivalent to stating that the second-order work rate is always positive.
- In the large (including cycles): $(\sigma_{ij} - \sigma_{ij}^*) \dot{\epsilon}_{ij}^I \geq 0$, where σ is on the yield surface ($f = 0$), and σ^* is inside or on the yield surface ($f = 0$).

These stability postulates lead to

- No softening is allowed; flow must be associative ($g = f$), where g is the plastic (or evolution) potential function.
- All surfaces $f = 0$ must be convex (i.e., $\frac{\partial^2 f}{\partial \sigma_{ij} \partial \sigma_{kl}}$ = positive (semi) definite).

which form the basis of existence and uniqueness (stress-boundedness) in initial IBVPs. Note that real materials do not have to satisfy the Drucker stability postulates, but if one's solution does not, then one should expect to have real numerical problems.

2.1.4 Existence and Uniqueness

A desirable feature for any mathematical theory describing the mechanical behavior of materials is that the solution

obtained is unique and stable. Uniqueness may be an assured property of the real world, but it must be demonstrated in the case of mathematical idealizations of real behavior. This topic has been addressed extensively in the literature (e.g., see Refs. 11 and 24) and thus will only be briefly reviewed here. The key concepts for uniqueness are the utilization of the principle of virtual work, Drucker's stability postulates in the small, normality, convexity, and associativity.

Consider a continuum element at any instant of time (t) and consider additional changes in loads or displacements such that two different changes in stresses and strains (state a and state b, where the superscripts on the stress and strain indicate these different states) are produced that satisfy both equilibrium and compatibility. Then one must conclude that for small deformations, the difference $(\dot{\sigma}_{ij}^a - \dot{\sigma}_{ij}^b)$ is in equilibrium with zero

body and traction rates, and the difference $(\dot{\epsilon}_{ij}^a - \dot{\epsilon}_{ij}^b)$ is compatible with zero displacement rate imposed over a specific area, A_u . Then, applying the principle of virtual work to these two difference sets, one obtains

$$\int_V (\dot{\sigma}_{ij}^a - \dot{\sigma}_{ij}^b) (\dot{\epsilon}_{ij}^a - \dot{\epsilon}_{ij}^b) dV = 0 \quad (21)$$

which can be finally written in terms of elastic-plastic (ep) relations, given that

$$\dot{\sigma}_{kl} = C_{kl ij}^{ep} \dot{\epsilon}_{ij} \quad (22)$$

and that C^{ep} is only a function of the same existing stress, strain, and inelastic strain before the changes,

$$\dot{\sigma}_{kl}^a - \dot{\sigma}_{kl}^b = C_{kl ij}^{ep} (\dot{\epsilon}_{ij}^a - \dot{\epsilon}_{ij}^b) \quad (23)$$

For Equation (21) to be satisfied (using Eq. (23)), the integrand must be zero; that is,

$$(\dot{\epsilon}_{kl}^a - \dot{\epsilon}_{kl}^b) C_{kl ij}^{ep} (\dot{\epsilon}_{ij}^a - \dot{\epsilon}_{ij}^b) = 0 \quad (24)$$

and uniqueness is trivially proved if $C_{kl ij}^{ep}$ is positive definite,

since then $\dot{\epsilon}_{ij}^a = \dot{\epsilon}_{ij}^b$.

It can be shown that

$$C_{kl ij}^{ep} = C_{kl ij}^e + C_{kl ij}^p = C_{kl ij}^e - \left(\frac{1}{\theta} \right) \bar{N}_{ij} N_{kl} \quad (25)$$

and

$$\begin{aligned} \theta &= \hat{h} - \left(\frac{\partial f}{\partial \sigma_{ij}} \right) \bar{N}_{ij} \geq 0 \\ \hat{h} &= - \left(\frac{\partial f}{\partial \epsilon_{ij}^p} \right) \left(\frac{\partial g}{\partial \sigma_{ij}} \right) - \left(\frac{\partial f}{\partial k} \right) \left(\frac{\partial k}{\partial \epsilon_{ij}^p} \right) \left(\frac{\partial g}{\partial \sigma_{ij}} \right) \end{aligned} \quad (26)$$

where the first term in $\hat{h}$ is associated with kinematic hardening and the second, with isotropic hardening and

$$\begin{aligned} \bar{N}_{ij} &= C_{ij mn}^e \left(\frac{\partial g}{\partial \sigma_{mn}} \right) \\ N_{kl} &= C_{kl pq}^e \left(\frac{\partial f}{\partial \sigma_{pq}} \right) \end{aligned} \quad (27)$$

The following sufficient conditions can be used to prove the positive definiteness of $C_{kl ij}^{ep}$. Below, each part will be taken separately since if each part is positive definite, then the sum is as well.

2.1.4.1 Elastic Part: $C_{kl ij}^e$

Given a hyperelastic material, $\sigma_{ij} = \sigma_{ij}(\epsilon_{kl})$, and the fact that a strain-energy density function, W , exists and the stress is normal to this function, that is,

Normality:

$$\sigma_{ij} = \frac{\partial W}{\partial \epsilon_{ij}} \quad (28)$$

then through differentiation, it can be shown

$$\dot{\sigma}_{ij} = \left(\frac{\partial \sigma_{ij}}{\partial \epsilon_{kl}} \right) \dot{\epsilon}_{kl} = \left(\frac{\partial^2 W}{\partial \epsilon_{ij} \partial \epsilon_{kl}} \right) \dot{\epsilon}_{kl} \quad (29)$$

and substituting for $\dot{\sigma}_{ij}$ into Drucker's stability in the small,

$\dot{\sigma}_{ij} \dot{\epsilon}_{ij} \geq 0$ (see Section 2.3.1),

Convexity:

$$\frac{\partial^2 W}{\partial \epsilon_{ij} \partial \epsilon_{kl}} \dot{\epsilon}_{kl} \dot{\epsilon}_{ij} \equiv \dot{\epsilon}_{kl} C_{kl ij}^e \dot{\epsilon}_{ij} \geq 0 \quad (30)$$

where convexity is true when a line segment between any two distinct points on the graph of the function lies above the graph between the two points (see Figure 10). Consequently, the positive semidefiniteness of $C_{kl ij}^e$ is shown.

2.1.4.2 Plasticity Part: C_{klij}^p

First, C_{klij}^p and hence C_{klij}^{ep} are symmetric only for the case of associative plasticity (i.e., when g (plastic potential) is equal to f (the yield surface), (i.e., $\bar{N} \equiv N$), resulting in the normality condition; that is, the tensor $\dot{\epsilon}_{ij}^p$ is normal to the yield surface, $f=0$).

$$\dot{\epsilon}_{ij}^p = \lambda \left(\frac{\partial f}{\partial \sigma_{ij}} \right) \quad (31)$$

Further, the yield surface must also be convex, that is,

$$\frac{\partial f}{\partial \sigma_{ij}} (\sigma_{ij} - \sigma_{ij}^*) \geq 0 \quad (32)$$

where σ_{ij}^* is any point inside or on the yield surface (see Figure 10). Note, convexity ensures reasonable continuum properties (uniqueness, etc.) in boundary value problems (see Ponter, Ref. 25). A similar argument can be made for viscoplastic materials wherein the consistency condition is not required (see Robinson and Swindeman, Ref. 26).

2.2 What Constitutes a State Variable?

In thermodynamics, a state variable is an independent variable of a state function. These include both external state variables, which are directly measurable (e.g., stress, strain, time, temperature, etc.), and internal state variables, which typically represent average quantities (e.g., dislocation arrangement, dislocation density, etc.) that define the salient features of the internal arrangement (internal state) of the material and the rather complex evolution of the microstructure of the material system when subjected to different histories of mechanical loads and temperature changes. These internal state variables are typically not directly measurable quantities, and the number and type of internal state variables (i.e., scalar, tensorial: second order, fourth order, etc.) is still a topic of current research (Refs. 27 to 29). The literature, however, generally supports the use of two internal state variables: (1) a second-order tensor, α_{ij} , often identified as the back (or internal) stress, which in some averaged sense represents the movement and/or arrangement of dislocations within the material and (2) a scalar, κ , often identified as the drag (or threshold) stress, which typically represents the density of dislocations.

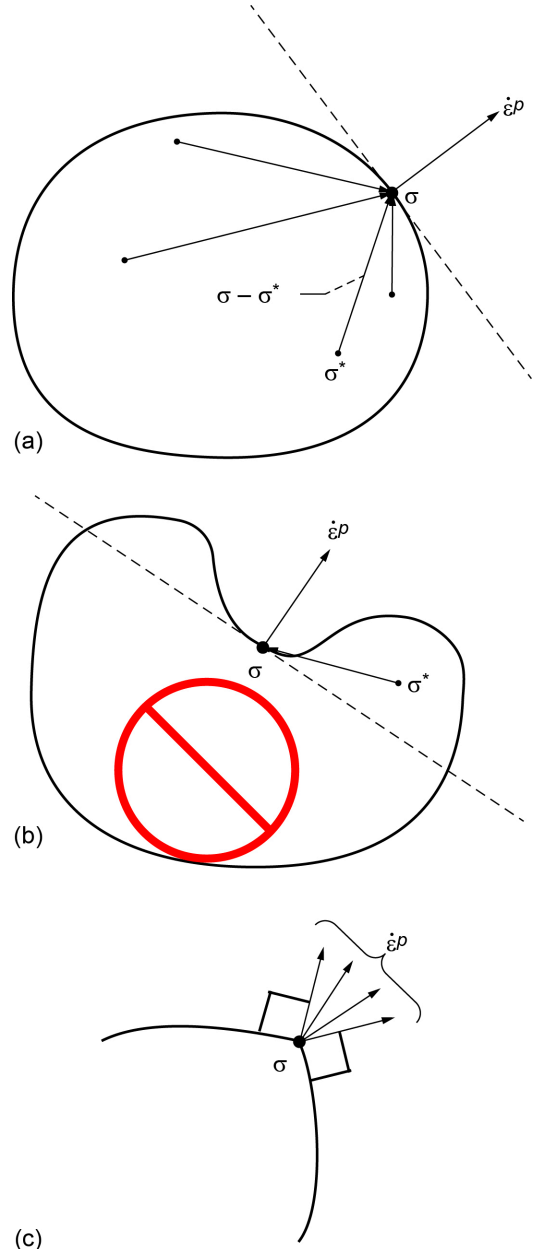


Figure 10.—Convex and concave yield surfaces.
 (a) Convex. (b) Concave (not allowable).
 (c) Corner.

These variables (both external and internal) are used as independent variables that define equations of state as well as flow and evolution equations for a given material, such as

Flow:

$$\dot{\epsilon}^I = f(\sigma, T, \alpha_1, \alpha_2, \dots, \alpha_n) \quad (33)$$

Evolutionary law:

$$\dot{\alpha}_i = g(\sigma, T, \alpha_1, \alpha_2, \dots, \alpha_n) \quad (34)$$

in which the inelastic strain rate is a unique function, f , of the current stress, temperature, and internal state variables (α_n), wherein the evolution (rate of change) of the internal state variable is defined by another function, g . Onat and Fardshisheh (Refs. 30 and 31) provide an excellent discussion on the mathematical as well as geometric interpretation (i.e., the state space, see Appendix A) of the various features involved in time-dependent modeling of material behavior using state variables.

The question of what constitutes a good state variable will be addressed through the following thought experiment (Ref. 30), wherein the isothermal, creep response of an isotropic material subjected to four different uniaxial stress levels (σ_1 , σ_2 , σ_3 , and σ_4) will be considered along with a single four-step creep test consisting of the same stress levels, wherein each stress level is held constant for the same period of time before the next load level is applied. The resulting creep-strain-versus-time plots are shown in Figure 11. The black lines in Figure 11 constitute the same creep response associated with σ_1 , σ_2 , σ_3 , and σ_4 , and the red lines would be the resulting four-step creep response given the assumed state variable. In the first case, see Figure 11(a), time (t , which previously was taken to be an external variable) is assumed now to be the single internal state variable (α) such that $\alpha = t$, and for each step in stress, at t_i , one would follow a vertical line to that stress state (denoted by the star) and impose the creep response at σ_{i+1} from that point until the next step up in stress. The resulting response curve for the four-step creep

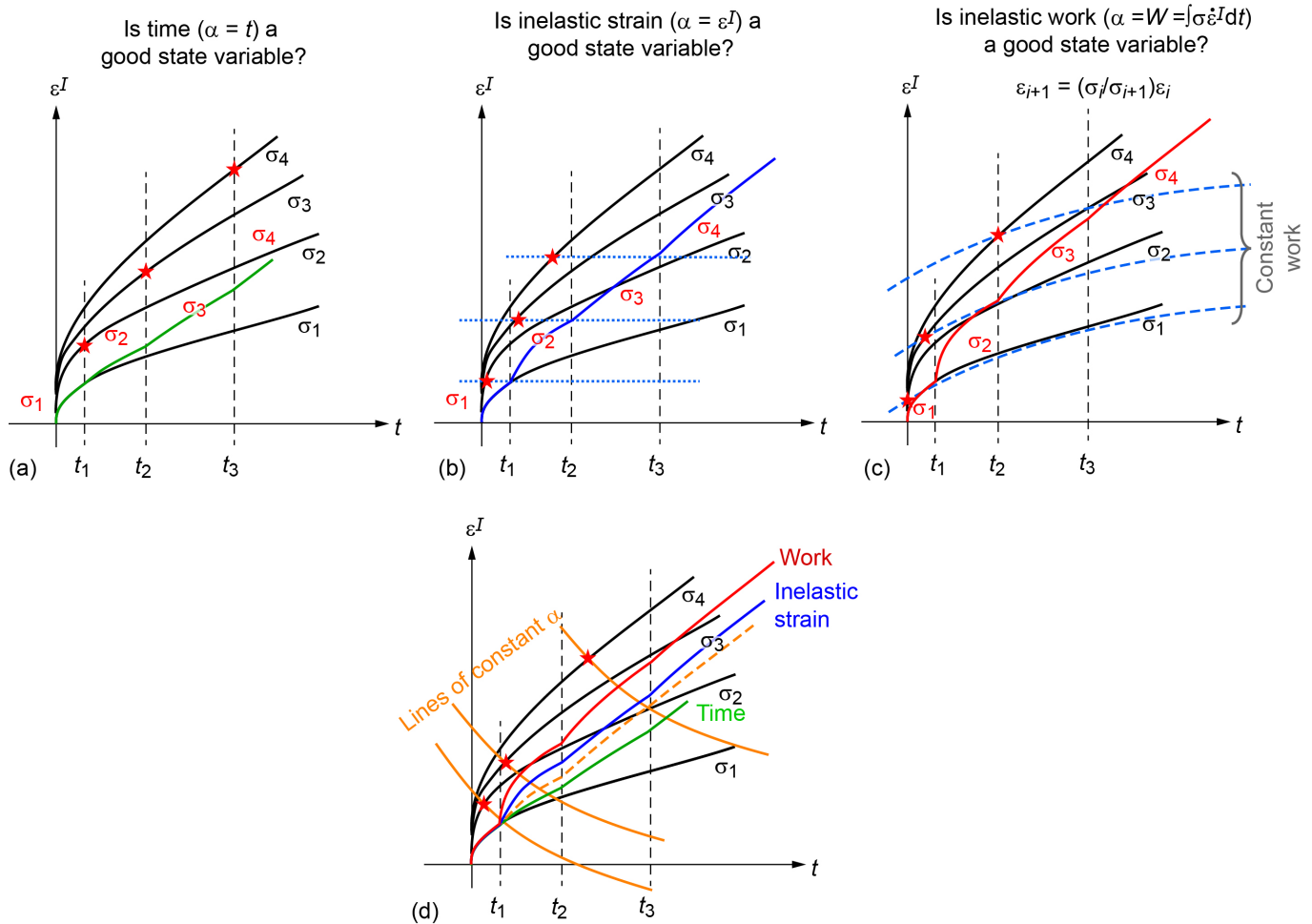


Figure 11.—Creep tests at different stress levels (σ_1 , σ_2 , σ_3 , and σ_4); superimposed lines of four-step creep test. (a) Assuming time. (b) Assuming inelastic strain. (c) Assuming constant inelastic work. (d) Assuming along lines of constant internal state variable, α .

test is shown by the green line in Figure 11(a). The next case assumes the inelastic strain to be an appropriate state variable, that is, $\alpha = \epsilon^I$, such that at each step in stress, at t_i , one would follow a horizontal line to the stress state σ_{i+1} (denoted by the star) and impose the creep response at σ_{i+1} from the point t_i until the next step up in stress occurs. Again, the resulting response curve for the four-step creep test is shown by the blue line in Figure 11(b). In the third case, inelastic work is taken to be the appropriate state variable, that is, $\alpha = W$, such that the creep response along lines of constant work, at each step, are taken to project the four-step creep test results, shown by the red line in Figure 11(c). In the fourth case, internal stress α is taken to be the appropriate state variable, such that the creep response along lines of constant α , at each step, are taken to project the four-step creep test results. The resulting response curve is shown by the orange dashed line Figure 11(d). Note in Figure 11(d) the other four-step creep response curves in which time, inelastic strain, and work are assumed to be appropriate state variables are also shown. Experience has shown that neither time, inelastic strain, nor work responses match actual test results, and thus a generic internal stress tensor is typically assumed, along with the well-accepted Bailey-Orowan-type (Refs. 32 and 33) evolution law, in which the hardening process (driven by the accumulation of inelastic strain) is in competition with a thermal recovery (softening) process proceeding in time. Figure 12 shows actual experimental results for a titanium alloy, TIMETAL[®] 21S (Titanium Metals Corporation), subjected to various stresses. If one examines these results considering the preceding thought experiment, one will agree that time, inelastic strain, and work do not make good state variables for predicting creep behavior. Moreover, it appears in Figure 12 that the second step response (for the three-step stress test, red dashed line) does not seem to coincide with the 110-MPa creep curve (which is the second step in the three-step creep test, green line) anywhere. This could be due to a variety of reasons; for example, specimen-to-specimen variability, the way the step test was conducted, having more than a single state variable (mechanism) active, and so forth. Clearly, reality is more complicated than a typical thought experiment. Yet according to Onat (Refs. 30 and 31), one typically needs to do some type of transient dip (step) tests to tease out information about internal state variables. For more details about visualizing internal state variables and different assumed functional forms relative to evolution equations, see Appendix A.

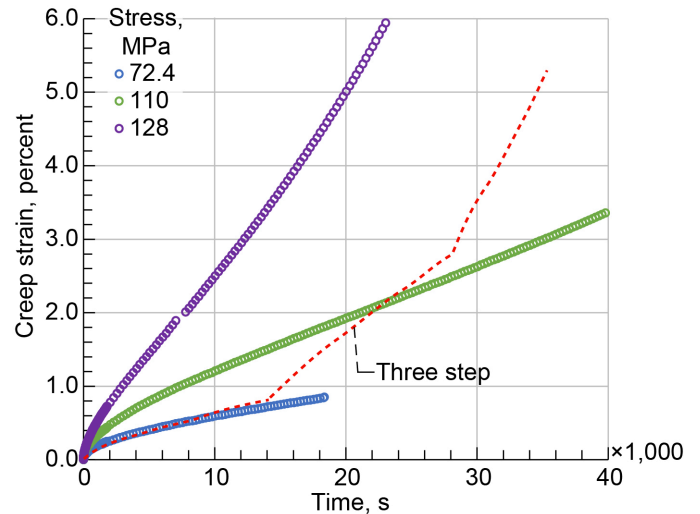


Figure 12.—Experimental creep strain-versus-time responses of TIMETAL[®] 21S for three stress levels and one three-step creep test using the same stress levels at 650 °C.

2.3 Classification of Testing: Exploratory, Characterization, and Validation

Three types of experimentation are necessary to support the rational formulation of constitutive equations, they are exploratory, characterization, and validation tests, as illustrated in Figure 13 and first discussed in (Ref. 34). Specification of typical test matrices, irrespective of which test category, will involve, either explicitly or implicitly, a priori selection of a modeling approach. To maximize the value of any given experimental program (for both current as well as yet-to-be-developed models), one must attempt to capture and document as much material response information (e.g., the complete fundamental response spectrum and not just specific pointwise parameter values, like Young's modulus, ultimate tensile strength, yield stress, etc.) as possible without prejudice to any particular model or idealization. This is required so that one could subsequently change and/or refine modeling approaches. Remember, the more sophisticated the idealization, the more costly the experimental program,³ yet the greater the applicability and higher the accuracy of the model. This in turn increases the demand on any informatics infrastructure as well.

2.3.1 Exploratory Tests

Exploratory tests are those that identify fundamental deformation and damage mechanisms (reversible and irreversible), illuminate salient material response features such as time-dependent

³This cost is associated with an increase in level of effort (setup, alignment, etc.) and resources (time, material, machining, etc.)—and thus dollars—particularly when testing at high temperature.

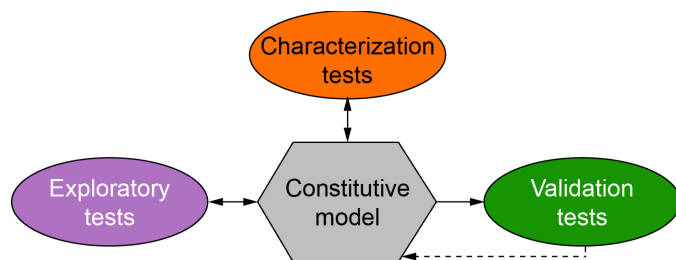


Figure 13.—Interaction between thermomechanical testing and constitutive model development.

and/or time-independent regimes, isotropic and/or kinematic hardening behavior, sensitivity to hydrostatic stress field, material asymmetry (i.e., difference in tension and compression behavior), and/or material isotropy or anisotropy. It is precisely these types of tests that are used to guide the mathematical structure of a model; for example, the partitioning of energy potential into that associated with stored energy and dissipated energy as well as the specific functional forms (power law, exponential, hyperbolic, etc.). An excellent example of this is in the area of unified or nonunified model adoption; that is, investigating whether the inelastic strain should either be divided into a time-independent (plastic) component and a time-dependent (creep) component (nonunified approach) or treated as a single entity (unified approach). To answer this question, creep-plasticity interaction tests (those in which a material is subjected to prior creep or plasticity history before being immediately subjected to the other type of subsequent loading) need to be conducted to ascertain the significance of the creep-plasticity interaction exhibited by a given material (see Section 3.2.1.3.4). Another example would be conducting asymmetry (tension and compression) tests to ascertain whether J_3 needs to be included in the yield or threshold surface definition. Clearly, the extent of exploratory testing can be greatly reduced for those materials that have significant history and prior knowledge associated with them, whereas new materials will require more in-depth analysis.

2.3.2 Characterization Tests

Characterization tests are those that provide the required deformation and damage information (database) for determining the material-specific mathematical functional forms and associated parameters to represent a particular material over a given range of conditions. For example, these types of tests typically involve tensile tests at various strain rates, creep and relaxation tests at various load levels, and cyclic tests at different R ratios, with all of these involving multiple repeats to understand the statistical material variability and performed over a wide range of temperature to understand the sensitivity to temperature. Remember, data content, which implies a mixture of tests (rather than numerous tests of one type) at different temperatures, is much more important than obtaining

merely a quantity of a single or few types of tests because as the content increases, the resulting model should be more robust and able to accurately represent a wider spectrum of loading than a model characterized with insufficient breadth. However, it is helpful to perform repeat tests of the same type to define the variability in response functions (e.g., tensile, creep, cyclic, etc.) and pointwise values (e.g., E , ν , σ_y , etc.) from material lot to lot. A detailed discussion of these types of tests (both uniaxial and multiaxial) and the associated parameter estimation is provided in Section 3.2.

2.3.3 Validation Tests

Validation tests, which are often structural (multiaxial) in nature, provide the prototypical response data, which enable validation of a constitutive model through comparison of structural response with predictions based on the model. Validation tests are typically outside the loading (e.g., stress, strain, temperature) and time domains investigated during characterization, and they typically involve inhomogeneous fields of stress, strain, and temperature. This contrasts with exploratory and characterization tests, which have stringent demands on homogeneity of stress, temperature, and statical determinacy. Results from validation tests ideally provide feedback for subsequent developmental and/or refinement efforts to a given model. Furthermore, validation tests become more and more complex and expensive as one moves up the building-block pyramid; unfortunately, the actual stress field is rarely directly measurable for comparison with model predictions and typically only limited strain-field information at elevated temperature is also available. However, with the advent and increased use of optical full-field measurement techniques, one can hope this will change soon. Consequently, one typically must be content with displacement or deflection measurements at some convenient locations within the structure, thereby providing comparison of just a few numbers with little or no useful feedback for further refinement of the model. Note that while the measurements can be converted to values of stress and strain, such a task typically requires an accurately characterized constitutive model to do so, which—paradoxically—is precisely what one is attempting to do.

As a result, material model validation tests are typically limited to biaxial (tension-compression) specimens or tension-torsion tubular tests where the material behavior is tested under controlled multiaxial loading scenarios and various environments. These tests are more complex and expensive (both in specimen preparation and test frame operation) to perform and often utilize full-field measurement techniques to obtain the various response fields (strain, temperature, etc.). They do, however, provide excellent feedback for model improvement. Note: many models in the literature have not been validated

under multiaxial loading conditions (Refs. 34 to 37), particularly at elevated temperature.

2.4 GVIPS (Generalized Viscoplasticity With Potential Structure) Model

GVIPS belongs to the category of phenomenological (or macroscopic) constitutive material representations based on continuum-mechanics theories (e.g., viscoelasticity and viscoplasticity), utilizing the principles of thermodynamics for irreversible processes. In this, the notion of internal state variables is utilized, which aims at the use of average descriptions for events associated with the rather complex evolution of the microstructure of a particular material system when subjected to different mechanical loads and temperature changes. The phenomenology emphasized in this classification stems from the need to distinguish these approaches from alternative and rather more fundamental approaches, using the multiscale methods (see Ref. 38) in which attempts are made to start from the elementary dynamical processes associated with microscopic defects (e.g., dislocations, grain boundaries, voids, etc., in metals) and proceed with the study of complicated interactions between numerous such processes to span the widely varying length scales involved; that is, from the “atomistic” (nano-) to the microscale to the mesoscale to the final macroscale of interest in typical engineering work (e.g., experimental testing for materials characterization and finite element (FE) analysis of structural components). Despite many advances in the use of direct simulations spanning the entire spectrum of length scales, the daunting tasks of processing the resulting (and typically unmanageably large) data sets from these simulations, and the uncovering of patterns and laws within them, which can provide an effective overall description of the behavior, are as yet largely unfulfilled goals of multiscale modeling. This then provides the main motivation for using phenomenological, physics-based, macroscopic models such as GVIPS. To further appreciate this observation, recall that for a typical metallic test piece (i.e., a few centimeters length scale), one will have to consider interactions between a prohibitively large number of microstructural features, with typically 10^6 to 10^{12} of dislocation lines per a cubic centimeter of the severely (plastically) deformed test piece at the end of the experiments. In addition, some of the physical laws presently in use for 3D dislocation dynamics cannot realistically account for the many, less understood, features of the evolving dislocation networks. For example, consider the magnified scans in Reference 39, which illustrate the increase in dislocation pattern complexity as the deformation of a metal under fatigue-type

uniaxial testing proceeds with strain-controlled cycles. However, this larger picture notwithstanding, several concrete and workable links between physical metallurgy, first-principles calculations, and multiscale modeling simulations concern the identification of important types of suitable state variables and the mathematical structure underlying their evolutions (Ref. 40). Indeed, such links have been motivations in the development of GVIPS models. In particular, the notion of multiplicity of mechanisms in the description reflects the vastly different interactions needed for an aggregate of individual state variables; these include many second-order tensors, each accounting for interactions at different scales: the short range between dislocations and other dislocations; and the long range between a well-developed dislocation cell or subgrain and other defects and interfaces. In addition to differences in microstructural length scales, the time rates of change governing their dynamics are also vastly different, hence the notion of the characteristic time (relaxation) spectrum in GVIPS.

Furthermore, the complete potentiality of the GVIPS framework leads to a hierarchy of representations, from the Gibb’s function (Grandparent) to the kinematic decomposition of the deformation and microstructure state equations (Parent) to the kinetic and evolution equations (Children) (see Figure 14). This is what guarantees that the overall response is always bounded in terms of the total and/or rate quantities: for example, from transient to steady-state creep or from a state of nonlinear hardening to a state of saturation in hardening. It is the intricate interaction of such unit processes⁴, with their vastly different energy contents and rate-limiting values that motivated the use of fundamental energy arguments that underlie the GVIPS models (hence the emphasis on complete-potential energy structure). Thus, the partitioning between energy storage (e.g., hardening) and energy dissipation (e.g., recovery and inelastic flows) that are used proceed with varying degrees of competition during the deformation of the material.

GVIPS is a coupled, fully associative unified viscoelastoplastic model that has been formulated (Refs. 1 to 4 and 12) (along with its more recent extension into the coupled deformation and damage regime, Ref. 6) with sufficient generality to permit systematic introduction of multiple mechanisms for both viscoelastic (time-dependent reversible) and viscoplastic (time-dependent irreversible) response components. This general multimechanism hereditary deformation model has been shown to accurately represent a wide spectrum of material response under different loading conditions for the case of titanium alloys (Refs. 2 to 5 and 41 to 44) and other alloys (e.g., 9Cr-1Mo, INCONEL[®] alloy 738 (Special Metals

⁴Note, energy measures provide the major consideration in dislocation dynamic theories; for example, (1) in the study of thermal activation, (2) mass and vacancies diffusion, (3) energy barriers such as jog

formation energy, and (4) rate dependence of many species (e.g., distributions of finite-strength obstacles).

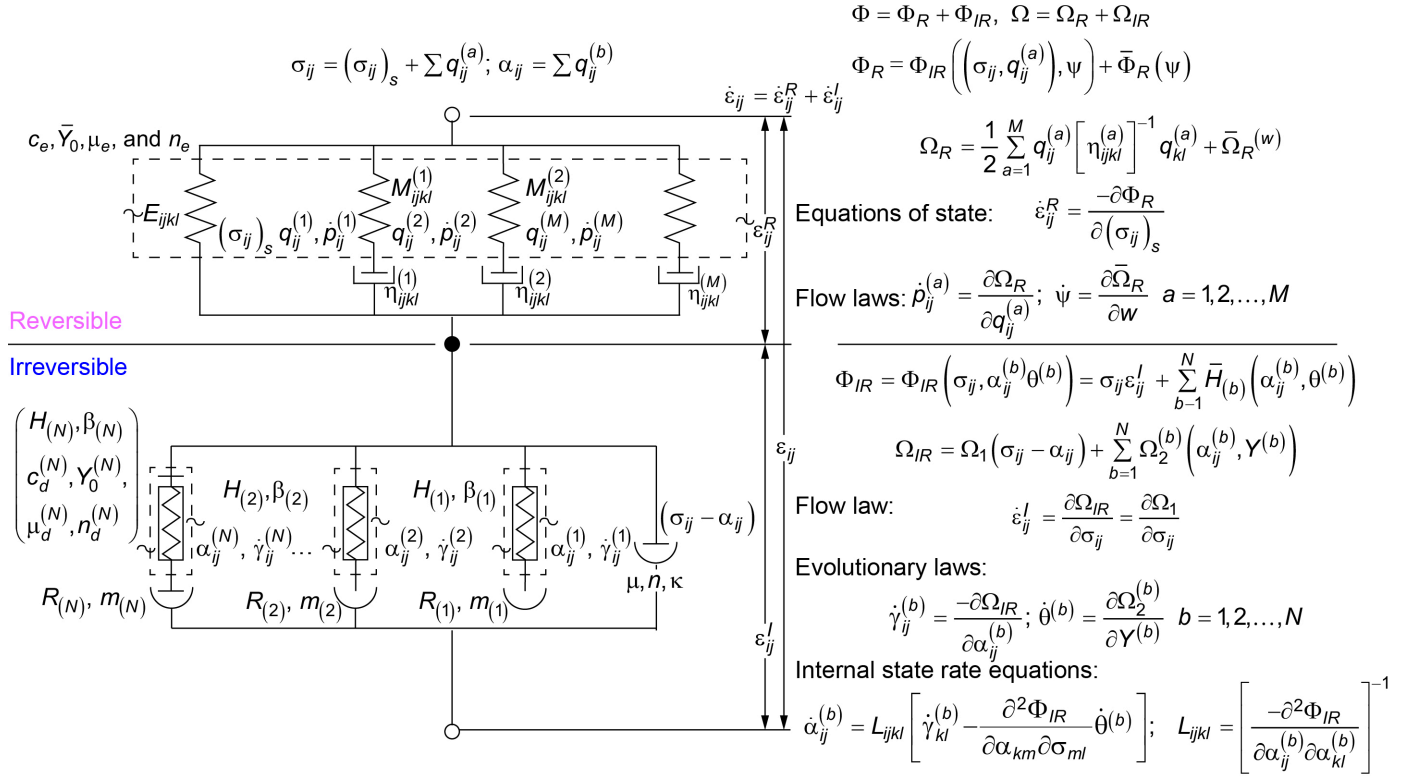


Figure 14.—Multimechanism deformation and damage GVIPS framework with key governing equations provided; see Reference 2 for more details.

Corporation), Waspaloy, Cu-4-Cr-2, and Haynes® 188 (Haynes International) (Refs. 45 and 46). Examples include (1) rate-dependent (effective) material tangent stiffness during initial loading or any subsequent reversed loading, (2) pure transient response (e.g., in creep or relaxation) within the reversibility region, (3) anelastic behavior upon stress reversal, irrespective of the load level, as well as (4) other response features common to unified viscoplastic formulations (e.g., rate sensitivity, creep-plasticity interaction, thermal recovery, etc.).

The following generalized isotropic material behavior constitutive model results:

$$\dot{\sigma}_{ij} = E_{ijkl} (\dot{\epsilon}_{kl} - \dot{\epsilon}_{kl}^I) + \dot{q}_{ij} \quad (35)$$

where Table I provides the definition of inelastic strain rate along with the associated flow and evolutionary laws. A key feature of this constitutive model as opposed to the plasticity and viscoplasticity types of approaches that have traditionally been applied to the analysis of metals (see Table I) is that in the GVIPS approach, the total strain (ϵ_{ij}) is partitioned into reversible (ϵ_{ij}^R) and irreversible (ϵ_{ij}^I) regimes, and within each of these regimes the stress is partitioned into equilibrium ($(\sigma_{ij})_s$ and $(\sigma_{ij} - \alpha_{ij})$) and nonequilibrium components ($q_{ij}^{(i)}$ and $\alpha_{ij}^{(i)}$), as shown in Figure 14.

In general, in previous models in the literature, the reversible (elastic) regime was assumed to be time independent, and the irreversible (plastic) strains were assumed to be time independent (at low temperatures) or more commonly at higher temperature, to be time dependent (viscoplastic). Research efforts have determined that strains in the reversible regime could be time independent or dependent (Refs. 47 to 50), depending upon the temperature. Therefore, concepts from viscoelasticity, which in general previously had not been applied to metals, need to be applied to the constitutive equations employed to analyze metals (Refs. 3 and 4). Furthermore, in the regime of time-dependent strains, because of the wide spectrum of rate dependence of the material in both the reversible and irreversible domains, multiple mechanisms (i.e., spring-dashpot sets in terms of mechanical analogs, see Figure 14) need to be considered. Again, the term “multimechanism” is used to emphasize the connotation associated with the activity of numerous features and components in different contexts, that is, from the underpinning physical metallurgy to the associated mathematical constructs, as well as to the manifestation of macroscopic phenomena and responses. On the physical side, one speaks of microstructural features spanning vastly different length scales, whose rates (velocities) of change are also different during the deformation process.

In previous work, studies determined that a single mechanism provides realistic results across approximately three decades of strain rate. If more than a three-decade strain rate range is likely to be encountered, more mechanisms should be used to fully capture the strain-rate-dependent behavior. The more mechanisms that are used, the more likely the characterized model is to appropriately model the behavior across all strain rates. Obviously, the more mechanisms that are present, the more model parameters and experiments that isolate these various mechanisms are required, so judgment must be employed in choosing the number of mechanisms.

Recently, the GVIPS model was extended to include damage (see Ref. 6) that accounts for two different types of softening mechanisms present in most materials; that is, stiffness reduction (e.g., voids, cracks, etc.) and/or strength reduction (e.g., gamma prime rafting, precipitate shearing, etc.). The associated equations that go with Figure 14 are similar to those in Table I, but they are now extended to include damage.

Equations of state with stiffness damage included:

$$\dot{\sigma}_{ij} = (1 - \psi) E_{ijkl} (\dot{\epsilon}_{kl} - \dot{\epsilon}_{kl}^I) + \dot{q}_{ij} \quad (36)$$

$$\dot{q}_{ij}^{(a)} = (1 - \psi) M_{ijkl}^{(a)} (\dot{\epsilon}_{kl} - \dot{\epsilon}_{kl}^I) - M_{ijkl}^{(a)} n_{klmn}^{(a)-1} q_{mn}^{(a)} - \frac{\dot{\psi}}{1 - \psi} q_{ij}^{(a)} \quad (37)$$

where the number of viscoelastic material parameters is $2+2M+4$ and include E , ν , and parameter pairs $(M^{(a)}, \rho^{(a)})$ per mechanism where $a = 1$ to M along with the four damage parameters, c_e , μ_e , n_e , and $\bar{Y}_0$, which influence the evolution equations associated with stiffness degradation (see the dashed box in the reversible portion of Figure 14).

$$\dot{\psi} = c_e \bar{Y} \quad (38)$$

$$\dot{Y} = \begin{cases} \frac{1}{\mu_e} \left[\left(\sqrt{\frac{\dot{\epsilon}_{ij} \dot{\epsilon}_{ij}}{\dot{\epsilon}_{ref}}} \right)^{1/n_e} - \frac{1}{\bar{Y}_0} \right] & \text{for } \epsilon \geq \bar{\epsilon}_{cut} \\ 0 & \text{otherwise} \end{cases} \quad (39)$$

where $\bar{\epsilon}_{cut}$ is a “cutoff” total strain value below which no damage occurs and $\dot{\epsilon}_{ref}$ is a reference strain rate.

The inelastic flow and internal stress evolution law, including strength reduction damage, are as follows:

$$\dot{\epsilon}_{ij}^I = \begin{cases} f(F) \Gamma_{ij} & \text{for } F \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (40)$$

$$\dot{\alpha}_{ij}^{(b)} = Q_{ijkl}^{(b)} \left\{ \dot{\epsilon}_{kl}^I - \frac{\left\{ R_b [G^{(b)}]^{m(b)} (1 - \theta) \right\}}{H_{(b)} \langle 1 - \sqrt{G^{(b)}} \rangle^{\beta(b)}} \pi_{kl}^{(b)} \right\} \quad (41)$$

$$\text{for } \pi_{kl}^{(b)} \cdot (\sigma_{kl} - \alpha_{kl}) \geq 0;$$

$$= 0 \text{ otherwise}$$

where

$$Q_{ijkl}^{(b)} = H_{(b)} \left(1 - \sqrt{G^{(b)}} \right)^{\beta(b)} \times \left\{ Z_{ijkl} + \frac{\beta(b)}{2k_{(b)}^2 \sqrt{G^{(b)}} [1 - (1 - \beta(b)) \sqrt{G^{(b)}}]} \alpha_{ij}^{(b)} \alpha_{kl}^{(b)} \right\} \quad (42)$$

and the evolution of strength reduction is

$$\dot{\theta}^{(b)} = c_d^{(b)} Y^{(b)} \quad (43)$$

$$\dot{Y}^{(b)} = \begin{cases} \frac{1}{\mu_d^{(b)}} \left[\left(\sqrt{\frac{2\dot{\epsilon}_{ij}^I \Lambda_{ijkl} \dot{\epsilon}_{kl}^I}{\dot{\epsilon}_{ref}}} \right)^{1/n_d^{(b)}} - \frac{1}{Y_0^{(b)}} \right] & \text{for } \epsilon^I \geq \bar{\epsilon}_{cut}^I \\ 0 & \text{otherwise} \end{cases} \quad (44)$$

where the material functions, F and $G^{(b)}$, π_{kl} , and the 3+5N irreversible viscoplastic material constants are given in Table I, and the 4N strength reduction constants are $c_d^{(b)}$, $\mu_d^{(b)}$, $n_d^{(b)}$, and $Y_0^{(b)}$, where $b = 1$ to N , and

$$\Lambda_{ijkl} = \left[\frac{1}{2} (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) - \frac{1}{3} (\delta_{ij} \delta_{kl}) \right] \quad (45)$$

is a deviatoric operator assuming an isotropic material.

In the case of material stiffness degradation, the quantity Ψ is the measure of the amount of stiffness damage and varies from 1 to ∞ (with 1 being the virgin undamaged state and ∞ being the fully damaged material). It is important to note that the evolution of damage is directly evaluated and does not require any iterative updates in the implicit integration scheme for the other stresses and internal variables (Ref. 51). In the present model, a single stiffness damage mechanism, which is identified by the dotted line in the reversible strain region (i.e., the upper region) of Figure 14, is utilized to influence all mechanism stiffnesses. Consequently, a total of four parameters, c_e , $\bar{Y}_0$, μ_e , and n_e , are introduced for characterizing

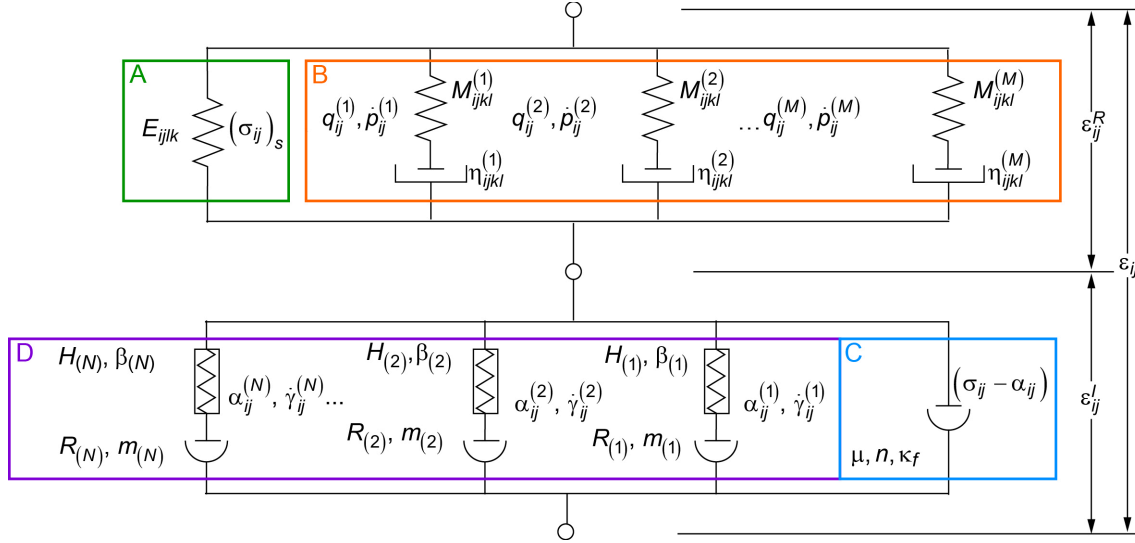


Figure 15.—Four basic blocks of GVIPS constitutive model without damage.

the stiffness damage, which in essence modifies all reversible moduli (i.e., E_{ijkl} and $M_{ijkl}^{(a)}$).

Similarly, in the case of irreversible material softening, or strength reduction, the quantity $\theta^{(b)}$ is the measure of the amount of strength reduction damage, for each mechanism (b), and varies from 1 to ∞ (where 1 is the virgin undamaged state and ∞ is the fully damaged material). Therefore, a total of four strength damage parameters (i.e., $c_d^{(i)}$, $Y_0^{(i)}$, $\mu_d^{(i)}$, and $n_d^{(i)}$) per irreversible mechanism are introduced. This is illustrated in the irreversible regime of Figure 14.

Again, as in the stiffness reduction portion of the model, an inelastic “cutoff” value $\bar{\epsilon}_{cut}^i$ is introduced and compared to the total inelastic strain, below which no strength damage is said to occur.

The use of multiplicity of mechanisms in GVIPS general classes (embedding the effects of many time and/or length scales) has also facilitated the ability to specialize GVIPS to simpler options (as well as those more restricted in scope). The general hereditary multimechanism GVIPS model is composed of four basic blocks, A, B, C, and D, as shown in Figure 15. Using these basic blocks, one can construct any number of specialized deformation models. These models range from the trivial purely elastic case, to the simple linear viscoelastic, to the classical rate-independent elastoplastic, to the rather elaborate forms of hereditary descriptions (involving viscous effects, nonlinear hardening, dynamic recovery, thermal or static recovery, relaxation, ratchetting or shakedown phenomena

under load cycles, etc.). The associated values for each model parameter needed to impose one of the four specializations discussed is given in Table II.

TABLE II.—GVIPS SPECIALIZATIONS WITHOUT DAMAGE

Name	Associated blocks		Model parameters
	Activate	Deactivate	
Elastic	A	B, C, D	All $M^{(a)} = 0$ All $\eta^{(a)} = \infty$ $\kappa_f = \infty$
Perfectly plastic	A, C	B, D	All $M^{(a)} = 0$ All $\eta^{(a)} = \infty$ All $H_{(b)} = 0$ All $R_{(b)} = 0$ $\mu = \text{small}$
Classical plastic with linear hardening	A, C, D	B	All $M^{(a)} = 0$ All $\eta^{(a)} = \infty$ $\mu = \text{small}$ $k_{(b)} = \infty$ All $R_{(b)} = 0$
Norton-Bailey creep	A, C	B, D	All $M^{(a)} = 0$ All $\eta^{(a)} = \infty$ All $H_{(b)} = 0$ All $R_{(b)} = 0$

2.5 Parameter Estimation

Now, given fundamental material response histories, the most important and often most difficult⁵ aspect of modeling the behavior of a given material is obtaining the required material functions (e.g., $f(F)$ and $g(G)$) and associated material parameters (see Table I). The difficulty associated with this process typically stems from not only the variety in mathematical forms for the material functions (e.g., power law, exponential, hyperbolic sine, etc.), but also the fact that given the material functions, there is often no unique set of material parameters for any given load path. Therefore, numerous iterations and difficult compromises are required before a final set of material parameters (for the assumed material functions) can be obtained.

Traditionally, characterization has been accomplished through basic trial-and-error procedures (i.e., graphical and/or mechanistic), which attempt to fit the predicted response from the constitutive model to the response exhibited by the experimental test data. In the case of idealized elastic, elastic-plastic, or steady-state creep material behavior, these approaches are quite successful; however, for more general and sophisticated constitutive models such as GVIPS, they are rather limited, difficult, and many times less than fruitful. This is particularly true when dealing with models that possess a very large number⁶ of material constants (see GVIPS model in Table I) that (1) are often lacking in their direct physical interpretation (this is not the case in the GVIPS model), (2) may have vastly different magnitudes, and (3) are highly interactive with each other. Further complications also arise when a large number of experimental tests of various types (i.e., stress-, strain-, or mixed-control) under transient and/or steady-state conditions are expected to be simulated. Thus, there is an obvious need for a systematic development of a general methodology for constitutive parameter estimation. To this end, software tools have been developed (e.g., COMPARE (Refs. 5, 6, and 49), Z-mat (Ref. 52), and Murthy (Ref. 53)) to enable a design engineer to easily and efficiently bridge this gap between constitutive theory and experimental test data, in which optimum material parameters are determined by minimizing the errors between experimental data and the correlated responses. Within

COMPARE, the estimation of material parameters is accomplished by casting the problem as a minimum-error, weighted-least-squares, multiobjective optimization problem, wherein this optimization problem is solved using the sequential quadratic programming (SQP) technique (see Figure 16). Also, COMPARE is sufficiently general to handle a comprehensive set of test data under arbitrary load-control variables, multiaxial stress and/or strain state, and both transient and steady-state response measurements.

It is important to realize from the outset that the resulting set of material parameters (regardless of the parameter estimation tool used to obtain them) is nonunique due to the nonlinear nature of the problem; however, if enough data content is provided, then it is generally believed that the final obtained parameters should be relatively independent of the initial guess and bounds provided. Here, data content is meant to imply not just quantity but also variety of experimental data provided, which will highlight and illuminate various issues such as multiaxiality, time scale, control mode (stress versus strain), and multiple deformation and damage mechanisms, to name a few. For example, in the case of time scales, if one only provided stress-strain tensile data, then one should not expect a model that possesses sufficient breadth to be able to accurately predict both creep and relaxation response because the time duration of these types of tests far exceeds that of a tensile test. Similarly, if one provides only creep or relaxation data, one should not expect the model to accurately represent the typically shorter time domain tensile behavior of a given material.

The order of characterization is as follows: (i) viscoelastic parameters, (ii) viscoplastic parameters, followed by (iii) damage parameters characterized in conjunction with refinement of viscoplastic parameters if damage mechanisms are to be included; it is then often helpful to conduct a final optimization run with tight limits to ensure that all parameters can be updated if necessary. A resulting set of viscoelastic deformation parameters (see Table III) and viscoelastoplastic deformation and damage parameters are shown in Table IV and Table V, respectively. These tables outline all of the material constants necessary to describe the full behavior of Ti-6-4 at elevated temperatures.

⁵Depending upon the sophistication of the constitutive model and environment; for example, room temperature versus elevated temperature and loading conditions.

⁶In the specific model presented, the total number of material parameters required is $5+5N+2M$, where N defines the number of viscoplastic mechanisms and M the number of viscoelastic, see Figure 15.

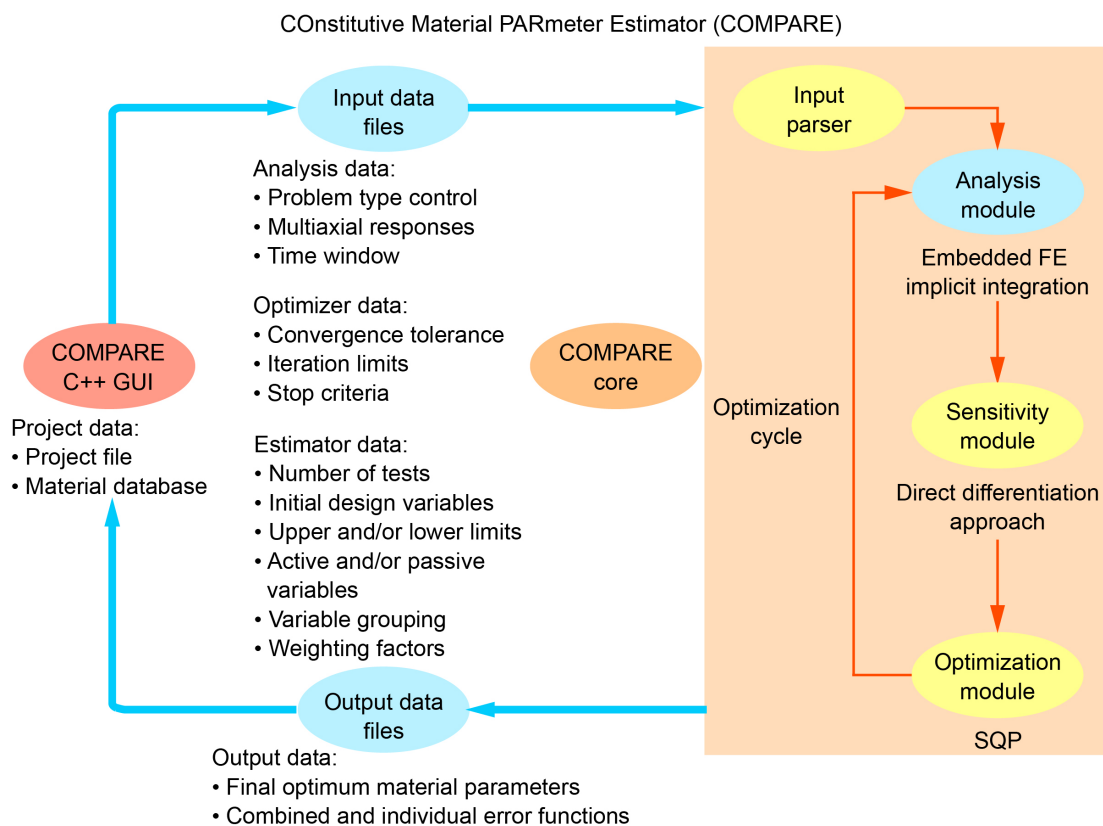


Figure 16.—Flow chart for COMPARE.

TABLE III.—SET OF CHARACTERIZED PARAMETERS FOR VISCOELASTIC DEFORMATION PORTION OF GVIPS MODEL: Ti-6-4

Constant	Temperature, °F (°C)				
	600 (316)	700 (371)	800 (427)	900 (482)	1,000 (538)
E_s , ksi (MPa)	1.28×10^4 (8.83×10^4)	1.09×10^4 (7.52×10^4)	9.54×10^3 (6.58×10^4)	7.16×10^3 (4.94×10^4)	2.66×10^3 (1.83×10^4)
E_{m1} , ksi (MPa)	1.78×10^3 (1.23×10^4)	2.35×10^3 (1.62×10^4)	2.91×10^3 (2.01×10^4)	3.05×10^3 (2.10×10^4)	3.82×10^3 (2.63×10^4)
ρ_1 , s ⁻¹	21,082	21,082	21,082	21,082	21,082
E_{m2} , ksi (MPa)	-----	297.744 (2.06×10^3)	454.565 (3.1×10^3)	735.838 (5.1×10^3)	1,121.079 (7.7×10^3)
ρ_2 , s ⁻¹	-----	3,394	3,394	3,394	3,394
E_{m3} , ksi (MPa)	-----	-----	353,500 (2.44×10^3)	694,779 (4.79×10^3)	3,666,691 (2.53×10^4)
ρ_3 , s ⁻¹		-----	290	290	290
E_{m4} , ksi (MPa)	-----	-----	-----	801,539 (5.53×10^3)	1,165,859 (8.04×10^3)
ρ_4 , s ⁻¹	-----	---	-----	56	56
$D = E_s + \Sigma E_{mi}$, ksi (MPa)	1.46×10^4 (1.01×10^5)	1.35×10^4 (9.31×10^4)	1.33×10^4 (9.17×10^4)	1.24×10^4 (8.5×10^4)	1.24×10^4 (8.55×10^4)

TABLE IV.—SET OF CHARACTERIZED PARAMETERS FOR VISCOPLASTIC
DEFORMATION PORTION OF GVIPS MODEL: Ti-6-4

Constant	Temperature, °F (°C)				
	600 (316)	700 (371)	800 (427)	900 (482)	1,000 (538)
κ , ksi (MPa)	25.4266 (175.31)	20.01187 (137.98)	13.6292 (93.97)	3.2542 (22.44)	1.580 (10.89)
n	9	8.875359	5.23094	2.10964	1.5
μ , ksi·s (MPa·s)	7.99×10^4 (5.51×10^5)	1.02×10^5 (7.03×10^5)	1.48×10^6 (1.02×10^7)	3.49×10^6 (2.41×10^7)	3.95×10^7 (2.72×10^8)
k_1 , ksi (MPa)	19.0028 (131.02)	15.11888 (104.24)	14.5706 (100.46)	10.7693 (74.25)	11.5067 (79.34)
m_1	1.921247	1	1.00592	2.37143	1.99597
β_1	1.004319	9.899237	8.50291	2.82444	2.68514
R_1 , s ⁻¹	8.67×10^{-6}	1.34×10^{-4}	4.22×10^{-4}	2.05×10^{-2}	8.34×10^{-2}
H_1 , ksi (MPa)	5.21×10^4 (3.59×10^5)	9.08×10^4 (6.26×10^5)	1.22×10^4 (8.41×10^4)	1.03×10^4 (7.10×10^4)	41.3 (284.75)
k_2 , ksi (MPa)	10.23959 (70.60)	14.99791 (103.41)	3.98683 (27.49)	16.1439 (111.31)	10.5588 (72.80)
m_2	1	1.817302	2.23399	1.00688	1
β_2	1.094341	1.001861	3.47164	7.76606	7.41099
R_2 , s ⁻¹	1.00×10^{-7}	1.13×10^{-7}	1.20×10^{-6}	1.75×10^{-3}	9.12×10^{-3}
H_2 , ksi (MPa)	374 (2.58×10^3)	2.31×10^3 (1.59×10^4)	1.72×10^3 (1.19×10^4)	9.36×10^4 (6.45×10^5)	3.01×10^4 (2.08×10^5)
k_3 , ksi (MPa)	-----	15.00021 (103.42)	15.6986 (108.24)	10.2503 (70.67)	10.0907 (69.57)
m_3	-----	1.091881	1.15705	1.00354	1
β_3	-----	4.80568	5.05322	2.6967	7.54667
R_3 , s ⁻¹	-----	4.26×10^{-7}	5.02×10^{-7}	1.59×10^{-3}	1.02×10^{-3}
H_3 , ksi (MPa)	-----	10.0 (68.95)	687 (4.74×10^3)	651 (4.49×10^3)	2,910 (2.01×10^4)
k_4 , ksi (MPa)	-----	-----	-----	-----	9.00185 (62.07)
m_4	-----	-----	-----	-----	1.78684
β_4	-----	-----	-----	-----	2.16416
R_4 , s ⁻¹	-----	-----	-----	-----	4.50×10^{-3}
H_4 , ksi (MPa)	-----	-----	-----	-----	5.33 (36.75)

TABLE V.—SET OF PRELIMINARY GVIPS DAMAGE
PARAMETERS ASSUMING BOTH STIFFNESS AND
STRENGTH REDUCTION MECHANISMS
ARE ACTIVE: Ti-6-4

Constants	Temperature, °F (°C)		
	800 (427)	900 (482)	1,000 (538)
Cutoff	0.05	0.04082	0.045
c_e	18.24602	0.56032	17.92172
$\bar{Y}_0$	0.363438	0.68218	0.250604
μ_e	0.147320	0.35287	0.241456
n_e	0.713635	0.76574	0.752975
Cutoff	0.005	0.002	0.003
$c_d^{(1)}$	0.938191	0.97327	-----
$Y_0^{(1)}$	0.925382	0.97191	-----
$\mu_d^{(1)}$	0.985915	0.99865	-----
$n_d^{(1)}$	0.109727	0.34328	-----
$c_d^{(2)}$	1.463858	0.10360	19887.65
$Y_0^{(2)}$	1.438048	0.10319	3.260156
$\mu_d^{(2)}$	0.965642	0.09998	0.022
$n_d^{(2)}$	1.959042	0.73605	0.275047
$c_d^{(3)}$	0.103319	0.10080	19904.23
$Y_0^{(3)}$	0.103672	0.10014	3.30446
$\mu_d^{(3)}$	1.022884	0.19326	0.022024
$n_d^{(3)}$	0.1	0.78482	0.379256
$c_d^{(4)}$	-----	-----	0.583012
$Y_0^{(4)}$	-----	-----	0.576292
$\mu_d^{(4)}$	-----	-----	1.043089
$n_d^{(4)}$	-----	-----	0.1

Note that 600 and 700 °F are not included in Table V because of an inadequate number of experiments to perform a complete correlation for the damage portion of the model. More loading experiments that exhibit damage to the specimen would be necessary to optimize a set of parameters.

3.0 Data Content and Its Impact on Modeling

3.1 Implication of Testing

One cannot overemphasize the importance of experimentalists and modelers working together when developing a given test matrix; often, model parameters that are required can be hard—if not impossible—to measure. Furthermore, it is imperative that modelers strive to understand exactly how experiments are conducted to ensure proper interpretation of the results. For example, the control mode may be load (stress) or displacement (strain) with the target value being either stress or strain and the temperature being either held fixed or changing. In the case of elevated-temperature testing, the control mode and rate of application has a major influence, especially in the case of thermomechanical deformation or fatigue, wherein load and/or temperature phasing can have a huge effect. One must also understand how the response is being measured (e.g., strain gage or extensometer) and whether feedback (as in the case of strain-controlled tests) to the test machine is utilized or merely crosshead (e.g., displacement) control is being utilized. This is particularly important when deformation and damage inhomogeneities occur and for elevated temperature testing because rate effects become more pronounced, such that the actual total strain rate throughout a given displacement-controlled tensile test can vary by orders of magnitude from start to finish.

Another excellent example is the classic creep test; it is conducted either on a deadweight machine for long-duration tests (days to years) or on servohydraulic or electromotor machines for short-term tests (hours to days). Regardless of duration, in both cases, these tests are typically load controlled and not stress controlled as often thought by the modeling community. This has significant implications: the strain accumulates and the cross-sectional area of the material changes, thus causing a difference between engineering stress (F/A_0) versus true (Cauchy) stress (F/A). Therefore, the purpose of this section is to briefly introduce the various types of test machines and measurement devices, specimen preparation, and potential issues one can encounter to get the reader thinking about the types of questions a modeler should ask when interacting with an experimentalist or before utilizing results they have obtained from the literature.

3.1.1 Test Machines and Measurement Devices

As much as possible, a given volume element within a specimen (gage length) should be subjected to a uniform field of stress, strain, and temperature. This can be quite difficult, depending upon the specimen design and the form of heating, but it is extremely important when performing material exploratory and/or characterization testing. Depending upon the type of loading desired, different types of test machines are utilized; dead-load weights are often utilized for long-duration creep testing, electromechanical (e.g., Instron®, Illinois Tool Works Inc.) or servohydraulic (e.g., MTS®, MTS Systems Corp.) for tensile, compression, or cyclic testing, and short-term creep and relaxation tests. Electromechanical and servohydraulic systems (see Figure 17) have different sizes of load cells (e.g., 20 kip (89 kN) or 50 kip (222 kN)), and each can have different types of grips (e.g., warm, water-cooled, or hydraulically actuated collet-type grips). The size of a load cell, grips (i.e., boundary conditions), and extensometer setting can all affect the accuracy of resolution such that if very accurate measurements are required and a 50-kip load cell is used instead of a 20-kip load cell, the resolution of the given load step will be affected. Tests typically run include elastic properties (e.g., stiffness and Poisson's ratio), static deformation and/or failure (e.g., creep and stress relaxation), crack growth, quasi-static response (e.g., tension, compression, shear, and bending), low-cycle and thermomechanical fatigue, and low- or high-cycle isothermal fatigue.

Elevated testing requires different types of heating methods; for example, resistance furnace (see Figure 17(a)), direct induction heating, or quartz lamp heating (Figure 18). Strains are measured using strain gages at lower temperatures whereas at higher temperatures, extensometers are typically used (see Figure 18(a)). In the case of transverse strain measurements, an optical micrometer is often used to obtain diametral strain measurements; more recently it has become more commonplace to measure strains using full-field measurement techniques or digital image correlation (DIC) (Ref. 54). An example of DIC is shown in Figure 19. DIC techniques add significant complexities that one should be aware of, such as camera setup, calibration, surface treatment (i.e., speckle pattern, which can affect the stiffness for some materials, though it is generally not an issue with metals or composites). This is particularly true at elevated temperatures. Full-field techniques typically cannot capture rigid-body motion because if the camera is moved slightly or the sample moves during testing, everything is assumed to be a strain. Remember, 2D imaging (a single camera) assumes no out-of-plane strain development occurs, whereas 3D imaging (a two-camera setup) can capture out-of-plane strain but has higher uncertainty and is more difficult to calibrate. Imaging for measurement often has a tradeoff between camera resolution, sampling (frame rate), and memory.



Figure 17.—Uniaxial electromechanical and servohydraulic test frames. (a) Instron® axial test rig with furnace oven capable of 1,400 °C. (b) MTS® axial test rig with environmental chamber for vacuum or inert environment testing.

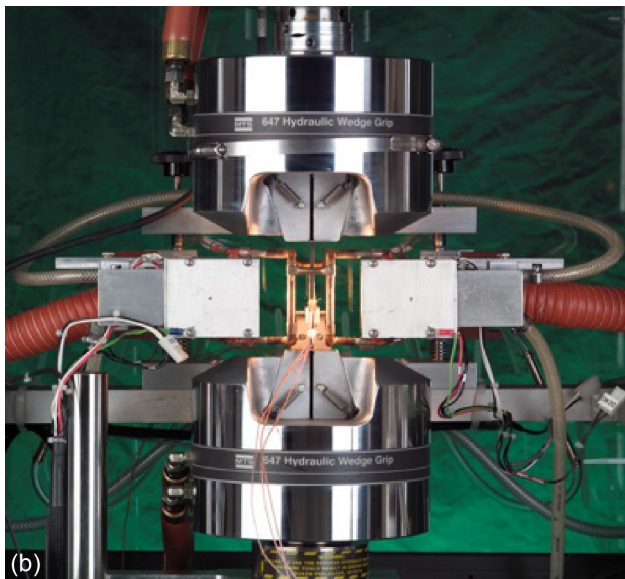
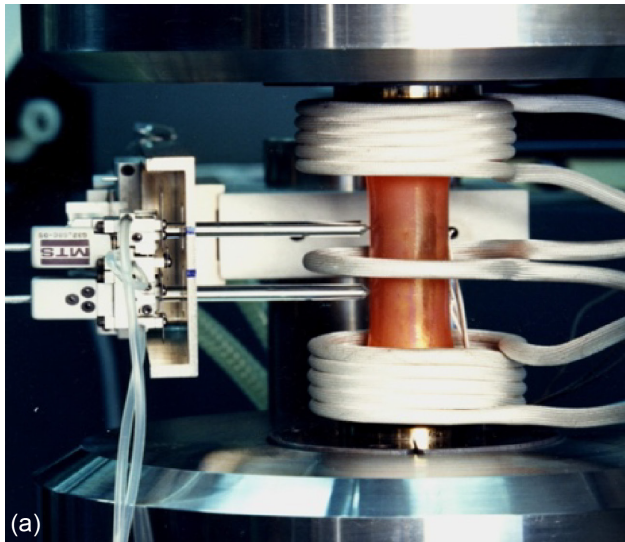


Figure 18.—Heating method examples. (a) Direct induction heating. (b) Quartz lamp system.

For example, to capture crack formation, a high-resolution camera is typically needed to get enough pixels at the crack tip for an accurate strain measurement, which limits the frame rate based on the memory available and the ability to write the images to the disc or hard drive. This is an important issue because the inability to write images instantly (the higher the resolution, the longer the write time) can change the synchronization of image time with the machine (load) frame time, which can throw off all results. Note, sampling rates for DIC are much lower than those for traditional strain measurement techniques; for example, ~ 1 kHz for strain gage measurements, but ~ 1 to 5 Hz for quasi-static tests because of the memory issue. Moreover, one must know where the crack will form to be able to focus the high-resolution area.

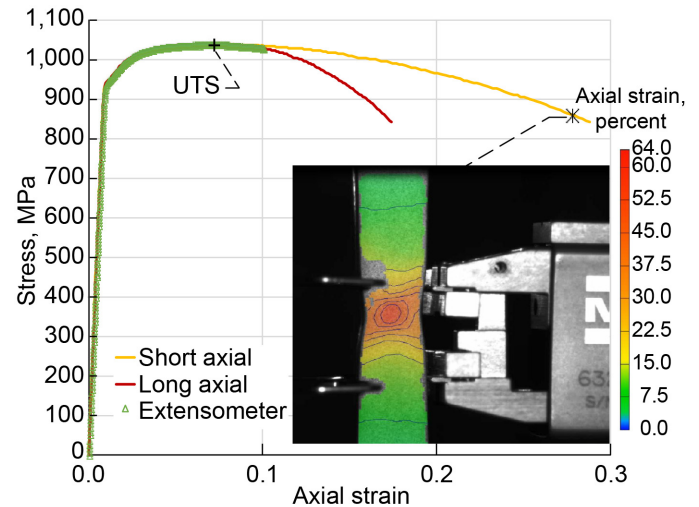


Figure 19.—Tensile test on rectangular specimen where strains are measured using extensometer and full-field technique (with short and long axial strain measurement extracted from full-field measurement within gage length). See Reference 14.

Remember, stresses are computed from forces, which are measured by load cells (dynamometers mounted in series with the specimens), with equations for isostatic cases. Further, it is extremely important that both the modeler and experimentalist work closely together and that modelers exert sufficient energy to fully understand the way a given test is conducted so that interpretation of the results can be made correctly and consistently. Many times, a given measurement may not be wrong, but the interpretation of those measurements might be (see Appendixes B and D of Ref. 14 for a case in point). For example, strain-controlled, thermomechanical fatigue tests are extremely difficult to run because any error in the calculation of thermal strain (coefficient of thermal expansion) could induce error because these typically are based on constant mechanical strain (where mechanical = total – thermal) rate control. Another classic example is a creep test, wherein the loading is always spoken of in terms of applied stress (even the definition of creep strain begins when the stress is held constant), when in actuality the typical test is run under load control and not stress control. Therefore, as time progresses and total strain accumulates, the actual stress increases in proportion to the total strain; for example, $\sigma = \sigma_0(1 + e)$ where σ_0 is the initially applied stress, F/A_0 , and e is the total strain. Note, most constitutive models assume the inelastic stress is driven by true Cauchy stress, which is equivalent to engineering stress only in the small-strain regime ($e < 0.02$ or 0.03). This can have significant impact on the characterization of a given model under creep loading, particularly a long-duration or multistep creep test wherein the accumulated strain can become large.

Similarly, when testing to failure, one needs to be aware of specimen necking and the location thereof, such that this large deformation will clearly impact the interpretation of the actual stress being applied and potentially the strain measurement, which is dependent upon the location of necking versus where the strain is actually being measured (i.e., dependent on the strain measuring device).

Lastly, the measurement of temperature is typically accomplished using one or more thermocouples, which are sensors consisting of two dissimilar metal wires joined at one end and connected to a temperature readout (thermometer) at the other. These are used to measure temperature at a given point. Optical IR (infrared) cameras are noncontact instruments used to obtain full-field temperature measurements over the surface of a given object. It is very important to have accurate temperature measurements, particularly when inelastic flow is occurring because such flow is highly dependent upon temperature. Therefore, one should be aware of the accuracy of the devices used to measure temperature. Typically, thermal gradients over a given gage length are limited to 1 °F or °C per unit of measure. Further, remember that thermocouples must be welded or soldered onto the specimen, which can induce a site for fracture initiation (as can other specimen-contacting instrumentation). Again, one must be aware of this potential when addressing failure loads and the like.

A multiaxial load system consists of a tension-torsion or in-plane biaxial testing machine equipped with four hydraulic actuators with 490-kN (110-kip) tensile and compressive capacity, arranged perpendicularly in a vertical load frame (see Figure 20). At NASA Glenn Research Center, one of the biaxial systems is also outfitted with an environmental chamber that permits testing in both high vacuum and inert environments. Representative specimens include plates, beams, shells, fasteners, fabrics, and built-up metallic and composite structures.

3.1.2 Specimen Preparation

The most common test is a simple uniaxial tension test in which the gage length of the specimen is subjected to a uniform stress and strain field. Sample design is stringently controlled, and designs of samples are regulated by standards (e.g., ASTM E8, ASTM E466, etc.). Figure 21 shows an example of rectangular dog-bone, cylindrical dog-bone, and thin-walled tubular specimen designs. Note the gage area is typically smaller in width or thickness than the remainder of the specimen so that it is subjected to higher stress; thus, the likelihood of failure within the gage length is not only higher, but the deformation in the gage is uniform, homogeneous, and easily measurable. The specimen design (e.g., the radius of curvature, R) is based on minimizing the stress concentration due to thinning of the gage area—again, to minimize deformation and failure outside of the gage area.

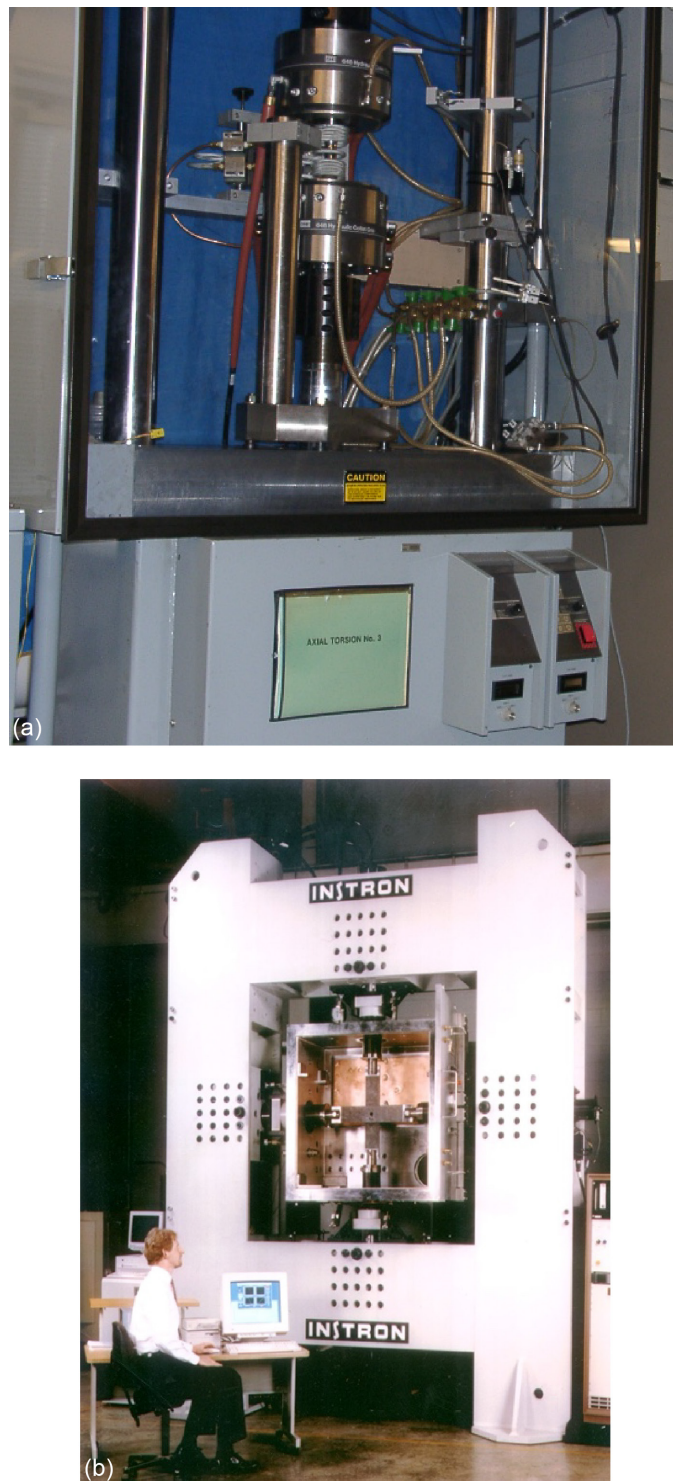


Figure 20.—Multiaxial test frames. (a) Axial-torsion (18,662 kg (50-kip) axial and 2,260 Nm (20,000 in.-lb) of torque) and induction heating system. (b) In-plane biaxial testing machine equipped with four hydraulic actuators with 490 kN (110,000 lb) capacity.

Tension-torsion specimens are often thin-walled tubular specimens, and in-plane material characterization or validation specimens are often cruciform specimens. The finish of a given specimen is very important for specimens that will be used to predict fatigue life, whereas deformation specimens do not need as smooth of a surface finish. Similarly, should a distinction be made for matching specimens to a given application versus modeling changes for a given application? For instance, if an additively

manufactured (AM) part was built for a fatigue application for which polishing the surface is not possible, should fatigue tests be conducted on specimens with the as-built configuration or should polished specimens and/or parts be tested and then modeled to determine a knockdown factor at the part-level testing? These types of questions need to be addressed before specifying a given test program. See earlier discussion on idealization consistency in Section 2.0, “Background: Constitutive Modeling.”

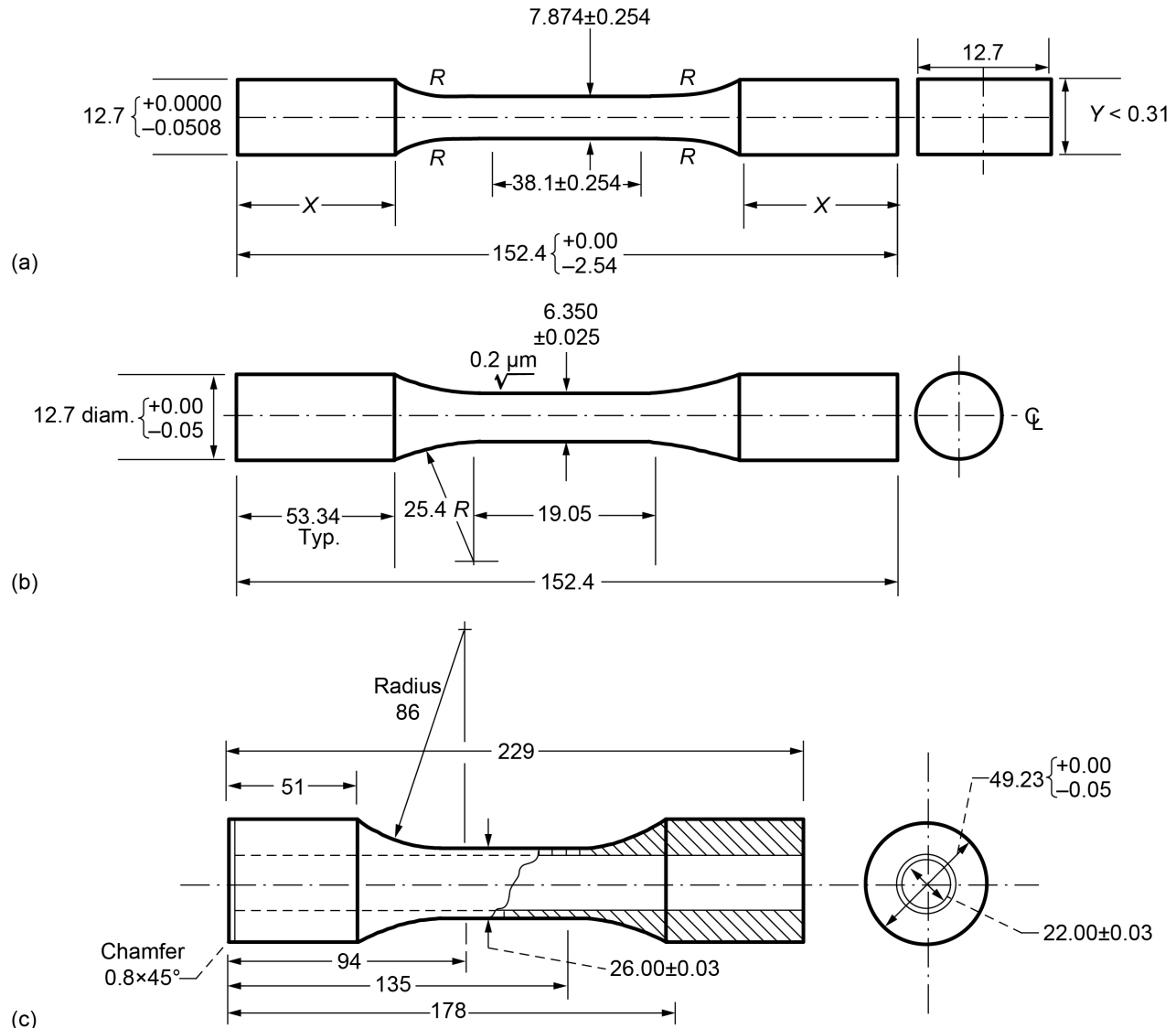


Figure 21.—Test sample designs. All dimensions are shown in millimeters. (a) Rectangular dog-bone. (b) Solid tubular dog-bone. (c) Tension-torsion thin-walled tubular specimen.

3.1.3 Potential Testing Issues

In this section, several topics will be discussed that could pose an issue if not discussed or understood by the practitioner.

- **Boundary conditions.** Boundary conditions (BCs) associated with test fixtures and clamping devices are important for a modeler to understand. For example, the results of a test will vary widely if a modeler (1) assumes a pinned BC when in reality it is clamped, (2) assumes perfect clamping when there is machine compliance, (3) assumes perfect clamping when there is damage in the gripped region due to clamping, or (4) assumes perfect clamping when stresses are introduced into the specimen from the fixture. Therefore, BCs should be taken seriously.
- **Elastic modulus testing.** Determination of the elastic modulus at various strain rates and temperatures is often done on the same sample because most of the test range can be established with one specimen, thus minimizing material. This can be done by simply loading and unloading the sample at various rates. It is important to stay lower than the proportional limit to prevent inelasticity and avoid creating history effects. This is simple at lower temperatures, where time dependency is minimal. However, it is more difficult at elevated temperatures, where rate effects play a significant role in influencing the apparent yielding or the proportional limit becomes much lower. These tests will also act as replicates for moduli taken from other types of tests.
- **Dynamic modulus testing.** To determine dynamic moduli, a fast rate (frequency) method is best used. Often, systems that create a resonance in a sample (vibration (1 to 10 kHz), ultrasonic (10 to 100 kHz), etc.) are used. These methods can be performed at various temperatures; however, special equipment is needed. Standard mechanical load frames could be used such as high-cycle fatigue (HCF) rigs (10 to 1,000 Hz) or perhaps even low-cycle fatigue (LCF) rigs (0.1 to 3 s⁻¹); however, the user would have to show that the rate was sufficiently high to omit viscous behavior. Experience has shown that a strain rate of 1×10⁻³ s⁻¹ is a suitable value for estimating the dynamic modulus at all but the highest temperature, and even there the error is small.
- **Exploratory testing.** Exploratory tests should be done to provide guidance for choosing the target limits for subsequent characterization tests. Exploratory tests for modulus were explained previously. Similarly, values of threshold stress and proportional limit should be found as a function of loading rate and temperature. Note that a complete set does not have to be ascertained in the exploratory region. All that

is needed is sufficient information to gain a good estimate of these values for conducting further tests. A good idea of the boundaries and limits for future tests is also valuable. For example, when conducting a tensile test at a slow strain rate, how long will it last? In the lower extreme (low loading rates, i.e., 1×10⁻⁶ s⁻¹), the test may have to be terminated for time reasons before failure or other desired values are attained. Experience has shown that a maximum time of 12 to 24 h is a good choice for total monotonic test times and for hold periods in creep or relaxation because it allows the test to run overnight and be terminated the next day. Most of the time dependency is recovered over this time. Minimal information is gained by using longer tests other than concatenated tests.

- **Loading rate issues.** There will always be a tradeoff between using a rate high enough to lock in time dependency yet not so high as to overshoot on the subsequent target value(s) for the remainder of the test, be it stress or strain. Such an overshoot, for example, in the case of a creep test, will induce overloads in the stress, thereby modifying the subsequent creep behavior. It could also cause an overshoot in the strain limit for stress relaxation tests. For other types of test frames, higher rates may be possible. The highest rates will also be influenced by the tuning of the machine (controller PID⁷ settings). For certain test rigs (e.g., screw-driven machines), a sufficiently high rate may not be attainable to lock in time dependency because of their slow response.

To lock in all viscoelastic behavior, loading rates should be done at very fast levels. This is especially true at the highest temperatures, where diffusion is prominent. A rate of 1×10⁻³ s⁻¹ (or its equivalent in load control) was used for most tests discussed in this report. This was the minimum rate that was thought to be usable, yet it allowed for a smooth transition from strain- to load-controlled modes on a servohydraulic test frame.

- **Control.** For both creep and stress relaxation tests, the test machine utilized must have good stability and the ability to maintain the target value (stress/load or strain) over a long time period. The target value must not drift with time, nor should there be oscillations in the target value in the short term. Both the load and strain measurements must have good stability and sufficient resolution to detect small and rapid changes in values. This is particularly important when performing viscoelastic tests that are conducted at small values of stress and strain. It is also important to record fast

⁷PID stands for proportional, integral, and derivative. These are the various settings that control things like setpoint over- and undershoot setpoint oscillation, speed at which the setpoint is achieved, stability

of setpoint over time, and so forth. All controllers (e.g., temperature, hydraulic, pneumatic, and electronic) use PID controls.

transients, such as during a mode switch, step change, or just loading under fast rates. The choice of resolution is sometimes paradoxical because small resolutions are often needed even though the test will be conducted to large values. For instance, this is true in a tensile test where high resolution is desired for calculating accurate values for the modulus, proportional limit (or transition at the knee of the curve), but larger ranges (higher load and strain capability) are needed to reach failure. Therefore, a compromise on the range of the measuring device may be needed.

- **Strain measurements.** If multiple strain measurements are desired at the same time, such as longitudinal strain using various gage lengths or the desire to measure transverse strains, the additional equipment necessary will cause complications, primarily from space limitations. Larger specimens may then be required, and this should be determined in advance because sample geometry may affect results. Moreover, larger samples require more material, so sufficient supply (preferably from the same stock) must be available. Larger material forms may also be needed if testing in other dimensions is required, for example, when testing for anisotropy in different dimensions.
- **Transverse strain measurements.** Several strain-measuring devices are available to choose from, each with its own advantages and disadvantages.
- **Strain gages.** These are the most common device. Depending on their size, multiple gages can be placed on the sample, measuring different directions. Strain gages are highly accurate and easy to use, but they are normally good only at lower temperatures (<300 °F).
- **Clip-on extensometers.** Such devices can also be used, yet they normally only measure strain in one direction. Versions for measuring strain at elevated temperatures are also available. Extensometers are easy to use but take up much more space than strain gages, which limits the number of extensometers that can be placed on a sample. More extensometers may require larger test samples. Extensometers also can induce failure at their contact points on the sample. They must also be compatible with the heating type used in the test.
- **Optical displacement measuring devices** are also available, but these are more expensive and more difficult to use than contact devices. They must have line of sight to the sample and therefore must be considered along with the heating (or cooling) system. However, they can provide a 2D strain measurement on the sample at multiple points (i.e., able to indicate strain inhomogeneities) and over the full field. Moreover, the strain field can be determined on a scale that is smaller than the assumed uniformity value within the gage. One drawback is that data processing requires much more time than the previous methods. Using such techniques

allows tracking Poisson's ratio during the test; however, the area where the transverse strain is measured has to be consistent with the area from the longitudinal direction (see Ref. 14). Moreover, the addition of strain localization (e.g., from damage) may result in strange behavior from the transverse strain and Poisson's ratio values.

An excellent example illustrating the inhomogeneity of local fields within the gage length of a cyclic test specimen is provided in (Ref. 55), wherein 10 microstrain gages were mounted within the gage length as well as an extensometer, which measured the entire gage response. It was shown that each microstrain gage measured a different cyclic response (the mean strain being different in magnitude as well as sign); however, the average of these 10 gages provided the same response as that obtained from the extensometer. Such work clearly demonstrates the strong connection between the measurement device and volume of material over which the average is measured.

- **Heating method.** The choice of heating methods must be considered in advance. Not only must the desired temperatures be obtainable, but they must also be constant and uniform over the gage section. The addition of any instrumentation touching the sample can lead to localized cold spots as well as crack initiation sites. If performing experiments with a changing temperature during the test, a heating system must be chosen that has low thermal mass and can vary the temperature quickly (e.g., direct induction heat) in a controlled fashion.
- **Data acquisition.** When conducting tests, the most information should be acquired from each sample. First, data acquisition rates should be variable during the test to capture transients without saving too much additional data during the rest of the test. This is possible with today's computer-controlled test systems, but it does require more complex programming. As far as tests go, the maximum value should be gained from each sample. For example, after viscoelastic (fully reversible) tests, tests can be run for viscoplastic and/or damage characterization. These additional tests can be performed later in time or concatenated to the previous test. The latter requires more complex programming and foresight. All tests should have additional steps that eventually lead to (near) failure. In this way, history effects can be characterized while minimizing material and machining costs. Such samples can also be used to validate the characterized model. One exception to this is if the sample is to be used for destructive testing (e.g., microscopy) after some loading step. It is best if the subsequent tests are recommended by the analyst to gain the maximum value from the validation. To fully exercise the model, the validation tests should be

well thought out and complex. Note that while any loading state is possible on paper, the instrumentation has limited capability. Complicated tests require sophisticated and costly equipment that is usually out of the bounds of resources (and the patience of the experimenter!).

3.2 Critical Characterization and Validation Testing Requirements

3.2.1 Deformation: Uniaxial

3.2.1.1 Preliminary Tensile Tests

Prior to detailed characterization testing, a set of monotonic uniaxial constant-strain-rate, strain-controlled, tensile tests should be conducted to gain an overall understanding of key features of the material response. These preliminary or exploratory tests, described in Table VI, provide initial understanding of the material and estimates for some of the key material parameters, such as the threshold limit κ (i.e., the stress delineating the boundary between reversible and irreversible response), rate dependence of elastic modulus, and maximum internal stress contribution, all of which will be useful in defining later tests. Furthermore, to gain insight into a material's temperature dependence, tests should be conducted at up to four temperatures, with the temperatures ranging from room temperature (where any rate, but not necessarily time dependency, of the material response is expected to be minimal) to relatively high homologous temperatures, for example $TH = 0.4$, where the material response would be expected to have a great deal of both rate and time dependence. Note these exploratory tests can also be considered characterization tests, as they provide initial data, which can be used to determine specific material constants as well. These tensile tests should cover a wide range of strain rates: for example, a very slow rate (e.g., $1 \times 10^{-6} \text{ s}^{-1}$), medium rate (e.g., $1 \times 10^{-4} \text{ s}^{-1}$), and relatively fast rate (e.g., $1 \times 10^{-3} \text{ s}^{-1}$). Also, for reasons to be discussed in Section 3.2.1.3.1, "Preliminary Characterization Using Exploratory Tensile Tests," an additional tensile test should be conducted at a strain rate of $0.5 \times 10^{-3} \text{ s}^{-1}$ for the case of the highest temperature. Conducting the tests at different strain rates provides insight into the overall rate dependence of the material as well as insight into the rate dependence of the initial elastic modulus. Schematics of these tests, which are discussed in detail later in Section 3.2.1.3, are shown in Figure 22.

At all rates examined, the proportional limit can be determined, with the lower limit at the slowest strain rate (i.e., $1 \times 10^{-6} \text{ s}^{-1}$) providing an excellent estimate of the threshold limit κ , as most of the time-dependent reversible deformation would be consumed at the slow strain rate. At the highest rate, the test should be carried out until material failure, thus illuminating both

TABLE VI.—PRELIMINARY TENSILE TESTS

[Repeat at four temperatures: T_1 , T_2 , T_3 , and T_4 .]

Type	Rate, s^{-1}	Limit value	Additional
Tension	1×10^{-6}	$\epsilon = 0.015$	Relaxation at final strain
Tension	1×10^{-4}	$\epsilon = 0.015$	Unload and hold at zero stress
Compression	1×10^{-4}	$\epsilon = 0.11$	
Tension	1×10^{-3}	Failure	
Tension	0.5×10^{-3}	Failure	High temperature only

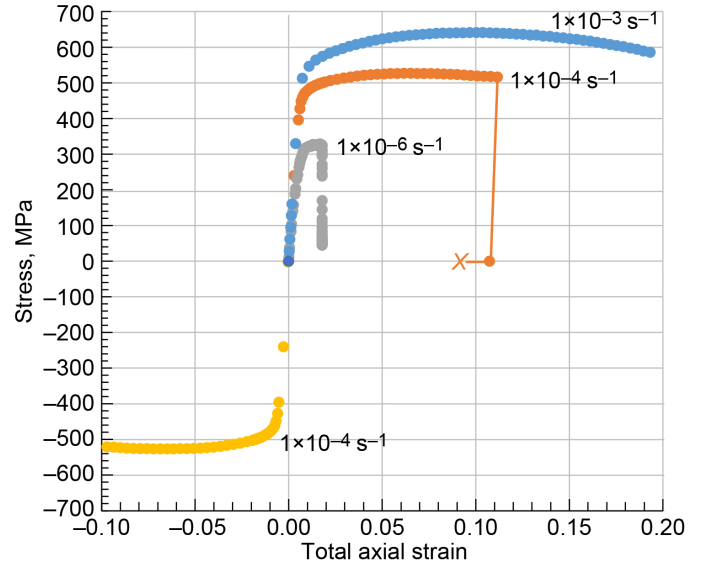


Figure 22.—Stress-strain curves for tensile tests at multiple strain rates.

the maximum and/or saturation stress and the failure strain at each temperature. For the deformation characterization, knowing the saturation stress will provide insight into the value of the sum of the internal drag stresses, $k_{(b)}$, because the effective stress ($\Sigma = \sigma - \alpha$) is equal to the equilibrium stress ($\sigma_{eq} = E_s(e - \epsilon^I)$) at this point, such that $\sqrt{3} \sum k_{(b)} = \sigma - E_s(e - \epsilon^I)$, where E_s is the time-independent modulus. The saturation stress value at the high strain rate should be used to make this determination because at the higher strain rate, most viscoplastic mechanisms are fully activated and none of the deformation mechanisms have been given sufficient time to relax out of the material during loading. Knowing the maximum stress, the strain to failure, and the strain levels where the maximum stress is reached and when the stress starts to decrease will be of value in the damage characterization. Carrying out the tensile tests until stress saturation (or the UTS of the material) is reached is particularly valuable because a recently added feature of the GVIPS model involves modifying the hardening functions to correctly predict the saturation of the material response. To properly characterize this feature of the

model, the saturation stress of the material across a variety of strain rates needs to be firmly established. Performing an additional compression test at the slowest strain rate (i.e., $1 \times 10^{-6} \text{ s}^{-1}$) enables one to assess if asymmetry (difference in tension and compression) exists in the material as well as the threshold stress value. If asymmetry exists, this would require the inclusion of J_3 in the functional form of the flow law, see Section 2.1.2, “Frame Indifference (Objectivity).”

The results obtained at the highest strain rate will also be useful in the characterization of the viscoplastic (irreversible) parameters. As will be discussed in more detail later by examining the variation of the saturation stress with temperature, insight can be gained into the temperature variation of the parameters in the flow law. Also, by utilizing the two high-strain-rate tests at elevated temperature, the values of the flow law parameters μ and n can be estimated (see Section 3.2.1.3, “Irreversible (Plastic or Viscoplastic)”). The highest temperature results will be used for this part of the characterization because the rate dependence of the results should be greatest at the highest temperature.

Examining the low strain and stress values of the various tests, one can determine if the material response, and thus the current value of the modulus, is a function of strain rate. Such an observation is key to the concept of using a combined viscoelastic-viscoplastic model for metals because traditional plasticity or viscoplasticity approaches assume the elastic modulus to be independent of strain rate, and thus are time independent. Experience has shown for titanium- and nickel-based superalloys, at higher temperatures the modulus typically does vary with strain rate, which is a behavior more representative of a viscoelastic material. The modulus at the highest strain rate can be assumed to approximate the dynamic modulus (D), which is the time-independent stiffness (E_s) (described in Section 3.2.1.2, “Reversible (Elastic or Viscoelastic)”) added to the summation of the modulus of each viscoelastic mechanism (E_m) (see Eq. (47)). The overall rate dependence of the material response can be used to assist in the characterization of the parameters of the viscoplastic flow law, which is discussed in Section 3.2.1.3.

At the medium and slow strain rates, the test will not be taken to failure but rather will involve unloading (1×10^{-4} line) and relaxation histories (1×10^{-6} line) in Figure 22, which will provide further insight into the time-dependent material response. For example, at the medium rate (1×10^{-4} line) the material can be loaded well into the range of material nonlinearity; for example, in the case of titanium alloys this is a total strain level of 1.5 percent with the alloy unloaded to zero stress and then held at zero stress for up to 24 h or, alternatively, unloaded to zero strain and held to let the stress relax. If the material does not experience

any strain (or stress) recovery after 24 h, this would be a strong indicator that for the temperature under consideration, there is no time-dependent reversible (anelastic) component to the material response. Any inelasticity in the material response could be assumed to be due purely to irreversible mechanisms. Alternatively, at the slow strain rate the material should be loaded to a strain level of 1.5 percent, and then the strain held at that level (i.e., $\dot{\epsilon} = 0$), resulting in stress relaxation. By loading at such a slow strain rate (i.e., $1 \times 10^{-6} \text{ s}^{-1}$), much of the time-dependent reversible deformation should be relaxed out of the material during the monotonic loading portion of the test. Therefore, during the stress relaxation portion of the test, the time-dependent irreversible deformation will most likely be operative. As a result, the stress level where the relaxation shuts down could be used as a refined estimate of the threshold stress κ , compared to the prior proportional limit estimate. Having an accurate estimate of the threshold stress is important so that during the viscoelastic (reversible) testing portion of the test program, the experimentalist can ensure that the testing is being conducted within the reversible regime.

The specific tests described above not only provide initial insights into the material response characteristics,⁸ which can be useful in designing future tests, but they can also be utilized later as part of the detailed characterization process. Further, for practical reasons and to keep required testing to a minimum, the specific tests outlined in Table VI and the following test matrices (Table VII to Table X) are designed to feed into multiple goals and tasks within the characterization and validation process.

In summary, the following has been determined from preliminary tensile tests (at each of the different test temperatures):

- Rate dependence of elastic modulus (all tests).
- Initial estimate of reversible and irreversible threshold stress (low-rate test).
- Whether or not the material has time-dependent reversible deformation (medium-rate test).
- Initial estimate of failure stress and strain (high-rate test).
- Maximum stress in material (if different from failure stress above).
- Estimate of maximum internal stress contribution (the sum of the κ_b) terms.

3.2.1.2 Reversible (Elastic or Viscoelastic)

If preliminary tensile testing has indicated the need to characterize a material’s viscoelastic behavior,⁹ several ($2+2M$) key material parameters need to be determined (see Table I). The threshold stress κ , which delineates the reversible from the

⁸These can also give insight into the required mathematical functional forms of the flow and evolutionary equations.

⁹To date, almost every metallic material that has been tested at NASA Glenn Research Center has shown viscoelastic behavior at some temperature.

irreversible domain, is the most fundamental material parameter and can be determined precisely using results from a set of creep and relaxation tests and the viscoelastic subtraction method (Refs. 4 and 56):

$$\kappa = \bar{\sigma} - E_s \epsilon^I \quad (46)$$

where $\bar{\sigma}$ is the stress level (higher than the proportional limit) where an irreversible creep test was conducted, E_s is the stiffness that would be present if the material was loaded at an infinitely slow speed (calculated from connecting the end point of reversible creep and relaxation tests, shown in Figure 23), and ϵ^I is the irreversible strain observed from the unloading portion of the creep test in Figure 23. Note the time-independent elastic stiffness E_s needs to be found as does the full relaxation spectrum of the material; that is, for each of the viscoelastic mechanisms introduced, the specific stiffness E_m and dashpot viscosities η_m or relaxation time per mechanism, $\rho_m = E_m/\eta_m$, need to be computed. Furthermore, because the dynamic modulus of the material D is equal to the sum of all reversible stiffnesses,

$$D = E_s + \sum_{i=1}^M E_m^{(i)} \quad (47)$$

it is apparent that given D and E_s one can easily solve for the sum of the nonequilibrium stiffnesses (E_m , or previously M in Figure 14 and Figure 15).

Vital tests in the characterization of the viscoelastic parameters involve a combination of creep and relaxation tests. The first set of tests involve both creep and relaxation tests, which can be used to determine E_s . Typically, one would choose a target stress value of approximately 50 percent of the estimated threshold value κ_{est} , which is equal to the proportional limit determined from the slow-strain-rate (i.e., $1 \times 10^{-6} \text{ s}^{-1}$) preliminary tensile test. For the relaxation tests, the material is loaded to a strain level that is equivalent to an elastic stress level lower than the estimated threshold stress κ . In the loading portion of each test, the material is loaded at a strain rate of approximately $1 \times 10^{-5} \text{ s}^{-1}$, which is midway between the medium and slow strain rates from the preliminary tensile tests. Utilizing this rate provides additional insight into the rate dependence of the material. In the constant stress (creep) portion of the test, the material is held at a constant stress ($0.5\kappa_{est}$) for a significant period of time (approximately 12 to 24 h) until the creep rate drops to zero, and an equilibrium strain value is reached (i.e., the point when the material time dependence stops). At this point, the material is assumed to be at both an equilibrium stress and strain, and all time-dependent mechanisms are no longer active in the material. Subsequently, the material is unloaded to zero stress and held there for at least 24 h, or until all strain is recovered, thus verifying that the test was indeed conducted in the fully reversible domain. A schematic of the stress-strain curve from a test of this type is shown in Figure 23. Next, strain relaxation tests are

conducted, wherein each test is loaded to the same target stress level (yet the loading rate for each test is different; e.g., 1×10^{-6} , 1×10^{-4} , and $1 \times 10^{-3} \text{ s}^{-1}$) as in the previous creep test ($0.5\kappa_{est}$) and then held at a constant strain until such time as the relaxation process (i.e., the reduction of stress at constant strain) shuts down. Once again, a period of 12 to 24 h should be sufficient. Again, after each relaxation period, the specimen is unloaded and strain is recovered prior to reloading at the different rate, thus ensuring reversibility. A schematic of the stress-strain curve of a test of this type is also shown in Figure 23. Stress-versus-time plots (not shown) can then be examined from these tests; here, all tests should begin at different stress levels (due to the different loading rates) and converge at the end to the same equilibrium stress value. If the stress-time curves for each of the different strain rates lie on top of each other, this result indicates that only one viscoelastic mechanism is required. However, if the results are different for one or more loading strain rates, then this would suggest that multiple viscoelastic mechanisms are present and must be accounted for, because during the loading process the faster internal clocks (relaxation spectra) fully released (dissipated) their energy, thus influencing the subsequent relaxation response.

To determine the time-independent elastic modulus E_s , a best-fit line is then drawn from the origin of the stress-strain curve through the end points of the creep and relaxation tests. The slope of this line (Relaxation test line in Figure 23) can be assumed to be the time-independent stiffness E_s . By using the results from both creep and relaxation tests to obtain E_s , different load histories are employed, and different mechanisms are used to get to the equilibrium point. In this way, a more accurate determination of the time-independent elastic stiffness can be made. Also, using the results from relaxation tests (or any test that has a strain-controlled loadup) at different strain rates to obtain E_s , enables one to infer the statistical scatter (mean and standard deviation) in the measurement of this parameter. Note, specimen-to-specimen variation is obtainable by repeating this set of experiments on other specimens, if time and resources permit.

Now, given E_s , the concept of viscoelastic subtraction that has been previously established (Ref. 4) can be used to determine the value of the threshold stress κ . To find this threshold value, a creep or relaxation test conducted at a stress level higher than the estimated value of the parameter is required. Stress levels of approximately 25 to 50 percent higher than the estimated value of κ determined earlier have been found to work reasonably well. Nevertheless, the value chosen should still be at a stress level lower than the proportional limit of the material and in the apparent elastic (reversible) range. Here, the creep test is held for the same length of time as the creep test at lower stress level that shut down, thus ensuring that all viscoelastic mechanisms have been fully activated and the time-dependent viscoelastic mechanisms most likely have released all of their energy (dissipated out). At this point, the material is unloaded and held at zero stress until the strain

recovery shuts down. If the creep stress was truly higher than the value of the threshold stress, some permanent irreversible strain will be left in the material. A plot of the stress-strain curve from this type of test is also shown in Figure 23, where the irreversible strain is clearly labeled. The product of the time-independent viscoelastic modulus and the irreversible strain can be subtracted from the applied creep stress to determine the threshold stress using Equation (46). Note that all three tests described are required to apply the viscoelastic subtraction method.

Clearly, $2+2M$ independent material parameters (where M is the number of individual reversible mechanisms) are required to fully characterize this material at a given temperature: E_s , ν , $M^{(a)}$, and $\rho^{(a)}$ (where $\rho^{(a)} = \eta^{(a)}/M^{(a)}$). The solution for E_s discussed previously makes the solution of $M^{(a)}$ straightforward:

$$M = E_D - E_S \text{ (single mechanism)} \quad (48a)$$

$$\sum_{a=1}^M M^{(a)} = E_D - E_S \text{ (M mechanisms)} \quad (48b)$$

If M multiple mechanisms are required, an initial estimate for each $M^{(a)}$ can be obtained by dividing the sum of all time-dependent moduli ($M^{(a)}$) by the total number of mechanisms (M); that is, $\sum_{a=1}^M M^{(a)} / M$. An estimate for the relaxation time ρ can be easily illustrated assuming only a

single mechanism, such that the solution of the governing equation for a three-element model (see Figure 14 and Refs. 3 and 4) is

$$\sigma(t) = \left(\frac{\sigma_o}{D} \right) E_s + \sigma_o \left(1 - \frac{E_s}{D} \right) e^{-t/\rho} \quad (49)$$

Thus, with a number of data points $(\ln(\sigma), t)$ taken from a relaxation test, one can extract the slope $(-1/\rho)$ from a plot of $\ln(\sigma)$ versus time, where the starting point is $(\ln(\sigma_o), 0)$ and the end point is $(\ln[(\sigma_o/D)E_s], t_{end})$. One can do this for each relaxation test at different loading rates, where the initial point $(\ln(\sigma_o), 0)$ will be different for each loading rate, but the final point $(\ln[(\sigma_o/D)E_s], t_{end})$ should be the same, because the target strain $e_o = \sigma_o/D$ is the same for all tests. Therefore, one should now have a reasonable starting estimate for $M^{(a)}$, and $\rho^{(a)}$, which can be further refined through iteration.

In general, the viscoelastic properties E_s , ν , $M^{(a)}$, and $\rho^{(a)}$ and the threshold stress κ can (and probably will) all vary with temperature. To determine the temperature dependence, the previously described series of tests need to be repeated at various temperatures. Clearly, these temperatures should be consistent with those used in the preliminary tensile tests. Similarly, as suggested previously, the specimen-to-specimen variability could be assessed if desired.

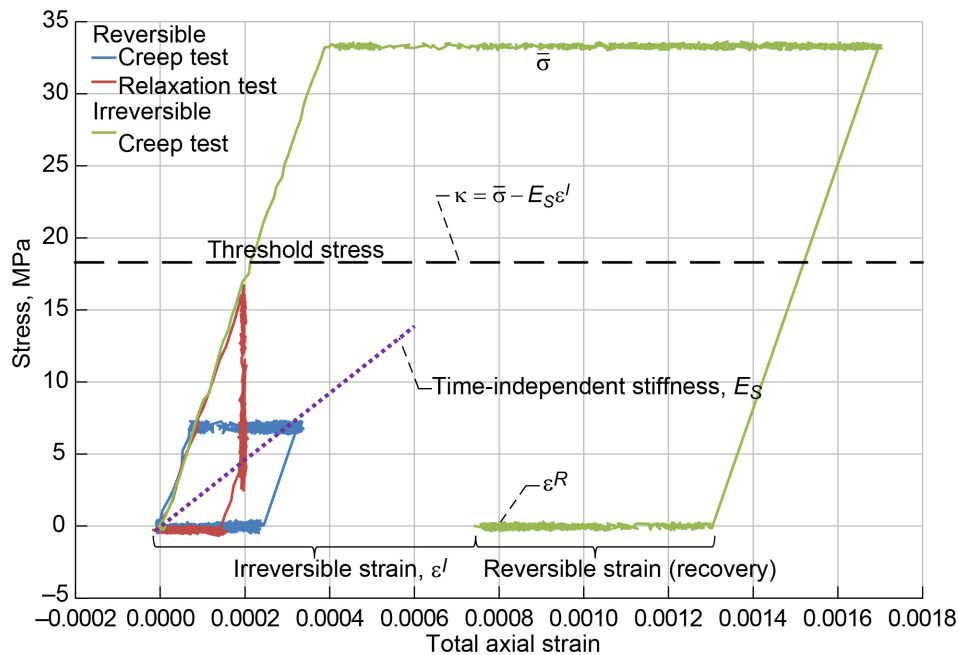


Figure 23.—Stress-strain curves for creep and relaxation tests for Ti-6-4. Pink relaxation and blue creep tests are lower than threshold value and thus in reversible regime. Green creep test is higher than threshold, irreversible regime. Here, all loading rates are $1 \times 10^{-3} \text{ s}^{-1}$ to lock in all time dependence.

TABLE VII.—VISCOELASTIC TEST MATRIX

[Repeat at four different temperatures: T_1 , T_2 , T_3 , and T_4 .]

Test	Type	Rate, s^{-1}	Limit/value	Additional
1	Creep	1×10^{-5}	$0.50 \cdot \kappa$	
2	Creep	1×10^{-5}	$1.50 \cdot \kappa$	Unload and hold
3	Relaxation	1×10^{-6}	$\sigma_{highest} = 0.50 \cdot \kappa$	
4	Relaxation	1×10^{-4}	$\sigma_{highest} = 0.50 \cdot \kappa$	
5	Relaxation	1×10^{-3}	$\sigma_{highest} = 0.50 \cdot \kappa$	

Table VII summarizes the test matrix required for viscoelastic characterization of the material response. Note the absolute minimum number of tests per temperature is three: one creep and one relaxation lower than the estimated threshold, along with a creep test with unload and hold higher than the estimated threshold. Because tests 1 and 3 to 5 are run at a stress lower than κ , and test 2 is run above κ , one sample can be used for all five of these tests. That will permit a more accurate calculation of κ . Scatter in κ as well as the other viscoelastic parameters can be documented by repeating this process with multiple specimens. Remember that κ is the threshold stress in shear and is related to that resulting from uniaxial tension, that is, $\kappa = Y/\sqrt{3}$ where Y is the threshold in tension, assuming a J_2 yield surface.

In summary, the following has been obtained from viscoelastic tests:

- Threshold stress κ , denoting the boundary between reversible and irreversible responses.
- Time-independent infinite elastic stiffness E_s .
- Data to determine number and character of individual viscoelastic mechanisms for each temperature (i.e., $M^{(a)}$ and $\rho^{(a)}$).

Figure 24 and Figure 25 show how plotting the dynamic modulus and the infinitely slow modulus (E_s) as a function of temperature (see left-side axis) and the threshold stress and proportional limit stress as a function of temperature (see right-side axis) produces a modeling map for a given material. This is shown in Figure 24 and Figure 25 for titanium alloy Ti-6-4 and Ni-based superalloy ME-3, respectively. Note the various reversible and irreversible zones as a function of stress and temperature along with the time-dependent and rate-dependent zones are identified in each figure, thus enabling identification of appropriate constitutive models (i.e., elastic, viscoelastic, viscoplastic) for the specific material of interest. By comparing Figure 24 and Figure 25, one can immediately see the advantages

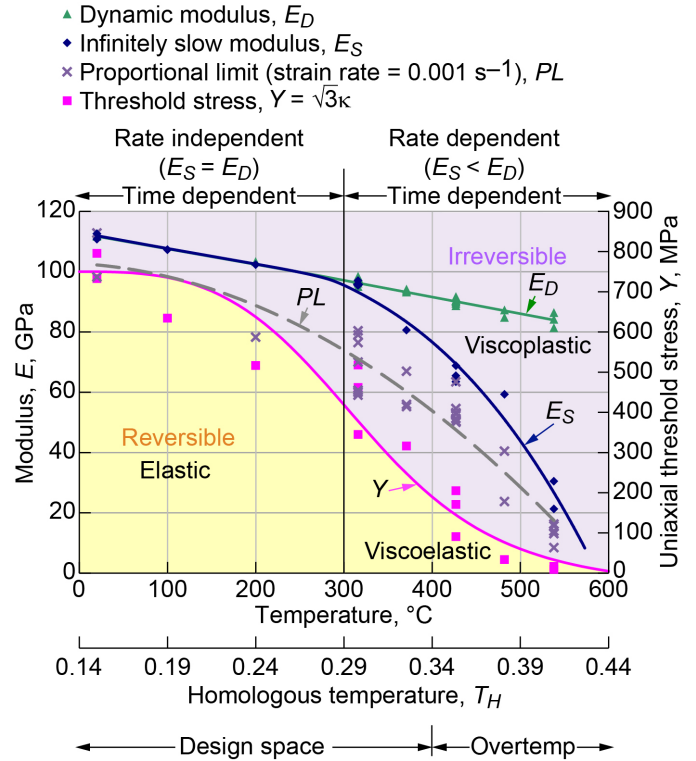


Figure 24.—Modeling map describing reversible and irreversible regimes for Ti-6-4 (adapted from Ref. 47).

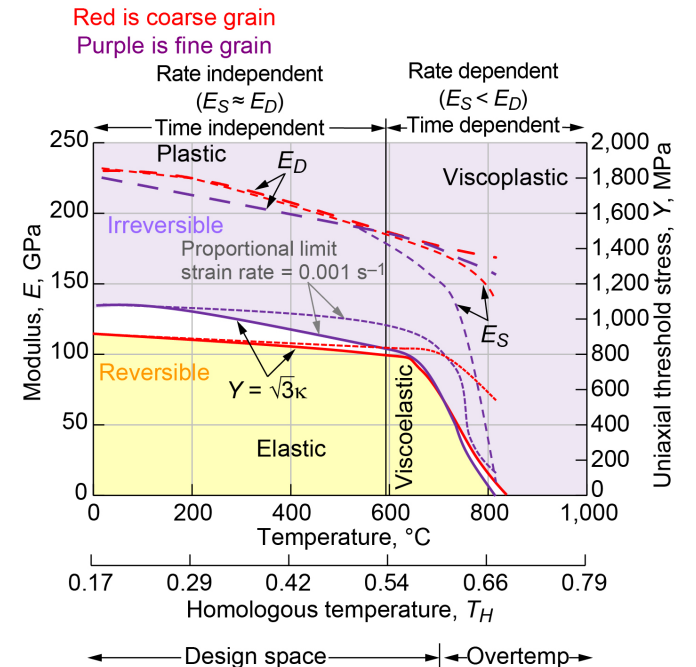


Figure 25.—Modeling map describing reversible and irreversible regimes for Ni-based superalloy ME-3.

of a superalloy over that of the titanium alloy with respect to properties and temperature capability. Also, in Figure 25, one can see that grain size also effects both E_s and κ , with the effect on E_s being the more dominant.

3.2.1.3 Irreversible (Plastic or Viscoplastic)

In the characterization of the irreversible (viscoplastic) portion of the GVIPS material model, several material constants must be determined (i.e., $3+5N$, where N is the number of individual irreversible mechanisms). The constants κ , n , and μ are associated with the flow law; the constants k_b , H_b , and β_b are associated with nonlinear hardening mechanisms; and the constants R_b and m_b are associated with thermal recovery mechanisms, as shown in the GVIPS section of Table I. To accomplish this, several different types of tests are required to determine each of the various parameters.

Each test should be repeated over a variety of temperatures described previously to enable determination of the full temperature dependence of the various material constants. Figure 26 illustrates schematically the assumed temperature dependence of each material parameter. Further, note that these parameters

are integrated so the values are not unique; however, certain test conditions reveal certain aspects/features of the material model (state space, see Appendix A) more so than others. The minimum required tests for the viscoplastic test matrix per temperature is summarized in Table VIII.

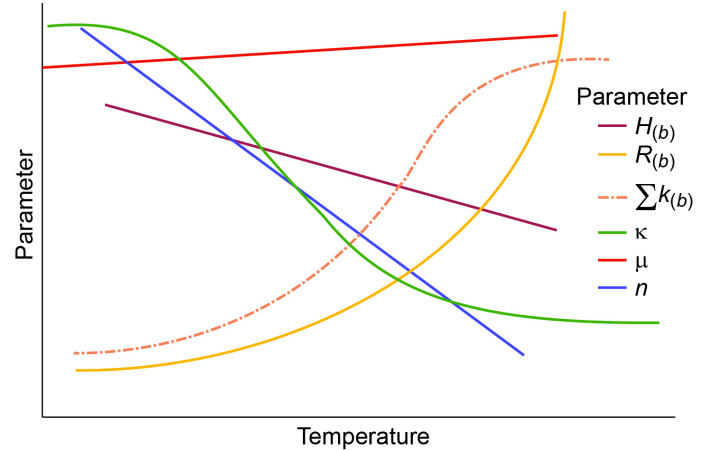


Figure 26.—Viscoplastic material parameter temperature dependence.

TABLE VIII.—VISCOPLASTIC TEST MATRIX
[Repeat at four different temperatures: T_1 , T_2 , T_3 , and T_4 .]

Test	Type	Loading strain rate, s^{-1}	Limit value	Additional comments
Previous	Tensile	1×10^{-6}	$\epsilon = 0.015$	Relaxation at final strain
Previous	Tensile	1×10^{-4}	$\epsilon = 0.015$	Unload and hold at zero stress
Previous	Tensile	1×10^{-3}	Failure	
Previous	Tensile	0.5×10^{-3}	Failure	High temperature only
9	Creep	1×10^{-4}	σ_1	
10	Creep	1×10^{-4}	$\sigma_2 > \sigma_1$	
11	Creep	1×10^{-4}	$\sigma_3 > \sigma_2$	
12	Step creep	1×10^{-4}	$\sigma_1, \sigma_2, \sigma_3$	
13	Relaxation	1×10^{-6}	$\epsilon = 0.015$	
14	Relaxation	1×10^{-3}	$\epsilon = 0.015$	
15	Step relaxation	1×10^{-6}	$\epsilon_1 < \epsilon_2 < 0.015$	
16	Step relaxation	1×10^{-3}	$\epsilon_1 < \epsilon_2 < 0.015$	
17	Plasticity-creep	1×10^{-4}	$\epsilon = 0.01, \sigma_3$	
18	Cyclic	1×10^{-3}	$1.2\epsilon_y$	Symmetric
19	Cyclic	1×10^{-3}	$5\epsilon_y$	Symmetric
20	Cyclic	1×10^{-3}	$1.2\sigma_y$	Symmetric
21	Cyclic	1×10^{-3}	σ_s	Symmetric
24	Cyclic	1×10^{-3}	$1.2\sigma_y$	0 to $1.2\sigma_y$
25	Cyclic	1×10^{-3}	σ_s	0 to σ_s

3.2.1.3.1 Preliminary Characterization Using Exploratory Tensile Tests

Given the results from the exploratory tensile tests, several of the material constants and their temperature dependence can be estimated. For example, the flow law can be manipulated to determine an approximate relationship (by ignoring the -1 in the flow law) between the saturated value of the internal stress α_s , the saturation stress σ_s (where the stress-strain curve flattens out), the inelastic strain rate (given that at saturation the inelastic strain rate $\dot{\epsilon}^I$ equals the total strain rate $\dot{\epsilon}$), and several of the material constants associated with the flow law (i.e., κ , μ , n). That is,

$$\alpha_s = \sigma_s - \sqrt{3}\kappa \left(\frac{3\mu\dot{\epsilon}}{\sqrt{3}\kappa} \right)^{1/(2n+1)} \quad (50)$$

Note that because the internal (back) stress at saturation is equal to the sum of each viscoplastic hardening mechanism thresholds, $k_{(b)}$ at saturation,¹⁰ the equation can be rearranged into the following uniaxial form:

$$\sigma_s = \sqrt{3} \sum k_{(b)} + \sqrt{3}\kappa \left(\frac{3\mu\dot{\epsilon}}{\sqrt{3}\kappa} \right)^{1/(2n+1)} \quad (51)$$

Thus, given the saturation stress results for the two high-strain-rate (1×10^{-3} and $0.5 \times 10^{-3} \text{ s}^{-1}$) tensile tests and that for the $1 \times 10^{-4} \text{ s}^{-1}$ rate at the highest temperature and the value of κ determined from the reversible testing, one can subtract Equation (51), evaluated at various strain rates, to eliminate the $\sum k_{(b)}$ term and obtain two equations that can be solved simultaneously to obtain reasonable estimates for the values of n and μ at this temperature. This procedure can be repeated for other temperatures. In this way, one can verify the fundamental assumption that temperature dependence resides only in the exponent n and the threshold κ (which was explicitly determined from the viscoelastic subtraction method).

$$\sigma_s^{(1)} - \sigma_s^{(2)} = \sqrt{3}\kappa \left[\left(\frac{3\mu(\dot{\epsilon}_1)}{\sqrt{3}\kappa} \right)^{1/(2n+1)} - \left(\frac{3\mu(\dot{\epsilon}_2)}{\sqrt{3}\kappa} \right)^{1/(2n+1)} \right] \quad (52)$$

$$\sigma_s^{(1)} - \sigma_s^{(3)} = \sqrt{3}\kappa \left[\left(\frac{3\mu(\dot{\epsilon}_1)}{\sqrt{3}\kappa} \right)^{1/(2n+1)} - \left(\frac{3\mu(\dot{\epsilon}_3)}{\sqrt{3}\kappa} \right)^{1/(2n+1)} \right] \quad (53)$$

¹⁰Under fast tensile loading, each internal stress mechanism will saturate, $\dot{\alpha} = 0$, when $Q_{ijkl} = 0$ (see Table I), which happens when $1 - \sqrt{G^b} = 0$, which occurs when either $\sqrt{3}\alpha_b = k_{(b)}$ or

An estimate of the value of $k_{(b)}$ for each mechanism can also be obtained by dividing the value of the sum of the $k_{(b)}$ terms (i.e., $\sqrt{3} \sum_{b=1}^N k_{(b)} = \sigma_{sat} - Y$) by the number of estimated mechanisms, N .

In conclusion, given saturated tensile tests at different strain rates, one can obtain an estimate for the sum of $k_{(b)}$, μ , and n , given κ and E_s from previous viscoelastic tests. Then one can obtain an initial guess for each $k_{(b)}$ by dividing the $\sqrt{3} \sum_{b=1}^N k_{(b)}$ by the number of viscoplastic mechanisms desired. Note the lower bound for a given $\sqrt{3}k_{(b)}$ is equal to the highest creep stress examined, thus ensuring that under creep steady state is achieved through the Bailey-Orowan competitive nature of hardening versus thermal recovery (see bracketed term in the evolution law, Table I or Eq. (41)) instead of through $Q_{ijkl} = 0$.

In summary, the following has been determined from tensile tests at different total strain rates:

- Given threshold stress κ and time-independent infinite elastic stiffness E_s at different temperatures and saturation stress associated with each total strain rate tensile test, the material parameters influencing the flow law, viscosity μ and exponent n can be determined.
- Similarly, an estimate of the value of $k_{(b)}$ for each mechanism at a given temperature can also be obtained by dividing the value of the sum of the $k_{(b)}$ terms (i.e., $\sqrt{3} \sum_{b=1}^N k_{(b)} = \sigma_{sat} - Y$) by the number of estimated mechanisms N , where $Y = \sqrt{3}\kappa$ for an assumed J_2 flow surface.

3.2.1.3.2 Relaxation Tests

Like the viscoelastic testing, a series of relaxation tests can also be performed here within the irreversible regime at various total strain rates to gain insight into the number of irreversible mechanisms N required to accurately represent the response spectrum. As before, the material is loaded to a specified strain level, where upon the strain is held constant for a period of time (e.g., 24 h). A good target strain level is 1.5 percent (or near the strain level where the saturation stress occurs), because by then the material will typically have been loaded well into the irreversible regime, where multiple viscoplastic mechanisms—if present—will have been activated.

To establish the number of possible mechanisms required, the relaxation response should be examined at a variety of loading strain rates, for example, 1×10^{-3} and $1 \times 10^{-6} \text{ s}^{-1}$ at a minimum.

$$\alpha_b = 2\sqrt{3}k_{(b)} / (2 + \beta), \text{ assuming current } g(G) \text{ form and uniaxial loading (i.e., } \sqrt{3} \sum_{b=1}^N k_{(b)} = \alpha_{sat} \text{).}$$

When examining the normalized-stress-versus-time plots, one can immediately see if more than a single mechanism is active at this temperature. An important point to note is that the relaxation test at the slow rate was already conducted as part of the exploratory tensile test. This specification is an example of how one test can be used for multiple purposes, which helps to hold down the number of total tests. Example stress-strain curves from such relaxation tests, but at different rates and strain targets, are shown in Figure 27, and the corresponding stress-time curves shown in Figure 28.

An additional test that can provide insight into the equilibrium response of the material is a step relaxation test. In this type of test, the material is loaded to a given strain level, and then the strain is held until stress relaxation shuts down. At this point, the material is reloaded until a new, higher strain level is reached, at which point the strain is held constant again until the relaxation stops. This process can be repeated multiple times (e.g., two more times) with the final relaxation strain level being 1.5 percent of the original or higher. The lowest strain level should be slightly higher than the proportional limit (for the applied strain rate and temperature), with the remaining two strain levels equally divided between this point and the final level of 1.5 percent. The process should be carried out both at the low strain rate of $1 \times 10^{-6} \text{ s}^{-1}$ and the high strain rate of $1 \times 10^{-3} \text{ s}^{-1}$. By carrying out this process, several goals can be achieved. First, the history dependence (both path and rate) of the relaxation behavior over an extended portion of the loading curve can be studied. In addition, the repeatability of the original exploratory tensile test with single relaxation history can be determined (it provides a replicate loading curve). The stress-versus-strain curve for the step relaxation tests are shown in Figure 29, and the stress-versus-time profile is shown in Figure 30.

Relaxation tests are especially important in understanding the state space trajectories that a given model needs to produce, such that the stress versus internal state variable projection can be deduced directly from such a test (see Appendix A). Another benefit of performing relaxation tests is that one can be assured (given an isotropic material) that the cross section (i.e., the specimen area) will not change during a given relaxation segment because the total strain is being held constant.

Given the stress-versus-time response data (see Figure 30) from a relaxation test, one can immediately obtain stress, stress rate, and inelastic strain rate data sets at each time point, where the inelastic strain rate comes directly from the time derivative of the uniaxial viscoelastic subtraction Equation (46), that is,

$$\dot{\epsilon}^I = \frac{\dot{\sigma}}{E_s} \quad (54)$$

given that κ and E_s are not functions of time (constants). Now, taking the uniaxial expression of the evolution law,

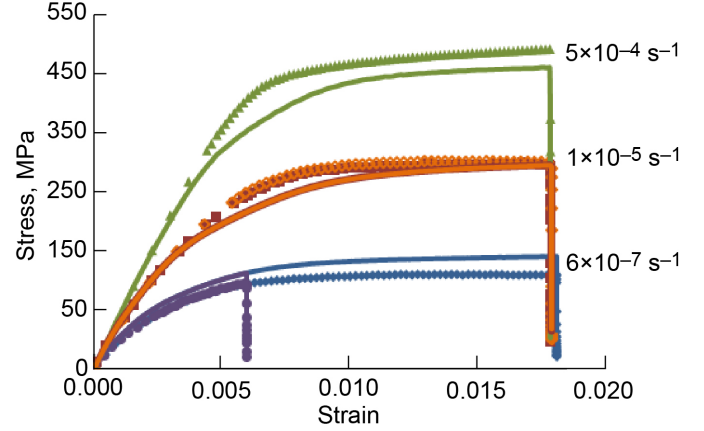


Figure 27.—Stress-strain plot of relaxation tests at multiple loading strain rates at 427 °C for Ti-6-4; experimental (data points) and simulation (curves) results.

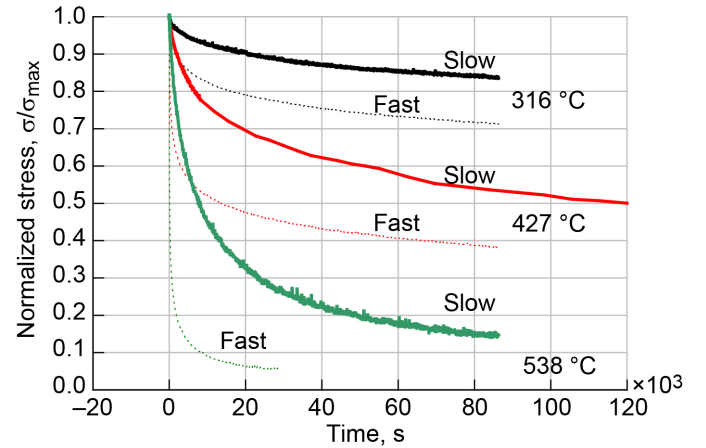


Figure 28.—Relaxation test, stress-versus-time space, performed at multiple loading rates and temperatures for Ti-6-4.

Equation (41), and considering only short-term response (the thermal recovery term is very small and so can be ignored), one gets

$$\frac{2}{3} \dot{\alpha}^{(b)} = Q^{(b)} \dot{\epsilon}_{(b)}^I \quad (55)$$

where the uniaxial $Q^{(b)}$ operator takes the form

$$Q^{(b)} = H_b^{(b)} \left\langle 1 - \frac{\alpha^{(b)}}{A^{(b)}} \right\rangle^{\beta_{(b)}} \left\{ \frac{\left[1 - \left(1 + 0.5\beta_{(b)} \right) \frac{\alpha^{(b)}}{A^{(b)}} \right]}{1 - \left(1 - \beta_{(b)} \right) \frac{\alpha^{(b)}}{A^{(b)}}} \right\} \quad (56)$$

so that

$$\frac{2}{3} \dot{\alpha}^{(b)} = Q^{(b)}(\alpha) \frac{\dot{\sigma}}{E_s} \quad (57)$$

Thus, rearranging the previous equation and integrating both sides gives

$$\int_{\alpha_1}^{\alpha_2} \frac{2}{3} [Q^{(b)}(\alpha)]^{-1} d\alpha = \frac{1}{E_s} \int_{\sigma_1}^{\sigma_2} d\sigma \quad (58)$$

Clearly, Q inverse is a very complex function and not easy to integrate analytically; therefore, a simplifying assumption of Q will be made so that the integration can take place analytically; that is,

$$Q^{(b)}(z^b) = H_b^{(b)} (1 - z^b)^{\beta_{(b)}} (1 - z^b) \quad (59)$$

where a change in variable is made such that

$$\int_{z_i}^{z_{i+1}} \hat{H}^{(b)} \langle 1 - z^b \rangle^{-(1+\beta_{(b)})} dz = \frac{1}{E_s} \int_{\sigma_i}^{\sigma_{i+1}} d\sigma \quad (60)$$

where $z^b = \alpha^{(b)}/A^{(b)}$ and $dz = 1/A^{(b)} d\alpha$ and $\hat{H}^{(b)} = \frac{2}{3} \left(\frac{A^{(b)}}{H_b^{(b)}} \right)$

and $A^{(b)} = \sqrt{3}k_{(b)}$.

Upon integrating, one gets the following result for data points $i+1$ and i :

$$\begin{aligned} \frac{\hat{H}^{(b)}}{\beta_{(b)}} (1 - \alpha_{i+1})^{(-\beta_{(b)})} - \frac{\hat{H}^{(b)}}{\beta_{(b)}} (1 - \alpha_i)^{(-\beta_{(b)})} \\ = \frac{1}{E_s} (\sigma_{i+1} - \sigma_i) \end{aligned} \quad (61)$$

and

$$\alpha_{i+1}^{(b)} = \sigma_{i+1} - \sqrt{3}\kappa \left[\frac{3\mu}{\sqrt{3}\kappa} \left(\dot{\epsilon}_{(b)}^I \right)_{i+1} \right]^{\frac{1}{2n+1}} \quad (62)$$

$$\alpha_i^{(b)} = \sigma_i - \sqrt{3}\kappa \left[\frac{3\mu}{\sqrt{3}\kappa} \left(\dot{\epsilon}_{(b)}^I \right)_i \right]^{\frac{1}{2n+1}} \quad (63)$$

Therefore, given multiple pairs of points of data (i.e., $[\sigma_{i+1}, (\dot{\epsilon}_{(b)}^I)_{i+1}]$ and $[\sigma_i, (\dot{\epsilon}_{(b)}^I)_i]$) from a relaxation curve, one can solve for the material parameters $H_b^{(b)}$ and $\beta_{(b)}$ for each mechanism, given previously obtained parameters κ , μ , n , and $k_{(b)}$ obtained from tensile tests and viscoelastic subtraction

¹¹Note the approximation accuracy for Q decreases as z and β increase. For example, if $\beta = 1.5$, the error goes from 1.25 to 12.7 percent as z

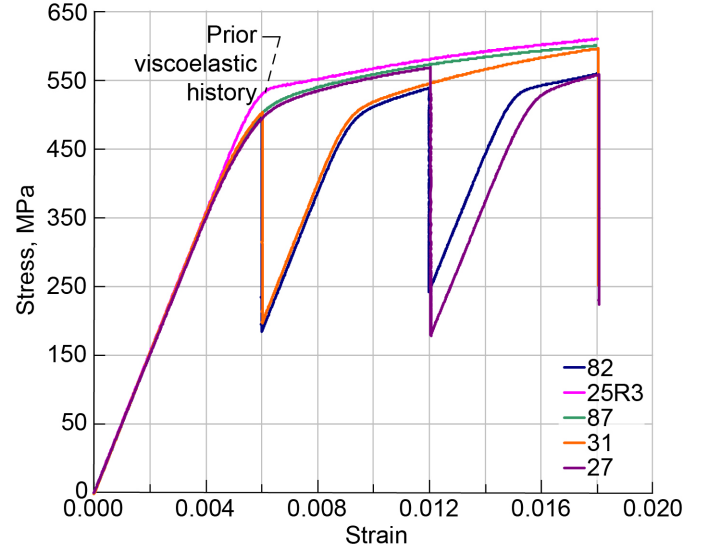


Figure 29.—Stress-strain curves for multistep relaxation tests on Ti-6-4 at 427 °C in the irreversible regime; sample numbers described in Reference 57.

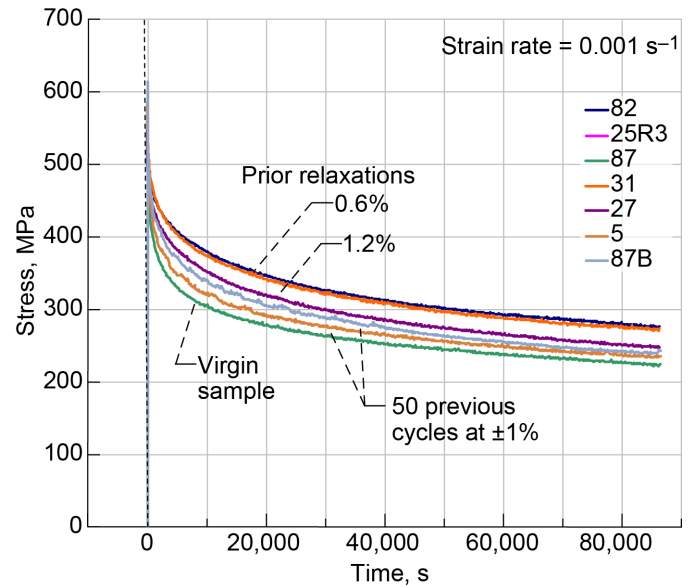


Figure 30.—Stress-time curves for the multistep relaxation tests of Figure 29.

method. Note, z must always be <1 and β must be ≥ 1 . Remember that these equations will provide merely reasonable estimates¹¹ of the various material parameters, which will need to be further revised through iteration once all model parameters are obtained, see Section 2.5.

goes from 0.01 to 0.1, whereas if $\beta = 3.0$, the error goes from 3.0 to 30.5 percent as z goes from 0.01 to 0.1.

In summary, the following has been determined from relaxation tests:

- Data sets $\left[\sigma_{i+1}, \left(\dot{\epsilon}_{(b)}^I \right)_{i+1} \right]$ from each relaxation curve for $i = 1$ to the number of points P , where $\left(\dot{\epsilon}_{(b)}^I \right)_{i+1} = \dot{\sigma}_i / E_s$ and $\dot{\sigma}_i = \frac{\sigma_{i-1} - \sigma_{i+1}}{t_{i-1} - t_{i+1}}$ are obtained for each viscoplastic mechanism.
- Given these data sets, the irreversible hardening modulus $H_b^{(b)}$ and hardening exponent $\beta_{(b)}$ can then be solved for each temperature investigated.
- Remember z (i.e., $\alpha^{(b)} / \sqrt{3} k_{(b)}$) must always be < 1 to assure a nonzero Q , and β must be ≥ 1 to maintain the functional form.

3.2.1.3.3 Creep Tests

Although creep tests are known to exhibit considerably more variability (scatter) than relaxation tests, they do provide needed insight into the long-term behavior of the material. Furthermore, because these tests are conducted over a long period of time, multiple long-term mechanisms are likely to be activated and thus can be characterized as well. The initial short-term mechanism associated with the transient portion of the creep response is also available to provide confirmation of the characterization of the hardening parameters, both in the flow and evolutionary laws. Both monotonic and step creep tests should ideally be conducted to ensure maximum coverage of the state space (see Appendix A). In the first type, conventional creep tests are performed, where the material specimen is loaded and then the stress (i.e., load) is held constant for a fixed amount of time. The creep tests should be performed at a minimum of three stress levels per temperature to ensure that the full spectrum of stresses over the temperature range of interest is characterized.

Table IX provides an example of how the temperature and stress levels should be overlapped to provide maximum benefit. The loading portion of the creep tests should be carried out at an equivalent intermediate strain rate, for example, $1 \times 10^{-4} \text{ s}^{-1}$, to capture the midpoint of the material response. The loadup portion of the creep tests is specified to be under strain control, in sharp contrast to traditional creep test, which are run under load (stress) control. This enables all loading portions of tests to be compared to previously run strain-controlled tensile tests to assess specimen repeatability and to obtain a more accurate statistical measure of Young's modulus, apparent yield, and so forth. However, upon reaching the desired creep stress, a smooth transition to load control must occur. A schematic of the stress-strain curves for typical creep tests conducted at three

TABLE IX.—STRESS AND TEMPERATURE CREEP TEST MATRIX

Stress	T_1	T_2	T_3	T_4
σ_1		✓		✓
σ_2 $\sigma_2 > \sigma_1$	✓	✓	✓	✓
σ_3 $\sigma_3 > \sigma_2$		✓		✓
σ_4 $\sigma_4 > \sigma_3$	✓	✓	✓	✓

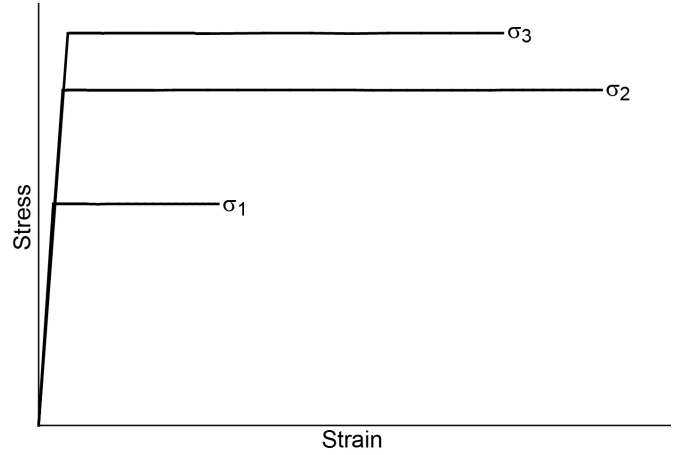


Figure 31.—Stress-strain curves for multiple creep tests in irreversible regime.

different stress levels is shown in Figure 31, and corresponding strain-time plots are shown in Figure 32.

The creep tests are primarily used to characterize the material constants primarily related to thermal recovery mechanisms, for example, R and m . However, the data from the primary creep portions of the tests can also be employed (like in relaxation tests, see Section 3.2.1.3.2) to assist in characterizing the hardening parameters, H and β as well. By examining the material response over a long period of time, all the viscoplastic mechanisms should have sufficient time to be activated. Insight into the equilibrium state of the material can also be obtained because after long time periods, any transient reversible behavior will have shut down.

To obtain an estimate for R and m , creep data sets $(\sigma, \dot{\epsilon}_s^I)$ at steady state ($\dot{\alpha} = 0$, $\dot{\sigma} = 0$) need to be obtained. To this end, using the uniaxial form of Equation (41),

$$\frac{2}{3} \dot{\alpha}^{(b)} = Q^{(b)} \left[\dot{\epsilon}_{(b)}^I - \frac{R_b^{(b)} (G^b)^{m^{(b)}} \frac{2}{3} \alpha_b}{H_b^{(b)} \langle 1 - \sqrt{G^b} \rangle^{\beta_{(b)}}} \right] \quad (64)$$

where $G^b = \alpha_b^2 / 3k_{(b)}^2$ and $\sqrt{G^b} = \alpha_b / \sqrt{3}k_{(b)}$. It is apparent that in the case of steady-state creep, where $Q \neq 0$, the Bailey-Orowan competitive term (the term in bracket) must be 0:

$$\left(\dot{\epsilon}_{(b)}^I\right)_s = (RH)^{(b)} \left\{ \frac{\left[\left(\alpha_b\right)_s / A^{(b)} \right]^{2m^{(b)}+1}}{\left\langle 1 - \left(\left(\alpha_b\right)_s / A^{(b)} \right) \right\rangle^{\beta_{(b)}}} \right\} \quad (65)$$

where

$$(RH)^{(b)} = \frac{2}{3} \left(\frac{R_b^{(b)}}{H_b^{(b)}} \right) \sqrt{3}k_{(b)} \quad \text{and} \quad A^{(b)} = \sqrt{3}k_{(b)} \quad (66)$$

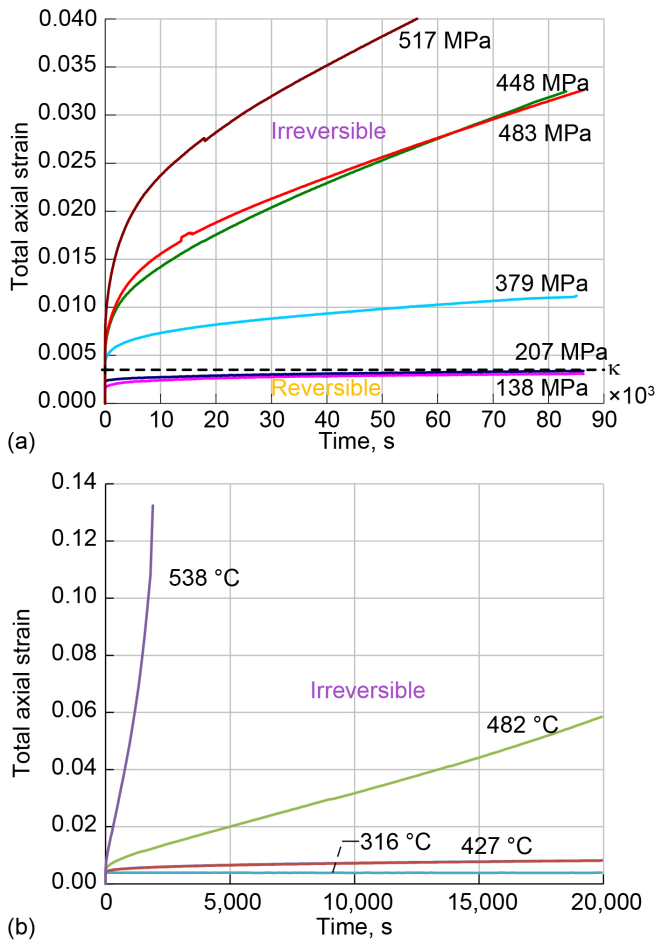


Figure 32.—Stress-temperature creep response of Ti-6-4. (a) Stress dependence at 427 °C. (b) Temperature dependence at strain rate of 0.001 s⁻¹ at 379 MPa.

Then, taking the log of both sides, one can obtain the following expression:

$$\log \left(\left(\dot{\epsilon}_{(b)}^I \right)_s \right) = \log (RH)^{(b)} + (2m^{(b)} + 1) \log \left(\frac{(\alpha_b)_s}{A^{(b)}} \right) - \beta \log \left(1 - \left(\frac{(\alpha_b)_s}{A^{(b)}} \right) \right) \quad (67)$$

Then, plotting $\log \left(\left(\dot{\epsilon}_{(b)}^I \right)_s \right)$ versus $\log \left((\alpha_b)_s / A^{(b)} \right)$, one can immediately obtain the ratio R_b/H_b term from the intersection of the y-axis and m from the slope of the resulting line. Knowing H_b and an R_b/H_b term for each mechanism (b), one can obtain an estimate for R_b .

Note, because $Q \neq 0$, it is implied that $[(\alpha_b)_s / A^{(b)}] < 1$, or

$$\frac{\alpha_s^{(b)}}{A^{(b)}} = \frac{\sigma_s}{A^{(b)}} - \frac{\sqrt{3}\kappa}{A^{(b)}} \left[\frac{3\mu}{\sqrt{3}\kappa} \left(\dot{\epsilon}_{(b)}^I \right)_s \right]^{\frac{1}{2n+1}} < 1 \quad (68)$$

This preceding requirement implies that each $k_{(b)}$ needs to be greater than the highest steady-state stress value obtained from typical creep tests.¹²

With respect to step creep tests, two types of creep tests should be performed: those with increasing levels of loading and those with long hold times at lower (e.g., zero) stress. Considering the first type of test, the material is loaded until a certain stress level is reached; held for a significant period of time to allow the material to reach the secondary creep regime, whereupon the load (stress) is increased rapidly up to another stress level; and then held once again for a period of time. This process is repeated until all prior creep stress levels have been traversed or the specimen fails. A good rule to follow is that the stress levels used should be those employed in the conventional (monotonic or single-step) creep tests. In this way, the step creep test helps to define the transient history response of the material, in that the response of the material after the step typically exhibits a response different from that of the material during a traditional creep test conducted at the same stress. Also, as discussed previously in Section 2.2, step tests can be used to identify which parameter (e.g., time, inelastic strain, inelastic work, or some internal variable) constitutes a reasonable state variable that contains sufficient information to predict the future response of the material. A schematic of the stress-versus-strain curve is given in Figure 33, shown along with Figure 12, which is presented again for the reader's convenience to show the stress-versus-time curve from the step creep test. Note that because of the history dependence

¹²Tensile test results that saturate (i.e., stress rate equal to 0) can also provide steady-state values (i.e., inelastic strain rate is equal to the

applied total strain rate); however, these values will currently be accounted for in the GVIPS model with Q going to 0.

of the material, the creep strain-versus-time plots from the subsequent step creep test do not directly overlap the creep strain-versus-time plots of the original monotonic creep tests.

The second type of step test (strain transient dip test) involves a traditional creep test under a specified load, such as σ_1 , taken into the steady-state regime, whereupon the load is partially reduced to σ_2 (e.g., $\sigma_2 = 0.75\sigma_1$) and the total strain is monitored in time. The frequently observed strain response consists of the creep rate becoming essentially zero immediately following the stress reduction and then eventually increasing to a new steady-state value corresponding to the reduced stress level, σ_2 . The time required ($\Delta\tau$) for the creep rate to increase is commonly referred to as the “hesitation period” and corresponds to the time it takes for a thermal recovery mechanism to reduce the internal stress to a value where the inelastic strain rate reactivates. A schematic illustrating this type of test is shown in Figure 34, where after a significant amount of time, ($\Delta\tau_1$, the time between points c and d), the specimen will continue creeping but now at a rate corresponding to the reduced stress, σ_2 , for example. The duration of this hesitation period is dependent upon the amount of the load drop invoked. Multiple load drops should be investigated at various temperatures to clearly define the temperature dependence of the thermal recovery mechanism, that is, identify the material parameters R and m in Table I. Finally, it is also typically observed that if one drops the load close to zero and holds for some time and then reloads the specimen back to the original load level of σ_1 , reinitiation of primary creep will most likely be observed.

One can use this type of test to characterize the material parameters (R_b and m_b) associated with the thermal recovery mechanism within the GVIPS model. Returning again to Equation (64) and realizing that when the stress is reduced to be within Region II of the state space (see Figure 34(c) and Appendix A)—that is, $\sigma < \sqrt{3}\kappa + \alpha$ —the inelastic strain rate is zero, and one obtains the following:

$$\frac{2}{3}\dot{\alpha}^{(b)} = -Q(\alpha)^{(b)} \left[\frac{R_b^{(b)} (G^b)^{m_b} \frac{2}{3}\alpha_b}{H_b^{(b)} \langle 1 - \sqrt{G^b} \rangle^{\beta_b}} \right] \quad (69)$$

where $G^b = \alpha_b^2 / 3k_{(b)}^2$ and $\sqrt{G^b} = \alpha_b / \sqrt{3}k_{(b)}$, and $Q(\alpha)^{(b)}$ is given in Equation (56). Again, making the variable substitution, $z^b = \alpha^{(b)} / A^{(b)}$, results in

$$\dot{z}^b = -Q(z) \frac{\left[R_b^{(b)} z^{2m_b+1} \right]}{H_b^{(b)} (1-z)^{\beta_b}} \quad (70)$$

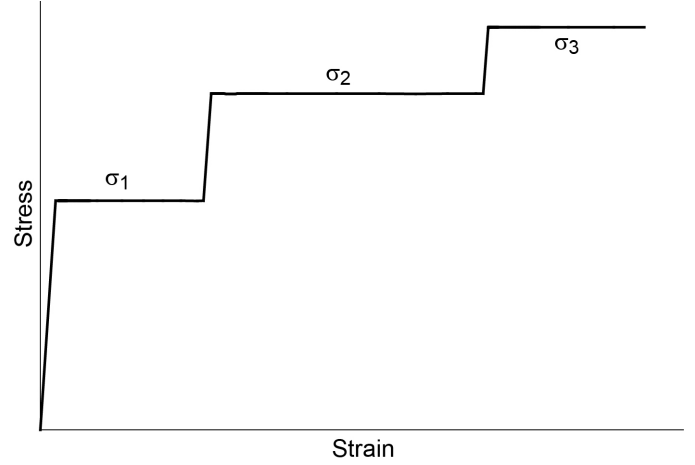


Figure 33.—Stress-versus-time plot for step creep test in irreversible regime.

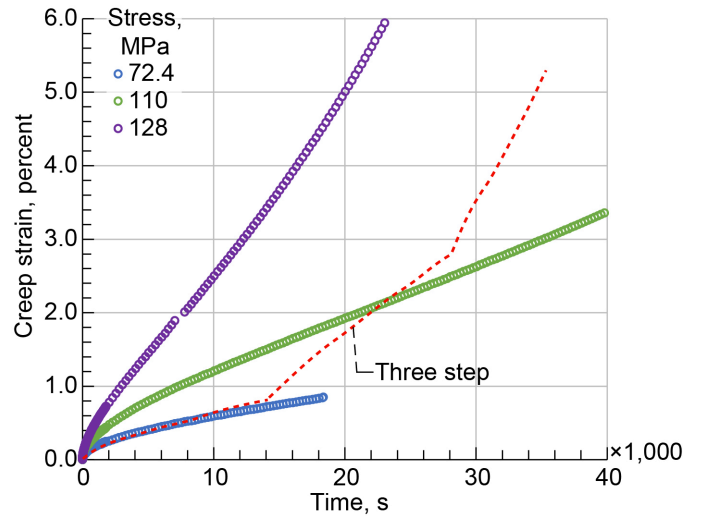


Figure 12.—Experimental creep strain-versus-time responses of TIMETAL® 21S for three stress levels and one three-step creep test using the same stress levels at 650 °C.

where

$$Q(z) = H_b^{(b)} (1-z)^{\beta_b} QF$$

and

$$QF = \frac{[1-cz]}{[1-dz]} \quad \text{for } c = 1 + 0.5\beta_b \text{ and } d = 1 - \beta_b$$

Clearly, because of the complexity of Q , the integration of Equation (70) will not be tractable analytically. The previous simplifying assumption to Q (see Eq. (59)) cannot be made here since α_b , most likely will not be small, however after

examination of the character of QF for various values of β_b and values of $z^b = \alpha^{(b)} / A^{(b)}$ (that run from 0 to 1) one observes that QF goes from 1 to -0.5 as z goes from 0 to 1, with a sign change happening somewhere between $0.3 < z < 0.5$ depending upon the value of β_b . Consequently, to enable analytical integration, a simplifying assumption will be made that QF is a constant equal to its average (~ 0.3) between $z = 0.2$ to 0.6 , where the sign would change. As a result, Equation (71) can be rewritten as either

$$\dot{z}^b = -QFR_b^{(b)} z^{2m_b+1} \quad (71a)$$

or

$$\int_{z_i}^{z_{i+1}} -z^{-(2m_b+1)} dz = QF \int_{t_i}^{t_{i+1}} R_b^{(b)} dt \quad (71b)$$

Solving Equation (71) leads to the following result:

$$\frac{1}{2m_b} \left[z_{i+1}^{-2m_b} - z_i^{-2m_b} \right] = QFR_b^{(b)} (t_{i+1} - t_i) = QFR_b^{(b)} \Delta\tau_i \quad (72)$$

where, from Figure 34 and the associated state space (see Figure 34(c)),

$$z_i = \frac{\sigma_i}{\sqrt{3}k_{(b)}} - \frac{\sqrt{3}\kappa}{\sqrt{3}k_{(b)}} \left[\frac{3\mu}{\sqrt{3}\kappa} (\dot{\epsilon}_i^I)_s \right]^{1/(2n+1)}$$

and

$$z_{i+1} = \frac{\sigma_{i+1}}{\sqrt{3}k_{(b)}} - \frac{\sqrt{3}\kappa}{\sqrt{3}k_{(b)}} \left[\frac{3\mu}{\sqrt{3}\kappa} (\dot{\epsilon}_{i+1}^I)_s \right]^{1/2n+1}$$

where $\alpha_i = z_i^* A$, and $\Delta\tau_1$ and $\Delta\tau_2$ indicate the time periods from c to d and e to f, respectively, in Figure 34(c). One would employ multiple test results (i) and some form of regression scheme to obtain an estimate of the model parameters R_b and m_b .

In summary, the following has been determined from creep tests:

- Using steady-state creep data, it was shown how to obtain R/H and m from a $\log\left(\left(\dot{\epsilon}_{(b)}^I\right)_s\right)$ -versus- $\log\left(\alpha_s^{(b)}/A\right)$ plot.
- Alternatively, if one has transient creep dip tests results, one could obtain R and m directly.

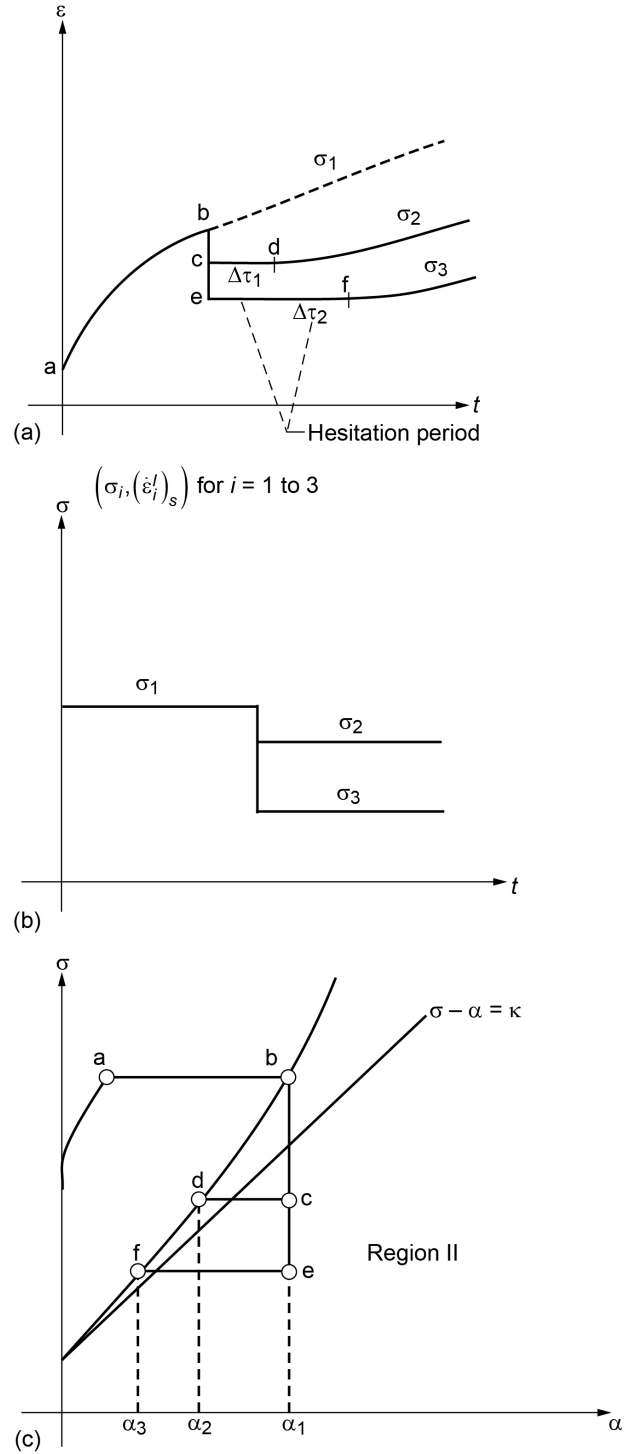


Figure 34.—Transient dip creep test. (a) Strain-time plot identifying the hesitation period, where $\Delta\tau_1$ and $\Delta\tau_2$ indicate the time periods from c to d and e to f, respectively. (b) Corresponding stress-time plot. (c) Corresponding state space (see Appendix A).

3.2.1.3.4 Plasticity-Creep Interaction

A key feature distinguishing unified viscoplastic models in general, and the GVIPS model specifically, from more traditional plastic (rate-independent) and creep (time-dependent) modeling approaches (nonunified) is that the irreversible material response is accounted for within a single inelastic strain measure instead of the partitioning of the inelastic strain into separate quantities (e.g., plastic and creep) having their own set of governing equations with no explicit interaction between them. For example, utilizing a classical approach, if creep loading and standard tensile loading are combined into a single test, the classical equations could not properly simulate the behavior. However, the GVIPS equations can model a loading scenario of this type, which makes a creep-plasticity interaction test an ideal test for model verification purposes. In a plasticity-creep test, the material is first loaded in tension until it is well into the range where irreversible deformation takes place. Next, the stress is then lowered to a particular level—a common selection being the level of the highest stress used in a previous creep test—and then the stress level is held for a significant time so that creeping occurs. An illustration of a stress-versus-strain profile of a test of this type (i.e., an overload test) is shown in Figure 35, where the material is loaded to a particular stress level, unloaded to a lower stress level, and then held at the lower stress for a given length of time. Figure 36 shows the significant difference observed between this overload test and a corresponding virgin creep test. Also, a step-creep test (red line in both figures) is shown as well, thus demonstrating the significant path dependency of inelastic behavior. Another variation on this test (creep-plasticity) can be performed, wherein the material can be first subjected to creep for a period of 6 to 12 h, and then the material can be loaded monotonically as in a tensile test.

In this type of test (plasticity-creep), the response of the material when subjected to a complex range of deformation mechanisms can be considered. Although the results from this test obviously could be used in the characterization portion of the material analysis, the true utility of this type of test is to serve as a set of validation results once the material constants are determined from the other tests. Because this test involves multiple deformation mechanisms and their interaction, simulating a test of this type and comparing the results to the experiments can help to verify that the determined material constants not only are valid for a variety of deformation modes but also that the characterized model (i.e., functional form) can correctly capture their interaction. Note that Section 2.3 explains the difference between characterization and validation tests.

3.2.1.3.5 Cyclic Tests

The final uniaxial tests that need to be conducted constitute a series of cyclic tests. Cyclic loading conditions are very common

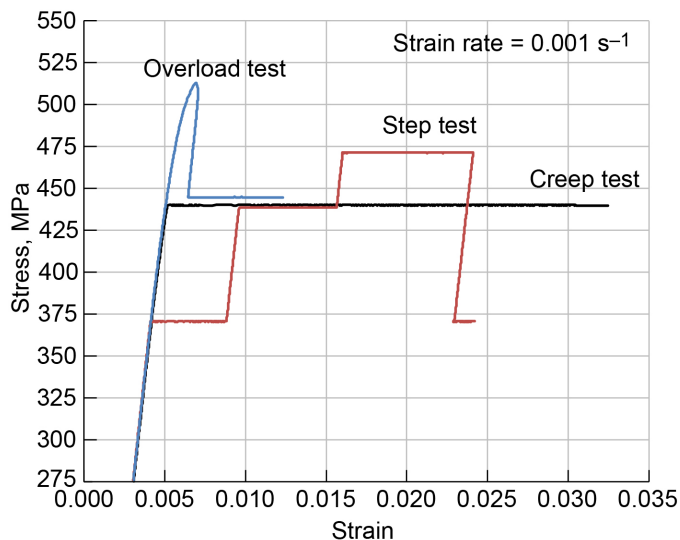


Figure 35.—Plasticity-creep interaction test, stress-versus-strain profile of Ti-6-4 at 427 °C.

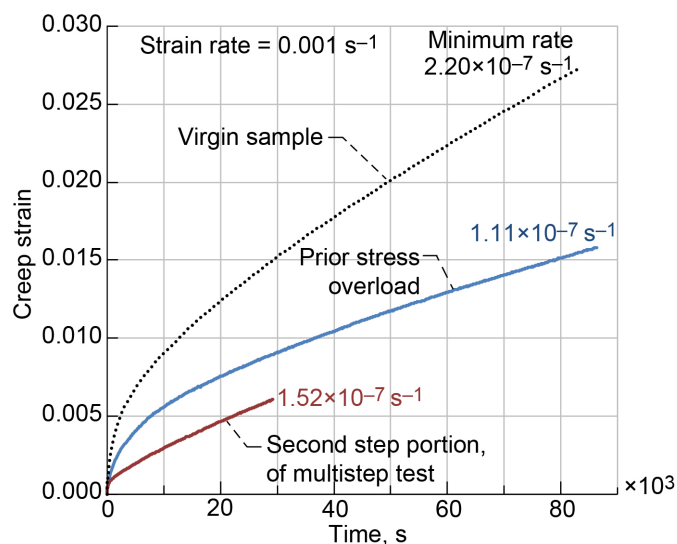


Figure 36.—Creep strain-versus-time plot of the results of Figure 35 (see Fig. 61 in Ref. 57 for original data).

in aerospace structures, and the ability to accurately simulate the cyclic loading response is a key requirement for a material model. In a high-cycle fatigue situation, the material typically reaches an equilibrium state. Therefore, the values of the state variables in the material model also need to reach a state of equilibrium. There are two possible uses of cyclic tests in the characterization and validation of material models. First, these tests can be used in the characterization process because the material constants can be correlated such that the full equilibrium state of the material can be accounted for. Furthermore, the more types of tests that are included in the characterization process, the more types of material loading conditions will be able to be correctly analyzed

by the material model. Alternatively, tests such as the extended tension test and long-term creep tests arguably also result in the material reaching an equilibrium state, so that simulating the cyclic tests after the material constants have been determined could be a reasonable way of validating that the determined material constants are adequate (see Ref. 58). The data from the cyclic tests are also useful in the characterization of the damage parameters. As will be discussed later during the process of cyclically loading the material, the stiffness and/or strength of the material can be reduced because of damage.

Several types of symmetric cyclic tests can be run (see Table VII). The tests should be conducted at the high strain rate of $1 \times 10^{-3} \text{ s}^{-1}$ to allow for a reasonable number of cycles to be completed in a reasonable¹³ amount of time. First, a strain-controlled test can be used where the material is loaded in tension and compression (cyclically) to a strain level slightly higher than the apparent yield strain and second, another test where the material is loaded cyclically in tension and compression to a much larger strain level (such as 5 times the apparent yield strain). Next, the material can be loaded in strain control to a level slightly higher than the apparent¹⁴ yield stress (plus or minus) and then in another test loaded in strain control to a level plus or minus the saturation stress. These tests can be repeated at more than one strain rate to ensure that the rate dependence of the equilibrium response is captured. Schematics of such cyclic tests are shown in Figure 37 and Figure 38.

Asymmetric tests can also be conducted. The stress-limited tests can be repeated where, instead of cycling the material between the tensile and compressive values, the material is only cycled between the tensile upper limit and zero; see Figure 39 for an example. Once again, the tests can be repeated at more than one strain rate.

No specific model parameters are characterized using cyclic test results; however, these types of tests enable refinement of limit stresses such as κ and $k(b)$. Note that for the current GVIPS model it is assumed that all cyclic hardening is of the kinematic type (i.e., movement of threshold surface), even though it is possible that the threshold surface could expand (i.e., isotropic hardening) as well, which would require κ to evolve. Isotropic hardening laws, according to literature, typically are dependent upon the amount of inelastic work accomplished in each cycle. Experience to date has not indicated the need for such a modification.

3.2.2 Deformation: Multiaxial

A multiaxial test program is a key ingredient in the formulation of robust, multiaxial, constitutive models, in that it can

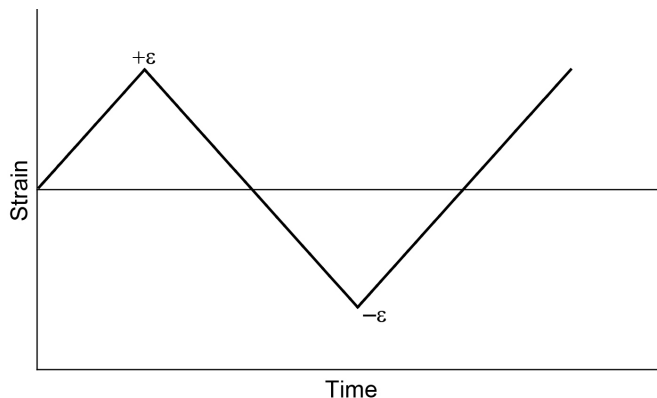


Figure 37.—Strain-versus-time plot for strain-limited symmetric cyclic test in irreversible regime.

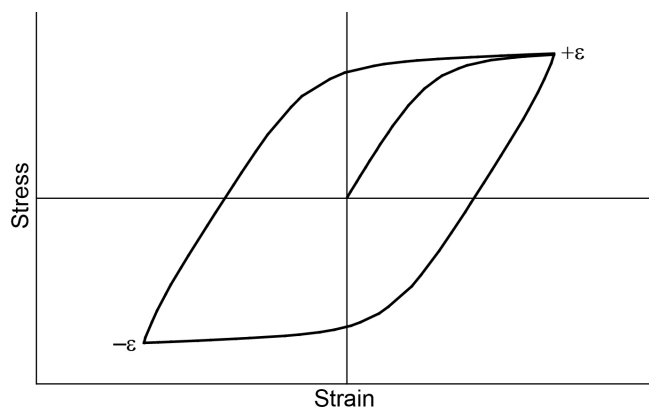


Figure 38.—Stress-versus-strain plot for strain-limited symmetric cyclic test in irreversible regime.

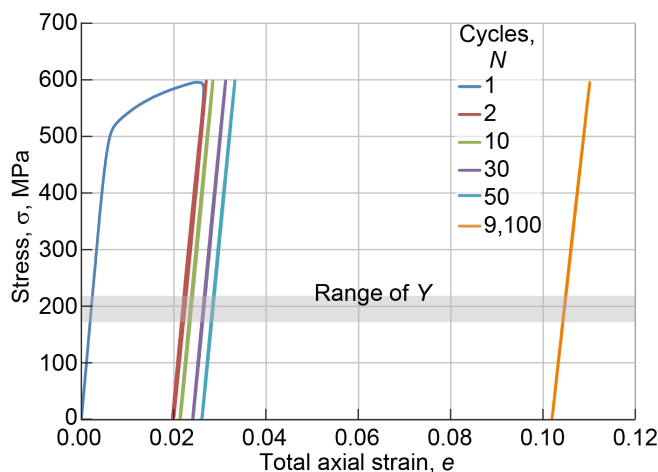


Figure 39.—Asymmetric cyclic test of Ti-6-4 at 427 °C and 10^{-3} s^{-1} strain rate between 0 and 600 MPa. Only selected cycles are shown; failure at $N_f = 9,735$ cycles (see Fig. 58 of Ref. 57).

¹³The term “reasonable” is hard to define in general because it depends upon several factors, including the ability of the testing machine, availability of funds, time allotted by a given project, and so forth.

¹⁴The term “apparent” is used because at high temperature the yield is highly rate dependent; this contrasts with the threshold stress, which is merely temperature dependent but not rate dependent.

combine all three types of testing (exploratory, characterization, and validation) and requires close interaction between experimentalist and theoretician in establishing and interpreting appropriate results. Furthermore, the examination of the multi-axial response of a material under various thermomechanical loading conditions expected to be experienced by the components (e.g., cyclic, nonisothermal, nonproportional, etc.) is necessitated for a complete characterization and validation of a material's response. Also, from a theoretical point of view, several assumptions need to be explored and either validated or invalidated: for example, the notion of normality or coaxiality of the viscoelastic theory (i.e., for stress levels within the reversible domain of the material response).

One key role of a multi-axial test program is to determine multi-axial flow and damage surfaces. The usefulness of determining these multi-axial surfaces can be best understood in the context of classical plasticity wherein the concept of a yield surface plays the central role in delineating the elastic and inelastic deformation regions of the material. The existence and description of this yield surface is precisely that which allows a consistent multi-axial statement of plastic flow (flow law) to be written. Mathematically, this is done by making use of the fundamental assumption that the yield surface (e.g., $F = f(\sigma_{ij}, \alpha_{ij}, T, \kappa) = 0$) has the properties of a potential function (i.e., normality), and thus leads to an associated form of the flow law through differentiation of F :

$$d\varepsilon_{ij}^I = \frac{\partial f}{\partial \sigma_{ij}} \quad (73)$$

where σ_{ij} , α_{ij} , ε_{ij}^I , T , and κ are the applied Cauchy stress, internal (back) stress, inelastic strain tensors, temperature, and yield (or threshold) stress, respectively. Traditionally, yield surfaces are typically mapped out using an equivalent inelastic strain definition and a rather arbitrary target value of permanent set. In fact, only multi-axial testing provides a quantitative description of F , which allows a correct mathematical form of the flow law to be deduced. To date, significant testing has been done in support of the concept of a yield surface and the normality condition, albeit at relatively low temperatures and for monolithic materials (e.g., Refs. 36, 59 to 62) and with respect to elevated temperature and metal matrix composites (Ref. 63).

At elevated temperatures, metallic alloys traditionally exhibit strong time-dependent deformation behavior, and the concept of a yield surface, in the classical sense, generally breaks down when moving into the realm of viscoplasticity. However,

analogous geometrical- and thermodynamic-based concepts such as a surface of constant inelastic strain rate (SCISR) or a surface of constant dissipation rate (SCDR) have been postulated¹⁵ to play the same central role in viscoplastic constitutive theories as yield surfaces do in classical plasticity. In fact, given a complete (i.e., fully associative) potential-based framework like GVIPS, a full multi-axial test program would allow for a consistent characterization and validation of the constitutive equations, since the identification of the Gibbs and dissipation potential would completely define the ensuing compliance operator as well as flow and evolution laws for the inelastic strain and internal state variables, respectively. This fundamental approach to the formulation of a consistent multi-axial theory is in stark contrast to the more typical ad hoc and generally inadequate approach of extending uniaxial constitutive theories for multi-axial conditions by simply placing bars over the pertinent variables and declaring them effective values.

Although, it has been shown (e.g., Lissenden and Arnold (Ref. 64)) that SCDRs are independent of the mathematical constructs of F , thereby making them the most theoretically meaningful measurements when desiring to quantify the state dependency of the dissipation and assess the applicability of the important normality condition (i.e., convexity of this potential). However, because of the presence of internal state (not measurable) variables in the definition of SCDRs, these surfaces are very difficult to measure, whereas surfaces of inelastic power (SCIPs) and surfaces of constant inelastic strain rate (SCISRs) can be determined from multiple probing experiments, on a single specimen, assuming a negligibly small change in material state occurs during each probe. The fact that SCISRs are not necessarily proportional to surfaces having a constant value of F is significant because the desirable convexity and normality features associated with constant F (i.e., $\Omega = \text{constant}$) surfaces are not guaranteed for these SCISRs. Note, both preceding definitions (see Ref. 65 for SCISRs, and see Ref. 66 for SCDRs) have been used to describe the form of a dissipation potential at elevated temperatures for monolithic materials.

Three key theoretical aspects need to be explored and verified in multi-axial space: (1) the concept of a reversible and irreversible threshold surface (see Section 3.2.1.2), (2) multi-axial reversibility for stress levels less than the threshold stress, and (3) proportional/nonproportional path dependency particularly at saturation. All of these can be investigated either using biaxial specimens and/or tension-torsion specimens. The first two can be investigated by performing a number of probing experiments along predetermined directions in either a tension-torsion (as depicted in Figure 40) or biaxial strain space; where each

¹⁵SCDRs are defined to be $\Sigma_{ij}\dot{\varepsilon}_{ij}^I$, where Σ_{ij} is the effective deviatoric stress tensor (i.e., $\Sigma_{ij} = S_{ij} - a_{ij}$, with S_{ij} being the applied (external)

deviatoric stress; a_{ij} , the back (internal) deviatoric stress); and $\dot{\varepsilon}_{ij}^I$, the inelastic strain rate tensor, whereas SCISRs are defined as $\sqrt{\dot{\varepsilon}_{ij}^I \dot{\varepsilon}_{ij}^I}$.

probe angle (e.g., every 15°) is represented by varying the ratio of either $\epsilon_{22}/\epsilon_{11}$ or $\epsilon_{12}/\epsilon_{11}$ (e.g., 0, 0.27, 0.57, 1.0, 1.7, 3.7, etc.). Note a ratio of -1 in the case of a biaxial specimen would represent the case of pure shear and may or may not be attainable because of local buckling, depending upon the dimensions of the specific specimen. Also, as illustrated in Figure 40, the order of the probing angles can be very important when trying to minimize the impact of prior loading (i.e., loading in the opposite direction to the prior probe (e.g., 1-2, 3-4, 5-6, etc.) helps keep the threshold surfaced centered around the origin). The results of these probing experiments can be used to map out the shape of the various surfaces as well as determine if the irreversible strain vector is normal to that surface as well (see Figure 41). From the current perspective, however, what one wants to document is whether for various multiaxial paths the viscoelastic subtraction method can be used to construct the reversible and irreversible threshold surface. To accomplish this one would merely need to follow along a given probe angle up to an effective strain (stress) level less than 75 percent of the uniaxial threshold (assuming a J_2 representation), then hold (creep) or relax until the monitoring variable shuts down, and then unload and wait again to confirm that all deformation is recovered (reversed). This can then be repeated along a variety of various probe angles. With this information an effective E_s can then be determined. The specimen can then be loaded along a given probe angle (e.g., probe angle 1) past the threshold value so that some irreversible strain would be incurred upon subsequent unload and hold. Then given this irreversible strain along each probe, the actual threshold value can be obtained using the viscoelastic subtraction method for that angle. These probing experiments could either be done using a biaxial specimen or a tension-torsion specimen since both will provide insight into the multiaxial behavior of the material.

Next, the response of the material (e.g., Ti-6-4) should be examined in the saturated condition, which is significantly beyond the initiation of irreversible strain. This is accomplished by pursuing multiple proportional load paths (see Table X) and associated multiple nonproportional load paths (see Table XI). In this way one should be able to discern not only the difference between proportional and nonproportional loading but also map out multiple points on various surfaces of inelastic power to uncover the shape of these surfaces in a fully saturated (limit) state, thereby enabling verification of the functional form of a given model (e.g., the current GVIPS model). The nonproportional tension-torsion load-path history given in Table XI and the proportional history given in Table X are depicted in Figure 42. These two types of histories are represented schematically in Figure 42, wherein the paths represented by the red lines are

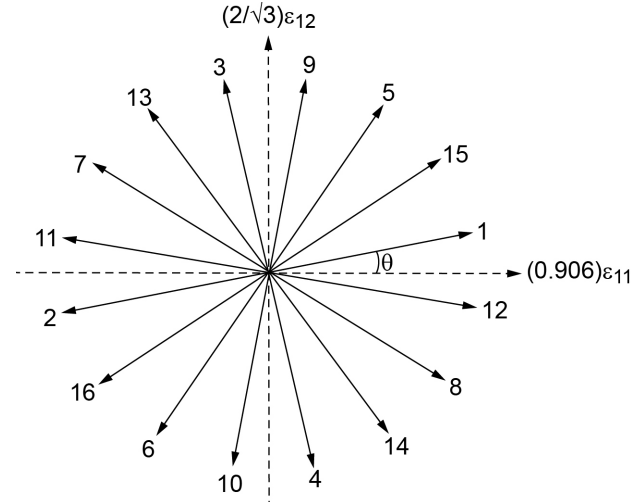


Figure 40.—Representative probe angles within the biaxial strain space.

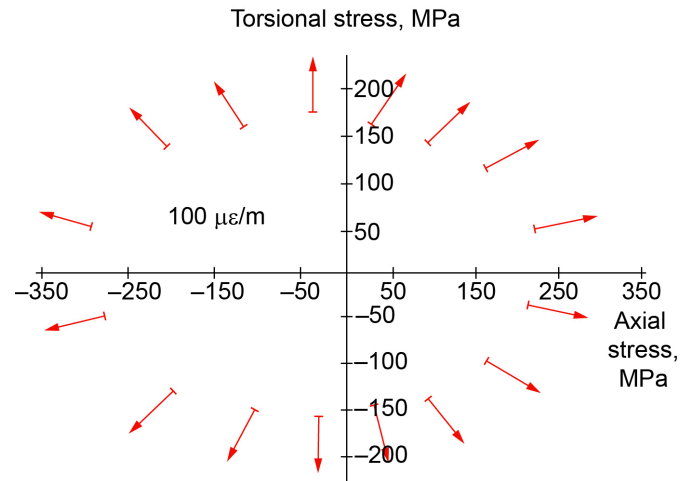


Figure 41.—SCISR and direction of the inelastic strain rate vectors for solution-annealed 316 stainless steel; target value of inelastic strain rate = $100 \mu\epsilon/m$ (from Ref. 67).

the nonproportional histories given in Table XI and the blue lines are the proportional paths given in Table X. Note the difference between proportional and nonproportional paths for a given material will be evident by comparing the stress results at endpoints a (achieved by proportional loading and that by nonproportional loading path), and subsequently at points b, c, d, e, and f for both types of loading paths. One could alternatively use an in-plane biaxial specimen but in this case one would not have the benefit of knowing directly the stress state at these end points (as is the case for a tension-torsion specimen) but would instead need a fully characterized and

TABLE X.—PROPORTIONAL LOAD
PATH SEGMENTS: SPECIMEN A

[Repeat at four different temperatures: T_1 , T_2 , T_3 , and T_4 .]

Step 1	$\dot{e}_{12} = \dot{e}_{11} = c$ $e_{12} = Re_{11}$; $R=1$	Continue until	$\sigma_{12} = \text{saturation}$ $\sigma_{11} = \text{saturation}$ point a
Step 2	Unload $e_{12} = e_{11}$ $\dot{e}_{12} = \dot{e}_{11} = -c$	Continue until $e_{12} = e_{11} = 0$	$\sigma_{12} = P_2$ $\sigma_{11} = P_1$
Step 3	Reload $e_{12} = 0.5e_{11}$ $\dot{e}_{12} = 0.5\dot{e}_{11}$	Continue until	$\sigma_{12} = \text{saturation}$ $\sigma_{11} = \text{saturation}$ or $e_{11} = X_3$; point d
Repeat Steps 2 and 3 with changing strain ratios, R .			

TABLE XI.—NONPROPORTIONAL LOAD
PATH SEGMENTS: SPECIMEN B

[Repeat at four different temperatures: T_1 , T_2 , T_3 , and T_4 .]

Step 1	$\dot{e}_{11} = c$; $\dot{e}_{12} = 0$ $\dot{e}_{12} = 0$	Continue until $e_{11} = X_3$	$\sigma_{11} = \text{saturation}$, NP_1 $\sigma_{12} = NP_2$
Step 2	$e_{11} = X_3$; $\dot{e}_{11} = 0$ $\dot{e}_{12} = 0$	Continue until $e_{12} = X_3$	$\sigma_{11} = NP_3 < NP_1$ $\sigma_{12} = NP_4 > NP_2$ = saturation?; point a
Step 3	$e_{11} = X_3$; $\dot{e}_{11} = 0$ $\dot{e}_{12} = -c$	Unload $e_{12} \Rightarrow 0$	$\sigma_{11} = NP_5$ $\sigma_{12} = NP_6 < NP_4$
Step 4	$\dot{e}_{12} = 0$; $e_{12} = 0$ $\dot{e}_{11} = -c$	Unload $e_{11} \Rightarrow 0$	$\sigma_{11} = NP_7$ $\sigma_{12} = NP_8$
Repeat Steps 1 through 4 for strain levels X_2 (point b) and X_1 (point c); see Figure 42.			

“validated” model to predict the resulting end stress state(s); see Section 2.3.3, “Validation Tests.”

3.2.3 Deformation Summary

Sections 3.2.1 and 3.2.2 have outlined the type of tests required to fully characterize the deformation model parameters for a typical viscoelastoplastic model, the GVIPS model being the example considered herein. Each of the subsections discussed how to obtain initial estimates of the specific model parameters from each type of experiment. For other simpler models like Robinson’s Viscoplastic model, more precise estimates can be deduced. Considering the complexity of the Q operator associated with the GVIPS model (which comes from the fully associated nature of the model), however, simplifying assumptions were needed to perform the required integrations, thus making the determination of the model parameters much more problematic and less precise. Further complicating this is the multimechanism aspect of the GVIPS model, which requires many more parameters, and thus tests and data points, to enable deduction of all the associated model parameters. This is

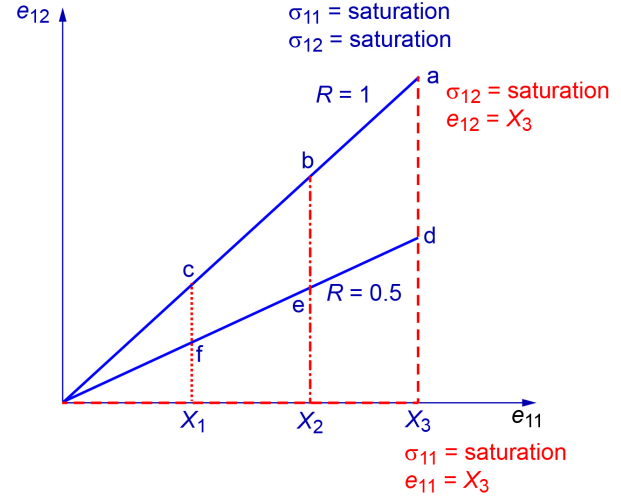


Figure 42.—Proportional and nonproportional loading paths. R is the loading ratio (i.e., $R = e_{12}/e_{11}$).

precisely why a parameter estimation tool like COMPARE (see Section 2.5) is so valuable and powerful. Lastly, although in each section the reader was shown how to deduce parameters from the various tests, typically only in the case of simpler models is such a hand-calculated characterization of a given model made anymore.

In summary, the following is determined from irreversible uniaxial tests:

- Parameters associated with viscoelastic behavior (e.g., κ , E_s , $M_{(a)}$, and $\rho_{(a)}$)
- Parameters for the flow law (κ , m , n)
- Parameters to determine hardening ($k_{(b)}$, $H_{(b)}$, $\beta_{(b)}$) and thermal recovery ($R_{(b)}$ and $m_{(b)}$)
- Long-term response of material under a variety of loading conditions
- Repeatability of selected tests
- Insight into history and path dependence of material

It is important that all the test data and associated metadata be stored properly to enable future development and refinement of models; see Appendix B for more detail regarding this. Finally, given the preceding test results, the full characterization of the deformation portion of a given viscoelastoplastic model at multiple temperatures can be made for a given material.

3.2.4 Damage

From the previous sections, viscoplastic deformation modeling is a mature topic that is based on the so-called internal state variable formalism in thermodynamics for irreversible processes. Inclusion of damage within constitutive equations,

however, is still an active area of research—with the experimental studies primarily still being of the exploratory type. For instance, until a certain point is reached in each loading event (e.g., UTS in a tensile test), the response of the material is primarily dictated by deformation and diffusional mechanisms (e.g., dislocation slip, thermal recovery, and dislocation boundary interactions). However, as discussed previously, at some point damage mechanisms that can be broadly categorized as stiffness degradation (which influences the stiffness moduli of each reversible mechanism) or strength degradation (which influences each viscoplastic (irreversible) mechanism) begin to come into play (see Figure 14).

One aspect that makes understanding damage difficult is that once past the UTS of a tensile experiment, things become problematic since necking and large strain versus small strain theory often must be accounted for. Also, for most types of tests performed, damage accumulates slowly followed by an exponential increase in damage in the last seconds or minutes. Additionally, in the case of fatigue of Ti-6-4 it was observed that the damage accumulated in strain-controlled tests was significantly different than anything else observed; that is, a few very long cracks, which had minimum effect on the mechanical properties measured. All this is troublesome from both a modeling and testing viewpoint.

In the current GVIPS model, the total strain is assumed to be the parameter that determines when the damage mechanisms become operational. To characterize the material constants linked to the evolution of damage, the material needs to be loaded beyond this strain level, $\bar{\epsilon}_{cut}$. An important point to note is that in the GVIPS approach, two types of damage are assumed to occur either singularly or simultaneously: (1) stiffness reduction, which is basically a viscoelastic (reversible) mechanism and (2) strength reduction, which is basically a viscoplastic (irreversible) mechanism.

3.2.4.1 Preliminary Tests

To begin the process of characterizing the damage response, tensile tests until failure should be conducted at strain rates of 1×10^{-3} and $1 \times 10^{-6} \text{ s}^{-1}$. Note that the test at the higher strain rate was already conducted as part of the deformation characterization process. From these tests, the failure stress and strain need to be recorded, as well as the maximum stress (UTS) and the strain at which the maximum stress occurs. This strain level can provide guidance in determining where the strain cutoff for the damage analysis occurs.

Given these tensile tests, two additional tensile tests at each of the two specified strain rates should be conducted with multiple unloads to zero stress (regardless of potential yielding on unloading) and reloads included as part of the test program. The unloads should be conducted at strain levels equal to and larger than the peak stress strain level. If the modulus on unloading is different from the initial loading modulus, stiffness reduction mechanisms can be assumed to be present, which at the very least contribute to the material softening. By tracking how much the unloading modulus changes as a function of strain, the level and progression of stiffness reduction damage can be quantified. The modulus change can be assumed to be directly related to the accumulation of stiffness reduction damage. If the unloading modulus after the stress peak is equal to the initial modulus of the material, the damage can be assumed to be solely due to strength reduction. An example of the stress-strain curves (with only partial unloading) from these types of tests and both the load and unload moduli are shown in Figure 43. Note the large difference between engineering stress (F/A_0) versus true stress (F/A) because when exploring damage, the strain accumulation is typically in the large-strain domain

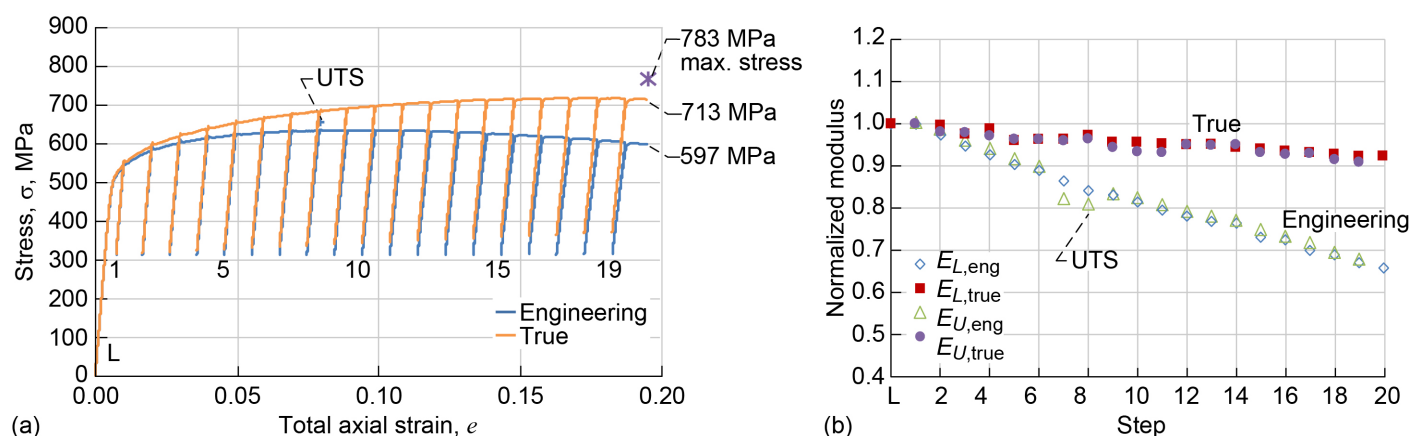


Figure 43.—Load-unload tensile test at strain rate of 10^{-3} s^{-1} for Ti-6-4 at 427 °C (from Ref. 14). L represents the initial load step. (a) Stress-strain curves (engineering and true) showing UTS. (b) Loading and unloading moduli, E_L and E_U , as function of loading step.

(see Appendix D in Ref. 14 for a more in-depth discussion related to Poisson's ratio and the influence of large strain).

Cyclic tests can also be used to begin the process of characterizing the progression of damage. A stress-limited cyclic test, or ratcheting test, where the stress is cycled between zero and a maximum stress, can be used to help characterize the amount of damage in a material. By tracking the separation between the corresponding load and unload portions of the cycle, both stiffness and strength reduction can be determined, in coordination with other test results. To provide a correlation to the tensile tests, the mean stress for these ratcheting tests should be set equal to the peak stress from the low-rate tensile test. The cyclic tests shown in Figure 44 and Figure 45 have a maximum stress of 344.75 MPa and was loaded at a rate of 6.895 MPa/s.

Figure 44 shows simulation results for the case of stiffness damage, the separation between the corresponding unload and reload portions of the cycle. Notice how the trend initially shows some hardening (i.e., plastic hardening) for the first three cycles, then peaks, and subsequently begins to steadily decrease (i.e., soften) as the cycles continue. This shows the competitive nature of the plastic hardening versus the stiffness damage (softening) with the stiffness damage steadily increasing until failure, as expected. Figure 45 shows simulation cyclic response of the material in the presence of strength degradation damage. Again, note that for the first 12 cycles the plastic hardening effects are seen, but once the peak is reached and the strength reduction (softening) begins to dominate, a drop shows in the curve with a subsequent horizontal portion signifying the residual (remaining life) strength state.

Stiffness reduction and strength reduction mechanisms as well as the combination of both stiffness and strength reduction mechanisms were examined using recent tension to failure and tension with intermittent unloads on Ti-6-4 (Ref. 14) to characterize these damage parameters (Ref. 68). Since a small number of tests with damage were run experimentally, the investigation into this portion of the model was relatively limited in comparison to that of viscoelastoplastic deformation. The experimental tests run at 1,000 °F (538 °C) (Ref. 14) contained the widest variety of tests with damage, and thus provided for the best proving ground for viscoelastoplastic damage modeling.

Considering only stiffness degradation (damage associated with the viscoelastic portion of the model), the COMPARE results fit quite well to the experimental data. Figure 46(a) shows a stress-strain plot of a tension-to-failure test with periodic unloading. Note the simulated response with no damage

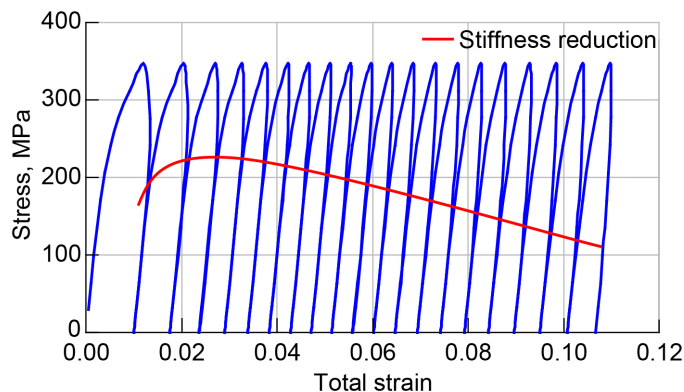


Figure 44.—Stress control cycles with stiffness degradation damage (from Ref. 6).

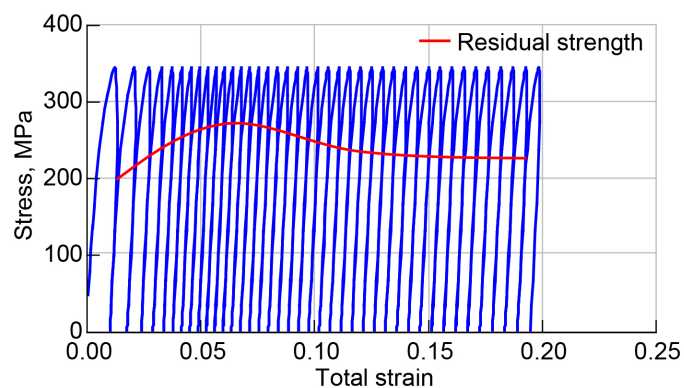


Figure 45.—Stress control cycles with strength degradation damage (from Ref. 6).

continues to climb to progressively higher stresses as the strain increases, deviating from the experimental data. However, with stiffness degradation only introduced, the influence of including damage in the viscoelastoplastic portion of the GVIPS model is immediately apparent, in that with each repeated unload and load cycle (after the third cycle), the specimen incurs damage, and the stiffness of the material degrades. This is not immediately noticeable when examining a stress-strain plot. However, examining the modulus-versus-load-cycle plot (see Figure 46(b)), a clear trend is evident. Where the experimental data, deformation-only data, and stiffness-only degradation data show how the damage alters the stiffness of the material. Note that the model portraying deformation only has essentially the same stiffness throughout the test. With stiffness degradation introduced, though, the downward trend is modeled quite well.

Next, the strength degradation mechanism within the model was activated to gain insight to whether this could better simulate the response in this region. With four viscoplastic mechanisms at 1,000 °F (538 °C), 16 new parameters were created for the strength degradation. However, focusing on short-term tensile tests meant that only the short-term mechanisms needed to be activated, which reduced the complexity by half. With the two appropriate mechanisms for the short-term tests characterized, the impact of introducing strength degradation instead of stiffness to the model is not immediately apparent; comparing Figure 46(a) to Figure 47(a) shows a similar fit. Yet when the modulus degradation is examined, the true benefit becomes apparent (see Figure 47(b)), as almost no stiffness degradation is predicted (as expected from the model idealization).

Lastly, Figure 48 provides results for the case when both stiffness and strength degradation are activated simultaneously. Note now the rate of damage (slope) simulated by the model is more closely aligned with the rate of damage incurred by the specimen, as evident in Figure 48. This allows for a better understanding of how damage is incurred over a given number of cycles, as it better matches the rate of damage. Although the model with stiffness degradation only may have a lower overall error when compared to the experimental data, the idea of “quality over quantity” is again important in deciphering the results.

Another type of test to determine the damage progression in a material is a creep test that goes into the tertiary creep regime. Wherein tertiary creep can be caused by rafting within the microstructure of metals, such a mechanism is associated with strength degradation within the GVIPS framework. If damage is not accounted for, a simulation of a constant-stress creep test will merely flatten out into a steady-state regime during high-strain loading. The creep test should be loaded at a strain rate of $1 \times 10^{-3} \text{ s}^{-1}$, with the creep stress being set to the mean stress from a cyclic ratcheting test. One key point to note in a long-range creep test is that the tests need to be conducted at constant stress as much as possible. In the tertiary creep range this constant stress level can be difficult to maintain since the cross-sectional area is changing during the loading process, which means that a constant-load test does not equate to a constant-stress test, except for a very short time period (small accumulation of strain). To ensure that constant stress is maintained in the material, the transverse strains need to be measured in the material which then enable the true stress to be calculated. From the creep tests, determining a good approximation of where tertiary creep begins (from the strain-versus-time plots) is important, as this point is what can be approximated as being the inelastic strain cutoff for the initiation of damage. Also, from this preliminary creep test the inelastic strain to failure is

determined and compared to the results from the tensile test at the equivalent strain rate, as the values should be comparable.

The damage observed in a creep test can be a combination of stiffness reduction and strength reduction. To determine the presence of stiffness reduction, an additional creep test can be conducted where the material can be unloaded (and possibly reloaded) at a series of strain levels in the tertiary creep range. By producing a stress-strain plot of a test of this sort, and by conducting unloading at several different strain levels, one can determine if a modulus reduction is taking place (i.e., the unloading modulus is lower than the initial loading modulus, and the modulus decreases as the strain increases). If the modulus

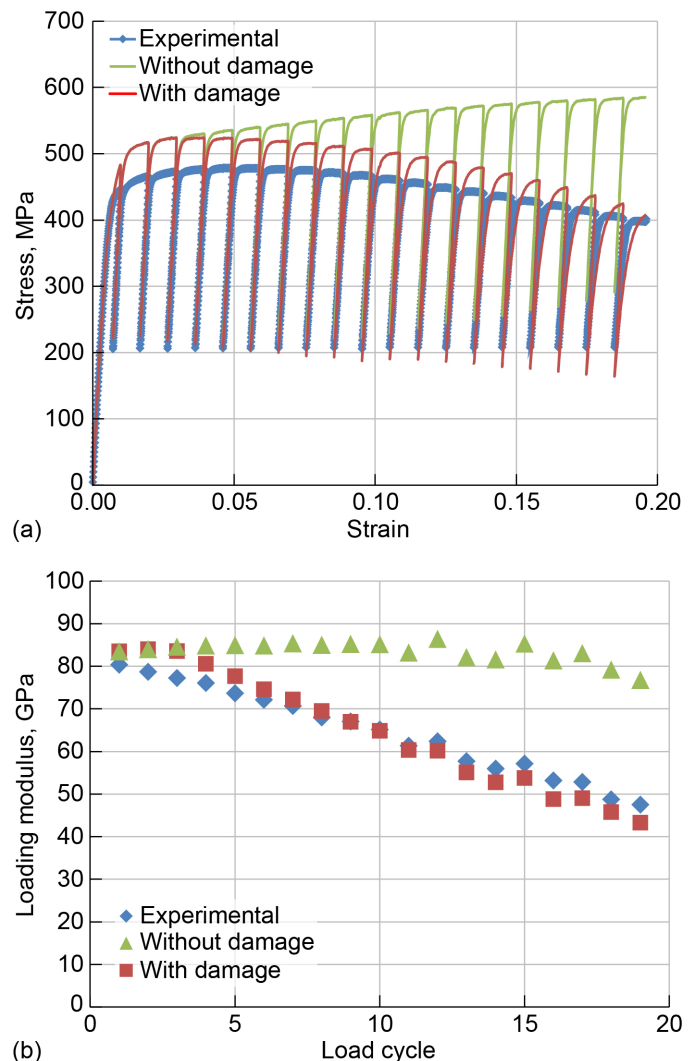


Figure 46.—Load-unload tensile test, at strain rate of 10^{-3} s^{-1} for Ti-6-4 at 538 °C. (a) Engineering stress-strain curves (GVIPS simulation with stiffness reduction damage only). (b) Loading modulus E_L as function of loading step (GVIPS simulation with stiffness reduction damage only).

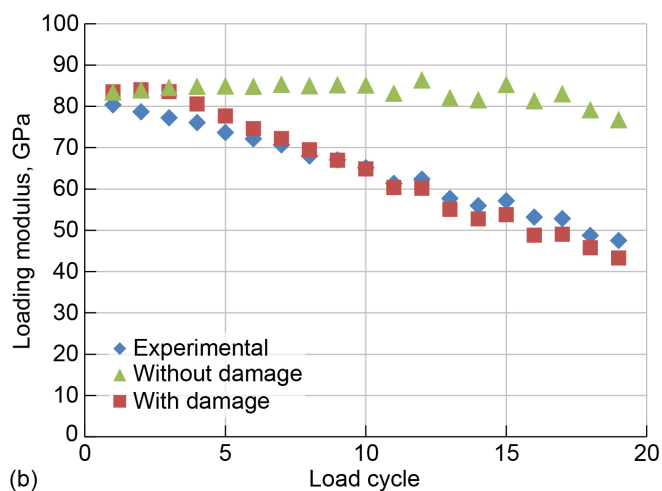
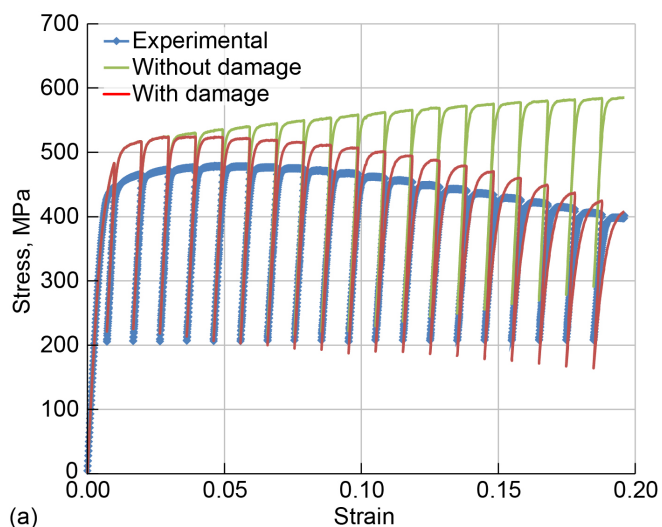


Figure 47.—Load-unload tensile test, at strain rate of 10^{-3} s^{-1} , for Ti-6-4 at 538 °C. (a) Engineering stress-strain curves (GVIPS simulation with strength reduction damage only). (b) Loading modulus E_L as function of loading step (GVIPS simulation with strength reduction damage only).

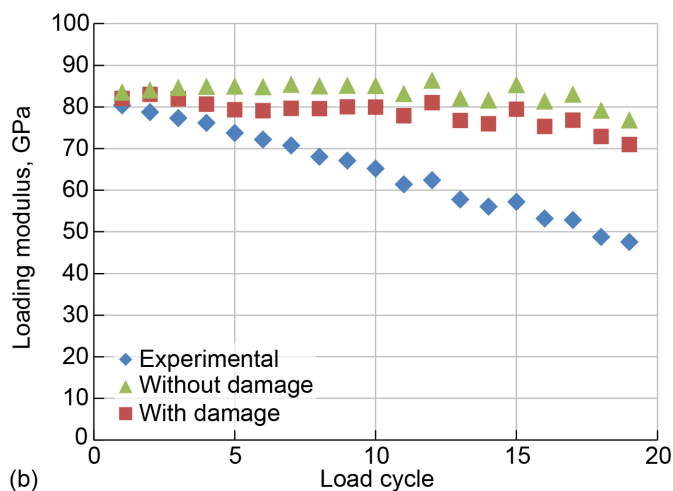
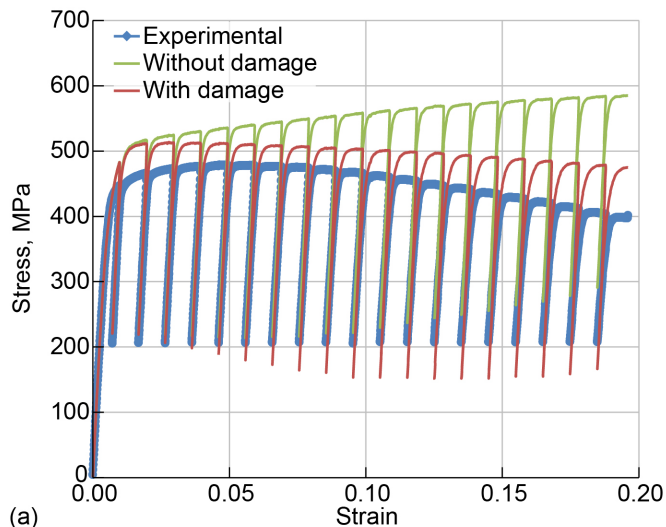


Figure 48.—Load-unload tensile test, at strain rate of 10^{-3} s^{-1} for Ti-6-4 at 538 °C. (a) Engineering stress-strain curves (GVIPS simulation has both stiffness and strength reduction active). (b) Loading modulus E_L as function of loading step (GVIPS simulation has both stiffness and strength reduction active).

reduction does take place, the material is at least partially subject to stiffness reduction damage. If no modulus reduction takes place, the damage is all strength reduction. Note that from the test data it is very hard to absolutely partition the damage into stiffness reduction and strength reduction because of noise in the data as well as variation between specimens. A trial-and-error approach in the material characterization needs to take place to determine the optimal partitioning of damage between stiffness reduction and strength reduction. The material should be unloaded at strain levels equal to 50, 75, 85, 90, and 95 percent of the failure strain. If due to the rapid progression of tertiary creep these exact strain levels cannot be utilized, as many as possible should be applied. The creep tests with unload can also provide insight into the repeatability of the creep tests by comparing to the original creep test. Example stress-strain and strain-time curves from a creep test of Ti-6-4 is shown in Figure 49. One observes that both the longitudinal and transverse strain exhibit tertiary creep, and if one uses engineering values, one would conclude that the damage is due to stiffness degradation. If true values are used, however, then it appears that strength reduction is active and stiffness reduction is minimal.

Simulation of the tertiary creep response using the GVIPS model is shown for two creep curves, loaded to the same stress level of 379 MPa (the mean stress in Figure 46 to Figure 48) at 482 and 538 °C (see Figure 50, wherein both damage stiffness and strength reduction are active in the simulation). Clearly, the correlation is excellent.

Relaxation tests can also be used to determine the damage progression in a material. Both in individual tests and in a step relaxation test (to help characterize history dependence), a material can be loaded to strain levels before, at, and after the peak stress (in an engineering stress sense), and then a relaxation can take place. By examining the stress-versus-time response during each of the relaxations, the progression of damage can be observed because as more damage accumulates, the lower the stress level will reach during the relaxation (primarily due to strength reduction). If sufficient strength reduction has taken place during relaxation, the stress level can even reach a level of zero stress. This result indicates that the equilibrium stress decreases as a function of damage, possibly even reaching a level of zero. Example stress-strain curves from even reaching a level of zero. Example stress-strain curves from multistep relaxation tests are shown in Figure 51. Change in the engineering modulus is approximately 10.5 percent.

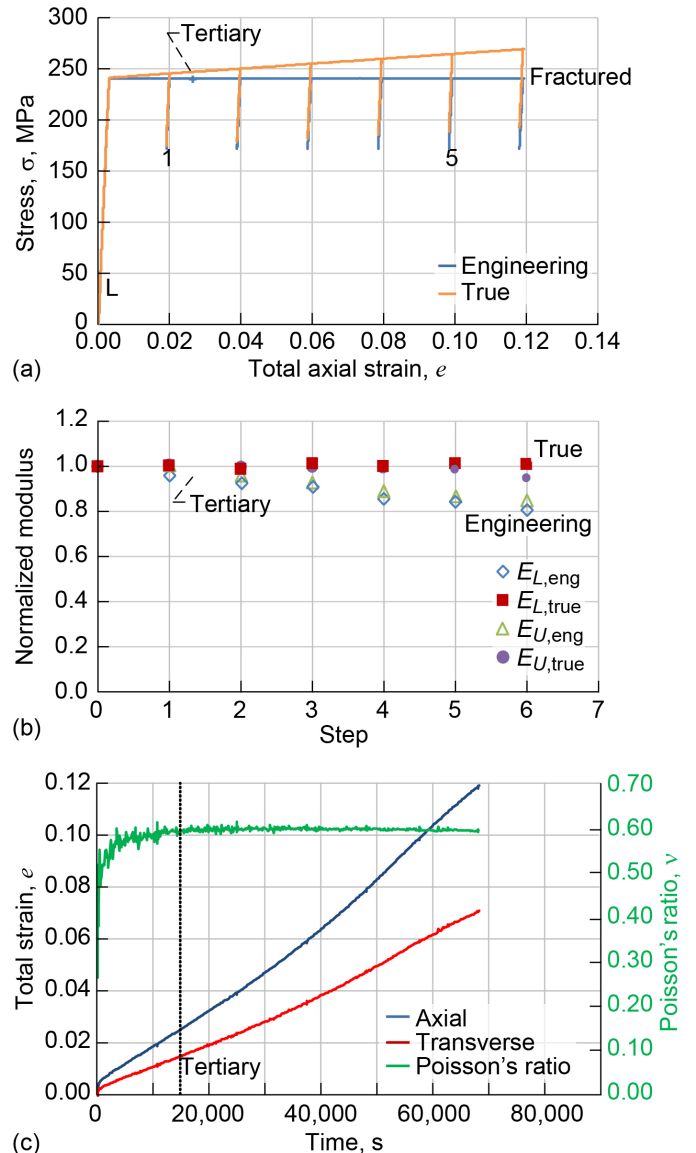


Figure 49.—Creep test of Ti-6-4 at 241 MPa and 482 °C at strain rate of 10^{-3} s^{-1} (from Ref. 14). (a) Stress-strain curves (engineering and true) showing loading-unloading history, with inception of tertiary creep. (b) Normalized loading and unloading moduli, E_L and E_U , as function of load step. (c) Strain-time curves (axial and transverse) and Poisson's ratio showing inception of tertiary creep.

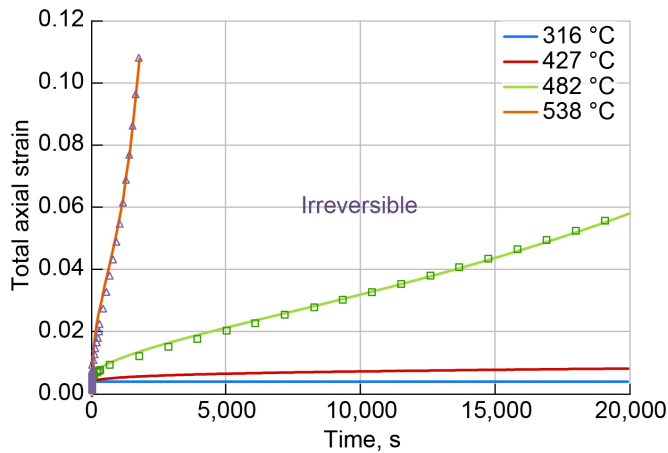


Figure 50.—Experimental (symbols) and GVIPS simulations (lines) of creep response at 379 MPa for Ti-6-4 at various temperatures.

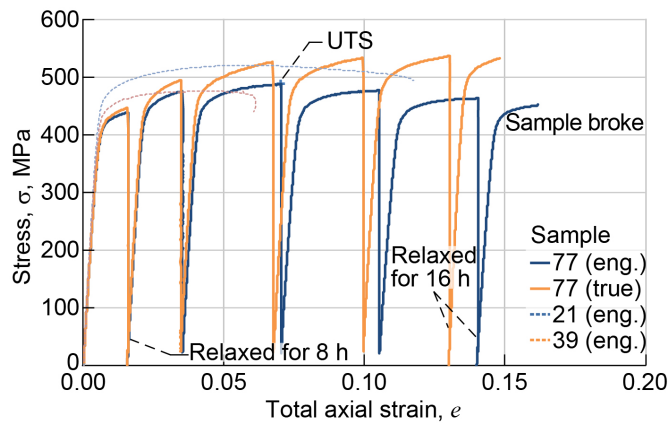


Figure 51.—Multistep relaxation test of Ti-6-4 at 538 °C and strain rate of 10^{-3} s^{-1} with stress-strain curves (engineering and true) showing UTS. (from Ref. 14).

3.2.4.2 Detailed Cyclic Tests

To supplement the preliminary tests (see Section 3.2.4.1), which provided initial insight into the damage progression behavior of a material, there are several additional cyclic tests that can provide detailed information on the damage progression, and ultimately life, of a metal (material).

First, examining the fully reversed cyclic tests (performed as part of the deformation characterization) can also provide insight into the damage progression of the material. In the case of strength reduction in the material, the maximum stress levels in the hysteresis loops will decrease until a steady-state condition is reached (see Figure 52). In the case of stiffness reduction, the hysteresis loop will rotate and flatten out until ultimately a completely flat loop, with near zero stresses, is obtained (Figure 53). This expansion and rotation of the hysteresis loop due to stiffness reduction can also be seen in a

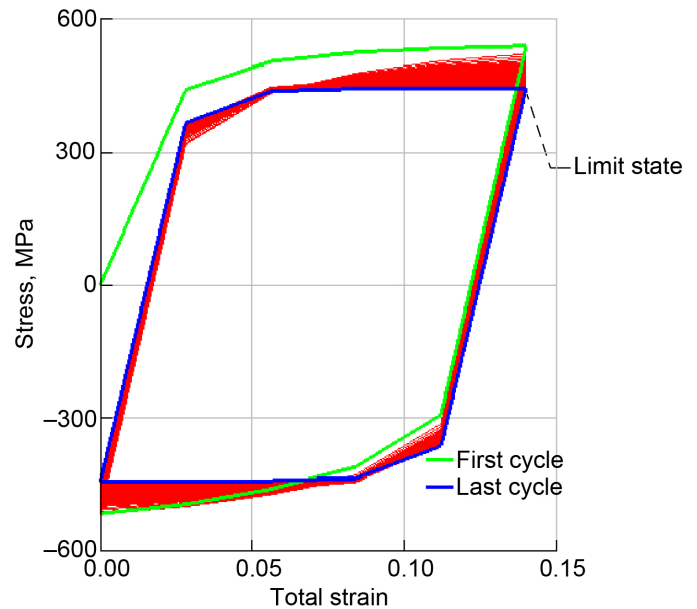


Figure 52.—Simulated hysteresis behavior, with strain control, assuming strength degradation damage mechanism (from Ref. 6).

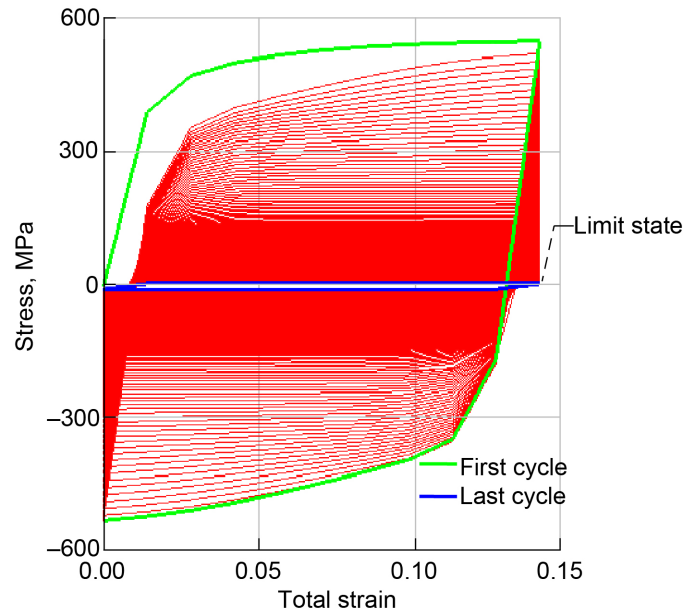


Figure 53.—Simulated hysteresis behavior, with strain control, assuming stiffness degradation damage (from Ref. 6).

fully reversed stress-controlled test. However, in this case, a full flattening out of the hysteresis loop is not likely to be obtained. By conducting the fully reversed tests (i.e., $R_\sigma = \sigma_{\min}/\sigma_{\max} = 0$) under stress control (i.e., load control) at several stress levels, a traditional S-N curve can also be generated. The stress levels should be equal to the proportional limit as well as the proportional limit plus 0.25, 0.5, and 0.75 times the difference

between the ultimate stress and the proportional limit. An example S-N curve is shown in Figure 54 for Ti-6-4 tested at 427 °C and an elastic stress-equivalent rate of 10^{-3} s^{-1} ; however, in this case a mean stress was applied, $R_\sigma = 0.1$.

Various types of asymmetric cyclic tests can also be conducted. An asymmetric ratcheting test, where the stresses are cycled between a high and low value with a nonzero mean stress can be conducted to gain insight into the mean stress effects on the material response (see Figure 55 for an example). Strain-controlled asymmetric tests can also be conducted, where the material is loaded to saturation and then cycled in strain control around a nonzero mean strain (see Figure 56 for an example).

A cyclic test version of the creep-plasticity interaction test can also be performed. For example, the material could be

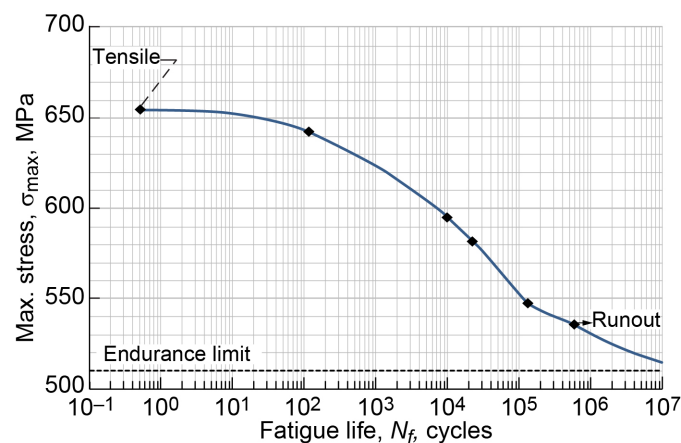


Figure 54.—Fatigue tests of Ti-6-4 at 427 °C and stress equivalent rate of 10^{-3} s^{-1} (from Ref. 14).

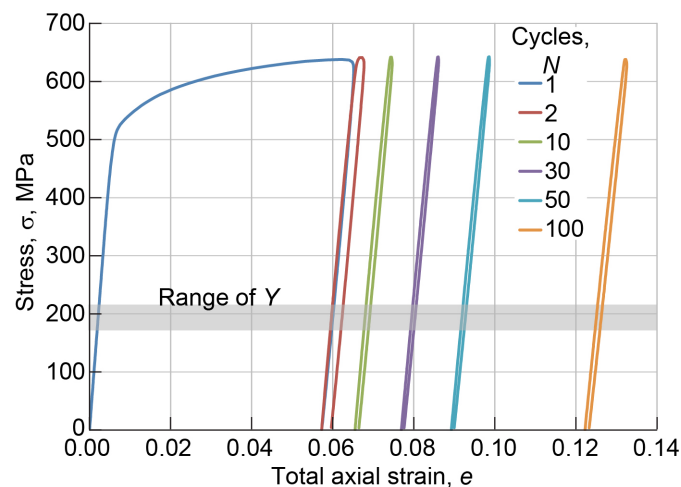


Figure 55.—Mean stress fatigue test of Ti-6-4, in load control between 0 and 640 MPa at 427 °C and 10^{-3} s^{-1} strain rate shown for selected cycles; failure at $N_f = 115$ cycles (from Ref. 47).

loaded in tension to a certain stress level, held in creep for a period of time, unloaded until zero stress, cycled down to a minimum strain level, and reloaded to the creep stress level again (Figure 57). A test of this type can help to validate the capability of a model, the GVIPS model, to account for all types of loading within one set of constitutive equations.

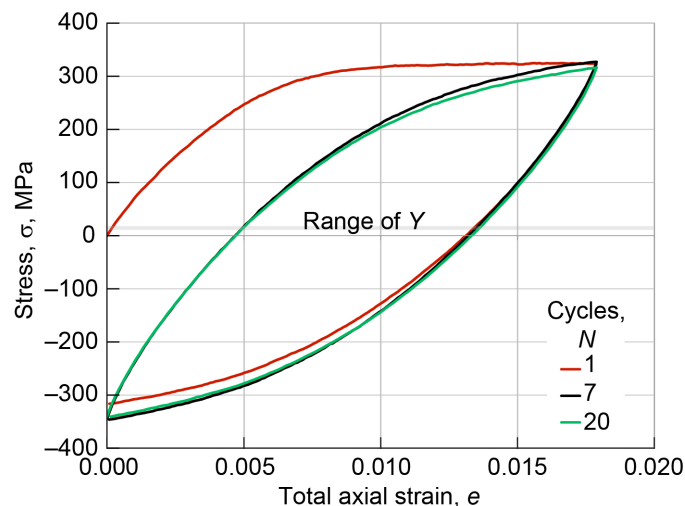


Figure 56.—Hysteresis loops for Ti-6-4 fatigued at 538 °C and 10^{-5} s^{-1} strain rate between 0 and 0.018 strain for selected cycles (from Ref. 47). Range of threshold stress Y is also shown.

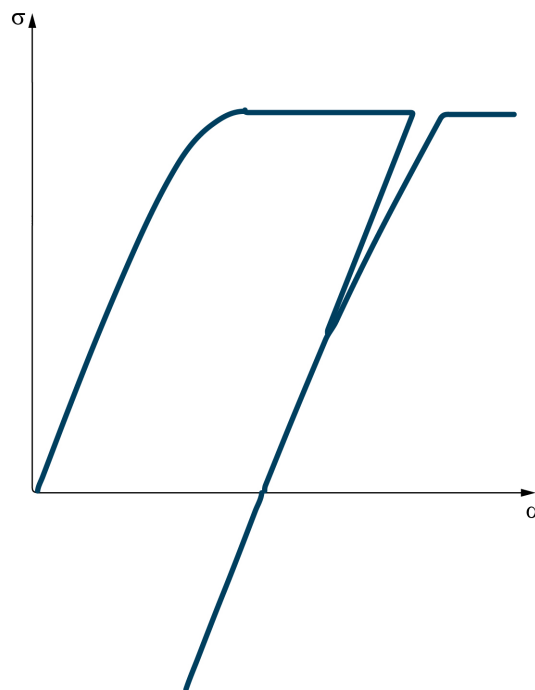


Figure 57.—Creep-plasticity interaction curve.

Table XII summarizes the tests required to characterize the uniaxial damage portion of the material model. Tensile, creep, relaxation, and cyclic tests are discussed. Similarly to what was specified for the viscoelastic and viscoplastic portions of the test matrix, the tests should be repeated at two to three temperatures different from the original reference temperature to characterize the temperature dependence of the material constants.

3.2.5 Damage Summary

Section 3.2.4 has outlined the type of tests required to characterize the additional damage model parameters for a typical viscoelastoplastic model. The GVIPS model was the example considered herein, assuming a fully characterized deformation model has already been determined up to the total strain cutoff value $\bar{\epsilon}_{cut}$ (which determines when stiffness reduction initiates) and the inelastic cutoff value $\bar{\epsilon}_{cut}^i$ (which dictates when strength reduction mechanism initiates as well) (see Section 2.4). Table XII indicates the tests, at a given temperature, that can be used to characterize the various damage model parameters, assuming they have been taken to failure. Remember, damage modeling is still in its infancy in comparison to the maturity of deformation modeling. Therefore, these types of tests should be considered only suggestions and are subject to change as progress advances in the area of damage characterization. It is strongly recommended that one utilizes a parameter estimation tool like COMPARE (see Section 2.5) to obtain an optimized set of deformation and damage model parameters, particularly when introducing multiple mechanisms as well. Lastly, it is even more important

that all the test data and associated metadata be stored properly to enable future development and refinement of models (see Appendix B) because of the immaturity of this field.

In summary, the following has been determined from damage uniaxial and cyclic tests:

- If the total applied strain $\epsilon > \bar{\epsilon}_{cut}$, then stiffness degradation is assumed active.
- GVIPS parameters associated with stiffness reduction are c_e , $\bar{Y}_0$, μ_e , and n_e .
- If the accumulated inelastic strain $\epsilon^I > \bar{\epsilon}_{cut}^i$, then strength degradation is assumed active.
- GVIPS parameters associated with strength reduction $\theta^{(b)}$ are $c_d^{(i)}$, $Y_0^{(i)}$, $\mu_d^{(i)}$, $n_d^{(i)}$, where i denotes each assumed irreversible mechanism.
- Tests in Table XII should be repeated for the various temperatures of interest to ensure full temperature dependency of the preceding damage parameters.

3.3 Structural Validation Testing

Actual aerospace structural components are subjected to thermomechanical service conditions involving temperature levels, thermal transients, and mechanical loads severe enough to result in significant nonlinear (e.g., inelastic or damage-induced) deformation. To be able to accurately predict the stress, strain, and temperature fields within these structural components, the developed constitutive equations are expressed in multiaxial

TABLE XII.—TESTS REQUIRED TO DETERMINE GVIPS DAMAGE MODEL PARAMETERS
[Repeat at four different temperatures: T_1 , T_2 , T_3 , and T_4 .]

Test	Type	Rate, s^{-1}	Limit value	Comments
1	Tensile	1×10^{-6}	$\epsilon = \text{failure}$	
2	Tensile with multiple unloads	1×10^{-6}	$\epsilon = \text{failure}$	
3	Tensile with multiple unloads	1×10^{-3}	$\epsilon = \text{failure}$	
4	Cyclic	1×10^{-3}	Load between zero stress and max. stress	Mean stress equal to ultimate stress from low-rate tensile test
5	Creep	1×10^{-3}	Stress equal mean stress from cyclic test	Load tertiary creep regime
6	Creep with unload	1×10^{-3}	Stress equal to mean stress from cyclic test	Unload at strain levels equal to 50%, 75%, 85% 90%, and 95% failure strain
7	Step relaxation	1×10^{-3}	Failure	Multiple relaxations and reloads after peak stress
8	Cyclic	$\dot{\epsilon}_1$	Strain at saturation stress	Symmetric
9	Cyclic	$\dot{\sigma}_1$	σ_s	0 to saturation stress

form and must be appropriate for all modes of mechanical and thermal loading expected to be experienced by the components (e.g., cyclic, nonisothermal, nonproportional, etc.). Although the deformation and damage characterization tests described previously are primarily uniaxial, some multiaxial tests should also be conducted to verify that the constitutive equations and the characterized material constants can accurately predict the response of a material under multiaxial loading conditions, which are more representative of the loading applied to actual structures. By conducting biaxial tests, controlled multiaxial loading conditions can be applied in a manner such that the constitutive equations can be rigorously verified and validated. Furthermore, the establishment and compilation (within a given database, see Appendix B) of the proposed experimental program will enable researchers to identify and evaluate the progression (evolution) of damage from an undamaged to damaged state, thus providing a wealth of knowledge for expanding diagnostic (inspection) and prognosis (life prediction) capability. Although not discussed herein, linkage of diagnostic methods, through the use of state awareness variables, is critical to provide information on the current load/damage state at a given point in time so that a physics-based prognostics model can independently determine the future response of the structure (see Ref. 69 for more details).

To verify the multiaxial deformation and damage constitutive models (fundamental to prognostic algorithms), a more complex subcomponent testing program is required. As indicated in Figure 58, the overall structure for a verification program to test the validity of the material constants related to damage (i.e., flaws) will be introduced into a biaxial plate specimen subject to biaxial loading (forces F_1 and F_2), and the progression of the damage, reaction force, residual strength, and inelastic strain during testing will be monitored. This type of test program represents the baseline configuration represented in Figure 58. Table XIII represents the proposed test matrix for in-plane loading of prognostically challenging (PC) and diagnostically challenging (DC) configurations (i.e., those with varying defects). Given the results of these fundamental studies, the performance of detection techniques, adequacy of an assumed state awareness variable(s), and prognosis methods can be evaluated.

The baseline plate configuration to be examined will be the classic problem of a plate with a single stress riser loaded uniaxially and biaxially under both monotonic and cyclic histories. This baseline case will be complemented by two other configurations with multiple stress risers of varying character (i.e., holes, slots, cracks, etc.), which might correspond to inherent structural attachment configurations and/or impact damage. A

variety of crack initiators will be examined, looking toward developing a significant database to characterize and validate proposed diagnostic and prognosis algorithms under relatively realistic conditions and to enable assessment of the uncertainties in the underlying failure mechanisms. Figure 58(a) illustrates the baseline (B) defect configuration, which itself is neither DC nor PC. However, when combined with the elevated-temperature load histories specified in Table XIII, the baseline configuration will become PC (B/PC). The second configuration (middle diagram) is considered to fall into the category of DC because of the inherent nonequal excitation intensities around the inflicted defects. This configuration is mildly PC. The third configuration, defects of varying character (location, size, geometry, sharp, smooth, etc.), is both diagnostically and prognostically challenging (DPC), irrespective of the applied load history, and will be used primarily for demonstration of the accuracy of a given prognosis algorithm (the one of specific concern herein is the GVIPS class model). A number of these tests should be performed blind; that is, with the inflicted damage state kept hidden from both diagnostic and prognosis practitioners.

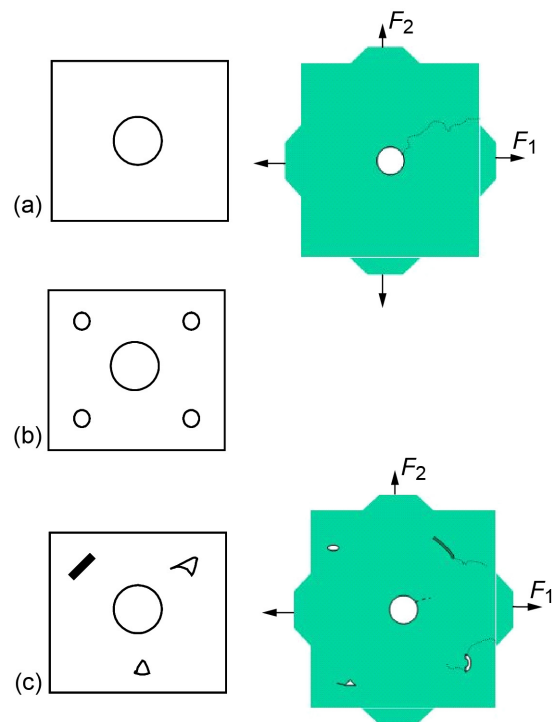
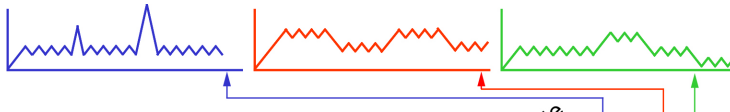


Figure 58.—Biaxially loaded plate with various inflicted defect configurations. (a) B: baseline or B/PC: baseline and prognostically challenging at elevated temperatures. (b) DC: diagnostically challenging. (c) DPC: diagnostically and prognostically challenging.

TABLE XIII.—EXAMPLE TEST MATRIX



Defect configuration		Uniaxial	Biaxial	Simple cyclic	Biaxial creep, combustor linear	Complex cyclic F1-Fixed	Cyclic-discrete F2-Variable overload	Stepped cyclic	Thermal
Biaxiality ratio		0	2	1	2	V	V	2	
Inflicted defect configuration	Baseline	✓	✓		✓				
	B/PC			✓	✓	✓	✓	✓	✓
	DC		✓		✓			✓	✓
	DPC		✓	✓	✓	✓	✓	✓	✓

Room temperature
 Room and elevated temperature
 Elevated temperature

With respect to imposed load history, a number of biaxiality ratios ($BR = F_1/F_2$) (see Table XIII) are anticipated; for example, $BR = 0$ (uniaxial), $BR = 1$ (equal biaxiality—which, e.g., occurs in a pressurized sphere), $BR = 2$ and/or 0.5 (representing the loading in the case of a linear cylindrical pressure vessel and combustor system), and lastly a varying ratio $BR = V$ (wherein one component is held fixed while the other is varied). Even though some monotonic load histories will be imposed, emphasis should be placed on fatigue-induced cracks, with more complex histories involving thermal cycles, periodic overloads, and stepped histories being conducted toward the end of the program for validation. The verification test program will commence with an examination of room-temperature tests, followed by some elevated-temperature tests (light blue and green cells in Table XIII). This will enable prognosis enhancements to be initiated if necessary and enable others to assess state awareness characterization and perform diagnostic down selection. The yellow cells in Table XIII will be conducted subsequently.

A key aspect of this experimental program will be the precise documentation of the order (both temporally and spatially) of flaw insertion (be it operator and/or load history induced), thus enabling meaningful interrogation of the data, aiming at state awareness identifications, that is, signature—structural manifestation—of the deformation and life behavior of the material. For example, envision the following test procedure:

1. Perform characterization tests: Fully known conditions—single stress riser (Figure 58(a)).
2. Apply all desired diagnostic techniques to virgin plate (no hole).

3. Induce hole of specified diameter, location, and character (thus overwhelming any inherent defects/flaws due to manufacturing)—again, apply all selected local/global diagnostic techniques to the plate.
4. Mechanically load plate and monitor initiation and propagation of crack(s) by taking a variety of spatial (full-field and discrete locations) measurements (strain-based, vibration) at various time intervals (intermittent and continuous) throughout the mission profile.

Repeat Steps 2, 3, and 4 for different load histories. In the case of validation tests (i.e., when multiple stress risers are present) the preceding steps will be repeated, but in between Steps 3 and 4, the following step will be inserted:

5. Impose predefined defects (with specific character) at specified locations; obtain all pertinent signatures.

This will simulate material randomness in that multiple initiation sites can occur at unexpected locations when mechanical loads are applied, thus challenging both diagnostic and prognosis methodologies and enabling a clear demonstration of capabilities. Note: although data for both diagnostic and prognostic algorithm verification will be collected (see Appendix B), the primary objective within such a program is to verify the prognostic capability of the GVIPS class model, given a known defect configuration.

Other structural tests (vanes, T- or W- stiffened plates, sandwich plates, etc.) can also be used for validation, and the most important aspect is the ability to measure the full strain-field response due to the various load histories, because deformations

and/or strains are the only thing one can measure and compare to the model predictions. It is also important to measure the full temperature field as well so that that specific temperature at a given location can be accounted for within the FE analysis, because it is the thermally induced stresses that are often missed. It is precisely these types of structural benchmark problems that will demonstrate the capability of a given constitutive model (prognosis model) and build confidence in such a model to accurately predict complex real-world problems.

4.0 Summary and Conclusions

This report has provided a brief review of experimentally observed hereditary and nonhereditary material behavior along with background information on standard as well as advanced internal state variable constitutive modeling at elevated temperature. A description of exploratory, characterization, and validation testing was presented and a detailed outline given of what constitutes “sufficient” data content (i.e., quality and quantity) for primarily enhancing, characterizing, and validating a particular sophisticated nonlinear time- and history-dependent (hereditary) class of constitutive models known as GVIPS (generalized viscoplasticity with potential structure). Material behavior in the reversible (or viscoelastic) regime and also the irreversible (or viscoplastic) regime at elevated temperature was presented for the identification of both deformation and damage model parameters. Examples illustrated were primarily for metallic materials. In all cases, both uniaxial and multiaxial tests were described, and the linkage between the specific tests and the

parameters within the model that can be characterized from the results of these tests was also discussed. The intent behind this document is to help both the modeler and experimentalist understand each other’s specific points of view and provide guidance for both model development and characterization. Emphasis has been placed on providing the mechanician with information regarding how tests are performed and what issues to be aware of when interpreting results. It is hoped that experimentalists will take away a new perspective on the types of information that modelers and analysts are looking for from them.

The case was made that the type and extent of materials information routinely collected is not always sufficient to meet the current, much less future, needs of the materials modeling community. If, however, one is diligent to capture the complete fundamental material response history (both uniaxial and multiaxial) and all associated pedigree information (both processing and test procedures), the likelihood of meeting both current and future needs is greatly enhanced. Although this may constitute a paradigm shift in an individual’s thinking regarding material data collection, analysis, and dissemination, particularly given the increase in expense and effort in obtaining such data, it is an essential one, particularly as both analyst and designer move toward utilizations of sophisticated, nonlinear time- and history-dependent (hereditary) constitutive models. Finally, generation of this materials data would be futile without an appropriately robust informatic infrastructure, as discussed, for handling the potentially massive amounts of such data and establishing the traceability between the data collected and model development.

Appendix A.—State Space Representation of Models

The state space representation, as suggested by Onat and Fardshisheh (Ref. 31), provides a consistent framework for geometrically describing constitutive models (hereditary material behavior). The construction of this state space (as discussed in detail by Arnold, Ref. 70) is accomplished by rewriting the one-dimensional (1D) mechanical rate form of Equations (4), (33), and (34) in standard vector form and plotting the corresponding $\mathbf{G}$ fields (or relaxation trajectories):

$$\dot{\mathbf{S}} = \mathbf{G}(\sigma, \alpha) + \mathbf{D}(\dot{\epsilon}) \quad (\text{A.1})$$

or

$$\begin{bmatrix} \dot{\sigma} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} -E f(\sigma, \alpha) \\ g(\sigma, \alpha) \end{bmatrix} + \begin{bmatrix} E \dot{\epsilon} \\ 0 \end{bmatrix} \quad (\text{A.2})$$

For the specific functional forms associated with the Robinson viscoplastic model (Ref. 71) along with the associated material parameters A , n , κ , H , β , R , and m , the functions $f(\sigma, \alpha)$ and $g(\sigma, \alpha)$ are

$$f(\sigma, \alpha) = \dot{\epsilon}^I$$

and

$$g(\sigma, \alpha) = \frac{H}{\alpha^\beta} \dot{\epsilon}^I - R |\alpha|^{m-\beta} \text{sgn}(\alpha)$$

where

$$\dot{\epsilon}^I = \begin{cases} AF^n & \text{for } F > 0 \\ 0 & \text{otherwise} \end{cases} \quad (\text{A.3})$$

$$\text{and } F = \frac{(\sigma - \alpha)^2}{\kappa^2} - 1$$

The relaxation trajectories are obtained from Equation (A.1) by imposing the condition of constant total strain (i.e., $\dot{\epsilon} = 0$), thus implying that $\dot{\mathbf{S}} = \mathbf{G}(\sigma, \alpha)$, wherein the $\mathbf{G}$ fields (or relaxation lines) can be mapped out as in Figure A.1. Note that a point (σ, α) in this space corresponds to a particular inelastic state of the material. Often, the state point is designated by the vector $\mathbf{S}$, and it is helpful to consider how the state point moves (evolves) in the state space under various conditions. The essence

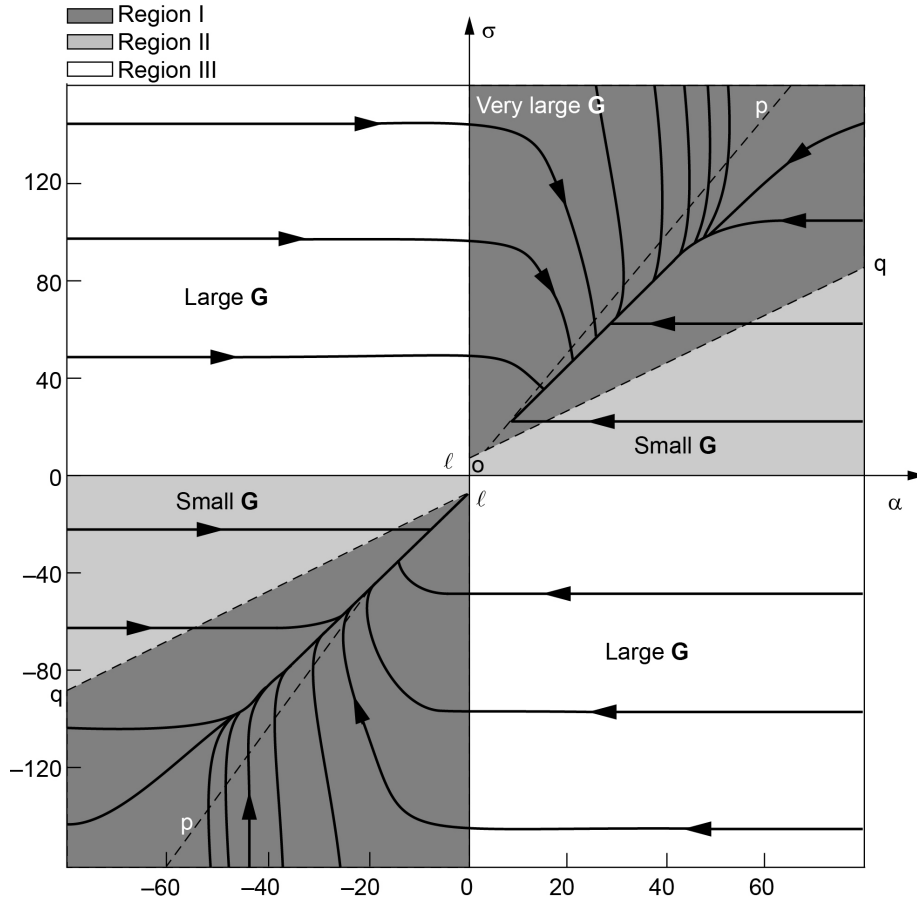


Figure A.1.—Full state space representing the Robinson model (Ref. 71).

of inelastic behavior is, of course, contained in the knowledge of the vector field $\mathbf{G}(\sigma, \alpha)$. With this field known, if the current state (σ, α) is known, then $\mathbf{G}$ is known at that point. If the current total strain rate $\dot{\epsilon}$ and modulus E are known, then $\mathbf{D}$ is known and from Equation (A.1) and (A.2), $\dot{\mathbf{S}}$ is known, and therefore the state point moves instantaneously through the state space (or equivalently, the material response is known). To this end, consideration is given to some geometrical features of the state space. If $\mathbf{G}(\sigma, \alpha)$ is bounded, then from Equation (A.1) or (A.2)—given that $\dot{\epsilon}$ is large enough— $\dot{\mathbf{S}} = \mathbf{D}(\dot{\epsilon}) = \begin{bmatrix} E\dot{\epsilon} \\ 0 \end{bmatrix}$, which implies that the state change under a sufficiently high strain rate follows vertical paths (i.e., elastic) in the state space. Another important feature in the state space corresponds to the concept of steady state (SS). The term then implies that $\dot{\mathbf{S}} = \begin{bmatrix} \dot{\sigma} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$; thus, from Equation (A.3) it is known that $\dot{\epsilon}^I = f(\sigma, \alpha) = \dot{\epsilon}$ and that $g(\sigma, \alpha) = 0$, which defines the locus of SS points in the state space; see path ℓ - p in Figure A.1. Given the preceding form for $g(\sigma, \alpha)$ it can be shown that this locus is

$$\alpha = \sigma - \kappa \sqrt{\left(\frac{R}{AH}\right)^{1/n} \alpha^{m/n} + 1} \quad (\text{A.4})$$

At each point on this locus, the corresponding $\mathbf{G}$ vector is vertical, and the inelastic strain rate (SS rate) is given by $\dot{\epsilon}^I = f(\sigma, \alpha) = AF^n$.

Line o - q in Figure A.1 is the line of maximum stress and in this case $\sigma - \alpha = k$, above which is Region I of the state space and below which is Region II of the state space. Region III is the second and forth quadrants.

Now, returning to Onat's more general system of Equations (33) and (34), Section 2.2, with two state variables (q_1, q_2) (i.e., $n = 1$), the uniaxial state space corresponding to this uniaxial theory is shown in Figure A.2, where the ordinate axis is associated with the applied stress ($q_1 = \sigma$) and the abscissa axis with the internal stress ($q_2 = \alpha$). The line ℓ - n in Figure A.2 denotes the locus of SS points (or attractive points), and the line ℓ - m denotes the onset of an inaccessible regime where the $\mathbf{G}$ field is so large that, regardless of the rate of loading, the trajectory of the state is along this line. Lines that are parallel to the q_1 axis correspond to lines of fast deformations because $\mathbf{D}$ is always parallel to the q_1 axis and dominates as $\dot{\epsilon}$ approaches infinity (elastic path). The state path o - a - b (denoted in Figure A.2) is that of a simple creep test. Note that at point b , $\mathbf{G}$ is parallel to the q_1 axis and colinear, but opposite in direction, to $\mathbf{D}$; point b corresponds to a state of steady creep ($\dot{\mathbf{S}} = 0$). Also, the simple creep test indicates

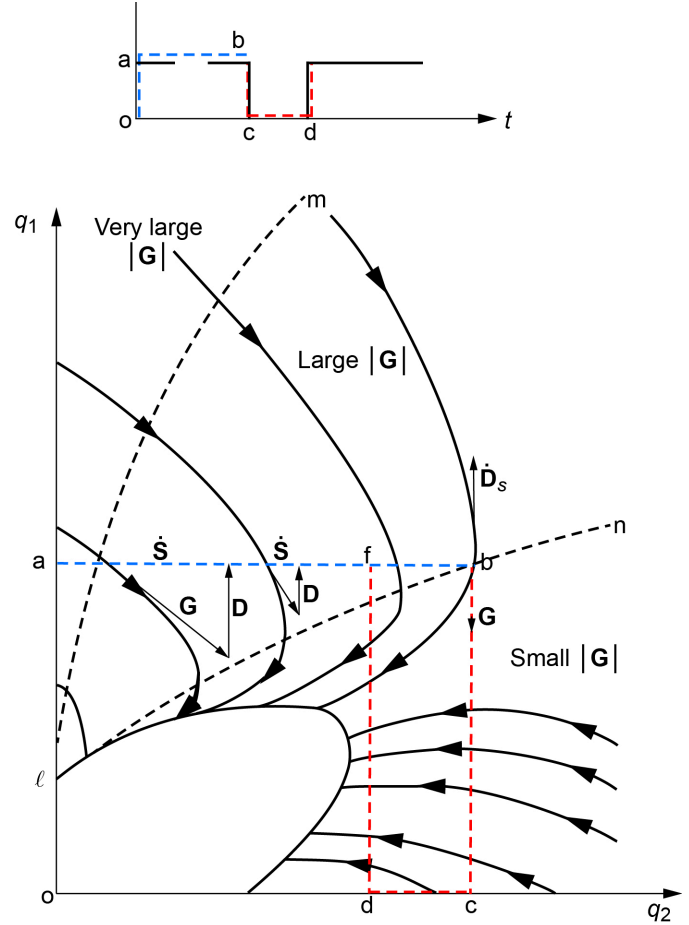


Figure A.2.—General state space. Adapted from Reference 31.

that if SS creep is to be attained, the magnitude of $\mathbf{G}$ must be large in the vicinity of point a , yet must decrease as the state point moves toward b in order for $\dot{\mathbf{S}}$ to vanish at point b . This is indicated by the vector diagram in Figure A.2.

The state path o - a - b - c - d - f (depicted as the blue and red lines in Figure A.2) represents that of an interrupted creep test. After unloading to zero stress (b - c), the state point moves toward the origin along (c - d), thus indicating state recovery (q_2 decreasing). Upon abrupt reloading (d - f), the state point moves to f , whereupon primary creep occurs until the state point arrives once again at point b , a state of SS creep. Experimental results, for metals, suggest that the $\mathbf{G}$ vector must be small below the dotted line ℓ - n (i.e., the locus of SS points) because (1) the amount of strain recovery is quite small (i.e., $f(q_1, q_2)$ small) during the period of unloading and (2) the required length of time at zero stress for any noticeable softening (state recovery) is long; therefore, $g(q_1, q_2)$ must also be small.

Lastly, it was speculated (see Onat (Ref. 30)) that a dead zone in which $\mathbf{G} = 0$ surrounds the origin, with the actual shape being debatable (the GVIPS (generalized viscoplasticity with potential

structure) and Robinson models assume a continuous diagonal zone, whose width is associated with κ , the threshold stress). The existence of such a zone is supported by the fact that (1) no appreciable creep deformations are observable in the presence of very small stresses and (2) the recovery observed under zero load is never complete (i.e., that even for very long periods at zero load, the point d never reaches the origin). It has been shown (Refs. 72 and 73) experimentally that in titanium- and nickel-based superalloys, a distinct viscoelastic region does indeed exist at elevated temperatures and an elastic zone exists at lower temperatures.

Clearly, the state space representation enables one to picture (given two state variables) the essential phenomenological features of time-dependent behavior. Not all features are accounted for in Figure A.2, most notably dynamic recovery (Figure 2) because only the first quadrant was shown; however, by considering a full state space representation (see Figure A.1), one can envision the trajectory (path) that would involve dynamic

recovery. It also provides insight when developing an exploratory test matrix for a given material. Most importantly, the state space concept provides a convenient format for comparing the various features (i.e., functional forms) of classical as well as the more modern viscoplastic constitutive theories consisting of two state variables because the second state variable in Equations (33) and (34) was left unspecified. This is clearly seen by comparing the state space representation of the classic standard four-element solid model shown in Figure A.3 and that of a three-element model, in which no SS creep is allowed; see Figure A.4. Note in the four-element model, the relaxation lines curve the line O-B-C represent the locus of SS points, and the line O-D is the locus of maxima stress in relaxation. In the case of a three-element model, however, the lines of relaxation ($\mathbf{G}$ fields) are straight, and upon reaching the locus O-C-B relaxation would stop as $\mathbf{G} = 0$. This contrasts with the four-element model where relaxation would continue until the stress reaches zero, thus illustrating the reversibility of the model.

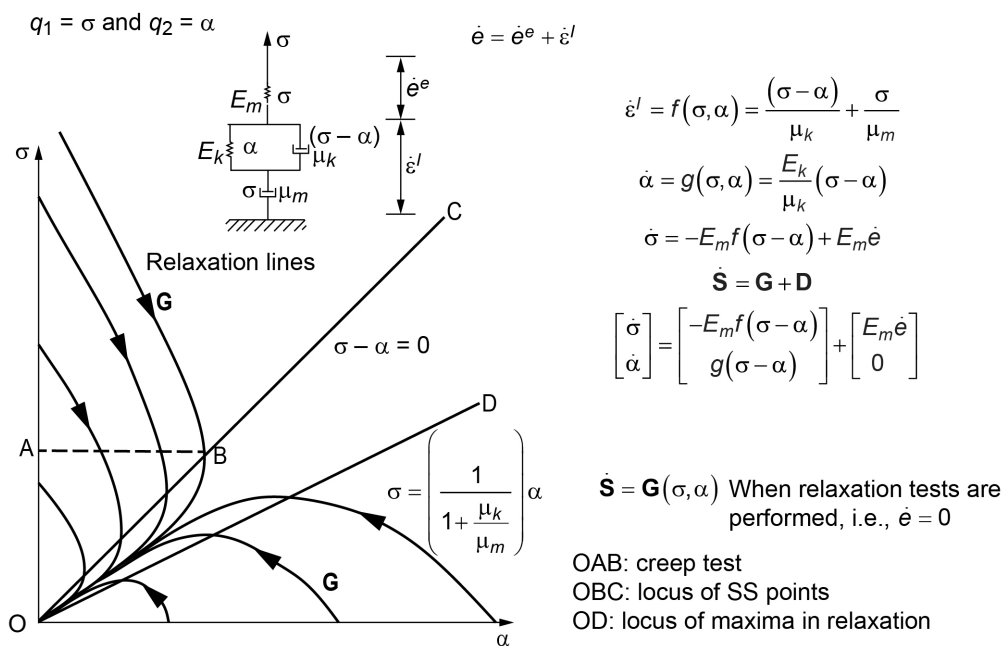


Figure A.3.—State space representation for four-element irreversible model.

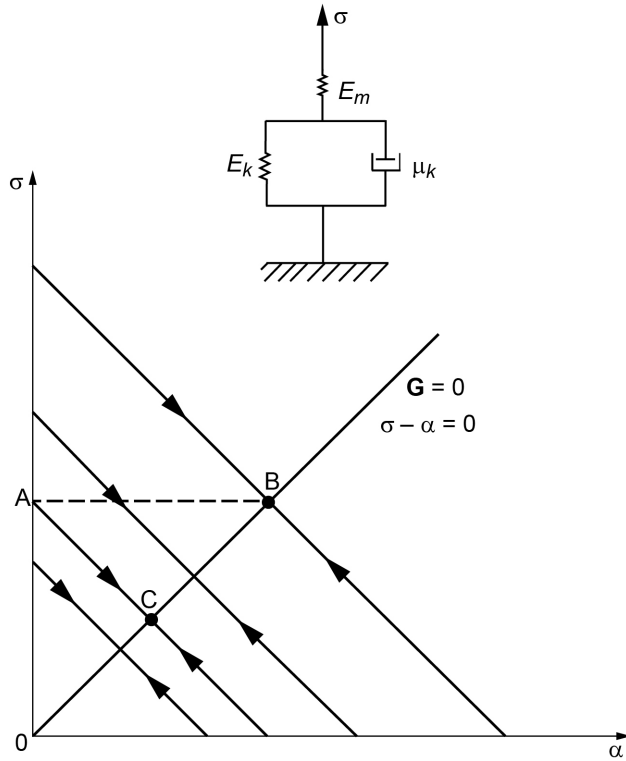


Figure A.4.—State space for three-element reversible model. Adapted from Reference 31.

Figure A.5 illustrates the creep process, including the stress history, a typical inelastic strain response and the state point path in the state space (A-B-C); see Figure A.5(b). Here, the loading is assumed to be fast enough to be virtually elastic (from the origin to point A). The stress is then held constant. The state point, at first (near A) moves rapidly toward B. The inelastic strain rate is also large but diminishes (i.e., primary creep) as the state point moves along the line A-C, and in time the state point eventually reaches C on the SS locus (line c-b) and thereafter remains stationary. The constant SS creep rate is then given by $f(\sigma, \alpha)$. Note the state point never leaves Region I under ordinary creep conditions. In Figure A.5 creep tests at four stress levels are shown with each creep path being higher up in the state space than the previous one.

Figure A.6 depicts constant-rate straining, as in a monotonic tensile test, and shows the strain-rate-dependent inelasticity predicted by the Robinson model (Ref. 71). State points for each constant strain rate are attracted to the corresponding points (P and Q) on the SS locus. Note for the state spaces shown in Figure A.6 to Figure A.10, the special case of when $\kappa = 0$ is shown; therefore, all G fields end and attractive lines start at the origin.

Figure A.7 shows that point P (on the SS locus) can be reached by several different paths, for example, path A constant

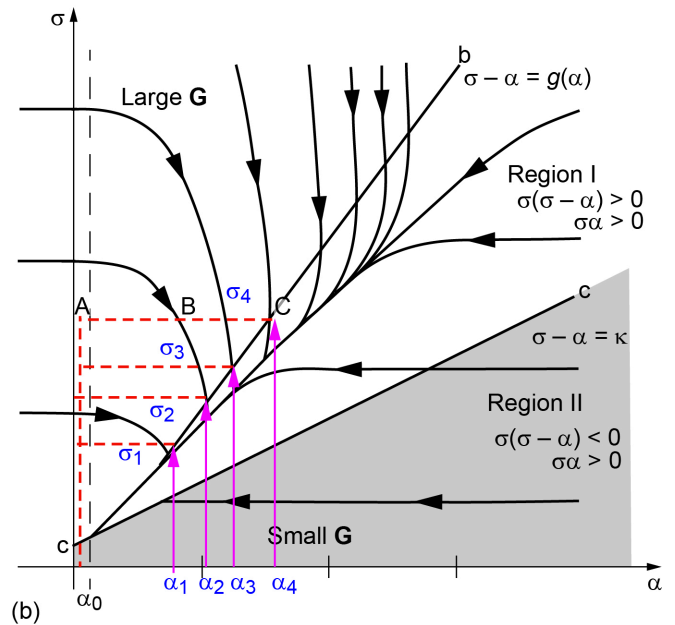
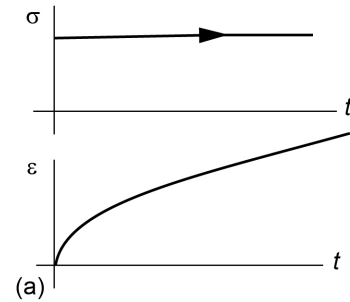


Figure A.5.—Creep response (indicated by the dashed red lines) within the state space. Note that $g(\alpha)$ is defined in Equation (A.4). (a) Stress-time and strain-time plots. (b) State space.

rate straining, by a creep path B and by stress relaxation, path C. Similarly, Figure A.8, Figure A.9, and Figure A.10 illustrate responses predicted correctly (in comparison to experiment) by the Robinson model (as well as GVIPS) but not predicted correctly by classical models given in Table I. Figure A.8 illustrates path dependency (O-B-C-D) of creep, showing that the primary creep rate produced by σ_2 (point C) would be significantly less than that produced initially by σ_2 without the prior history of σ_1 (O-B). Figure A.9 illustrates the stress recovery induced by a thermal recovery mechanism (e.g., thermal annihilation of dislocations) that takes place upon unloading. Note in Figure A.9 that no accompanying inelastic or viscoelastic strain recovery occurs with the change in state; however, upon reloading, reinitiation of primary creep will most likely happen. This state recovery was described before in Figure 34 but in the case of identification of a hesitation period when unloading from σ_1 to either σ_2 or σ_3 . The associated state space for this case is

repeated below in Figure 34(c), where $\Delta\tau_1$ and $\Delta\tau_2$ indicate the time periods from c to d and e to f, respectively, in Figure 34. Finally, Figure A.10 presents the often experimentally observed behavior of a complete reversal of the stress relaxation behavior, from classic decrease in stress with time to an actual increase in stress with time when a load path has been induced such that the state point is lower than the locus of SS (for nonzero α) yet higher than the line of maximum stress (i.e., $\sigma - \alpha = 0$).

In conclusion, the state space is an excellent framework to help a model developer understand both the capabilities of a given modeling framework and the required experimental features that a model must be able to exhibit graphically, thus enabling one to experiment with different material functional forms (i.e., $f(\sigma, \alpha)$ and $g(\sigma, \alpha)$) that provide the desired experimentally observed features. For additional insight into the use of the state space and detailed information regarding the utilization of the state space, the reader is referred to the works of Onat and Fardshisheh (Refs. 30 and 31).

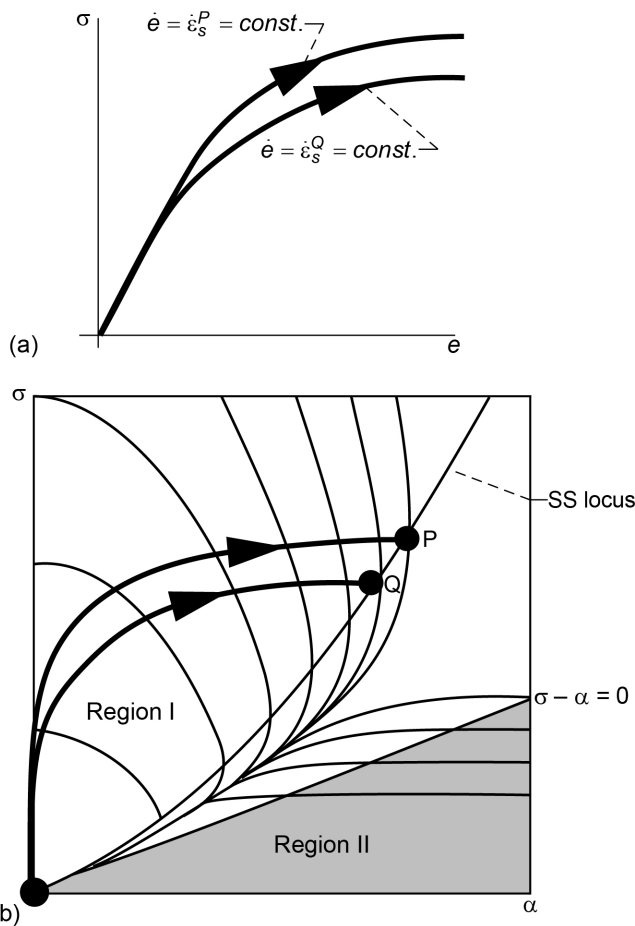


Figure A.6.—Constant-rate straining. (a) Stress-strain plot. (b) State space.

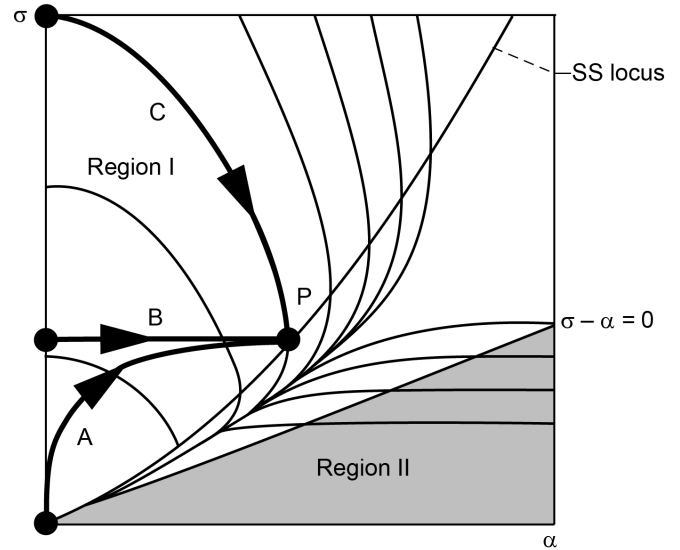


Figure A.7.—State point trajectories leading to Point P on the SS locus.

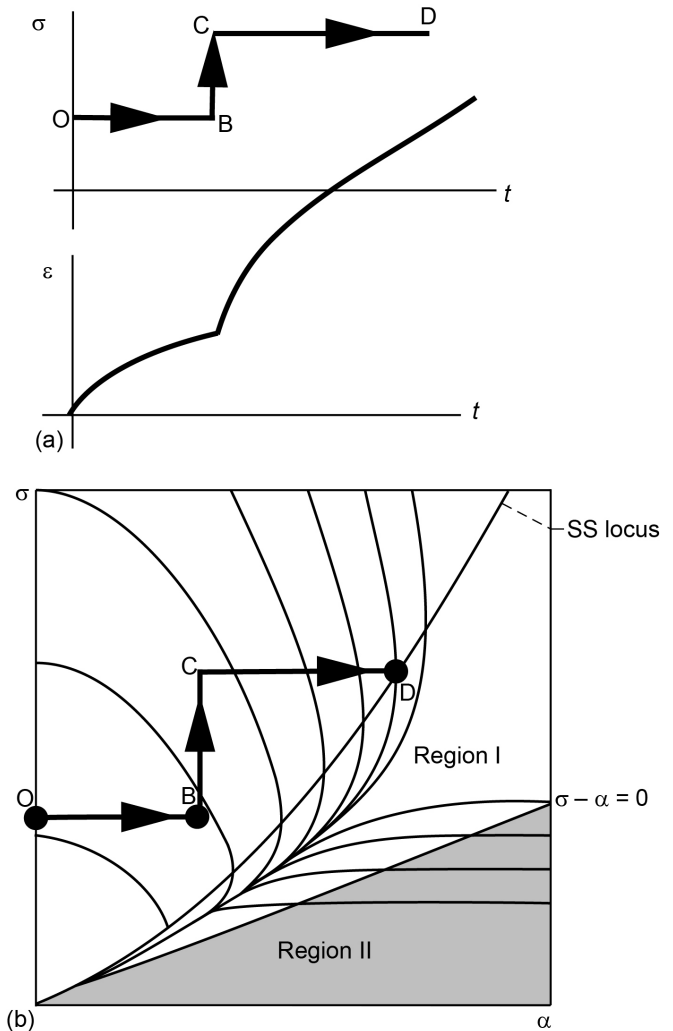


Figure A.8.—Response to stepwise variable stress test. (a) Stress-time and strain-time plots. (b) State space.

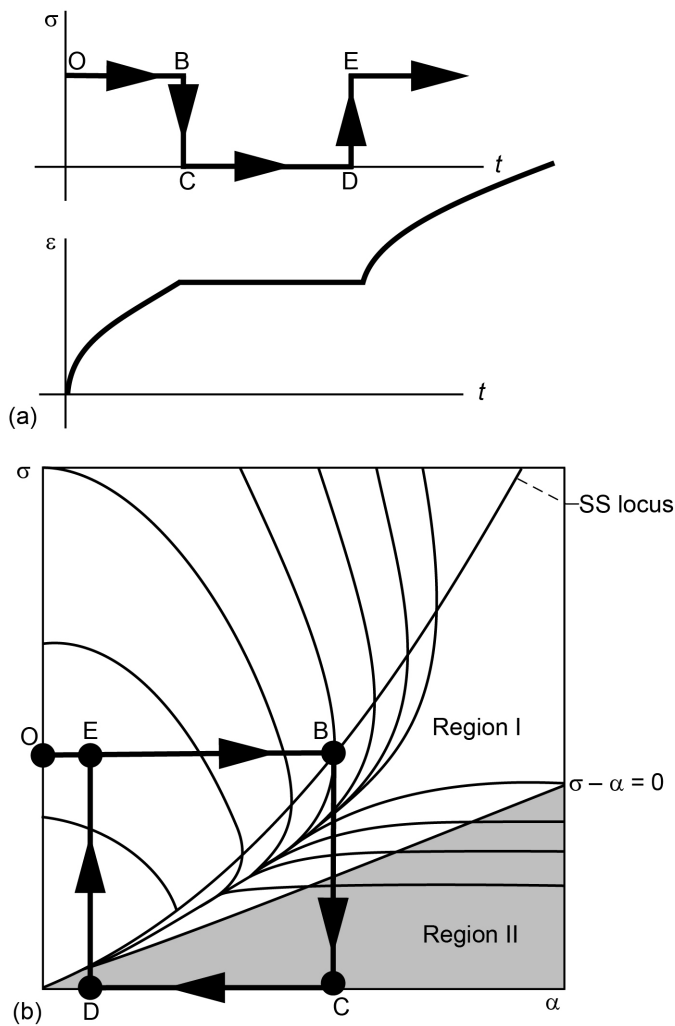


Figure A.9.—Response to interrupted creep test, showing state recovery. (a) Stress-time and strain-time plots. (b) State space.

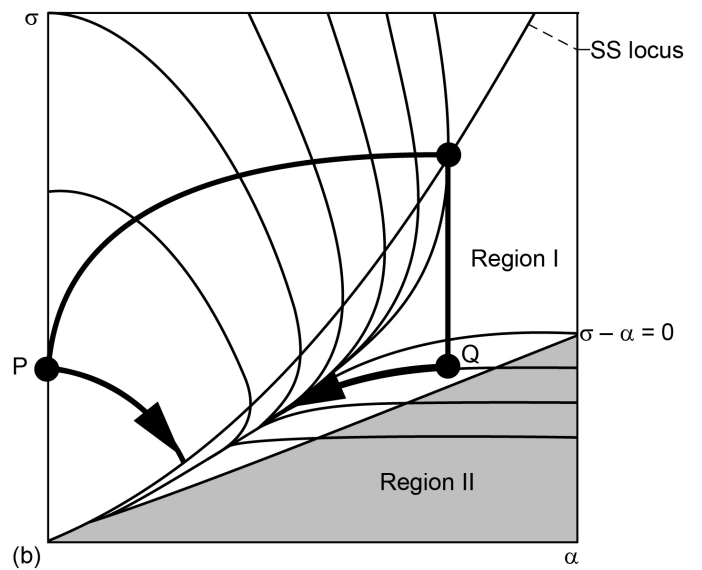
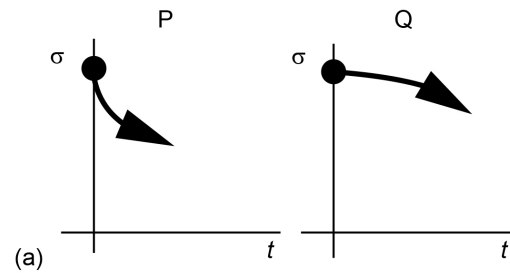


Figure A.10.—Qualitatively different relaxation responses from the same initial stress but different inelastic states (different state points P and Q). (a) Stress-time plots. (b) State space.

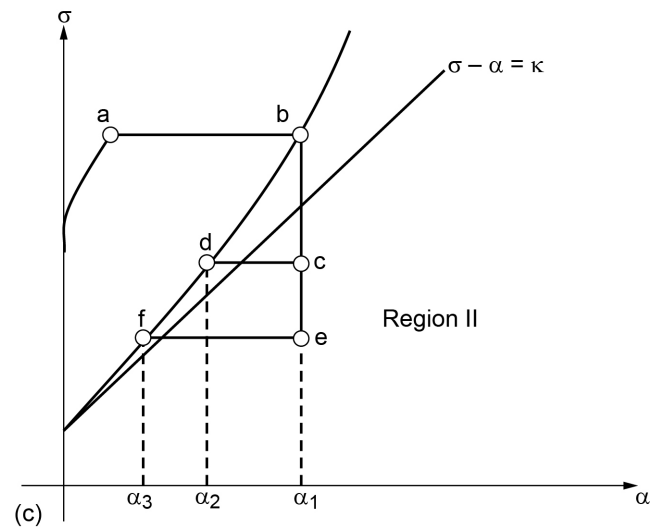


Figure 34(c).—State space response for the transient dip test in Figure 34.

Appendix B.—Information Management of Data

Informatics has been defined as the science concerned with gathering, manipulating, storing, retrieving, and classifying recorded information. A key aspect of informatics is its focus on understanding problems with which the information is associated and then applying information technology as needed to address those problems, highlighting the interconnection between modeling and data collection. Thus, it is important to describe the required informatic infrastructure necessary to enable knowledge capture, discovery, and dissemination of material and structural information throughout an organization, not only for use on current problems but also to maximize the future utility of the information.

Knowledge begins with data, yet data in and of itself has no meaning. Data only describes what is or what was. Information is data with meaning as it puts the data in context relative to other data. Knowledge is then information with intrinsic value, implications, or connections. Two types of knowledge have been defined (Ref. 74). The first is tacit (or hidden) knowledge that people carry with them. It comprises insight, intuition, skills, experience, and other knowledge that has not been formally shared. The second is explicit knowledge, which is known and defined as it has been captured in the form of reports, drawings, notes, etc. Tacit knowledge becomes explicit through sharing, thus the need to build an information infrastructure that cannot only capture, analyze, and disseminate explicit knowledge but also one that enables contextual linkage and association (most notably through the use of metadata) so as to enable the discovery of new knowledge as well. The existence of such a system (e.g., ANSYS GRANTA MI (Ref. 75); see Ref. 76 for description of associated schema) will then maximize the impact on both material and structural discipline practitioners and/or researchers. At the core of this system is the ability to capture the fundamental response data (with its full pedigree, chemistry, processing, heat treatment, testing information, etc.) and application potential (see Ref. 77) of a given material system (be it monolithic, composite, multifunctional, etc.) and not just specific predefined (generally accepted) values. As knowledge evolves, these and other yet unmined data can be processed and linked, thereby becoming information and ultimately knowledge.

This general data-information-knowledge pyramid is at the core of the material data lifecycle (see Figure B.1) proposed by the Material Data Management Consortium (MDMC),¹⁶ wherein data is captured and consolidated from external sources

and legacy databases, as well as internal (possibly proprietary) testing programs. Next, data is analyzed and integrated to create/discover useful information and then deployed (disseminated) to the people who need and use it. The continual maintenance of the whole system (the data and information generated as well as the relationships, or links, between them) is the last yet essential stage of the data lifecycle; see Reference 78 for an example of such a digital thread management tool. Note that the middle ring of Figure B.1 provides additional information regarding the type of data utilized and functions performed during each phase in the data lifecycle; the outermost ring details the individuals most likely responsible for these functions.

Supporting the various required activities throughout this data lifecycle requires the (preferably seamless) integration of a variety of software tools. These include data input, reduction/analysis, visualization, and reporting tools; material parameter estimation tools; product life management (PLM) tools; and structural analysis codes that utilize a central database. These tools should enable material and structural engineers to input, manage, and utilize information in as an efficient, reliable, and user-friendly way as possible. Finally, these tools should also enable enterprise-wide (even worldwide) solution or access.

At NASA Glenn Research Center, the material database schema illustrated (at a very high level) in Figure B.2 has been implemented (Ref. 76). Note that within each dotted area (i.e., collections) several associated lower-level data tables are grouped. For instance, within the Test data collection, one would find data tables for each category of test data (i.e., tensile, creep, relaxation, cyclic, crack growth, etc.) wherein each individual record would contain both the generally accepted pointwise values (such as E , σ , ν , G , etc.) and the complete response histories (functional date) for each individual test, along with images of failure surfaces and microstructures. Within the generic folders (not shown) within each table, one would find the corresponding consolidated (statistically rolled-up) temperature-dependent response of a given material for each test category of interest. Finally, after further consolidation and characterization of organization-specific algorithms and life modes, the corresponding material design allowables (e.g., stress, strain, elongation, etc.) would be stored for each material of interest within the model pedigree collection. Such information storage and linkage enables full traceability of each and every data item for both physics-based and data-driven models (Refs. 43 and 78 to 81).

¹⁶A group of aerospace and energy sector organizations (both industrial and governmental) that have joined forces to develop best practices and associated software tools to integrate material and structural information technology with the realities of practical product design and advanced research. This group was established

(through collaboration) with ASM International, NASA Glenn Research Center, and Granta Design Limited (now a part of ANSYS) in 2002 and is currently in its 20th year of operation. See <https://www.grantadesign.com/industry/collaborations/consortia/mdmc/>.

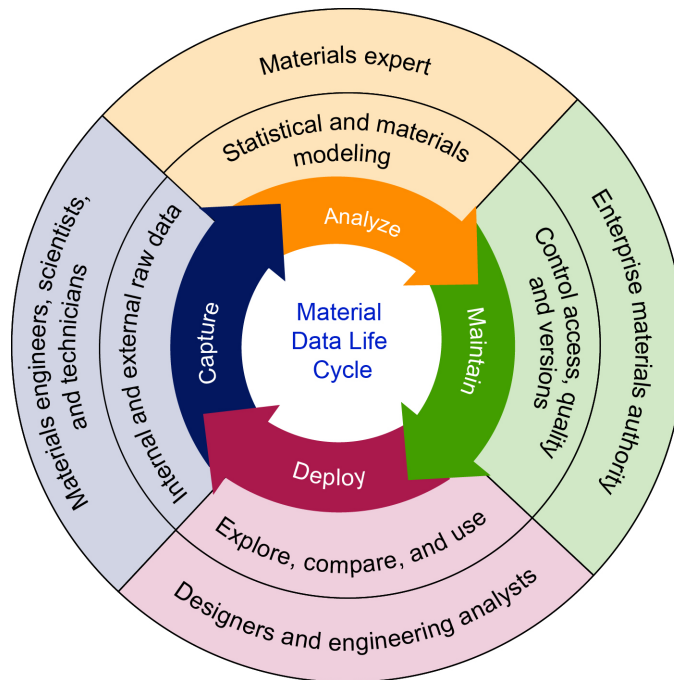


Figure B.1.—Four aspects of material data lifecycle.

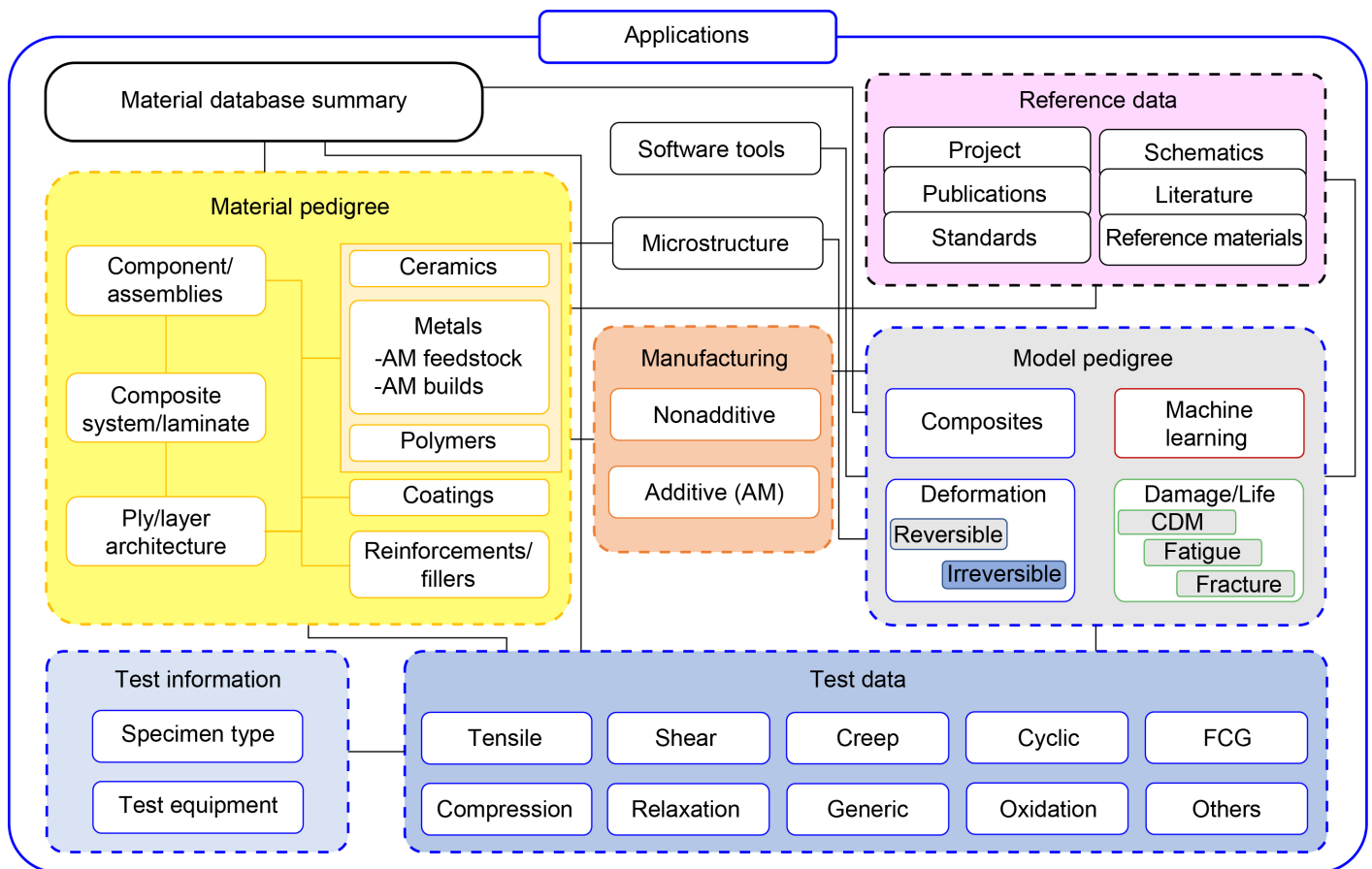


Figure B.2.—Possible data schema for material information (see Ref. 76).

The previously described informatic infrastructure, that is, the ANSYS Granta MI system, allows one to not be limited to the collection of material parameters and information, which merely satisfy the input requirements for conventional design and analysis methods alone. The type and extent of material information routinely collected today (in the form of material handbooks and other database systems) is not always sufficient to meet the current, much less future, needs of the material modeling community. Instead, we can now strive to collect (through exploratory, characterization, and validation testing) and capture to the largest extent possible the complete multiaxial response history (e.g., stress, strain, time, and temperature) for a given material. Clearly, a tradeoff exists between the cost of data acquisition, storage, maintenance, and dissemination and the current and future value of the data being collected. However, this trade is extremely difficult to make a priori as one often does not comprehend the importance of the data or information until after some time has elapsed and the window of opportunity has passed. Therefore, one should collect as much information as possible at the time to avoid an “if only I had ...” situation.

For example, in the case of isothermal tensile, relaxation, creep, and cyclic testing, the entire stress, total strain, and time history (both longitudinal and transverse if possible) should be captured (recorded) from the beginning to the end of the experiment, whereas in the case of thermomechanical fatigue (wherein temperature and mechanical load vary continuously), the fundamental response parameters stored at each time increment must be temperature, force, and displacement (instead of stress and strain). Storage of this fundamental data enables subsequent data postprocessing to occur, regardless of the model of choice. For instance, in the case of classic creep testing, typically creep strain (the strain at which the load was held constant) versus time is reported, as this is sufficient for characterizing a Norton-Bailey creep model (see Table I). However, unified viscoplastic models such as the GVIPS (generalized viscoplasticity with potential structure) model (see Table I) require total-strain-versus-time histories to properly characterize their associated parameters. This distinction is critical when load levels are sufficiently high to initiate inelastic straining during the loadup phase of the experiment because unified viscoplastic models (which inherently capture creep-plasticity interactions) need this information to accurately model the evolution of the internal (or back) stress within a given material. Clearly, this data requirement also impacts the

way a given test is conducted as well as instrumented and the level of effort involved in conducting the tests. For example, a classic creep test performed with a dead-load frame would be unable to provide meaningful measurements during the initial loadup region of the test. Similarly, tensile tests should be conducted under total strain rate control and not constant crosshead displacement control as the latter will allow orders of magnitude variation in strain rates within the gage length, depending upon the rate sensitivity of the material, temperature, and gradients if any, being applied.

With respect to the need to capture the multiaxial response of a material (note, even in the simplest case of uniaxially applied tension, the response is always multiaxial), it can be shown that just a 16 percent variation in Poisson’s ratio (e.g., 0.25 versus 0.3) along with a 10 percent variation of Young’s modulus in an isotropic material, can result in a 2, 24, and 14 percent variation in normal, off-diagonal, and shear material stiffness coefficients; see Reference 82 for additional example calculations. Clearly, because Poisson’s ratio is defined as the ratio of the strain transverse to the loading direction (22) versus that in the load (11) direction (e.g., $\nu = -e_{22}/e_{11}$) the need to accurately and consistently measure and record data from both directions is obvious (Ref. 14). It must be noted that to obtain such data experimentally requires a significant (at least an order of magnitude) increase in level of effort (setup, alignment, etc.) and resources (e.g., data collection system), particularly when testing at elevated temperatures. However, it is precisely this multiaxial information that will provide insight into the appropriateness of specific modeling idealizations and structural analyses; therefore, it is a necessary expense.

Finally, effective data management should reduce both current and future testing efforts if a more complex model is required. For example, if an organization has already conducted some of the preliminary tests (notable tensile and compressive loading) on a given batch of material, they may not need to replicate any of the data if these data are present in the information management system with sufficient pedigree and meta-data to enable confident assertion of the validity of the data, saving time and money. Also, information management tools can serve as both a great means for defining and tracking the progress of a test matrix and identifying outliers. Further, they can, in turn, maintain material inventory and identify out-of-calibration equipment, poor test setups, defective materials, and so forth, all of which will save significant time and resources.

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