On the effect of flow direction on impedance eduction: ONERA/NASA joint comparison

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Acoustic liners are typically mounted in aircraft inlets and aft-bypass ducts to absorb noise generated by the fan. The effect of flow direction on the liner behavior remains questionable despite numerous studies carried out in different laboratories. Resultingly, in support of a collaboration with the International Forum for Aviation Research (IFAR), the NASA Langley Liner Physics team has acquired a detailed database for use in the evaluation of flow-direction effects on the acoustic liner impedance eduction process. These data have been supplied to the IFAR partners for further investigations. The ONERA Liner team has investigated the effect of flow direction using these data and the ONERA impedance eduction technique. The eduction approach is based on the use of the linearized Euler equations, and thus natively takes into account a shear grazing flow. The main goal is to compare against results obtained previously by the NASA Langley team and evaluate the effect of modifying the effective Mach number and flow profile inside the eduction process. The results demonstrate that the flow profile choice is critical, as seemingly small variations lead to significant modifications of the educed impedance spectra.

Nomenclature

c_0	=	Speed of sound in air (m/s)
f	=	Frequency (Hz)
M	=	Grazing flow Mach number
M_{ave}	=	Grazing flow averaged Mach number
M_c	=	Grazing flow centerline Mach number
M_{SP}	=	Grazing flow setpoint Mach number
p	=	Acoustic pressure (Pa)
(U, V)	=	Axial and vertical mean velocities (m/s)
(u, v)	=	Axial and vertical acoustic velocities (m/s)
(x, y, z)	=	Axial, transverse and vertical coordinates (in)
j	=	Imaginary unit
$\mathcal{R}$	=	Acoustic reflection coefficient
ρ_0	=	Density of the mean flow (kg/m^3)
ω	=	Angular frequency (rad/s)
ζ	=	Normalized acoustic impedance
r	=	Normalized acoustic resistance (real part of ζ)
χ	=	Normalized acoustic reactance (imaginary part of ζ)

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I. Introduction

This paper presents the contributions of ONERA to Challenge #5 on acoustic liners conducted by the International Forum for Aviation Research (IFAR), which targets an improved understanding of flow direction effects on the acoustic impedance educed for a liner mounted in a grazing flow duct. This topic has been of great interest to the acoustic liner community. It remains unclear whether the liner impedance is independent of the flow direction. The impedance may be fully insensitive to flow direction while still responding to other changes in the aeroacoustic environment associated with flow reversal. Alternatively, the liner could possess a flow-direction–dependent impedance, regardless of other environmental changes. The purpose of IFAR Challenge #5 is to gain clarity on this issue.

In a previous work by Roncen et al. [1], impedance eduction results were examined using both numerical (synthetic) and experimental data for the NASA CT57 liner. These results were obtained by solving the linearized Euler equations (LEE), coupled with an impedance boundary condition. The objective was to investigate the influence of the shear grazing flow profile on the impedance eduction process for both upstream and downstream source locations, considering either a one-dimensional (1D) or two-dimensional (2D) flow profile. The study showed that the mismatch in the educed impedance between upstream and downstream source configurations could be partially attributed to wavenumber discrepancies, which were more pronounced when using an unadjusted 1D profile. Here, "unadjusted" refers to the measured 1D flow profile taken at the center of the duct (or vein), whose average differs from the actual mean flow in the full 3D bench. Additionally, for the 2D flow profile impedance eduction case, the uncertainty (expressed as credibility intervals) in the impedance was greater under upstream source conditions.

In Jones et al. [2], it was demonstrated that even small differences in Mach number (as little as 0.01) can lead to significant changes in the educed impedance spectrum, especially near the antiresonance. The authors suggested using a calibration liner (e.g., the CSQ3 liner) to determine an accurate average Mach number for impedance eduction.

To further investigate the matter, NASA recently shared with IFAR partners a database containing acoustic measurements and flow surveys at various conditions for four different liner samples. All measurements were performed by NASA Langley in the Grazing Flow Impedance Tube (GFIT) [2]. Impedance eduction results obtained by the NASA Langley Liner Physics team with their usual approaches (discussed later in this paper) are also included in the dataset. In doing this, IFAR partners can both educe the impedance using their own methods, while comparing to NASA impedance results.

The aim of the present paper is to use this detailed database as an input into ONERA's own impedance eduction process, including analysis of the flow profile, to further assess the influence of the Mach number. A description of the NASA setup and ONERA method is provided in Section II. Section III presents results obtained by ONERA. Section IV summarizes the key findings of this work.

II. Setup and method

A. NASA GFIT setup

The GFIT test bench (Figures 1 and 2) is used at NASA Langley to measure the acoustic characteristics of liners. This 2.0"×2.5" waveguide contains upstream and downstream source sections each consisting of 12 acoustic drivers. The surface of the test liner forms a portion of the upper wall of the GFIT, and 95 microphones are flush-mounted in the walls of the test window. Near-anechoic terminating diffusers are included at each end of the duct to control reflections and reduce overall flow noise. Among those 95 microphones, 53 are placed on the lower wall of the test section directly opposite the liner and are used for impedance eduction. To generate desired flow conditions, pressurized and heated air is supplied to the inlet of the GFIT, while a vacuum system is employed at the duct exit to evacuate air from the tube. A pitot-static probe mounted near the downstream termination of the GFIT is used to set the desired centerline Mach number M_{SP} (labeled setpoint Mach number). It has been observed that, since the probe is located far downstream, the true centerline Mach above the liner (denoted M_c) slightly differs from the setpoint value (see previous papers such as [2] for more discussion on this topic).

Two acoustic source levels (120 dB and 140 dB) and three values of M_{SP} (0.1, 0.2 and 0.3) are used, with the acoustic source placed sequentially upstream and downstream in each case. The acoustic excitation signal is a swept sine ranging from 400 to 3000 Hz, in 10 Hz increments [3].

B. Liner samples

The focus of the current paper is on a single liner sample selected among the four included in the dataset. This liner, labeled CSQ3, is a 16"-long liner containing a 19×304 array of 0.05"×0.05"×3.0" chambers. (Fig. 3). Due to the small cross section of each chamber, no facesheet is needed to provide sufficient resistance. With a chamber length-to-diameter ratio of 60 and a porosity of around 22%, this liner is expected to be relatively insensitive to changes in source sound pressure level (SPL) and grazing flow velocity compared to standard liners with a perforated facesheet.

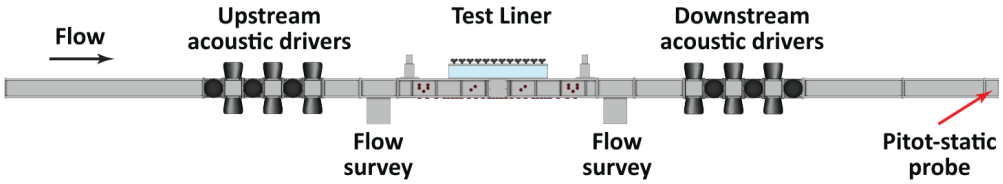


Fig. 1 Sketch of the NASA Grazing Flow Impedance Tube (GFIT).

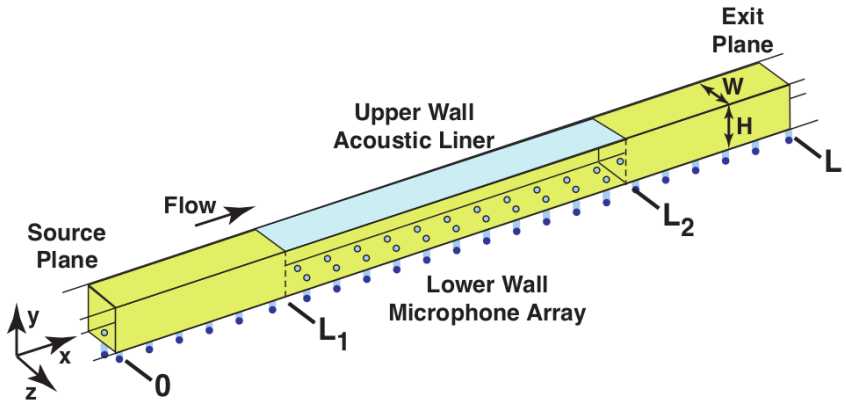


Fig. 2 Sketch of the GFIT test section.

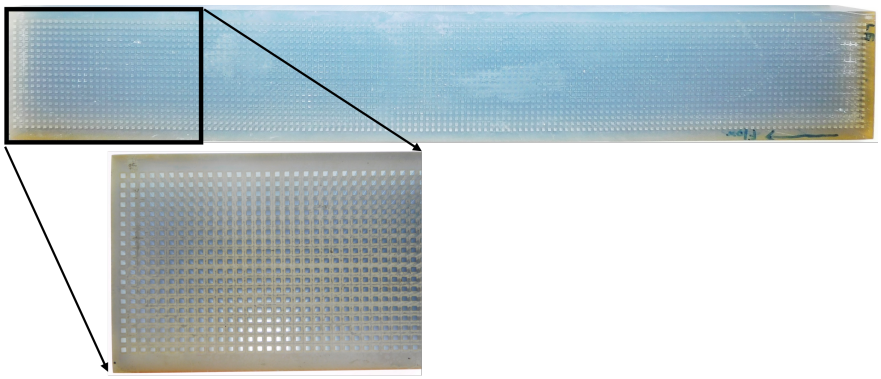


Fig. 3 CSQ3 liner sample (19×304 array).

C. Linearized Euler equations approach

To make the numerical simulations tractable, a simplified 2D case is considered. Figure 4 shows the generic computational domain used for eduction for the upstream source condition. Since the grazing flow always travels along the positive x direction, the domain is the same for upstream and downstream excitations, and the only necessary modification for switching the source position is to switch the inlet and outlet boundary conditions accordingly.

The total length (following the x axis) of the mesh is equal to 40" to accurately represent the complete GFIT test section. The leading edge of the liner respective to flow (labeled L_1 in Figure 2) is set at $x_{LE} = 8.2$". Since the liner is 16"-long, the trailing edge (labeled L_2 in Figure 2) is then set at $x_{TE} = 24.2$". To fit with the GFIT coordinate system as presented in Figure 2, we consider $x_0 = 0$" and $x_M = 40$".

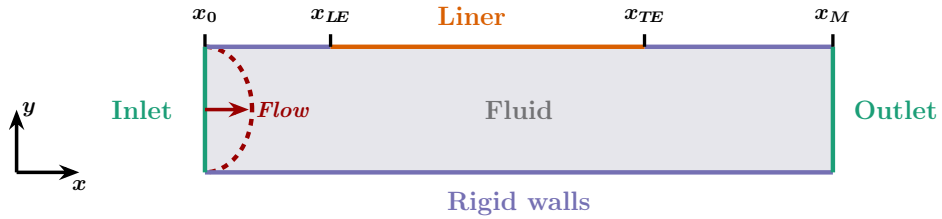


Fig. 4 Computational domain used for the eduction, upstream source position.

Due to numerical limitations, a 2D flow configuration is used in the simulations performed at ONERA. Although the actual flow is 3D, the lined section spans the entire duct width, which partly justifies the simplification. A goal of this study is to examine the influence of the flow profile used: either taken directly from a centerline measurement or adjusted such that its spatial average matches the experimental flow rate. Obtaining the numerical acoustic fields necessary for the impedance eduction strategy requires solving the equations of acoustic propagation in the presence of a grazing flow. This is done using the 2D harmonic linearized Euler equations (LEE). Written in non-conservative form with an $e^{j\omega t}$ time dependence and assuming homentropic flow, the LEEs are given in the frequency domain by [4]:

$$\mathbf{L}\boldsymbol{\varphi} = \boldsymbol{S}, \tag{1}$$

with $\boldsymbol{S}$ the source term used to impose pressure and velocity at the inlet, and where

$$\mathbf{L}\boldsymbol{\varphi} = \mathrm{j}\omega\boldsymbol{\varphi} + \boldsymbol{A}_x\frac{\partial\boldsymbol{\varphi}}{\partial x} + \boldsymbol{A}_y\frac{\partial\boldsymbol{\varphi}}{\partial y} + \boldsymbol{B}\boldsymbol{\varphi}, \tag{2}$$

and

$$\boldsymbol{A}_x = \begin{pmatrix} U & 0 & c_0 \\ 0 & U & 0 \\ c_0 & 0 & U \end{pmatrix},\ \boldsymbol{A}_y = \begin{pmatrix} V & 0 & 0 \\ 0 & V & c_0 \\ 0 & c_0 & V \end{pmatrix},\ \boldsymbol{B} = \begin{pmatrix} \partial_x U & \partial_y U & 0 \\ \partial_x V & -\partial_x U & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Components of vector $\boldsymbol{\varphi} = \left(u, v, \frac{p}{\rho_0 c_0}\right)^{\mathsf{T}}$ represent the acoustic perturbations around the sheared mean flow of axial and transverse velocities U and V, respectively. Due to the homentropy condition, the energy equation is replaced by the state equation $p = c_0^2\rho_0$.

As we consider a shear mean flow profile, the impedance boundary condition on the lined wall is the standard one,

$$p = \pm\rho_0 c_0 \zeta v, \tag{3}$$

where the sign is chosen depending on whether the upper or lower wall is considered.

A Discontinuous Galerkin (DG) scheme is chosen to spatially discretize Eq. 1 and the associated boundary conditions. In this method, discontinuities are allowed at the interface between two elements. An example of how such a method can handle discontinuities at hard-soft wall interfaces is found in [5, 6]. To ensure the connection between elements and proper application of the boundary conditions, a numerical flux is defined. An upwind numerical flux is used to ensure connection between interior cells, while a centered flux is chosen at the boundaries. The resulting sparse linear system is then solved using linear algebra tools (see previous papers [4, 7, 8] for more details).

The normalized impedance ζ of the liner sample is determined with an inverse eduction method based on the 2D harmonic LEE, and repeated at each frequency of interest. An assumed impedance is used to determine acoustic pressures at the GFIT microphone locations. These pressures are then compared to the experimental results in an iterative scheme until the error between the two is minimized. This optimization process is usually carried out using the reflection coefficient $\mathcal{R} = (\zeta - 1)/(\zeta + 1)$ instead of the impedance in the boundary condition 3. This avoids singularities, as this coefficient is properly bounded on the complex unit disk.

A numerical operator G_{LEE} that computes from any impedance the projection of the numerical field onto the observation space is created. By minimizing the numerical and experimental mismatch, the "true" impedance is educed.

The process is summarized in the equation below:

$$\zeta = \underset{X}{\operatorname{argmin}} \left(\left\| G_{\mathrm{LEE}}(X) \; - \boldsymbol{\varphi}_{\mathrm{meas}} \right\|_2 \right), \tag{4}$$

where X is an optimization variable. Note that the operator G_{LEE} depends on the flow profile and frequency.

In the present study, the observation space (denoted Ω) corresponds to the microphone locations, and the relevant observed field is the acoustic pressure. The eduction process is thus defined as the minimization of the following objective (or cost) function:

$$J_{\mathrm{cost}} = \sum_{\Omega} |p_{\mathrm{LEE}} - p_{\mathrm{meas}}|^2, \tag{5}$$

where p_{LEE} and p_{meas} denote respectively the numerical and experimental acoustic pressure fields.

The incident acoustic source coefficient and outlet impedance are generally taken as additional variables in the minimization above [4], but these have been removed from the explanation for simplicity.

D. Acoustic pressure measurements

As said previously, 53 microphones placed on the lower wall are typically used for impedance eduction at NASA. In the current study, ONERA selected the acoustic pressures from these 53 microphones as inputs for the eduction process described above. More details on the measurement procedure can be found in [2].

In order to evaluate the values of the source coefficient and outlet impedance (both complex numbers), which are necessary for conducting the eduction with the LEE method, a wavesorting procedure is applied. This procedure is usually performed following the two-microphone method [9]. Two microphone pairs (one on each side of the lined section) were selected among the 53 on the lower wall. Their (x, z) coordinates are listed in Table 1: the x positions are symmetric around the lined section and the z positions are chosen to have microphones placed as close as possible to the duct's centerline. According to [10], the distance between the two microphones in each pair is suitable to minimize errors in the relevant frequency range (400 – 2500 Hz). It is important to note that the wavesorting method employed here requires highly precise calibration of each microphone or a switching procedure to eliminate intrinsic phase errors. Since all the microphones used on the GFIT are not phase matched to each other and no switching procedure is applied, the resulting source coefficients and outlet impedances are strongly dependent on the initial calibration accuracy. To mitigate this issue, only the amplitudes of the computed coefficients are used as inputs for the eduction, while the complex phases are optimized simultaneously with the liner impedance.

Table 1 Microphones used for the wavesorting procedure.

Mic ID	x (in)	z (in)
Upstream #1	0	1.1
Upstream #2	2	1.1
Downstream #1	38	1.1
Downstream #2	40	1.1

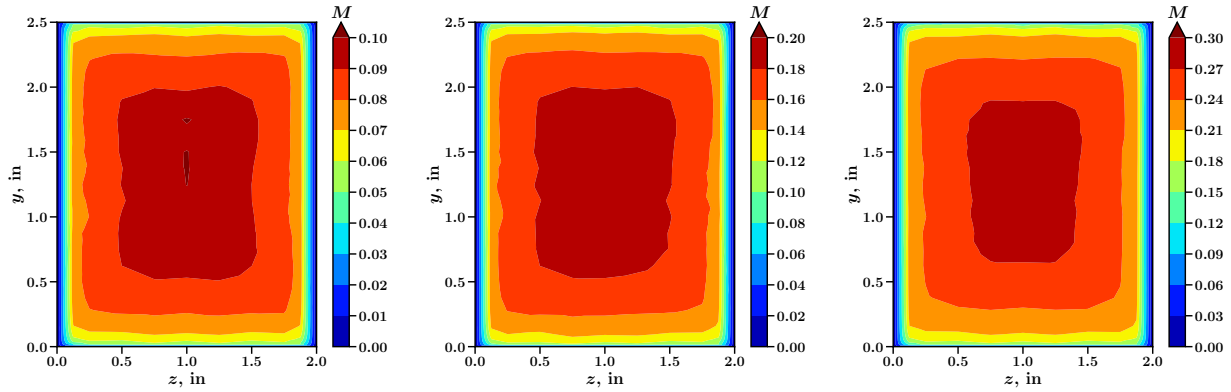
E. Flow profiles

Flow surveys were performed upstream and downstream of the lined section for each setpoint Mach number using a pitot probe. These surveys allowed detailed 2D flow profiles to be obtained and included in the shared dataset. Examples of such flow profiles, taken upstream of the test section for each of the three setpoint Mach numbers, are shown in Figure 5.

ONERA used these data as input for the impedance eduction. Indeed, the 2D LEE method requires a 1D flow profile as input (no variation of the flow in the propagating x direction is considered). Two sheared profiles were considered in this study. The first, labeled "1D-based shear profile", is based on flow surveys at z = 1", i.e., in the middle of the duct. The second one, labeled "2D-based shear profile", is based on the z-averaged profile included in the shared dataset. This profile was computed by the NASA Langley team using the full 2D flow surveys, averaged in the spanwise (z) direction, and is thus deemed more representative of the true 3D flow profile. The LEE process needs the flow profile to be a continuous function of y. Since they are computed using flow survey data, the shear profiles are initially discrete: to make them suitable for the eduction process, functions are created by interpolating the initial survey points using a classic spline fit. Values of y that correspond to top and bottom walls are forced to zero to ensure proper boundary conditions. Both fitted profiles are then used directly as input for the impedance eduction process.

Figure 6 shows a comparison between these two profiles for each selected setpoint Mach number. Due to the way it is computed, the 2D fit profile is closer to the original averaged Mach number used by NASA in their impedance eductions (labeled M_{ave}, dashed lines). The 1D fit is closer to the setpoint centerline Mach M_{SP}, but may not be a good representation of the near-wall behavior. The 2D fit is expected to give results that are closer to the NASA results, where the 2D-averaged Mach number was used to evaluate the wavenumbers that serve as input for the direct eduction approach [2].

It is important to note that the centerline Mach number observed on both flow profiles in Figure 6 is lower than the expected setpoint, particularly at higher flow speeds. This known behavior is mainly due to variations of the flow profile along the full length of the duct, as M_{SP} is set using only one probe in the downstream section. In the method used by the NASA team, these variations are partly taken into account by averaging the Mach values obtained in the upstream and downstream flow survey planes, and considering linear variations to compute an averaged Mach number over the liner [2]. In the present paper, these variations of the shear profiles along the x-direction are neglected and we consider that the upstream flow profile is representative over the whole duct length.



(a) M_{SP} = 0.1 **(b) M_{SP} = 0.2** **(c) M_{SP} = 0.3**

Fig. 5 Full 2D flow profiles, upstream of the liner, computed from the surveys at various M_{SP} values.

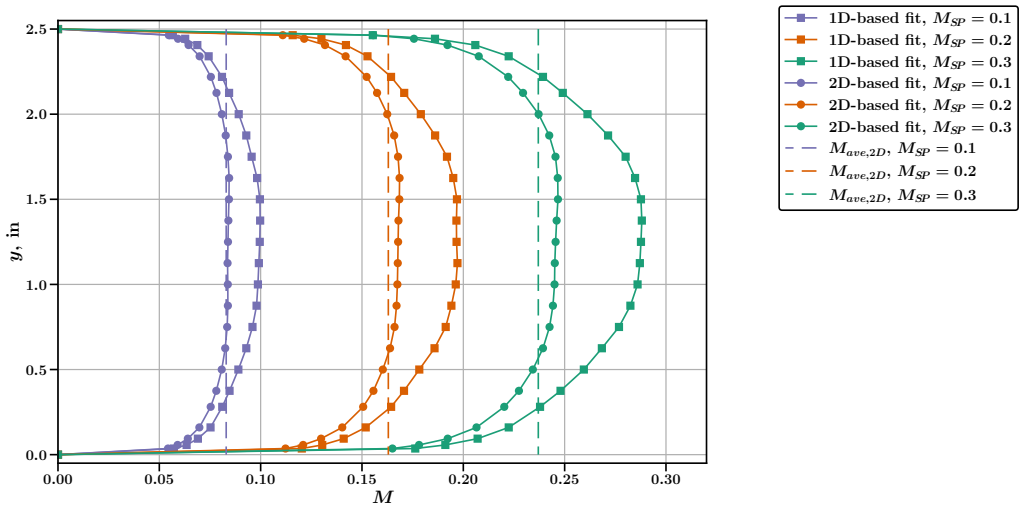


Fig. 6 1D shear flow profiles used for eduction. For each curve, markers represent survey points. The 2D-averaged Mach numbers originally used by NASA are plotted in dashed lines.

III. Results

A. Initial comparison without flow

Data from the GFIT test bench and NASA experiments were initially processed at NASA using two impedance eduction methods. The first approach is Prony's method [11], and the second is based on the Convected Helmholtz Equation (CHE) [12]. The eduction results were included in the dataset shared to the partners. In the present paper, only the results using Prony's method will be presented. In Prony's method, the liner is assumed to be of infinite length, and the sound field is considered unidirectional. The axial wavenumber is determined from acoustic pressure measurements using wall-mounted microphones opposite the uniform-impedance liner. This wavenumber is then input into Myers' impedance boundary condition [13] to compute the test liner's impedance, which is subsequently filtered to ensure smooth frequency-dependent behavior.

Figures 7 and 8 show the eduction results with Prony's method obtained at NASA compared to the results obtained using the LEE method, with no grazing flow ($M_{\mathrm{SP}} = 0$). The NASA and ONERA approaches give similar results when no grazing flow is considered, in both source configurations. The educed impedance values do not seem to be affected by the incident acoustic level, no matter the eduction method used. This is an expected result, since the CSQ3 liner should be pretty insensitive to SPL by design. The slight discrepancy in the antiresonant frequency is partly attributed to higher uncertainties in that frequency band, since the liner becomes very reflective.

B. Influence of the flow profile

The goal of this subsection is to process the cases with a grazing flow using the LEE approach, and compare its results with the ones obtained using Prony's method.

Figures 9 and 10 show the eduction results with Prony's method obtained at NASA, for setpoint Mach numbers of 0 and 0.2, and two SPL values: 120 dB and 140 dB.

Test conditions show minimal sensitivity around the resonance frequency (1000 Hz), as expected. However, as the frequency approaches the antiresonance region, near 2000 Hz where the liner is particularly reflective, resistance values vary significantly, especially in the upstream case where high values are observed. Additionally, a frequency shift between upstream and downstream conditions appears at Mach 0.2, with antiresonance occurring at a lower frequency in the downstream configuration. It is also worth noting that SPL has a negligible influence on these observations. These impedance discrepancies at the antiresonance frequency were also reported in [1] for the CT57 liner, under both upstream and downstream configurations at Mach 0.3, when using 2D flow profile-based impedance eduction.

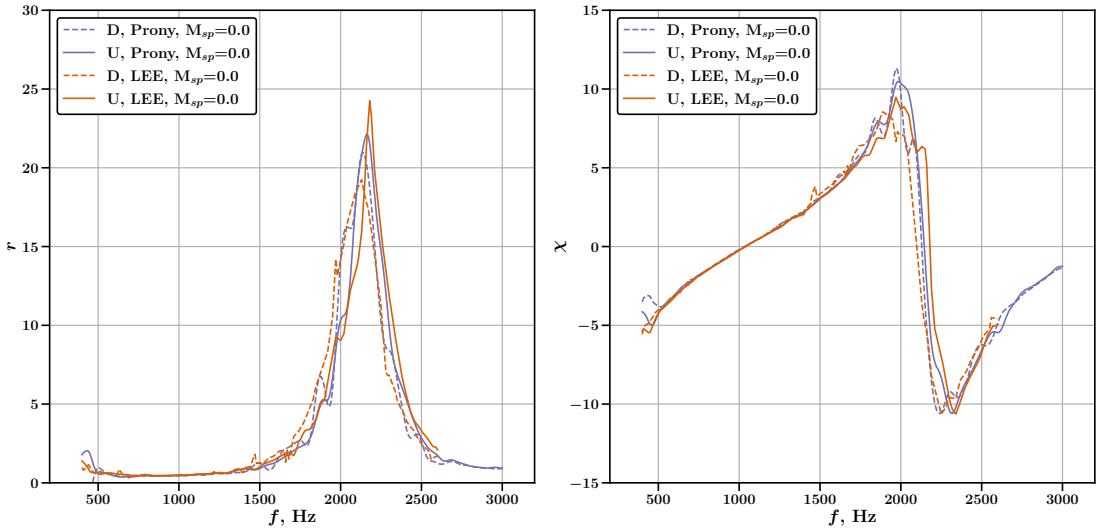


Fig. 7 Educed resistance (r) and reactance (χ) spectra for upstream (U) and downstream (D) sources; comparison between Prony and LEE approaches, case without flow, 120 dB source, CSQ3 liner

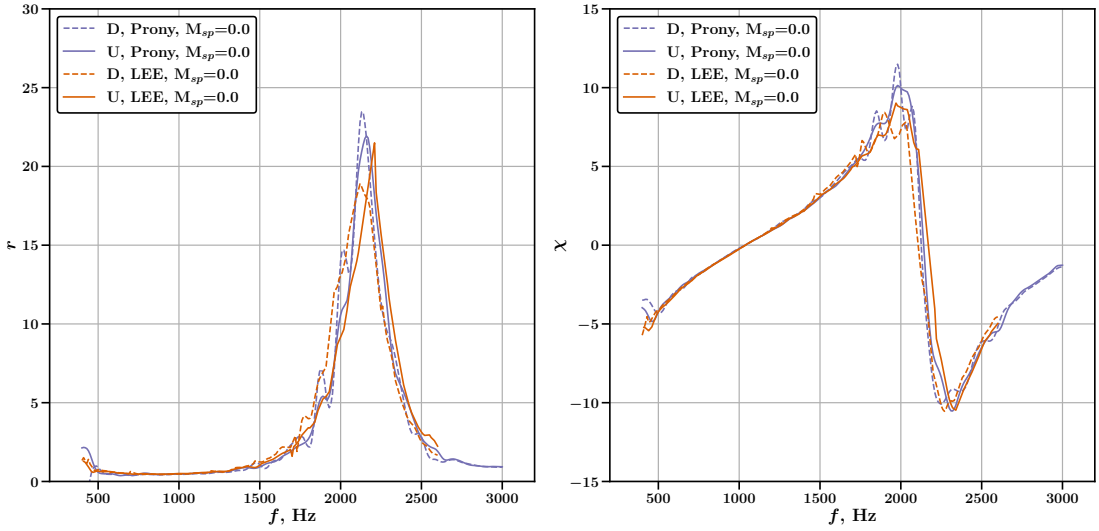


Fig. 8 Educed resistance (r) and reactance (χ) spectra for upstream (U) and downstream (D) sources; comparison between Prony and LEE approaches, case without flow, 140 dB source, CSQ3 liner

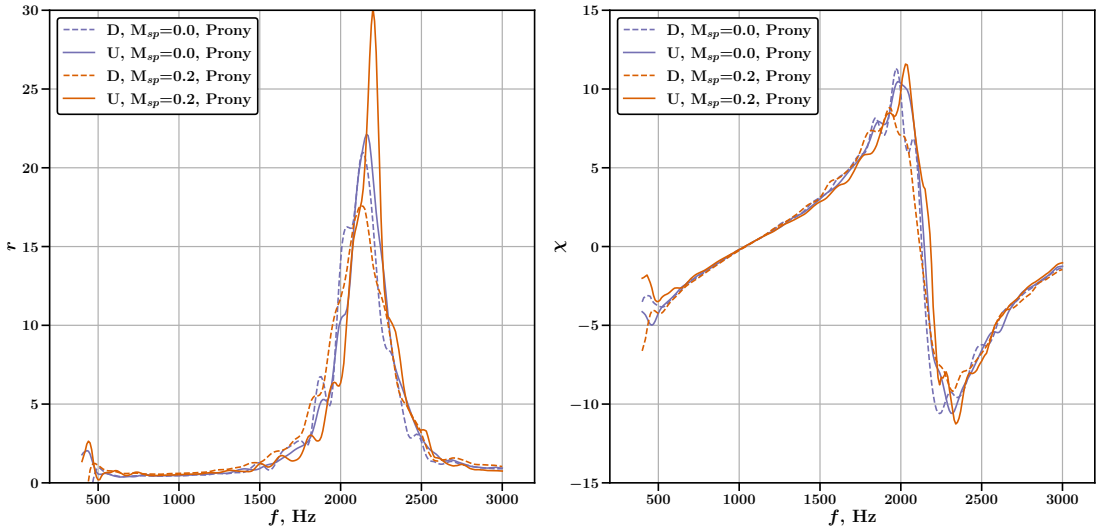


Fig. 9 Prony educed resistance (r) and reactance (χ) spectra for upstream (U) and downstream (D) sources for setpoint Mach numbers 0 and 0.2; 120 dB source, CSQ3 liner

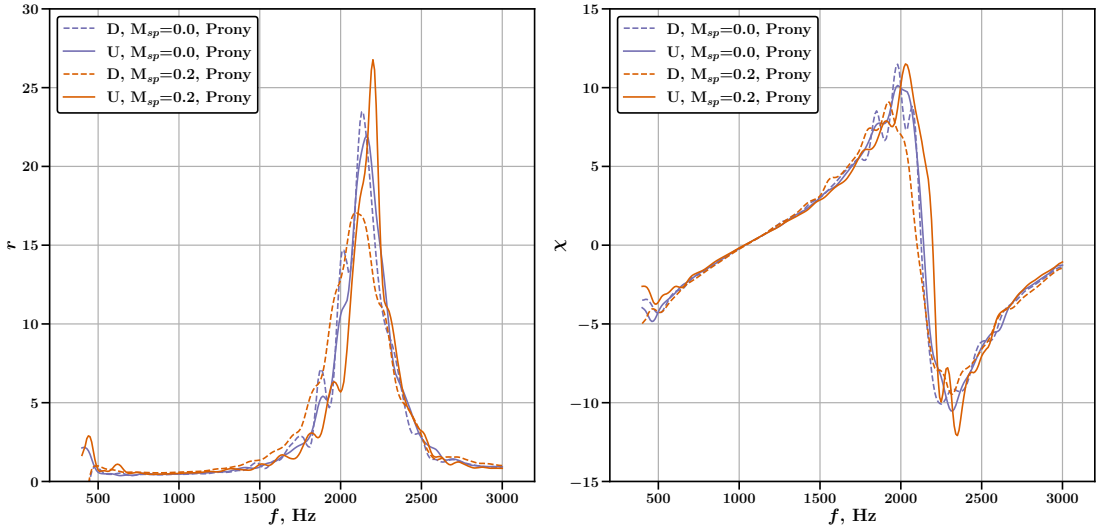


Fig. 10 Prony educed resistance (r) and reactance (χ) spectra for upstream (U) and downstream (D) sources for setpoint Mach numbers 0 and 0.2; 140 dB source, CSQ3 liner

The following figures (Figures 11 to 14) relate to the same experimental configurations processed by ONERA using the LEE and fitted shear flow profiles. Test conditions show minimal sensitivity around the resonance frequency (1000 Hz), as expected. With the 1D-based shear profile, the antiresonance shifts notably towards the higher frequencies in the downstream configuration, and towards the lower frequencies in the upstream configurations. This behavior is less notable when using the 2D-based shear profile: in this case, both antiresonances shift towards lower frequencies, but the discrepancy between the two peaks is reduced.

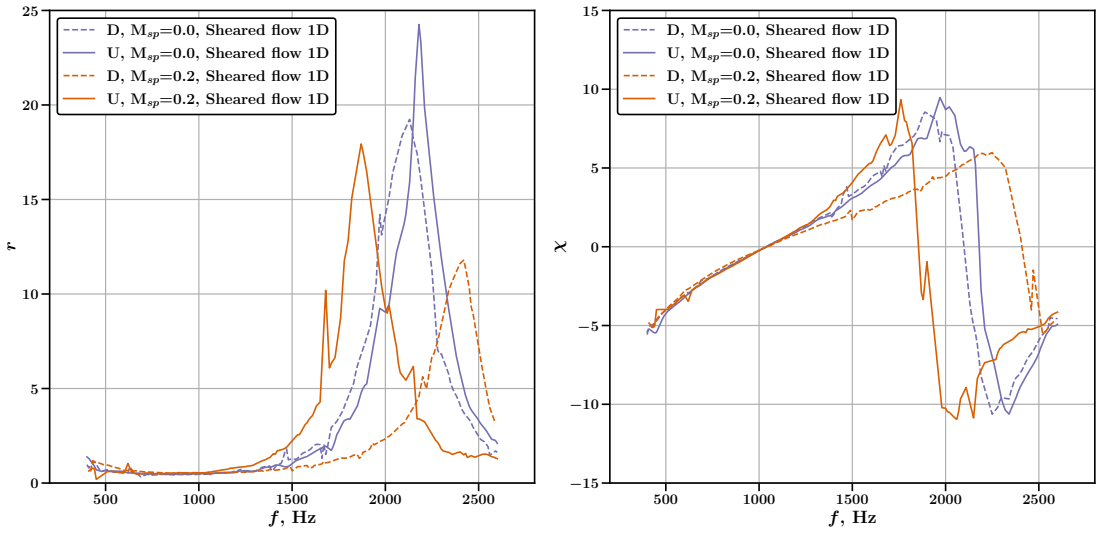


Fig. 11 LEE educed resistance (r) and reactance (χ) spectra for upstream (U) and downstream (D) sources for setpoint Mach numbers 0 and 0.2; 120 dB source, CSQ3 liner, 1D-based flow profile

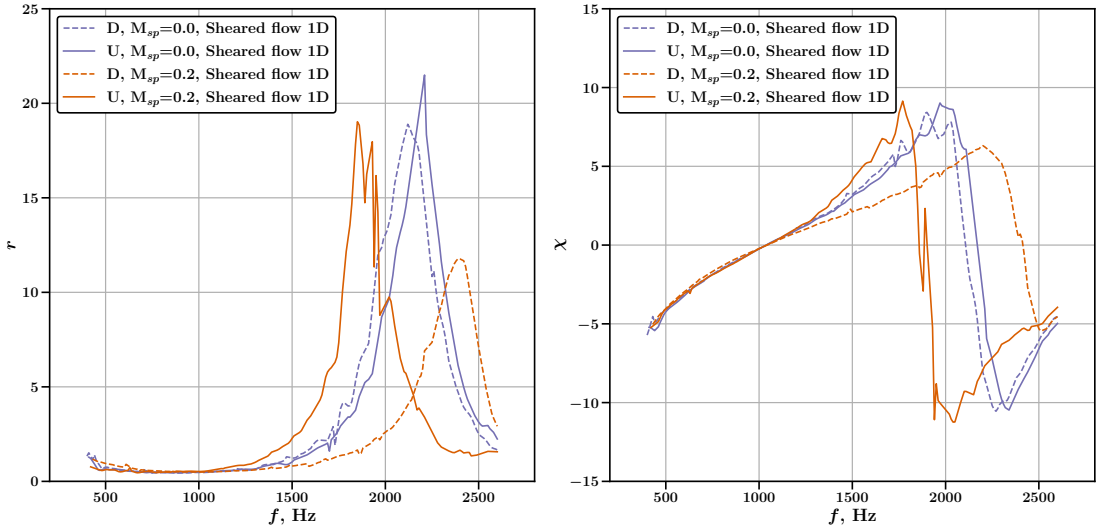


Fig. 12 LEE educed resistance (r) and reactance (χ) spectra for upstream (U) and downstream (D) sources for setpoint Mach numbers 0 and 0.2; 140 dB source, CSQ3 liner, 1D-based flow profile

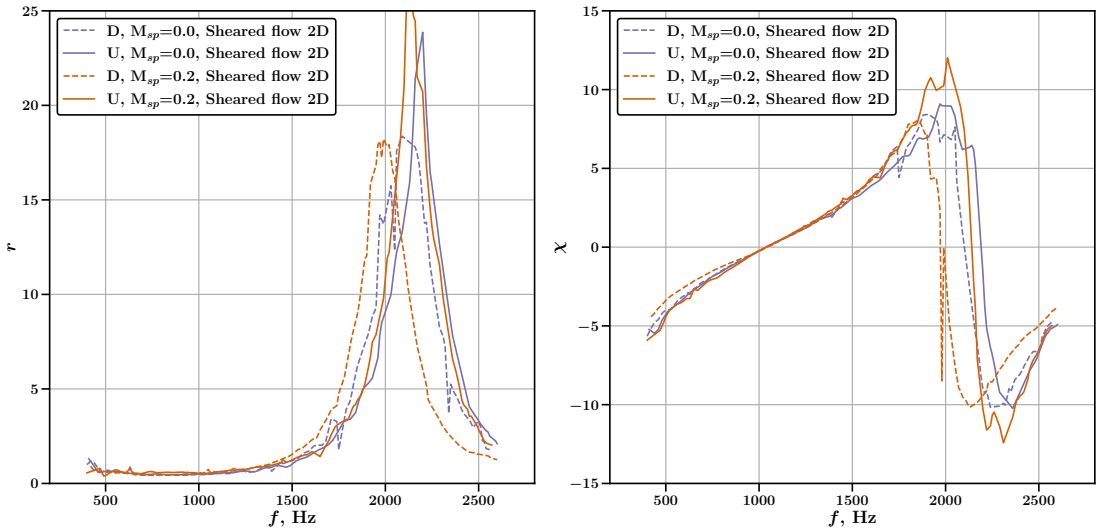


Fig. 13 LEE educed resistance (r) and reactance (χ) spectra for upstream (U) and downstream (D) sources for setpoint Mach numbers 0 and 0.2; 120 dB source, CSQ3 liner, 2D-based flow profile

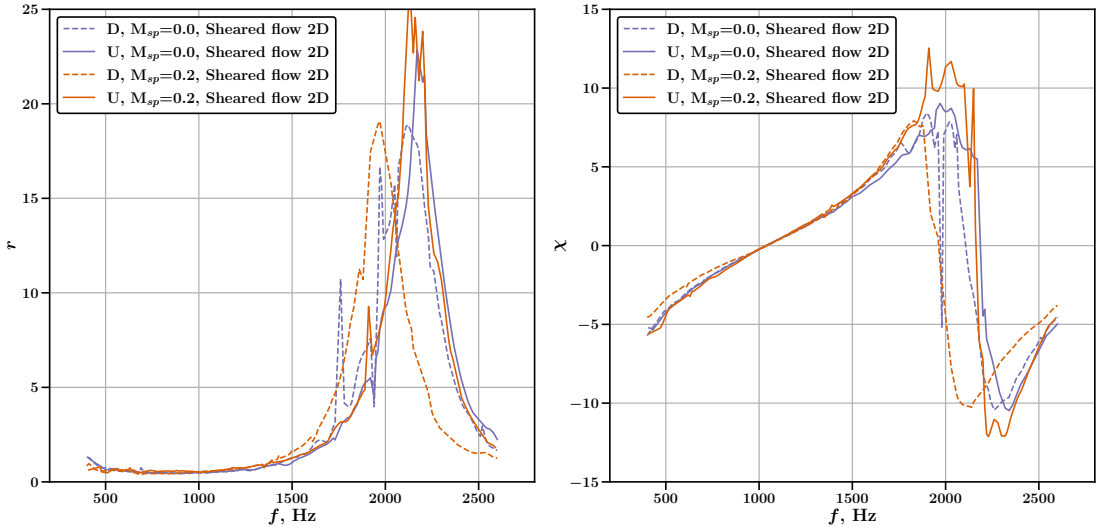


Fig. 14 LEE educed resistance (r) and reactance (χ) spectra for upstream (U) and downstream (D) sources for setpoint Mach numbers 0 and 0.2; 140 dB source, CSQ3 liner, 2D-based flow profile

A comparison between the different eduction methods illustrating the dispersion around the antiresonance that appears when considering a shear flow profile in the LEE approach is shown in Figure 15), for a setpoint Mach number of 0.2. The LEE method and the Prony method overlap very well in upstream source condition when a 2D-based profile is used. In downstream conditions, the agreement is still correct, but discrepancies around the antiresonance appear. On the other hand, the results obtained with the 1D-based profile are far from the initial results: using the 2D-based profile for the LEE method clearly causes the upstream and downstream antiresonance peaks to coalesce better than using the 1D-based profile. This is probably due to the modification of averaged Mach number linked with the profile change (see Fig. 5). As observed by Jones et al. [2], a slight change in the value of the bulk Mach number creates an important variation in impedance values. This behavior appears to be reaffirmed when the LEE are used in the eduction method.

It is however worth noting that when focusing around the resonance (Figure 16), the discrepancies between eduction methods are greatly reduced, highlighting the importance of understanding the physical phenomena involved and the limitations of each eduction method. In the case of the LEE method used by ONERA, when the acoustic absorption is high, the minimization process leading to the impedance seems to hardly be impacted by other conditions such as the variations of the flow profile, as there is often a clear solution to the eduction problem (i.e. a global minimum in the objective function). On the other hand, when there is little acoustic absorption (i.e. near the antiresonance) the acoustic reflection coefficient $\mathcal{R}$ used in the computation tends to 1, and a slight uncertainty on its value can lead to high uncertainties on the value of ζ.

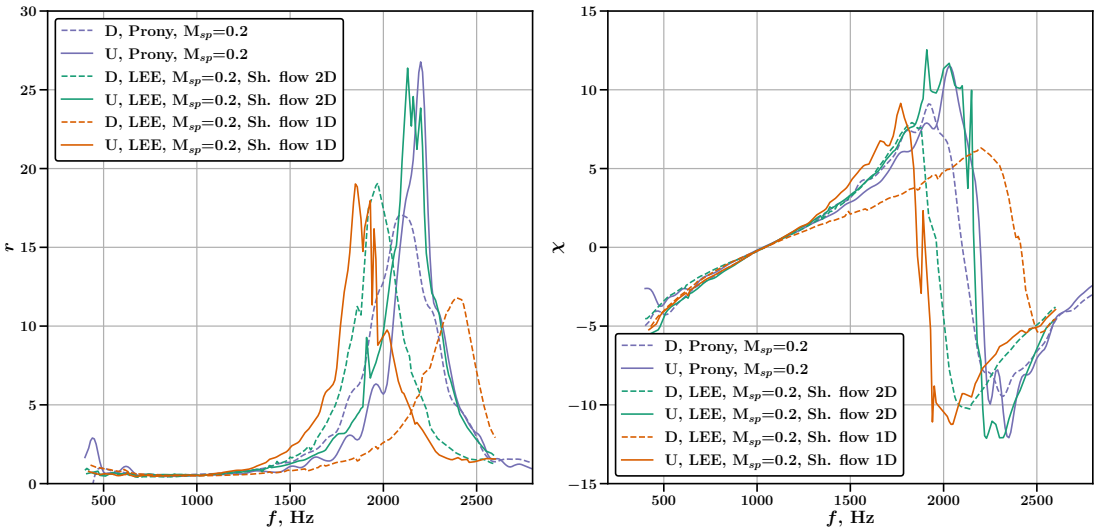


Fig. 15 Comparison of different eduction methods at Mach 0.2 and 140 dB, CSQ3 liner

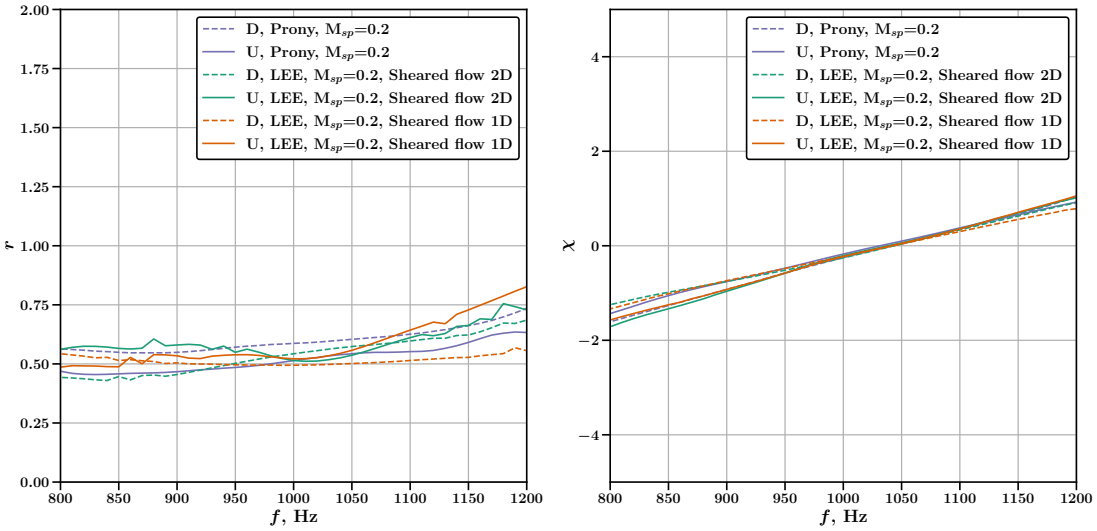


Fig. 16 Comparison of different eduction methods at Mach 0.2 and 140 dB, CSQ3 liner. Frequency range centered around the resonance of the acoustic liner.

IV. Concluding remarks

ONERA has used data shared by NASA as part of the IFAR Challenge #5 to perform impedance eductions. This challenge was created to assess the effect of flow direction on acoustic liner impedance. The ONERA method, based on the LEE, allows a direct way to account for the shear flow.

The following findings are worth noting:

- Impedance eduction done using the LEE on cases without grazing flow yields results that are very similar to results obtained by NASA in both upstream and downstream configurations.
- However, using a complete shear flow profile corresponding to the measured flow profile in the test bench gives very different results. The behavior is similar around the liner's resonance, but the difference between the upstream and downstream conditions appear clearly near the antiresonance. Moreover, the assumptions used for the shear flow profile appear to notably affect the observed behavior. Choosing a shear profile that is close to the 2D-averaged Mach number reduces discrepancies at antiresonance, even with high grazing flow ($M_{\mathrm{SP}} = 0.2$). This behavior appears to be quite independent of the eduction method, and mainly linked to the model chosen for the grazing flow.
- Discrepancies are reduced when evaluating resonance. This can be partly explained by the fact that near resonance, higher acoustic absorption mitigates the physical impact of grazing flow on the eduction process. Moreover, near resonance there is often a reduced group of possible solutions to the inverse problem (due to the high absorption) and so the method is less prone to errors and uncertainties. On the other hand, near antiresonance and when using the LEE method at ONERA, the acoustic reflection coefficient approaches 1; a slight error in its value can thus lead to high uncertainties in the value of the acoustic impedance in this frequency range.
- As shown in previous works, the choice of Mach number is a critical parameter for impedance eduction, particularly for eduction methods that use an averaged Mach number. The results shown here demonstrate that this extends to the LEE method that can take into account shear flow, with the advantage that the discrepancies are reduced as long as the shear profile is representative enough of the measured flow. Further work on this topic, using the full dataset shared by NASA, is needed to better understand the links between grazing flow direction and eduction results.

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