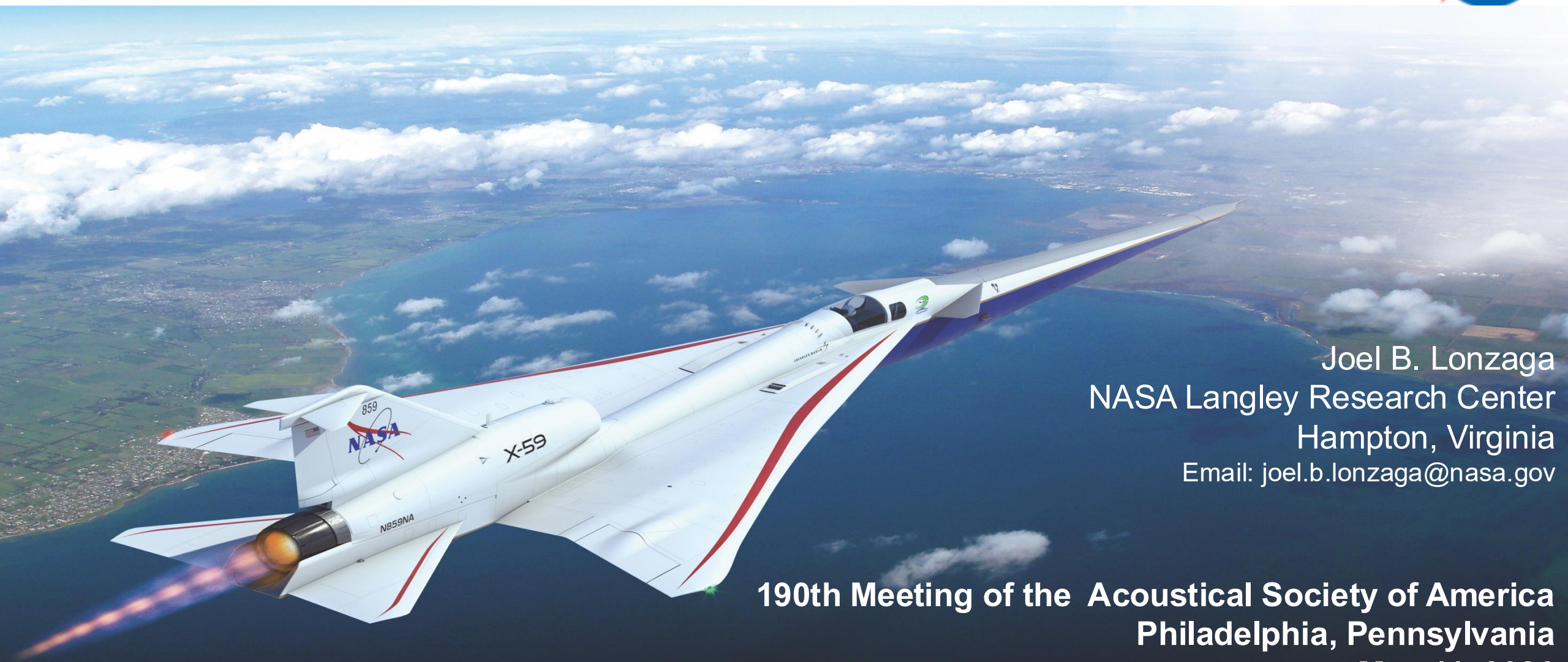


Turbulence-driven losses in sonic booms: A multiple scattering theory-based mean energy framework



Joel B. Lonzaga
NASA Langley Research Center
Hampton, Virginia
Email: joel.b.lonzaga@nasa.gov

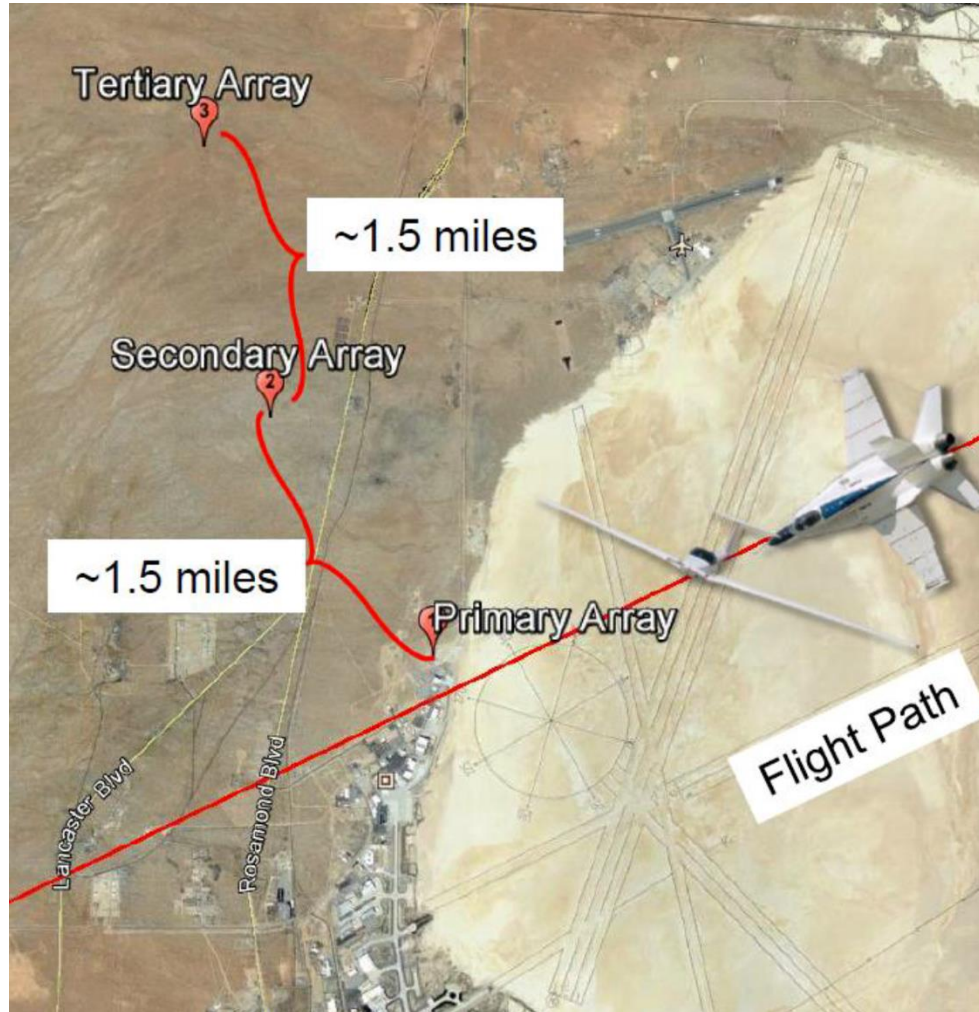
190th Meeting of the Acoustical Society of America
Philadelphia, Pennsylvania
May 11, 2026

- **Increasing loudness reduction with propagation distance through turbulence**
 - **Observed in flight tests**
 - **Need to account for turbulence effects**
- **KZK equation**
 - Parabolic equation approximation and conservation of the mean energy density
- **Multiple scattering theory**
 - Backscattering effects due to fluctuations
- **Conclusion**

Overprediction of Mean Acoustic Levels by PCBoom at the Ground



SonicBAT Flight Test at Edwards Air Force Base



- F-18 flight tracks were mostly from ENE (right to left)
- TG-14 was mostly flying under the F-18
 - Above the atmospheric boundary layer (ABL)
 - Altitude = 4,000-12,000 ft
 - Had a microphone on its wingtip
- 3 microphone arrays

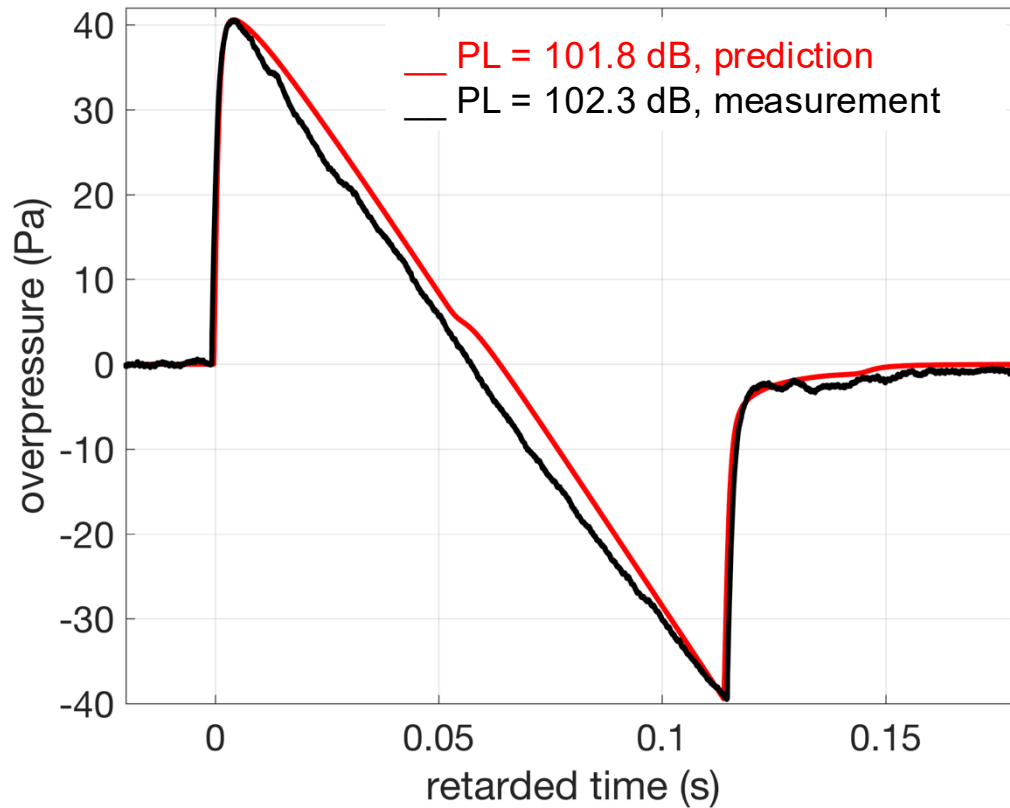
TG-14 glider



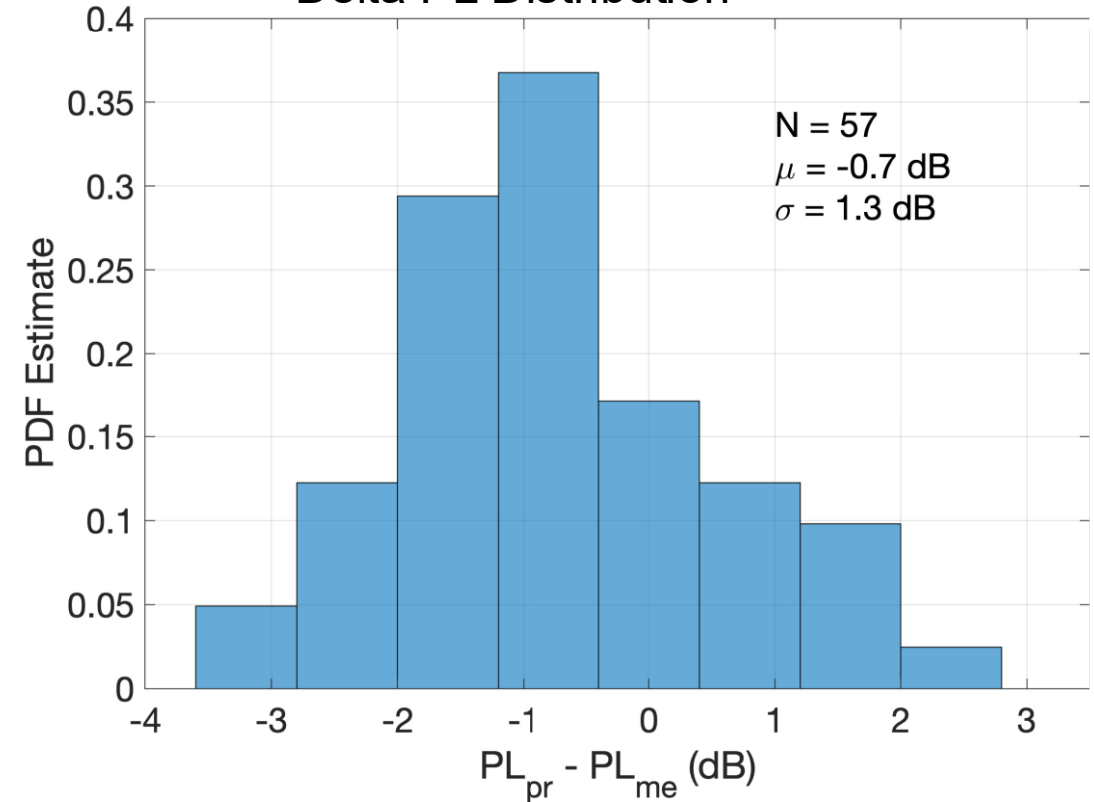
A Validation of PCBoom Without Turbulence



SonicBAT Flight 7, Pass 3

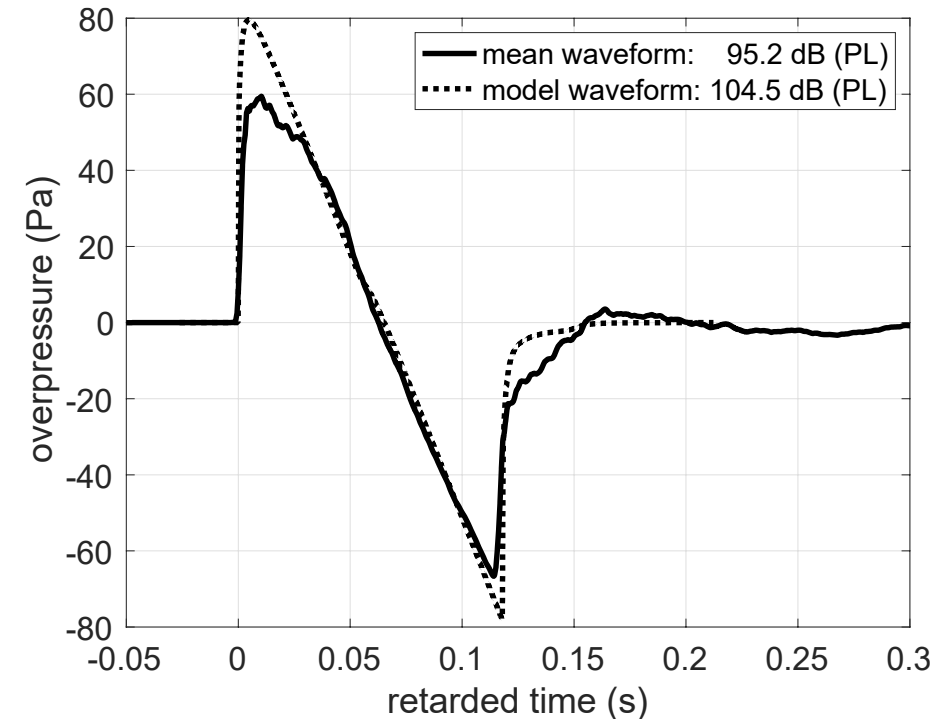
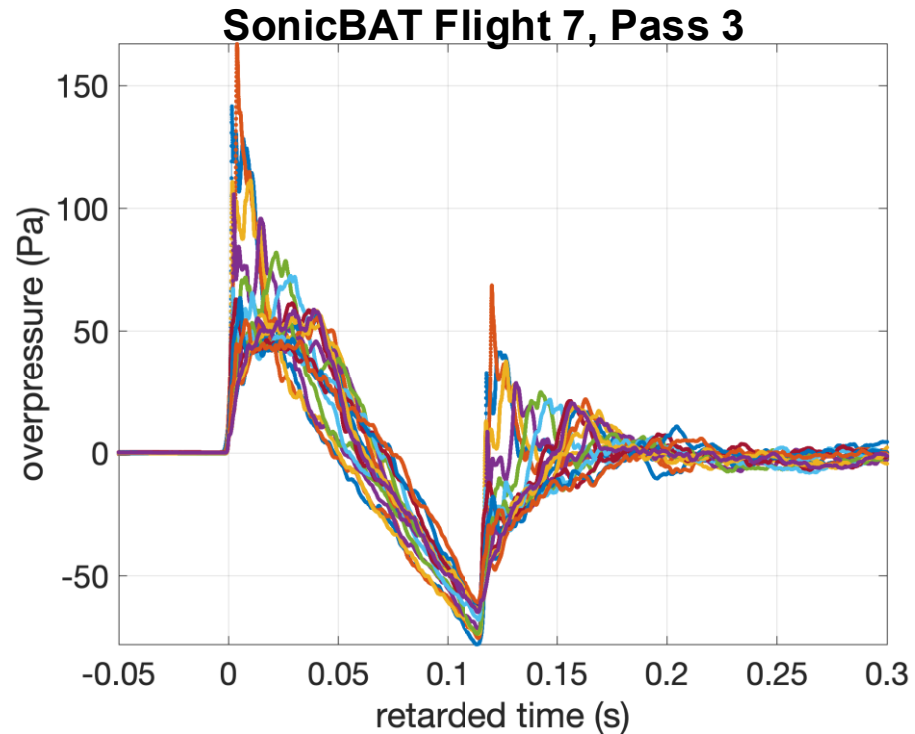


Delta PL Distribution



- Measured waveform and PL from microphone on wingtip of TG-14 above the ABL
- Delta PL: predicted PL - measured PL
- Predicted waveform and PL from PCBoom
- Nearfield signatures from CFD provided by Boeing
- Results published in *Ref. 1*

Comparison of a Model With Ground Recorded Data With Turbulence



Single Flight: Sonic Boom Arrivals

- Flight 7, Pass 3 in Edwards AFB (**high turbulence level**)
- Measured booms from 18 microphones within the linear primary array, each ~30 m apart
- Model waveform overpredicts loudness of the mean waveform

Loudness Reduction Due to Turbulence Versus Propagation Distance Through the ABL

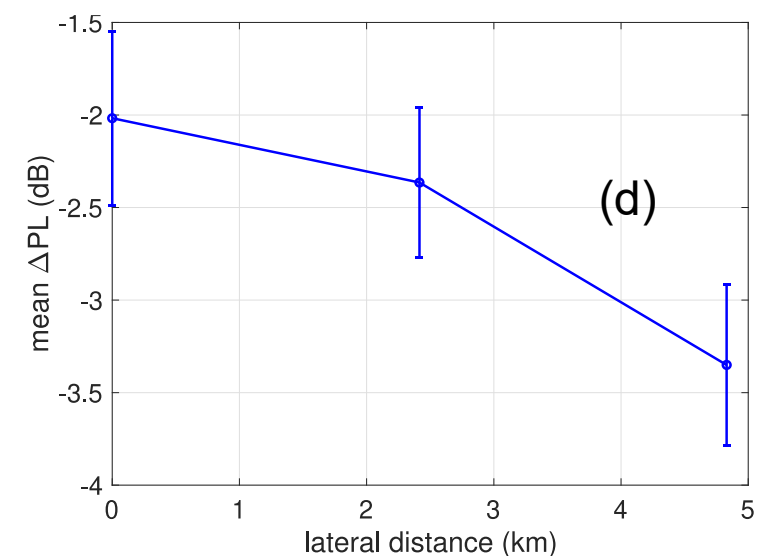
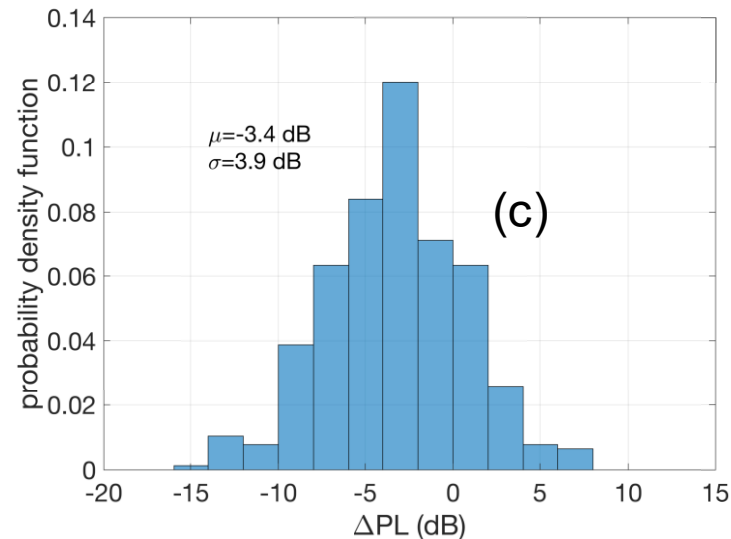
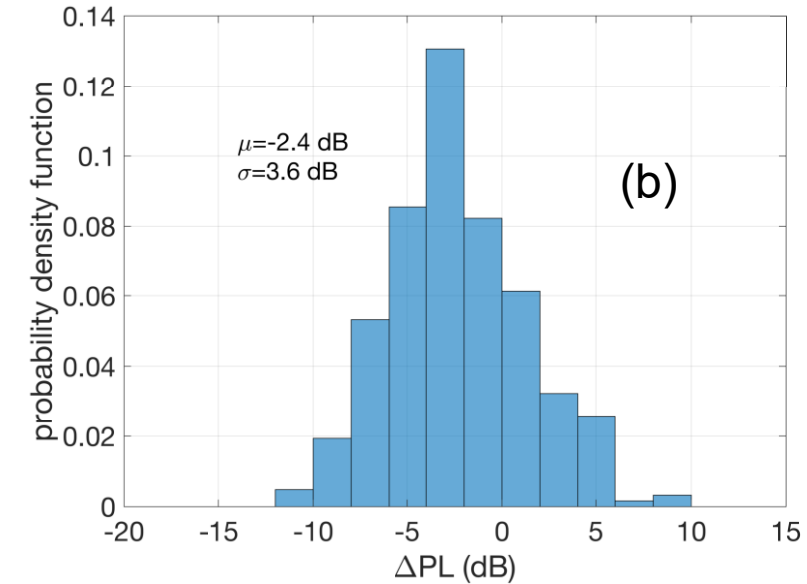
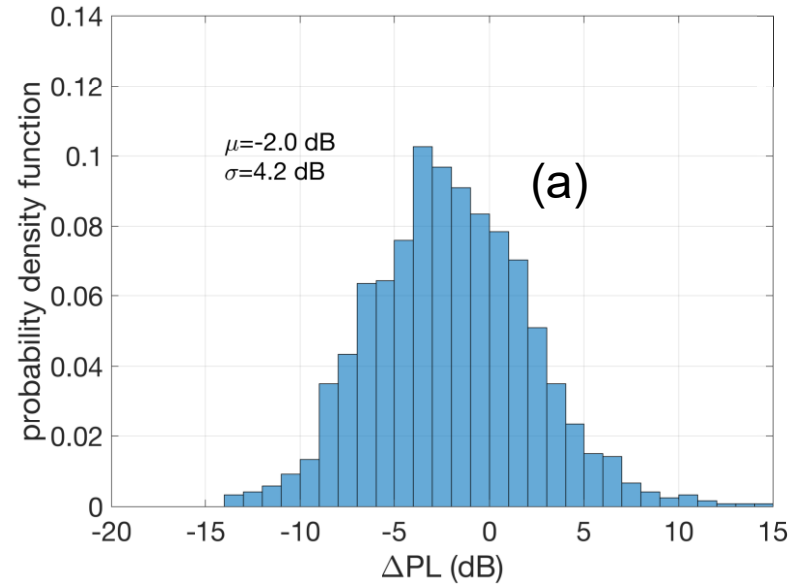


- Distribution of loudness reduction
 $\Delta PL = \text{mean } PL_{\text{meas}} - PL_{\text{pred}}$

(a) Primary array
(b) Secondary array
(c) Tertiary array

- Mean of loudness reduction increases with propagation distance through the ABL, Fig. (d), with uncertainty bars

- This dataset adds to a body of evidence that *sonic boom loudness reduction increases with propagation distance* [Ref. 2,3]



[2] Page, et al. (2020), “Quiet Supersonic Flights 2018 (QSF18) Test”

[3] Stout et al. (2021), “Evaluation of numerical predictions of sonic boom level variability due to atmospheric turbulence”

- KZK equation, a nonlinear parabolic equation
 - Linear parabolic equation (PE) and Burgers equation
 - Linear PE is a one-way approximation
 - No backscattering [e.g., Ref. 4,5]
 - No energy losses
- KZK Equation Solvers [e.g., Ref. 6]
 - Computationally expensive and slow
- KZK turbulence filters
 - Finite-impulse response
 - Signal processing-based surrogate models
 - Linear process
 - Nonlinearity is assumed negligible
 - Suitable for the low booms from X-59

KZK Equation:

$$\frac{\partial p}{\partial x} = (\mathcal{D} + \mathcal{T} + \mathcal{N} + \mathcal{A})p,$$

where

$$\mathcal{D} = \frac{\bar{c}}{2} \int_{-\infty}^{\tau} d\tau' \frac{\partial^2}{\partial \mathbf{r}_{\perp}^2}$$

is a mathematical operator associated with the diffraction effect,

$$\mathcal{T} = \frac{\tilde{n} + \tilde{m}_x}{\bar{c}} \frac{\partial}{\partial \tau} - \tilde{\mathbf{m}}_{\perp} \cdot \frac{\partial}{\partial \mathbf{r}_{\perp}},$$

is the turbulence effect operator,

$$\mathcal{N} = \frac{\beta p}{\bar{\rho} \bar{c}^3} \frac{\partial}{\partial \tau}$$

is the nonlinear effect operator, and

$$\mathcal{A} = \frac{\delta_d}{2\bar{c}^3} \frac{\partial^2}{\partial \tau^2} + \sum_{\nu} \frac{c'_{\nu}}{\bar{c}^2} \int_{-\infty}^{\tau} d\tau' e^{-\frac{(\tau-\tau')}{t_{\nu}}} \frac{\partial^2}{\partial \tau'^2}$$

is the absorption effect operator.

[4] Tappert (1977), “The parabolic approximation method,” in *Wave Propagation and Underwater Acoustics, Lecture Notes in Physics*,

[5] Ostashev & Wilson (2015). *Acoustics in Moving Inhomogeneous Media*

[6] Stout et al. (2021), “Evaluation of numerical predictions of sonic boom level variability due to atmospheric turbulence,”

- Splitting of the KZK Equation, method in [Ref. 7]
 - Burgers equation
 - $\frac{\partial p}{\partial x} = (N + A)p$
 - Parabolic Equation (PE)
 - $\frac{\partial p}{\partial x} = (D + T)p$
 - applies to low boom due to negligible nonlinearity in the atmospheric boundary layer
 - In the frequency domain, the PE becomes
 - $\frac{\partial P}{\partial x} = \left(\frac{i}{2\kappa_o} \frac{\partial^2}{\partial \mathbf{r}_\perp^2} - \tilde{\mathbf{m}}_\perp \cdot \frac{\partial}{\partial \mathbf{r}_\perp} - i\kappa_o \tilde{n}_e \right) P$
- Where:
- $P = P(\mathbf{r}, \omega)$
 - $\tilde{n}_e(\mathbf{r}) = \tilde{n}(\mathbf{r}) + \tilde{m}_x(\mathbf{r})$
 - $\kappa_o = \omega/\bar{c}$
- An analytical solution already exists for $\langle |P(\mathbf{r}, \omega)|^2 \rangle$ in PE approximation [Ref. 8]
 - $\langle |P(\mathbf{r}, \omega)|^2 \rangle = A_o^2(\omega) = \text{const}$, mean energy density is conserved
 - Any numerical solution must obey mean energy density conservation

[7] Bradley, et al. (2020), "Sonic booms in atmospheric turbulence (SONICBAT): The influence of turbulence on shaped sonic booms,"

[8] Rytov, et al. (1989), "Principles of Statistical Radiophysics 4: Wave Propagation Through Random Media"

Multiple Scattering Theory



• Mean Acoustic Pressure Field

- Using the full 3D wave equation, the acoustic pressure field is given by

$$P(\mathbf{r}) = P_o(\mathbf{r}) + 2\kappa_o^2 \int d^3\mathbf{r}' G_o(\mathbf{r}, \mathbf{r}') n'(\mathbf{r}') P(\mathbf{r}'); \quad G_o(\mathbf{r}, \mathbf{r}') = \frac{e^{i\kappa_o|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}; \quad \kappa_o = \omega/\bar{c}$$

- In turbulent media, the mean field is of interest. Recently its expression was obtained [Ref. 8]

$$\langle P(\mathbf{r}, \omega) \rangle = \int_{-\infty}^{\infty} \frac{dk_x}{2\pi} \frac{S(\omega) e^{ik_x x}}{D(k_x)}, \quad S(\omega) \text{ is the source strength}$$

- Dispersion relation: $D(k_x) = k_x^2 - \kappa_o^2 - \tilde{\Sigma}(k_x)$
 - Setting $D(k_e) = 0$ yields the effective wavenumber k_e

- Self-energy for the exponential correlation function:

$$\tilde{\Sigma}(k) = \sum_{v=1}^3 \frac{4\sigma_v^2 \kappa_o^4 \ell_v^2}{(k^2 - \kappa_o^2) \ell_v^2 - 2i\kappa_o \ell_v + 1}, \quad \ell_v = \text{correlation length},$$

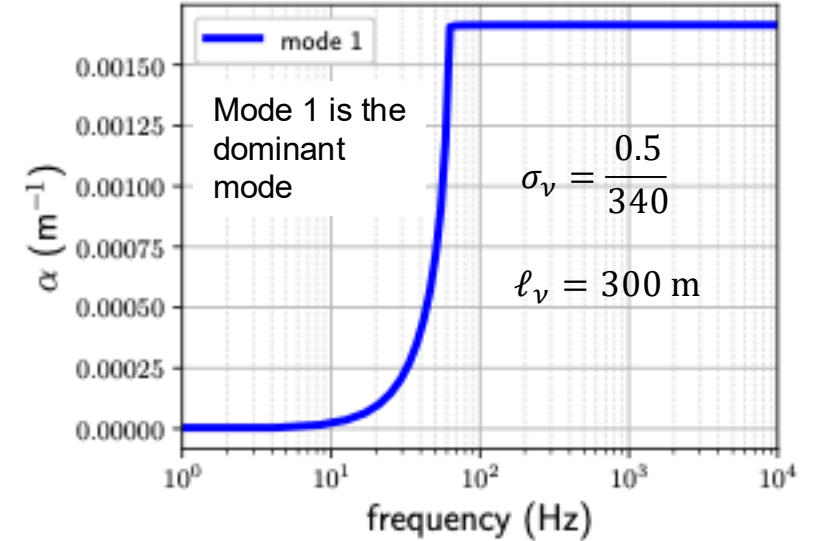
$$\sigma_v^2 = \text{variance}$$

- Solution of the mean field previously presented:

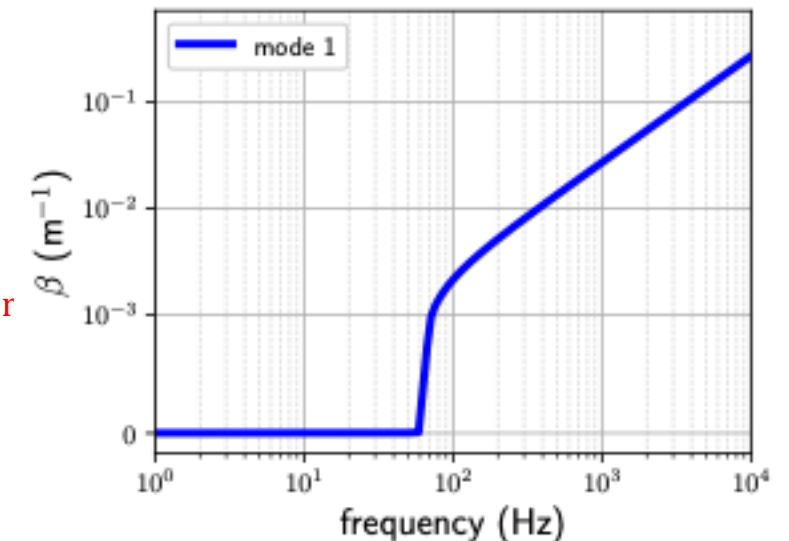
$$\langle P(\mathbf{r}, \omega) \rangle = A(\omega) e^{ik_e x}, \quad \text{where: } A(\omega) = \frac{iS(\omega)}{2\kappa_o}, \quad k_e = \kappa_o + \beta + i\alpha$$

- Only picks the dominant mode, ignoring one other mode

Modal attenuation



Modal phase shift



Bimodal Mean Acoustic Pressure Field

- A more accurate solution includes the two modes (roots of the dispersion eq.)

$$k_{e,\pm} = \kappa_o \sqrt{1 - \frac{1}{2\kappa_o^2 \ell_v^2} \left(a_v \mp \sqrt{a_v^2 + 16\kappa_o^4 \ell_v^2 \sigma_v^2} \right)}, \quad a_v = 1 - 2i\kappa_o \ell_v$$

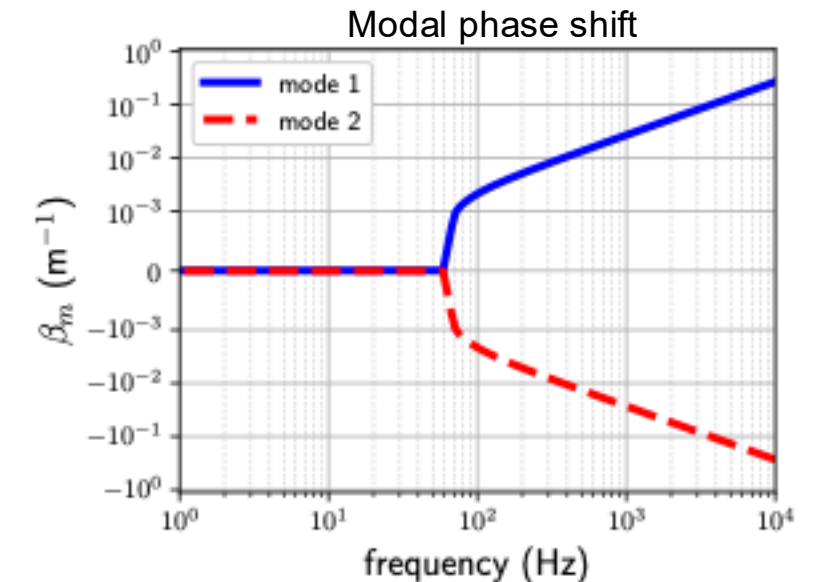
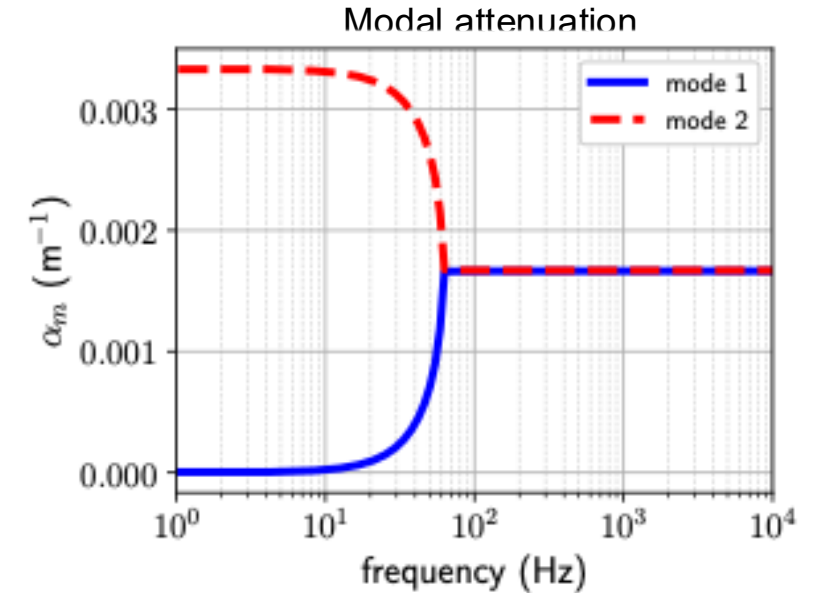
- Use contour integration in the upper half of the k_x – complex plane and residue theorem:

$$\langle P(\mathbf{r}, \omega) \rangle = \int_{-\infty}^{\infty} \frac{dk_x}{(2\pi)} \frac{S(\omega) e^{ik_x x}}{D(k_x)}$$

$$\langle P(\mathbf{r}, \omega) \rangle = \sum_{m=1}^2 \frac{2\kappa_o A(\omega)}{D'(k_{e,m})} e^{ik_{e,m} x}$$

where: $D'(k_{e,m}) = \frac{\partial D(k_x)}{\partial k_x}$ evaluated at $k_x = k_{e,m}$

- Parameters in plots: $\sigma_v = \frac{0.5}{340}$, $\ell_v = 300$ m



Effects of the Second Mode in the Mean Field

- Bimodal expression for the mean waveform

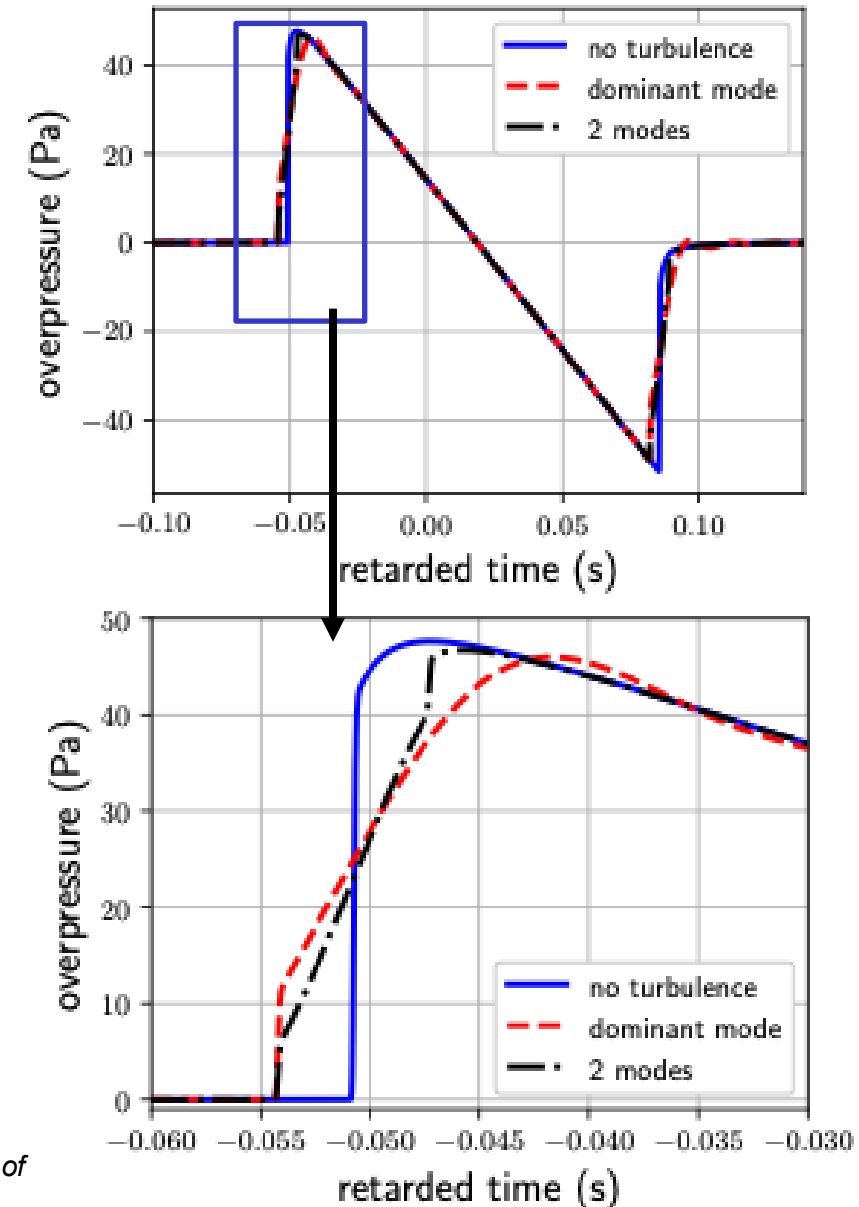
$$\langle p(\mathbf{r}, t) \rangle = \int_{-\infty}^{\infty} \frac{d\omega dk_x}{(2\pi)^2} \frac{S(\omega) e^{i(k_x x - \omega t)}}{D(k_x)} = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \sum_{m=1}^2 \frac{2\kappa_o A(\omega)}{D'(k_{e,m})} e^{i(k_{e,m} x - \omega t)}$$

where: $D'(k_{e,m}) = \frac{\partial D(k_x)}{\partial k_x}$ evaluated at $k_x = k_{e,m}$

- Plot features:

- Rounding and thickening of the shock by about 10 ms
 - Routinely observed in flight tests and lab experiment [Ref. 9, 10]
- Mode 2 effects appear insignificant on the waveform shape

- Parameters in plots: $\sigma_v = \frac{0.5}{340}$, $\ell_v = 300$ m, $x = 1$ km



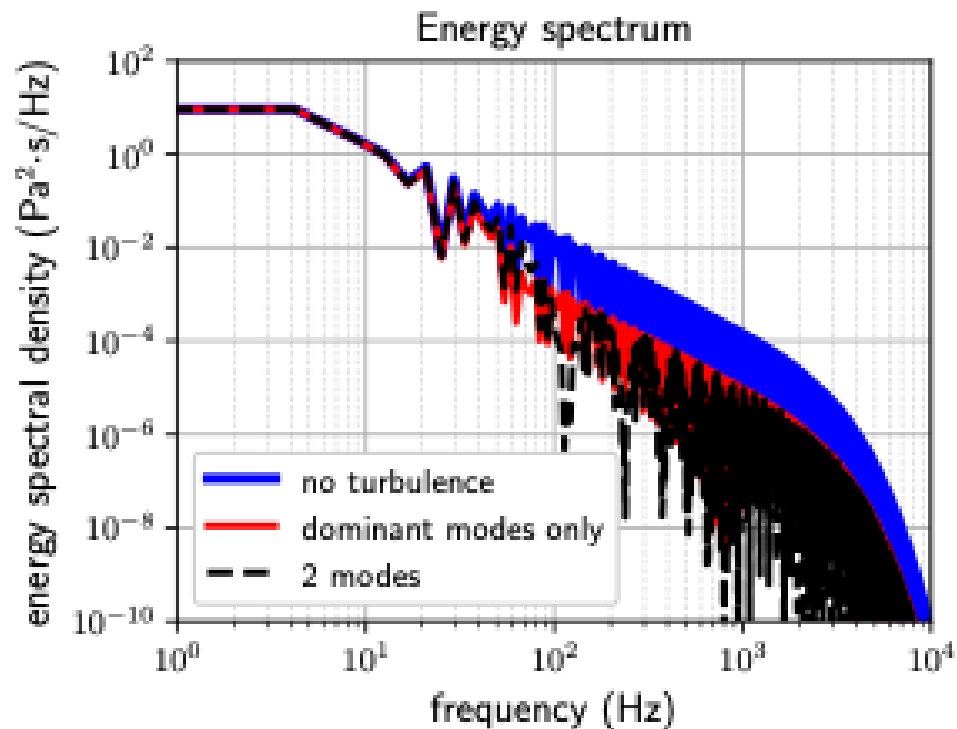
[9] Pierce (1971), "Statistical theory of atmospheric turbulence effects on sonic-boom rise times"

[10] Lipkens and Blackstock (1998), "Model experiment to study sonic boom propagation through turbulence. Part II. Effect of turbulence intensity and propagation distance through turbulence"

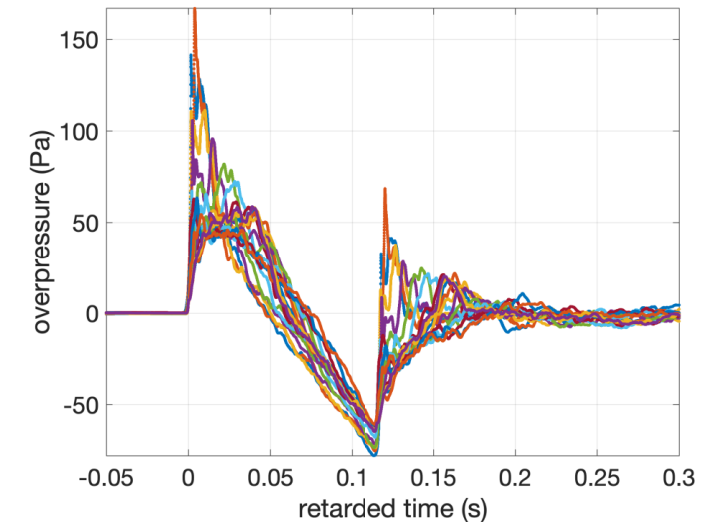
Modal Interference Effects in the Spectrum



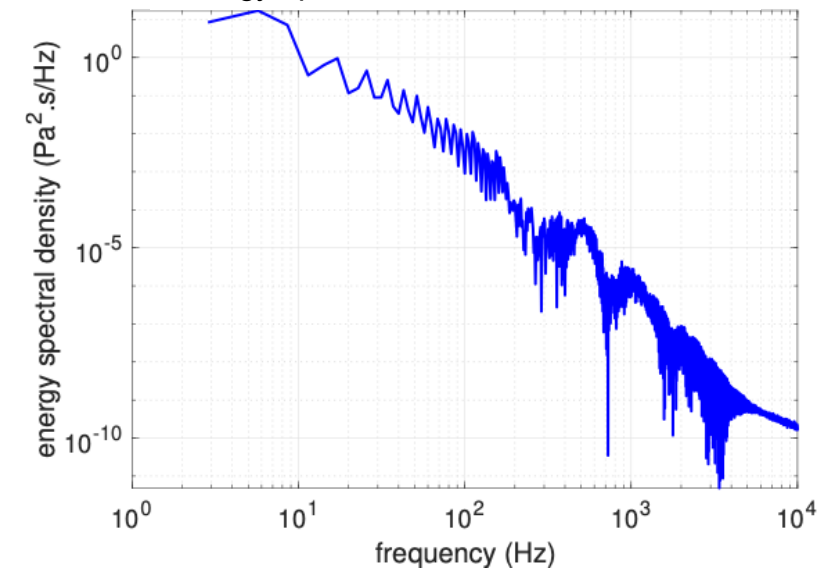
- Closely spaced dips (comb effects) in the model plots are a typical feature of impulsive signals
- In contrast, the more widely separated dips arise from interference between modes 1 and 2
 - Also observed in the mean field from a flight test



Observed Interference effects in a flight test



Energy spectrum of the mean waveform



Mean Energy Density

- Energy density is another quantity of interest for sound exposure level-based loudness metrics

$$E(\mathbf{r}, \omega) = P(\mathbf{r}, \omega)P^*(\mathbf{r}, \omega)$$

- Integral formulation of the full 3D wave equation

$$P(\mathbf{r}) = P_o(\mathbf{r}) + 2\kappa_o^2 \int d^3\mathbf{r}' G_o(\mathbf{r}, \mathbf{r}')n'(\mathbf{r}')P(\mathbf{r}'); \quad G_o(\mathbf{r}, \mathbf{r}') = \frac{e^{i\kappa_o|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}; \quad \kappa_o = \omega/\bar{c}$$

- Mean energy density (e.g., Ref. [11])

$$\langle P(\mathbf{r})P^*(\mathbf{r}) \rangle = \langle P(\mathbf{r}) \rangle \langle P^*(\mathbf{r}) \rangle + 4\kappa_o^4 \int d^3\mathbf{r}' d^3\mathbf{r}'' G_o(\mathbf{r}, \mathbf{r}')G_o^*(\mathbf{r}, \mathbf{r}'')\phi(\mathbf{r}' - \mathbf{r}'')\langle P(\mathbf{r}')P^*(\mathbf{r}'') \rangle$$

- In the $\mathbf{k}$ -space:

$$\langle \tilde{P}(\mathbf{k}')\tilde{P}^*(\mathbf{k} - \mathbf{k}') \rangle = \frac{\langle \tilde{P}(\mathbf{k}') \rangle \langle \tilde{P}^*(\mathbf{k} - \mathbf{k}') \rangle}{1 - \tilde{J}(\mathbf{k}, \mathbf{k}')}$$

- Energy coupling:

$$\tilde{J}(\mathbf{k}, \mathbf{k}') = 4\kappa_o^4 \int \frac{d^3\mathbf{k}''}{(2\pi)^3} \tilde{G}_o(\mathbf{k}'')\tilde{G}_o^*(\mathbf{k} - \mathbf{k}'')\Phi(\mathbf{k}'' - \mathbf{k}')$$

- The solution:

$$\langle P(\mathbf{r})P^*(\mathbf{r}) \rangle = \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}} \int \frac{d^3\mathbf{k}'}{(2\pi)^3} \frac{\langle \tilde{P}(\mathbf{k}') \rangle \langle \tilde{P}^*(\mathbf{k} - \mathbf{k}') \rangle}{1 - \tilde{J}(\mathbf{k}, \mathbf{k}')}$$

Modal Solution of the Mean Energy Density

- Final expression of the mean density:

$$\langle P(\mathbf{r})P^*(\mathbf{r}) \rangle = \sum_{m,n=1}^2 B_{mn} e^{i(k_{e,m} - k_{e,n}^*)x} + \sum_j C_j e^{ik_{c,j}x}$$

- Mode labels: $k_1 = k_{e,1}$, $k_2 = k_{e,2}$, $k_3 = k_{c,1}$, $k_4 = k_{c,2}$, $k_5 = k_{c,3}$
- Effective wavenumber k_e and scattering wavenumber k_s ,

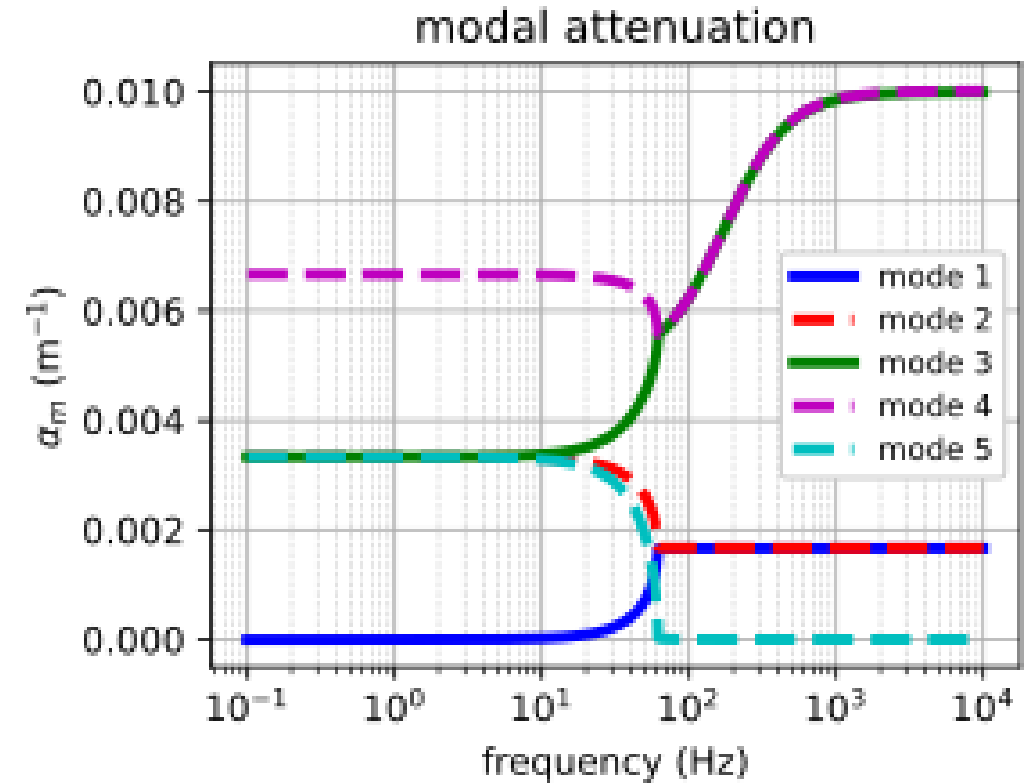
$$k_{e,m} = \kappa_o + k_{s,m}, \quad k_{s,m} = \beta_m + i\alpha_m$$

$$D(k_e) = k_e^2 - \kappa_o^2 - \tilde{\Sigma}(k_e) = 0$$

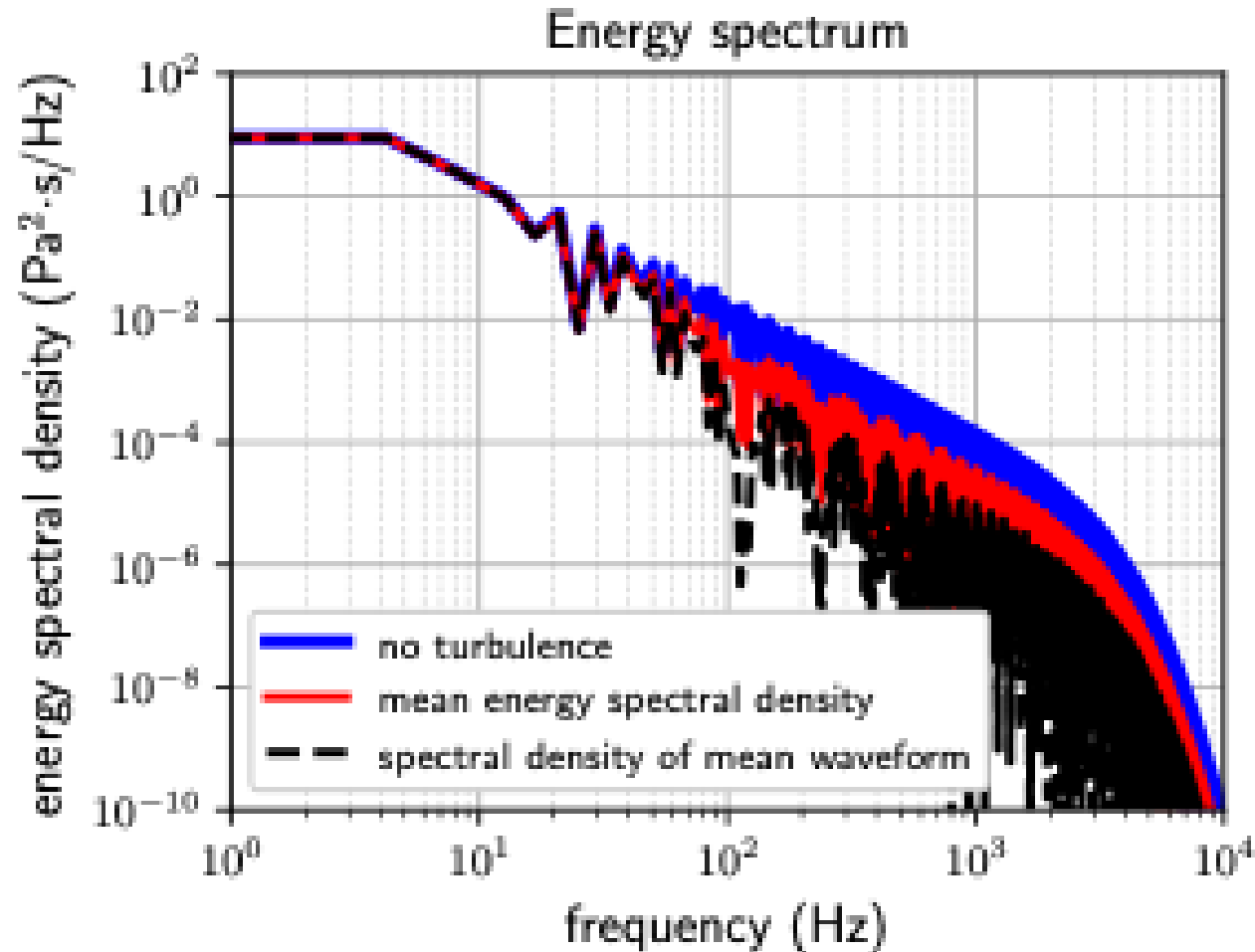
- Energy coupling equation and coupling wavenumber k_c :

$$E(k_{c,j}, k_{e,m}) = 1 - \tilde{J}(k_{c,j}\hat{\mathbf{n}}, k_{e,m}\hat{\mathbf{n}}) = 0$$

- Modal excitation coefficients: B_{mn} and C_j
- Mode 5 is unattenuated above the transition frequency of ~70 Hz



Prediction of Energy Losses



- Losses in the mean energy density is predicted to be less than the losses in the energy density of the mean waveform
- **Future work:**
 - Quantify the losses as a function of propagation distance through turbulence
 - Investigate how the interference effects from the two modes can be used to infer turbulence parameters

- Sonic boom data from a flight test indicate that loudness reduction increases with
 - turbulence strength
 - propagation distance through turbulence
- One model uses the KZK equation
 - One-way propagation model, ignoring backscattering
 - Combines
 - Burgers equation for the effects of atmospheric absorption and nonlinearity
 - Parabolic equation for the turbulence effects
 - Known in the literature to conserve mean energy density
 - Solvers are computationally expensive and slow
- Multiple scattering theory has been developed to account for backscattering effects
 - Model shows two modes exist for the mean field
 - Causes interference observed in flight test datasets
 - Ongoing development of the higher-order statistical moment, (e.g., energy density)