

BUFFET ONSET PREDICTION WITH THE LINEARIZED FREQUENCY-DOMAIN METHOD

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Abstract: Transonic buffet is a flow instability associated with shock-wave oscillation and boundary-layer separation that can cause unsteady aerodynamic loads, structural fatigue, and degraded aircraft performance. This paper presents a method to predict buffet onset using a linearized frequency-domain computational fluid dynamics approach. A reduced state-space aerodynamic model is constructed from frequency-domain data using the Loewner framework, and buffet stability is determined from its eigenvalues. Buffet onset is identified when the real part of the dominant fluid eigenvalue changes sign and becomes positive. To represent buffet conditions, the realization is allowed to retain physically meaningful unstable fluid poles, while a residue-based criterion is used to distinguish them from spurious poles introduced by interpolation. In combination with a projection of the aerodynamic outputs based on proper orthogonal decomposition (POD), the reduced model also provides the corresponding eigenvectors, enabling reconstruction of their spatial structure in physical space in addition to onset prediction. The approach is demonstrated for the OAT15A airfoil and the NASA Common Research Model. Results agree well with reference global stability analysis and show that the combined Loewner–POD framework robustly extracts the dominant buffet-driving fluid modes. The method provides an efficient reduced-order alternative to direct global stability analysis and can be extended toward combined flutter and buffet prediction.

1 INTRODUCTION

Transonic buffet is an aerodynamic instability of major relevance to aircraft design because it limits the usable flight envelope and may lead to increased vibration levels, passenger discomfort, and structural fatigue. It is generally associated with self-sustained shock-wave motion and shock-induced boundary-layer separation in transonic flow, producing unsteady aerodynamic loads over a broad range of spatial and temporal scales. Experimental investigations on

supercritical airfoils and realistic wing configurations have shown that buffet is a persistent and practically important phenomenon in transonic flight, both in canonical configurations and in aircraft-relevant geometries [1, 2]. For realistic wings, the phenomenon is further complicated by three-dimensional effects, spanwise variation, and buffet-cell structures, which make robust onset prediction especially important [3].

An important step in understanding buffet was to interpret its onset as a flow-stability problem. Early work on global instability demonstrated that the onset of flow unsteadiness can be linked to unstable eigenmodes of the linearized flow operator [4]. This viewpoint was subsequently established for transonic buffet on airfoils, where buffet onset was related to an unstable global mode of the flow [5], and was later extended to infinite unswept and swept wing configurations [6–8]. Further developments, including triglobal and full-aircraft analyses, have confirmed that buffet in realistic three-dimensional configurations can also be interpreted within an eigenvalue-based framework, although with increasingly rich modal behavior and significantly higher computational demands [3, 9]. This global-stability perspective has therefore become central to the modern interpretation of buffet onset.

Despite its physical clarity, direct global stability analysis (GSA) remains computationally demanding, especially for realistic three-dimensional and full-aircraft cases. Even when the dominant buffet mode can be extracted successfully, the associated eigenvalue problem remains very large, and repeated analyses over Mach number, angle of attack, or configuration changes can become prohibitively expensive [3, 9]. This computational expense motivates the development of reduced formulations that preserve the relevant buffet-driving dynamics while avoiding the cost of solving the full linearized computational fluid dynamics (CFD) eigenproblem. In this context, approaches based on reduced-order modeling and input-output representations are especially attractive, provided that they retain the fluid modes governing onset.

Related work has shown that transonic aeroelastic and aerodynamic instabilities can often be interpreted in terms of fluid modes. In the review by Gao and Zhang [10], transonic aeroelastic phenomena were reinterpreted from the perspective of dominant fluid modes, emphasizing that the relevant instability mechanisms cannot always be understood solely through structural modal coupling. In the same spirit, Nitzsche et al. [11] investigated fluid-mode flutter in transonic pre-buffet flows and showed that coupling with an elastic structure can destabilize a fluid mode in separated flow. More recent studies have further explored how reduced-order descriptions involving dominant fluid modes can represent pre-buffet dynamics and fluid-structure interaction in transonic flow [12]. This perspective is especially relevant for buffet onset in rigid configurations, where the instability is fundamentally aerodynamic and should therefore be identifiable directly from the fluid system.

A practical way to build such a reduced formulation is to start from frequency-domain aerodynamic data. Linearized frequency-domain methods provide unsteady aerodynamic information about a nonlinear steady equilibrium without requiring time-accurate simulations for every condition of interest and have already proved effective for viscous transonic flows [13]. Their relevance is not limited to methodology studies: they have also been adopted in industrial aeroelastic workflows, for example in linearized CFD- and computational structural mechanics (CSM)-based flutter processes for very flexible aircraft [14] and in gust-load processes based on the linearized frequency-domain (LFD) method [15]. More generally, frequency-domain formulations have become established in aeroelastic applications such as flutter analysis [16]. Building on this idea, reduced-order realizations of aerodynamic transfer functions have been

used to recast stability analysis as a linear generalized eigenvalue problem. In particular, the p - L framework introduced by Quero et al. [17] showed that a Loewner-based realization of frequency-domain aerodynamic data can be used for eigenvalue-based flutter prediction while preserving the true aeroelastic damping. Derivative-based tracking procedures were later introduced to robustly follow the relevant roots as system parameters vary [18].

The reduced-order framework described in Quero et al. [18] was subsequently extended to include fluid modes explicitly in the coupled aeroelastic analysis, making it possible to address flutter and buffet-related instabilities within a unified solver [19]. This coupled viewpoint is also consistent with the fluid-mode flutter interpretation discussed by Nitzsche et al. [11], where structural feedback can destabilize a fluid mode that is stable in the uncoupled flow. In the present work, the emphasis is placed differently. Here, the objective is to predict buffet onset of the fluid system itself, without dynamic structural coupling in the stability analysis. In the Common Research Model case considered in Section 3.2, however, the reference geometry includes static aeroelastic deformation, so that different angles of attack correspond to different equilibrium shapes. The reduced model is purely aerodynamic, and buffet onset is determined directly from its eigenvalues. It must retain physically meaningful unstable fluid poles associated with rigid buffet onset, while excluding spurious unstable poles introduced by interpolation. Although the present work is restricted to rigid configurations, the formulation can later be coupled with structural equations, extending it toward a unified reduced-order framework for flutter, rigid buffet, and aeroelastic buffet.

In the present work, a reduced aerodynamic state-space model is constructed from linearized frequency-domain CFD data through the Loewner framework, and buffet onset is assessed from the eigenvalues of that model. The dominant fluid mode is identified by means of a residue-based significance criterion, and buffet onset is detected when its real part changes sign and becomes positive. Because the number of states in the reduced aerodynamic realization is much smaller than in the full CFD system, the resulting eigenvalue analysis is considerably cheaper than direct global stability analysis while retaining the buffet-driving dynamics of interest. In addition, by combining the Loewner realization with a proper orthogonal decomposition (POD)-based projection of the frequency-domain aerodynamic outputs, the present framework provides access not only to the dominant buffet eigenvalue, but also to the corresponding fluid-mode eigenvector and its spatial reconstruction in physical space. The approach is demonstrated for the OAT15A airfoil and for the three-dimensional Common Research Model (CRM) configuration, showing that buffet onset can be predicted from reduced aerodynamic models derived from frequency-domain CFD data without structural coupling and without solving the large-scale linearized flow eigenvalue problem directly.

Beyond the conceptual unification, the methodology described here also has a practical advantage. Linearized frequency-domain CFD computations are already used in industrial workflows for flutter analysis, aeroelastic loads, and gust-loads [14–16]. The present framework indicates that the same frequency-domain information can also be exploited for buffet prediction, so that flutter, buffet, and load-related analyses can be treated in a harmonized reduced-order setting rather than through separate methodologies. In combination with structural feedback, this opens the possibility of using the same underlying LFD data and reduced aerodynamic model to address pre-buffet, buffet, and flutter behavior within a common framework.

2 BUFFET-MODE IDENTIFICATION FROM FREQUENCY-DOMAIN DATA

The objective of the present work is to predict transonic buffet onset for a rigid aerodynamic configuration from linearized frequency-domain aerodynamic data. In contrast with coupled aeroelastic formulations, the instability of interest here is defined as a property of the fluid system alone. Buffet onset is therefore sought directly from a reduced-order realization of the aerodynamic transfer matrix, without structural coupling and without solution of the full linearized eigenvalue problem.

A CFD-based description is required because transonic buffet is governed by flow features such as shock motion, shock-induced boundary-layer separation, and their interaction with the underlying steady flow field. These effects are not represented with sufficient fidelity by classical linear aerodynamic theories such as the doublet-lattice method [20], which are well suited to subsonic unsteady aerodynamics but not to transonic buffet physics. At the same time, the prediction of buffet onset does not require nonlinear time-accurate simulations of finite-amplitude oscillations, since onset is associated with the loss of stability of an equilibrium flow state. For that reason, a linearized CFD formulation in the frequency domain is sufficient for the present purpose, provided that it accurately captures the dominant buffet-driving fluid dynamics.

Let α_0 , M_∞ , and Re denote the angle of attack, Mach number, and Reynolds number defining the steady operating condition about which the flow is linearized. For each operating point, the linearized frequency-domain solver provides the harmonic aerodynamic response to prescribed input perturbations over a set of reduced frequencies k . These data are used to construct a reduced-order aerodynamic model whose eigenvalues are interpreted as fluid, or aerodynamic, poles. The central hypothesis of the present work is that buffet onset can be identified from the evolution of the dominant aerodynamic pole as the operating condition changes.

In this framework, buffet onset is defined as the point at which the dominant fluid mode loses stability, that is, when the real part of its eigenvalue changes sign and becomes positive. The associated imaginary part provides the buffet frequency. Introducing the dominant aerodynamic eigenvalue $\lambda_{a,\text{dom}}$, the onset condition is written as

$$\text{Re}(\lambda_{a,\text{dom}}) = 0. \quad (1)$$

The goal is therefore to determine, from frequency-domain aerodynamic data alone, a reduced-order model that captures the dominant buffet dynamics, identifies the relevant fluid mode, and retains physical unstable poles without keeping spurious ones.

As in GSA, the aim is not to reproduce the full eigenspectrum, but to capture the poles that govern buffet onset. Because the approach is based on a reduced aerodynamic realization, the resulting stability analysis is of much lower dimension than the original linearized flow problem. The lower dimensionality of the resulting stability analysis does not imply that the overall cost is always lower than that of Krylov-based global stability analysis, since the reduced model must first be identified from a set of LFD samples, and each frequency-domain solve is itself non-negligible. The practical efficiency therefore depends on how many sampled frequencies are needed to recover the dominant buffet-driving poles with sufficient robustness. If only a single leading mode is sought at one operating point, iterative eigenvalue extraction may be competitive. Additionally, the present formulation can leverage existing industrial workflows that already employ LFD data, such as flutter or gust-load analysis. In that case, the combined Loewner–POD framework can reuse the existing frequency-domain samples and turn them into

a reduced dynamical model whose poles can be inspected directly. The ability to reuse existing LFD data is particularly relevant because conventional frequency-domain loads and flutter workflows do not in general reveal automatically whether a fluid mode has become unstable; by contrast, the present realization enables buffet onset to be detected directly from the available LFD data, without requiring a separate global stability analysis or costly time-domain simulations.

2.1 Linearized frequency-domain aerodynamic data

The reduced-order aerodynamic model developed in the present work is constructed from aerodynamic data obtained with a linearized frequency-domain (LFD) solver. The LFD approach is based on the assumption that the unsteady response to a sufficiently small harmonic perturbation about a nonlinear steady flow state is predominantly linear and can therefore be represented by its first harmonic in the frequency domain [13, 21, 22]. Such a first-harmonic linear representation makes the method particularly suitable for onset prediction problems, for which the relevant dynamics are governed by infinitesimal perturbations of a steady equilibrium.

In the present paper, the steady flow state is denoted by $\mathbf{W}_0$, whereas $\mathbf{W}(t)$ and $\mathbf{W}(i\omega)$ denote the corresponding time-domain and frequency-domain flow states, respectively, with t denoting time, ω the angular frequency, and i the imaginary unit. Following the standard LFD formulation, the semi-discrete Reynolds-averaged Navier-Stokes equations can be written as

$$\frac{d}{dt}(\mathbf{M}\mathbf{W}) + \mathbf{R}(\mathbf{W}, \mathbf{x}, \dot{\mathbf{x}}) = \mathbf{0},$$

where $\mathbf{M}$ is the integration matrix associated with the discrete control volumes, $\mathbf{R}$ is the residual vector of the spatially discretized flow equations, $\mathbf{x}$ denotes the grid-node coordinates, and $\dot{\mathbf{x}}$ the grid-node velocities [13, 21, 22]. The equations are linearized about the steady state $(\mathbf{W}_0, \mathbf{x}_0)$, and a harmonic response with angular frequency ω is assumed.

The harmonic excitation is described here in terms of a vector of generalized input coordinates $\mathbf{u}(i\omega) \in \mathbb{C}^{n_u}$, whose components may correspond to rigid-body motions, elastic modal amplitudes, or a selected combination of both. The corresponding harmonic displacement field on the aerodynamic surface is obtained from these generalized coordinates through the structural basis and the spline transfer to the aerodynamic mesh. Denoting by Φ the matrix of selected rigid-body and elastic modes and by $\mathbf{H}_{ag}$ the spline matrix from the structural to the aerodynamic surface, the aerodynamic-surface displacement is written as

$$\mathbf{x}(i\omega) = \mathbf{H}_{ag}\Phi\mathbf{u}(i\omega).$$

Thus, $\mathbf{u}(i\omega)$ represents the generalized excitation amplitudes, whereas $\mathbf{x}(i\omega)$ is the induced harmonic motion of the aerodynamic surface entering the LFD equations. In the present framework, multiple input components can therefore be considered.

In the motion-induced case considered here, the first-harmonic LFD problem can be written in the form used in the literature as [13, 21]

$$\left(i\omega \mathbf{M} + \left.\frac{\partial \mathbf{R}}{\partial \mathbf{W}}\right|_0\right) \mathbf{W}(i\omega) = -\left(\left.\frac{\partial \mathbf{R}}{\partial \mathbf{x}}\right|_0 + i\omega \left.\frac{\partial \mathbf{R}}{\partial \dot{\mathbf{x}}}\right|_0 + i\omega \mathbf{W}_0 \left.\frac{\partial \mathbf{M}}{\partial \mathbf{x}}\right|_0\right) \mathbf{x}(i\omega),$$

where the subscript 0 indicates evaluation at the steady state. Substituting the relation between

generalized coordinates and aerodynamic-surface motion yields

$$\left(i\omega \mathbf{M} + \frac{\partial \mathbf{R}}{\partial \mathbf{W}} \Big|_0 \right) \mathbf{W}(i\omega) = - \left(\frac{\partial \mathbf{R}}{\partial \mathbf{x}} \Big|_0 + i\omega \frac{\partial \mathbf{R}}{\partial \dot{\mathbf{x}}} \Big|_0 + i\omega \mathbf{W}_0 \frac{\partial \mathbf{M}}{\partial \mathbf{x}} \Big|_0 \right) \mathbf{H}_{ag} \Phi \mathbf{u}(i\omega).$$

In the notation of the present paper, this equation is written compactly as

$$\mathbf{L}_a(i\omega) \mathbf{W}(i\omega) = \mathbf{B}_u(i\omega) \mathbf{u}(i\omega),$$

with

$$\mathbf{L}_a(i\omega) := i\omega \mathbf{M}_0 + \frac{\partial \mathbf{R}}{\partial \mathbf{W}} \Big|_0,$$

and

$$\mathbf{B}_u(i\omega) := - \left(\frac{\partial \mathbf{R}}{\partial \mathbf{x}} \Big|_0 + i\omega \frac{\partial \mathbf{R}}{\partial \dot{\mathbf{x}}} \Big|_0 + i\omega \mathbf{W}_0 \frac{\partial \mathbf{M}}{\partial \mathbf{x}} \Big|_0 \right) \mathbf{H}_{ag} \Phi.$$

Accordingly, $\mathbf{L}_a(i\omega)$ denotes the frequency-dependent linearized aerodynamic operator, whereas $\mathbf{B}_u(i\omega) \mathbf{u}(i\omega)$ represents the forcing induced by the prescribed generalized harmonic motion. Equivalent formulations are also used in more recent implementations of LFD solvers for motion-induced and gust-induced airloads [16, 22].

For each prescribed angular frequency ω , solving the LFD system yields the complex first-harmonic flow solution $\mathbf{W}(i\omega)$. The aerodynamic quantities of interest are then obtained by post-processing of this solution. Denoting by $\mathbf{y}(i\omega) \in \mathbb{C}^{n_y}$ the vector of selected aerodynamic outputs, one may write

$$\mathbf{y}(i\omega) = \mathbf{C}_y(i\omega) \mathbf{W}(i\omega) + \mathbf{D}_y(i\omega) \mathbf{u}(i\omega),$$

where $\mathbf{C}_y(i\omega)$ is the output operator associated with the selected aerodynamic quantity and $\mathbf{D}_y(i\omega)$ accounts for possible direct contributions of the prescribed harmonic input to the output. Such a contribution may arise, for example, when the output depends not only on the flow-state perturbation but also on geometric terms associated with the harmonic motion or surface deformation.

The aerodynamic transfer matrix $\mathbf{Q}(i\omega) \in \mathbb{C}^{n_y \times n_u}$ is then defined through the input-output relation

$$\mathbf{y}(i\omega) = \mathbf{Q}(i\omega) \mathbf{u}(i\omega). \quad (2)$$

Accordingly, $\mathbf{Q}(i\omega)$ can be written in the form

$$\mathbf{Q}(i\omega) = \mathbf{C}_y(i\omega) \mathbf{L}_a(i\omega)^{-1} \mathbf{B}_u(i\omega) + \mathbf{D}_y(i\omega).$$

Thus, $\mathbf{Q}(i\omega)$ represents the aerodynamic transfer matrix induced by the LFD solution and the selected aerodynamic post-processing. Depending on the application, the output vector $\mathbf{y}(i\omega)$ may represent, for example, pressure coefficients, integrated force or moment coefficients, generalized aerodynamic forces, or surface-based aerodynamic quantities.

In the present work, the transfer matrices $\mathbf{Q}(i\omega)$ are evaluated over a set of angular frequencies and subsequently used as the input data for the reduced-order realization described in Section 2.2. In terms of reduced frequency, the sampled ranges are $k = [0 : 0.025 : 1]$ for the

OAT15A airfoil and $k = [0 : 0.1 : 6]$ for the CRM configuration. The role of the LFD solver is therefore to provide frequency-domain aerodynamic data of sufficiently high fidelity to capture the dominant buffet-driving fluid dynamics, while avoiding the cost of nonlinear time-accurate simulations over many oscillation periods at each frequency. This computational advantage is one of the reasons why LFD methods have become attractive for practical analyses of flutter and gust-induced airloads, and in the present context it also enables a reduced-order stability analysis for buffet onset.

Note that the LFD equations have been written in terms of the angular frequency ω , consistent with the standard formulation of the harmonic linearized problem. For the reduced-order identification and the presentation of aerodynamic transfer data in the remainder of the paper, this frequency variable is recast in terms of the reduced frequency $k = \omega l_{\text{ref}}/U_\infty$, where l_{ref} is the chosen reference length and U_∞ is the freestream velocity. Accordingly, once the aerodynamic transfer matrix has been computed, the sampled transfer data may be expressed equivalently as functions of ω or k . In the present work, the notation in terms of reduced frequency k is adopted.

2.2 Combined Loewner–POD framework

POD projection and reconstruction of aerodynamic modes

Let $\mathbf{u}(ik) \in \mathbb{C}^{n_u}$ denote the vector of amplitudes of the imposed harmonic inputs at reduced frequency k , and let $\mathbf{y}(ik) \in \mathbb{C}^{n_y}$ denote the corresponding vector of aerodynamic output amplitudes obtained from the linearized frequency-domain solver. The aerodynamic transfer matrix $\mathbf{Q}(ik) \in \mathbb{C}^{n_y \times n_u}$ is defined by the input-output relation provided in Eq. 2.

Thus, each entry of $\mathbf{Q}(ik)$ represents the complex response of one aerodynamic output component to one harmonic input component at reduced frequency k . Depending on the selected application, the input vector $\mathbf{u}(ik)$ may contain one or several generalized harmonic motions, such as rigid-body heave or pitch motions, or selected elastic modal coordinates. In the present framework, multiple input components can therefore be considered. In practice, however, a realization based on a single representative degree of freedom is often sufficient, since the dominant fluid-mode pole generally leaves a clear signature in the corresponding aerodynamic transfer function. For physically meaningful rigid-body or elastic motions, it is not expected that the excitation be exactly orthogonal to the dominant fluid mode, so that its associated pole remains observable in the realized model.

In the present work, the aerodynamic transfer matrix is used to construct a reduced-order aerodynamic model directly from frequency-domain data by means of the Loewner framework. Unlike classical rational-function approximations, the Loewner realization does not require an a priori choice of pole locations and is therefore well suited to transonic flows, where complex poles associated with fluid modes are essential for representing buffet-related dynamics [19].

When the aerodynamic output is high-dimensional, as for distributions of surface pressure coefficients, the realization can be first projected onto a low-dimensional basis obtained by proper orthogonal decomposition (POD). In the present formulation, the POD basis is extracted directly from the aerodynamic transfer functions in the frequency domain, rather than from time-domain snapshots. This viewpoint is consistent with the frequency-domain Karhunen-Loeve/POD formulation proposed by Kim [23] and with the frequency-domain POD-based reduced-order aerodynamic modeling of Hall et al. [24]. More generally, Dergham et al. [25] showed that frequency-domain, or “frequential,” snapshots can be used to construct POD-type

reduced-order models not only for stable systems, but also for unstable flows without marginal modes. This point is especially relevant here, since buffet onset is associated with the appearance of unstable aerodynamic poles.

For each sampled reduced frequency k_ℓ , $\ell = 1, \dots, N_k$, and each input channel $m = 1, \dots, n_u$, let $\mathbf{y}_m(ik_\ell) \in \mathbb{C}^{n_y}$ denote the corresponding aerodynamic output vector associated with a unit excitation in input component m . These vectors are assembled into the snapshot matrix

$$\mathbf{Y} = [\mathbf{y}_1(ik_1) \ \mathbf{y}_1(ik_2) \ \cdots \ \mathbf{y}_1(ik_{N_k}) \ \mathbf{y}_2(ik_1) \ \cdots \ \mathbf{y}_{n_u}(ik_{N_k})] \in \mathbb{C}^{n_y \times n_s},$$

where $n_s = n_u N_k$ is the total number of frequency-domain samples. To obtain a real-valued POD basis while retaining the information contained in the complex frequency response, the matrix $\mathbf{Y}$ is replaced by

$$\mathbf{Y}_S = [\text{Re}(\mathbf{Y}) \ \text{Im}(\mathbf{Y})] \in \mathbb{R}^{n_y \times 2n_s}.$$

The POD basis is then obtained from the singular-value decomposition

$$\mathbf{Y}_S = \Phi_{\text{POD}} \Sigma \mathbf{V}^T \approx \Phi_r \Sigma_r \mathbf{V}_r^T,$$

where $\Phi_{\text{POD}} \in \mathbb{R}^{n_y \times n_y}$ contains the POD modes. The retained basis is denoted by $\Phi_r \in \mathbb{R}^{n_y \times n_{\text{POD}}}$, where $n_{\text{POD}} \ll n_y$ is selected from the singular-value decay. The corresponding truncated singular-value matrix and right singular-vector matrix are $\Sigma_r \in \mathbb{R}^{n_{\text{POD}} \times n_{\text{POD}}}$ and $\mathbf{V}_r \in \mathbb{R}^{2n_s \times n_{\text{POD}}}$, respectively. The original aerodynamic outputs are then projected according to

$$\tilde{\mathbf{y}}(ik) = \Phi_r^T \mathbf{y}(ik).$$

Substituting the definition of $\mathbf{Q}(ik)$ into the previous expression gives

$$\tilde{\mathbf{y}}(ik) = \Phi_r^T \mathbf{Q}(ik) \mathbf{u}(ik),$$

so that the transfer matrix in POD coordinates is

$$\tilde{\mathbf{Q}}(ik) = \Phi_r^T \mathbf{Q}(ik), \quad \tilde{\mathbf{Q}}(ik) \in \mathbb{C}^{n_{\text{POD}} \times n_u}.$$

The Loewner realization is subsequently constructed for $\tilde{\mathbf{Q}}(ik)$ rather than for the original high-dimensional transfer matrix. In this way, the interpolation is carried out in a reduced output space that preserves the dominant spatial structures of the aerodynamic response while substantially reducing the realization cost. At the same time, this projection is not used only for dimensionality reduction: it also provides the basis for reconstructing the dominant fluid mode in physical space once the reduced-space realization has been constructed. The corresponding full-field aerodynamic vector is recovered by lifting the reduced output through the POD basis,

$$\hat{\mathbf{y}}_j = \Phi_r \tilde{\mathbf{y}}_j. \quad (3)$$

This reconstruction capability is essential in the present work, because it shows that the combined Loewner–POD framework provides not only the dominant buffet eigenvalue, but also the associated fluid-mode eigenvector in physical space. The POD basis defines the reduced output space in which the aerodynamic transfer matrix is represented and the Loewner realization is constructed, while also enabling reconstruction of the dominant fluid-mode eigenvector.

Loewner realization of the reduced aerodynamic transfer matrix

Upon application of the Loewner framework to the reduced transfer matrix $\tilde{\mathbf{Q}}(ik)$ at a given steady operating condition defined by the angle of attack α_0 , Mach number M_∞ , and Reynolds number Re , a generalized aerodynamic state-space model is obtained in the form

$$\begin{aligned} \mathbf{E}_{a0}(\alpha_0, M_\infty, Re) &\in \mathbb{R}^{n_a \times n_a}, & \mathbf{A}_{a0}(\alpha_0, M_\infty, Re) &\in \mathbb{R}^{n_a \times n_a}, \\ \mathbf{B}_{a0}(\alpha_0, M_\infty, Re) &\in \mathbb{R}^{n_a \times n_u}, & \mathbf{C}_{a0}(\alpha_0, M_\infty, Re) &\in \mathbb{R}^{n_{POD} \times n_a}. \end{aligned}$$

These matrices define a generalized state-space realization that interpolates the reduced transfer matrix on the imaginary axis, that is, in the frequency domain [19]. Note also that the steady CFD solver can converge to a steady solution even when that solution is linearly unstable. The reduced aerodynamic realization may therefore be constructed about a steady base flow corresponding to an unstable operating condition. If the same condition is simulated with an unsteady solver, small perturbations will grow, so that the steady solution is no longer dynamically sustained. As finite-amplitude oscillations develop, the mean flow departs from the original steady solution, and the response may then either evolve away from the steady state or settle into a limit-cycle oscillation.

The set of eigenvalues $\lambda(\mathbf{E}_{a0}, \mathbf{A}_{a0})$ such that

$$\det(\lambda \mathbf{E}_{a0} - \mathbf{A}_{a0}) = 0$$

is computed numerically. For clarity, the explicit dependence on (α_0, M_∞, Re) is omitted in the following.

From the computed eigenvalues, only a subset within a prescribed numerical tolerance λ_∞ is retained,

$$\lambda_f = \{\lambda \in \lambda(\mathbf{E}_{a0}, \mathbf{A}_{a0}) \mid |\lambda| \leq \lambda_\infty\}.$$

Here, λ_∞ is chosen based on the maximum frequency covered by the frequency-domain data used for identification. In the applications of Section 3, this corresponds to $\lambda_\infty = 1$ for the OAT15A airfoil and $\lambda_\infty = 6$ for the CRM case.

In contrast with the earlier coupled formulation, unstable poles are not removed a priori. Instead, sufficiently dominant unstable poles, according to a criterion defined below, are retained, whereas the remaining unstable poles are stabilized by mirroring them through the imaginary axis. Denoting by λ_a the set of poles used for the residualized realization, one writes

$$\lambda_a = \lambda_s \cup \lambda_{u,\text{keep}} \cup \lambda_{u,\text{flip}},$$

where λ_s is the set of stable poles, $\lambda_{u,\text{keep}}$ is the set of unstable poles retained as physically meaningful, and $\lambda_{u,\text{flip}}$ is the set of unstable poles that are mirrored according to

$$\lambda \mapsto -\text{Re}(\lambda) + i \text{Im}(\lambda).$$

With the total number n_{a0} of retained poles, a least-squares fit to the reduced transfer matrix is imposed in the frequency domain. Using a polynomial residualization of degree 2, the transfer matrix is written as

$$\sum_{m=1}^{n_r} \frac{\mathbf{R}_{r,m}}{ik - \lambda_{r,m}} + \sum_{n=1}^{n_c} \left[\frac{\mathbf{R}_{c,n}}{ik - \lambda_{c,n}} + \frac{\bar{\mathbf{R}}_{c,n}}{ik - \bar{\lambda}_{c,n}} \right] + \mathbf{Q}_0 + \mathbf{Q}_1 ik - \mathbf{Q}_2 k^2 = \tilde{\mathbf{Q}}(ik), \quad (4)$$

where n_r is the number of real poles satisfying $\text{Im}(\lambda_{a,m}) = 0$ and n_c is the number of complex-conjugate pairs, so that $n_{a0} = n_r + 2n_c$. The matrices $\mathbf{R}_{r,m}$, $\mathbf{R}_{c,n}$, and $\bar{\mathbf{R}}_{c,n}$ are the residues associated with the poles $\lambda_{r,m}$, $\lambda_{c,n}$, and $\bar{\lambda}_{c,n}$, respectively. The polynomial term up to second order in reduced frequency accounts for the behavior at infinity, so that no descriptor matrix is required in the final residualized realization [19].

Upon solving the least-squares problem defined by Eq. 4, the associated standard state-space matrices are obtained from the residue-pole representation by rank factorization of the residue matrices. For a real pole, let

$$\mathbf{R}_{r,m} = \mathbf{U}_m \boldsymbol{\Sigma}_m \mathbf{V}_m^*,$$

denote its singular-value decomposition, where $*$ denotes the conjugate transpose. The rank r_m is determined from the singular values in $\boldsymbol{\Sigma}_m$ using a prescribed numerical tolerance. The corresponding state-space submatrices are

$$\begin{aligned} \mathbf{A}_{r,m} &= \lambda_{r,m} \mathbf{I}_{r_m}, & \mathbf{B}_{r,m} &= (\mathbf{T}_m^T \boldsymbol{\Sigma}_m \mathbf{T}_m) (\mathbf{V}_m \mathbf{T}_m)^*, \\ \mathbf{C}_{r,m} &= \mathbf{U}_m \mathbf{T}_m, & m &= 1, \dots, n_r, \end{aligned}$$

where $\mathbf{T}_m$ extracts the first r_m columns.

For a complex-conjugate pair, let

$$\mathbf{R}_{c,n} = \mathbf{U}_n \boldsymbol{\Sigma}_n \mathbf{V}_n^*, \quad \bar{\mathbf{R}}_{c,n} = \bar{\mathbf{U}}_n \boldsymbol{\Sigma}_n \mathbf{V}_n^T.$$

The corresponding state-space submatrices are

$$\begin{aligned} \mathbf{A}_{c,n} &= \begin{bmatrix} \lambda_n \mathbf{I}_{r_n} & \mathbf{0} \\ \mathbf{0} & \bar{\lambda}_n \mathbf{I}_{r_n} \end{bmatrix}, \\ \mathbf{B}_{c,n} &= \begin{bmatrix} (\mathbf{T}_n^T \boldsymbol{\Sigma}_n \mathbf{T}_n) (\mathbf{V}_n \mathbf{T}_n)^* \\ (\mathbf{T}_n^T \boldsymbol{\Sigma}_n \mathbf{T}_n) (\mathbf{V}_n^T \mathbf{T}_n) \end{bmatrix}, & \mathbf{C}_{c,n} &= [\mathbf{U}_n \mathbf{T}_n \quad \bar{\mathbf{U}}_n \mathbf{T}_n], \end{aligned}$$

with r_n the rank of $\boldsymbol{\Sigma}_n$.

The global state-space matrices are then assembled as

$$\begin{aligned} \mathbf{A}_{ac} &= \text{blkdiag}(\mathbf{A}_{r,1}, \dots, \mathbf{A}_{r,n_r}, \mathbf{A}_{c,1}, \dots, \mathbf{A}_{c,n_c}), \\ \mathbf{B}_{ac} &= [\mathbf{B}_{r,1}^T \cdots \mathbf{B}_{r,n_r}^T \mathbf{B}_{c,1}^T \cdots \mathbf{B}_{c,n_c}^T]^T, \\ \mathbf{C}_{ac} &= [\mathbf{C}_{r,1} \cdots \mathbf{C}_{r,n_r} \mathbf{C}_{c,1} \cdots \mathbf{C}_{c,n_c}], \end{aligned}$$

where $\text{blkdiag}(\cdot)$ denotes the matrix formed by placing its arguments on the diagonal and zeros elsewhere.

Finally, the state-space matrices are transformed so that only real entries remain,

$$\mathbf{A}_a = \mathbf{T} \mathbf{A}_{ac} \mathbf{T}^*, \quad \mathbf{B}_a = \mathbf{T} \mathbf{B}_{ac}, \quad \mathbf{C}_a = \mathbf{C}_{ac} \mathbf{T}^*,$$

where

$$\mathbf{T} = \begin{bmatrix} \mathbf{I}_{n_{ar}} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{n_{ac}} \otimes \mathbf{J} \end{bmatrix}, \quad \mathbf{J} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -i & i \end{bmatrix},$$

with

$$n_{ar} = \sum_{m=1}^{n_r} r_m, \quad n_{ac} = 2 \sum_{n=1}^{n_c} r_n, \quad n_a = n_{ar} + n_{ac} \leq n_{a0}.$$

The resulting matrices $(\mathbf{A}_a, \mathbf{B}_a, \mathbf{C}_a)$, together with the polynomial term $(\mathbf{Q}_0, \mathbf{Q}_1, \mathbf{Q}_2)$, define the residualized reduced-order aerodynamic model used in the remainder of the paper.

Identification of unstable fluid modes and buffet-onset criterion

The poles of the reduced aerodynamic model are the eigenvalues of $\mathbf{A}_a$. They are interpreted as fluid, or aerodynamic, poles. Their associated right and left eigenvectors, ϕ_j and ψ_j , satisfy

$$\mathbf{A}_a \phi_j = \lambda_{a,j} \phi_j, \quad \psi_j^* \mathbf{A}_a = \lambda_{a,j} \psi_j^*.$$

If the realization is built in POD coordinates, the corresponding full-field aerodynamic eigenvector is reconstructed according to Eq. 3.

$$\hat{\mathbf{y}}_j = \Phi_r \tilde{\mathbf{y}}_j = \Phi_r \mathbf{C}_a \phi_j.$$

The reduced number of aerodynamic states reduces the cost of extracting these poles relative to GSA applied to the full-size problem. At the same time, matching the transfer function on the imaginary axis ensures that the poles dominating the buffet response are captured by the realization.

To quantify the contribution of each aerodynamic pole to the transfer matrix, residues are computed by means of the generalized two-sided Rayleigh quotient. For each pole j ,

$$\mathbf{R}_j = \frac{(\mathbf{C}_a \phi_j) (\psi_j^* \mathbf{B}_a)}{\psi_j^* \phi_j}.$$

The significance of pole j is measured by

$$\delta_j = \frac{\|\mathbf{R}_j\|_2}{|\operatorname{Re}(\lambda_{a,j})|}, \quad j = 1, \dots, n_a.$$

This criterion combines residue magnitude with distance to the imaginary axis. In the present work, the dominant fluid mode is identified as the pole with maximum δ_j . Selecting the dominant pole in this way is more robust than choosing the least-stable pole solely from its real part [19]. When this dominant pole is associated with buffet onset, its corresponding eigenvector is interpreted as the buffet mode. In that case, the reconstructed quantity $\hat{\mathbf{y}}_j$ represents the spatial structure of the buffet mode in the selected aerodynamic output.

A key point is that the reduced aerodynamic realization is allowed to contain physically unstable poles. However, not every unstable pole produced by the interpolation is necessarily physical. To distinguish physically meaningful unstable poles from spurious ones, the unstable poles are filtered according to a normalized significance criterion. Unstable poles are those satisfying

$$\operatorname{Re}(\lambda_{a,j}) > 0.$$

Among these, only unstable poles with sufficiently large normalized significance are retained,

$$\frac{\delta_j}{\delta_{\max}} > \delta_u, \quad \operatorname{Re}(\lambda_{a,j}) > 0, \quad j = 1, \dots, n_a, \quad (5)$$

where

$$\delta_{\max} = \max_j \delta_j$$

and $\delta_u \in (0, 1)$ is a prescribed threshold. In the present work, the unstable-pole retention threshold is set to $\delta_u = 0.5$ for the OAT15A airfoil and $\delta_u = 0.02$ for the CRM configuration. These values are case dependent and reflect differences in the relative separation between physically relevant unstable poles and spurious unstable poles in the two applications. In both cases, the procedure was found to remain effective over a finite interval of threshold values, indicating that the identification of the dominant buffet-driving mode is not tied to a single finely tuned choice of δ_u . A more systematic sensitivity analysis of this parameter is beyond the scope of the present work and is left for future study.

Unstable poles satisfying the condition in Eq. 5 are retained as physically meaningful buffet poles. The remaining unstable poles are mirrored to the stable half-plane before the final residualized realization is constructed [19]. This selective treatment is consistent with the implementation used in the present work and allows buffet-related unstable poles to be preserved while suppressing spurious unstable interpolation poles.

Once the dominant unstable fluid mode has been identified, buffet onset is determined directly from the eigenvalues of the reduced aerodynamic model. The onset condition is reached when the real part of the corresponding eigenvalue changes sign and becomes positive, see Eq. 1. The corresponding imaginary part of $\lambda_{a,\text{dom}}$ provides the linear buffet frequency, and the associated eigenvector defines the buffet mode. In a nonlinear time-marching simulation, the observed oscillation frequency may differ from this value once the flow develops finite-amplitude limit-cycle oscillations [26]. Because the criterion is evaluated on a reduced-order realization built from linearized frequency-domain data, the resulting procedure offers a computationally efficient alternative to solving the full linearized computational fluid dynamics eigenvalue problem directly. Evaluating the criterion on the reduced aerodynamic realization also allows buffet onset to be treated as a purely aerodynamic instability of the reduced model without requiring structural coupling as in [19].

3 APPLICATION CASES

3.1 OAT15A airfoil

The OAT15A airfoil is considered as a representative two-dimensional transonic buffet test case [1]. It is a well-established benchmark for shock-buffet studies and is particularly suitable here because reference experimental and numerical data are available in the literature. In the present work, the configuration is used as a rigid aerodynamic case for the assessment of the proposed buffet-onset prediction method.

The aerodynamic data are based on Reynolds-averaged Navier-Stokes simulations on a two-dimensional unstructured mesh. The OAT15A mesh contains approximately 1.5×10^5 grid points. In the outer boundary-layer and wake region, an approximately isotropic spacing of $\Delta x \approx 0.5\%$ of the chord is employed, while wall-tangent and wall-normal stretching ratios are kept below 1.2. The farfield boundary is located at a distance of 100 chord lengths from the airfoil in order to minimize numerical reflections. A representative view of the computational mesh is shown in Fig. 1.

In the reference setup, the flow solution is obtained with the DLR TAU solver [27, 28]. The steady Reynolds-averaged Navier-Stokes (RANS) equations are discretized in space using a

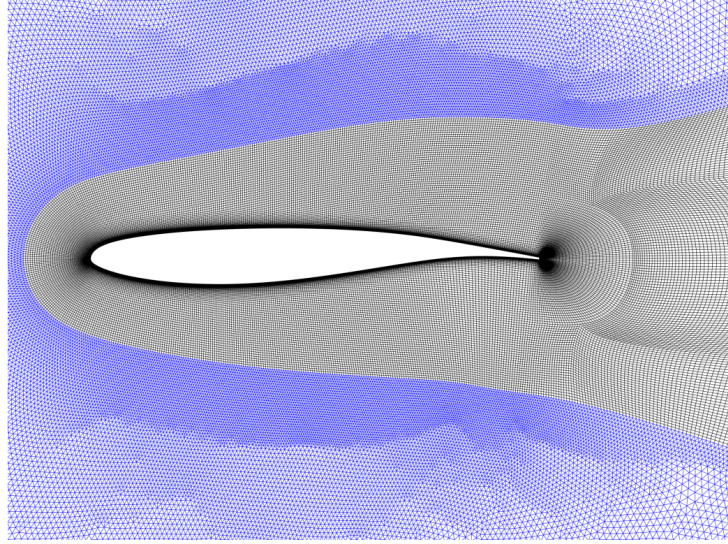


Figure 1: OAT15A airfoil mesh [11].

second-order Jameson–Schmidt–Turkel (JST) scheme with matrix dissipation [28, 29]. Gradients are computed using a Green–Gauss formulation, while the turbulence-model equation is treated with a second-order average of flux. The compressibility-corrected negative Spalart–Allmaras turbulence model is employed [30]. Prescribed surface motions entering the LFD right-hand side are propagated to the surrounding volume mesh by radial-basis-function interpolation. The steady solution is advanced in pseudo-time with a standard backward Euler scheme, and steady convergence is reached down to a density-residual norm of 10^{-12} . For unsteady RANS (URANS), a physical time-step size of 10^{-5} s is used. Each time step is converged by up to 50 lower–upper symmetric Gauss–Seidel (LU-SGS) subiterations, terminated either by a Cauchy criterion on C_D or by reaching the maximum number of subiterations. In all cases considered here, the time steps converged within 50 subiterations. This temporal discretization corresponds to approximately 5000 time steps per buffet cycle.

For the OAT15A case, the Reynolds number is fixed to $Re = 3 \times 10^6$ and the freestream temperature to 273 K. At the representative Mach number of $M_\infty = 0.73$, the flow exhibits the characteristic transonic features associated with pre-buffet conditions, including a recompression shock and shock-induced separation effects whose sensitivity to angle of attack is analyzed in detail in the reference study. In the present work, the linearized frequency-domain excitation is introduced through a prescribed pitching motion about the 40% chord point.

Figure 2 shows the evolution of the dominant fluid eigenvalue, defined as the pole with maximum δ_j , for $\alpha = 2.8^\circ, 3.0^\circ, 3.1^\circ, 3.2^\circ$, and 3.4° . Buffet onset is predicted between $\alpha = 3.1^\circ$ and 3.2° , in excellent agreement with the dominant eigenvalue obtained from GSA. The Loewner realization is built here from the lift coefficient associated with a prescribed pitching degree of freedom, and the same dominant eigenvalue is also recovered from other aerodynamic quantities (such as drag or moment coefficient) and from other physically meaningful input excitations. The recovery of the same dominant eigenvalue from different output quantities and input excitations indicates that the identified buffet pole is insensitive to the particular output quantity and input excitation used for the realization.

The time-domain estimate shown at $\alpha = 3.2^\circ$ is obtained with the matrix pencil method. This method represents the scalar response signal as a sum of exponentially damped complex modes

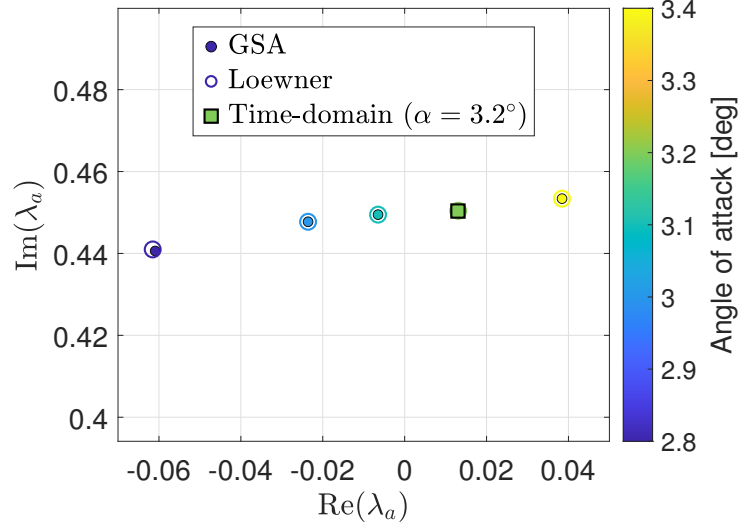


Figure 2: Dominant fluid eigenvalue for the OAT15A airfoil from the Loewner–LFD model, GSA, and time-domain analysis.

and identifies the corresponding poles from a reduced generalized eigenvalue problem formed from shifted Hankel matrices [31, 32]. The matrix pencil method first identifies the discrete-time poles z_j of the sampled signal. The corresponding continuous-time eigenvalues are then obtained from $\lambda_j = \log(z_j)/\Delta t$.

3.2 CRM configuration

The second application case is the NASA Common Research Model (CRM), used here as a representative three-dimensional wing shock-buffet configuration of direct relevance to transport aircraft aerodynamics. The CRM is a generic commercial wide-body aircraft configuration with design Mach number 0.85 and nominal lift coefficient 0.5. The wing has an aspect ratio of 9, a taper ratio of 0.275, and a quarter-chord sweep angle of 35° . For the wind-tunnel model, the mean aerodynamic chord is 0.189 m, the span is 1.586 m, and the reference area is 0.280 m². The configuration considered here, following Timme [3], is a wing-body-tail variant with a 0° tail setting angle, while pylon, nacelle, and blade-sting support are excluded. In addition, the static aeroelastic wing deformation is imposed according to measurements from the European Transonic Wind tunnel (ETW).

The free-stream Mach number is $M_\infty = 0.85$, and the Reynolds number based on the reference chord is $Re = 5.0 \times 10^6$. The analysis targets angles of attack in the range 3.5° to 4.0° , which bracket the onset of shock buffet. According to Timme [3], the instability onset occurs just above 3.7° , with an associated Strouhal number of approximately 0.39.

The computational mesh used in the reference study is generated with the SOLAR mesh generator and follows accepted industrial practice for full-aircraft configurations. The baseline half-model mesh contains approximately 6.2×10^6 points, including about 1.7×10^5 points on solid walls. A viscous wall resolution of $y^+ < 1$ is kept. The hemispherical farfield boundary is located 100 semi-span lengths away from the aircraft, and a symmetry boundary condition is imposed in the fuselage centre plane.

The flow is computed with the DLR-TAU Reynolds-averaged Navier-Stokes solver using a second-order vertex-centred finite-volume discretization. Inviscid fluxes are evaluated with a central scheme with matrix artificial dissipation, gradients for viscous fluxes and source terms

are computed with the Green-Gauss theorem, and turbulent closure is provided by the negative Spalart-Allmaras model. Steady base flows are obtained with backward Euler time stepping, lower-upper symmetric Gauss-Seidel iterations, local time stepping, and geometric multigrid acceleration.

The CRM case exhibits the characteristic three-dimensional features of wing shock buffet. The steady surface-pressure distributions shown in Timme [3] for $\alpha = 3.5^\circ$, 3.75° , and 4.0° reveal a distinct spanwise shock pattern together with a shock-induced reverse-flow region in the mid-semi-span sector, located just outboard of the Yehudi break. The distribution for $\alpha = 3.75^\circ$ is reproduced in Figure 3. With increasing angle of attack, the shock moves upstream and the reverse-flow region expands spanwise, consistent with buffet onset physics. Timme [3] reported that a single unstable oscillatory global mode becomes unstable between $\alpha = 3.70^\circ$ and 3.75° , thereby triggering shock-buffet onset. The associated mode is characterized by a spanwise-localized cellular structure combining shock oscillation and downstream shear-layer pulsation, which is representative of three-dimensional buffet on a finite swept wing.

The CRM is therefore a suitable three-dimensional validation case: it is geometrically and physically much richer than the two-dimensional airfoil benchmark considered in Section 3.1, yet it is sufficiently documented in the literature to allow comparison with a reference global-stability analysis. Figure 4 compares the pitch-pitch component of the generalized aerodynamic-force matrix obtained from the Loewner realization with the reference linearized frequency-domain result. In the present case, close agreement is obtained over the full reduced-frequency range only when the unstable aerodynamic pole associated with buffet is retained in the realization. If that unstable pole is removed or stabilized, the characteristic local peak and phase variation in the response are no longer reproduced. The loss of agreement when that unstable pole is removed or stabilized confirms that, for buffet conditions in a rigid aerodynamic configuration, the reduced model must be allowed to contain physically meaningful unstable fluid poles.

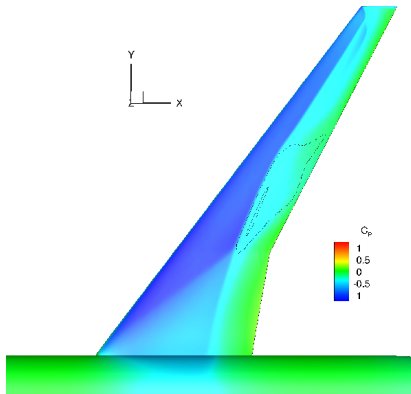


Figure 3: Steady pressure over the CRM wing at Mach 0.85, angle of attack 3.75° .

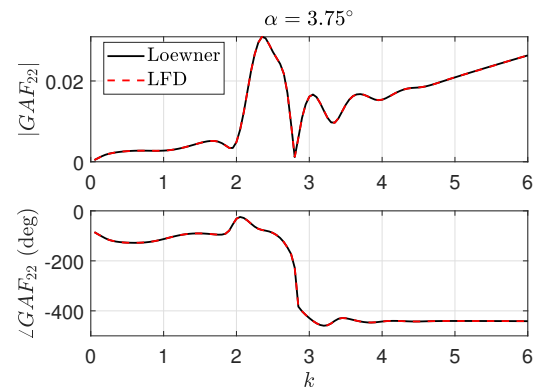


Figure 4: Pitch-pitch GAF component vs reduced frequency: Loewner realization vs LFD reference. Mach 0.85, angle of attack 3.75° .

To assess the robustness of the proposed procedure, different aerodynamic output quantities were used to construct the reduced aerodynamic realization, including selected generalized aerodynamic-force (GAF) components and the lift coefficient. In all cases, the same dominant fluid eigenvalue was identified. This indicates, as in the two-dimensional case discussed in Section 3.1, that the identified buffet-driving mode is not specific to a particular output quantity or input forcing, but is instead a dominant feature of the aerodynamic system. Figures 5 and 6 compare representative subsets of the fluid eigenvalues obtained from the Loewner realization and from global stability analysis (GSA) for the CRM configuration at two angles of attack in the vicinity of buffet onset. The comparison shows that the LFD-based Loewner model captures the dominant buffet-relevant eigenvalues of the three-dimensional flow. In particular, the eigenvalues closest to the imaginary axis are recovered with good agreement. At $\alpha = 3.70^\circ$ in Figure 5, the relevant modes remain stable, although a subset already lies relatively close to the imaginary axis.

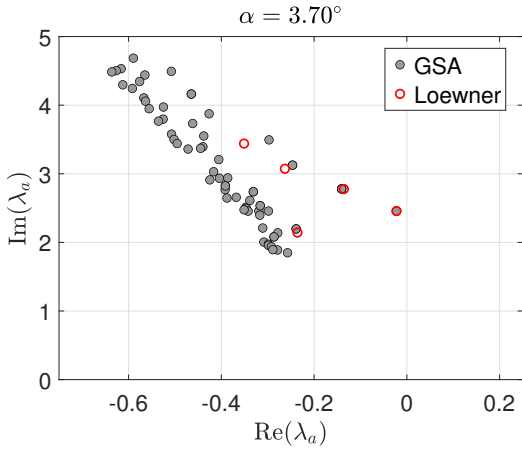


Figure 5: Fluid eigenvalues at $\alpha = 3.70^\circ$, showing agreement between GSA and the Loewner realization in the near-critical stable regime.

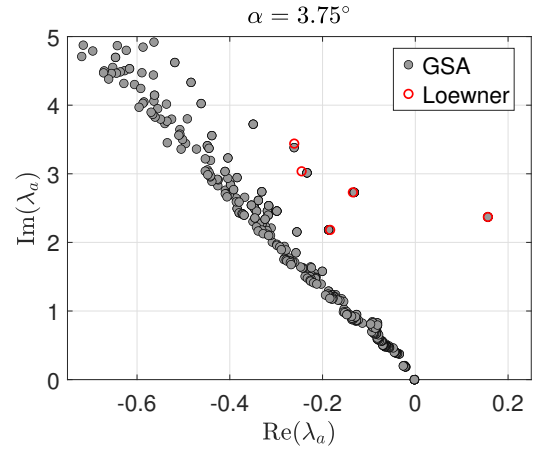


Figure 6: Fluid eigenvalues at $\alpha = 3.75^\circ$, showing agreement between the Loewner realization and GSA near instability.

At $\alpha = 3.75^\circ$ shown in Figure 6, the spectrum becomes unstable, as indicated by the change in sign of the real part of the eigenvalue associated with the dominant buffet fluid mode, which becomes positive. At the same time, the reduced aerodynamic model continues to reproduce the dominant buffet-driving eigenvalues. These results support the use of the Loewner realization as a reduced-order surrogate for extracting the fluid modes governing buffet onset in realistic three-dimensional configurations.

In addition to the eigenvalue comparison, the spatial structure of the dominant buffet mode is examined through the corresponding eigenvectors. As discussed in Section 2.2, the reduced aerodynamic realization constructed from LFD data with the combined Loewner and POD procedure provides not only the dominant fluid poles, but also the associated modal distributions in physical space. In the present formulation, the POD basis is used both to define the reduced output coordinates on which the Loewner realization is built and to reconstruct the full-field eigenvector from the reduced modal representation. For the CRM configuration, this allows a direct comparison of the buffet-mode eigenvector obtained from the reduced model with the reference eigenvector from global stability analysis.

The dimensionality reduction enabled by POD is substantial for the CRM configuration. For the surface-based output corresponding to the pressure-coefficient distribution, the full output dimension prior to POD is 167,190 which is reduced to 30 retained POD coordinates. For the volume-based output corresponding to a field quantity, the corresponding full output dimension is 6,164,948, again reduced to 30 retained POD coordinates. In both cases, the Loewner realization is then constructed in the reduced POD space. The corresponding full-field eigenvector is then recovered through Eq. 3.

Figures 7 and 8 compare, at $\alpha = 3.75^\circ$, the real part of the pressure-coefficient perturbation associated with the dominant buffet mode on the wing surface, obtained from the combined Loewner–POD model and GSA, respectively. In both cases, the dominant modal activity is concentrated in the same outboard wing region, with similar shock-aligned and downstream structures. This agreement indicates that the reduced-order model reproduces the surface signature of the buffet mode with good fidelity.

Figures 9 and 10 show, again at $\alpha = 3.75^\circ$, the corresponding real part of the streamwise momentum perturbation in the off-surface flow field downstream of the wing. For the volume-based reduced realization and the corresponding eigenvector reconstruction, the POD basis is constructed from the streamwise momentum field, and 30 POD modes are retained. The reconstructed mode from the combined Loewner–POD model captures the same coherent volumetric structures as the GSA reference, including the dominant localized perturbation structure in the separated-flow region behind the shock and its downstream development. This confirms that the reduced-order model preserves not only the dominant buffet eigenvalue, but also the physically relevant eigenvector structure associated with that mode.

Overall, these comparisons show that the proposed combined Loewner–POD framework recovers not only the dominant buffet eigenvalue, but also the eigenvector structure. The method reproduces both the temporal behavior, through the unstable eigenvalue, and the spatial organization of the mode, through the corresponding eigenvector. The recovery of both the dominant eigenvalue and the associated eigenvector structure is an important result, since the physical interpretation of buffet onset depends not only on the location of the dominant pole in the complex plane, but also on the associated flow structure.

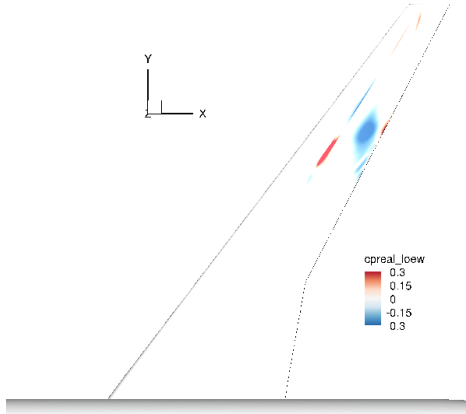


Figure 7: Real part of the CRM buffet-mode surface pressure perturbation at $\alpha = 3.75^\circ$ from the combined Loewner-POD model.

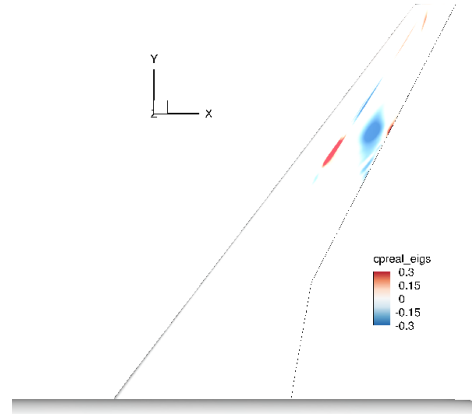


Figure 8: Real part of the CRM buffet-mode surface pressure perturbation at $\alpha = 3.75^\circ$ from GSA.

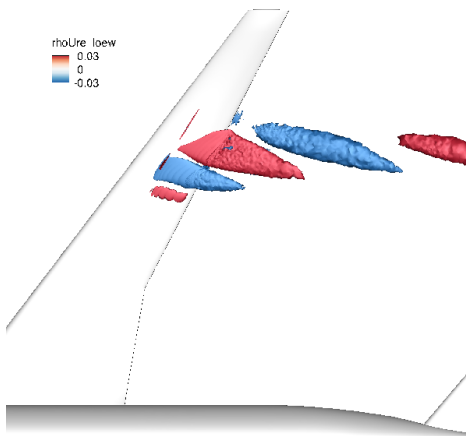


Figure 9: Real part of the CRM buffet-mode streamwise momentum perturbation at $\alpha = 3.75^\circ$ from the combined Loewner-POD model.

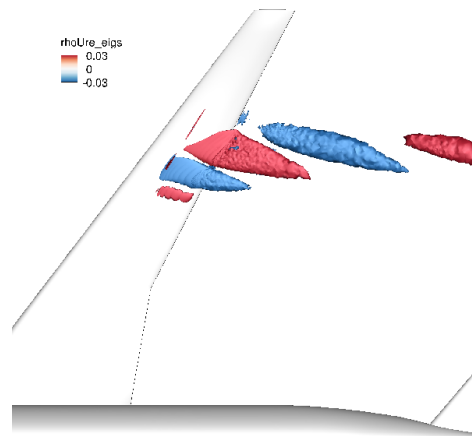


Figure 10: Real part of the CRM buffet-mode streamwise momentum perturbation at $\alpha = 3.75^\circ$ from GSA.

4 CONCLUSIONS

A method for predicting transonic buffet onset from linearized frequency-domain CFD data by means of a reduced-order aerodynamic realization has been presented. To the authors' knowledge, this work is the first application of the Loewner framework to buffet prediction for a realistic three-dimensional aircraft configuration. The approach constructs a Loewner-based state-space model from frequency-domain transfer data and determines buffet onset directly from the eigenvalues of that reduced aerodynamic system. In this framework, the dominant buffet-driving fluid mode is identified through a residue-based significance criterion, and onset is detected when the real part of the corresponding eigenvalue changes sign and becomes positive. A central feature of the method is that physically meaningful unstable fluid poles are intentionally retained in the realization, while spurious unstable poles introduced by interpolation are filtered out.

The results for the OAT15A airfoil and the CRM configuration show that the proposed procedure recovers the dominant buffet-relevant eigenvalues with good agreement relative to reference global stability analysis. For the OAT15A case, the method predicts buffet onset in the expected angle-of-attack range and is consistent with both global-stability and time-domain estimates. For the three-dimensional CRM configuration, the reduced model reproduces the dominant buffet-driving poles near onset and captures the change in stability of the dominant fluid mode as the flow moves into the unstable regime. The CRM results further show that retaining the unstable buffet pole is essential for reproducing the frequency-domain aerodynamic response with sufficient fidelity.

Beyond eigenvalue prediction, the combined Loewner–POD formulation also provides access to the eigenvector associated with the dominant buffet mode. The agreement of the reconstructed modal distributions with the corresponding global-stability results, both on the wing surface and in the off-surface flow field, demonstrates that the proposed framework recovers the dominant buffet mode in a stronger sense than by onset prediction alone, since it preserves not only the unstable eigenvalue responsible for buffet onset but also the associated flow structure in physical space.

Overall, the results show that linearized frequency-domain data can be used to build reduced aerodynamic models that retain the key dynamics governing buffet onset. While the reduced eigenanalysis is performed on a much lower-dimensional system than in direct global stability analysis, the overall computational efficiency remains case dependent because it depends on the number of LFD samples required. In practical settings, the current approach is particularly advantageous when LFD data are already available from flutter or gust-load workflows, since the combined Loewner–POD framework can then reuse those samples and reveal the destabilization of fluid modes without a separate global stability analysis or costly time-domain simulations.

From a practical standpoint, the method supports a more harmonized treatment of buffet and flutter prediction. Previous work already showed that Loewner-based reduced-order models constructed from frequency-domain aerodynamic data can be used in a unified flutter/buffet setting with structural feedback [19], while LFD computations are already employed in industrial processes for gust loads and flutter assessment [14–16]. The present results suggest that the same LFD data used in such workflows can also be exploited for buffet prediction. This possibility of reusing the same LFD data enables a common frequency-domain aerodynamic database and reduced-order modeling framework to be used for gust-loads, flutter, and buffet, rather than relying on separate analysis strategies. With structural feedback included, this

perspective naturally extends to the prediction of pre-buffet and buffet behavior in a unified reduced-order environment.

As an outlook, the methodology should be extended to additional configurations and flow regimes in order to further assess its robustness across a broader class of buffet phenomena, including more complex three-dimensional aircraft geometries and additional operating conditions. A particularly promising direction is its combination with nonlinear reduced-order models for buffet dynamics. Recent work on transonic buffet modeling via invariant manifolds has shown that nonlinear reduced models can capture the post-onset evolution, late transients, limit-cycle behavior, and reconstruction of the full flow field for buffet flows. In this perspective, the present combined Loewner–POD framework could provide an efficient linear modal description near the unstable equilibrium, while invariant-manifold-based or related nonlinear reduced models could extend that description into the nonlinear regime beyond onset. Such a combination could lead to reduced-order models that describe both buffet onset and the nonlinear dynamics beyond onset [26].

5 REFERENCES

- [1] Jacquin, L., Molton, P., Deck, S., et al. (2009). Experimental study of shock oscillation over a transonic supercritical profile. *AIAA Journal*, 47(9), 1985–1994. <https://doi.org/10.2514/1.30190>.
- [2] Masini, L., Timme, S., and Peace, A. J. (2020). Analysis of a civil aircraft wing transonic shock buffet experiment. *Journal of Fluid Mechanics*, 884, A52. <https://doi.org/10.1017/jfm.2019.906>.
- [3] Timme, S. (2020). Global instability of wing shock-buffet onset. *Journal of Fluid Mechanics*, 885, A37. <https://doi.org/10.1017/jfm.2019.1001>.
- [4] Crouch, J. D., Garbaruk, A., and Magidov, D. (2007). Predicting the onset of flow unsteadiness based on global instability. *Journal of Computational Physics*, 224(2), 924–940. <https://doi.org/10.1016/j.jcp.2006.10.035>.
- [5] Crouch, J. D., Garbaruk, A., Magidov, D., et al. (2009). Origin of transonic buffet on aerofoils. *Journal of Fluid Mechanics*, 628, 357–369. <https://doi.org/10.1017/S0022112009006673>.
- [6] Crouch, J. D., Garbaruk, A., and Strelets, M. (2019). Global instability in the onset of transonic-wing buffet. *Journal of Fluid Mechanics*, 881, 3–22. <https://doi.org/10.1017/jfm.2019.748>.
- [7] Paladini, E., Beneddine, S., Dandois, J., et al. (2019). Transonic buffet instability: From two-dimensional airfoils to three-dimensional swept wings. *Physical Review Fluids*, 4(10), 103906. <https://doi.org/10.1103/PhysRevFluids.4.103906>.
- [8] He, W. and Timme, S. (2021). Triglobal infinite-wing shock-buffet study. *Journal of Fluid Mechanics*, 925, A27. <https://doi.org/10.1017/jfm.2021.678>.
- [9] Sansica, A. and Hashimoto, A. (2023). Global stability analysis of full-aircraft transonic buffet at flight Reynolds numbers. *AIAA Journal*, 61(10), 4437–4453. <https://doi.org/10.2514/1.J062808>.

- [10] Gao, C. and Zhang, W. (2020). Transonic aeroelasticity: A new perspective from the fluid mode. *Progress in Aerospace Sciences*, 113, 100596. <https://doi.org/10.1016/j.paerosci.2019.100596>.
- [11] Nitzsche, J., Ringel, L. M., Kaiser, C., et al. (2019). Fluid-mode flutter in plane transonic flows. In *International Forum on Aeroelasticity and Structural Dynamics*. Savannah, Georgia, USA. IFASD 2019. <https://elib.dlr.de/127989> (accessed April 2026).
- [12] Dou, Z., Gao, C., and Zhang, W. (2024). Dynamic characteristic of transonic aeroelasticity affected by fluid mode in pre-buffet flow. *Journal of Fluid Mechanics*, 999, A24. <https://doi.org/10.1017/jfm.2024.764>.
- [13] Thormann, R. and Widhalm, M. (2013). Linear-frequency-domain predictions of dynamic-response data for viscous transonic flows. *AIAA Journal*, 51(11), 2540–2557. <https://doi.org/10.2514/1.J051896>.
- [14] Stickan, B., Meyer, T., Thormann, R., et al. (2024). Linearized CFD and CSM based flutter process for very flexible aircraft. In *International Forum on Aeroelasticity and Structural Dynamics*. The Hague, The Netherlands. IFASD 2024.
- [15] Weigold, W., Stickan, B., Travieso-Alvarez, I., et al. (2017). Linearized unsteady CFD for gust loads with TAU. In *International Forum on Aeroelasticity and Structural Dynamics*. Como, Italy. IFASD 2017.
- [16] Jacobson, K. E., Stanford, B. K., Wood, S. L., et al. (2020). Flutter analysis with stabilized finite elements based on the linearized frequency-domain approach. In *AIAA Scitech 2020 Forum*. Orlando, Florida, USA. <https://doi.org/10.2514/6.2020-0403>.
- [17] Quero, D., Vuillemin, P., and Poussot-Vassal, C. (2021). A generalized eigenvalue solution to the flutter stability problem with true damping: The p-L method. *Journal of Fluids and Structures*, 103(103266). <https://doi.org/10.1016/j.jfluidstructs.2021.103266>.
- [18] Quero, D., Vuillemin, P., and Poussot-Vassal, C. (2022). Improved mode tracking for the p-L flutter solution method based on aeroelastic derivatives. In *International Forum on Aeroelasticity and Structural Dynamics*. Madrid, Spain. IFASD 2022. <https://elib.dlr.de/187885> (accessed April 2026).
- [19] Quero, D., Kaiser, C., and Nitzsche, J. (2024). A general solver for the prediction of flutter and buffet onset. In *International Forum on Aeroelasticity and Structural Dynamics*. The Hague, The Netherlands. IFASD 2024. <https://elib.dlr.de/205025> (accessed April 2026).
- [20] Albano, E. and Rodden, W. P. (1969). A doublet-lattice method for calculating lift distributions on oscillating surfaces in subsonic flows. *AIAA Journal*, 7(2), 279–285. <https://doi.org/10.2514/3.5086>.
- [21] Kaiser, C., Thormann, R., Dimitrov, D., et al. (2015). Time-linearized analysis of motion-induced and gust-induced airloads with the DLR TAU code. In *Deutscher Luft- und Raumfahrtkongress (DLRK) 2015*. Rostock, Germany. DLRK 2015. <https://www.dglr.de/publikationen/2015/370189.pdf> (accessed April 2026).

- [22] Kaiser, C., Quero, D., Nitzsche, J., et al. (2024). Analysis of linearized motion- and gust-induced airloads with a next-generation computational fluid dynamics solver. In *International Forum on Aeroelasticity and Structural Dynamics*. The Hague, The Netherlands. IFASD 2024.
- [23] Kim, T. (1998). Frequency-domain Karhunen–Loeve method and its application to linear dynamic systems. *AIAA Journal*, 36(11), 2117–2123. <https://doi.org/10.2514/2.315>.
- [24] Hall, K. C., Thomas, J. P., and Dowell, E. H. (1999). Reduced-order modelling of unsteady small-disturbance flows using a frequency-domain proper orthogonal decomposition technique. In *37th AIAA Aerospace Sciences Meeting and Exhibit*. Reno, Nevada, USA. <https://doi.org/10.2514/6.1999-655>.
- [25] Dergham, G., Sipp, D., Robinet, J.-C., et al. (2011). Model reduction for fluids using frequential snapshots. *Physics of Fluids*, 23(6), 064101. <https://doi.org/10.1063/1.3590732>.
- [26] Vojković, T., Quero, D., Jayaraj, R., et al. (2026). Transonic buffet modeling via invariant manifolds. <https://doi.org/10.48550/arXiv.2602.10363>.
- [27] Gerhold, T. (2005). Overview of the hybrid RANS code TAU. In *MEGAFLOW — Numerical Flow Simulation for Aircraft Design*. Springer, pp. 81–92. https://doi.org/10.1007/3-540-32382-1_5.
- [28] Schwamborn, D., Gerhold, T., and Heinrich, R. (2006). The DLR TAU-Code: Recent applications in research and industry. In *Proceedings of the European Conference on Computational Fluid Dynamics (ECCOMAS CFD 2006)*. Egmond aan Zee, The Netherlands. <https://elib.dlr.de/22421> (accessed April 2026).
- [29] Jameson, A., Schmidt, W., and Turkel, E. (1981). Numerical solution of the Euler equations by finite volume methods using Runge–Kutta time-stepping schemes. In *AIAA 14th Fluid and Plasma Dynamics Conference*. Palo Alto, California, USA. <https://doi.org/10.2514/6.1981-1259>.
- [30] Allmaras, S. R., Johnson, F. T., and Spalart, P. R. (2012). Modifications and clarifications for the implementation of the Spalart–Allmaras turbulence model. In *7th International Conference on Computational Fluid Dynamics (ICCFD7)*. Big Island, Hawaii, USA. https://www.iccfd.org/iccfd7/assets/pdf/papers/ICCFD7-1902_paper.pdf (accessed April 2026).
- [31] Hua, Y. and Sarkar, T. K. (1990). Matrix pencil method for estimating parameters of exponentially damped/undamped sinusoids in noise. *IEEE Transactions on Acoustics, Speech, and Signal Processing*, 38(5), 814–824. <https://doi.org/10.1109/29.56027>.
- [32] Sarkar, T. K. and Pereira, O. (1995). Using the matrix pencil method to estimate the parameters of a sum of complex exponentials. *IEEE Antennas and Propagation Magazine*, 37(1), 48–55. <https://doi.org/10.1109/74.370583>.

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