



Emergence of Complex Modes in Lightly Damped Structural Systems

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Insights from an ISS Cargo Carrier Modal Test SCLV 2026

**Arya Majed (ASD), Joel Sills (NASA/LaRC),
Greg Walsh (NASA/GSFC), Ed Henkel (ASD)**



1. Introduction

- In a modal test involving an unpressurized ISS Cargo Carrier with various cargo mass configurations all was going as predicted by pretest analysis until ...
- For a particular cargo mass configuration, the measurements indicated a mode-veering behavior involving two closely spaced target modes. The identified mode shapes exhibited dominant imaginary modal components (near-quadrature behavior), while the corresponding eigenvalues remained primarily split in frequency (imaginary direction in the complex plane).
- The configuration was slightly modified to transition out of mode-veering. Whereas the eigenvalues had previously separated primarily in frequency (imaginary component), the modified configuration produced an eigenvalue split primarily in decay rate (real component), now resulting in strongly complex-modes.
- **The observations suggested that strongly complex modes can emerge in lightly damped linear structures through modal coupling alone, without requiring repeated symmetric modes or nonlinear behavior.**



2. Methods/Tools

- During modal testing, a mathematical framework was devised involving the two-mode damped interaction problem. The resulting damped interaction characteristic equation was quartic in the eigenvalue and contained both even and odd powers, unlike the undamped characteristic equation which contains only even powers due to eigenvalue symmetry. The goal was to see whether the observed modal behavior could be mathematically reproduced.
- A combination of perturbation theory, MIT's symbolic calculator (Maxima), and the MSC/NASTRAN complex eigensolver were utilized to reduce the quartic under the light-damping asymptotic assumption. This reduction effectively transformed the local interaction problem from the full complex eigenvalue domain to a reduced perturbation-coordinate domain governed by the local split parameter s .
- The full methodology for this *Reduced Perturbation Framework* will be documented in a forthcoming NASA monograph titled ***Emergence of Complex Modes in Lightly Damped Structural Systems***, planned for release following this initial disclosure.



3. Objectives

- In this short presentation format, we will provide a set of useful equations, relationships, and physical interpretations direct from the damped two-mode interaction problem.
- We will demonstrate that the results from the two-mode interaction problem are in direct agreement with observed test data.
- Practical methods for estimating the parameters driving the relations direct from modal test data are provided.
- We finalize with two examples demonstrating weak and strong interactions.



4. Parameter Definition and Description

Parameter	Definition	Description
ω – uncoupled central frequency	$\omega = \frac{\omega_1 + \omega_2}{2}$	Mean / central modal frequency parameter of the two nearby <u>uncoupled</u> modes. This is the common frequency region where the interaction occurs.
δ – uncoupled separation	$\delta = \frac{\omega_2 - \omega_1}{2}$	Baseline half-frequency <u>uncoupled</u> separation parameter. This is the measure of how close the two modes are before interaction. It can be derived from pretest analysis or test (so long as the modes are uncoupled or weakly interacting).
ϵ – coupling strength	$\begin{bmatrix} c_{11} & \epsilon \\ \epsilon & c_{22} \end{bmatrix}$	Modal coupling strength quantifies how strongly the two modes interact. The cross-coupling term can be caused by structural asymmetries, mass distribution, joints, etc. and can vary from one test configuration to the next.
s – perturbation parameter	$s = \frac{\lambda_1 - \lambda_2}{2}$	Local eigenvalue perturbation parameter measuring how the actual <u>coupled</u> eigenvalues move away from the central eigenvalue

Note: These parameters will become crystal clear in the given examples



Case 1 – Weak Coupling: Approximate Relations

(s is dominant imaginary)

Define

$$\alpha = \sqrt{\delta^2 - \frac{\epsilon^2}{4}}$$

$$s = \pm i\alpha$$

Eigenvalues

plus conjugates:

$$\lambda_{1,2} \approx -\zeta\omega + i(\omega \pm \alpha)$$

$$\lambda_{1,2} \approx -\zeta\omega - i(\omega \pm \alpha)$$

Mode shapes

For $s = +i\alpha$:

$$\phi_+ \approx \begin{bmatrix} 1 \\ -i \frac{2(\alpha + \delta)}{\epsilon} \end{bmatrix}$$

For $s = -i\alpha$:

$$\phi_- \approx \begin{bmatrix} 1 \\ -i \frac{2(\delta - \alpha)}{\epsilon} \end{bmatrix}$$

$$|\Delta\lambda| = 2\sqrt{\delta^2 - \frac{\epsilon^2}{4}}$$

(estimate for actual eigenvalue separation as a function of uncoupled separation and coupling strength)

Notes:

- Eigenvalues split mainly in frequency.
- The two modes remain separated.
- **Mode-shape ratio is purely imaginary, giving quadrature behavior.**
- This is the mode veering / weak interaction regime.



Case 2 – Transition Boundary: Approximate Relations (*s is zero*)

Eigenvalues

plus conjugates:

Mode shape

$$s = 0$$

$$\lambda_{1,2} \approx -\zeta\omega + i\omega$$

$$\lambda_{1,2} \approx -\zeta\omega - i\omega$$

$$\frac{\phi_2}{\phi_1} \approx -\frac{2}{2\delta}(i\delta) = -i$$

$$\phi \approx \begin{bmatrix} 1 \\ -i \end{bmatrix}$$

$$|\Delta\lambda| = 0$$

Notes:

- Eigenvalues locally merge.
- Frequency split goes to zero.
- Mode-shape ratio is purely imaginary, giving quadrature behavior.



Case 3 – Strong Coupling: Approximate Relations (s is real)

Define

$$\beta = \sqrt{\frac{\epsilon^2}{4} - \delta^2}$$
$$s = \pm\beta$$

Eigenvalues

plus conjugates:

$$\lambda_{1,2} \approx (-\zeta\omega \pm \beta) + i\omega$$

$$\lambda_{1,2} \approx (-\zeta\omega \pm \beta) - i\omega$$

$$|\Delta\lambda| = 2\sqrt{\frac{\epsilon^2}{4} - \delta^2}$$

Mode shapes

For $s = +\beta$:

$$\phi_+ \approx \begin{bmatrix} 1 \\ -\frac{2\beta}{\epsilon} - i\frac{2\delta}{\epsilon} \end{bmatrix}$$

For $s = -\beta$:

$$\phi_- \approx \begin{bmatrix} 1 \\ \frac{2\beta}{\epsilon} - i\frac{2\delta}{\epsilon} \end{bmatrix}$$

Notes:

- Eigenvalues split mainly in rate of decay.
- Frequencies lock.
- **Mode-shapes become intrinsically complex and take on both a real and imaginary part; i.e., can take on any phase and no longer quadrature.**



Estimating Coupling Strength Directly From Test

Two equations:

1. Estimate the perturbation parameter as a function of measured poles of interacting modes

$$s = \frac{\lambda_1 - \lambda_2}{2}$$

2. Estimate coupling strength:

$$\epsilon = 2\sqrt{s^2 + \delta^2}$$



Example – Cargo Carrier Cargo Configuration 1

Note: For demonstration purposes only
actual test data cannot be utilized for confidentiality

For **Configuration 1**, from the test poles:

$$\lambda_1 = -1.2315 + i123.15$$

$$\lambda_2 = -1.2717 + i127.17$$

So:

$$s = \frac{\lambda_1 - \lambda_2}{2}$$

$$s = \frac{0.0402 - i4.02}{2}$$

$$s = 0.0201 - i2.01$$

Note: According to above calculations, s is dominant imaginary (Case 1).

From pretest analysis (FEM), we can get an estimate for uncoupled separation

$$\delta_{pretest} = \frac{127.86 - 122.52}{2}$$

$$\delta_{pretest} = 2.67$$

Next, estimate the coupling strength:

$$\epsilon = 2\sqrt{2.67^2 - 2.01^2}$$

$$\epsilon \approx 3.52 \text{ rad/s}$$

$$\epsilon < 2\delta$$

$$3.52 < 5.34$$

Using:

$$\frac{\phi_2}{\phi_1} = -\frac{2}{\epsilon}(s + i\delta)$$

For the upper branch:

$$s = i2.01$$

$$\frac{\phi_2}{\phi_1} = -\frac{2}{3.52}(i2.01 + i2.67)$$

$$= -\frac{2}{3.52}(i4.68)$$

$$\frac{\phi_2}{\phi_1} \approx -i2.66$$

$$\phi \approx \begin{bmatrix} 1 \\ -i2.66 \end{bmatrix}$$

$$\zeta \approx \frac{-\text{Re}(\lambda)}{\omega}$$

Mode 1:

$$\zeta_1 \approx \frac{1.2315}{123.15}$$

$$\zeta_1 \approx 1.00\%$$

Mode 2:

$$\zeta_2 \approx \frac{1.2717}{127.17}$$

$$\zeta_2 \approx 1.00\%$$

Configuration 1 exhibits:

- weak modal interaction
- quadrature phase relationship
- frequency splitting

This is the **pre-transition / mode-veering regime**.

This is another confirmation on pre-transition/mode-veering.

Approximate Solutions Using the Reduced Perturbation Framework



Example – Cargo Carrier Cargo Configuration 2

Note: For demonstration purposes only
actual test data cannot be utilized for confidentiality

For **Configuration 2**, from the test poles:

$$\lambda_1 = -0.40 + i125.16$$

$$\lambda_2 = -2.10 + i125.16$$

So:

$$s = \frac{\lambda_1 - \lambda_2}{2}$$

$$s = \frac{(-0.40 + i125.16) - (-2.10 + i125.16)}{2}$$

$$s = 0.85 \text{ rad/s}$$

Note: According to above calculations, s is real (Case 3).

Next, estimate the coupling strength:

$$\epsilon = 2\sqrt{0.85^2 + 2.67^2}$$

$$\epsilon \approx 5.60 \text{ rad/s}$$

$$\epsilon > 2\delta$$

$$5.60 > 5.34$$

This is another confirmation on strong-coupling / complex-mode regime.

Using:

$$\frac{\phi_2}{\phi_1} = -\frac{2}{\epsilon}(s + i\delta)$$

For the upper branch:

$$= -\frac{2}{5.60}(0.85 + i2.67)$$

$$\frac{\phi_2}{\phi_1} \approx -0.30 - i0.95$$

So:

$$\phi \approx \begin{bmatrix} 1 \\ -0.30 - i0.95 \end{bmatrix}$$

$$\zeta \approx \frac{-\text{Re}(\lambda)}{\omega}$$

Mode 1:

$$\zeta_1 \approx \frac{0.40}{125.16}$$

$$\zeta_1 \approx 0.32\%$$

Mode 2:

$$\zeta_2 \approx \frac{2.10}{125.16}$$

$$\zeta_2 \approx 1.68\%$$

Configuration 2 exhibits:

- Strong coupling
- Mode shape with real and imaginary parts
- frequency lock
- Eigenvalue split in real part (different decay rates)

This is strongly complex modes in otherwise low damped structure.

Arbitrary phase
Quadrature phase relation is gone



Concluding Remarks

- The Reduced Perturbation Framework confirmed that highly complex modes can emerge naturally in lightly damped linear structural systems through modal coupling alone, without requiring nonlinearities, isolation systems, or repeated symmetric modes.
- The nature of the modal interaction is governed by the perturbation parameter s : imaginary s corresponds to weak coupling, producing mode veering, frequency splitting, and near-quadrature modal behavior, while real s corresponds to strong coupling, producing decay-rate splitting and fully complex mode shapes with significant real and imaginary components.
- The Reduced Perturbation Framework provides a compact and physically interpretable mathematical basis for identifying, understanding, and predicting the emergence of complex modes directly from modal test data and pretest analytical models.
- The framework further provides a direct physical interpretation of complex modal behavior in terms of modal phase relationships, eigenvalue splitting mechanisms, and coupling-induced energy exchange between interacting modes.