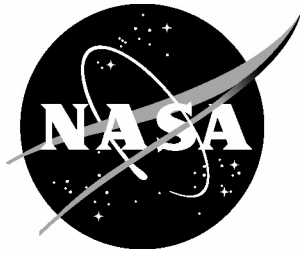


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August 2026

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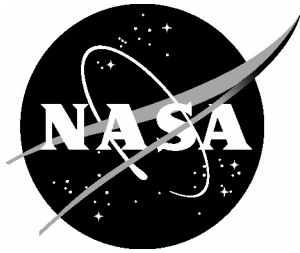
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Abstract

Reynolds stress budgets computed from a direct numerical simulation of the spanwise-periodic flow over a Gaussian bump are analyzed to gain insight into turbulence modification under different pressure-gradient regimes in three subregions. The four nonzero Reynolds stress components are defined in the local orthogonal coordinate system at a given location on the bump surface. The first subregion starts upstream of the bump under a mild adverse pressure gradient (APG) that becomes progressively stronger toward the bump. The second subregion succeeds the first subregion near the bump foot and contains a strong favorable pressure gradient (FPG) until very near the bump apex. The third subregion is the strong APG section that begins near the apex and lasts until the flow separation point. The mild/strong APG reduces the mean shear rate near the wall, which has a dampening effect on turbulence production while the strong FPG leads to an order of magnitude increase in near-wall turbulence production as a result of the much-enhanced mean shear rate. The mean shear rate of the buffer zone between inner and outer parts of the decelerated flow past the apex also gives way to enhanced production. The turbulent kinetic energy (TKE) gets redistributed internally among the normal stress components via the pressure-strain correlation. For the shear stress, pressure strain is generally a consuming term. The reduction in near-wall TKE production in the APG sections leads to decreases in pressure-strain amplitudes for all normal stresses. For the shear stress, a similar correlation between production and pressure strain also exists in the mild APG section, but a more intricate behavior is found in the strong APG section. In the strong FPG region, the much-amplified near-wall production of turbulence leads to substantial enhancements of the pressure-strain correlation for all components. As the flow goes through different pressure-gradient regimes, the decreasing/increasing deficits or surpluses between energizing (production and/or pressure strain) and consuming (dissipation and/or pressure strain) terms near the wall are balanced by the relevant spatially-redistributive terms that proportionally either diminish or amplify in magnitude. The budget surpluses resulting from the much-amplified near-wall turbulence production in the strong FPG section lead to the formation of an internal layer beneath the accelerated boundary layer, where new near-wall peaks emerge for the wall-normal and shear stresses as the original peaks of the other components strengthen substantially. The paper describes the rather complex flow dynamics that eventually leads to boundary-layer separation.

1 Introduction

Strong pressure gradients are known to alter the balance between production and dissipation of turbulent kinetic energy (TKE) in a turbulent boundary layer (TBL), and subsequently lead to profound changes in the nature of turbulence and thus strongly impact the overall behavior of the flow. An adverse pressure gradient (APG) generally suppresses near-wall turbulence and can potentially cause flow separation as a result of the near-wall momentum depletion depending on the severity of the APG. A favorable pressure gradient (FPG), on the other hand, usually enhances near-wall turbulence but can also lead to partial or

full relaminarization depending on several factors, such as the strength and spatial extent of the FPG, and whether the local Reynolds number is sufficiently low [21]. In the outer part of the TBL away from the wall, the APG effect is normally destabilizing while that of the FPG is stabilizing. This is because of the respective enhancement or reduction of turbulence production associated with the changes to the mean shear rate caused by the pressure gradient. The APG creates an inflection point which is destabilizing, and vice versa.

The Reynolds stress transport equations describe the generation, spatial transport and viscous dissipation of the relevant turbulent stress components in a turbulent flow, and therefore provide one of the fundamental bases for the understanding of the associated flow dynamics. Thus, an analysis of the various individual terms that make up the transport equations, also known as the budget terms, is one of the several avenues that can provide insight into turbulence modification in a boundary layer subjected to pressure gradients. While there have been numerous experimental and computational investigations concerning canonical turbulent flows subjected to pressure gradients, analyses of the Reynolds stress and/or the TKE budgets in such flows at relatively high Reynolds numbers are generally few and far between. Such budget data at high Reynolds numbers are particularly desirable for efforts concerned with further improvement of turbulence models based on the TKE or Reynolds stress transport equations. However, it is practically impossible to directly measure all of the budget quantities in an experiment; such quantities can be determined only via high-fidelity simulations in the form of a direct numerical simulation (DNS) or a highly-resolved large-eddy simulation (LES) at Reynolds numbers accessible by the available computing capability.

Some recent noteworthy computational studies related to budget analyses in flat-plate TBLs under moderate APG conditions include the highly-resolved LES of Pozuelo et al. [17] that achieved a momentum-thickness Reynolds number, Re_θ , of up to about 8700, the DNS of Gungor et al. [10] that went up to $Re_\theta = 8000$, as well as the DNS of Rkein and Laval [18] that also had about the same maximum Re_θ . Other similar work under stronger APG conditions include the study by Coleman et al. [3], who performed DNS of flat-plate TBLs undergoing separation caused by APGs of varying severity and reached a maximum Re_θ of about 9000 prior to separation, and the study by Kitsios et al. [11] that achieved a maximum $Re_\theta = 13800$ in a similar DNS of a TBL nearing separation on a flat surface due to a strong APG. To our knowledge, these reported Re_θ values are among the highest achieved to date in the DNS or LES of TBLs developing under moderate to strong APGs. The budget analyses in the investigations by Pozuelo et al. [17], Coleman et al. [3] and Kitsios et al. [11] were limited to just the TKE and did not extend to the individual Reynolds stress components. These works primarily examined various other aspects of turbulence modification by the varying APGs, such as the effect on statistical and spectral properties; two-point correlations; effects on the viscous sublayer, logarithmic layer and wake region of the mean velocity streamwise profiles; and alterations in length scales associated with near-wall turbulence. The study by Gungor et al. [10] focused mostly on the spectral features of turbulence production and energy transfer among the Reynolds stress components via the pressure-strain correlation in a flat-plate TBL under a moderate APG, and also presented a relatively brief budget analysis for all Reynolds stress components. In the work of Rkein and Laval [18], a more detailed analysis of the budgets of all Reynolds stresses and the TKE was performed, but only under a moderate APG condition. One of the main observations in these prior analyses of moderate to strong APG TBLs was the formation of well-defined peaks in the outer region of the Reynolds stress profiles, which became more pronounced as the APG was increased. These outer peaks were found to be the consequences of excess production over dissipation away from the wall. Another observation was that as the APG increased, the weakening near-wall structures due to the momentum defect caused the flow to act less like a zero pressure gradient (ZPG) TBL and more like a mixing layer.

In contrast, similar budget analyses in canonical TBLs subjected to FPGs are relatively scarce. Work related to this subject includes the study by Schiavo et al. [20], who performed a highly resolved LES of the incompressible TBL in a convergent-divergent channel with pressure gradients and separation and analyzed the TKE budgets at different locations corresponding to the FPG and APG stations. The inlet Reynolds

number based on the wall-friction velocity and the channel half-height was set to two separate values of $Re_\tau = 170$ and 615, which can be considered relatively low. The TKE balances at the FPG and APG stations were examined both in physical and wavenumber space, and proper orthogonal decomposition was employed to evaluate the impact of the most energetic modes on turbulence production, viscous dissipation and spatial redistribution processes. Wang et al. [27] examined the TKE budgets as part of the DNS data analysis for a supersonic flat-plate TBL subjected to a moderate FPG induced by external expansion waves. Similarly, Nicholson et al. [15] studied the budgets of the Reynolds stresses and the TKE, in addition to those of the internal energy and the turbulent heat flux, using the DNS database of a hypersonic TBL subjected to FPGs. Wang et al. [27] as well as Nicholson et al. [15] observed that in the supersonic or hypersonic TBLs that they examined, the FPG had a dampening effect on the near-wall turbulence generation, as well as its dissipation and transport relative to the ZPG case. This particular observation in compressible flows is in direct contradiction to our observation in the case of a subsonic TBL subjected to a strong FPG, as considered in the present work.

At the time of this writing, other than the study of Schiavo et al. [20], we were unable to find any specific studies in which the Reynolds stress and/or the TKE budgets were generated and examined in detail for a canonical subsonic TBL subjected to a strong FPG in the absence of compressibility effects, commonly encountered in many practical low-speed flow applications. The seeming absence of research on this particular subject provides one of the main motivations behind the present work, in which the DNS data from a canonical smooth-body flow separation problem under assumed spanwise-periodic conditions [26] is used to scrutinize the evolution of the four nonzero Reynolds stresses and their budgets. This benchmark test case, commonly known as the “speed-bump flow” [23], was the subject of a detailed experimental investigation campaign [5, 4, 6, 7, 8] over the past few years. The problem setup is designed to investigate the interaction of an incoming low-speed TBL with the favorable and adverse pressure gradients generated by a wall-mounted Gaussian bump geometry, at varying Reynolds numbers based on the length of the experimental model. The main features of the problem include the strong FPG section over the windward side of the bump that generates a substantial flow acceleration, and the strong APG over the leeward side that leads to either incipient or strong flow separation depending on the Reynolds number. Prior to the strong FPG section, the incoming flow starts off with an initially mild APG that becomes progressively stronger toward the bump foot. Thus, the flow configuration contains three distinctly different pressure-gradient regimes upstream of separation. Additional complications arise due to the mean streamline curvature effects resulting from the body contour. The analysis of the Reynolds stress budgets in the present work provides insight into turbulence modification under different pressure-gradient regimes in the three subregions upstream of separation for the present canonical test case that contains complex phenomena similar to those encountered in many practical applications. The primary motivation behind the detailed analysis of the DNS data is to provide physical insight for advancing modeling approaches for complex turbulent flows in the context of Reynolds-averaged Navier-Stokes [14, 19] or wall-modeled large-eddy simulation [2, 12, 28, 1] frameworks.

The paper is organized as follows. Section 2 presents the Reynolds stress transport equations and explains the physical meanings of the various budget terms. It also provides the details of transforming the budgets, originally defined in the Cartesian coordinate system, to the local orthogonal coordinate system at a given location on the bump surface. The information regarding the DNS from which the present budget analysis data was acquired, and the analysis of the budget terms over the aforementioned three subregions of the Gaussian bump flowfield are given in Section 3. Section 4 summarizes the main findings and provides the concluding remarks.

2 Reynolds stress transport equations

Although a compressible flow solver was used to perform the simulation of the Gaussian bump flow at a freestream Mach number of 0.2 [26], the incompressible form of the Reynolds stress transport equations is considered in the budget analysis. The reason for this is twofold. Firstly, at the given Mach number, any compressibility effects are expected to be small enough to not skew any major findings. Secondly, the incompressible form of the transport equations helps reduce the number of quantities that need to be computed and stored for the budgets. As the DNS was performed on graphics processing units with a limited amount of memory, minimizing the number of quantities needed for the budgets provided the least amount of complication from a computational point of view.

The velocity field is decomposed as: $\tilde{u}_i = U_i + u_i$, where $\tilde{u}_i$ represents the total or instantaneous value of velocity component i , U_i is the mean value of velocity component i , and u_i is corresponding fluctuation. Similar decompositions can be performed for other flowfield variables. The Reynolds stress tensor is given by $R_{ij} = \overline{u_i u_j}$, where the overline indicates averaging in time and along the statistically-homogeneous spanwise direction of the present flow. The transport equations of the Reynolds stress components for an incompressible flow are expressed in the following form

$$\frac{\partial \overline{u_i u_j}}{\partial t} = P_{ij} + \varepsilon_{ij} + C_{ij} + D_{ij}^v + T_{ij} + D_{ij}^p + \Pi_{ij} \quad (1)$$

where

$$P_{ij} = - \left(\overline{u_i u_k} \frac{\partial U_j}{\partial x_k} + \overline{u_j u_k} \frac{\partial U_i}{\partial x_k} \right) \quad (\text{production}), \quad (2a)$$

$$\varepsilon_{ij} = -2\nu \overline{\frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k}} \quad (\text{viscous dissipation}), \quad (2b)$$

$$C_{ij} = -U_k \overline{\frac{\partial u_i u_j}{\partial x_k}} \quad (\text{mean-flow convection}), \quad (2c)$$

$$D_{ij}^v = \nu \overline{\frac{\partial^2 u_i u_j}{\partial x_k \partial x_k}} \quad (\text{viscous diffusion}), \quad (2d)$$

$$T_{ij} = - \overline{\frac{\partial u_i u_j u_k}{\partial x_k}} \quad (\text{turbulent transport}), \quad (2e)$$

$$D_{ij}^p = - \frac{1}{\rho} \left(\overline{\frac{\partial u_i p}{\partial x_j}} + \overline{\frac{\partial u_j p}{\partial x_i}} \right) \quad (\text{pressure diffusion}), \quad (2f)$$

$$\Pi_{ij} = \frac{1}{\rho} \left(\overline{p \frac{\partial u_i}{\partial x_j}} + \overline{p \frac{\partial u_j}{\partial x_i}} \right) \quad (\text{pressure-strain correlation}), \quad (2g)$$

p is the pressure fluctuation, ρ and ν represent the mean density and the kinematic viscosity, respectively. The above transport equations are derived from the governing Navier-Stokes equations by assuming an incompressible flow. The various terms that appear on the right-hand side of Eq. (1) constitute the budgets of the Reynolds stress tensor defined in the Cartesian coordinate system. The indices $i = 1, 2, 3$ represent the x (axial), y (vertical), z (spanwise) directions, respectively. Due to the spanwise homogeneity of the present flow configuration, $\overline{u_1 u_3} = \overline{u_2 u_3} = 0$. Hence, the four nonzero Reynolds stress components are $\overline{u_1 u_1}$, $\overline{u_2 u_2}$, $\overline{u_3 u_3}$, and $\overline{u_1 u_2}$. Eq. (1) is exact and describes how the turbulent stresses evolve and redistribute due to production, dissipation, convection, diffusion and pressure interactions. This equation is valid for turbulent flow over a flat or curved surface and does not explicitly contain any terms associated with streamline curvature, but that effect is captured in the DNS and is accounted for in the magnitudes of various quantities that appear in the transport equations.

The production term is the generation of the turbulent stress by the interaction between the mean velocity gradients and the turbulent stresses, while the dissipation term is the conversion of the turbulent stress into internal energy by the action of viscous stresses on the smallest scales of turbulence. The pressure-strain correlation term represents the internal redistribution of energy among the turbulent stress components by the action of pressure fluctuations. The mean-flow convection term is the spatial advection of the turbulent stress by the mean velocity field. The viscous diffusion, pressure diffusion and turbulent transport terms denote the spatial redistribution of the turbulent stress via the actions of molecular mixing, pressure fluctuations and turbulent motions, respectively. Table 1 provides the abbreviations of transport equation terms that will be utilized in the plots throughout this paper. It should be noted that adding up the pressure diffusion and pressure-strain correlation terms provides the velocity-pressure-gradient correlation that was used in some budget analyses [24, 13, 22].

Abbreviation	Transport equation term
PROD	Production
VDISS	Viscous dissipation
CNVT	Mean-flow convection
VDIFF	Viscous diffusion
TTRS	Turbulent transport
PDIFF	Pressure diffusion
PSCR	Pressure-strain correlation
VPGR	Velocity-pressure-gradient correlation (= PDIFF + PSCR)
SUM	Sum of all budget terms

Table 1. Abbreviations for the Reynolds stress transport equation terms.

As will be seen in the upcoming analysis, while production is generally an energizing term for all components except the spanwise stress (due to the spanwise homogeneity of the present flow configuration leading to zero production of $\overline{u_3 u_3}$), it can also be consuming in certain parts of the flowfield for wall-normal and shear stress components when it backscatters energy to the mean flow. Pressure-strain correlation can be either energizing or consuming in nature, depending on the stress component and the specific spatial location in the flowfield. Viscous dissipation is generally a consuming term for all components. The remaining terms of the transport equations represent the various spatial redistribution mechanisms described above.

To derive the budget terms of the Reynolds stress components defined in the local orthogonal coordinate system defined at a given location on the bump surface, we make use of the following procedure. Let $\vec{t} = (t_x, t_y, t_z)$ and $\vec{n} = (n_x, n_y, n_z)$ be the local surface tangent and wall-normal unit vectors, respectively, at a given surface location. Note that because of the uniform surface profile along z in the present problem, $t_z = n_z = 0$. The local streamwise (or wall-parallel) component, u_s , and the local wall-normal component u_n , of the fluctuating velocity field are given by

$$u_s = u_1 t_x + u_2 t_y + u_3 \overset{0}{t_z} = u_1 t_x + u_2 t_y \quad \text{and} \quad u_n = u_1 n_x + u_2 n_y + u_3 \overset{0}{n_z} = u_1 n_x + u_2 n_y. \quad (3)$$

As we have a stationary grid, the local surface tangent and wall-normal vectors do not vary in time. The streamwise (or wall-parallel), wall-normal, and shear components of the Reynolds stress tensor in the local

orthogonal coordinate system are, respectively, given by

$$R_{ss} = \overline{u_s u_s} = \overline{(u_1 t_x + u_2 t_y)(u_1 t_x + u_2 t_y)} = \overline{u_1 u_1} t_x t_x + \overline{u_2 u_2} t_y t_y + 2\overline{u_1 u_2} t_x t_y, \quad (4a)$$

$$R_{nn} = \overline{u_n u_n} = \overline{(u_1 n_x + u_2 n_y)(u_1 n_x + u_2 n_y)} = \overline{u_1 u_1} n_x n_x + \overline{u_2 u_2} n_y n_y + 2\overline{u_1 u_2} n_x n_y, \quad (4b)$$

$$R_{sn} = \overline{u_s u_n} = \overline{(u_1 t_x + u_2 t_y)(u_1 n_x + u_2 n_y)} = \overline{u_1 u_1} t_x n_x + \overline{u_2 u_2} t_y n_y + \overline{u_1 u_2} (t_x n_y + t_y n_x). \quad (4c)$$

The spanwise component of the Reynolds stress in the local orthogonal coordinate system is the same as that in the original Cartesian coordinate system, which is denoted as R_{33} , and therefore no transformation is needed for that component. The time derivatives of the transformed Reynolds stress components can be expressed as

$$\frac{\partial \overline{u_s u_s}}{\partial t} = t_x t_x \frac{\partial \overline{u_1 u_1}}{\partial t} + t_y t_y \frac{\partial \overline{u_2 u_2}}{\partial t} + 2t_x t_y \frac{\partial \overline{u_1 u_2}}{\partial t}, \quad (5a)$$

$$\frac{\partial \overline{u_n u_n}}{\partial t} = n_x n_x \frac{\partial \overline{u_1 u_1}}{\partial t} + n_y n_y \frac{\partial \overline{u_2 u_2}}{\partial t} + 2n_x n_y \frac{\partial \overline{u_1 u_2}}{\partial t}, \quad (5b)$$

$$\frac{\partial \overline{u_s u_n}}{\partial t} = t_x n_x \frac{\partial \overline{u_1 u_1}}{\partial t} + t_y n_y \frac{\partial \overline{u_2 u_2}}{\partial t} + (t_x n_y + t_y n_x) \frac{\partial \overline{u_1 u_2}}{\partial t}. \quad (5c)$$

Substitution of Eq. (II) into the above relations provides the transformed budget terms of $\overline{u_s u_s}$, $\overline{u_n u_n}$ and $\overline{u_s u_n}$. For example, the transformed production terms become

$$\mathcal{P}_{ss} = t_x t_x P_{11} + t_y t_y P_{22} + 2t_x t_y P_{12} \quad (\text{production of } R_{ss}), \quad (6a)$$

$$\mathcal{P}_{nn} = n_x n_x P_{11} + n_y n_y P_{22} + 2n_x n_y P_{12} \quad (\text{production of } R_{nn}), \quad (6b)$$

$$\mathcal{P}_{sn} = t_x n_x P_{11} + t_y n_y P_{22} + (t_x n_y + t_y n_x) P_{12} \quad (\text{production of } R_{sn}). \quad (6c)$$

Analogous expressions are obtained for all other budget terms in a similar manner. Each individual budget term of the TKE transport equation is obtained by adding up the corresponding terms of the normal stress components defined either in the original Cartesian or the local orthogonal coordinate system, and dividing the sum by two.

3 Spanwise-periodic Gaussian bump flow at $Re_L = 4$ million

3.1 Simulation details and comparison with experimental data

The DNS performed by Uzun and Malik [26] considered a spanwise-periodic configuration with a uniform two-dimensional Gaussian profile along the span, given by $y(x) = h \exp(-(x/x_0)^2)$, where $x_0 = 0.195L$, $h = 0.085L$ is the bump height, and L is the length of the experimental model that is taken as the reference length scale. The Reynolds number based on the upstream freestream velocity, U_∞ , and L is $Re_L = 4$ million, while the Reynolds number based on h is 340000. The momentum-thickness Reynolds number of the incoming boundary layer (at the domain inlet where $x/L = -0.6$) is $Re_\theta \approx 3314$. The freestream Mach number is 0.2. The periodic domain span was set to $0.08L$.

Figure I depicts the computational schematic and also shows the three subregions considered for the Reynolds stress budget analysis. The inflow boundary was placed at $x/L = -0.6$. The physical domain was terminated at $x/L = 1$. The region from $x/L = 1$ to 2 (not shown) covers the sponge zone. The gradual grid stretching applied within the sponge zone damps out the turbulent structures before they reach the outflow boundary at $x/L = 2$, on which standard characteristic outflow boundary conditions were imposed. Viscous isothermal boundary conditions were applied on the lower boundary, which contains the Gaussian bump profile. The wall temperature was set the same as the freestream temperature. The outer boundary in the vertical direction was placed at $y/L = 1$, on which a nonreflecting characteristic boundary condition was

applied. The domain was discretized using 23040 points along the streamwise direction, 672 points in the vertical direction and 3072 uniform points along the spanwise direction. The total number of grid points was about 47.6 billion. The DNS was performed using a flow solver developed for graphics processing units. The code employs explicit high-order finite-difference and explicit time-integration schemes. Further information about the computational methodology and other simulation details are available in Uzun and Malik [26].

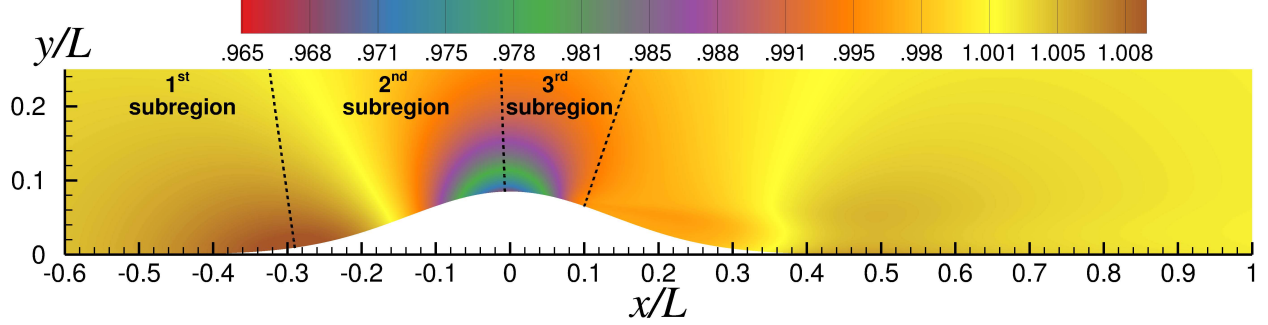


Figure 1. Domain schematic. The top boundary is at $y/L = 1$ and the outflow boundary is at $x/L = 2$. The region from $x/L = 1$ to 2 is the sponge zone. Contours denote the mean pressure normalized by its freestream value.

Although the statistical dataset for the mean velocity field and the Reynolds stress components was time-averaged over $16L/U_\infty$, the quantities needed for the evaluation of the Reynolds stress budgets were gathered over a sample size covering about $3.9L/U_\infty$. This was because the decision to examine the budgets was made during the latter stages of the simulation. A longer time sample was not feasible due to limited computational resources. During the budget analysis, it was determined that although the budget quantities converged reasonably well in the near-wall region of the attached flow that contains small structures with relatively short time scales, some quantities did not sufficiently converge in the outer part of the attached flow as well as in the separated flow and the recovery region further downstream. Therefore, the budget analysis of the present study is constrained to the attached flow region upstream of separation only. In that region, the largest streamwise and spanwise spacings are $\Delta s^+ \approx 9.6$, and $\Delta z^+ \approx 6.4$, respectively. The wall-normal grid spacing, Δn^+ , on the wall is generally less than 1 unit. The wall-normal spacing gradually increases with distance from the wall and becomes similar to the local streamwise spacing near the boundary-layer edge. The sums of the budget terms will be shown to indicate the residual magnitudes relative to the largest budget terms.

To provide an assessment of the level of agreement between the present DNS results and the available experimental data obtained under similar flow conditions, Figure 2 provides the surface pressure and skin-friction coefficient distributions along the bump surface, which are respectively defined as

$$C_p = \frac{p_w - p_\infty}{\frac{1}{2}\rho_\infty U_\infty^2} \quad \text{and} \quad C_f = \frac{\tau_w}{\frac{1}{2}\rho_\infty U_\infty^2} \quad (7)$$

where ρ_∞ , p_∞ , U_∞ , respectively, are the freestream density, pressure and velocity, p_w is the mean wall pressure and τ_w is the mean wall shear stress. In Figure 2 the C_p and C_f distributions from the DNS are compared with the experimental measurements [4, 8] taken along the model centerline at the same Re_L value of 4 million. To account for the differences between our spanwise-periodic simulation setup assuming free air conditions [26] and the actual experimental configuration with end and top wall effects, p_∞ of the DNS surface C_p is chosen such that the DNS C_p at $x/L = -0.5903$ matches the experimental value. This particular x/L location corresponds to the experimental surface pressure tap location that is nearest to the

DNS domain inlet at $x/L = -0.6$. The agreement between the computational results and the experimental measurement in the attached flow region upstream of separation is found to be satisfactory. The separated flow region shows greater differences between the computational results and the experimental data. This is due to the fact that the three-dimensional effects arising from the end walls and any blockage caused by the top wall at $y/L = 0.5$ in the experiment were not represented in the spanwise-periodic calculations that assumed free air conditions.

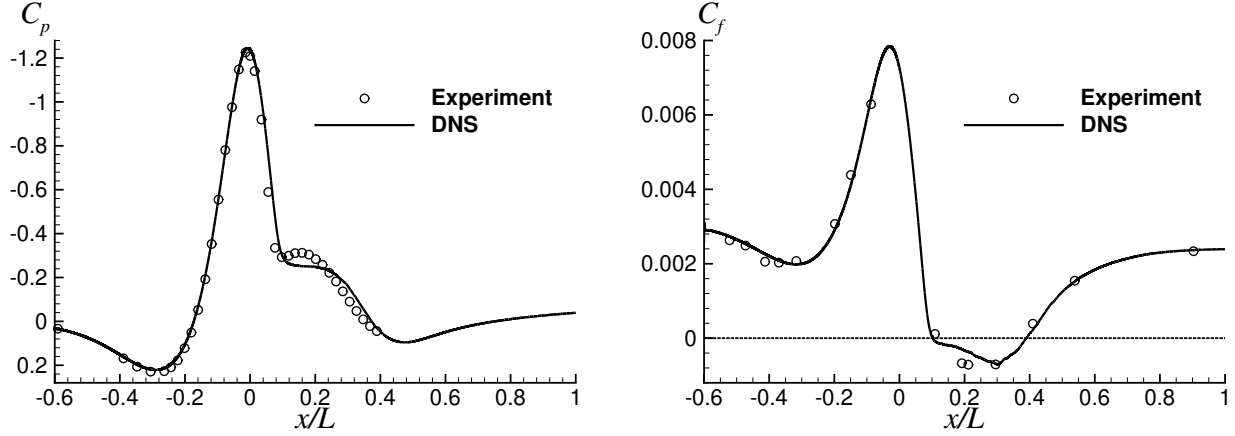


Figure 2. Streamwise variations of C_p and C_f and comparisons with the experimental data.

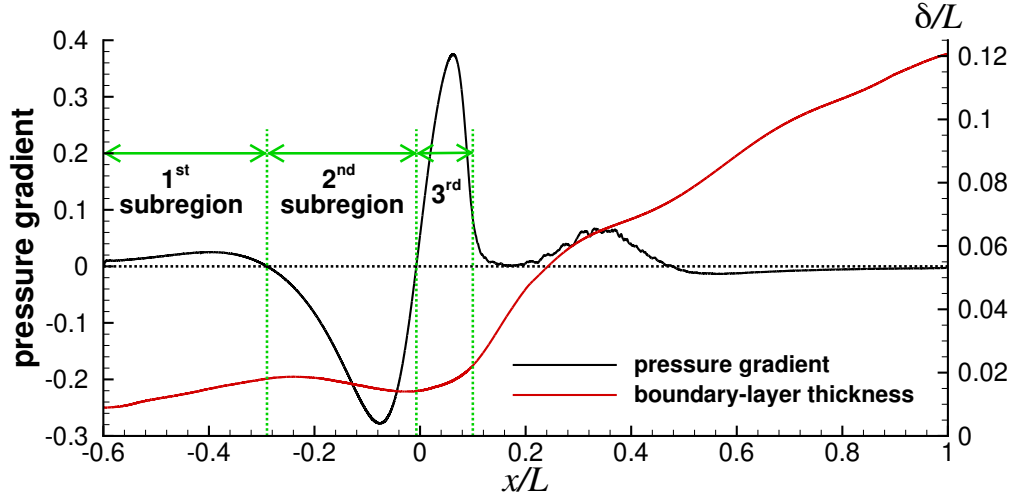


Figure 3. Variations of the (p_w/p_∞) gradient and the mean boundary-layer thickness along the bump surface.

Figure 3 depicts the variations of the mean pressure gradient, $\partial(p_w/p_\infty)/\partial s$, and the normalized mean boundary-layer thickness, δ/L , determined via the method proposed by Griffin et al. [9], along the bump surface. The three subregions for the budget analysis are also indicated in the figure. As seen here, a mild APG exists at the computational domain inlet where $x/L = -0.6$. The initial APG becomes progressively stronger as the incoming flow nears the bump foot. The incoming boundary layer experiences mostly a linear growth in δ/L within the initial APG section. The initial APG section ends at $x/L \approx -0.29$ and is succeeded

by a strong FPG that lasts until $x/L \approx -0.007$, which is positioned slightly upstream of the bump apex. The strong FPG section is then followed by a strong APG starting in the vicinity of the bump apex until the flow separation point at $x/L \approx 0.1$. The flow acceleration due to the strong FPG causes a reduction in δ/L over the windward side of the bump, while the deceleration past the bump apex due to the strong APG leads to rapid increase in δ/L . The region where $0.1 < x/L < 0.39$ contains the separated shear layer and the recirculating flow beneath it. Recovery from separation begins after the flow reattachment point at $x/L \approx 0.39$. Note that the structure of the viscous flow past the onset of separation is characterized by a free shear layer and the circulating region beneath it. The boundary-layer thickness continues to grow in the separation and recovery regions as the large-scale structures generated within the free shear layer subsequently reattach and the flow evolves further downstream.

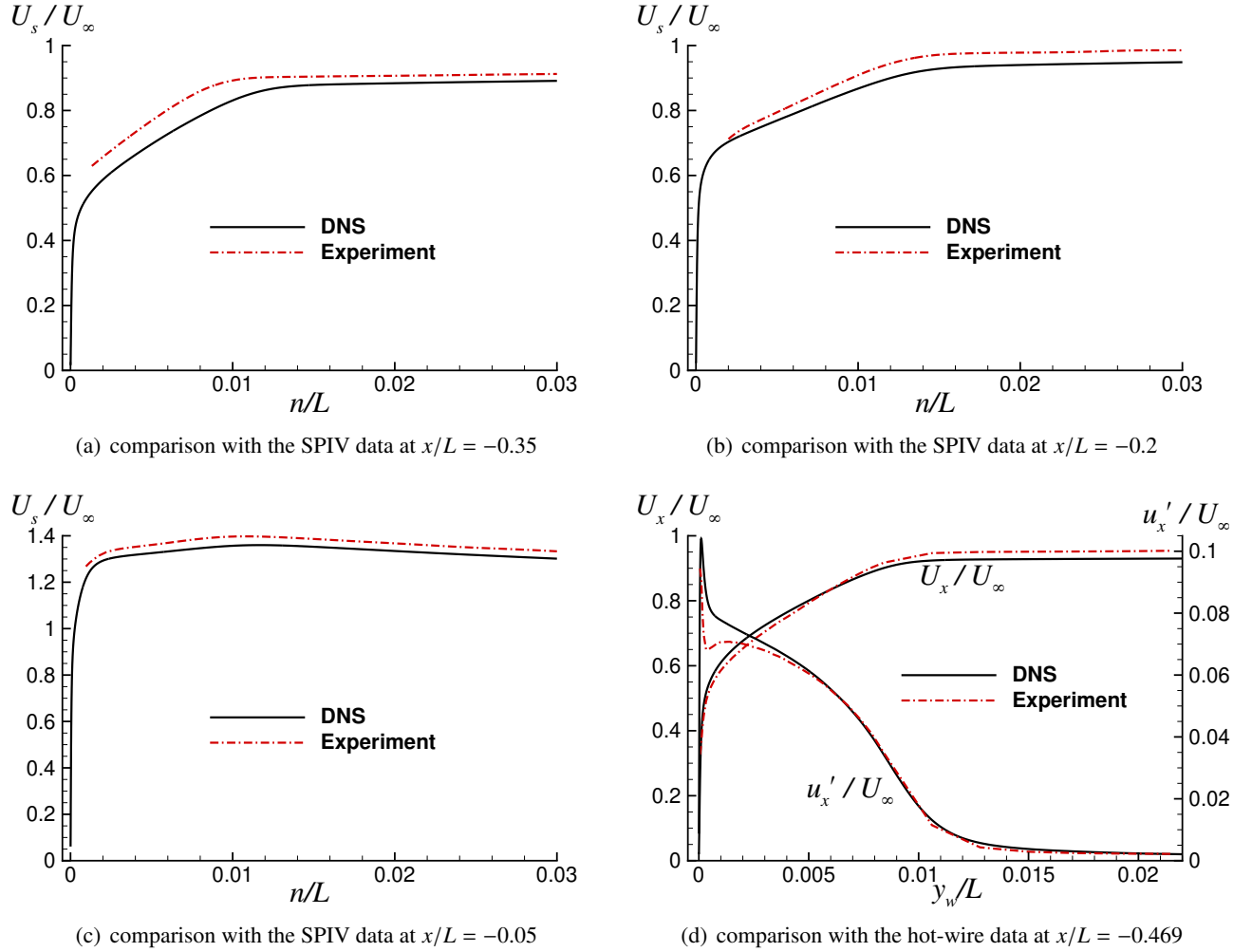


Figure 4. Comparisons between the DNS data and the experimental measurements.

The comparisons between the DNS mean streamwise velocity profiles and the corresponding experimental measurements acquired via the stereoscopic particle image velocimetry (SPIV) technique [6, 7, 8] at three x/L stations over the bump windward side are shown in Figures 4(a) through (c). These profiles are taken along the local wall-normal direction. Here, the wall-normal distance, n , is normalized by L , while the mean streamwise velocity, U_s , is normalized by U_∞ . It should be noted that the SPIV measurements were

originally taken on a Cartesian mesh, and the inherent limitations of the technique lead to erroneous measurements near the wall, which are discarded in the comparisons to be shown. The Cartesian mean velocity component measurements were interpolated onto a wall-normal line at a given x/L station, after which a coordinate transformation was applied to obtain the corresponding streamwise velocity defined in the local orthogonal coordinate system. As seen in Figures 4(a) through (c), the profiles display more qualitative than quantitative agreement between the two datasets. At $x/L = -0.35$, the DNS boundary layer is found to be slightly thicker than that in the experiment, while the stations at $x/L = -0.2$ and -0.05 show a closer agreement in the boundary-layer thickness. It is also observed that the SPIV profiles generally possess greater magnitudes relative to the DNS values. This difference is believed to be associated with the tunnel blockage effect, which is not accounted for in the DNS that assumed free air conditions. The boundary layer-edge velocity, U_e , is higher in the experiment due to the blockage effect. Hence, scaling the mean velocity profiles by U_e rather than U_∞ would be expected to produce a better collapse of the two datasets.

The comparisons of the normalized mean axial velocity component, U_x/U_∞ , and the normalized root-mean-square axial turbulence intensity component, u'_x/U_∞ , profiles between the DNS and the hot-wire anemometry data [7, 8] at $x/L = -0.469$ are shown in Figure 4(d). These profiles are plotted in terms of the normalized vertical distance from the wall, y_w/L , and show reasonable overall similarity between the DNS and the experiment, with some notable differences. Once again, the boundary-layer edge velocity of the experimental profile is higher due to the tunnel blockage effect. The axial turbulence intensity profile comparison shows good agreement further from the wall and some differences very near the wall, with the experimental near-wall peak value being slightly lower than that of the DNS. Despite the observed boundary-layer edge velocity differences, the C_f comparison in Figure 2 gives a closer agreement between the two datasets since the shown C_f is based on a reference freestream velocity far upstream of the bump that is not disturbed by the presence of the bump.

3.2 Analysis of the Reynolds stress budgets

We now examine the Reynolds stress budgets in the three subregions of the attached flow upstream of separation, as indicated in Figure 3 and discuss the main findings. To reiterate, in the upcoming analysis, the four nonzero Reynolds stress components are defined in the local surface-normal coordinate system at a given location on the bump surface. We should also reiterate here that the DNS captures both pressure-gradient and streamline curvature effects. These two effects overlap in different parts of the present flow configuration and can have additive or opposing effects on turbulence, depending on whether the curvature is convex or concave and the nature of the pressure gradient. Hence, when the pressure gradients are particularly strong, as is the case here, any potential curvature effects become obscured. A previous analysis of the Gaussian bump flow at the lower Reynolds number of $Re_L = 2$ million [25] indicated that the streamline curvature effect is secondary to the much stronger pressure-gradient effect. The present $Re_L = 4$ million case has a boundary-layer thickness distribution over the Gaussian bump that is similar to that in the lower Re_L case. In the outer region, where we expect the curvature effect to be most relevant, the mean flow gradients resulting from the streamline curvature are similar between the two Re_L cases. These observations suggest that the previous findings regarding the streamline curvature effect at the lower Re_L case also apply in the present Re_L case. Thus, in the upcoming analysis, the changes occurring to the budget terms in different subregions of the flow are considered to be primarily due to the pressure-gradient effect.

3.2.1 Examination of the initial APG region

In the initial APG region, where $-0.6 < x/L < -0.29$, the incoming TBL first interacts with a mild APG that gets progressively stronger as the flow nears the bump foot. Hence, one would expect the Reynolds stress budgets of the incoming TBL that develops under a mild APG condition to be similar to those of

a ZPG TBL. The first station chosen for the analysis in this region is at $x/L = -0.49$, where the local momentum-thickness Reynolds number is $Re_\theta \approx 4500$.

Figure 5 provides the comparison of the mean streamwise velocity and Reynolds stress component profiles in wall units at $x/L = -0.49$ with the ZPG turbulent boundary-layer DNS data of Sillero et al. [22]* at $Re_\theta = 4500$. Here, $n^+ = nu_\tau/\nu_w$, $U_s^+ = U_s/u_\tau$, where n is the wall-normal distance, U_s is the mean streamwise (or wall-parallel) velocity in the local orthogonal coordinate system, $u_\tau = \sqrt{\tau_w/\rho_w}$ is the local friction velocity, τ_w is the wall shear stress, and ρ_w and ν_w , respectively, are the density and kinematic viscosity at the wall. The Reynolds stress components are normalized by u_τ^2 . At the selected station, the skin-friction coefficient of the present mild APG TBL is about 11.5% lower than the corresponding value for the ZPG TBL at the same Re_θ [22]. Figure 5 shows that the mean velocity profiles are nearly identical when plotted in wall units. The Reynolds stress profiles in wall units also have very similar shapes between the two cases, but the present mild APG stress values are somewhat higher in the outer region compared to those of the ZPG case, the reason for which is not clear given the nearly identical velocity profiles.

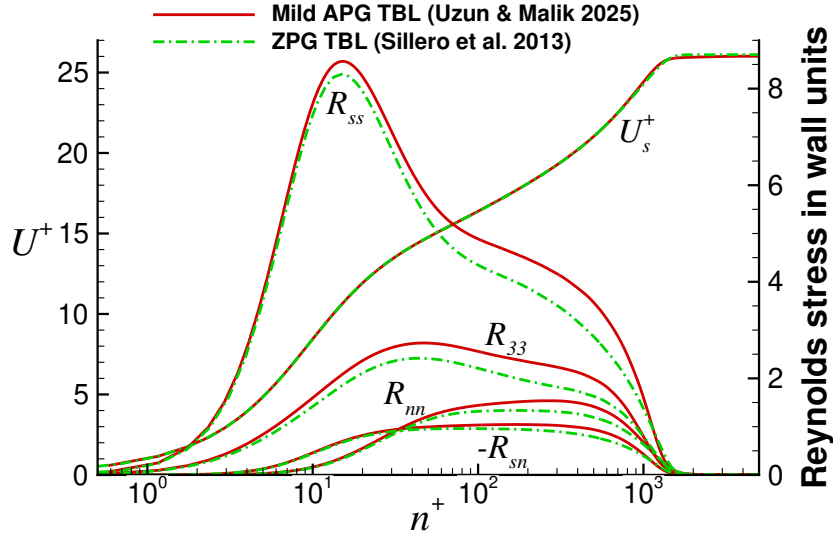


Figure 5. Comparison of mean streamwise velocity and Reynolds stress component profiles at $Re_\theta = 4500$ with the ZPG TBL data from Sillero et al. [22].

Figure 6 compares the budgets of the four Reynolds stress components between the present mild APG TBL and the ZPG TBL of Sillero et al. [22] at $Re_\theta = 4500$. All budget terms are normalized by the inner scaling. Note that the mean-flow convection is essentially zero or negligibly small relative to the other terms. The same statement can also be made about the residuals, except for those of the present mild APG wall-normal stress budget. Comparison against the corresponding ZPG budget suggests that the imbalance is likely caused by the insufficient convergence of the relevant budget terms, of which the velocity-pressure-gradient correlation and viscous dissipation become dominant in the vicinity of the imbalance. The residual peak is found to be about 10% of the velocity-pressure-gradient correlation. Nevertheless, the comparisons show similar shapes and magnitudes for all budget terms between the two cases. Hence, the main features of the present mild APG TBL budgets are very similar to those of the ZPG case at the same Re_θ . The observed differences in some budget terms could originate from several sources such as the mild APG effect in the present case, differences in numerics between the two simulations and different statistical sample sizes. A more detailed budget analysis at $x/L = -0.49$ is provided next.

*Their dataset is freely available for download at: https://torroja.dmt.upm.es/turbdata/blayers/high_re/

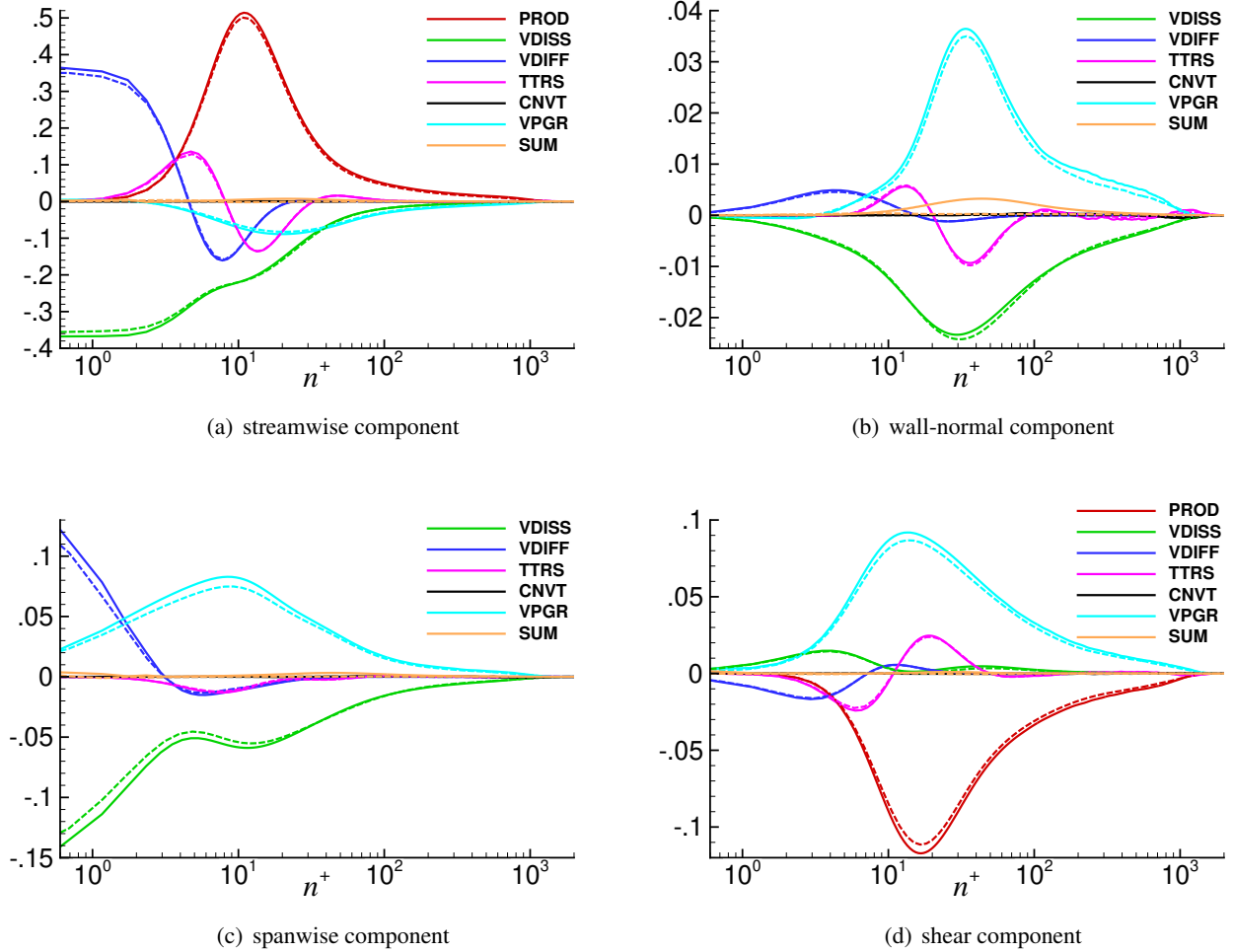


Figure 6. Comparison of the budgets between the present mild APG TBL (solid lines) and the ZPG case from Sillero et al. [22] (dashed lines) at $Re_\theta = 4500$. All budget terms are normalized by u_τ^4/ν_w .

Figures 7 and 8, respectively, plot the budgets of the four Reynolds stress components and the TKE in the near-wall and outer regions of the present mild APG boundary layer at $x/L = -0.49$. The budgets in the near-wall region are again normalized by u_τ^4/ν_w , while those in the outer region are normalized by u_τ^3/δ following Spalart [24]. The local boundary-layer thickness at this station has a value of about $0.012L$. The wall-normal distance is given as n^+ for the near-wall region budgets, and in terms of n/δ for the outer region budgets. As in Spalart [24], the near-wall region is defined by $n^+ \leq 50$, and the outer region by $n/\delta \geq 0.2$. For all normal stress components and the TKE, the budget terms with a positive value indicate a source or gain, while those with a negative value indicate a sink or loss. The opposite statement is true for R_{sn} since R_{sn} is generally a negative quantity. The budget terms that are either negligibly small or identically zero are omitted. One such term in all near-wall budgets is the mean-flow convection.

Figures 7(a) and (e) show that the near-wall TKE production mainly takes place in the form of R_{ss} . The production of R_{33} is identically zero while that of R_{nn} is negligible relative to that of R_{ss} . The TKE production peaks in the early buffer layer at $n^+ \approx 11$, while the peak production of R_{sn} occurs slightly later in the buffer layer at $n^+ \approx 17$. The ratio between the peak TKE production to the dissipation at $n^+ \approx 11$ is

found to be about 1.79, which is very close to the value of 1.81 observed in the case of a turbulent channel flow [16]. Production exceeds dissipation around the production peak. The excess TKE generated there is transported mostly toward the wall by the turbulent transport and viscous diffusion terms, and also toward the outer region via the turbulent transport but in a relatively small amount.

The pressure-strain correlation serves as an internal energy redistribution mechanism among the Reynolds stress components. It is traceless and thus zero for the TKE. For R_{sn} , pressure strain is the main sink over much of the boundary layer, and provides the balance against the primary source terms. As seen in Figures 7 and 8, this term is active over nearly the entire boundary layer for all Reynolds stress components, although the magnitudes beyond $n/\delta \approx 0.8$ are relatively low. It drains energy from R_{ss} over much of the boundary layer and transfers it to the other two normal components. For R_{ss} , pressure strain becomes very slightly positive and hence a very weak or negligible source in the very near-wall region. For R_{33} , pressure strain is a source throughout the boundary layer, while for R_{nn} , it is a sink in the near-wall region where $n^+ \lesssim 12$, and a source elsewhere. For R_{nn} , the energy drain by pressure strain in the near-wall region is counter-balanced by the energy supply from the pressure diffusion. Pressure diffusion is negligible for R_{ss} and identically zero for R_{33} because of the spanwise homogeneity. Hence, pressure diffusion in the TKE transport equation becomes a source over the viscous sublayer and the early part of the buffer layer, but with a considerably lower magnitude relative to the other more dominant source terms in the same region. For R_{sn} , pressure diffusion is the primary source very near the wall, where it is counter-balanced by pressure strain.

Much of the dissipation of the TKE in the near-wall region occurs in R_{ss} , with the rest of it taking place in R_{33} . The dissipation peaks of these two normal components, as well as that of the TKE, are located at the wall. For R_{nn} , dissipation in the near-wall region is much smaller than those of R_{ss} and R_{33} , and its peak occurs in the buffer layer at $n^+ \approx 30$. Hence, the near-wall region shows strong anisotropy in the split of TKE dissipation among the three normal stress components. Dissipation and viscous diffusion are the most dominant terms for the R_{ss} and the TKE transport equations in the viscous sublayer and counter-balance each other in that region. A similar observation can be made about R_{33} except that there is also a considerable contribution coming from pressure strain to balance against dissipation in close proximity of the wall for R_{33} . For R_{nn} , viscous diffusion is a much weaker secondary source near the wall and its magnitude in the viscous sublayer is comparable to that of dissipation. As noted above, the very near-wall balance for R_{nn} occurs mainly between pressure diffusion and pressure strain. For R_{ss} , R_{33} and the TKE, with distance from the wall, dissipation and viscous diffusion decrease in magnitude. The ramp up in production and positive turbulent transport with wall distance provides additional balance against the dissipation for R_{ss} and the TKE. For R_{33} , on the other hand, it is pressure strain that balances dissipation further from the wall, with turbulent transport becoming a secondary sink in the buffer layer region. Viscous diffusion turns into a sink at a short distance from the wall within the viscous sublayer, at $n^+ \approx 3$ for R_{33} , and $n^+ \approx 4.5$ for R_{ss} and the TKE, and drops to a negligible level beyond $n^+ \approx 25$. For R_{sn} , dissipation is a much weaker secondary sink compared to pressure strain in the near-wall region. Viscous diffusion becomes a weak secondary source for R_{sn} in the viscous sublayer, with its magnitude being about the same as that of dissipation. Both terms drop to negligible levels within the buffer layer and beyond. As noted above, for R_{sn} , the main source near the wall is pressure diffusion, counter-balanced by pressure strain. As the late buffer layer or the early logarithmic layer is reached, production is counter-balanced by dissipation for the TKE, while production is primarily balanced by pressure strain for R_{sn} .

Examining the outer region budgets shown in Figure 8, we find that production is the primary source for R_{ss} , R_{sn} and the TKE over roughly the first half of the outer region (i.e., until $n/\delta \approx 0.5$ – 0.6), and pressure strain for R_{nn} and R_{33} . These source terms decrease in magnitude with wall-normal distance. The production of R_{nn} is small but negative, and hence it is a weak sink in the outer part of the boundary layer. Such negative production is an outcome of the mean velocity gradient due to the mild APG. Beginning at around $n/\delta \approx 0.45$ – 0.5 , turbulent transport becomes a secondary source for all stress components and the TKE, and its magnitude exceeds those of production and pressure strain as the boundary-layer edge is

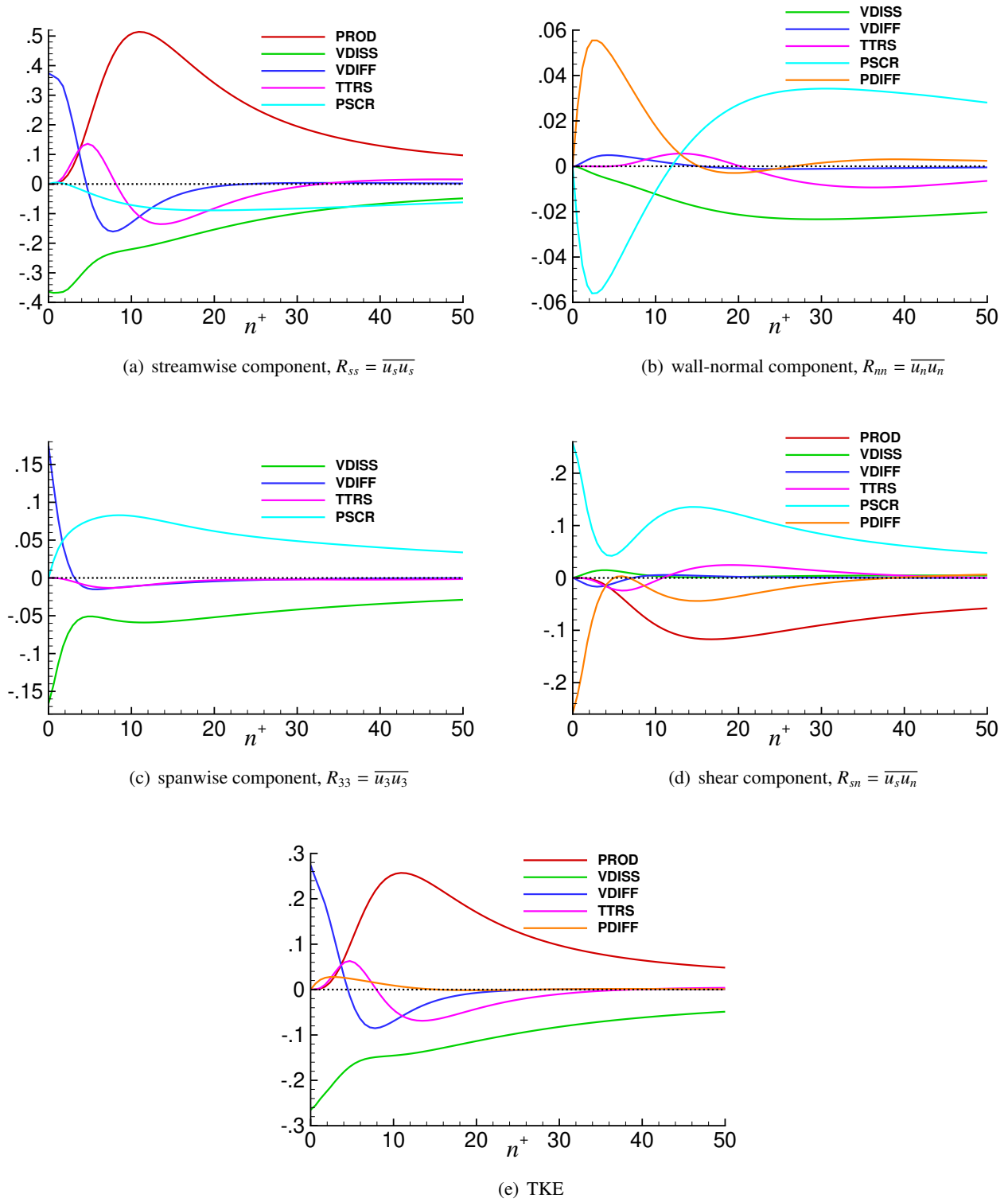


Figure 7. Reynolds stress and TKE budgets in the near-wall region at $x/L = -0.49$. All budget terms are normalized by u_τ^4/ν_w .

approached. Also starting at around $n/\delta \approx 0.45$ – 0.5 , pressure diffusion becomes a sink for both R_{nn} and R_{sn} , but its effect is rather weak in the TKE budget. Over the initial part of the outer region, turbulent transport is a relatively weak sink in all budgets, while pressure diffusion is a weak source for R_{nn} and R_{sn} , and again has a negligible effect for the TKE. Mean-flow convection is also generally very small below $n/\delta \approx 0.35$ – 0.4 , beyond which it becomes a sink in all budgets, although being nearly negligible for R_{sn} .

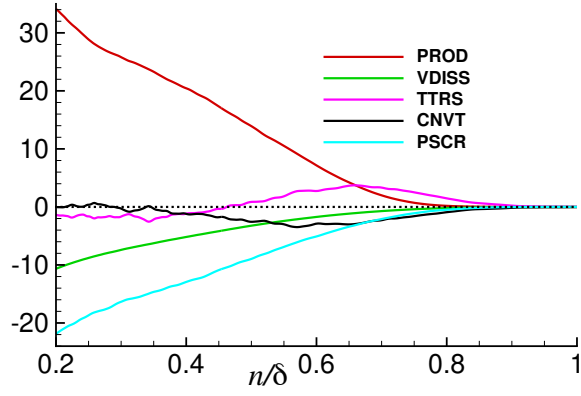
Dissipation is an important sink for both R_{nn} and R_{33} in much of the outer region, but it is of secondary importance for R_{ss} since pressure strain is a more dominant consuming mechanism than dissipation for that component. The dissipation magnitudes generally decrease with the wall-normal distance. Examining the dissipation magnitudes among the three normal stress components, we find that they are quite close to one another, indicating a much more isotropic split of the TKE dissipation in the outer part of the boundary layer. For R_{sn} , dissipation is quite low over the entire outer region. For R_{sn} , pressure strain acts as the main sink against production until $n/\delta \approx 0.5$, after which pressure diffusion becomes another sink against production and turbulence transport. As the boundary-layer edge is approached, pressure strain momentarily turns into a weak source from $n/\delta \approx 0.7$ to 0.8 , beyond which it becomes a weak sink again, counter-balanced by pressure diffusion very near the boundary-layer edge. Viscous diffusion is negligibly small in all outer region budgets.

The above features of the present mild APG turbulent boundary-layer budgets are qualitatively very similar to those identified in the budgets of a ZPG TBL [24] and a turbulent channel flow [13]. As seen earlier, our mild APG turbulent boundary-layer budgets also have good overall quantitative agreement with the ZPG budgets of Sillero et al. [22] at the same Re_θ . One of the key observations from the above analysis is the existence of a self-sustaining mechanism that drains energy from the mean flow and splits it among the turbulent stress components. To reiterate, the TKE production takes place mainly in the form of the streamwise stress production, the main constituents of which are the turbulent shear stress and the gradient of the mean streamwise velocity along the wall-normal direction. The TKE gets redistributed among the normal stress components via the pressure-strain correlation, which serves as the main source for the generation of wall-normal and spanwise stress components. The production of the turbulent shear stress, on the other hand, is contingent upon the wall-normal stress and the gradient of the mean streamwise velocity along the wall-normal direction. In this manner, the production of R_{ss} , R_{nn} and R_{sn} become intertwined with one another.

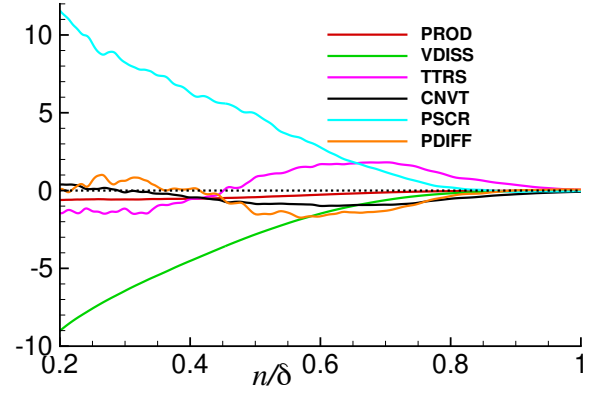
We now move onto the analysis of the budgets in the first subregion, over which the incoming boundary layer becomes subjected to a progressively-strengthening APG until $x/L \approx -0.29$. The changes occurring to the turbulent stresses and their budgets in this region are examined by comparing the profiles at the three streamwise stations of $x/L = -0.49$, -0.4 and -0.3 . For the upcoming discussion, it is informative to note that the present bump geometry has convex surface curvature in the range where $-0.138 \lesssim x/L \lesssim 0.138$ because of the symmetry about the apex, and concave curvature elsewhere. Thus, the initial APG section that lasts until $x/L \approx -0.29$ also contains concave mean streamline curvature.

Figure 9 shows the normalized mean streamwise and wall-normal velocity components, respectively denoted as U_s and U_n , and the Reynolds stress component profiles at the three stations within the initial APG section. The wall-normal distance, n , is scaled by L . Note that the boundary-layer thickness distribution, δ/L , was given in Figure 3. As seen in Figure 9(a), the mild APG causes a reduction in U_s across the entire boundary layer as the incoming flow approaches the bump. We also see that the magnitudes of U_n within the boundary layer are quite low relative to those of U_s at the three stations. As the flow nears the bump foot, the mean flowfield adjusts to the progressively-strengthening APG effect as well as to the concave mean streamline curvature; the shapes and magnitudes of the U_n profiles over the three stations illustrate the associated changes in the mean wall-normal velocity component.

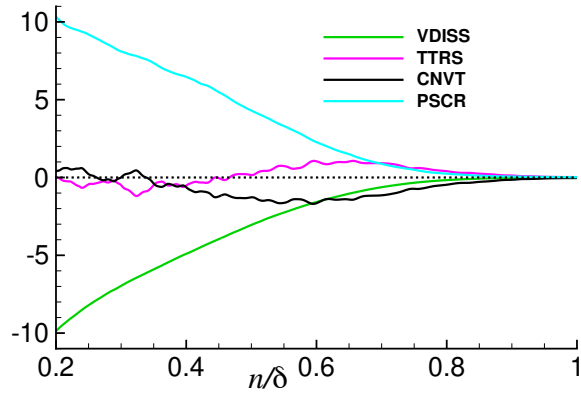
In Figure 9(b), we observe that the initial peaks of R_{ss} and R_{33} , which are positioned fairly close to the wall, decrease in strength as the incoming boundary layer interacts with the progressively-strengthening APG. The drop in the R_{ss} peak between the first and the third stations is more substantial relative to that in



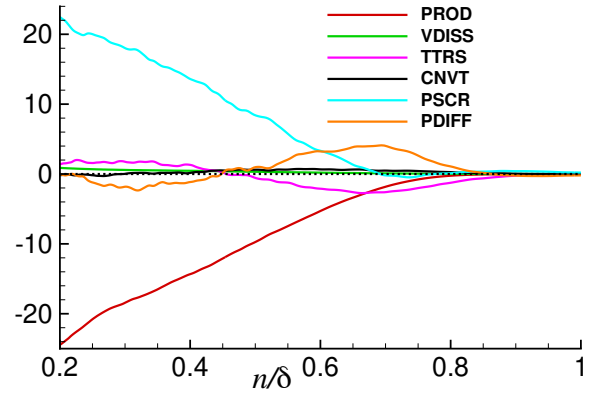
(a) streamwise component, $R_{ss} = \overline{u_s u_s}$



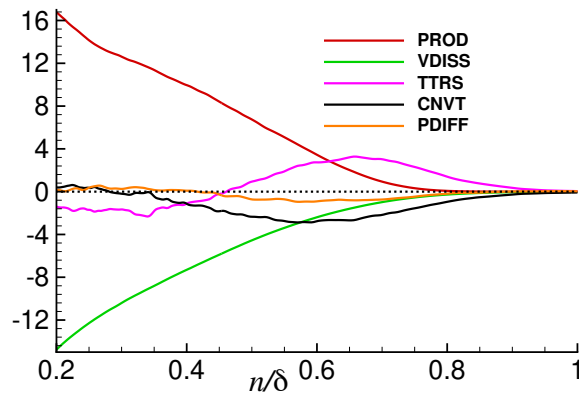
(b) wall-normal component, $R_{nn} = \overline{u_n u_n}$



(c) spanwise component, $R_{33} = \overline{u_3 u_3}$



(d) shear component, $R_{sn} = \overline{u_s u_n}$



(e) TKE

Figure 8. Reynolds stress and TKE budgets in the outer region at $x/L = -0.49$. All budget terms are normalized by u_τ^3/δ .

the R_{33} peak. Furthermore, the peak magnitude decrease for R_{33} between the second and the third stations is considerably less than that between the first and the second stations. At the first station, the R_{ss} peak is at $n/L \approx 1.05 \times 10^{-4}$, while the R_{33} peak is at $n/L \approx 3.2 \times 10^{-4}$. With the growing boundary-layer thickness in the initial APG section, the peaks of R_{ss} and R_{33} gradually shift outward over the three stations, while their respective near-wall stress levels decrease. At the first station, the respective peaks of R_{nn} and R_{sn} are positioned further away from the wall. These two components also experience a decrease in their respective near-wall levels as the flow moves between the first and the third stations. The profiles in the outer region, as depicted in Figure 9(c), show an outward spread due to the growth of the boundary-layer thickness. The peaks of R_{nn} and R_{sn} , which are initially located in the outer part of the boundary layer, shift outward and strengthen as the flow interacts with the APG. New outer peaks of the other two components also begin to develop as the flow moves deeper into the APG.

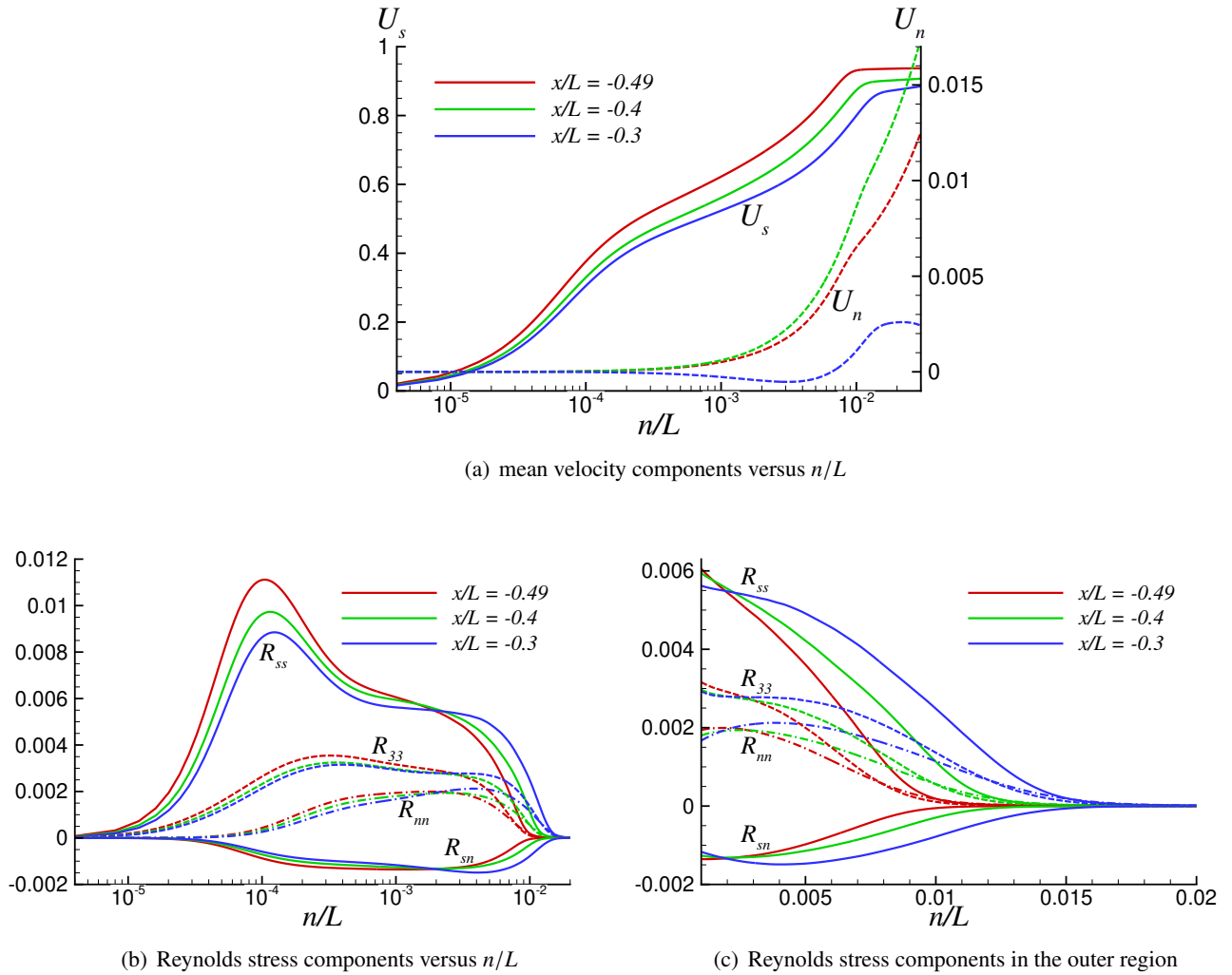


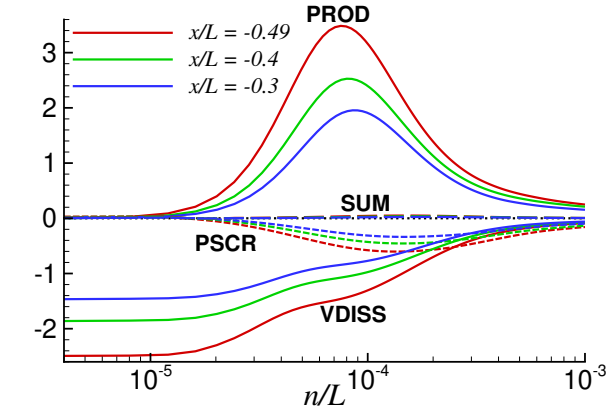
Figure 9. Mean velocity and turbulent stress profiles in the initial APG region. The normalization is based on U_∞ .

The corresponding evolutions of the Reynolds stress budgets over the three stations are shown in Figures 10 through 13. Here, all budget terms are normalized by the outer scaling, U_∞^3/L , to enable a relative

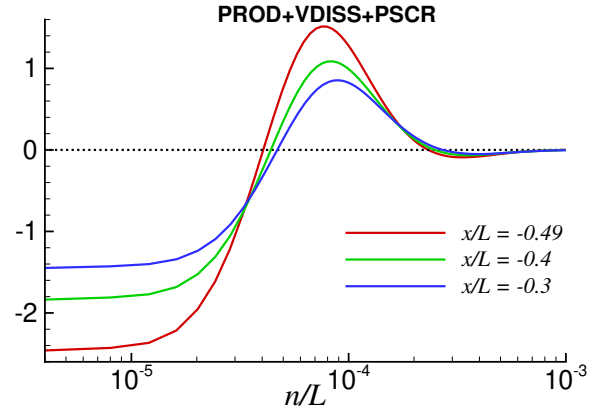
magnitude comparison among the three streamwise stations. The normalized wall-normal distance is given by n/L . The budgets very near the wall, defined by $n/L < 1 \times 10^{-3}$, and those further from the wall, defined by $n/L > 1 \times 10^{-3}$ are examined separately. Furthermore, the budget terms that are either energizing or consuming in nature are grouped together, and are shown separately from the spatially-redistributive terms. These plots show relatively low residuals in the R_{ss} and R_{sn} budgets (up to about a few percent of the respective dominant term), while parts of the R_{nn} and R_{33} budgets contain maximum residuals as high as 16% of the respective dominant term in the vicinity of the imbalance. Figures 10(a) and 13(a) show that the near-wall productions of R_{ss} and R_{sn} decrease by a substantial amount between the first and the third stations as the incoming flow becomes subjected to the progressively-strengthening APG. This is because the deceleration caused by the APG reduces the mean shear rate magnitude near the wall, which, in turn, has a dampening effect on production. The reduction in the near-wall productions of R_{ss} and R_{sn} then leads to decreases in the consuming terms, which are dissipation and pressure strain for both components, as seen in Figures 10(a) and 13(a). The energizing and consuming terms for the other normal stress components, R_{nn} and R_{33} , in the near-wall region also get dampened as a result of the reduction in R_{ss} production, as seen in Figures 11(a) and 12(a). For R_{33} , pressure strain is the sole energizing term while dissipation is the sole consuming term. For R_{nn} , pressure strain is a consuming term very near the wall that is more dominant than dissipation, and becomes an energizing term further from the wall.

Figures 10(b) through 13(b) show the sums of the energizing and consuming terms in the near-wall region for all components. Note that the sums shown in Figure 11(b) include the production of R_{nn} even though that term is generally negligible. Also recall that the production of R_{33} is identically zero because of the spanwise homogeneity. Between the first and the third stations, the near-wall deficit between the energizing and consuming terms for all components decreases considerably, largely due to the decay of the dominant consuming terms in that region. For all normal stress components, a surplus-containing portion exists at a short distance from the wall, which results from the energizing terms surpassing the consuming terms there. For R_{ss} , with outward distance, the surplus-containing portion is succeeded by a deficit-containing portion with a relatively small magnitude. The initial surplus amplitudes decrease between the first and the third stations because of the faster decay rate of the energizing terms relative to that of the consuming terms for all normal stress components. The deficit portion succeeding the surplus portion in the R_{ss} budget also weakens between the first and the third stations. For R_{sn} , due to pressure strain surpassing production in much of the near-wall region, its deficit portion extends further from the wall, as compared to those of the normal components, while its initial surplus-containing portion has a relatively low magnitude, as seen in Figure 13(b). This surplus portion gradually weakens as the flow interacts with the APG. For all components, with further outward distance from the surplus-containing portion, the net difference between the energizing and consuming terms region in the outermost part of the near-wall region drops to levels that are several orders of magnitude smaller than the near-wall levels.

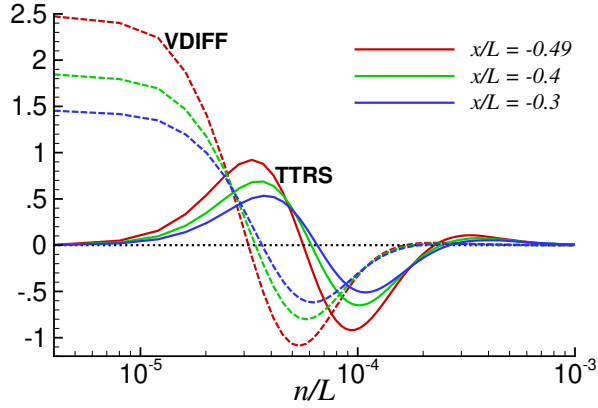
Figures 10(c) through 13(c) show the dampening of the near-wall spatially-redistributive terms at the three stations. The decreasing deficits between the energizing and consuming terms near the wall for all stress components are balanced by the relevant spatially-redistributive terms that also proportionally diminish in magnitude. As the near-wall deficit decreases, the amount of energy transported to the near-wall region by the spatially-redistributive terms also decreases. Conversely, as the surplus-containing portion weakens, a reduced amount of energy is transported away from that section. For R_{ss} , viscous diffusion and turbulent transport are the main near-wall energy redistribution mechanisms. Turbulent transport also convects some energy toward the outer part of the near-wall region where a deficit between the energizing and consuming terms exists for R_{ss} , as seen in Figure 10(b). For R_{33} , viscous diffusion is the main mechanism of redistribution to the very near-wall region. For R_{nn} and R_{sn} , pressure diffusion is the main redistributive term very near the wall, supplemented by a secondary contribution from the viscous diffusion. Pressure diffusion also transfers energy in the direction away from the wall in parts of the near-wall region for these two components. With increasing wall-normal distance, turbulent transport also becomes an important re-



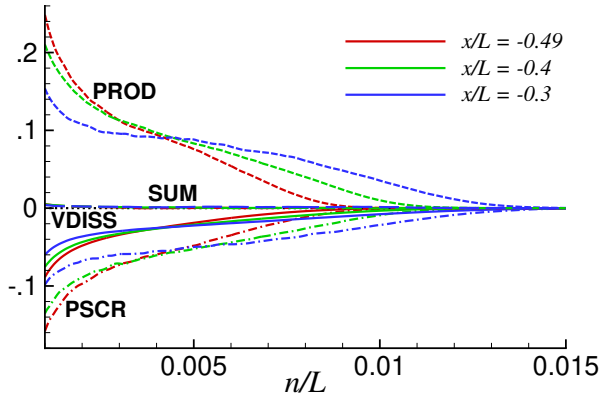
(a) energizing/consuming terms in near-wall region



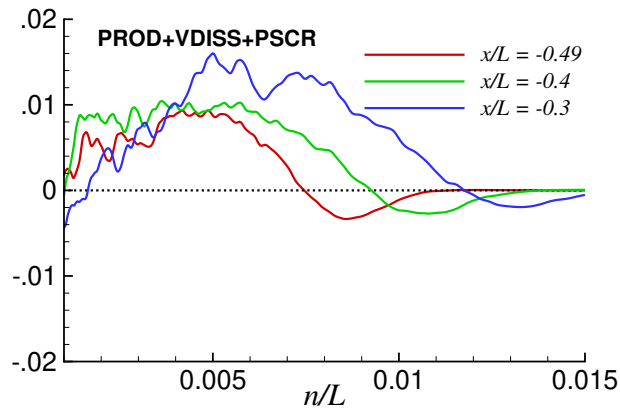
(b) energizing/consuming term sums in near-wall region



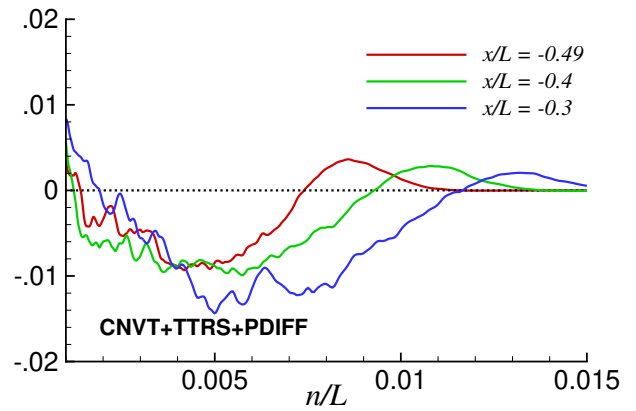
(c) spatially-redistributive terms in near-wall region



(d) energizing/consuming terms in outer region

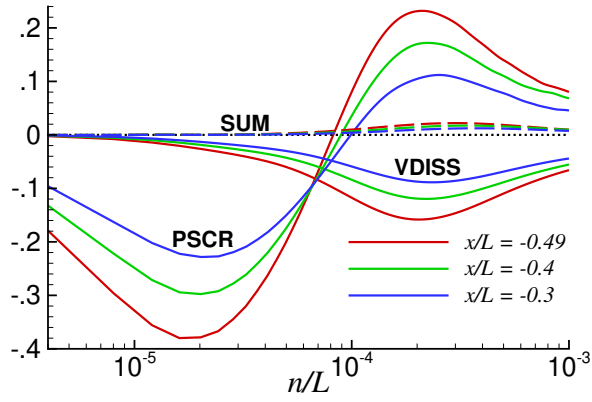


(e) energizing/consuming term sums in outer region

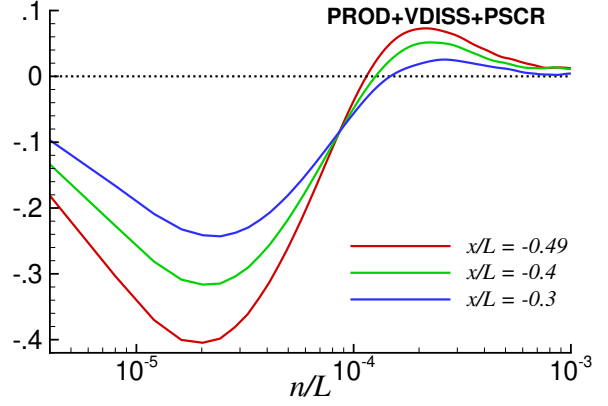


(f) spatially-redistributive term sums in outer region

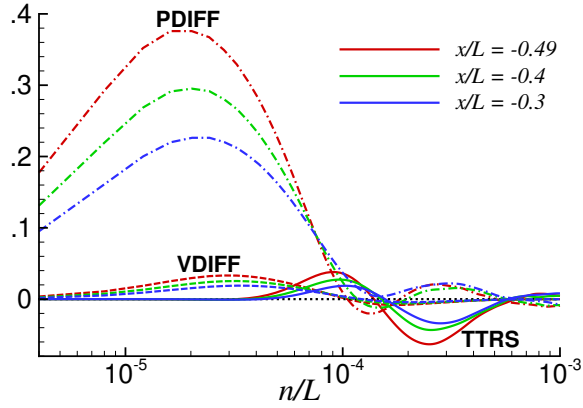
Figure 10. Evolution of R_{ss} budget terms in the initial APG region. All budget terms are normalized by U_∞^3/L .



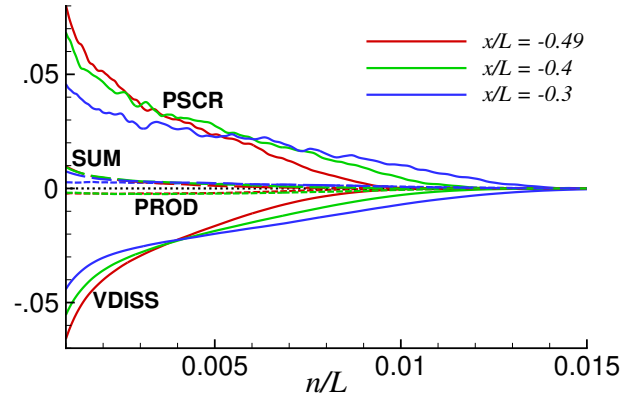
(a) energizing/consuming terms in near-wall region



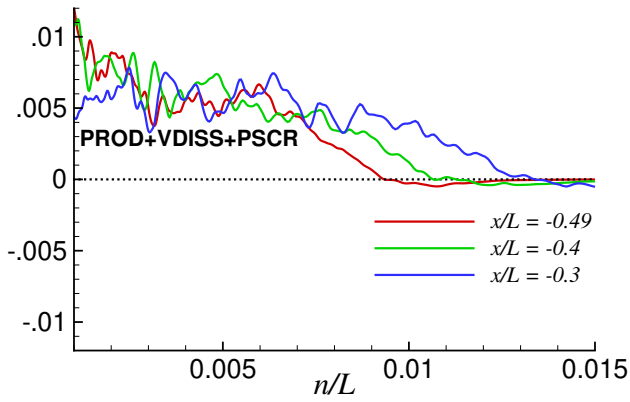
(b) energizing and consuming term sums in near-wall region



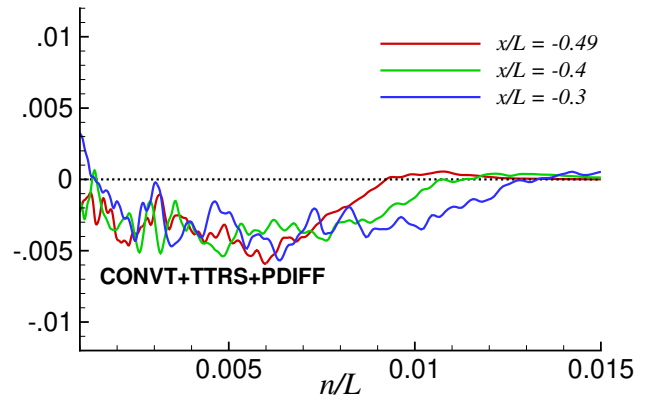
(c) spatially-redistributive terms in near-wall region



(d) energizing/consuming terms in outer region

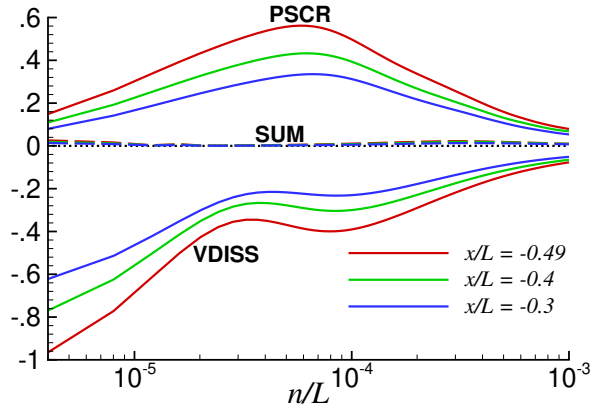


(e) energizing and consuming term sums in outer region

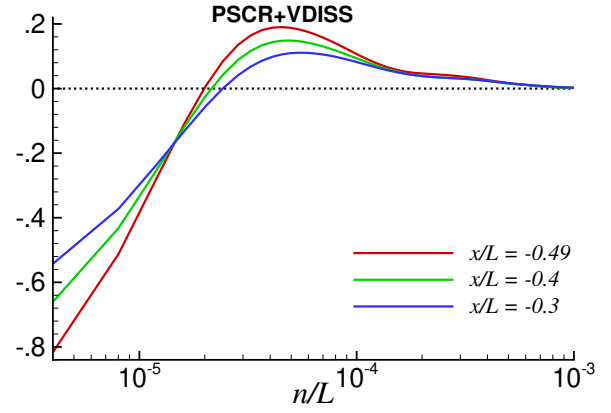


(f) spatially-redistributive term sums in outer region

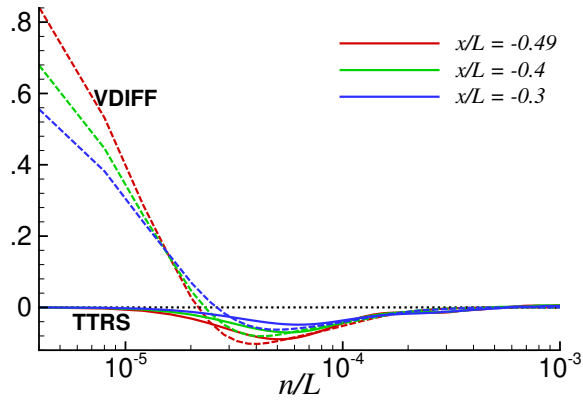
Figure 11. Evolution of R_{nn} budget terms in the initial APG region. All budget terms are normalized by U_∞^3/L .



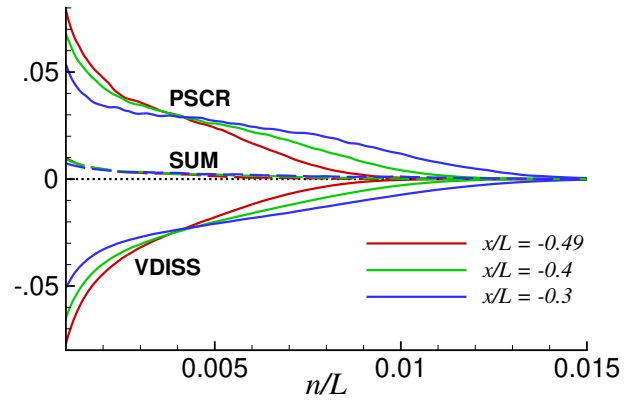
(a) energizing/consuming terms in near-wall region



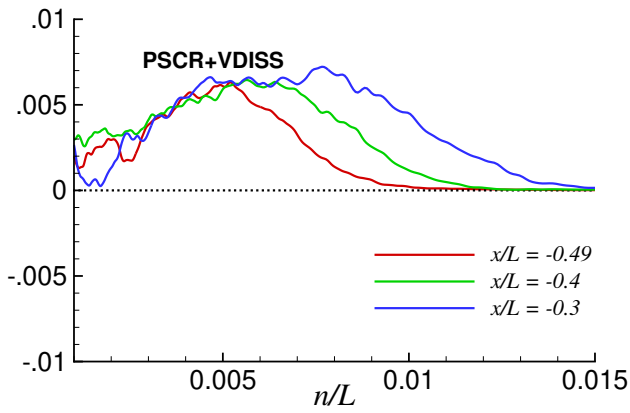
(b) energizing and consuming term sums in near-wall region



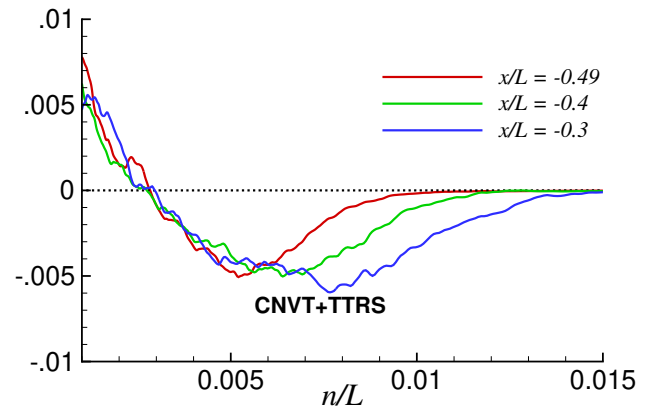
(c) spatially-redistributive terms in near-wall region



(d) energizing/consuming terms in outer region

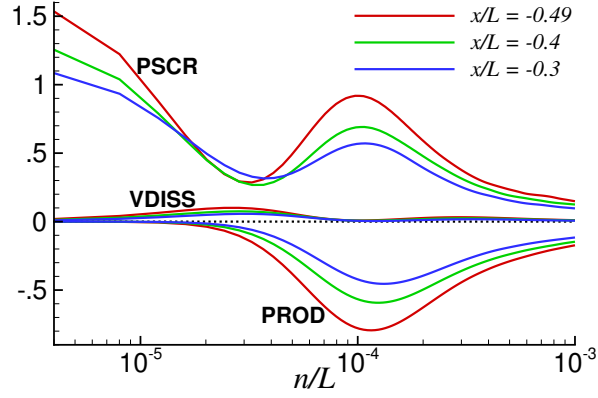


(e) energizing and consuming term sums in outer region

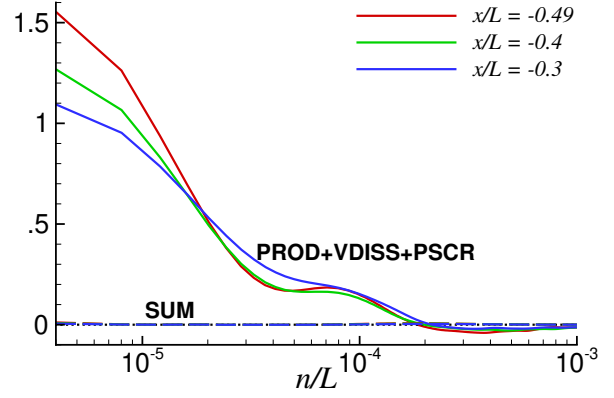


(f) spatially-redistributive term sums in outer region

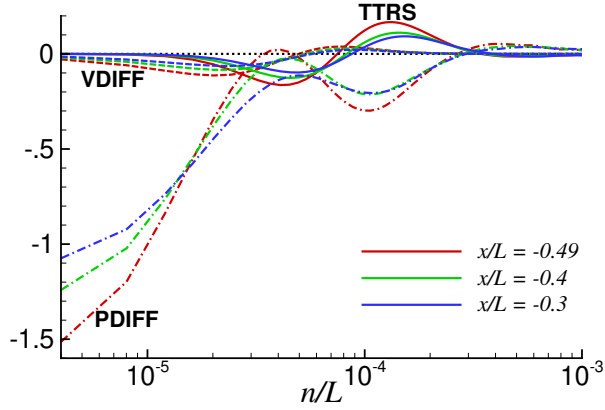
Figure 12. Evolution of R_{33} budget terms in the initial APG region. All budget terms are normalized by U_∞^3/L .



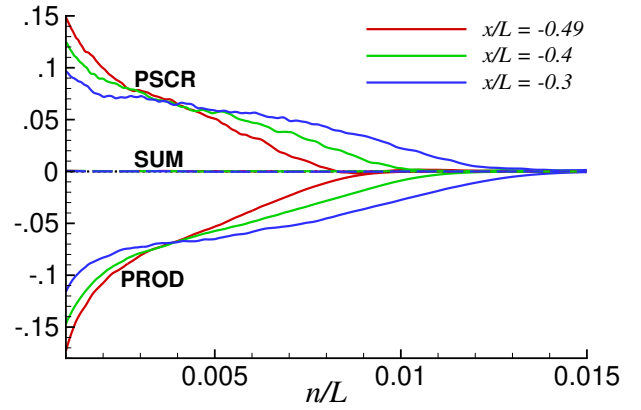
(a) energizing/consuming terms in near-wall region



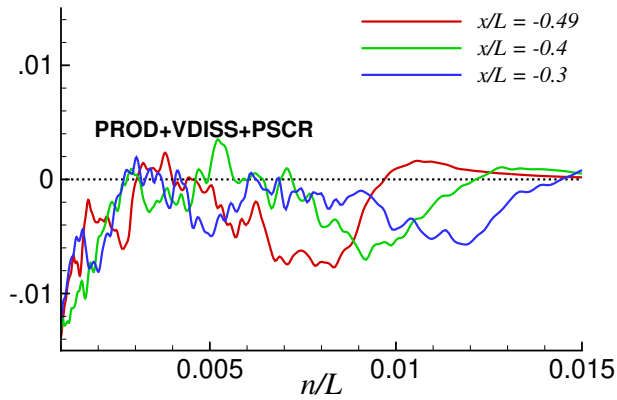
(b) energizing and consuming term sums in near-wall region



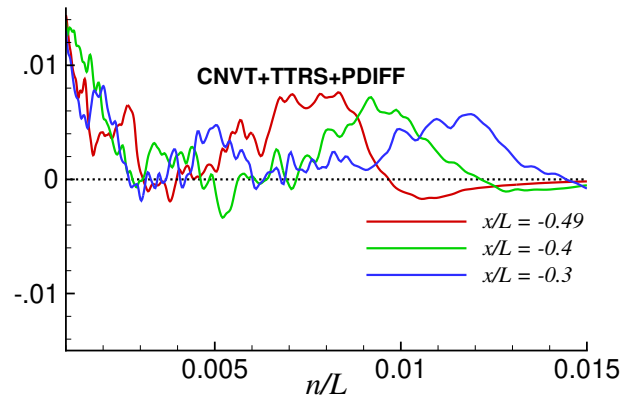
(c) spatially-redistributive terms in near-wall region



(d) energizing/consuming terms in outer region



(e) energizing and consuming term sums in outer region



(f) spatially-redistributive term sums in outer region

Figure 13. Evolution of R_{sn} budget terms in the initial APG region. All budget terms are normalized by U_∞^3/L .

distributive mechanism for R_{nn} and R_{sn} . For all stress components, the spatially-redistributive terms in the outermost part of the near-wall region are several orders lower in magnitude relative to the near-wall levels. The weakening of the near-wall R_{ss} and R_{33} peaks, and the decreases in the near-wall stress levels of all stress components, as seen earlier in Figure 9, are caused by the reduction in the near-wall R_{ss} and R_{sn} productions, which then lead to the dampening of all other budget terms in the same region.

The energizing and consuming terms in the outer part of the boundary layer at the three x/L stations over the initial APG region are provided in Figures 10(d) through 13(d), while the sums of these terms are provided in Figures 10(e) through 13(e). The sums of the spatially-redistributive terms in the same region are provided in Figures 10(f) through 13(f). For R_{sn} , although dissipation in the outer region is negligible, it is included in the sums shown in Figure 13(e). For R_{ss} , pressure diffusion in the outer region is negligible relative to the other redistributive terms; nevertheless, it is included in the sums shown in Figure 10(f). Because of the present statistical sample size constrained by the available computational resources, the individual spatially-redistributive terms were found to possess oscillatory behavior in the outer part of the boundary layer, which contains large structures with long time scales. Hence, the spatially-redistributive terms are added together in order to illustrate their net effect more clearly. It should also be noted that the sum of the spatially-redistributive terms may not exactly balance the sum of the energizing and consuming terms at certain locations due to the limited statistical sample size. Nevertheless, the overall trends of the profiles can still be useful in deducing the flow behavior in the outer part of the boundary layer.

As seen in Figure 10(d), at the first station of $x/L = -0.49$, the production of R_{ss} in the outer part of the boundary layer decreases with wall-normal distance. The decay rate of R_{ss} production with wall-normal distance at the other two stations is reduced as the incoming flow moves deeper into the APG. This is associated with the destabilizing effect of the progressively-strengthening APG on the outer part of the boundary layer. There is also the additional destabilizing concave mean streamline curvature effect that comes into play as the flow nears the bump. As noted earlier, the surface curvature is concave over the entire initial APG section, and remains so until $x/L \lesssim -0.138$. Figure 10(d) also shows that the production of R_{ss} , as well as its two consuming terms, spread further outward as the boundary-layer thickness grows over the three stations. As seen in Figure 10(e), at $x/L = -0.49$, the net difference between the energizing and consuming terms creates a surplus-containing portion, beyond which a comparatively small deficit-containing portion exists near the boundary-layer edge. As the flow moves between the first and the third stations, the surplus levels increase and cover a wider portion due to the boundary-layer thickness growth. Figure 10(f) shows the sums of the spatially-redistributive terms at the three stations, which balance the surplus or deficit portions of the R_{ss} budgets in the outer region. Some of the excess energy in the surplus portion is moved toward the deficit portion that exists in the outermost part of the boundary layer, while the rest is transported downstream by the spatially-redistributive terms. The outward spread of the R_{ss} levels between the first and the third stations and the emergence of a new outer stress peak, as depicted earlier in Figures 9(b) and (c), are correlated with the growing surplus levels in the outer region R_{ss} budgets depicted in Figure 10(e).

The energizing and consuming terms in the outer region R_{nn} budgets, plotted in Figure 11(d), show that both pressure-strain correlation and dissipation decrease with wall-normal distance at the first station. The decay in both quantities with wall-normal distance at the other two stations occurs at a reduced rate as the incoming flow moves deeper into the APG. The profiles extend further outward with the boundary-layer thickness growth. The enhanced turbulence generation (in the form of R_{ss} and R_{sn} production) in the outer part of the boundary layer induced by the APG enables supplies energy toward R_{nn} via the pressure-strain correlation and subsequently prompts the counteracting response from dissipation. It should also be remarked that the production of R_{nn} in the outer region, although being relatively low in magnitude relative to pressure-strain and dissipation, is negative due to the nature of the mean velocity gradient in the outer part of the boundary layer at the first two stations, indicating backscatter of energy to the mean flow. However, at the third station, production becomes positive with a rather small magnitude, which is an outcome of the

mean velocity gradient resulting from the concave mean streamline curvature.

The net difference between the energizing and consuming terms in the outer region R_{nn} budgets at the three stations is shown in Figure 11(e). The spatially-redistributive term sums that balance the energizing and consuming term sums at the same stations are provided in Figure 11(f). All three stations have a surplus in much of the outer region, succeeded by a nearly-balanced or a weak deficit portion near the boundary-layer edge. Over the initial part of the outer region, similar surplus levels are found among the three stations even though a clear trend is not apparent due to the oscillatory behavior of the profiles. The surplus levels extend further outward as the flow moves deeper into the APG. Other than the weak deficit portion found near the boundary-layer edge, there is not another deficit portion in the outer region R_{nn} budgets at the three stations. Hence, a small fraction of the excess energy is transported toward the weak deficit portion found near the boundary-layer edge, while the rest is transported downstream by the net action of the spatially-redistributive terms. Thus, the surplus energy levels generated in the outer region R_{nn} budgets as the flow moves between the first and the third stations are responsible for the outward spread of the R_{nn} levels and the strengthening of the original stress peak, as seen in Figures 9(b) and (c).

The energizing and consuming terms in the outer region R_{33} budgets are depicted in Figure 12(d). The trends in the pressure-strain and dissipation profiles seen here are very similar to those of R_{nn} discussed above. The enhanced turbulence generation induced by the APG in the outer part of the boundary layer supplies energy toward R_{33} by the pressure-strain correlation and prompts the counteracting response from the dissipation of R_{33} . Figure 12(e) shows the net difference between the energizing and consuming terms, while Figure 12(f) provides the sums of the spatially-redistributive terms that balance the sums of the energizing and consuming terms. As seen in Figure 12(e), surplus energy is present over the entire outer region at the three stations. The boundary-layer edge section remains nearly-balanced and there is no deficit portion in the outer region R_{33} budgets at the three stations. Thus, the excess energy in the surplus portion is transported downstream by the net action of the spatially-redistributive terms. Note again that due to the limited statistical sample size, the sum of the spatially-redistributive terms may not appear to exactly balance the sum of the energizing and consuming terms at certain $(x/L, n/L)$ locations in Figures 12(e) and 12(f), such as in the vicinity of $n/L = 1 \times 10^{-3}$. The outward spread of the R_{33} levels between the first and the third stations and the emergence of a new outer stress peak, as seen earlier in Figures 9(b) and (c), result from the surplus energy levels found in the outer region R_{33} budgets.

Figure 13(d) shows the outer region distributions of the energizing production term and the consuming pressure-strain correlation in the R_{sn} budgets at the three stations. Dissipation is omitted here since it is generally negligible in the outer region. The trends in the production and pressure-strain profiles of R_{sn} in the outer region are very similar to those of R_{ss} seen earlier. The decay rate of R_{sn} production with wall-normal distance is reduced as the incoming flow gets deeper into the APG. Once again, this is associated with the destabilizing effect of the progressively-strengthening APG on the outer part of the boundary layer, and the mean velocity gradient resulting from the concave mean streamline curvature as the flow nears the bump. The pressure-strain correlation provides the counteracting response to the changes occurring in the production term. The net difference between the energizing and consuming terms is provided in Figure 13(e), which also includes dissipation. This figure shows the formation of a deficit portion near the boundary-layer edge, preceded by a surplus-containing portion. Due to the oscillatory behavior of the profiles, a clear trend among the three stations is not evident over the initial part of the outer region. Nevertheless, it appears that the main surplus-containing portion shifts outward as the flow moves between the first and the third stations, and similar amounts of total surplus energy are found in the outer region among the three stations. The sums of the spatially-redistributive terms that balance the net difference between the energizing and consuming terms is provided in Figure 13(f). These terms transport energy to the relatively weak deficit portion near the boundary-layer edge and other weak deficit pockets that may exist over the initial part of the outer region. The rest of the surplus energy in the outer region is transported downstream. Hence, given similar total surplus levels among the three stations, the outward spread of the R_{sn} levels and the strengthening of the

original stress peak, as seen in Figures 9(b) and (c), result from the net action of the spatially-redistributive terms on the surplus energy.

We conclude the analysis of the initial APG region by an examination of the budgets normalized by the inner scaling of u_τ^4/ν_w , as depicted in Figure 14 at the three selected stations for all stress components and the TKE. As seen here, other than pressure diffusion and pressure strain that do not appear to achieve a tight collapse for the wall-normal and shear components in the near-wall region, the remaining budget terms of these two components, as well as all budget terms of the other stress components show a more reasonable overall collapse. For the TKE, pressure-correlation is identically zero while pressure diffusion is relatively small compared to the other terms; hence, the overall collapse of the TKE budgets also seems reasonable.

3.2.2 Examination of the strong FPG region

Next, we examine the development of the four Reynolds stress components and their respective budgets as the boundary layer interacts with the strong FPG over the windward side of the bump. Figure 15 depicts the normalized mean velocity component and the Reynolds stress component profiles at the four stations that cover the region from $x/L = -0.3$ until $x/L = -0.007$. The first station is located slightly upstream of the start of the FPG region (which is at $x/L \approx -0.29$), while the last station is where the C_p peak is found. The strong FPG causes a substantial acceleration of the incoming TBL toward the bump apex. As seen in Figure 15(a), the boundary-layer edge U_s rises by about 62 percent between the start and the end of the acceleration region. The strong flow acceleration over the curved surface leads to a negative U_n distribution over much of the boundary layer, except very near the wall where U_n remains nearly zero, as also depicted in Figure 15(a). The U_n profiles strengthen with streamwise distance over much of the acceleration region. Toward the end of acceleration very near the bump apex, the U_n distribution takes positive values that are much smaller than those of U_s . Another outcome of the strong acceleration is the significant rise in the wall skin friction, as depicted earlier in Figure 2.

In Figure 15, we observe that the original peaks of R_{ss} and R_{33} strengthen very significantly while new near-wall peaks emerge in R_{nn} and R_{sn} profiles as the flow accelerates toward the apex. These changes to the Reynolds stress profiles indicate the formation of an internal layer beneath the accelerated boundary layer, which was identified and examined in the earlier analysis of this flow at the lower Re_L value of 2 million [25]. It is also interesting to note that the original peaks of R_{nn} and R_{sn} at $x/L = -0.3$, which are positioned further away from the wall relative to the R_{ss} and R_{33} peaks at the same station, strengthen between $x/L = -0.3$ and -0.2 before starting to decay along with the outer level stress levels as the flow moves deep into the acceleration region. Meanwhile, the levels in the outer region for the other two components drop monotonically (except near the boundary-layer edge) over the entire FPG region.

Figures 16 through 19 provide the evolution of the four Reynolds stress component budgets over the strong FPG region. All budget terms are normalized by U_∞^3/L to enable a relative magnitude comparison of the budget terms among the selected streamwise stations. The budgets very near the wall (defined by $n/L < 1 \times 10^{-3}$) and those further from the wall (defined by $n/L > 1 \times 10^{-3}$) are shown separately. As in the preceding analysis, the energizing or consuming terms are grouped together and are shown separately from the spatially-redistributive terms. These plots show that the dominant budget terms in the near-wall region get substantially amplified during the flow acceleration while some budget imbalances form at certain n/L locations as the flow moves toward the apex. Relative to the respective dominant term in the vicinity of the imbalance, the maximum residual near the apex is about 4% for both R_{ss} and R_{sn} , 11% for R_{nn} and 6% for R_{33} . Similar percentage-wise imbalance levels can be found upstream of the apex as well.

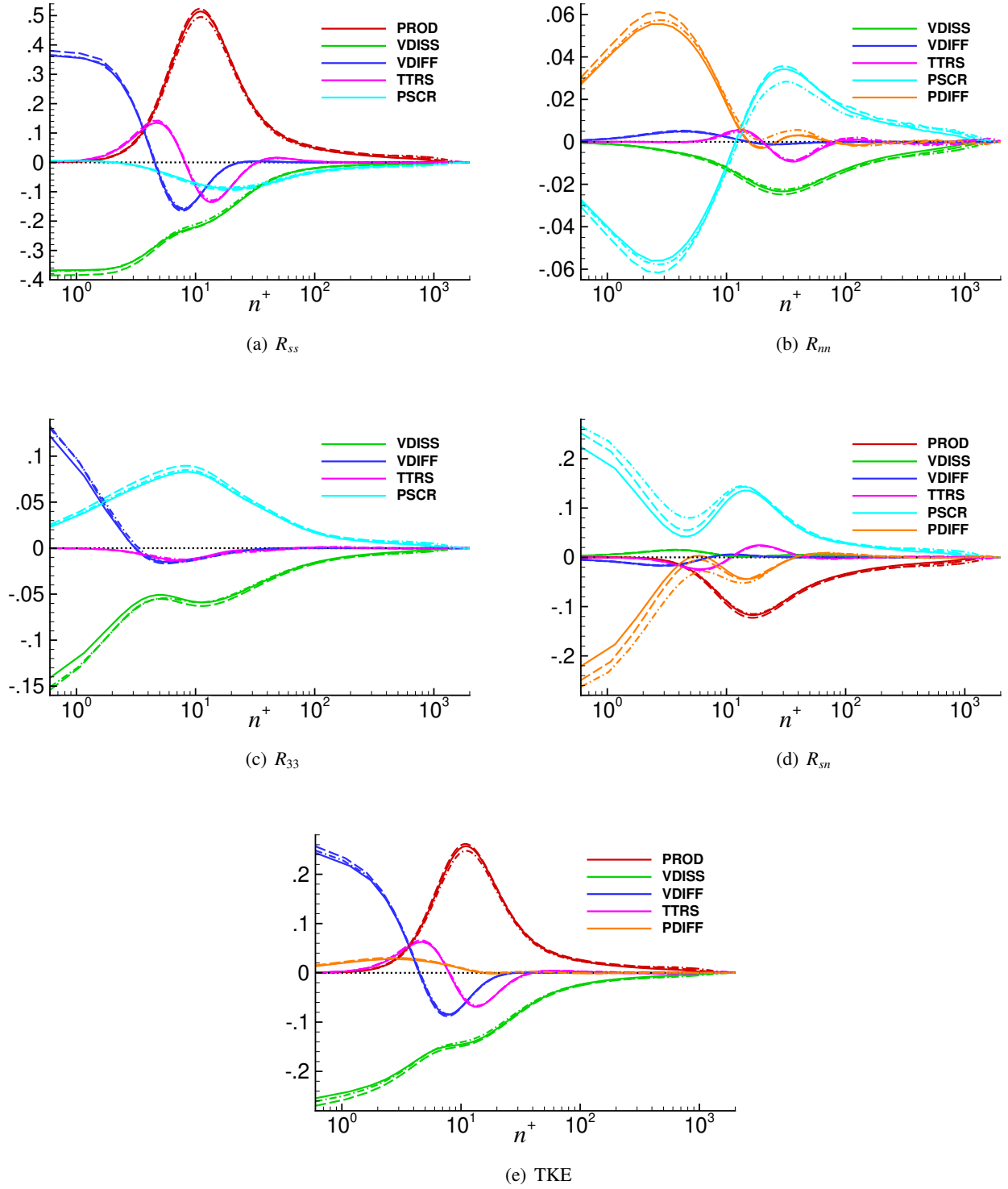


Figure 14. Reynolds stress and TKE budgets normalized by the inner scaling (u_τ^4/ν_w) in the initial APG region. Solid lines indicate the budget terms at $x/L = -0.49$; dashed lines indicate the budget terms at $x/L = -0.4$; dash-dotted lines indicate the budget terms at $x/L = -0.3$.

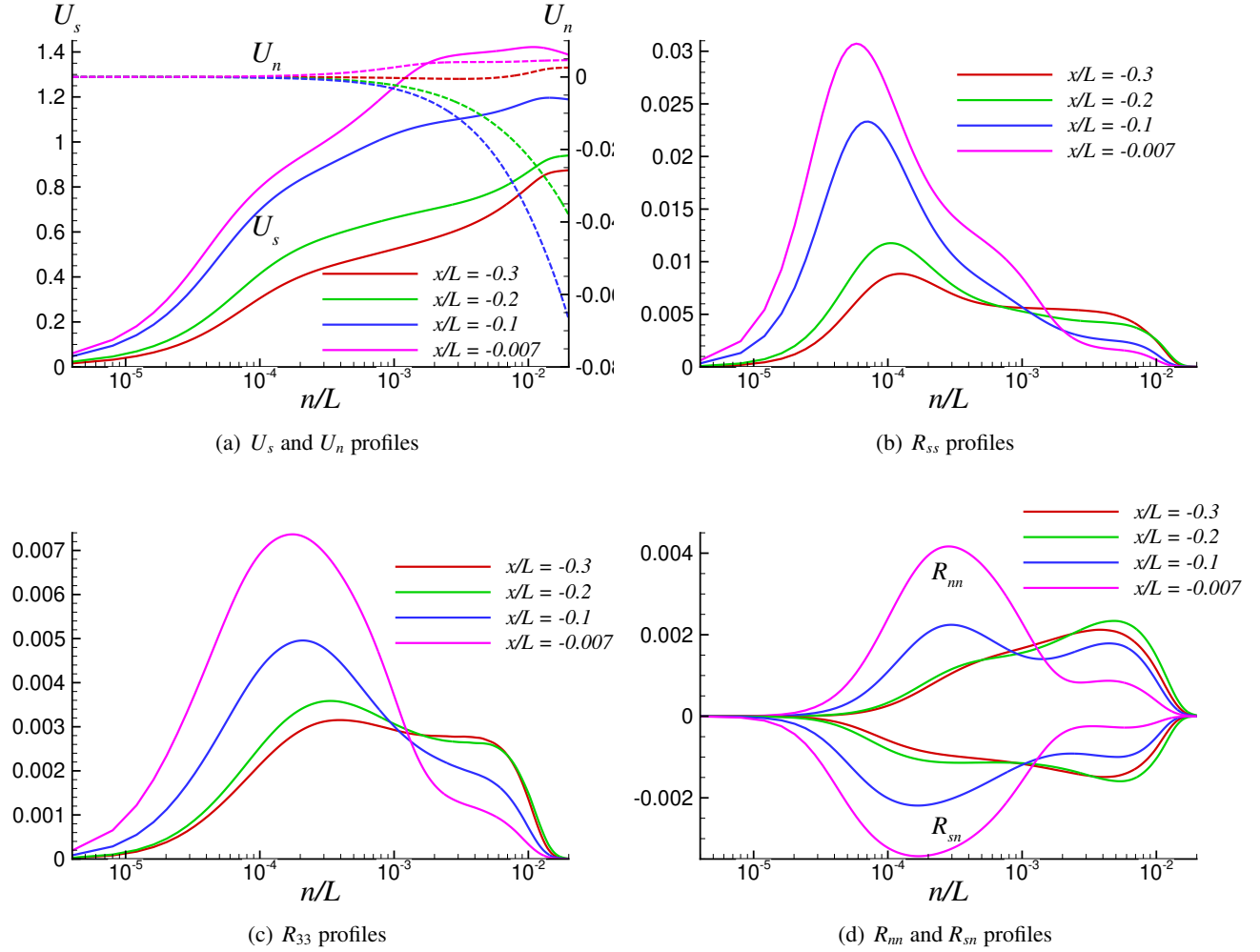
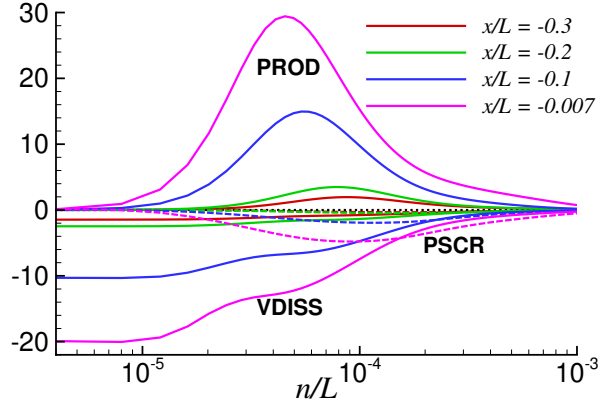
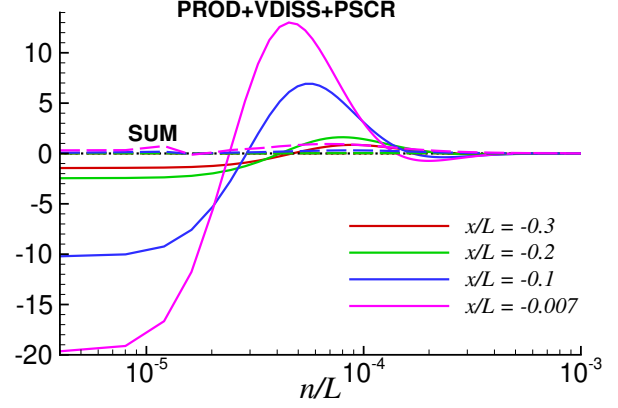


Figure 15. Mean velocity and turbulent stress profiles in the strong FPG region. The normalization is based on U_∞ .

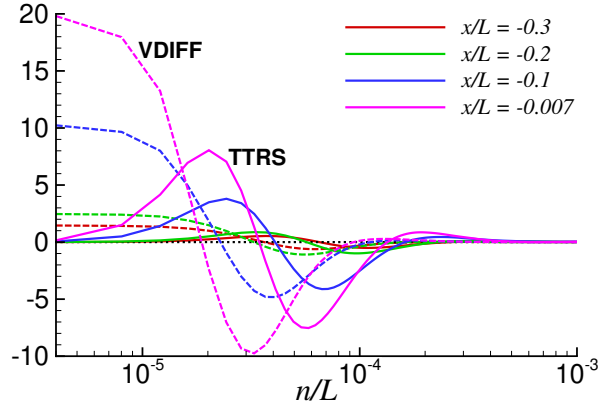
Figures 16(a) and 19(a) show that the near-wall production of R_{ss} and R_{sn} increases by an order of magnitude between the start and the end of the FPG. The increase in the near-wall mean shear rate due to flow acceleration is the main cause for such a remarkable growth in production. As seen in Figures 16(a) and 19(a), the near-wall consuming terms for R_{ss} and R_{sn} , which are dissipation and pressure strain, also increase dramatically in response to the much-amplified production of these two components. The near-wall budgets of R_{nn} and R_{33} , provided in Figures 17(a) and 18(a), show the associated growth in the energizing and consuming terms for those two components. For R_{33} , pressure strain is the main energizing term while dissipation is the sole consuming term. For R_{nn} , pressure strain is a consuming term very near the wall that is more dominant than dissipation, and becomes an energizing term further from the wall. It is also observed that the changes to the near-wall budget terms in the early stages of the flow acceleration, as depicted by the profiles at $x/L = -0.3$ and -0.2 , are not as drastic as those occurring in the latter stages. This is associated with how rapidly the near-wall mean shear rate rises due to flow acceleration, which in turn determines the rate of increase in the near-wall productions of R_{ss} and R_{sn} with streamwise distance over the strong FPG region.



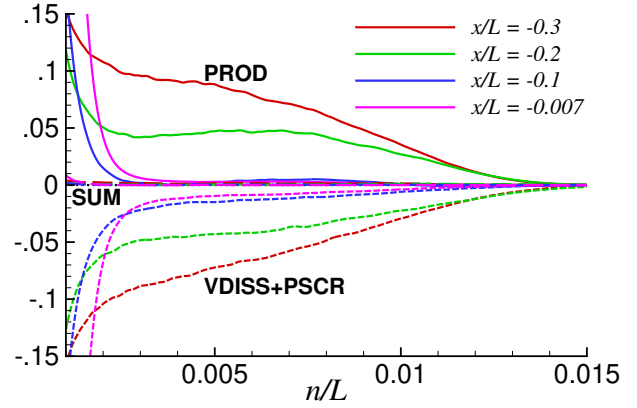
(a) energizing/consuming terms in near-wall region



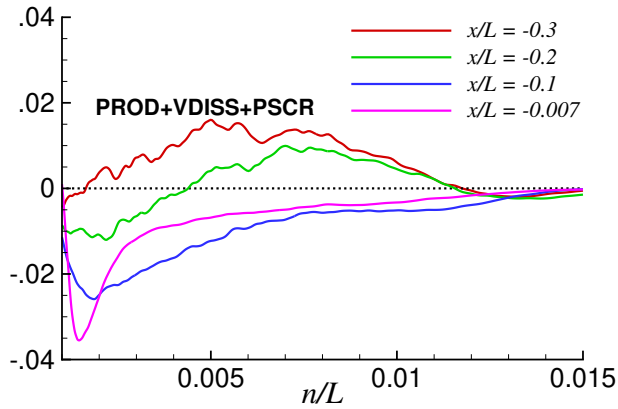
(b) energizing/consuming term sums in near-wall region



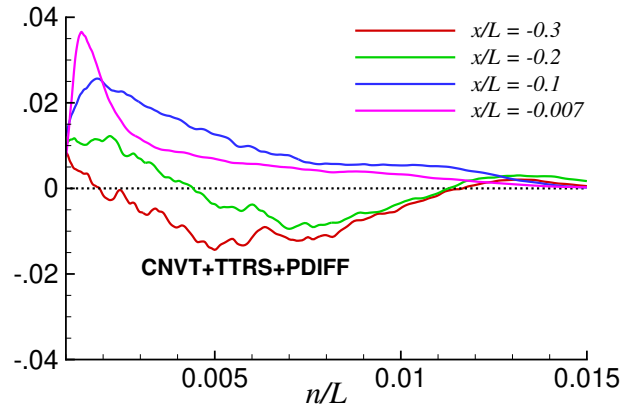
(c) spatially-redistributive terms in near-wall region



(d) energizing/consuming terms in outer region

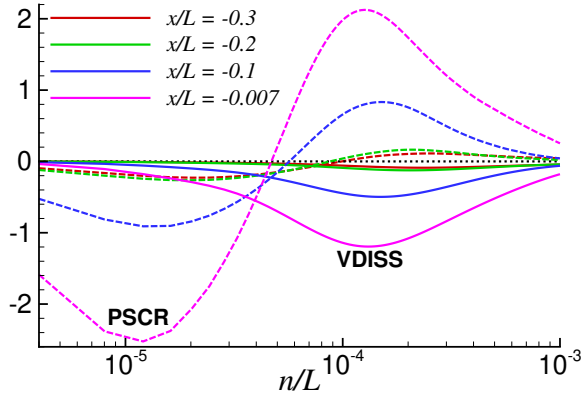


(e) energizing/consuming term sums in outer region

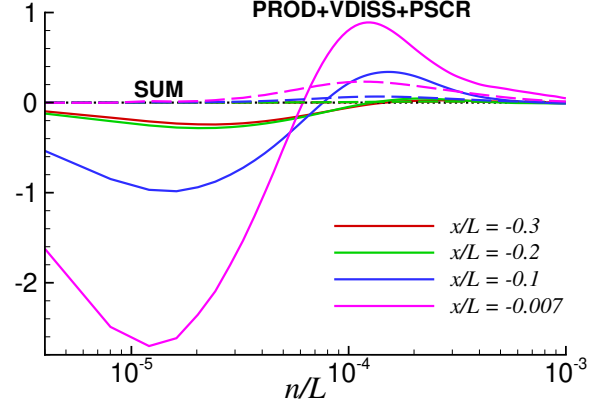


(f) spatially-redistributive term sums in outer region

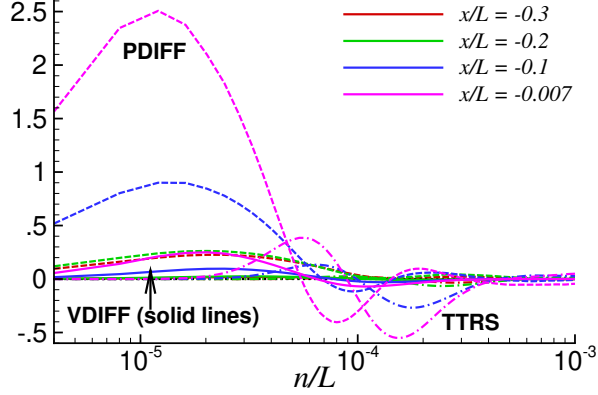
Figure 16. Evolution of R_{ss} budget terms in the strong FPG region. All budget terms are normalized by U_∞^3/L .



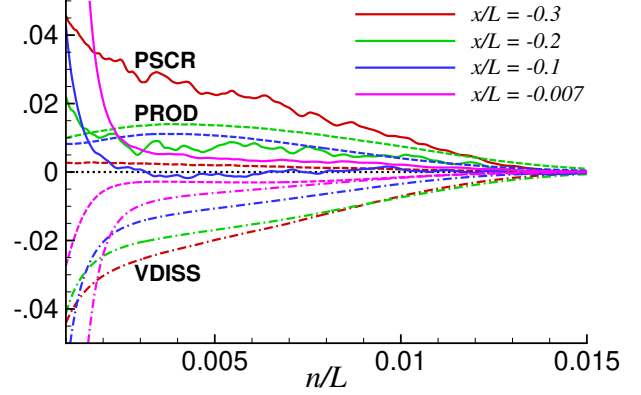
(a) energizing/consuming terms in near-wall region



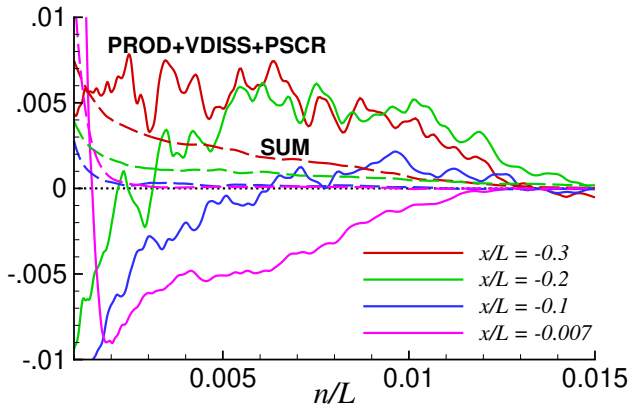
(b) energizing/consuming term sums in near-wall region



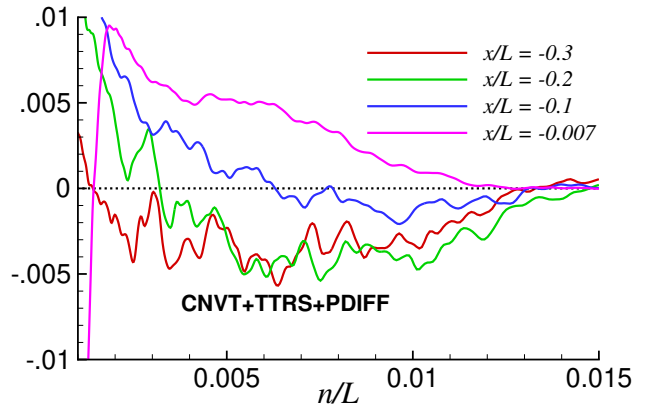
(c) spatially-redistributive terms in near-wall region



(d) energizing/consuming terms in outer region

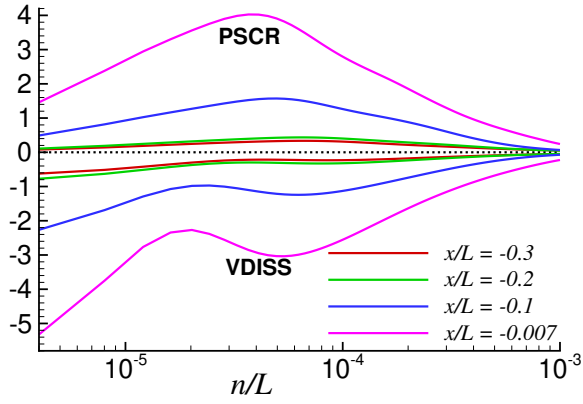


(e) energizing/consuming term sums in outer region

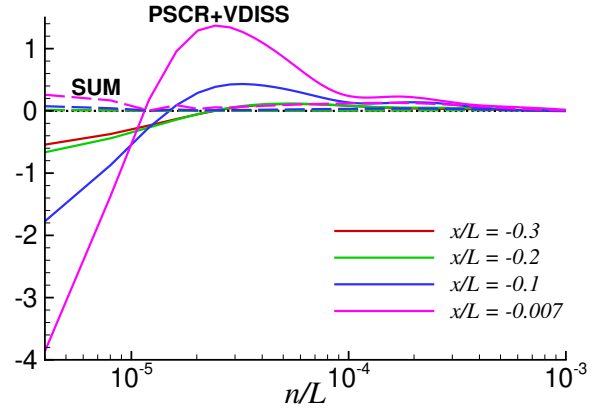


(f) spatially-redistributive term sums in the outer region

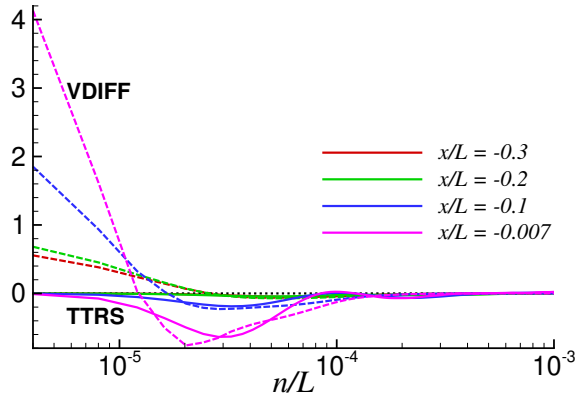
Figure 17. Evolution of R_{nn} budget terms in the strong FPG region. All budget terms are normalized by U_∞^3/L .



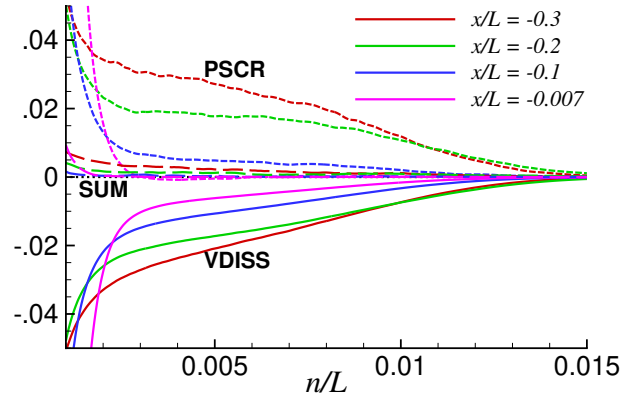
(a) energizing/consuming terms in near-wall region



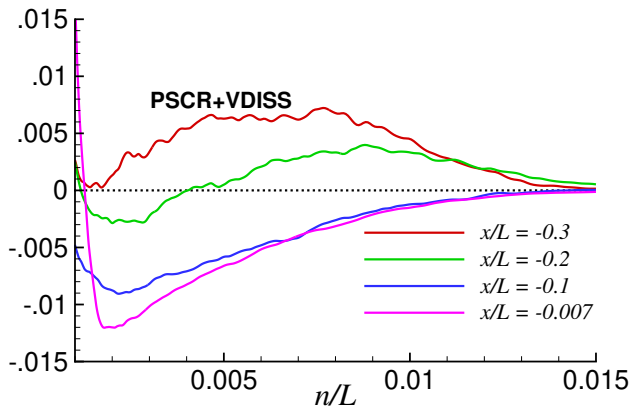
(b) energizing/consuming term sums in near-wall region



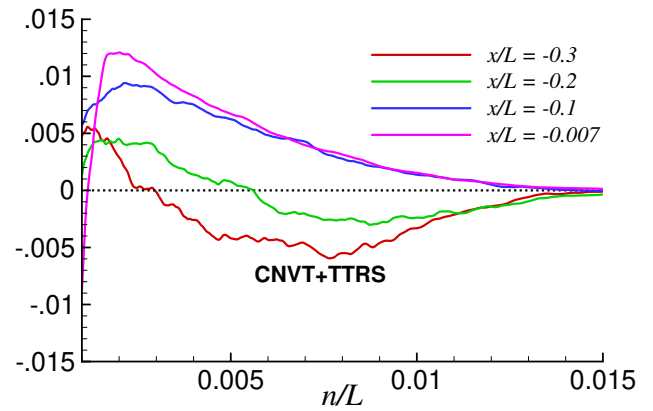
(c) spatially-redistributive terms in near-wall region



(d) energizing/consuming terms in outer region

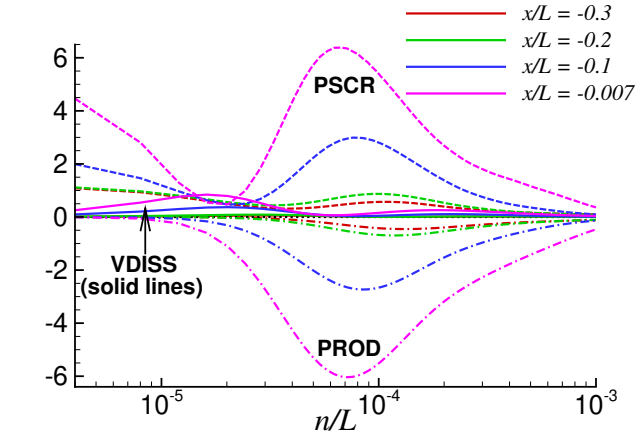


(e) energizing/consuming term sums in outer region

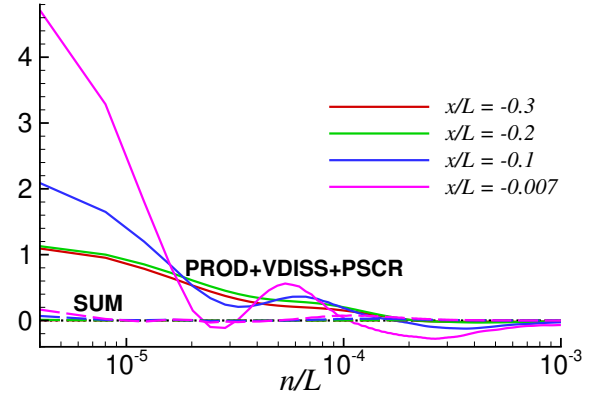


(f) spatially-redistributive term sums in outer region

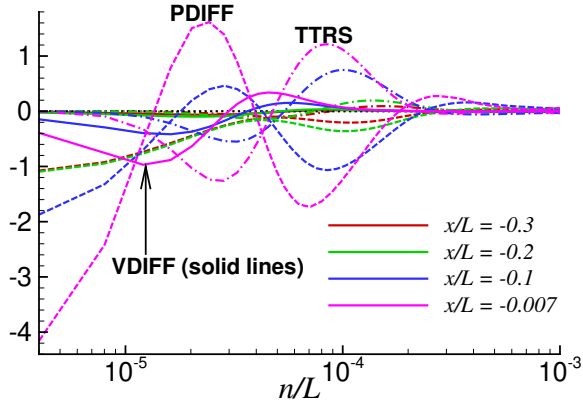
Figure 18. Evolution of R_{33} budget terms in the strong FPG region. All budget terms are normalized by U_∞^3/L .



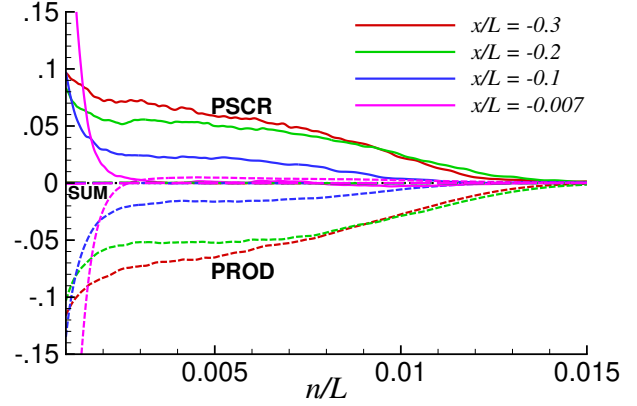
(a) energizing/consuming terms in near-wall region



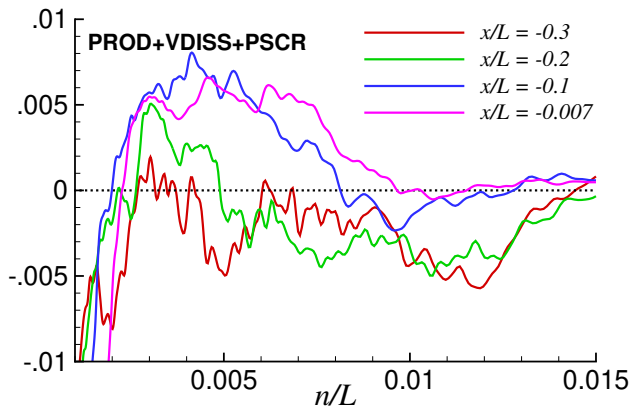
(b) energizing/consuming term sums in near-wall region



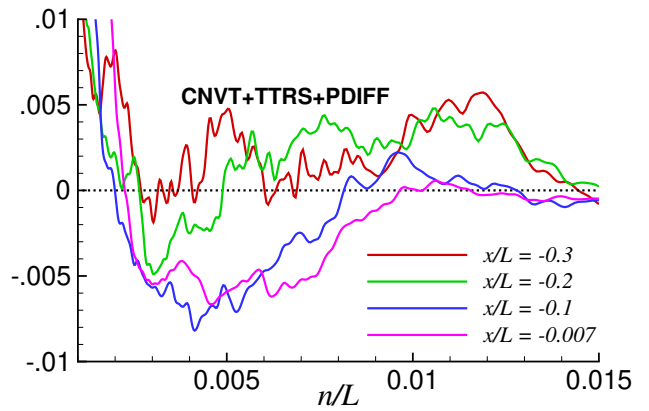
(c) spatially-redistributive terms in near-wall region



(d) energizing/consuming terms in outer region



(e) energizing/consuming term sums in outer region



(f) spatially-redistributive term sums in outer region

Figure 19. Evolution of R_{sn} budget terms in the strong FPG region. All budget terms are normalized by U_∞^3/L .

Figures 16(b) through 19(b) show the sum of the energizing and consuming terms in the near-wall region. Although the near-wall production of R_{nn} is generally negligible, it is nevertheless included in the sums shown in Figure 17(b). For R_{33} , recall that production is identically zero because of the spanwise homogeneity. For all components, very near the wall, the initial deficit between the energizing and consuming terms at the start of the acceleration grows about an order of magnitude by the end of the acceleration as the increase in consuming term amplitudes overwhelms any energizing terms in that region. For all normal stress components, with increasing wall distance, the energizing terms surpass the consuming terms, and this leads to the formation of a surplus portion at a short distance from the wall. Furthermore, the surplus peak values increase remarkably between the start and the end of the acceleration region. With increasing wall-normal distance, the surplus portion in the R_{ss} budget is followed by a weak deficit portion before the imbalance between the energizing and consuming terms in the outermost part of the near-wall region drops to levels that are several orders of magnitude smaller than those observed much closer to the wall. The budgets of the other two normal stress components also experience similar drops to their respective imbalances between the energizing and consuming terms in the outermost part of the near-wall region. Meanwhile, for R_{sn} , a secondary deficit peak forms at a short distance from the wall, mainly due to the consuming pressure-strain correlation exceeding production there. With further outward distance, the secondary deficit peak is succeeded by a flat section in which the energizing and consuming terms are nearly balanced at the streamwise stations of $x/L = -0.3$ and -0.2 . This flat section then evolves into a growing-surplus portion in the latter stages of acceleration. For R_{sn} , the profile at $x/L = -0.007$ also shows the formation of a trough between the primary and secondary deficit peaks, wherein a weak surplus is achieved.

Figures 16(c) through 19(c) show the relative amplifications of the near-wall spatially-redistributive terms over the acceleration region. Such magnifications of the redistributive terms occur to counteract the surplus or deficit portions forming between the energizing and consuming terms for a given stress component. The growing deficit between the energizing and consuming terms very near the wall for all stress components is balanced by the increased energy supply from the spatially-redistributive terms. The spatially-redistributive terms transport energy available further from the wall, either generated by production or attained via pressure strain, toward the very near-wall region via different mechanisms for a given stress component. For R_{ss} , viscous diffusion and turbulent transport are the main mechanisms that redistribute the energy in the near-wall region. For R_{33} , turbulent transport also convects some energy from the surplus portion toward the outer part of the near-wall region where a weak deficit between the energizing and consuming terms exists. For R_{33} , viscous diffusion is the sole mechanism of redistribution to the very near-wall region since turbulent transport is a sink in that region. For R_{nn} and R_{sn} , pressure diffusion is the dominant redistributive term very near the wall where there is relatively less secondary contribution coming from the viscous diffusion. For these two components, pressure diffusion also transfers energy in the direction away from the wall in parts of the near-wall region. Turbulent transport is a relevant mechanism in convecting energy to close proximity of the wall for R_{nn} and R_{sn} as well. For all components, in the outermost part of the near-wall region, the spatially-redistributive terms generally drop to levels that are several orders of magnitude lower than those found much closer to the wall.

The n/L positions of R_{ss} and R_{33} peaks, and of their respective budget surplus peaks at the four streamwise stations are provided in Table 2. Table 3 provides the corresponding information for R_{nn} and R_{sn} , but only for the last two stations since the near-wall stress peaks of those two components clearly emerge in the latter stages of acceleration, as seen in Figure 15. Table 2 shows that, based on the n/L scaling of the wall-normal distance, the R_{ss} and R_{33} peaks, and their respective budget surplus peaks gradually shift closer to the wall with streamwise distance. This is due to the steepest part of the mean streamwise velocity gradient, and thus the production peak of R_{ss} , moving closer to the wall, as depicted by the U_s profiles in Figure 15(a), and by the R_{ss} budgets in Figure 16(a). For both R_{ss} and R_{33} , the stress peaks are located outward of the respective budget surplus peak. At a given x/L , although the budget surplus of R_{33} peaks earlier than that of R_{ss} , the near-wall R_{ss} peak is considerably closer to the wall than the R_{33} peak. This is related to how much

x/L	R_{ss} peak n/L	R_{ss} budget surplus peak n/L	R_{33} peak n/L	R_{33} budget surplus peak n/L
-0.3	1.23×10^{-4}	8.98×10^{-5}	3.95×10^{-4}	5.65×10^{-5}
-0.2	1.04×10^{-4}	8.08×10^{-5}	3.35×10^{-4}	5.42×10^{-5}
-0.1	7.14×10^{-5}	5.62×10^{-5}	2.10×10^{-4}	3.27×10^{-5}
-0.007	5.82×10^{-5}	4.53×10^{-5}	1.75×10^{-4}	2.44×10^{-5}

Table 2. Near-wall position of R_{ss} and R_{33} peaks, and of their respective budget surplus peaks.

x/L	R_{nn} peak n/L	R_{nn} budget surplus peak n/L	R_{sn} peak n/L	R_{sn} budget surplus peak n/L
-0.1	2.96×10^{-4}	1.51×10^{-4}	1.66×10^{-4}	3.68×10^{-4}
-0.007	2.84×10^{-4}	1.24×10^{-4}	1.70×10^{-4}	2.64×10^{-4}

Table 3. Near-wall position of R_{nn} and R_{sn} peaks, and of their respective budget surplus peaks.

energy is left over at a given n/L location for R_{ss} or R_{33} once the excess energy in the surplus portion is redistributed by the relevant spatially-redistributive terms. Table 3 shows that the emerging near-wall peak of R_{nn} moves a bit inward between the two x/L stations, while that of R_{sn} shows a very slight outward shift between the same stations. The respective budget surplus peaks of these two components shift toward the wall at the same time. For R_{nn} , the stress peak is located outward of its budget surplus peak, while the stress peak of R_{sn} is located inward of its budget surplus peak. These observations are again outcomes of the excess energy redistribution by the relevant spatially-redistributive terms.

Figures 16(d) through 19(d) show the energizing and consuming terms in the outer part of the boundary layer at the four x/L stations in the acceleration region, while the sums of these terms are provided in Figures 16(e) through 19(e). The sums of the spatially-redistributive terms in the same region are provided in Figures 16(f) through 19(f). Since dissipation and pressure strain are both consuming terms for R_{ss} , they are added together for clarity in Figure 16(d). For R_{sn} , dissipation in the outer region is generally negligible and hence it is omitted in Figure 19(d), but included in Figure 19(e). For R_{ss} , pressure diffusion is generally small relative to the other terms; nevertheless, it is included in the sums shown in Figure 16(f).

As noted earlier, oscillatory behavior is present in some budget quantities in the outer part of the boundary layer due to the limited statistical sample size. At the first station of $x/L = -0.3$, the production of R_{ss} in the outer part of the boundary layer decreases with wall-normal distance, as seen in Figure 16(d). Once the flow starts accelerating, the production of R_{ss} in the outer part of the boundary layer decays along both the streamwise direction and the wall-normal direction. This is because the strong FPG decreases the gradient of the streamwise velocity along the wall-normal direction in the outer region, as can be seen in Figure 15(a). This is a stabilizing effect of the strong FPG in that region. Production of R_{ss} therefore drops to a very low level in the late stages of acceleration. The consuming term sum magnitudes in the outer region also decrease in tandem with the production decay caused by the strong acceleration. In the late stages of acceleration, however, the consuming term sum magnitudes exceed those of production. As seen in Figure 16(e), the net difference between the energizing and consuming terms creates a surplus over much of the outer region at $x/L = -0.3$. This initial surplus portion then decays and turns into a deficit with streamwise distance over the acceleration region, ultimately leading to the drop in R_{ss} levels observed in the outer part of the boundary layer, as depicted in Figure 15(b). Figure 16(f) shows the sums of the spatially-

redistributive terms at the four stations, which balance the surplus or deficit portions of the R_{ss} budgets in the outer part of the boundary layer. Some of the excess energy in the surplus-containing portion is moved toward the outer deficit portion in the initial stages of acceleration while the rest is transported downstream. In the latter stages of acceleration, the entire outer region contains a deficit, which is balanced by energy transported from upstream locations by the spatially-redistributive terms.

Examining the evolution of the energizing and consuming terms in the outer part of the boundary layer for R_{nn} , depicted by the four x/L stations in Figure 17(d), we see that the pressure-strain correlation, which is an energizing term, drops considerably over the first two stations, due to stabilizing FPG effect. At the third station, pressure strain becomes mostly inactive. Pressure strain then starts to increase near the apex as the strong FPG effect comes to an end and the strong APG effect is about to begin. Figure 17(d) also shows a relatively significant increase in the positive production of R_{nn} between $x/L = -0.3$ and -0.2 . As noted earlier, the surface curvature is concave until $x/L \lesssim -0.138$, after which it becomes convex until $x/L \lesssim 0.138$. Hence, the mean velocity gradient resulting from the concave mean streamline curvature in the initial stages of flow acceleration leads to a rise in the positive production of R_{nn} between the first two stations. The local surface curvature becomes convex at the third station. Comparing the production profiles between the second and the third stations, we see that the outer region production of R_{nn} at the third station has decreased relative to that at the second station. At the fourth station, production of R_{nn} becomes negative, indicating a backscatter of energy to the mean flow.

Comparing the dissipation profiles of R_{nn} between the first and the second stations in Figure 17(d), we find a reduction in dissipation over the initial outer region as the flow moves between those two stations. Dissipation then stabilizes over the remainder of the outer region and remains at similar levels between the two stations. Dissipation of R_{nn} continues to decrease monotonically with streamwise distance over much of the outer region for the next two stations. Figure 17(e) shows the net difference between the energizing and consuming terms, while Figure 17(f) provides the sums of the spatially-redistributive terms that provide balance against the net difference between the energizing and consuming terms for the outer region R_{nn} budgets. Between the first two stations, the initial surplus decreases by some amount over the initial part of the outer region, and then remains about the same or slightly increases with further outward distance. There is also a weak deficit portion near the boundary-layer edge at the first two stations, although the chosen horizontal range of Figure 17(e) may not clearly depict this particular feature. Hence, in the early stages of acceleration, a small fraction of the outer region surplus goes toward the weak deficit portion near the boundary-layer edge, while the rest is transported downstream. With similar surplus levels between the first and the second stations over much of the outer region, downstream transport of excess energy leads to the increase in the outer region R_{nn} levels along with the peak value, as seen earlier in Figure 15(d). As the flow moves deep into the acceleration region, the surplus acquired in the initial stages of acceleration gradually turns into a deficit. At the third station, there is still some small surplus present in the outermost region. At the fourth station, a deficit resides in much of the outer region, followed by a nearly-balanced short section near the boundary-layer edge. The growing deficit observed at the third and the fourth stations correlates with the R_{nn} decrease in the outer region at the same stations. The strong deficit forming between the energizing and consuming terms over much of the outer region in the late stages of acceleration is balanced by the energy transported from upstream locations.

The energizing and consuming terms in the outer region R_{33} budgets, depicted in Figure 18(d), reveal a mostly monotonic decay in the energizing pressure-strain correlation and in the consuming dissipation term as the flow acceleration takes place over the windward side. However, between the first two stations, the pressure-strain distribution over the latter part of the outer region remains about the same or becomes slightly stronger, despite the decrease over the initial part of the outer region. A similar observation can also be made about the dissipation profiles between the two stations. As the flow nears the apex, pressure strain becomes mostly negligible over much of the outer region while there is still some active dissipation taking place. The net difference between the energizing and consuming terms at the four stations is shown

in Figure 18(e). At the first station, a surplus exists over much of the outer region. As the flow reaches the second station, the surplus over the initial part of the outer region decreases significantly while slightly increasing further outward. This is related to the behavior of the pressure-strain correlation distributions at the two stations. Between the first two stations, the total amount of surplus energy at the second station decreases relative to that at the first station. As the flow moves further into the acceleration region, the diminishing surplus turns into a growing deficit over the entire outer region. These developments correlate with the decrease in the outer region R_{33} levels over the acceleration region seen earlier in Figure 15(c). Figure 18(f) plots the sums of the spatially-redistributive terms for the outer region R_{33} budgets. There is no deficit portion in the outer part of the boundary layer at the first and the second stations, while a relatively weak deficit pocket appears to emerge over the early part of the outer region at the second station, as seen in Figure 18(e). Hence, the excess energy in the outer region at the first station is transported downstream by the spatially-redistributive terms. Downstream transport continues at the second station with the leftover energy after the aforementioned deficit pocket is balanced. The growing deficits emerging between the energizing and consuming terms in the outer part of the boundary layer, as the acceleration takes place, are balanced by the energy transported from upstream locations.

The distributions of the energizing production term and the consuming pressure-strain correlation in the outer region R_{sn} budgets are shown in Figure 19(d). Recall that dissipation in this region is generally negligible for R_{sn} and is therefore omitted. It is observed that as the flow moves between the first two stations, there is some decrease in production along with an accompanying drop in pressure strain, over the initial part of the outer region. Over the remainder of the outer region, production at the second station slightly increases relative to that at the first station. A similar observation also holds true for pressure strain. As the flow moves deeper into the acceleration region, both production and pressure strain decay substantially. At the last station, it is observed that production turns into a consuming term as it becomes positive since R_{sn} is generally a negative quantity, and this indicates backscatter of energy to the mean flow. At the same station, pressure strain is quite small over the first half of the outer region, but turns into an energizing term over the second half as it becomes negative. The net difference between the energizing and consuming terms depicted in Figure 19(e), which also includes dissipation, shows a surplus in sections of the outer region at the first station. Comparing the first two stations, we find that some parts of the initial surplus decrease in magnitude or turn into a deficit, while other parts grow in magnitude. As seen in Figure 15(d), the outer peak of the R_{sn} profile shifts from $n/L \approx 4 \times 10^{-3}$ to 5.4×10^{-3} as the flow moves between the first two stations. Figure 19(e) shows that the surplus levels inboard of $n/L = 5.4 \times 10^{-3}$ show some decrease from the first to the second station. However, there is additional surplus available further outward and not too far from $n/L = 5.4 \times 10^{-3}$ at the second station. It therefore appears that the available nearby surplus and the action of the spatially-redistributive terms are responsible for the increase in the peak R_{sn} value between the first two stations. As the flow moves deeper into the acceleration region, the outer region surplus decreases and turns into a deficit, leading to a subsequent decay of R_{sn} levels in the outer part of the boundary layer. The sums of the spatially-redistributive terms that balance the net difference between the energizing and consuming terms in the outer region R_{sn} budgets are provided in Figure 19(f). A fraction of the surplus energy available in parts of the outer region at the first two stations goes toward the deficit portion near the boundary-layer edge, and other deficit pockets that may exist elsewhere, while the rest is transported downstream. As observed previously in the budgets of other stress components, the strong deficits emerging in the outer part of the boundary layer as the acceleration takes place are balanced by the energy transported from upstream locations.

The budgets normalized by the inner scaling of u_t^4/ν_w at the stations of $x/L = -0.2, -0.1$ and -0.007 are shown in Figure 20. The budgets at $x/L = -0.3$ are omitted here due to the fact the TBL does not yet fully experience the strong FPG effect at that station. For both R_{ss} and the TKE, the collapse of all budget terms at the stations of $x/L = -0.2$ and -0.1 , located roughly in the middle of the strong acceleration region, is found to be quite good. Between the same two x/L stations, R_{nn} and R_{sn} do not have a good collapse for

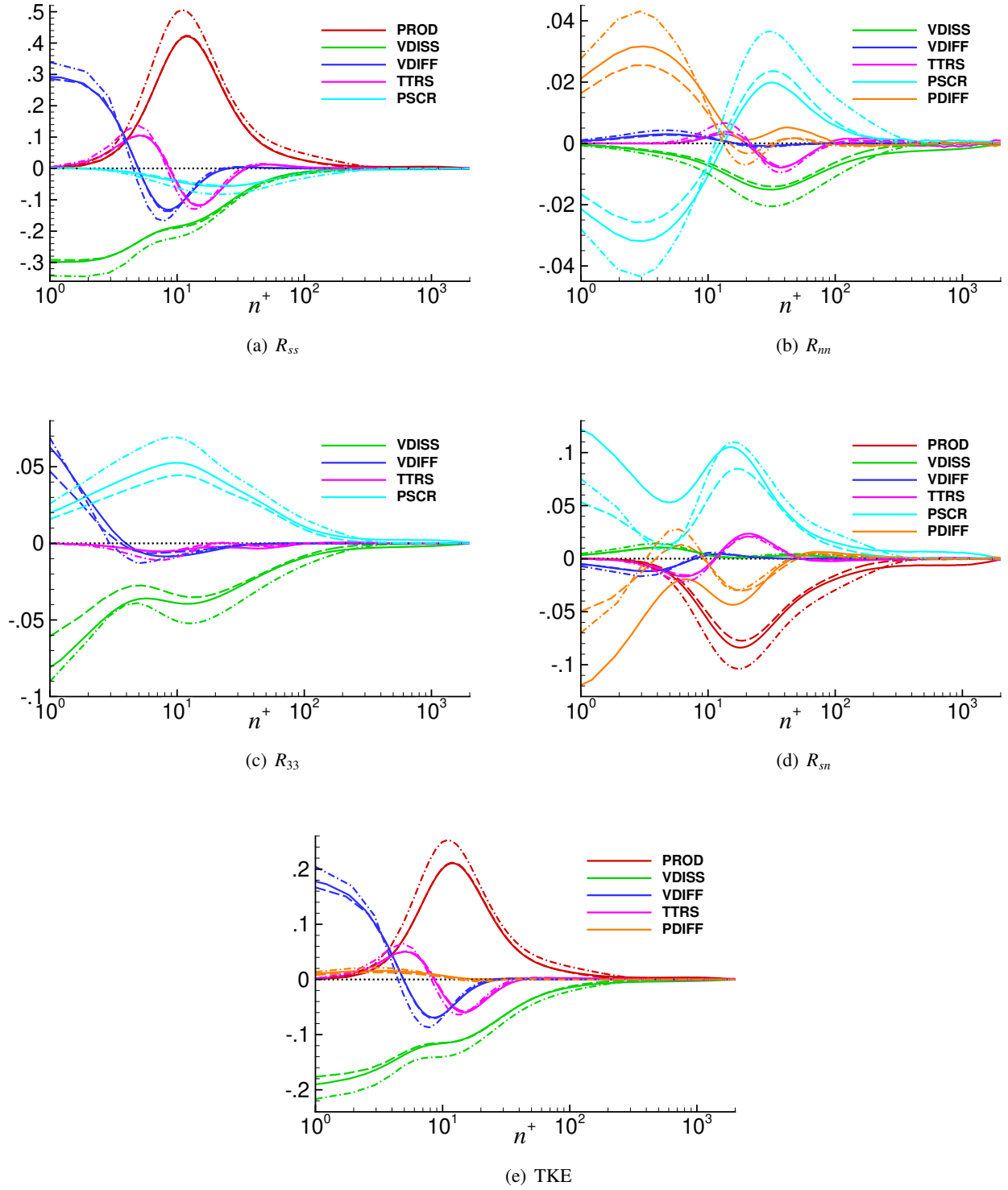


Figure 20. Reynolds stress and TKE budgets normalized by the inner scaling (u_τ^4/ν_w) in the strong FPG region. Solid lines indicate the budget terms at $x/L = -0.2$; dashed lines indicate the budget terms at $x/L = -0.1$; dash-dotted lines indicate the budget terms at $x/L = -0.007$.

pressure strain and pressure diffusion in the near-wall region, although the collapse improves further from the wall for R_{sn} . The remaining budget terms of these components show a better collapse. For R_{33} , the budgets at the two stations also show an improved collapse with further distance from the wall. The budgets at the station of $x/L = -0.007$, where the strong FPG effect vanishes and the strong APG is about to take over, for the most part do not collapse well onto the budgets at the prior stations.

3.2.3 Examination of the strong APG region

The final part of our analysis deals with the strong APG section that starts slightly upstream of the apex and terminates at the flow separation point where $x/L \approx 0.1$. Figure 21 shows the normalized mean velocity component and the Reynolds stress component profiles at the three stations where $x/L = -0.007, 0.05$ and 0.1 , respectively. As seen in Figure 21(a), the strong APG reduces U_s both near the wall and in the outer part of the boundary layer, and this leads to the formation of an intermediate buffer zone that forms the origin of the detached shear layer. At the first station, which is very slightly upstream of the apex, U_n is mostly negligible relative to U_s . As the boundary layer decelerates toward separation, U_n starts to take increasingly positive values across the entire boundary layer, except very near the wall where it remains close to zero. Such development of the mean velocity component profiles over the strong APG region indicates movement of the emerging shear layer in the outward direction.

As previously noted in Tables 2 and 3, and as can be seen in the Reynolds stress profiles depicted in Figure 21, at the initial station where $x/L = -0.007$, the R_{ss} peak is closest to the wall at $n/L = 5.82 \times 10^{-5}$, while those of R_{33} and R_{sn} are positioned further from the wall at approximately the same location where $n/L \approx 1.7\text{--}1.75 \times 10^{-4}$, and the R_{nn} peak is the furthest from the wall at $n/L = 2.84 \times 10^{-4}$. As the flow moves toward separation, the R_{ss} peak starts to weaken along with a drop in the near-wall R_{ss} levels while a new outer R_{ss} peak develops. A similar phenomenon also occurs in the R_{33} and R_{sn} profiles. The R_{nn} peak, on the other hand, initially positioned the furthest from the wall, gradually shifts outward into the emerging buffer layer while the near-wall R_{nn} levels decrease. This difference in the evolution of the R_{nn} profiles relative to those of the other stress components was verified by an examination of the intermediate stress profiles between $x/L = -0.007$ and $x/L = 0.05$, omitted here for brevity. As the upcoming analysis will show, these changes to the stress profiles are associated with the drastic reduction in the production of the TKE and shear stress near the wall, and the mean flow gradient of the emerging buffer layer giving way to enhanced turbulence production at some distance from the wall.

The evolutions of the Reynolds stress budgets over the three stations are provided in Figures 22 through 25. All budget terms are again normalized by the outer scaling, U_∞^3/L . Figures 22(a) through 25(a) provide the energizing and consuming term distributions over the entire boundary layer at the three stations, while Figures 22(b) through 25(b) show the same quantities over the region where $n/L > 1 \times 10^{-4}$ to provide their details within the aforementioned buffer zone. These plots show that the near-wall residuals of all budgets drop to nearly zero as the flow deceleration takes place from very near the apex until the flow separation point. Further from the wall where the buffer zone emerges, some imbalances are evident. Relative to the respective dominant term in the vicinity of the imbalance, the maximum residual at $x/L = 0.1$ is about 4% for R_{ss} , 16% for R_{nn} , 26% for R_{33} and less than 1% for R_{sn} . The strong APG rapidly decreases the near-wall mean flow gradient magnitudes and hence causes a drastic drop in the productions of R_{ss} and R_{sn} over a relatively short distance. At the point of separation, the near-wall production essentially goes to zero because of the nonexistent mean velocity gradient there. In Figures 22(a) and 25(a), we also observe production rising at some distance away from the wall where the buffer zone is located. The details of that region are provided in Figures 22(b) and 25(b). As seen in Figure 21(a), the mean velocity gradient forming within the buffer zone is responsible for the rising production there. As the center of the buffer zone moves away from the wall between the second and the third stations, the location of the production peak contained therein also shifts outward.

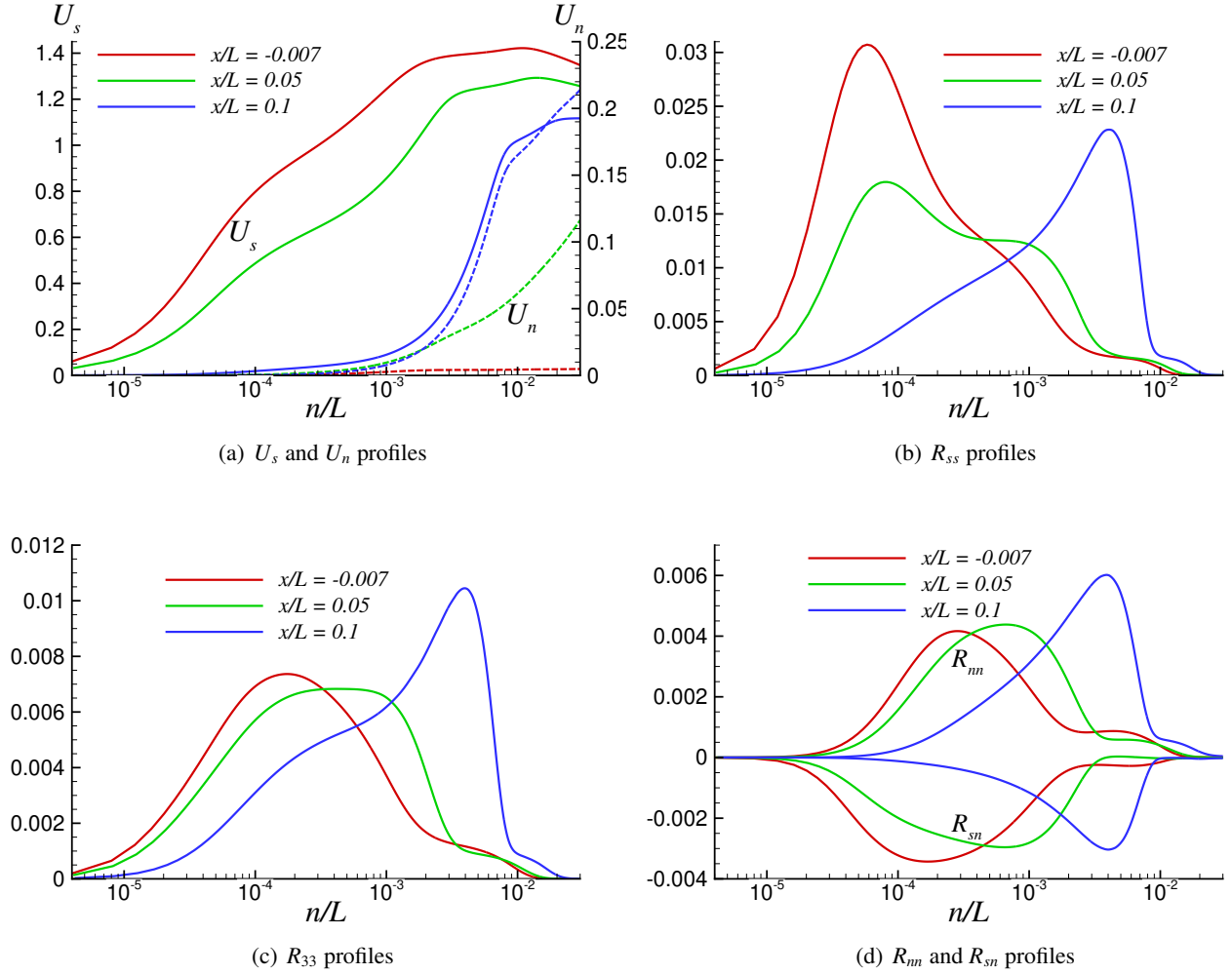
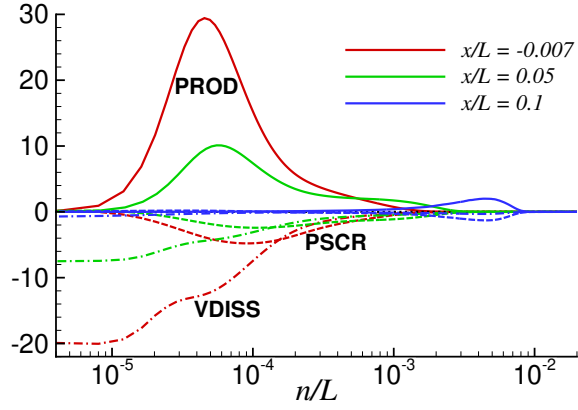
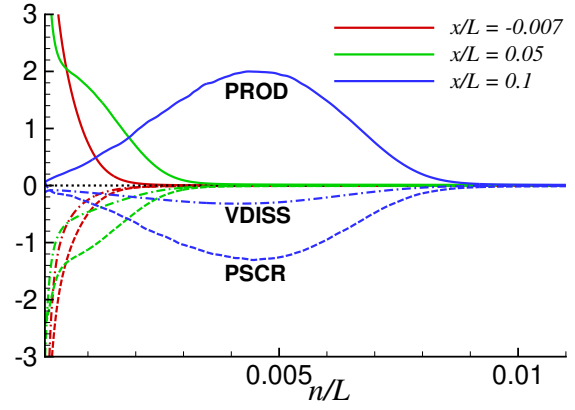


Figure 21. Mean velocity and turbulent stress profiles in the strong APG region. The normalization is based on U_∞ .

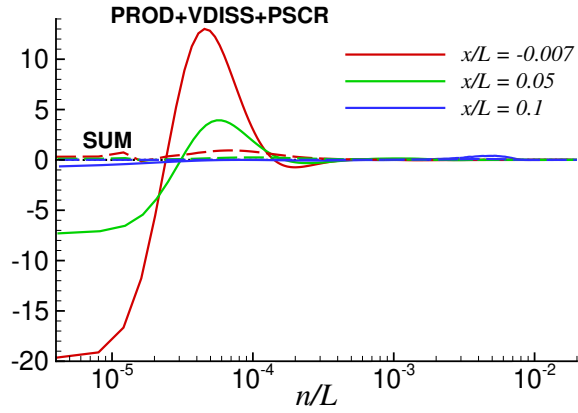
The drastic reduction in the near-wall production of R_{ss} by the strong APG also significantly dampens the consuming dissipation and pressure strain terms in close vicinity of the wall, as seen in Figure 22(a). For R_{sn} , the near-wall dissipation profiles also decay in similar manner, as depicted in Figure 25(a), while the pressure-strain profiles exhibit a more intriguing trend near the wall. Between the first two stations, the pressure-strain magnitude is about the same on the wall, but the levels in close proximity of the wall show an increase from the first to the second station. Between the second and the third stations, pressure strain shows some reduction very near the wall, but its decrease along the wall-normal direction at the third station occurs at a relatively slower rate compared to that at the second station. Hence, there exists a particular section of the near-wall region over which the pressure-strain levels at the third station exceed those at the second station, as seen in Figure 25(a). This behavior is different than what was observed in the earlier analysis of the comparatively mild initial APG region. Other than the severity of the pressure gradient, another difference between the two APG regions is the presence of convex mean streamline curvature over the strong APG section. At the third station where the flow is about to separate from the wall, the production peak of R_{sn} gets leveled out, and hence there is no secondary pressure strain peak away from the wall to counteract the



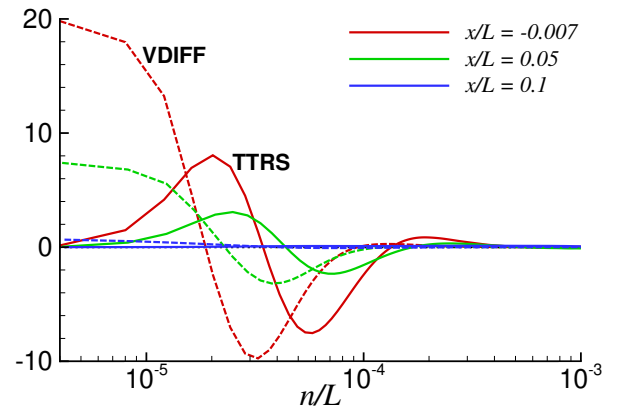
(a) energizing/consuming terms over full boundary layer



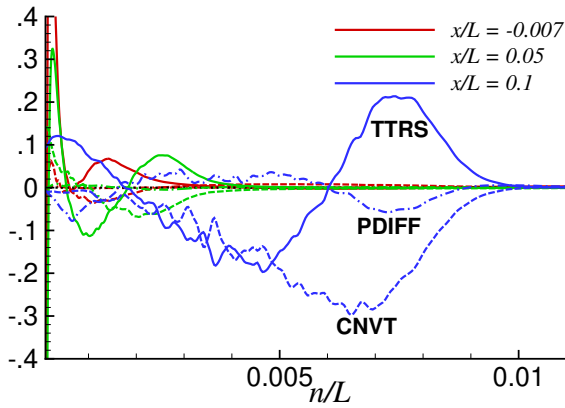
(b) energizing/consuming terms where $n/L > 1 \times 10^{-4}$



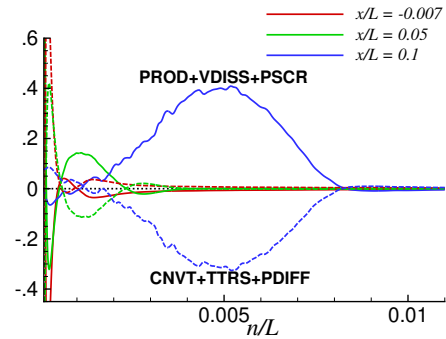
(c) energizing/consuming term sums over full boundary layer



(d) spatially-redistributive terms where $n/L < 1 \times 10^{-3}$

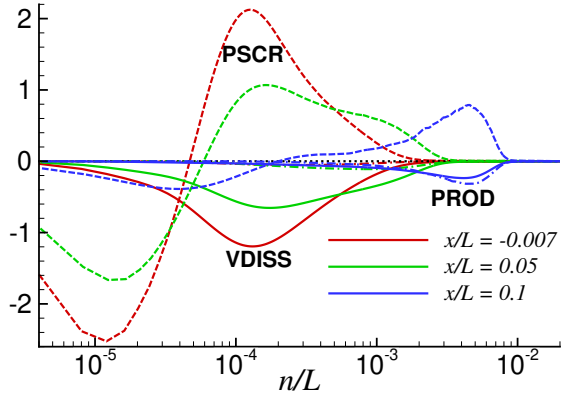


(e) spatially-redistributive terms where $n/L > 1 \times 10^{-4}$

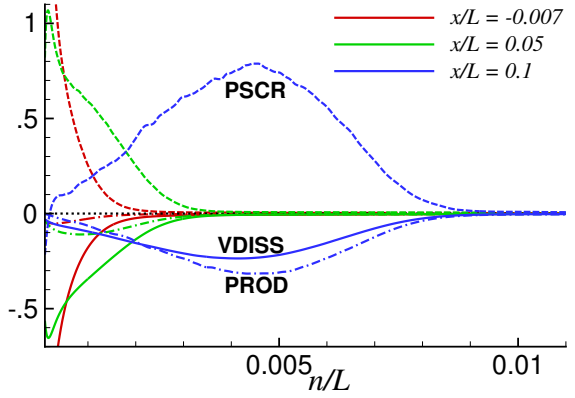


(f) energizing/consuming and spatially-redistributive term sums where $n/L > 1 \times 10^{-4}$

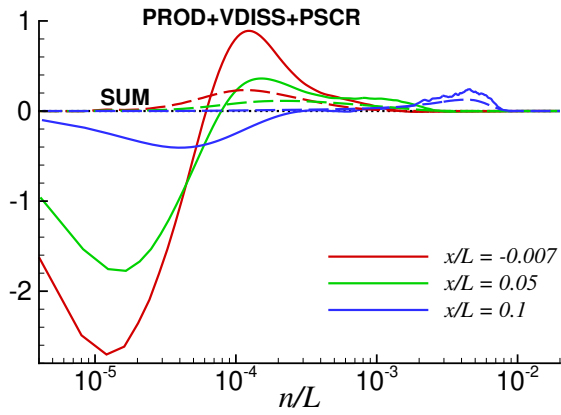
Figure 22. Evolution of R_{ss} budget terms in the strong APG region. All budget terms are normalized by U_∞^3/L .



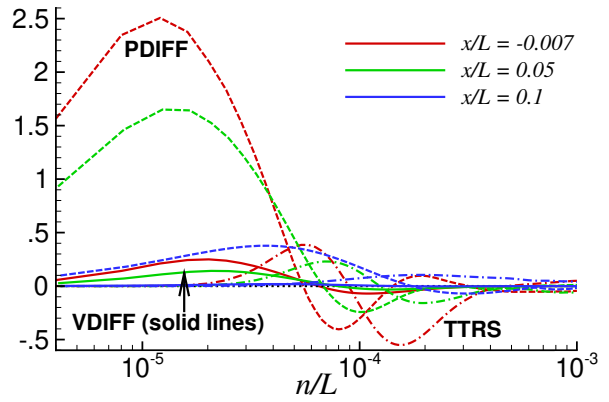
(a) energizing/consuming terms over full boundary layer



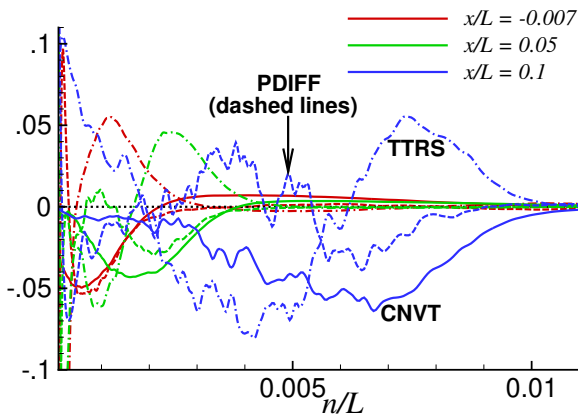
(b) energizing/consuming terms where $n/L > 1 \times 10^{-4}$



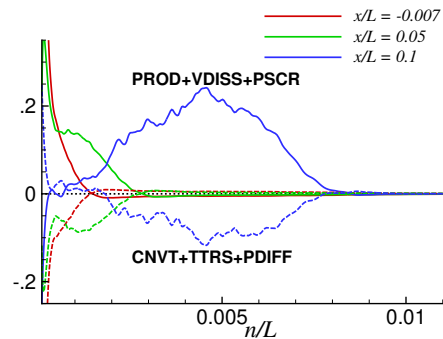
(c) energizing/consuming term sums over full boundary layer



(d) spatially-redistributive terms where $n/L < 1 \times 10^{-3}$

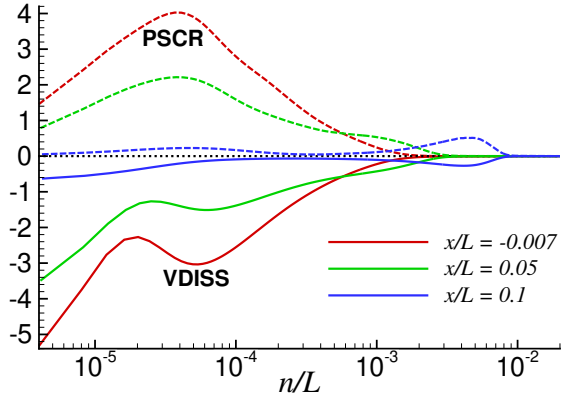


(e) spatially-redistributive terms where $n/L > 1 \times 10^{-4}$

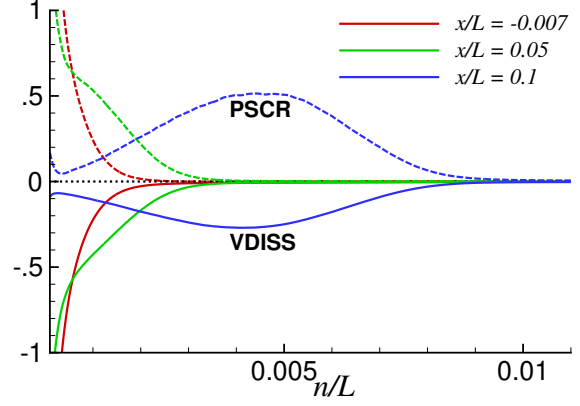


(f) energizing/consuming and spatially-redistributive term sums where $n/L > 1 \times 10^{-4}$

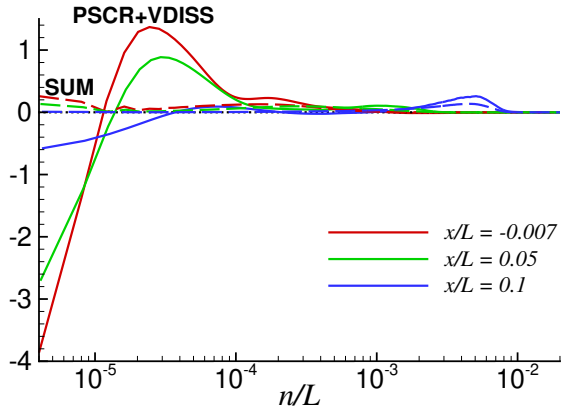
Figure 23. Evolution of R_{nn} budget terms in the strong APG region. All budget terms are normalized by U_∞^3/L .



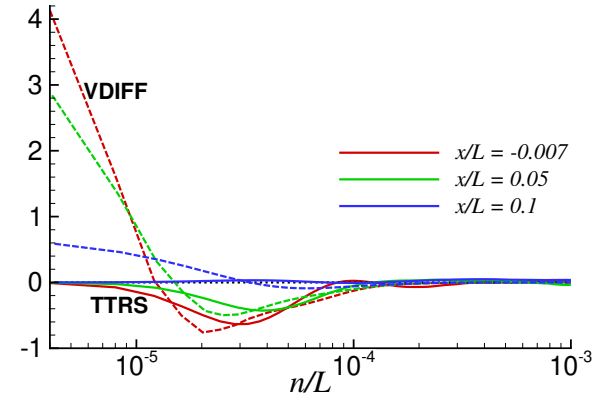
(a) energizing/consuming terms over full boundary layer



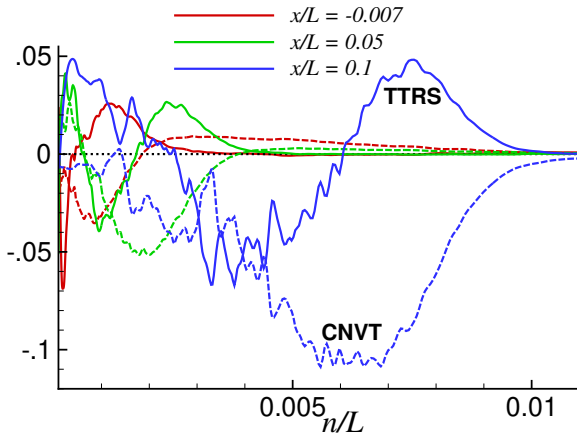
(b) energizing/consuming terms where $n/L > 1 \times 10^{-4}$



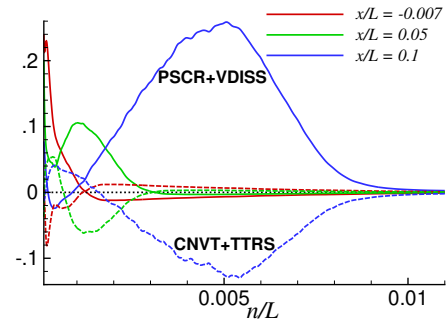
(c) energizing/consuming term sums over full boundary layer



(d) spatially-redistributive terms where $n/L < 1 \times 10^{-3}$

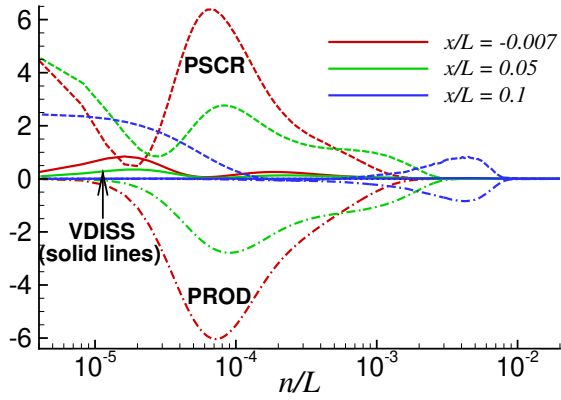


(e) spatially-redistributive terms where $n/L > 1 \times 10^{-4}$

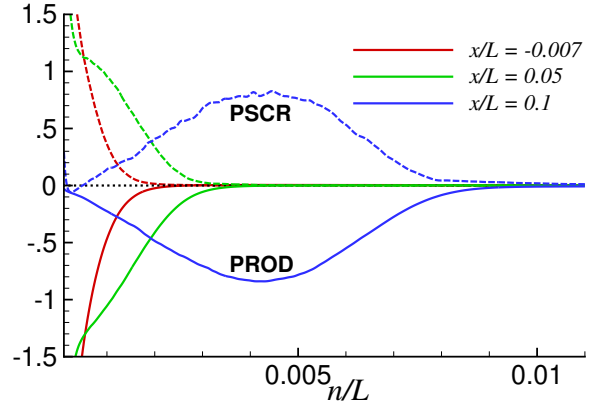


(f) energizing/consuming and spatially-redistributive term sums where $n/L > 1 \times 10^{-4}$

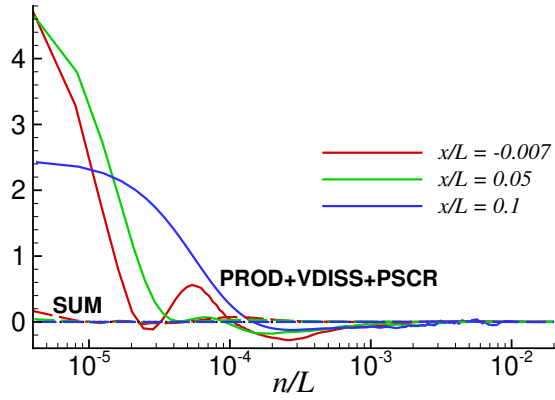
Figure 24. Evolution of R_{33} budget terms in the strong APG region. All budget terms are normalized by U_∞^3/L .



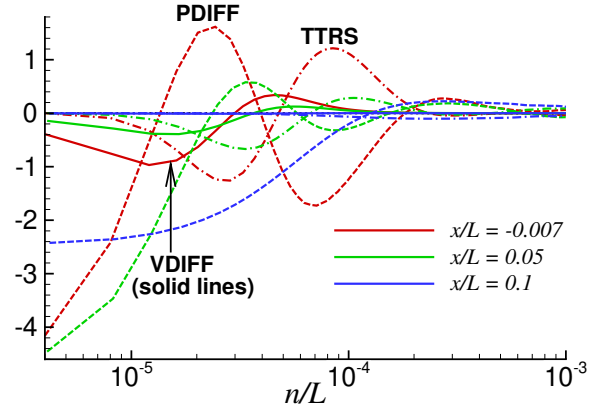
(a) energizing/consuming terms over full boundary layer



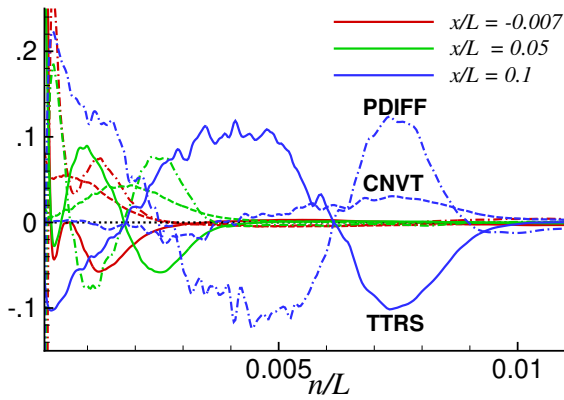
(b) energizing/consuming terms where $n/L > 1 \times 10^{-4}$



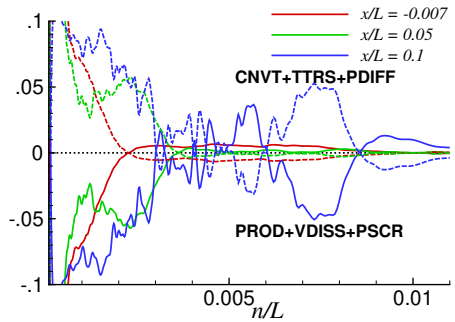
(c) energizing/consuming term sums over full boundary layer



(d) spatially-redistributive terms where $n/L < 1 \times 10^{-3}$



(e) spatially-redistributive terms where $n/L > 1 \times 10^{-4}$



(f) energizing/consuming and spatially-redistributive term sums where $n/L > 1 \times 10^{-4}$

Figure 25. Evolution of R_{sn} budget terms in the strong APG region. All budget terms are normalized by U_∞^3/L .

production peak, unlike at the earlier stations. At the second and the third stations, as production ramps up further away from the wall due to the mean velocity gradient of the buffer zone, pressure strain also increases in the same region as the main consuming term for R_{sn} , as seen in Figures 25(a) and (b). A similar observation regarding production and pressure strain within the buffer zone is also made for R_{ss} , for which dissipation is a secondary consuming term, as seen in Figure 22(b). For R_{sn} , dissipation is only relevant in the near-wall region and negligible elsewhere.

Figures 23(a) and 24(a) show that the energizing and consuming terms for R_{nn} and R_{33} in the near-wall region decrease substantially over the strong APG region. This is qualitatively similar to the previous observation made in the analysis of the initial APG section, but the dampening effect of the strong APG on these terms is now much more severe. The diminishing near-wall production of R_{ss} as the flow approaches the separation point gradually turns off the energy directed toward the other normal components via the pressure-strain correlation, and also causes a considerable reduction in the near-wall dissipation of all normal stress components. For R_{33} , pressure strain is the only energizing term since production is identically zero due to the spanwise homogeneity. As noted in the earlier observations, for R_{nn} , the near-wall production is negligible, while pressure strain is consuming very near the wall and becomes energizing with further distance from the wall. The peculiarity observed earlier in the near-wall behavior of R_{sn} pressure-strain profiles over the strong APG region does not appear to cause any curious behavior in the pressure-strain profiles of the other components. As observed in Figure 23(b), between the second and the third stations, we find growing negative production of R_{nn} within the buffer zone, indicating backscatter of energy to the mean flow. At the first station, there is also some negative production of R_{nn} , but at a much lower level compared to those at the other two stations. Such negative production of R_{nn} is caused by the local mean flow gradients arising from the combined effect of the strong APG and the convex mean streamline curvature. Hence, for R_{nn} , production along with dissipation are the two consuming terms within the buffer zone, while pressure strain is the sole energizing term. For R_{33} , as in the near-wall region, pressure strain is energizing while dissipation is consuming within the buffer zone.

Figures 22(c) through 25(c) provide the sums of the energizing and consuming terms over the entire boundary layer at the three stations for all stress components. For the normal stress components, we observe a substantial reduction in the initial deficit near the wall, as well as in the initial surplus located at a short distance from the wall as the flow travels over the strong APG section. At the separation point, the near-wall surplus vanishes for the normal stress components. For R_{sn} , due to the peculiar near-wall region behavior of pressure strain noted earlier, there is a deficit increase between the first and the second stations in the very near-wall region. Between the second and the third stations, there is some deficit reduction very near the wall accompanied by a deficit increase occurring at a short distance from the wall, as depicted in Figure 25(c). For R_{sn} , the initial surplus found in close proximity of the wall also shows some decrease between the first and the third stations. However, unlike the normal stress components, there is still some near-wall residual surplus in the R_{sn} budget at the point of separation. This is because there is some small production of R_{sn} taking place due to the local mean flow gradients, and its magnitude exceeds those of the consuming pressure strain and dissipation. Further from the wall, where the buffer zone is located, there is a surplus forming for the normal stress components because of the energizing terms becoming more dominant than the consuming terms there. The surplus acquired in that region is depicted in Figures 22(f) through 24(f). For R_{sn} , production and pressure strain are nearly balanced in the buffer zone, and therefore there is no strong surplus formation there, as seen in Figure 25(f).

Figures 22(d) through 25(d) plot the near-wall profiles of the spatially-redistributive terms at the three stations, while Figures 22(e) through 25(e) show the same terms over the region where $n/L > 1 \times 10^{-4}$ to provide the details within the buffer zone. The relevant near-wall spatially-redistributive terms identified previously for a given normal stress component experience diminishing amplitudes over the three stations due to the reduction in the energizing terms. The simultaneous decrease in the near-wall surpluses lessens the energy available to be transported toward the deficit portions. For R_{sn} , the two relevant spatially-redistributive

terms near the wall, which are viscous diffusion and turbulent transport, also experience damping. However, pressure diffusion is the main redistributive term that provides balance against the near-wall budget deficit of R_{sn} . The growing deficit in much of the near-wall region between the first two stations, caused by the behavior of pressure-strain correlation noted earlier, is therefore balanced by an increasing pressure diffusion. Between the second and the third stations, pressure diffusion decreases very near the wall, but shows an increase further from the wall to counter-balance the growing deficit in that region. Hence, the near-wall behavior of pressure diffusion is directly tied to that of pressure-strain correlation since adding up these two terms would give a zero velocity-pressure-gradient correlation in the near-wall region for R_{sn} .

In the region where $n/L > 1 \times 10^{-4}$, as depicted in Figures 22(e) through 24(e), for all normal stress components, as the flow past the apex moves toward the separation point, the mean-flow convection becomes a dominant sink within the buffer zone, while turbulent transport acts a source close to the wall and over the latter half of the buffer zone, and as a sink in between. The same observation about turbulent transport also holds for R_{sn} , as seen in Figure 25(e). The mean-flow convection for R_{sn} acts as mostly a sink over the shown region at the three stations, except over a short segment closer to the wall where it becomes a weak source. Near the point of separation, pressure diffusion becomes a tertiary redistributive mechanism for R_{ss} (after mean-flow convection and turbulent transport), and plays a contrary role to that of turbulent transport; it acts a source where turbulent transport is a sink, and vice versa. For R_{nn} , pressure diffusion is active at all three stations, as seen in Figure 23(e), and also appears to act in opposition to turbulent transport. For R_{33} , recall that pressure diffusion is identically zero everywhere because of the spanwise homogeneity. For R_{sn} , pressure diffusion also acts against turbulent transport with comparable strength, as seen in Figure 25(e).

The separate sums of the energizing and consuming terms, and those of the spatially-redistributive terms over the range where $n/L > 1 \times 10^{-4}$ are shown in Figures 22(f) through 25(f). Due to the limited statistical sample size, the net difference between the energizing and consuming terms does not appear to be exactly balanced by the net effect of the spatially-redistributive terms at certain locations. Nevertheless, Figures 22(f) through 24(f) show a substantial rise in the buffer zone budget surpluses between the second and the third stations for both R_{ss} and R_{33} , and a more modest increase for R_{nn} (also see Figures 22(c) through 24(c)). For R_{sn} , due to the near balance between production and pressure-strain, as depicted in Figure 25(b), there is not a strong surplus or deficit formation within the emerging buffer zone. Hence, we only find small pockets containing a weak deficit or surplus in that region, as seen in Figure 25(f).

To reiterate, just upstream of the strong APG section where $x/L = -0.007$, the original peaks of R_{ss} , R_{nn} and R_{33} are at $n/L = 5.82 \times 10^{-5}$, 2.84×10^{-4} and 1.75×10^{-4} , respectively. As seen in Figures 22(c) through 24(c), there is a drastic reduction in the near-wall surpluses of all normal stress components as the flow interacts with the strong APG. The vanishing near-wall surpluses over the strong APG section are responsible for the decay of the original peaks of R_{ss} and R_{33} . For R_{nn} , however, there is surplus energy available within the emerging buffer zone, as seen in Figure 23(c), that is not too far off from its peak location. Hence, for this particular component, the surplus energy in the nearby buffer zone enables a gradual outward shift of the original peak as the flow decelerates toward separation. At the second station where $x/L = 0.05$, the budget surpluses generated within the buffer layer lead to the formation of a relatively flat portion in the R_{ss} and R_{33} profiles, which provides the first hint of the formation of an outer peak in these two components, as seen in Figure 21. For R_{nn} , the original stress peak has shifted to $n/L = 6.68 \times 10^{-4}$ as the flow reaches the second station. As seen in Figure 23(c), at the second station, there is surplus energy available in the vicinity of the R_{nn} peak at $n/L = 6.68 \times 10^{-4}$, which enables further strengthening of this peak as it shifts outward. At the third station where the flow is on the verge of detachment from the surface, all normal stress components have formed a strong peak at $n/L \approx 4 \times 10^{-3}$. This is an outcome of the continued increase in the buffer zone budget surpluses between the second and the third stations.

For R_{sn} , the initial surplus generated at a short distance from the wall shows some decrease in its magnitude between the first and the third stations. Although some residual near-wall surplus exists near the point of separation, there is a part of the near-wall region where the deficit has increased from the first to

the third station, as depicted in Figure 25(c). As the available excess energy is transported by pressure diffusion toward the near-wall region to counter-balance the increased deficit there, as shown in Figure 25(d), the original R_{sn} peak decays along with the near-wall stress levels since there is not any leftover energy to maintain those levels. Similar to what was previously observed for the normal stress components, production increasing within the emerging buffer zone leads to the formation of an outer peak for R_{sn} . However, because of the near balance between production and pressure strain for R_{sn} , its outer peak strength remains nearly the same between the second and the third stations. In contrast, the normal stress components show a much more substantial increase in the outer peak strength between the two stations. This is due to the strong budget surpluses acquired for the normal stress components in the buffer zone.

Figure 26 provides the budgets normalized by the inner scaling of u_τ^4/ν_w at three stations within the strong APG region, which are at $x/L = 0.025, 0.05$ and 0.075 , respectively. We observe that the inner scaling does not generally achieve a tight collapse between the first two stations, while the third station profiles display major deviations from those of the first two. As the flow approaches separation at $x/L \approx 0.1$, u_τ becomes relatively small at the third station where the budget profiles possess much higher amplitudes near the wall and also in the buffer zone.

4 Summary and Concluding Remarks

An analysis of the Reynolds stress budgets for a TBL past a Gaussian bump provides insight into the modification of turbulence under different pressure-gradient regimes in three separate subregions of the flowfield upstream of separation. The first subregion starts off with an initially mild APG upstream of the bump. The initial APG becomes progressively stronger as the flow nears the bump. The first subregion ends in the vicinity of the bump foot and is succeeded by the second subregion in which a strong FPG exists until very near the bump apex. The third subregion is the strong APG section that starts in the vicinity of the bump apex and lasts until flow separation that occurs at a short distance downstream of the apex.

In all subregions, the positive production of the TKE mainly takes place in the form of the streamwise turbulent stress. The APG present within the first and the third subregions reduces the mean shear rate magnitude near the wall, which, in turn, has a dampening effect on the production of the TKE and that of the turbulent shear stress. The degree of near-wall production dampening in the APG sections of the flowfield is directly proportional to the strength of the APG. Hence, in the third subregion characterized by a strong APG, the near-wall productions of the TKE and the turbulent shear stress vanish as the flow approaches the point of separation where the near-wall mean shear rate essentially becomes zero. In the relatively weaker initial APG section, the peak productions of the TKE and the turbulent shear stress get nearly halved as the initially mild APG becomes progressively stronger toward the bump foot. Conversely, the strong FPG of the second subregion leads to an order of magnitude increase in the near-wall productions of the TKE and the turbulent shear stress between the start and the end of the acceleration region. This is a consequence of the enhanced shear rate magnitude near the wall due to the strong flow acceleration caused by the strong FPG, which also has the effect of nearly a fourfold increase in the surface skin friction between the start and the end of the acceleration region. In a similar vein, the mean flow gradient of the buffer zone emerging between the inner and outer regions of the strongly-decelerated flow past the bump apex gives way to enhanced TKE and turbulent shear stress production away from the wall as the flow moves toward separation. The production of wall-normal stress within this buffer zone is negative, due to the nature of the mean flow gradient arising from the combination of strong APG and the convex mean streamline curvature, indicating backscatter of energy from this particular component toward the mean flow.

The TKE gets redistributed internally among the three normal stress components via the pressure-strain correlation. Throughout much of the boundary layer in all subregions, pressure strain is generally a consuming term for the streamwise component, and an energizing one for the spanwise component. For the

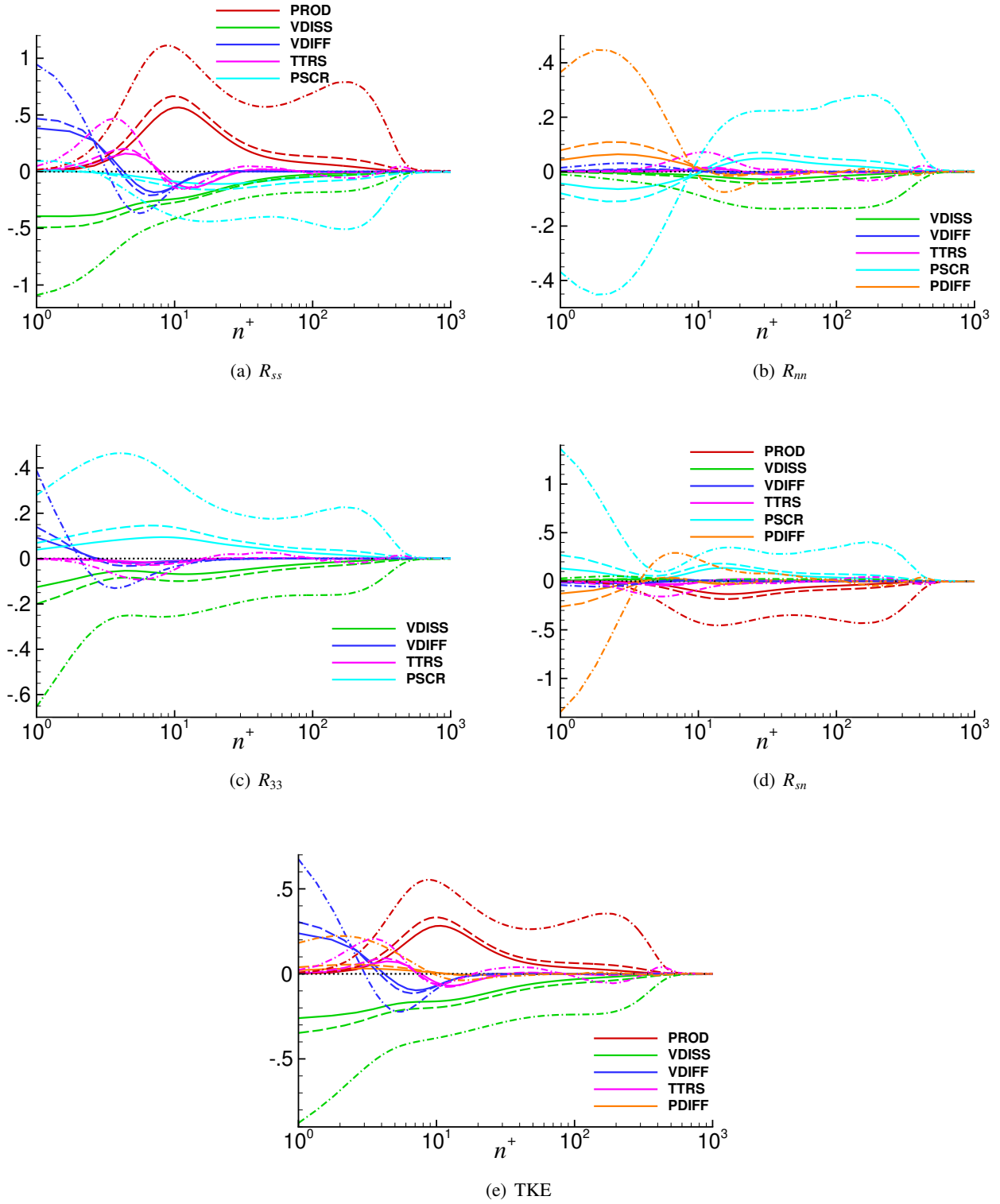


Figure 26. Reynolds stress and TKE budgets normalized by the inner scaling (u_τ^4/ν_w) in the strong APG region. Solid lines indicate the budget terms at $x/L = 0.025$; dashed lines indicate the budget terms at $x/L = 0.05$; dash-dotted lines indicate the budget terms at $x/L = 0.075$.

wall-normal component, pressure strain becomes consuming in the near-wall region and energizing elsewhere. For the shear stress component, pressure strain is generally a consuming term. The reduction in the near-wall TKE production within the APG sections leads to decreases in the amplitudes of the pressure-strain correlation for each normal stress component. For the shear stress component, pressure strain also exhibits a generally decreasing trend in response to the reduced near-wall production in the APG sections, except in certain parts of the very near-wall region downstream of the bump apex, where it momentarily increases between successive streamwise stations as the flow moves toward separation. In contrast, in the strong FPG region, the much-amplified near-wall productions of the TKE and the turbulent shear stress lead to substantial enhancements of the pressure-strain correlation for all components. Similarly, within the buffer zone of the strongly-decelerated boundary layer in the third subregion, pressure-strain magnitudes rise in parallel with the increasing production. Viscous dissipation, which is generally a consuming term for all components, decreases along with the reductions in near-wall production of the TKE and the shear stress in the APG regions, while increasing substantially in response to the enhanced near-wall production in the strong FPG section. Within the buffer zone of the decelerating flow past the bump apex, viscous dissipation also increases for the normal stress components as the TKE production ramps up, but remains negligible for the shear stress. The formation of new peaks in TKE production and dissipation in the outer part of TBLs subjected to moderate to strong APGs was also identified in the studies by Kitsios et al. [11], Coleman et al. [3], Pozuelo et al. [17] and Rkein and Laval [18]. Rkein and Laval [18] additionally observed that the new turbulent shear stress production peak that emerges in the same region is primarily balanced by pressure-strain since dissipation remains negligible there, similar to what was found in the present study.

At the start of each subregion, the net difference between the energizing and consuming terms for each stress component creates a deficit-containing portion very near the wall, succeeded by a surplus-containing portion at a short distance from the wall. The APG of the first subregion weakens these deficit and surplus portions by a considerable amount, while the strong FPG of the second subregion substantially strengthens them by almost an order of magnitude. The strong APG of the third subsection again reduces the near-wall deficits of all normal components considerably, but causes a more intricate behavior for the shear stress component due to the aforementioned peculiarity of the pressure-strain correlation near the wall. The strong APG also significantly weakens the near-wall surplus of the shear stress component, while completely eliminating those of the normal stress components. In the outer part of the incoming TBL upstream of the bump, the net difference between the energizing and consuming terms generates surplus-containing portions for each stress component, which persist over the entire first subregion. This is an outcome of the destabilizing effect of the APG in the outer part of the boundary layer and also that of the concave mean streamline curvature that comes into play closer to the bump. As the flow enters the second subregion and starts accelerating toward the bump apex, the outer region surpluses gradually transform into deficits due to the stabilizing effect of the strong FPG away from the wall, and that of the convex mean streamline curvature in the latter stages of acceleration. The buffer zone emerging within the strongly-decelerated boundary layer in the third subregion generates a strong surplus for the normal stress components but not for the shear component, because of the near balance between the energizing production term and the consuming pressure-strain term for the shear stress within the buffer zone.

As the flow goes through the different pressure-gradient regimes, the decreasing or increasing deficits between the energizing and consuming terms near the wall for all stress components are balanced by the relevant spatially-redistributive terms that proportionally either diminish or amplify in magnitude. The near-wall surpluses that weaken in the APG regions and strengthen in the strong FPG section determine how much excess energy is available for transport toward the deficit portions. For the streamwise stress component, viscous diffusion and turbulent transport are the main energy redistribution mechanisms that balance the near-wall deficit. Turbulent transport also convects some energy toward the outer part of the near-wall region where a deficit between the energizing and consuming terms exists for the streamwise component. For the spanwise stress component, viscous diffusion is the main redistribution mechanism that

balances the deficit in the near-wall region while turbulent transport acts a sink in the entire near-wall region. For the wall-normal and shear components, pressure diffusion is the main redistributive term very near the wall, supplemented by a secondary contribution from the viscous diffusion. Pressure diffusion also transfers energy in the direction away from the wall in parts of the near-wall region for these two components. With increasing wall-normal distance within the near-wall region, turbulent transport also becomes an important redistributive mechanism for the wall-normal and shear components.

The decreasing near-wall surpluses due to the APG effect within the first subregion lead to the reductions observed in the near-wall levels of all stress components along with the weakening peaks of the streamwise and spanwise stress components, which are initially positioned fairly close to the wall. The budget surpluses resulting from the destabilizing effect of the APG in the outer region, as well as from the destabilizing concave mean streamline curvature effect that comes into play as the flow nears the bump, cause an outward spread of all stress levels in the thickening boundary-layer. Furthermore, new outer peaks in the streamwise and spanwise stress components begin to form as the flow moves deeper into the APG, while the original peaks of the wall-normal and shear stress components, initially located in the outer part of the TBL, strengthen further. Once the flow enters the second subregion containing a strong FPG, the much-enhanced near-wall productions of the TKE and the shear stress and the resulting budget surpluses lead to the formation of an internal layer beneath the accelerated boundary layer. Within this internal layer, the original peaks of the streamwise and spanwise stress components strengthen very significantly while new near-wall peaks emerge for the wall-normal and shear components in the latter stages of flow acceleration toward the apex. To our knowledge, the formal connection between an internal layer generated beneath a strongly-accelerated TBL and the turbulent stress budgets has been established for the first time in the present analysis.

In the outer part of the accelerated TBL, the surpluses hold nearly steady for the wall-normal and shear components over the initial stages of acceleration but start to monotonically decrease for the streamwise and spanwise stress components over much of the outer region. Hence, the outer levels of the wall-normal and shear components strengthen over the initial stages of the strong acceleration along with their peaks, while those of the streamwise and spanwise stress components decrease over the same region. As the flow moves deep into the acceleration region and nears the apex, the combined stabilizing effect of the strong FPG and that of the convex mean streamline curvature results in outer region deficits, which then lead to the decay observed in the outer stress levels for all components. As a final note, in the canonical configuration of the Gaussian bump flow, the TBL goes through complex transformations that have been captured in the direct numerical simulation at a relatively high Reynolds number [26] and thoroughly analyzed in this paper. It is clear that the flow in a thin region buried deep in the TBL is characterized by rather intricate dynamics.

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References

- [1] R. Agrawal, S. T. Bose, and P. Moin. Nonequilibrium wall model for large eddy simulations of complex flows exhibiting turbulent smooth body separation. *Physical Review Fluids*, 9(12):124603, December 2024. [doi:10.1103/PhysRevFluids.9.124603](https://doi.org/10.1103/PhysRevFluids.9.124603).
- [2] S. T. Bose and G. I. Park. Wall-modeled large-eddy simulation of complex turbulent flows. *Annual Review of Fluid Mechanics*, 50:535–561, 2018. [doi:10.1146/annurev-fluid-122316-045241](https://doi.org/10.1146/annurev-fluid-122316-045241).

- [3] G. N. Coleman, C. L. Rumsey, and P. R. Spalart. Numerical study of turbulent separation bubbles with varying pressure gradient and Reynolds number. *Journal of Fluid Mechanics*, 847:28–70, 2018. [doi:10.1017/jfm.2018.257](https://doi.org/10.1017/jfm.2018.257)
- [4] P. Gray, I. Gluzman, F. Thomas, and T. Corke. Experimental characterization of smooth body flow separation over wall-mounted Gaussian bump. AIAA Paper 2022-1209, AIAA SciTech 2022 Forum, San Diego, California & Virtual, January 2022. [doi:10.2514/6.2022-1209](https://doi.org/10.2514/6.2022-1209)
- [5] P. Gray, I. Gluzman, F. Thomas, T. Corke, M. Lakebrink, and K. Mejia. A new validation experiment for smooth-body separation. AIAA Paper 2021-2810, AIAA Aviation 2021 Forum, Virtual Event, August 2021. [doi:10.2514/6.2021-2810](https://doi.org/10.2514/6.2021-2810)
- [6] P. Gray, I. Gluzman, F. Thomas, T. Corke, M. Lakebrink, and K. Mejia. Characterization of separated flow over the Boeing bump. AIAA Paper 2022-3342, AIAA Aviation 2022 Forum, Chicago, Illinois & Virtual, June–July 2022. [doi:10.2514/6.2022-3342](https://doi.org/10.2514/6.2022-3342)
- [7] P. Gray, M. Lakebrink, F. Thomas, T. Corke, I. Gluzman, and J. Straccia. Experimental and computational evaluation of smooth-body separated flow over Boeing bump. AIAA Paper 2023-3981, AIAA Aviation 2023 Forum, San Diego, California & Online, June 2023. [doi:10.2514/6.2023-3981](https://doi.org/10.2514/6.2023-3981)
- [8] P. D. Gray. *An Experimental Investigation of Smooth-Body Flow Separation over a Tapered Gaussian Bump*. PhD thesis, University of Notre Dame, Notre Dame, Indiana, December 2023.
- [9] K. P. Griffin, L. Fu, and P. Moin. General method for determining the boundary layer thickness in nonequilibrium flows. *Physical Review Fluids*, Vol. 6, 024608, February 2021. [doi:10.1103/PhysRevFluids.6.024608](https://doi.org/10.1103/PhysRevFluids.6.024608)
- [10] T. R. Gungor, Y. Maciel, and A. G. Gungor. Energy transfer mechanisms in adverse pressure gradient turbulent boundary layers: Production and inter-component redistribution. *Journal of Fluid Mechanics*, 948:A5, September 2022. [doi:10.1017/jfm.2022.679](https://doi.org/10.1017/jfm.2022.679)
- [11] V. Kitsios, A. Sekimoto, C. Atkinson, J. A. Sillero, G. Borrell, A. G. Gungor, J. Jiménez, and J. Soria. Direct numerical simulation of a self-similar adverse pressure gradient turbulent boundary layer at the verge of separation. *Journal of Fluid Mechanics*, 829:392–419, September 2017. [doi:10.1017/jfm.2017.549](https://doi.org/10.1017/jfm.2017.549)
- [12] A. Lozano-Durán and H. J. Bae. Machine learning building-block-flow wall model for large-eddy simulation. *Journal of Fluid Mechanics*, 963:A35, May 2023. [doi:10.1017/jfm.2023.331](https://doi.org/10.1017/jfm.2023.331)
- [13] N. N. Mansour, J. Kim, and P. Moin. Reynolds-stress and dissipation-rate budgets in a turbulent channel flow. *Journal of Fluid Mechanics*, 194:15–44, September 1988. [doi:10.1017/S0022112088002885](https://doi.org/10.1017/S0022112088002885)
- [14] J. G. Marvin and G. P. Huang. Turbulence modeling — Progress and future outlook. In *Fifteenth International Conference on Numerical Methods in Fluid Dynamics*, pages 35–56. Springer, Berlin, Heidelberg, 1997. [doi:10.1007/BFb0107077](https://doi.org/10.1007/BFb0107077)
- [15] G. L. Nicholson, J. Huang, L. Duan, M. M. Choudhari, B. Morreale, and R. D.W. Bowersox. Budgets of Reynolds stresses and turbulent heat flux for hypersonic turbulent boundary layers subject to pressure gradients. AIAA Paper 2022-1059, AIAA SciTech 2022 Forum, San Diego, California & Virtual, January 2022. [doi:10.2514/6.2022-1059](https://doi.org/10.2514/6.2022-1059)

- [16] S. B. Pope. *Turbulent Flows*, pages 281–285. Cambridge University Press, Cambridge, United Kingdom, 2000.
- [17] R. Pozuelo, Q. Li, P. Schlatter, and R. Vinuesa. An adverse-pressure-gradient turbulent boundary layer with nearly constant $\beta \simeq 1.4$ up to $Re_\theta \simeq 8700$. *Journal of Fluid Mechanics*, 939:A34, April 2022. [doi:10.1017/jfm.2022.221](https://doi.org/10.1017/jfm.2022.221)
- [18] H. Rkein and J.-P. Laval. Direct numerical simulation of adverse pressure gradient turbulent boundary layer up to $Re_\theta \simeq 8000$. *International Journal of Heat and Fluid Flow*, 103:109195, October 2023. [doi:10.1016/j.ijheatfluidflow.2023.109195](https://doi.org/10.1016/j.ijheatfluidflow.2023.109195)
- [19] C. L. Rumsey, J.-R. Carlson, T. H. Pulliam, and P. R. Spalart. Improvements to the quadratic constitutive relation based on NASA juncture flow data. *AIAA Journal*, 58(10):4374–4384, 2020. [doi:10.2514/1.J059683](https://doi.org/10.2514/1.J059683)
- [20] L. A. C. A. Schiavo, W. R. Wolf, and J. L. F. Azevedo. Turbulent kinetic energy budgets in wall bounded flows with pressure gradients and separation. *Physics of Fluids*, 29(11):115108, 2017. [doi:10.1063/1.4992793](https://doi.org/10.1063/1.4992793)
- [21] M. L. Shur, P. R. Spalart, M. Kh. Strelets, and A K. Travin. Direct numerical simulation of the two-dimensional speed bump flow at increasing Reynolds numbers. *International Journal of Heat and Fluid Flow*, 90:108840, August 2021. [doi:10.1016/j.ijheatfluidflow.2021.108840](https://doi.org/10.1016/j.ijheatfluidflow.2021.108840)
- [22] J. A. Sillero, J. Jiménez, and R. D. Moser. One-point statistics for turbulent wall-bounded flows at Reynolds numbers up to $\delta^+ \approx 2000$. *Physics of Fluids*, 25(10):105102, October 2013. [doi:10.1063/1.4823831](https://doi.org/10.1063/1.4823831)
- [23] J. P. Slotnick. Integrated CFD validation experiments for prediction of turbulent separated flows for subsonic transport aircraft. NATO Science and Technology Organization, Meeting Proceedings RDP, STO-MP-AVT-307, October 2019.
- [24] P. R. Spalart. Direct simulation of a turbulent boundary layer up to $R_\theta = 1410$. *Journal of Fluid Mechanics*, 187:61–98, February 1988. [doi:10.1017/S0022112088000345](https://doi.org/10.1017/S0022112088000345)
- [25] A. Uzun and M. R. Malik. High-fidelity simulation of turbulent flow past Gaussian bump. *AIAA Journal*, 60(4):2130–2149, 2022. [doi:10.2514/1.J060760](https://doi.org/10.2514/1.J060760)
- [26] A. Uzun and M. R. Malik. Direct numerical simulation of flow past a Gaussian bump at a high Reynolds number. *Theoretical and Computational Fluid Dynamics*, 39(4):Article Number 30, 2025. [doi:10.1007/s00162-025-00749-x](https://doi.org/10.1007/s00162-025-00749-x)
- [27] X. Wang, Z. Wang, M. Sun, Q. Wang, and Z. Hu. Effects of favorable pressure gradient on turbulence structures and statistics of a flat-plate supersonic turbulent boundary layer. *Physics of Fluids*, 32(2):025107, 2020. [doi:10.1063/1.5143433](https://doi.org/10.1063/1.5143433)
- [28] D. Zhou and H. J. Bae. Sensitivity analysis of wall-modeled large-eddy simulation for separated turbulent flow. *Journal of Computational Physics*, 506:112948, June 2024. [doi:10.1016/j.jcp.2024.112948](https://doi.org/10.1016/j.jcp.2024.112948)